1. 1.1 Overview and learning outcomes

In the video below, Dr Mohsen Motahari will explain what you will learn in Week 1.

View transcript

Welcome to the first week of the asset pricing module. In this week, we're going to give a general overview of asset pricing, including various definitions and a thorough discussion of what really is an asset, what constitutes the price of an asset, and how do we go about finding this price. We will describe general asset classes, characteristics in each case, and the ultimate goal of each asset pricing model to derive a mathematically and economically justified methodology of asset pricing as a function of time. We expect the result to be consistent with historical prices to a good degree and with a robust and clear explanation of the assumptions and limitations of the underlying theory. Ultimately, this path will lead us to a detailed discussion of the inherent randomness embedded within every asset and within the market as a whole.

It's very important in this introductory week, to get a good grasp of the stochasticity that derives the market and how this uncertainty is not only unavoidable but is actually used one way or another in order to derive the fair price of any asset. We will give an introduction to these mathematical techniques used to describe this randomness, namely probability theory and stochastic calculus, and how these established tools can be used to analyze historical prices, and predict the future prices of assets within a certain confidence level. A particular tool for this is Brownian motion, which is the most fundamental stochastic process used widely in finance. Finally, we will review the most fundamental ideas within asset pricing theory. These ideas will be explained in more detail in later weeks, but we will discuss here the need to have these in order to derive existing theories of asset pricing. We will mention the capital asset pricing model and what assumptions are used in the background of this theory. We will separate between systematic and specific risk and related performance. We will discuss arbitrage and how the absence of it is at the core of most theories in finance. We will review the efficient market hypothesis, and finally, we will discuss different types of investment and their respective objectives.

I hope you enjoy this first week and don't forget two things. First, no mathematical model is ever 100 per cent correct. We just try to do our best and quantify the uncertainty around the market within a certain set of assumptions. Second, what we always need to model mathematically is first, the expected pay-off of an asset, and second, the discount factor we use in order to bring these random cash flows to date. If we get a real grasp of these ideas, we will have accomplished our objectives in this first introductory week.

Learning outcomes

- Define what assets and asset classes are.
- Understand the first steps into deriving fair value of assets.
- Use probability theory and stochastic calculus in relation to assets.
- Describe the Capital Asset Pricing Model and its assumptions.
- Understand the ‘Efficient Market Hypothesis’.

1. Introduction

We will begin this week by providing general definitions for assets and asset classes, which may serve as a quick recap to those who have seen these phrases used throughout this programme or in other educational or career endeavours. Because it can be difficult to properly define such broad concepts, we will provide some examples as to what each of these concepts mean and how they can be further broken down. Gaining an understanding of both of these concepts will enable us to delve further into the nature of assets and asset pricing later in the week.

2. Defining an asset

So beginning this Week 1, we'd like to ask ourselves, what really is an asset?

Asset

Any instrument, financial or otherwise, tangible or not, which when held by investors is expected to maximise their wealth and their utility.

It's very important here to understand that obviously an asset is a very generic concept.

Tangible versus intangible assets

It's important again to clarify some of the terminologies that are used.

Tangible asset

A tangible asset is anything that has physical form, such as land or property, or precious metals or equipment.

Intangible asset

Intangible assets are instruments like equities, bonds, patents, trademarks, even copyrights – anything that doesn't have a physical form but represents some value to the investor or to the individual.

A quick note on terminology

You will have noticed that in the above example, we do not speak of financial assets only, but in general, we consider an asset as literally anything that has any value, to anyone. In this course, we will mainly focus on investment assets, however, we should keep in mind that the terminologies, by and large, extend to a great variety of concepts outside the financial world.

3. Asset classes

What is an asset class?

Asset class

A group of assets and securities that behave in a similar way and have very similar characteristics. Their price is driven by common risk factors and they will usually face similar types of risk.

Well-known asset classes

Again, the definition provided above is a rather generic definition of asset classes. We can make it more precise by providing certain examples. Well-known and often-mentioned asset classes include equities, fixed income, money markets and commodities.

Stock market

Equities

Within equities, we will have various types of stocks representing different companies. We'll have different sectors, different regions and so on. Within this asset class, we have further subcategories. Examples include sectors like pharmaceuticals, banks, retail etc and regions such as Europe, Americas, Asia and the like.

Person analysing the stock market.

Fixed income

If we look now at fixed income, another major asset class popular with investors, the building blocks here are the bonds. These are promises from one party to the other to repay the full capital back as well as pay a certain regular income, called coupon. These types of instruments can get more complex in terms of the actual payoff structure. We have subcategories here as well, such as government bonds, corporate bonds, high or low yield bonds etc. Classical fixed income debt securities have long maturity dates, from a few months to many years.

Person stacking up money coins.

Money markets

Within money markets, we will have various types of bonds, bonds of different maturity, bonds of different coupon payment structure, different sophistication, etc. Money markets are strictly speaking fixed income, but with very short expiry dates, from one to just a few days. For this reason, they are considered cash-like instruments.

Stack of boxes in a product warehouse.

Commodities

Within the asset class we call commodities, we will have things like precious metals, oil, livestock and many others.

Equities, bonds and alternative investments

Here is a nice recap of just a small number of asset classes; equities, bonds and alternative investments. Within the alternative investments category, we have assets like commodities, but also real estate, hedge funds, cryptocurrencies and others.

Equities

- industries
- countries
- styles.

Bonds

- government bonds
- foreign sovereign bonds
- corporate bonds:
  - investment grade
  - high yield (junk).

Alternatives

- commodities:
  - energy
  - industrial metals
  - precious metals
  - agricultural
- real estate
- hedge funds.

