Examining Hotelling's law

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I. INTRODUCTION

We will discuss the cellular auto-meta based model which captures the competition between different shops selling goods to the Customers at the same or different prices and located at the different locations. In the rational world, each store(shop) tries to gain the maximum Customer share to make the maximum profit. For that owner can change the location of the store and move to the more profitable(more Customers) location or can change the price of the good or can do both. We assume that store owners tries to gain more Customer share to gain long term benefits rather than short term benefits. Such competitions in real world can exist between any number of stores. We will see through simulation how the actions of these store owners can lead stores to decide the best location for the stores and prices for the goods. **According to Hotelling's law, there is an undue tendency for competitors to imitate each other in quality of goods, in location and in other essential ways**[3].

II. MODEL

A cellular auto-meta based model for this problem consists of the grid of cells, where each cell either represents a Customer or a shop. If the cell represents the shop then cell will have value equal to total number of stores plus store number(each is assigned to a unique number) in the grid. If the cell represents the Customer then the value in the cell represents which store that Customer chooses to buy goods. Dynamics of the grid changes after every time instance as Customer's choice of the store depends on both distance of the Customer from all stores and price of all stores. At any time instance, state of the grid can tells the locations of all the stores, choice of store for each Customer and the total Customer share that each store has. Based on these values in one state, stores and Customers take the new decision which will be reflected in the next state. Our goal is to get the inference about model after long enough simulation of above process.

A cellular auto-meta based model depends on rules that changes one state to next state. Before deriving the rules, it is important to talk about the necessary assumption for the modelling,

- Initial locations of the stores are randomly initialized.
- All the Customer assigned to the particular store based on its distance from the store and price of the goods at the store.
- No two stores can be located at the same location.
- Each Customer will buy goods from one and only one store.
- When multiple stores located at the different locations have equal value(distance between store and Customer + price of goods) to Customer then Customer will choose the store randomly out of these multiple stores.
- Optimization move of one seller is independent from other sellers.
- To increase the Customer share, each store use some strategy, here, we assume that all the store owners are using same kind of strategy at any point of time.
- At any time step, actions by store will be dependent on states of previous time step.

Rules

Considering above assumptions we will define the rules for the simulation of cellular auto-meta. Our cellular auto-meta consists of two types of cells, so we need to define two types of rules, one type of rules for Customers and other types of rules for stores. Rules for Customers are trivial, while rules for stores depends on the strategy through which store owner tries to increase the Customer share. Here, we try to see the impact of three such strategies on the simulation,

- **First Strategy** : Store owner only changes the location of the store.
- **Second Strategy** : Store owner only changes the prices of the goods.
- **Third Strategy** : Store owner alternatively changes the prices of the goods and location of the store.

First we define the rules for both, Change in location and Change in price.

A. Rules for Changing Location

- Customer chooses the store which has the minimum sum of distance between store and Customer and price of the goods at the store.
- In any instance, to increase the Customer share, store owner tries to randomly move in one of the four direction and calculate store's new Customer share, if the Customer share has increased then store owner moves his/her store into that direction, otherwise look for the next random directions to move. Here, four possible direction for store movement are up, down, left and right.
- Price of the goods at all stores remains constant.

- All stores can move simultaneously based on information of previous state.
- If in any instance, any particular store is cornered by other stores then store will remain in its current position until it has a place to move.
- If any store doesn't have neighbouring position with higher Customer share then that store will remain to its current position.

B. Rules for changing Price

- When store has Customer share which is greater than $\frac{1}{NumberOfStores}$, store owner will increase the price of the good by constant amount which very small compare to original cost of the good.
- When store has Customer share which is smaller than or equal to $\frac{1}{NumberOfStores}$, then store owner either decrease the price with constant amount or does not change the price. Store owner take this decision randomly.
- Decrease or increase in the price will be constant and same for all the stores, and very small compare to original price of the good.
- Locations of all the store will not change.
- Multiple stores can not change the price at the same instance of time.

III. RESULTS

Strategy 1: Location movement only

For 2 stores

We will consider 2 cases, one in which price is same for both stores and other in which price is different for both.

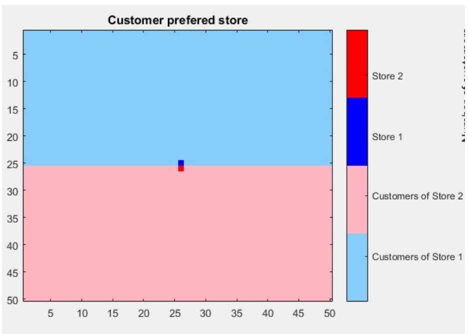

FIG. 1: Final state of Grid for **Same initial prices** and Strategy 1

Here, the initial prices are same for both the stores, so Customer will choose the store which is closer to him/her. As we can see that for same initial prices, store comes to

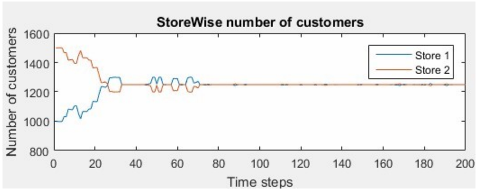

FIG. 2: Share of customers and Prices of stores for **Same initial prices** and Strategy 1

