Applied Soft Computing Journal 96 (2020) 106610

Contents lists available at ScienceDirect

Applied Soft Computing Journal

journal homepage: www.elsevier.com/locate/asoc

Forecasting of COVID19 per regions using ARIMA models and polynomial functions

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ARTICLE INFO

Article history:
Received 10 June 2020
Received in revised form 30 July 2020
Accepted 2 August 2020
Available online 6 August 2020

Keywords:
Covid-19 epidemic
Forecast
ARIMA model
Geographic region

ABSTRACT

COVID-2019 is a global threat, for this reason around the world, researches have been focused on topics such as to detect it, prevent it, cure it, and predict it. Different analyses propose models to predict the evolution of this epidemic. These analyses propose models for specific geographical areas, specific countries, or create a global model. The models give us the possibility to predict the virus behavior, it could be used to make future response plans. This work presents an analysis of COVID-19 spread that shows a different angle for the whole world, through 6 geographic regions (continents). We propose to create a relationship between the countries, which are in the same geographical area to predict the advance of the virus. The countries in the same geographic region have variables with similar values (quantifiable and non-quantifiable), which affect the spread of the virus. We propose an algorithm to performed and evaluated the ARIMA model for 145 countries, which are distributed into 6 regions. Then, we construct a model for these regions using the ARIMA parameters, the population per 1M people, the number of cases, and polynomial functions. The proposal is able to predict the COVID-19 cases with a RMSE average of 144.81. The main outcome of this paper is showing a relation between COVID-19 behavior and population in a region, these results show us the opportunity to create more models to predict the COVID-19 behavior using variables as humidity, climate, culture, among others.

1. Introduction

In December 2019 in Wuhan, China started the pandemic of COVID-19, commonly known as Coronavirus, which has caused havoc around the world. World Health organization reported on June 7 [1], the virus is in 216 Countries, there are 6 750 521 active cases, and it has produced 395 779 deaths. For this reason, scientists around the world have been focused on topics such detect it [2], prevent it [3], cure it [4], and predict it [5–13]. To predict the coronavirus different schemes has been applied, for example in [11] proposes an approach, which is based Composite Monte Carlo enhanced by deep learning and fuzzy rule induction to predict the COVID-19, [14] detailed models for forecasting the course of the pandemic, these models demonstrate the utility of parsimonious models for early-time data. Using the official data forecasting, [15] studied the spread of COVID-19, they realized forward prediction and backward inference of the epidemic. [16] applied mathematical models and time-series to describe the outbreak among passengers and crew members on Princess Cruises Ship.

A model using early forecasting from Small Dataset is proposed by [6]. In [5] the authors proposed used ARIMA models to predict the spread around the world, in specific they use two models ARIMA (1,2,0) and ARIMA (1,0,4). An ARIMA (1,1,2) model is selected to fit the predictions in Italy by [8,10] proposed ARIMA models to predict cases and deaths per 3 countries Italy, Turkey, and Spain. A ARIMA (2,2,2) model is used by [9] to predict the spread in India. Models for different regions of Italy are proposed by [7]. [13] uses (ARIMA) model to analyze two data sets and predict the daily new confirmed cases for the 7-day period. In [12], the authors have studied 15 countries (USA, Spain, Italy, France, Germany, United Kingdom (UK), Turkey, Iran, China, Russia, Brazil, Canada, Belgium, Netherlands and Switzerland) to predict the spread of the Coronavirus in these countries.

As we can see, advanced prediction models used ARIMA [17] to predict the spread of Coronavirus, this is because the ARIMA

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https://doi.org/10.1016/j.asoc.2020.106610
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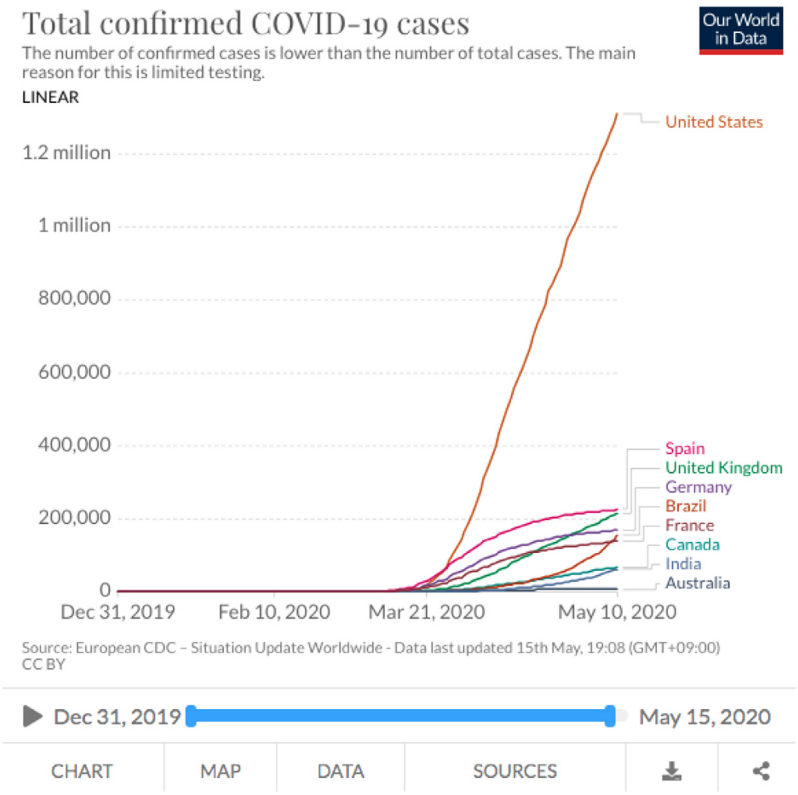