Indices in asset classes

We now discuss the idea of an index. Within every asset class, we will have a number of assets that are very similar in nature and it will have a number of indices. An index shows the performance of a basket of assets or a portfolio. The majority of the indices are systematic in the sense that they follow a specific set of rules in terms of how they are constructed.

Types of indices

We have three different types of indices depending on what kind of return we are considering.

Total return index

This index will capture the performance of an investment, including dividends received and including any interest on cash. Essentially, all income is considered whether it comes from growth, dividends or another source (hence the terminology 'total return').

Excess return index

This index measures the performance of a portfolio minus the risk-free rate or general benchmark (so we measure the excess above the benchmark).

Price return index

The price return index shows the performance of the basket, but here we assume that all dividends are paid out to the investor. So, essentially, this reflects an investment whereby the investor will actually 'cash out', as we say, all the dividends paid and these dividends do not count when we consider the performance as they are not reinvested.

Discretionary versus systematic indices

There's another way that we can categorise indices. We can have discretionary or systematic indices:

Discretionary

Discretionary is the index whereby the rules are either not published or they change all the time without the approval of the investor.

Systematic

Systematic is an index whereby the rules of how the index is constructed are published and they are known to the investor.

1. Introduction

From the outset, we want to define what we're trying to achieve, in order for us to understand how we're going to achieve this when using asset pricing theory later this week. So, we will begin by discussing the randomness in the market. Before proceeding, we will have a brief discussion on mathematics and reality, which will introduce ideas that you should keep in mind as we begin to explore the mathematical side of asset pricing theory.

We will then explore the two mathematical techniques used to model market randomness, probability theory and stochastics, and see how they can be further used to analyse historical prices and predict, within a certain confidence level, future prices of various assets. This lesson should provide a significant mathematical background to our final lesson later this week, which will discuss asset pricing theory objectives and fundamentals in addition to how investors use this knowledge to make investments.

2. Defining market randomness

Inherent randomness and the physical world

It is important that we understand the random nature of the assets. This means that, as with any other aspect in life, we cannot predict future asset prices with a 100 per cent certainty. So, we don't know what the price of any asset will be in the future. We can only have an estimation with a certain probability. The financial markets exist and investors are allowed to participate in the markets by buying and selling assets exactly because of this inherent randomness that we have. So here, we want to draw comparisons with the physical world and theories that attempt to consider how the whole universe works, such as in Physics.

The difference between physical models and financial models

Financial models try to explain the evolution of markets, whereas models in physics try to know how nature works. In physics, a deterministic model could a priori stand and exist correct. However, this does not hold in a financial market.

A diagram showing the gravitational movement of the planets: m subscript 1 in a red circle - defined as F subscript 1 - pointing to a larger blue circle with m subscript 2 in it, labelled as F subscript 2. Both are connected by a line labelled r
An example in physics is the theory of gravity, developed by Sir Isaac Newton and later generalised by Albert Einstein. This is a model that explains the movement of planets and in general it is a model that attempts to provide insights into the gravitational forces between two or more bodies. The model dictates that once we know for instance the related data of two planets today, then we can predict with a 100 per cent certainty the movement of these planets forever into the future. So, this is what we call a deterministic model because there is no inherent randomness in the whole system and the future is predictable, once the initial conditions are provided. However, as mentioned already, while this type of model can exist in physics, it cannot exist in finance. A deterministic universe can be a priori imagined and exist but a deterministic market will collapse in no time.

Graphical representation of random markets

We can see what I mean by random markets in this graph. Here, we have a simple plot of equity prices, bond prices and commodity prices for the last few decades. At any time point, from 1982 to 2016, no one would have been able to predict the future price of equities or bonds or commodities. But, what we do in asset pricing theory is that we look into the past and this is a plot of the past. We collect all these data, all these past prices for various assets. This is the one big tool that we have at our disposal in finance theory, information from the past. An asset price is random, we don't know its price in the future but we do know how this randomness has been realised in the past. By collecting these data and then by using some sophisticated mathematical models, our goal is to predict the asset prices in the future within a certain probability confidence.

If we want to be a little bit more formal, we would say that asset pricing theory is the use of standard probability theory together with a collection of assumptions about the nature of assets and the markets where these assets operate; with the ultimate goal of both explaining past behaviour as well as predicting, within certain probability limits, the future evolution of these assets.

Graphical representation of random markets: a simple plot of equity prices, bond prices and commodity prices (represented by blue, red and purple lines) from 1982 to 2016.

The ultimate goal of asset pricing theory

Hence, we come to the heart of the problem in asset pricing theory. The ultimate goal of any asset pricing theory is to:

- understand and describe the dynamics of assets
- explain investors' behaviours based on these dynamics
- predict future movements under certain assumptions and with certain statistical confidence.

How do we assign probability?

We therefore have, by the very definition of the problem at hand, to use tools that take into account the randomness of the market and try to do the best we can in order to predict – not with certainty, as this is impossible, but assign probabilities to – various potential events. So, how do we do all that?

View answer

These considerations very naturally guide us into the one tool we have at our disposal, in order to come up with asset pricing theories: this is nothing else other than **probability theory and stochastic calculus**, the well-established mathematical ideas that have been used for decades in other scientific fields.

Key points

So, to recap – and this should be a major takeaway message from this first lecture – we need to be reasonably comfortable with elements of probability theory and stochastic calculus, in order to grasp the fundamentals of asset pricing theory.

To put you at ease, you do not need a strong mathematics background for this introductory course and if you have no previous experience using probability theory, this is OK too. We will cover some elements during the course and we will leave more complicated concepts out of scope here.

The important thing is to always remember that our primary goal, whatever the model at hand, is to assign probabilities into all possible future outcomes. Once this is done there are lots of tools at our disposal to analyse this set of probabilities and derive predictions on assets' evolutions and investors' behaviours.

