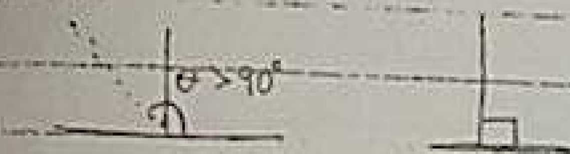
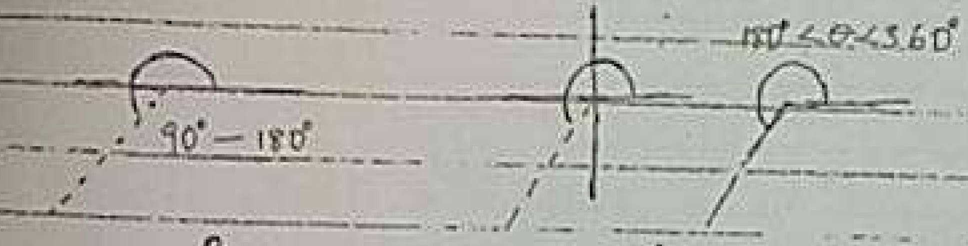
Obtuse angle: is an angle between 90° and 180° i.e $90° \le \theta < 180°$



Reflex angle: Is an angle between 180° & 360° i.e $180° \le \theta < 360°$



Radian measurement

In degree, a full revolution correspond to 360°. In radians a full revolution correspond to 2π radian. Radians are a unit of measurement for angles such that 2π radians corresponds to a rotation through an entire circle.

$360° = 2\pi$ rad

$\frac{360°}{2} = \frac{2\pi \text{ rad}}{2}$

$180° = \pi$ rad

$\frac{180°}{180°} = \frac{\pi \text{rad}}{180°}$

$1° = \frac{\pi \text{rad}}{180°}$ 180°

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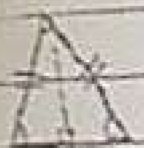
Proof that

$\boxed{\frac{0}{0}} \neq 2$

undefine $\neq 2$

Ratios of 30° and 60°



By pythagoras theorem:

$2^2 = 1^2 + h^2$

$4 = 1 + h^2$

$4 - 1 = h^2$

$3 = h^2 \Rightarrow h = \sqrt{3}$

1000+

$x = 1, y = \sqrt{3}, r = 2$

$\sin 30° = 1/2$

$\cos 30° = \sqrt{3}/2$

$\tan 30° = \dfrac{\sin 30°}{\cos 30°}$

$= 1/2 \div \sqrt{3}/2$

$= 1/2 \times 2/\sqrt{3}$

$\tan 30° = 1/\sqrt{3}$

$\sin 60° = \sqrt{3}/2$

$\cos 60° = 1/2$

$\tan 60° = \sqrt{3}$

~~& Note:~~

~~(i) $\sin 30° = \cos 60°$~~

~~(ii) $\sin 60° = \cos 30°$~~

~~In general, $\sin\theta$ = cos~~

Note: (i) $\sin 30° = \cos 60°$

(ii) $\sin 60° = \cos 30°$

In general, $\sin\theta = \cos(90° - \theta)$

$\cos\theta = \sin(90° - \theta)$

for $0° \le \theta < 90°$

Ratios of 45°

$\sin 45° = 1/\sqrt{2}$

$\cos 45° = 1/\sqrt{2}$

$\tan 45° = 1$

θ	0°	30°	45°	60°	90°	180°	360°
sin	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1	0	0
cos	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0	−1	1
tan	0	$1/\sqrt{3}$	1	$\sqrt{3}$	—	0	0

SIGNS OF THE RATIOS

2nd Quadrant	1st Quadrant
$90° \le \theta \le 180°$	$0 \le \theta < 90$
+ — $\sin\theta > 0$	$\sin\theta \ge 0$ or positive
$\cos\theta < 0$	$\cos\theta \ge 0$
$\tan\theta < 0$	$\tan\theta \ge 0$
$180° \le \theta < 270°$	$270° \le \theta \le 360°$
$\sin\theta < 0$	$\sin\theta < 0$
$\cos\theta < 0$	$\cos\theta > 0$ — +
+ — $\tan\theta > 0$	$\tan\theta < 0$
3rd Quadrant	4th Quadrant

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For negative angle $(-\theta)$

1. $\sin(-\theta) = -\sin\theta$
2. $\cos(-\theta) = \cos\theta$
3. $\tan(-\theta) = -\tan\theta$

Example 3: Express the following in terms of trig ratios of acute angle and then simplify.

a) $\tan 225°$ b) $\sin {}^{3\pi}/_4$ c) $-\cos 120°$ d) $-\sin(-250°)$ e) $\cos(-{}^{\pi}/_4)$

Solution:

• $\tan 225° = \tan(225 - 180°)$
$= 45° = 1$

• $\sin {}^{3\pi}/_4 =$

convert ${}^{3\pi}/_4$ to degree ${}^{3\pi}/_4 \times {}^{180}/_{\pi}$
$= 135°$

$\sin {}^{3\pi}/_4 = \sin(135°)$
$= \sin 45$
$= {}^{1}/_{\sqrt{2}}$

• $\cot(120) = \dfrac{-1}{\tan(120°)}$

$\tan 120° = -\tan(120° - 90°)$
$= -\tan 30°$
$= -{}^{1}/_{\sqrt{3}}$
$= \sqrt{3}$

• $-\sin(-250°) = -(-\sin 250°)$
$= \sin 250°$
$= -\sin(250° - 180°)$
$= -\sin 70°$

• $\cos(-{}^{\pi}/_4) = \cos(-\theta) = \cos\theta$
Convert $\cos(-{}^{\pi}/_4) = \cos {}^{\pi}/_4$
$= \cos 45° = {}^{1}/_{\sqrt{2}}$

13/07/2015

Cases when $\theta > 360°$

If $\theta > 360°$ then $\sin\theta = \sin(\theta - 360n)$
$\cos\theta = \cos(\theta - 360n)$
$\tan\theta = \tan(\theta - 360n)$

where n is the number of revolution.