Fig. 1. Example of the available information on "Our World in Data".

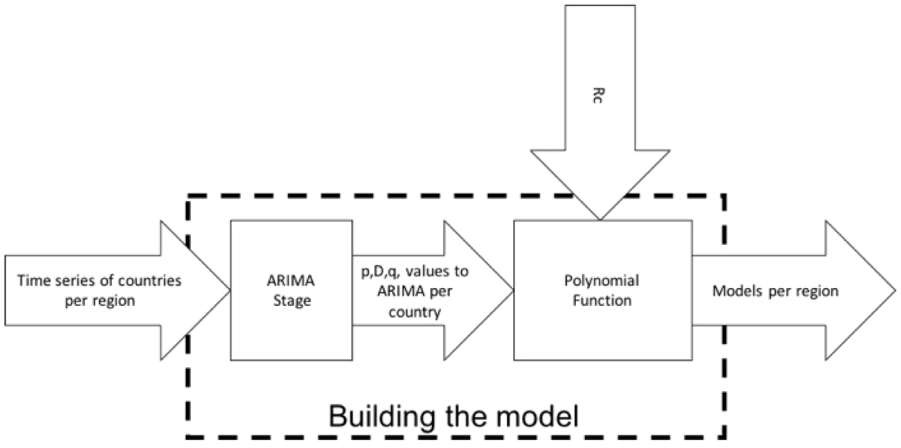


Fig. 2. Block diagram of Building the model.

models give results in terms of its predictive performance. The models give us the possibility to predict the virus behavior, it could be used to make future response plans. There are countries, which have faced the COVID-19 in the same way, (Brazil and Sweden) but with totally different consequences. The differences between these countries are geographical, demographic, economic, public health, cultural, poverty, among others. These differences have caused that Brazil has 694,116 cases while Sweden has 45,133 cases on June 8. As we can see, Brazil has 15.3 times more cases than Sweden. For this fact, in this work, we propose to create a relationship between the countries and two variables more to predict the COVID-19 behavior. These variables are the geographic region and the total population in the country. The geographic regions are North America, South America, Africa, Asia and Europe. The countries in the same geographic region (continent) different variables with similar values such as quantifiable data (climate, humidity, natural regions, etc.) and other non-quantifiable (cultural similarities, similar gastronomy, among others).

We propose an algorithm to performed and evaluated the Auto-Regressive Integrated Moving Average (ARIMA) model for 145 countries, which are distributed in 6 geographic regions. The ARIMA models using the available information until April 25 2020. Next, the information is divided into 2 sets, the first set is used to create the ARIMA models and the second set is used

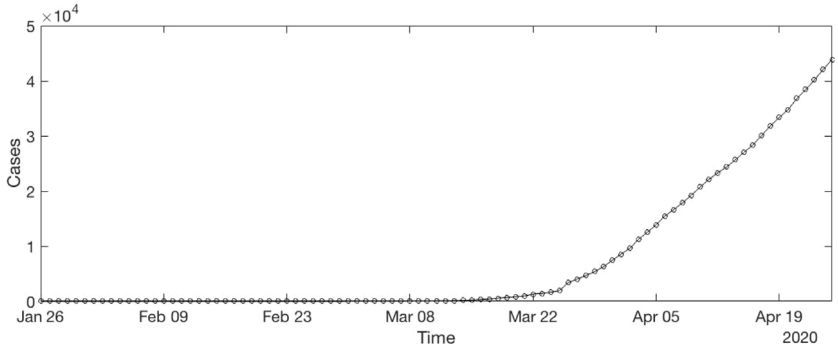


Fig. 3. COVID-19 time series of Canada.

to calculate the RMSE between the real data and predict data. The First set uses 90% of the data and the second set uses 10% of the data. Then, the calculated parameters of ARIMA models, the population per 1M people per country, the number of cases per country are used to create polynomial functions, which are able to predict the ARIMA parameters. These polynomial functions generate models for the next geographic regions: North America, South America, Africa, Oceania, Asia, and Europe.