As a simplification, in many cases and in many different settings, the underlying probabilities are assumed from the outset and then further analysis is provided in order to demonstrate how these assumptions affect asset movements.

3. Probability theory and stochastics

We will now discuss the probability theory and stochastic calculus.

We always assume that in the background, there is some probability space triplet:

$W = \{\Omega, {}_{F}, P\}$:

$$\Omega$$

is a sample space containing all possible outcomes.

$$F$$

is the set of all considered events where each event contains other outcomes or is null (this is the information we can have in the market at time 't')

$$P$$

is the probability measure, a map between probabilities and events (a function that assigns probabilities to certain events in the future). It is equivalent to answering the question, ‘What is the probability that the weather is going to be good tomorrow?’ The event would be the response, ‘The weather is going to be good tomorrow’ and this function is going to tell us the actual probabilities.

These basic definitions raise a few philosophical questions on how prices really do change.

Random variables

We now introduce the idea of a random variable. As a reminder to everybody who has done a basic course in mathematics, a random variable is nothing but from the sample space Ω to the space of real numbers.

Icon of two dice.
Continuing with our previous example, the temperature that we will have tomorrow is a random variable because tomorrow is an event that has not happened yet. We don't know which event in the sample space will happen but we know that one event out of the many will happen. And then if this event happens, then we can say that the temperature is going to be, let's say, 10°C or 20°C etc.

So anything that happens in the future is a random variable, which leads us into considering the idea of a stochastic process.

Stochastic process

A stochastic process is a collection of random variables that actually evolves sequentially into the future. So, it is a set of random variables and the parameterisation is done in time. Temperature at a certain point is one random variable. However, if we think of the temperature as a whole, then it is a collection of random variables from today to the future, so it is a stochastic process. We can see how these ideas relate to finance. We can see immediately how we can think of stochastic processes of the stock price of a certain company or the market price of oil, etc. All these variables are variables for which we don't have the exact number because we're not living in the future yet.

Discrete versus continuous stochastic variables

We separate between discrete and continuous stochastic variables.

Discrete variables

We consider discrete modelling when we are only trying to make assumptions for the price of an asset at certain times.

Continuous stochastic variables

We consider continuous modelling when we assume that the asset price changes continually.

Liquid versus illiquid assets

So, just as a recap, assets are stochastic processes and we're trying to model these assets. Assets can be described as liquid or illiquid.

Screenshot of stocks.

Liquid assets

Many market participants and prices will be quoted in exchanges. Prices change dynamically, almost continually. Here we should think of listed stocks, listed bonds and the like. Hence, for liquid assets, continuous modelling is many times optimal.

House keys.

Illiquid assets

There is no official exchange and there are not many market participants. Prices don’t officially change very often. Here, we think of house prices for example, or art, or even stocks that are not listed. Hence, for illiquid assets, discreet modelling is often optimal.

Keep in mind that, in reality, many other factors come into play as well when we decide the type of stochastic process we choose in modelling an asset.

Brownian motion

We will now discuss the most fundamental and popular type of continued stochastic process, Brownian motion. We're going to be using this quite often in asset pricing and finance in general.

Here, the assets move with a certain uncertainty defined by very precise rules. For example, assets are not allowed to jump, but there is an inherent volatility.

What we call a Geometric Brownian motion is a variation of Brownian motion that always stays positive (used heavily in the modelling of stocks, for example) – this is an example of a ‘diffusion’.

So, it is a very ideal process to use when we try to model something that cannot go below zero. We will be discussing Brownian motion as we go along in this module in general as we look at many underlying assets of limited liability.

A graph representing Brownian motion: x axis is time going from 0 to 1.5, the graph shows a line across demonstrating certain uncertainty (it jumps and dips).

1. Introduction

As mentioned previously, it was vital that we familiarised ourselves with the mathematical techniques of stochastics and probability theory before we delved deeper into the fundamentals of asset pricing. Now that we have done so in the previous lesson, we are ready to explore asset pricing theory. We will begin with a brief introduction to the core objectives of asset pricing theory. With these objectives in mind, we will then describe the fundamentals of asset pricing theory, specifically looking at the various models that will be explained in more detail in later weeks. Here, we revisit the Capital Asset Pricing Model, which you may have seen in other modules on this programme, and the market risk premium. Finally, to conclude the week, we will discuss different types of investments that investors choose to maximise their wealth based on asset pricing theory.

2. Asset pricing theory objectives

The core objectives of asset pricing theory

We now turn to the objectives of asset pricing theory. Given our discussion up until this moment, we can summarise that the core objectives, when we do asset pricing and portfolio theory, are the following:

- Try to understand the random nature of assets and examine their past behaviour. This is done by collecting data and performing quantitative data analysis. This is mainly achieved by representing an asset as a random variable whose price we cannot know with 100 per cent probability in the future.
- Create mathematical models that explain this past behaviour as well as predict the future movements of assets, keeping in mind that no model is perfect and serves as best effort only.
- Based on the above, make investment decisions with the objective of optimising wealth within certain risk tolerances and preferences.

Core equations of asset pricing theory

So, turning our discussion back around to be a little bit more mathematical, asset pricing theory can be summarised into two equations.

$$Pt = E\left[m_{t+1}\chi_{t+1}\right]$$

$$m_{t+1} = f(\text{data, parameters})$$

where

Pt

is the price of the asset at time 't',

x^{t+1}

is the asset payoff at a future time '(t+1)' and

m_{t+1}

is a stochastic discount factor. The letter 'E' will be used to define expectations of random variables such as the stochastic factor or other entities that are not known as the current 't'.