Example 4: Express the following in terms of trig. ratios of acute angles.

a. $\cos(-1200°)$ b. $\sin(-1560°)$
c. $\cos(555°)$ d. $\tan(-740°)$

Soln:

a. $\cos(-1200°) = \cos(1200°)$

$\cos(1200°) = -\cos(1200° - 360(3))$
$= -\cos(1200° - 1080°)$
$= -\cos(120°)$
$= -\cos(180° - 120°)$
$-\cos 60°$
$= -{}^{1}/_{2}$

b. $\sin(-1560^\circ) = -\sin(1560^\circ)$

$= -\sin(1560^\circ - 360(4))$

$= -\sin(1560^\circ - 1440^\circ)$

$= -\sin(120^\circ)$

$= -\sin(180^\circ - 120^\circ)$

$= -\sin 60^\circ$

$= -\sqrt{3}/2$

c. $\cos(585^\circ) = \cos(585 - 360(1))$

$= \cos(585 - 360)$

$= \cos(225^\circ)$

$= -\cos(225^\circ - 180^\circ)$

$= -\cos(45^\circ)$

$= -1/\sqrt{2}$

d. $\tan(-840^\circ) = -\tan(840^\circ)$

$= -\tan(840^\circ - 360(2))$

$= -\tan(840^\circ - 720^\circ)$

$= -(-\tan(180^\circ - 120^\circ))$

$= \tan 60^\circ$

$= \underline{\sqrt{3}}$

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TRIGONOMETRIC EQN.

1. If $\sin x = y$, then $x = \sin^{-1}(y)$
2. If $\cos x = y$, then $x = \cos^{-1}(y)$

Example 5: Find d values of θ from 0° to 360° which satisfy.

a. $\sin\theta = -\sqrt{3}/2$

b. $\cos(\theta + 30^\circ) = -1/2$

c. $\cos^2\theta = 2\cos\theta$

Soln.

a. $\sin\theta = -\sqrt{3}/2$

$\theta = \sin^{-1}(-\sqrt{3}/2)$

$= -\sin^{-1}(\sqrt{3}/2)$

$\therefore -60^\circ = 300^\circ$

In d 3rd quadrant, $180^\circ + \theta = -60^\circ$

$\theta = 180^\circ + 60^\circ$

$= 240^\circ$

$\therefore \theta = 240^\circ; 300^\circ$

b. $\cos(\theta + 30^\circ) = -1/2$

$\theta + 30^\circ = \cos^{-1}(-1/2)$

$= \cos^{-1}(1/2) = 60$ $\quad 180 - 60$

$\theta + 30^\circ = 120^\circ$ $\quad = 120$

$\theta = 120^\circ - 30^\circ$

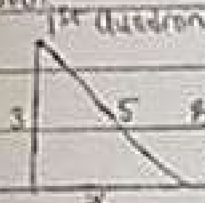
$\theta = 90^\circ$

$1^{st}/3^{rd}$

1. If $\sin\theta = 3/5$ without using table or calculator, d values of (i) $\cos\theta$ (ii) $\tan\theta$ — 23/07/15

2) Suppose $\cos\theta = -4/5$ and θ is obtuse. Find d value of $\sec\theta$, $\sin\theta$ and $\cot\theta$

Soln
1st quadrant



$\sin\theta = {}^{opp}/_{hyp} = 3/5$

By Pythagoras theorem,

$5^2 = 3^2 + x^2$

$x^2 = 5^2 - 3^2$

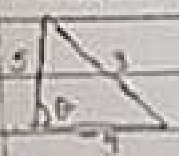
$= 25 - 9 = 16$

$x = \sqrt{16}$ ∴ $x = 4$

(i) $\cos\theta = {}^{adj}/_{hyp} = 4/5$

(ii) $\tan\theta = {}^{opp}/_{adj} = 3/4$

2nd quadrant



(i) $\cos\theta = -4/5$

(ii) $\tan\theta = 3/-4$

b) $\cos\theta = -4/5$

$\sec\theta = 1/\cos\theta$

$= 1/-4/5 = -5/4$

$\sin\theta = -3/5$

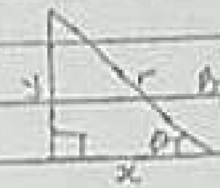
$\cot\theta = \frac{\cos\theta}{\sin\theta}$

$= -4/5 \div -3/5$

$= -4/1 \times -1/3 = \mp 4/3$

TRIGONOMETRIC IDENTITIES

Consider d right-angled triangle



Recall $\sin\theta = y/r$, $\cos\theta = x/r$ & $\tan\theta = y/x$

By Pythagoras theorem

$r^2 = x^2 + y^2$

$x^2 + y^2 = r^2$ —— (1)

dividing each term of eqn(1) by r^2

$x^2/r^2 + y^2/r^2 = r^2/r^2$

$(x/r)^2 + (y/r)^2 = 1$ —— (2)

substituting $\cos\theta = x/r$ and $\sin\theta = y/r$ into eqn(2) we have

$(\cos\theta)^2 + (\sin\theta)^2 = 1$

$\boxed{\cos^2\theta + \sin^2\theta = 1}$ —— (1)

Note: The above identity is true for any value of θ

Divide eqn (1) by $\sin^2\theta$

$\frac{\cos^2\theta}{\sin^2\theta} + \frac{\sin^2\theta}{\sin^2\theta} = 1/\sin^2\theta$

$\left(\frac{\cos\theta}{\sin\theta}\right)^2 + 1 = (1/\sin\theta)^2$

$(\cot\theta)^2 + 1 = (\operatorname{cosec}\theta)^2$

$\boxed{\cot\theta^2 + 1 = \operatorname{cosec}^2\theta}$ —— (2)

Divide eqn(1) by $\cos^2\theta$

$\frac{\cos^2\theta}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta} = 1/\cos^2\theta$

$\boxed{1 + \tan^2\theta = \sec^2\theta}$ —— (3)

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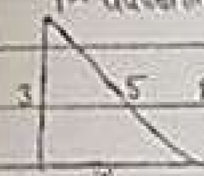
1. If $\sin\theta = 3/5$, without using table or 23/07/15
calculator, find the values of (i) $\cos\theta$ (ii) $\tan\theta$
b) Suppose $\cos\theta = -4/5$ and θ is reflex. Find the value of $\sec\theta$, $\sin\theta$ and $\cot\theta$