The results are evaluated using RMSE. The main contributions can be summarized as follows:

- We propose an algorithm to calculate the best ARIMA parameters per country with low RMSE.
- The algorithm to calculate the best parameter of ARIMA is tested with 10% of the original data.
- Our approach is analyzing 145 countries, almost 10 times more than another proposed scheme.
- The approach starts analyzing particular cases (countries) to create a general case (geographic region).
- Our approach is able to show a relation between the prediction error and other variables. In this work, a relation between the prediction error and the population per 1M people is shown.

The organization of the paper is as follows. Section 2 briefs the databases used. Section 3 presents the proposed approach. The paper ends with the Results, a Discussion section and Conclusion.

2. Databases

The time series created in this work using the data of "Our World in Data" [18], which is completely open access. They collect the data from the European Centre for Disease Prevention and Control (ECDC), the WHO, Johns Hopkins, United Nations, World Bank, Global Burden of Disease, Blavatnik School of Government, etc. They standardized names of countries using "Our World in Data" [18] standard entity names, they discarded detected inconsistencies in the original data, detailed documentation for each country is available [18]. Multiple time series for a country are collected, the complete COVID-19 dataset only includes the most complete number of people tested, confirmed cases and deaths. The data on the coronavirus pandemic is updated daily. "Our World in Data" has 77 charts on COVID-19. Fig. 1 shows an example of one chart. The data of charts contain information from 207 countries. Then, we can explore the statistics on COVID-19 for the countries in the world. This work uses the available information until May 28. The consulted chart is "Total and daily confirmed COVID-19 cases", which is used to create the time series per country called "Total confirmed COVID-19 cases". The time series start on the day when each country presented the first case of COVID-19 and finish on May 28. This fact means the length of time series are different per each country. For example, the time series of Canada starts on January 26 (123 days until May 28), Egypt time series starts on February 15 (103 days until May 28), time series of China starts on December 31 (149 days until May 28), Italy time series starts on January 31 (118 days until May 28), time series of Australia starts on January 25 (124 days until May 28), Brazil time series starts on February 26 (92 days until May 28), etc.

This paper proposes a model per geographic region. The countries are separated in 6 regions which are North America (13 countries), South America (12 countries), Africa (43 countries), Asia (40 countries) and Europe (33 countries). To create the models per region, we use the "total population in the age groups" available in the website of United Nations [19]. The population in the age groups is added to generate a total population. The values for each country are shown in the Tables in Appendix A.

3. Proposed approach

The proposed approach consists of two stages "Building the model" and "Evaluating the model". These stages are applied 6 times, one time per region. We use the time series "Total confirmed COVID-19 cases". The first stage "Building the model" requires the time series per country, which starts on the day when each country presented the first case of COVID-19 and it finishes on April 25. The second stage "Evaluating the model" requires the information of COVID-19 on May 28. Then the forecasting between May 12 and May 28 is calculated and compared with the real values. In the following subsections, the proposed approach is explained in a general way and using an example. The example calculates the "*p, D, q*" values of ARIMA to Canada and builds the North America model.

3.1. Building the model

Fig. 2 shows a block diagram of this stage, which consists of the ARIMA and polynomial functions. The inputs of this stage are time series of the countries per region and Rc, which are explained in Section 3.1.1 (Arima Stage) and 3.1.2 (Polynomial Functions), respectively.

3.1.1. ARIMA stage

We use the time series "Total confirmed COVID-19 cases" per country. Then, we have a time series presented in the following equations:

$$y = \{y_t, t \epsilon T\} \tag{1}$$

$$T = \{T_1, T_2, T_3, \ldots, T_{1+n}\} \tag{2}$$

In Eq. (1), y means the total confirmed cases per day presented in a country. In Eq. (2), T_1 means the day when each country