A stochastic factor is the mathematical tool which we are going to use in order to discount the payoff from the future to today. This will, in general, be a function of data and some model parameters. We see here that, ‘finding’ the correct factor involves analysing data and, of course, model parameters.

The above paradigm is a generic way of describing an asset pricing theory. It is based on the idea of an asset (or a portfolio of assets) that is going to pay a certain amount in the future (payoff), the structure of which is normally pre-defined. What is not normally defined is the stochastic discount factor, the way used to bring the payoff amount into today.

A lot of work is going to be done in order to model this discount factor and many such models will be discussed here.

The ‘operator epsilon’, the expected value of a random variable, is a standard probability theory metric which will be discussed later on in detail.

3. Fundamentals of asset pricing theory

Present value model

We begin exploring the fundamentals of asset pricing theory by discussing the present value model.

This idea says that an asset is nothing more than a right on certain future cash flows and therefore, if we want to find the price of the asset today, this price will depend on these future payments.

Essentially, this means that if we know with a certain confidence (or even with 100 per cent confidence in some cases) the future payoff one would receive by holding this asset, then this should by itself determine the price of the asset today, when discounted appropriately.

This model is the basis of all work we will be doing and summarises all asset pricing theory into finding the correct payoff structure and discounting this into today.

More details will be given in the next lecture and in this course.

Efficient market hypothesis

We now turn to another fundamental idea, the efficient market hypothesis.

The first thing about this concept is that it is a hypothesis, so we should never forget that this a model that cannot be proven.

With this hypothesis, we assume that we live in a market with no transaction cost and where all information is available to all investors. Further, expectations of investors are homogenous, and finally, investors are rational within the marketplace.

Now, if we assume all that, then we can make the hypothesis that markets are efficient, in the sense that all the information about an asset is reflected 100 per cent on the assets price.

This hypothesis is the basis for providing a market benchmark and this benchmark is the market cap weighted portfolio.

Market cap portfolio

It is a portfolio whereby the assets are weighted according to their capitalisation in comparison to the whole market. These will determine the weights of its asset within the market cap portfolio and this portfolio is what we call the optimal portfolio for an investor.

To an investor, this means that it makes no sense to call for a different portfolio and that is because all the information is reflected in the price of the asset today. So, the full information is reflected there in the price and therefore the price alone will determine how much of this asset one should hold. If you like, the price of the asset becomes a relative number that shows the importance of this asset as compared to the whole market. An investor will therefore hold the market cap portfolio and one riskless asset.

As mentioned before, this is a hypothesis – a simplified version of reality – and therefore we do not expect it to hold in reality. We don't really expect this to hold in practice because we know that information is not freely available to everybody. We know that markets are not frictionless as well. So, the assumptions do not hold entirely, but nevertheless, we want to put this hypothesis to the test.

Here we are looking at two different types of returns. The first one, is market-cap weighted which is represented by the blue line. This is exactly how the S&P is defined. We are also looking at the performance of another index – the red line – which we again call the S&P, but equally weighted.

A graph with a blue line representing market-cap weighted (this is how the S&P is defined). Another index is shown by a red line, which is also called the equally weighted S&P index. The blue line follows the red line closely but is always below the red line.

	S&P500 - Market cap weighted	S&P500 - Equally weighted
Annualised return*	4.34%	7.65%
Annualised volatility	14.95%	17.20%

Across 18 years of history, the market cap weight would provide a 4.34 per cent annualised return with close to 15 per cent annualised volatility, and the equally weighted S&P 500 Index would provide a higher return of 7.65 per cent. However, this comes with a higher volatility of 17.2 per cent. In order to compare investment strategies, we always have to take into account the annual volatility.

The point being made here is that having the market cap weighted portfolio is just one idealised situation based on the efficient market hypothesis. There is no other alternative for the investor. In reality, however, there are alternatives and that's why there are many different types of investments and different types of ways to calculate an index. However, the main takeaway is that the efficient market hypothesis helps you provide a certain benchmark and it is very fundamental in deriving certain pricing models, as we will see later on.

No-arbitrage theory

We turn our attention to another fundamental idea within asset pricing, namely no-arbitrage.

The Investopedia definition of arbitrage is ‘the simultaneous purchase and sale of an asset to profit from a difference in the price. It is a trade that profits by exploiting the price differences of identical or similar financial instruments on different markets or in different forms.’

More formally, we say that arbitrage is the simultaneous buying and selling of securities, currencies or commodities in different markets or in derivative forms, in order to take advantage of different prices for the same asset.

Based on these definitions of arbitrage, what do you think no-arbitrage means in the financial sense?

View answer

No arbitrage means that there is no ‘free lunch’ in the market: if markets are efficient, one cannot profit instantaneously.

No-arbitrage theory is really a very powerful idea that leads into pricing by simply being able to calculate the present value for an asset by two different distinct methods. Because there is no arbitrage, these two (or more) methods must be equal today and by equating these, one can solve for the asset price. Option pricing theory is a typical application.

Again, this is a powerful idea but, still, it is a model and hence a simplification of reality in order to accommodate modelling.

We will explain in more detail, give more precise definitions and use the concept of no arbitrage heavily within this course.

4. CAPM and the market risk premium

When talking about the fundamentals of asset pricing theory, two concepts we must include are the capital asset pricing model and the market risk premium.

CAPM: The cornerstone of financial theory

We now move on to discuss the capital asset pricing model, and subsequently, the market risk premium. This will be mentioned again during the course of this module but we give here an intuitive introduction. If you have seen this model before, either in this programme or elsewhere in your academics, please reflect on what you remember.