1st quadrant



$\sin\theta = \frac{opp}{hyp} = 3/5$

By Pythagoras theorem,

$5^2 = 3^2 + x^2$

$x^2 = 5^2 - 3^2$

$= 25 - 9 = 16$

$x = \sqrt{16} \therefore x = 4$

i) $\cos\theta = \frac{adj}{hyp} = 4/5$

ii) $\tan\theta = \frac{opp}{adj} = 3/4$

2nd quadrant

(i) $\cos\theta = -4/5$

(ii) $\tan\theta = -3/4$

b) $\cos\theta = -4/5$

$\sec\theta = 1/\cos\theta$

$= 1/-4/5 = -5/4$

$\sin\theta = -3/5$

$\cot\theta = \frac{\cos\theta}{\sin\theta}$

$= -4/5 \div -3/5$

$= -4/1 \times -1/3 = 4/3$

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TRIGONOMETRIC IDENTITIES

Consider a right-angled triangle

Recall $\sin\theta = y/r$, $\cos\theta = x/r$ & $\tan\theta = y/x$

By Pythagoras theorem

$r^2 = x^2 + y^2$

$x^2 + y^2 = r^2$ —— (1)

dividing each term of eqn(1) by r^2

$x^2/r^2 + y^2/r^2 = r^2/r^2$

$(x/r)^2 + (y/r)^2 = 1$ —— (2)

substituting $\cos\theta = x/r$ and $\sin\theta = y/r$ into eqn (2) we have

$(\cos\theta)^2 + (\sin\theta)^2 = 1$

$\boxed{\cos^2\theta + \sin^2\theta = 1}$ —— (1)

Note: The above identity is true for any value of θ

Divide eqn (1) by $\sin^2\theta$

$\frac{\cos^2\theta}{\sin^2\theta} + \frac{\sin^2\theta}{\sin^2\theta} = 1/\sin^2\theta$

$\left(\frac{\cos\theta}{\sin\theta}\right)^2 + 1 = (1/\sin\theta)^2$

$(\cot\theta)^2 + 1 = (\operatorname{cosec}\theta)^2$

$\boxed{\cot^2\theta + 1 = \operatorname{cosec}^2\theta}$ —— (2)

Divide eqn (1) by $\cos^2\theta$

$\frac{\cos^2\theta}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta} = 1/\cos^2\theta$

$\boxed{1 + \tan^2\theta = \sec^2\theta}$ —— (3)

Q.2 Solve d eqn for θ between 0° and 360° inclusive

a) $\cos^2\theta + \sin\theta + 1 = 0$

b) $2\tan^2\theta + 5 = 7\sec\theta$

c) $2\cot^2\theta + 8 = 7\operatorname{cosec}\theta$

Soln:

a) $\cos^2\theta + \sin\theta + 1 = 0$

$\cos^2\theta + \sin^2\theta = 1 \implies \cos^2\theta = 1 - \sin^2\theta$

$1 - \sin^2\theta + \sin\theta + 1 = 0$

$\sin^2\theta - \sin\theta - 2 = 0$

$\sin^2\theta + \sin\theta - 2\sin\theta - 2 = 0$

$\sin\theta(\sin\theta + 1) - 2(\sin\theta + 1) = 0$

$(\sin\theta + 1)(\sin\theta - 2) = 0$

either $\sin\theta + 1 = 0$ or $\sin\theta - 2 = 0$

$\sin\theta = -1$ or $\sin\theta = 2$

$\theta = \sin^{-1}(-1)$

$\theta = -90^\circ$

at angle $\alpha = 90^\circ$

3^{rd} Quadrant

$\theta = 360^\circ - \alpha$

$= 360^\circ - 90^\circ$

$= 270^\circ$

b) $2\tan^2\theta + 5 = 7\sec\theta$

$1 + \tan^2\theta = \sec^2\theta$

$\tan^2\theta = \sec^2\theta - 1$

$2(\sec^2\theta - 1) + 5 = 7\sec\theta$

$2\sec^2\theta - 2 + 5 - 7\sec\theta = 0$

$2\sec^2\theta + 3 - 7\sec\theta = 0$

$2\sec^2\theta - 7\sec\theta + 3 = 0$

$2\sec^2\theta - 6\sec\theta - \sec\theta + 3 = 0$

$2\sec\theta(\sec\theta - 3) - 1(\sec\theta - 3) = 0$

$(\sec\theta - 3)(2\sec\theta - 1) = 0$

either $\sec\theta = 3$ or $2\sec\theta = 1$ i.e $\sec\theta = 1/2$

$1/\cos\theta = 3$ or $1/\cos\theta = 1/2$

$\cos\theta = 1/3$ or $\cos\theta = 2$ – not possible.

$\cos\theta = 1/3$ – cos is in 1^{st} & 4^{th} Quadrant

$\theta = \cos^{-1}(1/3)$

$= 70.53^\circ$

at angle $\alpha = 70.53^\circ$

1^{st} Quadrant:

$\theta = 360^\circ - 70.53^\circ = 289.47^\circ$

$\theta = 70.53^\circ, 289.47^\circ$

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c) $2\cot^2\theta + 8 = 7\operatorname{cosec}\theta$

$\cot^2\theta + 1 = \operatorname{cosec}^2\theta$

$\cot^2\theta = \boxed{\operatorname{cosec}^2\theta - 1}$

$2(\operatorname{cosec}^2\theta - 1) + 8 = 7\operatorname{cosec}\theta$

$2\operatorname{cosec}^2\theta - 2 + 8 = 7\operatorname{cosec}\theta$

$2\operatorname{cosec}^2\theta - 7\operatorname{cosec}\theta + 6 = 0$

$2\operatorname{cosec}^2\theta - 4\operatorname{cosec}\theta - 3\operatorname{cosec}\theta + 6 = 0$

$2\operatorname{cosec}\theta(\operatorname{cosec}\theta - 2) - 3(\operatorname{cosec}\theta - 2) = 0$

$(\operatorname{cosec}\theta - 2)(2\operatorname{cosec}\theta - 3) = 0$

either $\operatorname{cosec}\theta = 2$ or $2\operatorname{cosec}\theta = 3$

i.e $1/\sin\theta = 2$ or $2/\sin\theta = 3$

$\rightarrow \sin\theta = 1/2$ or $\sin\theta = 2/3$

If $\sin\theta = 1/2$, then $\theta = \sin^{-1}(1/2) = 30°$

ref angle: $\alpha = 30°$

1st Quadrant: $\theta = 30°$ | 2nd Quadrant: $\theta = 180° - 30°$, $\theta = 150°$

If $\sin\theta = 2/3$, then $\theta = \sin^{-1}(2/3) = 41.81°$

ref angle: $\alpha = 41.81°$

1st Quadrant: $\theta = 41.81°$ | 2nd Quadrant: $\theta = 180° - 41.81° = 138.19°$