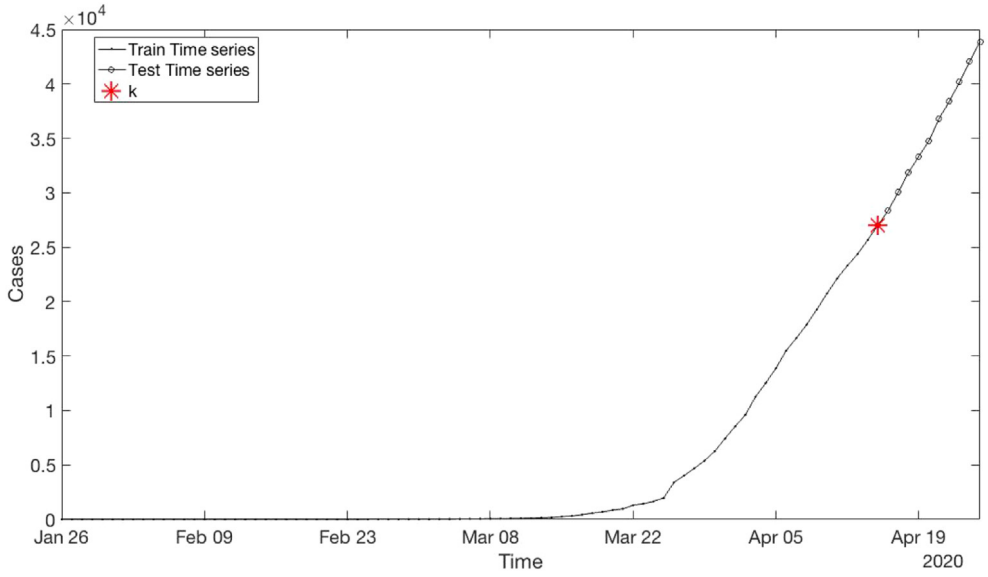


Fig. 4. Canada time series separated into training and testing time series.

detected the first COVID-19 patient. Then D_{1+n} represents the number of days elapsed until April 25. For example, Canada detected the first patient on January 26. Then, we can rewrite Eq. (2) as shown Eq. (3).

$$T = \{Jan26, Jan27, Jan28, \ldots, Apr25\} \tag{3}$$

The Fig. 3 shows the time series of Canada. To compute the best parameters of ARIMA, the time series is separated into training time series and testing time series. To train time series is created using 90% of the data from the original time series. At the same time, test time series is created using 10% of the data from the original time series. The Fig. 4 shows the example using time series of Canada.

Then, we have two time series (*Train* and *Test*). The next equations presented in a formal way these time series.

$$k = round(n * .9) \tag{4}$$

$$Train = \{y_t, t \epsilon U\} \tag{5}$$

$$U = \{T_1, T_2, T_3, \ldots, T_k\} \tag{6}$$

$$Test = \{y_t, t \epsilon V\} \tag{7}$$

$$V = \{T_{1+k}, T_{2+k}, T_{3+k}, \ldots, T_{1+n}\} \tag{8}$$

In Eq. (4), the k value is calculated, this value is the threshold to separate the data between *Train* and *Test*. Eqs. (5) and (7) present the *Train* and the *Test* time series, respectively. Then, Algorithm 1 is applied to calculate the best parameters of an autoregressive integrated moving average (ARIMA).

ARIMA is a statistical analysis, it uses time series data. The ARIMA predicts future values by examining the differences between values in the time series. An ARIMA model consists of 3 components Auto regression (AR), Integrated (I), and Moving average (MA). Each component is a parameter. To represent these parameters, ARIMA models use a standard notation p, D, and q. This standard notation indicates the type of ARIMA model used. Where p means the number of lag observations, D means the degree of difference, and q means the order of the moving average, for further details refer to [14,20,21].

Algorithm 1. Calculating the best parameters of ARIMA.

```
j←1
k←Train( (length of Test-1)
f ←length of Test+1
for px = 0 to 5 do
    for dx = 0 to 5 do
        for qx= 0 to 5 do
            Model←ARIMA(px, dx, qx, Train)
            Forecast←Model(f)
            If Forecast(1)>k    true then
                coef←k - Forecast(1)
            Else false
                coef← k - Forecast(1)
            Forecast←Forecast(2: length of Test)+coef
            vector(j) ←RMSE(Forecast,Test)
            Pval(j) ←px
            Dval(j) ←dx
            Qval(j) ←qx
            j←j+1
x←index of min(vector)
p←Pval(x)
D←Dval(x)
q←Qval(x)
```

In the previous Algorithm, Root Mean Square Error (RMSE) [22] measures the stability between the original data and forecast data, RMSE is calculate using Eq. (9).