The capital asset pricing model

The CAPM describes the relationship between systematic risk and expected returns. It is widely used in finance for pricing risky securities and for generating expected returns, and is derived using the principle of diversification using simplified assumptions. The most famous people who have developed this model include: Markowitz, Pinter and Mosin.

Assumptions	Resulting equilibrium conditions
Investors are risk averse and investments are limited to financial instruments.	All investors hold the same portfolio of risky assets: the market portfolio.
There are no taxes or transaction costs in the market.	The market portfolio contains all securities within the market and the weight of each security is its market value as a percentage of the total market.
Information is free and available to all investors.	The risk premium on the market as a whole depends on the average risk aversion of all market participants.
Investors are rational players trying to optimise their expected returns while minimising their risk, and the investors' expectations are homogeneous.	The risk premium on each individual security is a function of its covariance with the market.

Expected value of return given its risk – the CAPM formula

Expected value of return

$\overline{r_a} = r_f + \beta_a \left(\overline{r_m} - r_f\right)$

Where:

$r_f = \text{Risk free rate}$

$\beta_a = \text{Beta of the security}$

$\overline{r_m} = \text{Expected market return}$

We said that the capital asset pricing model describes expected returns of an asset, provided we know the risk of this asset. So, let's try to think a little bit about how that works by discussing this equation we see. The first part of the equation is the time value of money, the 'risk-free rate'. The second part of the equation is the inherent risk in the investment. Obviously, we don't know if the market portfolio is going to deliver more than the risk-free rate. But, that is what we expect on average. 'Beta' is a measure, not only of volatilities, but also of what we call a 'systematic risk', which is the risk that comes from the market as a whole. So, essentially, this parameter 'beta' of the securities is going to determine how risky this asset is compared to the market. We will discuss this more later on.

Market risk premium

The risk premium on the market portfolio will be proportional to its risk and the degree of risk aversion of the investor:

$E\left(r_m\right) - r_f = \bar{A}\sigma_M^2$ where σ_M^2 is the variable of the market portfolio and $\bar{A}$ is average degree of risk aversion across investors.

Alpha versus beta

Let's pause here to differentiate between 'alpha' and 'beta' and what they measure.

Alpha

‘Alpha’ measures the excess above the market return. It is a measure of performance based on a benchmark. So, if ‘alpha’ is equal to 1, that will mean that our asset or portfolio beats the benchmark by 1 per cent. This is a very important parameter when fund managers measure their performance.

Beta

Beta’ is a measure of the systematic risk of a certain security (compared to the market). For example, if ‘beta’ is equal to 1, then the asset has the same volatility as the market.

5. Making investments based on asset pricing theory

We now discuss types of investments. All investors are looking to maximise their wealth but there are many different types of benchmarks and different types of risk appetites.

Investors can look for undervalued companies (hence exploiting the inefficiency of the market), or can look for assets that pay good dividends or both.

Equally investors can have an absolute benchmark, where they want to maximise the value of their investments in absolute terms, or they want to take a relative view whereby they want to beat a certain benchmark index.

There are many ways to draw a strategy for portfolio management, depending on the investor's goal and risk tolerance. We will give some examples here.

Active versus passive investment

Active investment

Active investment tries to outperform the return of a benchmark, or optimise absolute returns by making active and very frequent decisions. These decisions will try to optimise returns, minimise risk or both. These funds will charge high fees as well as a performance fee in order to compensate for the fund manager's efforts and in return of the outperformance.

Passive investment

Passive investment aims to closely track a certain index or a certain market in a cost efficient way, with low transaction costs and low fees. These funds will typically concentrate on low risk and are looking to capture a certain market as best as possible.

How can portfolio managers achieve investment goals?

Other than active / passive differentiation the question remains how does a portfolio manager achieve his or her investment goals? We provide a few examples below.

Value investing

This investment involves researching the market to identify opportunities that are underpriced according to certain valuation models. So, we have a model, and our model is telling us that this particular asset price should be a certain level but we see that actually, the market price of this asset is a lot less.

Momentum investing

Here, we research the market to identify long-term averages and signals for momentum periods. Then, we invest accordingly, when there is a momentum or when the market is below a certain long-term mean. We can buy the asset and wait for it to revert back to this mean.

Short investing

We identify assets that are overpriced and go short with this particular market, with the anticipation of profiting when the market will come down.

1. Weekly summary

Here are the main concepts we covered in Week 1:

- We defined what an asset is, and explained that there could be tangible and intangible assets.
- We defined asset classes loosely as assets that behave similarly, differentiating between four common types of asset classes: equities, fixed-income, money markets and commodities.
- Each asset class has a number of indices, which show the performance of said asset class and are most often systematic.
- Each index can differ based on the type of return we consider, and we explained the differences between a total return index, an excess return index and a prize return index.
- It is important to keep in mind that although most indices are systematic, we can also have discretionary indices whereby rules are either unpublished or changing without investor approval.
- The financial market exists and investors buy and sell assets due to the fact that assets and the market are inherently random: we never know exactly what the future price of an asset will be.
- In asset pricing theory, we combine assumptions based on the nature of assets and markets with standard probability theory, with the goal of explaining past behaviour and predicting future evolution of those assets.
- No mathematical model is ever 100 per cent ‘correct’ - we can only aim to quantify uncertainty around the markets while working with a set of assumptions, and assign probabilities.
- We use probability theory and stochastic calculus to assign probabilities to future outcomes in the market.
- We can have continuous (when the price is always changing) or discrete (when we are measuring a price at a certain time) stochastic processes, with the most fundamental continuous stochastic process being Brownian motion.
- The core objectives of asset pricing theory are to examine historical and future behaviour of assets, create mathematical models to explain past behaviour and predict future behaviour and based on this data, make smart investment decisions - here, we provided the fundamental equations of asset pricing.
- The stochastic discount factor, the way used to bring the payoff amount into today, is not normally defined and will need to be modelled.
- When creating mathematical models, we must first model the expected payoff of an asset and then the discount factor we’d use to bring random cash flows into the present time.
- The present value model summarises all asset pricing theory into finding the correct payoff structure and discounting this into today.
- If all assumptions in the efficient market hypothesis are true, then we can make the hypothesis that markets are efficient, in the sense that all the information about an asset is reflected 100 per cent on the asset. This can help us form the market cap portfolio, the market benchmark.
- No-arbitrage theory leads to pricing by equating the present value of an asset by two distinct methods (these methods will be equal due to no arbitrage).
- The CAPM describes the relationship between systematic risk and expected returns. When viewing the expected return equation, the first part of the equation is the time value of money, the ‘risk-free rate’. The second part of the equation is the inherent risk in the investment.
- Alpha measures the excess return above market return, and beta measures the systematic risk of a portfolio compared to the market as a whole.
- Active investment tries to outperform the return of a benchmark, or optimise absolute returns by making active and very frequent decisions. Passive investment aims to closely track a certain index or a certain market in a cost-efficient way.