$\theta = 30°, 41.81°, 138.19°, 150°$

31/07/015

Examples:

a) If $z = a\sec\theta$, simplify (i) $z^2 - a^2$ (ii) $\frac{1}{\sqrt{a^2 - z^2}}$

b) If $x = \operatorname{cosec}\theta$ simplify

(i) $\frac{x}{x^2-1}$ (ii) $\frac{\sqrt{1-x^2}}{x}$

c) If $t = \tan\theta$, simplify (i) $\sqrt{t^2+1}$ (ii) $\frac{t}{\sqrt{1+t^2}}$

Soln:

a) $z = a\sec\theta$

$z^2 - a^2 = (a\sec\theta)^2 - a^2$

$= a^2\sec^2\theta - a^2$

$= a^2(\sec^2\theta - 1)$

$= a^2\tan^2\theta$

$= (a\tan\theta)^2$

(ii) $\frac{1}{\sqrt{a^2 - z^2}} = \frac{1}{\sqrt{a^2 - a^2\sec^2\theta}}$

$= \frac{1}{\sqrt{a^2(1-\sec^2\theta)}}$

consider: $1 + \tan^2\theta = \sec^2\theta$

$1 - \sec^2\theta = -\tan\theta$

$\frac{1}{\sqrt{a^2(1-\sec^2\theta)}} \Rightarrow \frac{1}{\sqrt{a^2\tan^2\theta}}$

$\therefore \frac{1}{\sqrt{1-\sec^2\theta}}$

b) $x = \operatorname{cosec}\theta$

i) $\frac{x}{x^2-1} = \frac{\operatorname{cosec}\theta}{\operatorname{cosec}^2\theta - 1}$

$= \frac{\operatorname{cosec}\theta}{\cot^2\theta}$

$= \frac{1}{\sin\theta} \div \frac{\cos^2\theta}{\sin^2\theta}$

$= \frac{1}{\sin\theta} \cdot \frac{\sin^2\theta}{\cos^2\theta}$

$= \frac{1}{\cancel{\sin\theta}} \cdot \frac{\cancel{\sin\theta}\sin\theta}{\cos\theta\cos\theta}$

$= \frac{\sin\theta}{\cos\theta\cos\theta} = \frac{\sin\theta}{\cos\theta} \cdot \frac{1}{\cos\theta} = \tan\theta\sec\theta$

i) $\cancel{\frac{\sqrt{x^2-1}}{x} = \frac{\operatorname{cosec}\theta}{\sqrt{\operatorname{cosec}^2\theta-1}}}$

$\cancel{= \frac{\operatorname{cosec}\theta}{\sqrt{\cot^2\theta}}}$

$\cancel{= \frac{1}{\sin\theta} \div \frac{\cos^2\theta}{\sin^2\theta}}$

ii) $\frac{\sqrt{x^2-1}}{x} = \frac{\sqrt{\operatorname{cosec}^2\theta-1}}{\operatorname{cosec}\theta}$

$= \frac{\sqrt{\cot^2\theta}}{\operatorname{cosec}\theta} = \frac{\cot\theta}{\operatorname{cosec}\theta}$

$= \frac{\cos\theta}{\sin\theta} \div \frac{1}{\sin\theta}$

$= \frac{\cos\theta}{\sin\theta} \cdot \frac{\sin\theta}{1}$

$= \cos\theta$

c) $t = \tan\theta$

i) $\sqrt{t^2+1} = \sqrt{\tan^2\theta+1}$

$= \sqrt{\sec^2\theta}$

$= \sec\theta$

ii) $\frac{t}{\sqrt{1+t^2}} = \frac{\tan\theta}{\sqrt{1+\tan^2\theta}}$

$= \frac{\tan\theta}{\sec\theta}$

$= \frac{\sin\theta}{\cos\theta} \div \frac{1}{\cos\theta}$

$= \frac{\sin\theta}{\cancel{\cos\theta}} \cdot \frac{\cos\theta}{1}$

$= \sin\theta$

Example 9: simplify the following expressions

a) $\sqrt{(1-\sin A)(1+\sin A)}$ b) $\operatorname{cosec} x \tan x$ c) $(\cot\theta)\sqrt{1-\cos^2\theta}$

Soln

a) $\sqrt{(1-\sin A)(1+\sin A)} = \sqrt{1-\sin^2 A}$

$= \sqrt{\cos^2 A}$

$= \cos A$

b) $\operatorname{cosec} x \tan x = \frac{1}{\sin x} \cdot \frac{\sin x}{\cos x}$

$= 1/\cos x$

$= \sec x$

c) $(\cot\theta)\sqrt{1-\cos^2 x} = \frac{\cos\theta}{\sin\theta} \cdot \sin\theta$

$= \cos\theta$

Example:10 prove the following identities

a) $\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = 2\operatorname{cosec}\theta$

b) $\frac{\operatorname{cosec}\theta}{\cot\theta+\tan\theta} = \cos\theta$

c) $\cos^4\theta - \sin^4\theta + 1 = 2\cos^2\theta$

Soln.

a) $\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta}$

$= \frac{\sin^2\theta + (1+\cos\theta)^2}{(1+\cos\theta)(\sin\theta)}$

$= \frac{\sin^2\theta + 1 + 2\cos\theta + \cos^2\theta}{\sin\theta + \cos\theta\sin\theta}$

$= \frac{2 + 2\cos\theta}{(1+\cos\theta)(\sin\theta)}$

$= \frac{2(1+\cos\theta)}{(1+\cos\theta)(\sin\theta)} = \frac{2}{\sin\theta} = 2\operatorname{cosec}\theta$

b) $\dfrac{\operatorname{cosec}\theta}{\cot\theta + \tan\theta} = \cos\theta$

$$\frac{\operatorname{cosec}\theta}{\cot\theta + \tan\theta} = \frac{1}{\sin\theta} \div \left(\frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}\right)$$

$$= \frac{1}{\sin\theta} \div \left(\frac{\cos^2\theta + \sin^2\theta}{\sin\theta\cos\theta}\right)$$

$$= \frac{1}{\sin\theta} \div \frac{1}{\sin\theta\cos\theta}$$

$$= \frac{1}{\sin\theta} \times \frac{\sin\theta\cos\theta}{1}$$

$$= \cos\theta$$

c) $\cos^4\theta - \sin^4\theta + 1 = 2\cos^2\theta$

$= (\cos^2\theta)^2 - (\sin^2\theta)^2 + 1 = 2\cos^2\theta$

$= (\cos^2\theta - \sin^2\theta)(\cos^2\theta + \sin^2\theta) + 1$

$= \cos^2\theta - \sin^2\theta + 1$ ~~(b) x = a sec θ + tan θ:~~

$= \cos^2\theta - (1 - \cos^2\theta) + 1$

$= \cos^2\theta - 1 + \cos^2\theta + 1$

$= 2\cos^2\theta$

Example 11 Eliminate θ from the following eqn.