$$RMSE = \sqrt{n \sum_{i=1}^{n} (X_i - S_i)^2} \tag{9}$$

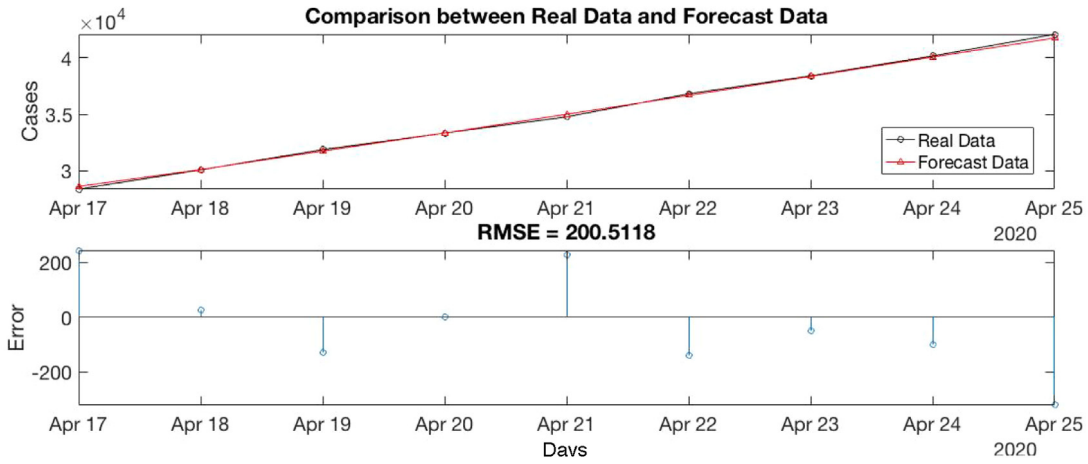


Fig. 5. Comparison between Real data and Forecast Data of Canada.

Table 1
Data of North America.

Country	Population per million people (ppMp)	Total confirmed COVID-19 cases per 1M people		ARIMA Parameters			RMSE average 640.61
		April 25	May-11	p	D	q	
Belize	0.390	45.269	45.269	0	2	0	2.830E−07
Canada	37.411	1162.546	1824.167	3	2	3	2.611E+02
Cuba	11.333	113.450	155.916	5	1	3	2.128E+01
Dominican Republic	10.739	529.964	953.825	5	0	3	6.851E+01
Grenada	0.112	133.311	186.635	5	1	3	2.452E−01
Guatemala	17.581	24.001	58.720	2	2	5	1.671E+01
Haiti	11.263	6.314	15.961	3	3	1	2.392E+00
Honduras	9.746	59.669	199.099	4	3	1	1.719E+01
Jamaica	2.948	97.259	169.528	5	3	3	7.446E+00
Mexico	127.576	99.835	271.630	2	3	0	1.614E+02
Nicaragua	6.546	1.660	2.415	5	5	3	3.859E−01
Panama	4.246	1237.146	1957.927	3	1	5	2.141E+01
United States	329.065	2690.383	4017.488	4	1	3	7.750E+03

In our example, the algorithm 1 is applied into Train Canada Time series. Then, we have *p, D, q* values, *p* is 3, *D* is 2 and *q* is 4. Fig. 5 shows a comparison between the Real data (Test Time series) and Forecast Data of Canada.

This process is applied to each country. Appendix A presents the *p, D, q* values of the ARIMA model and the RMSE to each analyzed country.

3.1.2. Polynomial functions

The ARIMA stage calculated the *p, D, q* values to each country. In this stage, we use these values, the information of cases confirmed COVID-19 per million people on April 25, and the Population per million people (ppMP). Table 1 shows an example of the obtained values in the previous stage, which are used in this stage.

This stage calculates the polynomial function for ARIMA parameters (*p, D, q*). This fact means, we must calculate 3 polynomial functions, one for each ARIMA Parameter. Then, we create the Matrixes V_p, V_D, V_q, which have the *p, D, q* values for the countries in the region. *Rc* is a matrix, each one of the elements of *Rc* belongs to one country. These elements are calculated per country as follows: the numbers of confirmed COVID-19 cases on April 25 multiply per the population (ppMp).

$$V_p = \begin{bmatrix} p_{C1} \\ p_{C2} \\ \vdots \\ p_{Ck} \end{bmatrix} \tag{10}$$

$$V_D = \begin{bmatrix} D_{C1} \\ D_{C2} \\ \vdots \\ D_{Ck} \end{bmatrix} \tag{11}$$

$$V_q = \begin{bmatrix} q_{C1} \\ q_{C2} \\ \vdots \\ q_{Ck} \end{bmatrix} \tag{12}$$

$$Rc = \begin{bmatrix} Rc_{C1} \\ Rc_{C2} \\ \vdots \\ Rc_{Ck} \end{bmatrix} \tag{13}$$

$$Ck = 1, 2, 3, \ldots, \ Number\ of\ countries\ in\ the\ region \tag{14}$$

In our example k=1, 2, 3, …, 13. Then we need to apply the Algorithm 2 on V_p, V_D, V_q.

The Algorithm 2 creates the vectors *t* and *d*. The vector *t* starts in 1 and finishes in the maximum value of *Rc*. The vector *d* has the information of the ARIMA parameters. Then, we need to calculate a polynomial p(t) of degree *n*, that is the best fit for the data in the vector *d*, as shown Eq. (15).

$$p(t) = p_1 t^n + p_2 t^{n-1} + p_3 t^{n-2} + \cdots + p_n t + p_{n+1} \tag{15}$$

Algorithm 2. Creating the vector to calculate a polynomial.