WEEK 1 ASSET PRICING WEBINAR SOLUTIONS

1. If markets are completely efficient, discuss the appropriate role of a portfolio manager. Compare this with the duties of portfolio managers in real life.

If security markets are completely efficient, portfolio managers will not be able to earn above-market returns. In this case, the portfolio manager has several responsibilities. First, the portfolio manager should seek optimal diversification while minimizing transaction costs. Second, the portfolio manager should help clients understand and quantify their risk tolerances and return needs. Finally, the portfolio manager should monitor both the clients' needs and circumstances and changes in the capital markets.
In real life, portfolio managers not only perform the above, they also focus on creating superior returns for their clients, compared to the market (in particular on the active types of investments). In general, the EMH is a hypothesis that is convenient to make in order to derive some of the most fundamental results in finance.

2. Consider a portfolio of 250 shares of firm A worth $30/share and 1500 shares of firm B worth $20/share. You expect a return of 4% for stock A and a return of 9% for stock B.

a. What is the total value of the portfolio, what are the portfolio weights and what is the expected return?

Portfolio value = 250 ($30) + 1500 ($20) = $37,000

Portfolio weights are

Xa = 250 ($30) / 37000 = 20%
Xb = 1500 ($20) / 37000 = 80%

Expected return = 0.20 (4%) + 0.08 (9%) = 8%

b. Suppose firm A's share price falls to $24 and firm B's share price goes up to $22. What is the new value of the portfolio? What return did it earn?

New Portfolio value = 250 ($24) + 1500 ($22) = $39,000

Return is : ($39000 - $37500) / 37500 = 4%

3. Suppose you estimate that stock A has a volatility of 32% and a beta of 1.42, whereas stock B has a volatility of 68% and a beta of 0.75. Suppose the risk-free rate is 2% and you estimate the market's expected return as 10%. Which stock has a higher return?

E(rA) = 0.02 + 1.42 (0.08) = 13.36%
E(rB) = 0.02 + 0.75 (0.08) = 8%

Based on the CAPM stock A has higher expected return than stock B.
(Sometimes this is referred to as cost of capital to the firm)
Also note that the beta of stock A is higher and that is why this stock has higher market risk. Beta is a measurement of the systematic risk of the stock compared to the market.
On the other hand, the volatility we see in total is higher for stock B, so stock B has more total risk (systematic and specific).

4. Describe CAPM.

CAPM describes the relationship between systematic risk and expected returns. The expected return of an asset is calculated by taking into account only one parameter, which is the market risk.
It has a number of assumptions such as: no taxes, no transaction costs, free information for all investors, investors are rational, Investor's expectation are homogeneous.
This is the formula;

$$\overline{r_a} = r_f + \beta_a(\overline{r_m} - r_f)$$

Where:
r_f = Risk free rate
β_a = Beta of the security
$\overline{r_m}$ = Expected market return

5. What is the difference between a day-trader and a long-term investor in terms of management of their trades?

Difference between Active Vs Passive Management

Asset Pricing
WEEK 1 Summary

Asset

Any instrument, financial or otherwise, tangible or not, which when held by investors is expected to maximise their wealth and their utility.

- Utility is a term in economics that refers to **the total satisfaction received from consuming a good or service**

Asset class

A group of assets and securities that behave in a similar way and have very similar characteristics. Their price is driven by common risk factors and they will usually face similar types of risk.

Well-known asset classes;

1. Equities

Within equities, we will have various types of stocks representing different companies. We'll have different sectors, different regions and so on. Within this asset class, we have further subcategories. Examples include sectors like pharmaceuticals, banks, retail etc and regions such as Europe, Americas, Asia and the like.

2. Fixed income

Example; bonds.
They are promises from one party to the other to repay the full capital back as well as pay a certain regular income, called coupon. These types of instruments can get more complex in terms of the actual payoff structure. We have subcategories here as well, such as government bonds, corporate bonds, high or low yield bonds etc. Classical fixed income debt securities have long maturity dates, from a few months to many years.

3. Alternatives

Such as commodities like energy
precious metals, oil, livestock and many others.
Real estates

What is an index?

An index shows the performance of a basket of assets or a portfolio. The majority of the indices are systematic in the sense that they follow a specific set of rules in terms of how they are constructed.

Types of indices;
There are 3 different types

1. Total return index

This index will capture the performance of an investment, including dividends received and including any interest on cash. Essentially, all income is considered whether it comes from growth, dividends or another source (hence the terminology 'total return').