a) $x = a\cos\theta$, $y = b\sin\theta$

by using $\cos^2\theta + \sin^2\theta = 1$

$\Rightarrow x^2 = a^2\cos^2\theta$, $y^2 = b^2\sin^2\theta$

$\Rightarrow \dfrac{x^2}{a^2} = \cos^2\theta$, $\dfrac{y^2}{b^2} = \sin^2\theta$

$\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = \cos^2\theta + \sin^2\theta \Rightarrow \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$

COMPOUND ANGLE

Let A & B be two different angles, then

$\sin(A+B) = \sin A\cos B + \sin B\cos A$

$\sin(A-B) = \sin A\cos B - \sin B\cos A$

also $\cos(A+B) = \cos A\cos B - \sin A\sin B$

$\cos(A-B) = \cos A\cos B + \sin A\sin B$ —— 2

Now:

$$\tan(A+B) = \frac{\sin(A+B)}{\cos(A+B)}$$

$$= \frac{\sin A\cos B + \sin B\cos A}{\cos A\cos B - \sin A\sin B}$$

$$= \frac{\dfrac{\sin A\cos B}{\cos A\cos B} + \dfrac{\sin B\cos A}{\cos A\cos B}}{\ldots}$$

$$\tan(A+B) = \frac{\dfrac{\sin A}{\cos A} + \dfrac{\sin B}{\cos B}}{1 - \dfrac{\sin A\sin B}{\cos A\cos B}}$$

Finally

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

Similarly: $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A\tan B}$

Example 12: Evaluate the following without using table/cal.

a) $\sin 75^\circ$ b) $\cos 15^\circ$ c) $\tan 105^\circ$ d) $\cos(5\pi/12)$

A) $\sin(30^\circ + 45^\circ)$

soal:

a) $\sin 75^\circ = \sin(45^\circ + 30^\circ)$

$= \sin 45^\circ \cos 30^\circ + \sin 30^\circ \cos 45^\circ$

$= 1/\sqrt{2} \cdot \sqrt{3}/2 + 1/2 \cdot 1/\sqrt{2}$

$= 1/\sqrt{2}\,(\sqrt{3}/2 + 1/2)$

$= 1/\sqrt{2}\left(\frac{\sqrt{3}+1}{2}\right)$

$= \frac{\sqrt{3}+1}{2\sqrt{2}}$

b) $\cos 15^\circ = \cos(45^\circ - 30^\circ)$

$= \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ$

$= 1/\sqrt{2} \cdot \sqrt{3}/2 + 1/\sqrt{2} \cdot 1/2$

$= 1/\sqrt{2}\,(\sqrt{3}/2 + 1/2) = 1/\sqrt{2}\left(\frac{\sqrt{3}+1}{2}\right)$

$= \frac{\sqrt{3}+1}{2\sqrt{2}}$

c) $\tan 105^\circ = \tan(60^\circ + 45^\circ)$

$= \frac{\tan 60^\circ + \tan 45^\circ}{1 - \tan 60^\circ \tan 45^\circ}$

$= \frac{\sqrt{3}+1}{1-(\sqrt{3})(1)}$

$= \frac{\sqrt{3}+1}{1-\sqrt{3}}$

d) $\cos(5\pi/24) =$

2) $\sin(120^\circ + 45^\circ)$

$= \sin 120^\circ \cos 45^\circ + \sin 45^\circ \cos 120^\circ$

But $\sin(120^\circ) = \sin 60^\circ$

$\cos(120^\circ) = -\cos 60^\circ$

$= \sin 60^\circ \cos 45^\circ - \sin 45^\circ (\cos 60^\circ)$

$= \sqrt{3}/2 \cdot 1/\sqrt{2} - 1/\sqrt{2} \cdot 1/2$

$= \frac{\sqrt{3}-1}{2\sqrt{3}}$

OR

$\sin(120^\circ + 45^\circ) = \sin((60^\circ + 60^\circ) + 45^\circ)$

$= \sin(60^\circ + 60^\circ)\cos 45^\circ + \sin 45^\circ \cos(60^\circ + 60^\circ)$

$= [\sin 60^\circ \cos 60^\circ + \sin 60^\circ \cos 60^\circ] \cos 45^\circ + \sin 45^\circ$

$(\cos 60^\circ \cos 60^\circ - \sin 60^\circ \sin 60^\circ)$

$= (2 \sin 60^\circ \cos 60^\circ)\cos 45^\circ + \sin 45 (\cos 60^\circ - \sin 60^\circ)$

$= (2 \cdot \sqrt{3}/2 \cdot 1/2)\,1/\sqrt{2} + 1/\sqrt{2}\,(1/2)^2 - (\sqrt{3}/2)^2$

$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{\sqrt{2}}(1/4 - 3/4)$

$= \sqrt{3}/2\sqrt{2} + 1/\sqrt{2}\,(-2/4)$

$= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}-1}{2\sqrt{2}}$

Example 2 Show that (a) $\sin(\pi/2 - x) = \cos x$ (b) $\cos(-x) = \cos x$
(c) $\cot(\pi/2 + x) = -\tan x$ (d) $\sin(x+\pi) = -\sin x$

Soln.

a) $\sin(\pi/2 - x) = \sin \pi/2 \cos x - \sin x \cos \pi/2$

$= \sin 90^\circ \cos x - \sin x \cos 90^\circ$

$= (1)\cos x - \sin x(0)$

$= \cos x - 0$

$= \cos x$

b) $\cos(-x) = \cos(0 - x)$

$= \cos 0 \cos x + \sin 0 \sin x$

$= (1)\cos x + (0)\sin x$

$= \cos x$

~~(c) $\cot(\pi/2 + x) = -\tan x$~~

~~$\cot\theta = \frac{\cos\theta}{\sin\theta}$~~

~~$\cos(\pi/2 + x) - \ldots$~~

(c) $\cot(\pi/2 + x) = -\tan x$

$\cot\theta = \frac{\cos\theta}{\sin\theta}$

$\cot(\pi/2 + x) = \frac{\cos(\pi/2 + x)}{\sin(\pi/2 + x)}$

$= \frac{\cos \pi/2 \cos x - \sin \pi/2 \sin x}{\sin \pi/2 \cos x + \sin x \cos \pi/2}$