For p, D, q **do**
  parameter← V_p or V_D or V_q
  maxval←max value of Rc
  i← indexes of Rc are sorted from minimum to maximum value
  Rc←sort Rc according to i
  parameter←sort parameter according to i
  k←1
  For i = 1 **to** maxval **do**
    t(i) ← i
    If i <= Rc(k) **true, then**
      d(i) ← parameter (k)
    else
      k←k+1
  d(i) ← parameter (k)

$$\begin{pmatrix} t_1^n & t_1^n & \dots & 1 \\ t_1^n & t_1^n & \dots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ t_1^n & t_1^n & \dots & 1 \end{pmatrix} \begin{pmatrix} p_1 \\ p_2 \\ \vdots \\ p_{n+1} \end{pmatrix} = \begin{pmatrix} d_1 \\ d_2 \\ \vdots \\ d_m \end{pmatrix} \quad (16)$$

n is degree of polynomial

To solve Eq. (15), we present the problem as shown the Eq. (16). Then, t is used to form Vandermonde matrix [15] V with *n+1* columns and m rows. Where m is the length of d. After to solve the Eq. (16), we find the values of P_1, P_2,..., P_{n+1}. To calculate the best values of P_n, we propose the Algorithm 3.

Algorithm 3 Calculating polynomial function.

For p, D, q **do**
  For i = 1 **to** 100 **do**
    P_n←solve the equation 16 using i value as degree of polynomial
    y←evaluate the polynomial using P_n and t
    y← round(y)
  rmse (i)←calculate the RMSE between y and d
  polynomial degree←index of min(rmse)

At the beginning of the Algorithms 2–3, we can take one of the ARIMA parameters, this fact means that the Algorithms 2 and 3 must be applicate 3 times, one time per ARIMA parameter. After the Algorithms 2 and 3 are applied the values to North America model are: 14 is degree of polynomial to parameter p, 15 is degree of polynomial to parameter D, and 47 is degree of polynomial to parameter q. The Eq. (18) shows an example of the polynomial created for ARIMA parameter D. Fig. 6 shows the polynomial functions for ARIMA parameters.

$$\begin{aligned} p_{NAd}(t) = {} & p_1 t^{15} + p_2 t^{14} + p_3 t^{13} + p_4 t^{12} + p_5 t^{11} \\ & + p_6 t^{10} + p_7 t^9 + p_8 t^8 + p_9 t^7 + p_{10} t^6 \\ & + p_{11} t^5 + p_{12} t^4 + p_{13} t^3 + p_{14} t^2 + p_{15} t \\ & + p_{16} \end{aligned} \quad (17)$$

The next Eqs. (18)–(20) show the polynomial functions for North America, the polynomials of all regions are available in Appendix B.

North America

$$p_{NAp}(t) = p_1 t^n + p_2 t^{n-1} + p_3 t^{n-2} + \cdots + p_n t + p_{n+1}; n = 14 \quad (18)$$

$$p_{NAd}(t) = p_1 t^n + p_2 t^{n-1} + p_3 t^{n-2} + \cdots + p_n t + p_{n+1}; n = 15 \quad (19)$$

$$p_{NAq}(t) = p_1 t^n + p_2 t^{n-1} + p_3 t^{n-2} + \cdots + p_n t + p_{n+1}; n = 47 \quad (20)$$

3.2. Evaluating the model

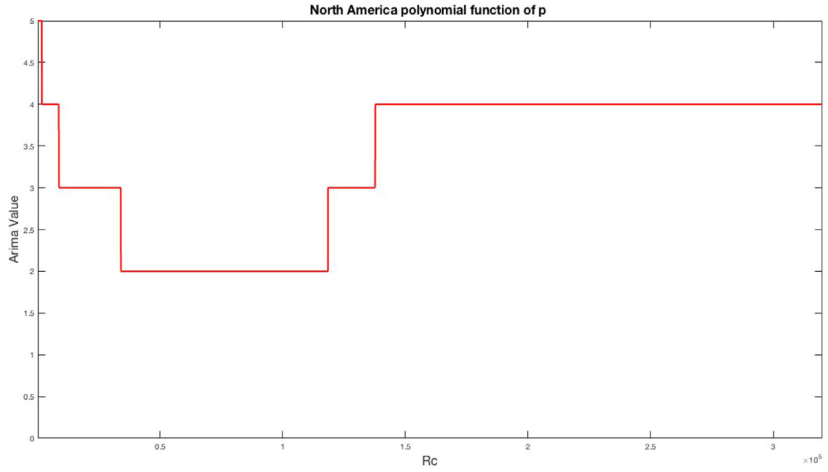
The Fig. 7 shows the block diagram of the Evaluating model stage. This stage has 3 inputs *Ec*, time series of the country until May 11, and days to predict. *Ec* has the value of Total confirmed COVID-19 cases multiply per the population for the country to be evaluated in the region. In this evaluation, we use the data on May 11 and the polynomial functions creates in the previous stage. The Algorithm 4 is used to calculate the ARIMA parameters.