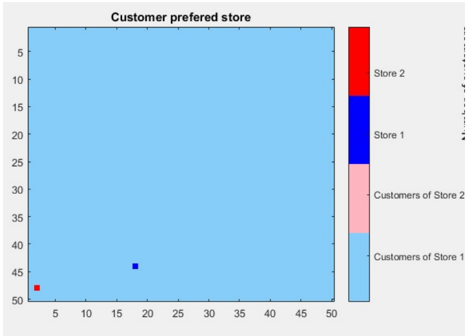

FIG. 3: Final state of Grid for **Different initial prices** (92:blue and 109:red) and Strategy 1

middle of the grid as that's the only way they can get guaranteed half share of customers.

While in case of different prices that is not the case, stores won't get stabilized in the middle because price is also a factor. Here, **store with higher price will want to come to the middle to attract more customers but at the same time it can not get much closer to other store due to its high price, because if both are almost near to each other than all customers would prefer store with lower price. So If the prices of the stores are different then only way to get the stable state is when store with lower price gets complete customer share and store with higher price can not able to find the neighbouring location with higher customer share than it's current customer share.**

Strategy 2: Price movement only

For 2 stores

In this case we will fix the location of both the stores and only change the prices of the goods according to rules.

FIG. 4: Share of customers and Prices of stores for **Different initial prices** (92:blue and 109:red) and Strategy 1

FIG. 5: Final state of Grid for Strategy 2

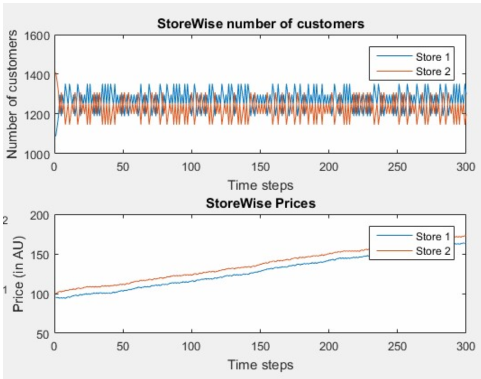

FIG. 6: Share of customers and Prices of stores for Strategy 2

Initially stores have different prices and locations, now one which is far from middle of the grid will decrease its price to get its share of customers ($\frac{1}{NumberOfStores}$). After achieving almost equal share of customer stores will start oscillatory behavior of share of customers by increasing prices when it is dominant and keeping prices constant or decreasing it when it is not dominant. This will result into exchanging the state of dominance in next iteration. So in any iteration one store will increase the price while the other store will decrease it or keep it constant, hence on an average there will be increase in price as shown in figure 6.

Strategy 3: Price and Location movement alternatively

In this case, stores will alternatively change the prices and locations. Initial locations and prices are chosen randomly. Figures 7 and 8 represent this case.

For 2 stores

The most important factor that determines the size of oscillations is the distance between two stores. If the distance is small then oscillations are larger and if the distance is large then oscillations are small. This is the reason why we are getting oscillations of different amplitudes

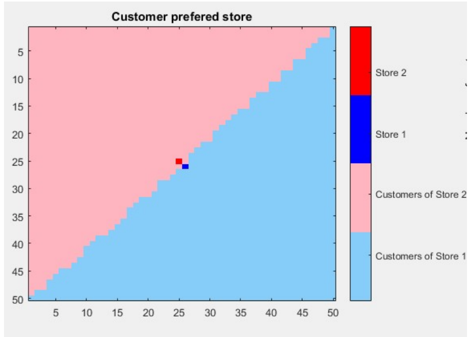

FIG. 7: Final state of Grid for Strategy 3

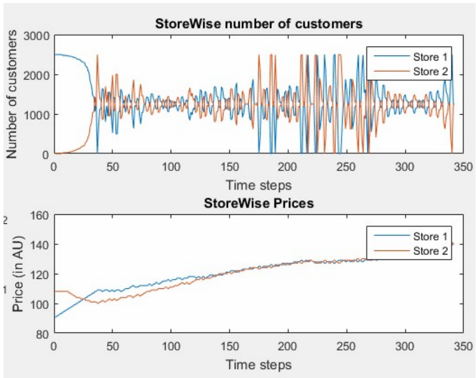

FIG. 8: Share of customers and Prices of stores for Strategy 3

in figure 8.

For 3 stores

As Evident from Fig. (9) and (10), There is an equilibrium, when considering the system of competition between 3 stores, but here all three stores don't get close to each other as compared to case of 2 Store and also, they try to align in some linear fashion, as opposed to case of 2 Stores where they try to align in Median Position.