$= \frac{(0)\cos x - (1)\sin x}{(1)\cos x + \sin x(0)}$

$= \frac{-\sin x}{\cos x} = -\tan x$

d) $\sin(x+\pi) = -\sin x$

$\sin(x+\pi) = \sin x \cos \pi + \sin \pi \cos x$

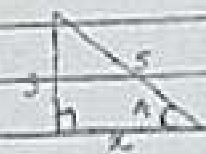
$= \sin x(-1) + (0)\cos x$

$= -\sin x$

Example 1: If $\sin A = 3/5$ and $\cos B = -12/13$, where A is acute and B is obtuse, find the values of (a) $\sin(A+B)$ (b) $\cos(A-B)$

c) $\tan(A-B)$

Solution: By Pythagoras



$5^2 = 3^2 + x^2$

$x^2 = 5^2 - 3^2$

$= 25 - 9$

$x^2 = 16$

$x = 4$

$13^2 = y^2 + (-12)^2$

$y^2 = 13^2 - (-12)^2$

$y = 169 - 144$

$= 25$

$y = 5$

$\cos B = -12/13$, $\sin B = 5/13$, $\tan B = -5/12$

a) $\sin(A+B) = \sin A \cos B + \sin B \cos A$

$= (3/5)(-12/13) + (5/13)(4/5)$

$= \frac{-36+20}{65} = -16/65$

b) $\cos(A-B) = \cos A \cos B + \sin A \sin B$

$= 4/5(-12/13) + (3/5)(5/13)$

$= \frac{-48+15}{65}$

$= -33/65$

K

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2/3° 4°, 3/4, 5°

c) $\tan(A-B) = \dfrac{\tan A - \tan B}{1+\tan A \tan B}$

$= \dfrac{3/4 - (-5/12)}{1+(3/4)(-5/12)}$

$= \dfrac{9+5}{12} \div \left(\dfrac{16}{16} - \dfrac{5}{16}\right)$

$= \dfrac{14}{12} \div \dfrac{11}{16}$

$= \dfrac{14}{12} \times \dfrac{16}{11} = \dfrac{56}{33}$

Example 4: Solve the following eqn: for $0° \le \theta \le 360°$

a) $\sin\theta\cos 10° + \cos\theta\sin 10° = 0.5$

b) $\sin(\theta+45°) = \sqrt{2}\cos\theta$

Soln:

a) $\sin\theta\cos 10° + \cos\theta\sin 10° = -0.5$

$\sin(\theta+10°) = -0.5$

$\theta + 10° = \sin^{-1}(-0.5)$

$\theta + 10° = -30°$

ref. angle, $\alpha = 30°$

3rd quadrant: $180° + \alpha = \theta + 10°$

$180° + 30° = \theta + 10°$

$210° = \theta + 10$

$360° - \alpha = \theta + 10°$

$360° - 30° = \theta + 10°$

$330° = \theta + 10°$

$\therefore \theta = 200°, 320°$

(b) $\sin(\theta+45°) = \sqrt{2}\cos\theta$

$= \sin\theta\cos 45° + \sin 45°\cos\theta = \sqrt{2}\cos\theta$

$= \dfrac{\sin\theta\cos 45°}{\cos\theta} + \dfrac{\sin 45°\cos\theta}{\cos\theta} = \dfrac{\sqrt{2}\cos\theta}{\cos\theta}$

$= \dfrac{1}{\sqrt{2}}\tan\theta + \dfrac{1}{\sqrt{2}} = \sqrt{2}$

$\sqrt{2} \times \dfrac{1}{\sqrt{2}}(\tan\theta + 1) = \sqrt{2} \times \sqrt{2}$

$\tan\theta + 1 = 2$

$\tan\theta = 1$

$\theta = \tan^{-1}(1)$

$\theta = 45°$

$\alpha = 45°$

4th quadrant

$\theta = 45°, 225°$

Example 5: prove the following identities:

a) $\cos(A+B)\cos(A-B) = \cos^2 A - \sin^2 B$

b) $\dfrac{\cos(A-B)}{\cos(A+B)} = \dfrac{1+\tan A\tan B}{1-\tan A\tan B}$

Soln: a) $\cos(A+B)\cos(A-B)$

$= (\cos A\cos B - \sin A\sin B)(\cos A\cos B + \sin A\sin B)$

$= \cos^2 A\cos^2 B + \cos A\cos B\sin A\sin B - \cos A\cos B\sin A\sin B - \sin^2 A\sin^2 B$

$= \cos^2 A - \cos^2 A\sin^2 B - \sin^2 A + \cos^2 A\sin^2 B$

$= \cos^2 A - \sin^2 B$

(b) $\dfrac{\cos(A-B)}{\cos(A+B)} = \dfrac{\cos A\cos B + \sin A\sin B}{\cos A\cos B - \sin A\sin B}$

$\dfrac{\dfrac{\cos A\cos B}{\cos A\cos B} + \dfrac{\sin A\sin B}{\cos A\cos B}}{\dfrac{\cos A\cos B}{\cos A\cos B} - \dfrac{\sin A\sin B}{\cos A\cos B}}$

$= \dfrac{1+\tan A\tan B}{1-\tan A\tan B}$

Example 6: express as a single trigonometric ratio.

a) $\frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x$

b) $\cos 15$

c) $\dfrac{1-\tan 45°}{1+\tan 45°}$ d) $\dfrac{\sqrt{3}+\tan x}{1-\sqrt{3}\tan x}$

Sol.

a) $\frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x = \sin 30^\circ \cos x - \cos 30^\circ \sin x$

$= \sin(30^\circ - x)$

b) $\frac{\cos 15^\circ + \sin 15^\circ}{\sqrt{2}} = \frac{1}{\sqrt{2}}\cos 15^\circ + \frac{1}{\sqrt{2}}\sin 15^\circ$