We apply the Algorithm 4 using the values of Canada on May 11 and the functions $P_{Nap}(t)$, $P_{Nad}(t)$, and $P_{Naq}(t)$ Eqs. (18)-(20). Canada belongs to North America, so we use the functions of North America to calculate the ARIMA parameters. To calculate another country, we must use the functions which belong to the region of the country.

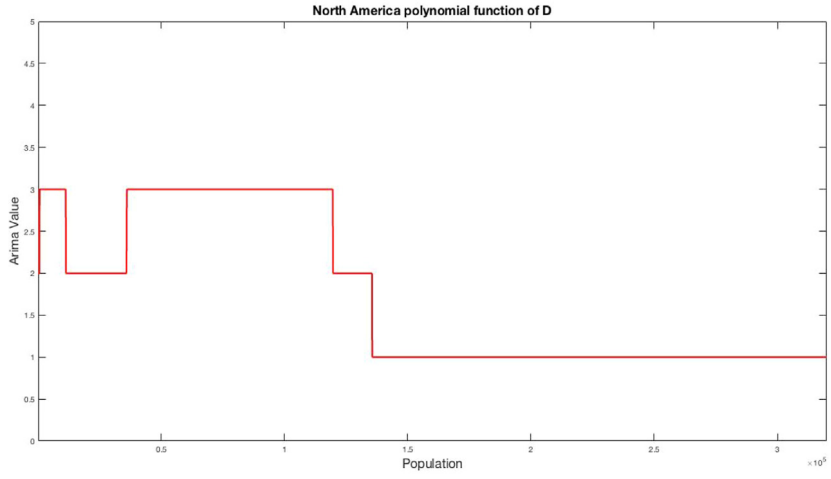
Algorithm 4. Calculating the ARIMA parameters.

For p, D, q **do**
.... y←evaluate the polynomial using the functions of the region and E_c
.... y← round(y)
.... ARIMA parameters ← y

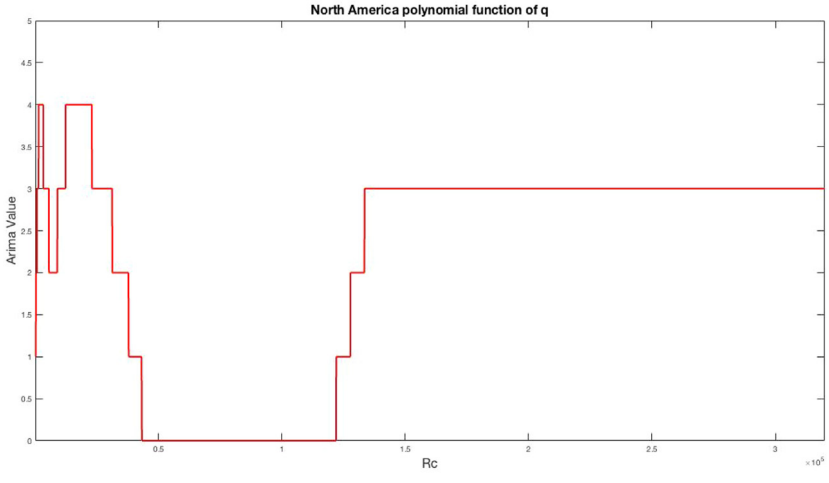
After we applied the Algorithm 4, the Canada ARIMA parameters are *p=3, D=2, q=3*. The next step is the prediction using



(A) Parameter p.



(B) Parameter D.



(C) Parameter q.

Fig. 6. Polynomial Functions for North America Model.

the Algorithm 5. The Canada results using the model of North America are shown in Figs. 8–9.

The Eq. (9) is applied to calculate the RMSE between the forecast values and real values. Fig. 8 shows a comparison between the real and forecast signals and Fig. 9 shows the forecast of Canada with confidence interval of 95%.