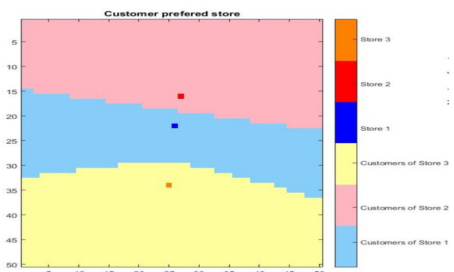

FIG. 9: Final state of Grid for Strategy 3

Fig. (10) shows the corresponding Price and Customer Base Plots for 3 stores, while they move from a random

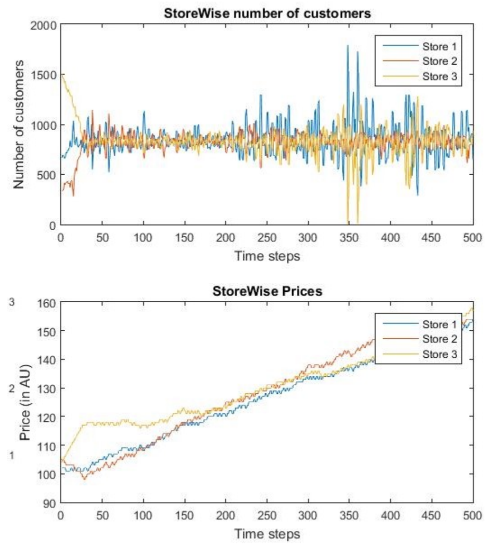

FIG. 10: Share of customers and Prices of stores for Strategy 3

initial condition to a state as seen in Fig. (9) and also for the duration where store in middle oscillates in either direction with hope to increase its customer base, while other stores respond with equivalent responses in form of movement of price or location of both, in order to maintain the stable customer base, or even try to increase customer base.

For 4 stores

System of 4 Stores behaves very uniquely, in sense that, it shows a equilibrium in which one can seen the system as competition between two 2-Stores System. So, 2-2 stores are in Local Competition with each other and such systems are in competition with each other globally. Though, there is always competition from every direction for each store, pairing them makes understanding of system dynamics much easy.

FIG. 11: Final state of Grid for Strategy 3

Fig. (12) shows the price and customer base of stores for case of 4 stores which moves from random initial condition to state somewhat similar to that shown in Fig. (11), and for further time steps, where each store tries to change their location and price in order to increase their customer base, and they still remain in equilibrium due to the equivalent responses from other stores.

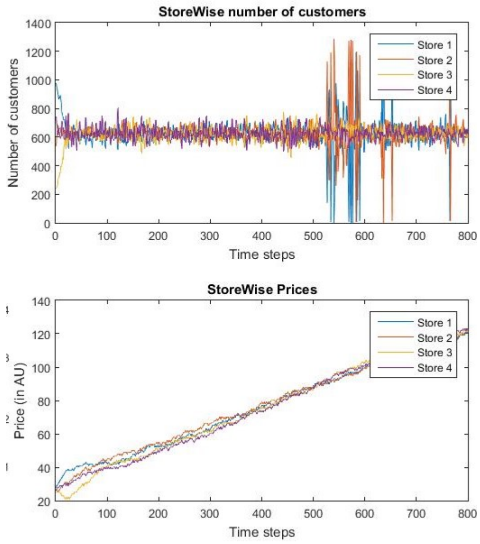

FIG. 12: Share of customers and Prices of stores for Strategy 3

As a result, location becomes stable (even if some store try to change its location to increase its customer base, other stores can respond by changing location or price or both, which forces the store which made the first move to stay in equilibrium location), and price keeps on increasing, as rules in model state that stores should increase price when they have a certain threshold customer base.

IV. CONCLUSIONS

In conclusion we have observed collective behavior of stores competing with each other obeying rules independently from other stores. In strategy 1, stores will try to move towards the center to get maximum benefit. We can see this kind of strategy applied in real life by local vendors, for example vendors who are selling food items on a beach, try to locate the position where they can find maximum customers, that is the middle position. In strategy 2, stores will try to get maximum customers by changing price. This kind of strategy applied by various telecommunication companies like Airtel, Jio, Vodafone etc. When Jio entered the market with lower price, large amount of customers switched to Jio, which forced other companies to decrease their the price. When all companies reach at the stable state that means almost get equal amount of customer share, all companies collectively slowly start increasing the prices until some company significantly decreases the price or new company enters into market. In strategy three, both the stores will be closer to the center, but considerably apart from each other to have volatility in price. In all strategy, we can also include other factors like quality of goods, reputation of stores, service provided by the stores etc instead of using just location and price.

[1] Ottino, B., Stonedahl, F. and Wilensky, U. (2009), *NetLogo Hotelling's Law model*, http://ccl.northwestern.edu/netlogo/models/Hotelling'sLaw, Center for Connected Learning and Computer-Based Modeling, Northwestern University, Evanston, IL.
[2] Wilensky, U. (1999), *NetLogo*, http://ccl.northwestern.edu/netlogo/, Center for Connected Learning and Computer-Based Modeling, Northwestern University, Evanston, IL.
[3] Hotelling, Harold. (1929). *"Stability in Competition."*, The Economic Journal 39.153: 41-57. http://www.jstor.org/stable/2224214