$= \cos 45^\circ \cos 15^\circ + \sin 45^\circ \sin 15^\circ$

$= \cos(45^\circ - 15^\circ)$

$= \cos 30^\circ$

$= \frac{\sqrt{3}}{2}$

c) $\frac{1 - \tan 45^\circ}{1 + \tan 45^\circ} = \frac{\tan 45^\circ - \tan 45^\circ}{\tan 45^\circ + \tan 45^\circ} \quad 2\tan 45^\circ = 0$

d) $\frac{\sqrt{3} + \tan x}{1 - \sqrt{3}\tan x} = \frac{\tan 60^\circ + \tan x}{1 - \tan 60^\circ \tan x}$

$= \tan(60^\circ + x)$

DOUBLE ANGLE FORMULAE

04/09/15

Recall

1. $\sin(A \pm B) = \sin A \cos B \pm \sin B \cos A$
2. $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$
3. $\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \pm \tan A \tan B}$

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Suppose $A = B$

1. $\sin(A + A) = \sin 2A$

$= \sin A \cos A + \sin A \cos A$

$= 2\sin A \cos A$

$\sin 2A = 2\sin A \cos A$ — (i)

2. $\cos 2A = \cos(A + A)$

$= \cos A \cos A - \sin A \sin A$

$= \boxed{\cos^2 A - \sin^2 A}$

$\therefore \boxed{\cos 2A = \cos^2 A - \sin^2 A}$

OR $\cos 2A = \cos^2 A - \sin^2 A$

$= \cos^2 A - (1 - \cos^2 A)$

$= \cos^2 A - 1 + \cos^2 A$

$= 2\cos^2 A - 1$

$= 2(1 - \sin^2 A) - 1$

$= 2 - 2\sin^2 A - 1$

$= \boxed{1 - 2\sin^2 A}$

$\therefore \cos 2A = \cos^2 A - \sin^2 A$

$\cos 2A = 2\cos^2 A - 1$

$\cos 2A = 1 - 2\sin^2 A$

3. $\tan 2A = \tan(A + A)$

$= \frac{\tan A + \tan A}{1 - \tan A \tan A}$

$= \frac{2\tan A}{1 - \tan^2 A} \quad \therefore \tan 2A = \frac{2\tan A}{1 - \tan^2 A}$

Evaluate the following

a) $2\sin 15° \cos 15°$

b) $\dfrac{2\tan 22\frac{1}{2}°}{1-\tan^2}$

c) $\dfrac{1-\tan^2 30°}{2\tan 30°}$

d) $\dfrac{1-2\cos^2 25°}{1-2\sin^2 65°}$

Solution

a) $2\sin 15° \cos 15° = 2\sin 2(15°)$

$= 2\sin 30°$

$= 2(\frac{1}{2})$

$= 1$

b) $\dfrac{2\tan 22\frac{1}{2}°}{1-\tan^2(22\frac{1}{2})°} = \tan 2(22\frac{1}{2})° = \tan 45°$

$= 1$

c) $\dfrac{1-\tan^2 30°}{2\tan 30°} = \dfrac{1}{\frac{2\tan 30°}{1-\tan^2 30°}}$

$= \dfrac{1}{\tan 2(30)°}$

$= \dfrac{1}{\tan 60°}$

$= \dfrac{1}{\sqrt{3}} \times \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{\sqrt{3}}{3}$

(4)

d) $\dfrac{1-2\cos^2 25°}{1-2\sin^2 65°} = \dfrac{-(-1+2\cos^2 25°)}{1-2\sin^2 65°}$

$= \dfrac{-(2\cos^2 25°-1)}{1-2\sin^2 65°}$

$= \dfrac{\cos 2(25)°}{\cos 2(65)°}$

$= \dfrac{-\cos 50°}{\cos 130°}$

$= \dfrac{-\cos 50°}{-\cos 50°} = 1$

e) $\sec 22\frac{1}{2}° \csc 22\frac{1}{2}° = \frac{1}{\cos 22\frac{1}{2}°} \times \frac{1}{\sin 22\frac{1}{2}°}$

$= \dfrac{1}{\sin 22\frac{1}{2}° \cos 22\frac{1}{2}°}$

$= \dfrac{1}{\frac{1}{2}\sin(2\times 22\frac{1}{2})}$

$= \dfrac{1}{\frac{1}{2}\sin 45°}$

$= \dfrac{1}{\frac{1}{2}\times\frac{1}{\sqrt{2}}} = \dfrac{1}{\frac{1}{2\sqrt{2}}}$

$= 2\sqrt{2}$

Example 2: Find the value of $\sin 2\theta$ and $\cos 2\theta$ when:

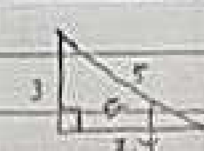
a) $\sin\theta = 3/5$, b) $\cos\theta = 12/13$ c) $\sin\theta = \sqrt{3}/2$.

Hence, find $\tan 2\theta$ in each case.

Soln:

a) $\sin\theta = 3/5$

$\sin 2\theta = 2\sin\theta\cos\theta$ $\quad 2(3/5) = 6/5$



$5^2 = 3^2 + x^2$

$25 = 9 + x^2$

$x^2 = 25 - 3^2$

$x^2 = 16$ $\quad x = 4$

1^{st} Quadrant 2^{nd} Quadrant

$\cos\theta = 4/5$ or $\cos\theta = -4/5$ $\sin 2\theta = 2\sin\theta\cos\theta$

then: $\sin 2\theta = 2\sin\theta\cos\theta$ $= 2\times 3/5 \times -4/5$

$= 2\times 3/5 \times 4/5$ $= -24/25$

$= 24/25$

$\cos 2\theta = 2\cos^2\theta - 1$

$= 2(\cos\theta)^2 - 1$

$= 2(4/5)^2 - 1$

$= 2\times 16/25 - 1$

$= 32/25 - 25/25$

$= \frac{32-25}{25} = 7/25$

$\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = \frac{24/25}{7/25} = \frac{24}{7}$ (1^{st} Quadrant)

$\tan 2\theta = -24/7$ (2^{nd} Quadrant)

b & c exercise

Example 1: Solve for θ from 0° to 360°

a) $\sin 2\theta = \sin\theta$ (b) $4\tan\theta\tan 2\theta = 1$

Soln:

a) $\sin 2\theta = \sin\theta$

$\sin 2\theta - \sin\theta = 0$

$2\sin\theta\cos\theta - \sin\theta = 0$

$\sin\theta(2\cos\theta - 1) = 0$

either $\sin\theta = 0$ or $2\cos\theta - 1 = 0$

if $\sin\theta = 0$

$\Rightarrow \theta = \sin^{-1}(0) = 0^\circ$

$\theta = 0^\circ, 360^\circ, 180^\circ$

if $2\cos\theta - 1 = 0$, then

$\theta = \cos^{-1}(1/2)$

$= 60^\circ$

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4th Quadrant: $360 - \alpha$