2. Excess return index

This index measures the performance of a portfolio minus the risk-free rate or general benchmark (so we measure the excess above the benchmark).

3. Price return index

The price return index shows the performance of the basket, but here we assume that all dividends are paid out to the investor. So, essentially, this reflects an investment whereby the investor will actually 'cash out', as we say, all the dividends paid and these dividends do not count when we consider the performance as they are not reinvested.

Another way to categorise indices is;

a. Discretionary

Discretionary is the index whereby the rules are either not published or they change all the time without the approval of the investor.

b. Systematic

Systematic is an index whereby the rules of how the index is constructed are published and they are known to the investor.

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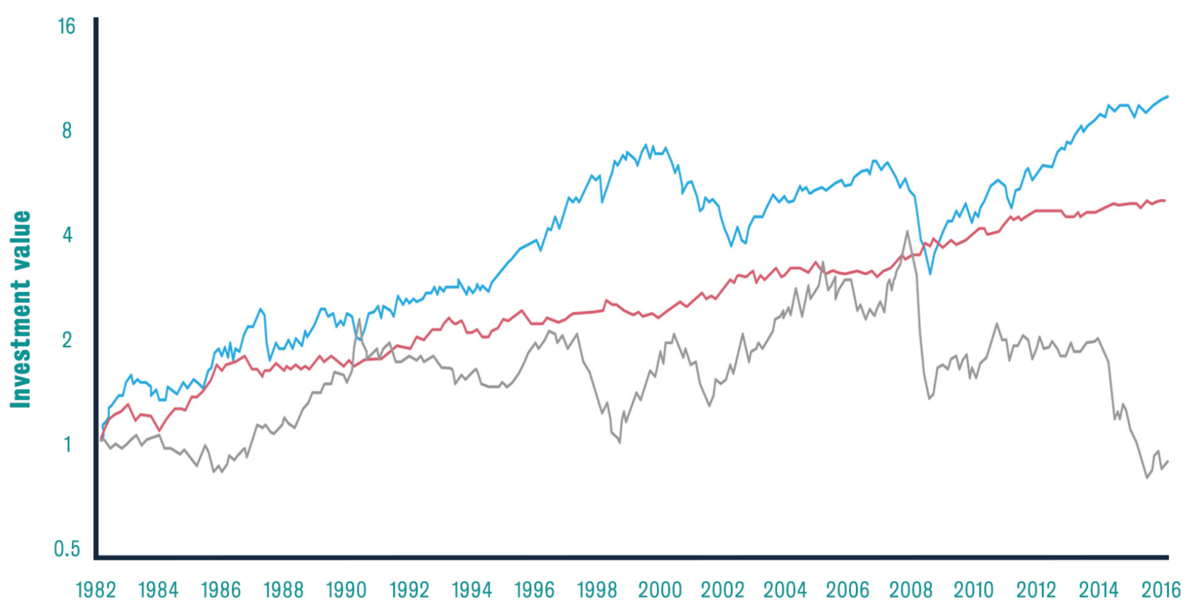
Inherent randomness of the financial market

It is important that we understand the random nature of the assets. This means that, as with any other aspect in life, we cannot predict future asset prices with a 100 per cent certainty. So, we don't know what the price of any asset will be in the future. We can only have an estimation with a certain probability. The financial markets exist and investors are allowed to participate in the markets by buying and selling assets exactly because of this inherent randomness that we have.

Graphical representation of random markets

simple plot of equity prices, bond prices and commodity prices



At any time point, from 1982 to 2016, no one would have been able to predict the future price of equities or bonds or commodities. But, what we do in asset pricing theory is that we look into the past and this is a plot of the past. We collect all these data, all these past prices for various assets. This is the one big tool that we have at our disposal in finance theory, information from the past. An asset price is random, we don't know its price in the future but we do know how this randomness has been realised in the past.

So to sum up;
asset pricing theory is the use of standard probability theory together with a collection of assumptions about the nature of assets and the markets where these assets operate; with the ultimate goal of both explaining past behaviour as well as predicting, within certain probability limits, the future evolution of these assets.

The ultimate goal of asset pricing theory;

- understand and describe the dynamics of assets
- explain investors' behaviours based on these dynamics
- predict future movements under certain assumptions and with certain statistical confidence.

Probability theory and stochastics

We always assume that in the background, there is some probability space triplet:

$$W=\{\Omega,\mathcal{F},P\}$$

Ω = (Omega) is a sample space containing all possible outcomes.

$\mathcal{F}$ = is the set of all considered events where each event contains other outcomes or is null (this is the information we can have in the market at time ’t')

P = is the probability measure, a map between probabilities and events (a function that assigns probabilities to certain events in the future). It is equivalent to answering the question, ‘What is the probability that the weather is going to be good tomorrow?’ The event would be the response, ‘The weather is going to be good tomorrow’ and this function is going to tell us the actual probabilities.

Random variables

As a reminder to everybody who has done a basic course in mathematics, a random variable is nothing but from the sample space Ω to the space of real numbers.

Example; the temperature that we will have tomorrow is a random variable because tomorrow is an event that has not happened yet. We don't know which event in the sample space will happen but we know that one even out of the many will happen. And then if this event happens, then we can say that the temperature is going to be, let's say, 10°C or 20°C etc.
So anything that happens in the future is a random variable, which leads us into considering the idea of a stochastic process.

Stochastic process;

A stochastic process is a collection of random variables that actually evolves sequentially into the future.
So to continue our example; if we think of the temperature as a whole, then it is a collection of random variables from today to the future, so it is a stochastic process.

stochastic variables have 2 types;

a. Discrete variables;

We consider discrete modelling when we are only trying to make assumptions for the price of an asset at certain times.