$\theta = 360^\circ - 60^\circ$

$= 300^\circ$

$\therefore \theta = 0, 60^\circ, 180^\circ, 300^\circ, 360^\circ$

b) $4\tan\theta\tan 2\theta = 1$

$4\tan\theta\left(\frac{2\tan\theta}{1-\tan^2\theta}\right) = 1$

$\frac{8\tan^2\theta}{1-\tan^2\theta} = 1$

$8\tan^2\theta = 1 - \tan^2\theta$

$8\tan^2\theta + \tan^2\theta - 1 = 0$

$9\tan^2\theta - 1 = 0$

$3^2\tan^2\theta - 1 = 0$

$(3\tan\theta)^2 - 1^2 = 0$

$(3\tan\theta - 1)(3\tan\theta + 1) = 0$

either $3\tan\theta - 1$ or $3\tan\theta + 1 = 0$

$\tan\theta = 1/3$ or $\tan\theta = -1/3$

$\theta = \tan^{-1}(1/3)$ $\theta = \tan^{-1}(-1/3)$

$\theta = 18.43^\circ$ $\theta = -18.43^\circ$

1st angle $\alpha = 18.43^\circ$

3^{rd} Quadrant: $180 + \alpha$

$\theta = 180^\circ + 18.43^\circ$

$= 198.43^\circ$

2^{nd} Quadrant $180 - \alpha$

$\theta = 180^\circ - 18.43^\circ$

$= 161.57^\circ$

4^{th} Quadrant: $360^\circ - \alpha$

$\theta = 360^\circ - 18.43^\circ$

$\therefore \theta = 18.43, 161.57, 341.57$ ... 341.57°

Example 4 Eliminate θ from the following

a) $x = \cos\theta$, $y = \cos 2\theta$

b) $x = 2\sin\theta$, $y = 3\cos 2\theta$

c) $x = 2\cos 2\theta$, $y = 3\cos\theta$

Soln

a) $\cos 2\theta = 2\cos^2\theta - 1$

$y = 2x^2 - 1$

b) $x = 2\sin\theta$, $y = 3\cos 2\theta$

$\cos 2\theta = 1 - 2\sin^2\theta$

$\sin\theta = x/2$, $\cos 2\theta = y/3$

$y/3 = 1 - 2(x/2)^2$

$y/3 = 1 - \frac{2x^2}{4}$

$y/3 = 1 - x^2/2$

$y/3 + x^2/2 = 1$

c) $x = 2\cos 2\theta$, $y = 3\cos\theta$

$\cos 2\theta = x/2$, $\cos\theta = y/3$

$\cos 2\theta = 2\cos^2\theta - 1$

$x/2 = 2(y/3)^2 - 1$

$x/2 = 2y^2/9 + 1 = 0$

Example 5 prove the following identities:

a) $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

b) $\frac{\cos 2\theta}{\cos\theta + \sin\theta} = \cos\theta - \sin\theta$

c) $\sin x = \frac{2\tan\frac{1}{2}x}{1+\tan^2\frac{1}{2}x}$

Soln:

a) $\sin 3\theta = \sin(2\theta + \theta)$

$= \sin 2\theta\cos\theta + \sin\theta\cos 2\theta$

$= 2\sin\theta\cos\theta\cos\theta + \sin\theta(1 - 2\sin^2\theta)$

$= 2\sin\theta\cos^2\theta + \sin\theta - 2\sin^3\theta$

$= 2\sin\theta(1 - \sin^2\theta) + \sin\theta - 2\sin^3\theta$

$= 2\sin\theta - 2\sin^3\theta + \sin\theta - 2\sin^3\theta$

$= 3\sin\theta - 4\sin^3\theta$

Similarly: $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$ — Exercise

b) $\frac{\cos 2\theta}{\cos\theta + \sin\theta} = \frac{\cos^2\theta - \sin^2\theta}{\cos\theta + \sin\theta}$

$= \frac{(\cos\theta + \sin\theta)(\cos\theta - \sin\theta)}{\cos\theta + \sin\theta}$

$= \cos\theta - \sin\theta$

c) $\cos 2x = 2\cos^2 x - 1$

$\sin 2x = 2\sin x\cos x$

$\sin(2x/2) = 2\sin x/2 \cos x/2$

$\sin x = 2\sin\frac{1}{2}x\cos\frac{1}{2}x$

$\sin x = 2\sin\frac{1}{2}x\cos\frac{1}{2}x \cdot \frac{\cos\frac{1}{2}x}{\cos\frac{1}{2}x}$

$= \frac{2\sin\frac{1}{2}x}{\cos\frac{1}{2}x} \cdot \cos^2\frac{1}{2}x$

$= 2\tan\frac{1}{2}x \cdot 1/\sec^2\frac{1}{2}x$

$= \frac{2\tan\frac{1}{2}x}{\sec^2\frac{1}{2}x}$

$= \frac{2\tan\frac{1}{2}x}{1+}$ multipl[illegible] simplify $\cos x = \dfrac{1-\tan^2 \frac{1}{2} x}{1+\tan^2 \frac{1}{2} x}$

* Set of irrational Number

is any no that is not rational e.g $\sqrt{2}, \pi$ etc

- Set of real no

The set contain | all rational or irrational no.

- All number line

$\longleftarrow \cdots -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \cdots \longrightarrow$

more calculation is based on $\mathbb{R}$

division by zero is not allow. e.g $8/0$ is undefined

$0/0$ is indeterminate.

positive real no have root of $16 \sqrt{16} = \pm 4$

negative " " " no root in $\mathbb{R}$

e.g. $\sqrt{-1}$ we can take d sequence root of -ve.

no in general -ve real number do not have even root but positive, +0, -1 (real no has odd root have $\sqrt[3]{-1} = (-1)^{1/3}$

OPERATION OF REAL NUMBER

If a,b,c are d real number, then:

i) $a+b = b+a$; the commutative law of addition

ii) $ab = ba$, the commutative law of multiplication

iii) $(a+b)c = a+(b+c)$; the association law of addition.

iv) $(ab)c = a(bc)$; The association law of multiplication

v) $a(b+c) = ab+ac$; The distributive law of multiplication & addition.

CF4