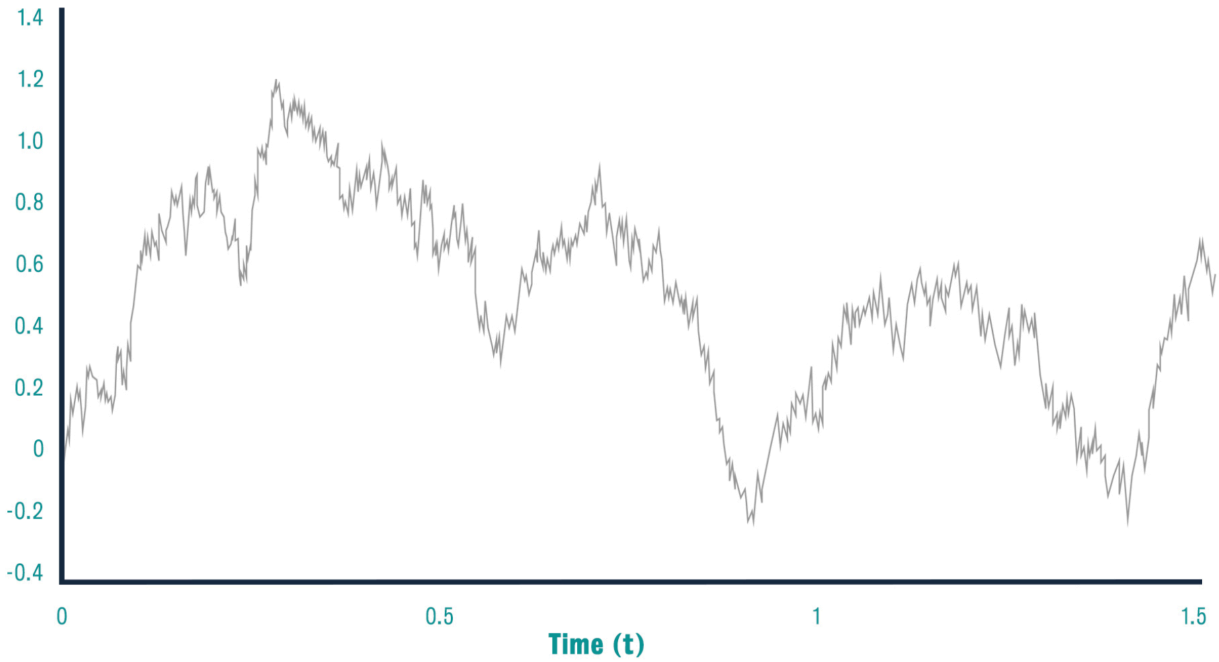
b. Continuous stochastic variables

We consider continuous modelling when we assume that the asset price changes continually.

As an example;

Brownian motion

continued stochastic process



Assets liquidity;

Liquid assets

Many market participants and prices will be quoted in exchanges. Prices change dynamically, almost continually. Here we should think of listed stocks, listed bonds and the like. Hence, for liquid assets, continuous modelling is many times optimal.

Illiquid assets

There is no official exchange and there are not many market participants. Prices don’t officially change very often. Here, we think of house prices for example, or art, or even stocks that are not listed. Hence, for illiquid assets, discreet modelling is often optimal.

Fundamentals of asset pricing theory;

- Present value model

if we know with a certain confidence (or even with 100 per cent confidence in some cases) the future payoff one would receive by holding this asset, then this should by itself determine the price of the asset today, when discounted appropriately.

- Efficient market hypothesis

The first thing about this concept is that it is a hypothesis, so we should never forget that this a model that cannot be proven.

With this hypothesis, we assume that we live in a market with no transaction cost and where all information is available to all investors. Further, expectations of investors are homogenous, and finally, investors are rational within the marketplace.
Now, if we assume all that, then we can make the hypothesis that markets are efficient, in the sense that all the information about an asset is reflected 100 per cent on the assets price.

- No-arbitrage theory

Arbitrage is the simultaneous buying and selling of securities, currencies or commodities in different markets or in derivative forms, in order to take advantage of different prices for the same asset.
So, No arbitrage means that there is no ‘free lunch’ in the market: if markets are efficient, one cannot profit instantaneously.

We now move on to discuss the capital asset pricing model, and subsequently, the market risk premium.

The capital asset pricing model;

The CAPM describes the relationship between systematic risk and expected returns. It is widely used in finance for pricing risky securities and for generating expected returns, and is derived using the principle of diversification using simplified assumptions.

Expected value of return

$$\overline{r_a} = r_f + \beta_a \left(\overline{r_m} - r_f\right)$$

Where:

$$r_f = \text{Risk free rate}$$

$$\beta_a = \text{Beta of the security}$$

$$\overline{r_m} = \text{Expected market return}$$

Types of investing;

Active investment

Active investment tries to outperform the return of a benchmark, or optimise absolute returns by making active and very frequent decisions. These decisions will try to optimise returns, minimise risk or both. These funds will charge high fees as well as a performance fee in order to compensate for the fund manager's efforts and in return of the outperformance.

Passive investment

Passive investment aims to closely track a certain index or a certain market in a cost efficient way, with low transaction costs and low fees. These funds will typically concentrate on low risk and are looking to capture a certain market as best as possible.

Alpha

'Alpha' measures the excess above the market return. It is a measure of performance based on a benchmark. So, if 'alpha' is equal to 1, that will mean that our asset or portfolio beats the benchmark by 1 per cent. This is a very important parameter when fund managers measure their performance.

Beta

Beta’ is a measure of the systematic risk of a certain security (compared to the market). For example, if ‘beta’ is equal to 1, then the asset has the same volatility as the market.