4. Results

This section presents the results for each region analyzed. Table 2 shows the average RMSE per region. In the table, the RMSE is calculated between the forecast and the real values. Fig. 10 presents a comparison using RMSE and the forecast for one

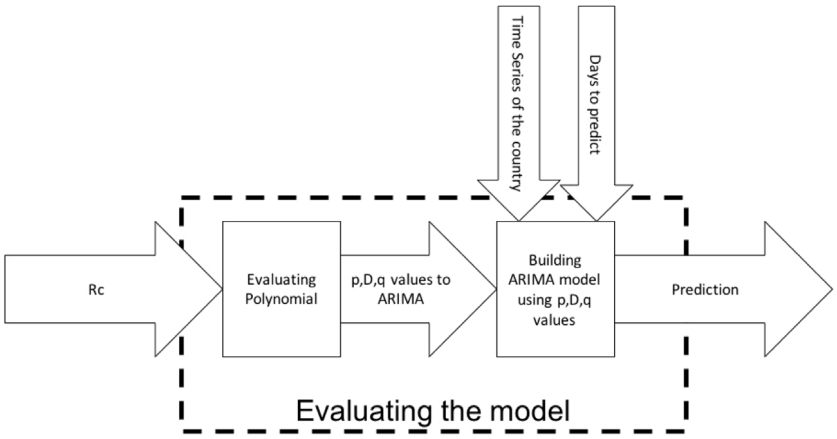


Fig. 7. Block Diagram of Evaluating the model.

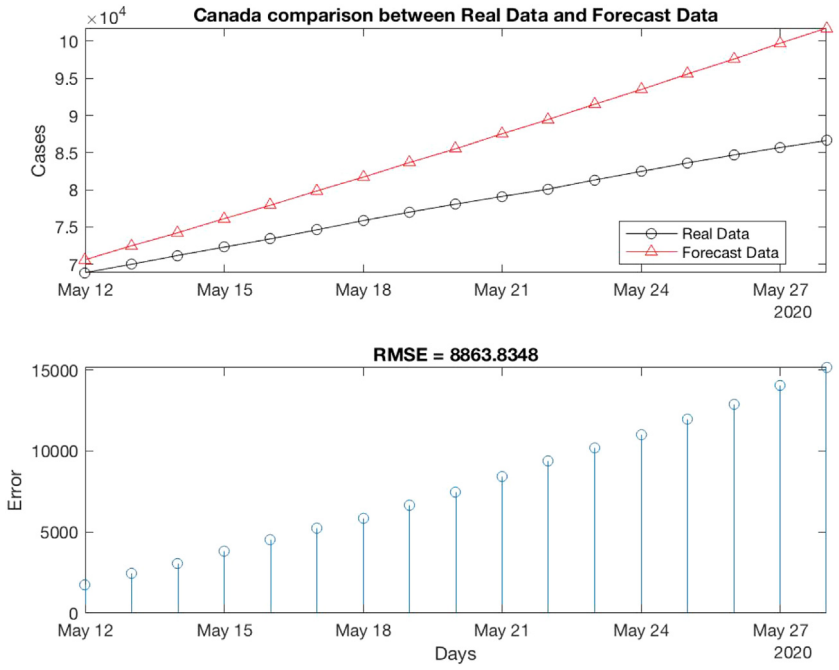


Fig. 8. Comparison between Real Data and Forecast Data.

country per each region in the following way: (a) North America, (b) South America, (c) Africa, (d) Oceania, (e) Asia, and (f) Europe.

Algorithm 5. Calculating the Forecast using ARIMA.

days ←days between May 11 and May 27
k←Cases on May 11
Model←ARIMA(p, d, q, days)
Forecast←Model(f)
If *Forecast*(1)>*k* **true then**
coef←*k* - *Forecast*(1)
Else false
coef← *k* - *Forecast*(1)
Forecast←Forecast(2: days)+coef

5. Discussion

The Appendix A presents the results per country before to create the geographic models. These results belong to each country in the different regions. As we mention in Section 3.1.1, the time series are separated into modeling (90% of the signal) and testing (10% of the signal). Below, we will discuss each region in particular.

North America region has 13 countries; this region presents a RMSE average of 640.61. The RMSE average of this region is the most bigger between the regions. This fact appears, because the United States presents the most bigger RMSE between the 145 countries (7749.99), this country has the largest number of population in the region (329.06 ppMp). On the other hand, Belize presents the lowest RMSE. The United States has almost 96 times the population of Belize (0.39 ppMp).

Europe region consists of 33 countries. In this experiment, the countries, which present a ppMp major than 45 ppMp presents biggest RMSE. Spain presents an RMSE of 1892.33 with a 46.73 ppMp, Italy has 60.55 ppMp and presents an RMSE of 566.88,

Table 2
Average of RMSE results.

Region	Average of RMSE between original and forecast signal in training stage	Average of RMSE between original and forecast signal from May 12 to May 27
North America	640.61	3.6051e+04
South America	104.78	2.0828e+04
Africa	13.80	1.4913e+03
Oceania	6.79	161.2570
Asia	89.46	3.52964e+03
Europe	218.59	2.88212e+04
[a]Average	144.81	1.2723e+04

[a]The average is calculated using the 145 countries.

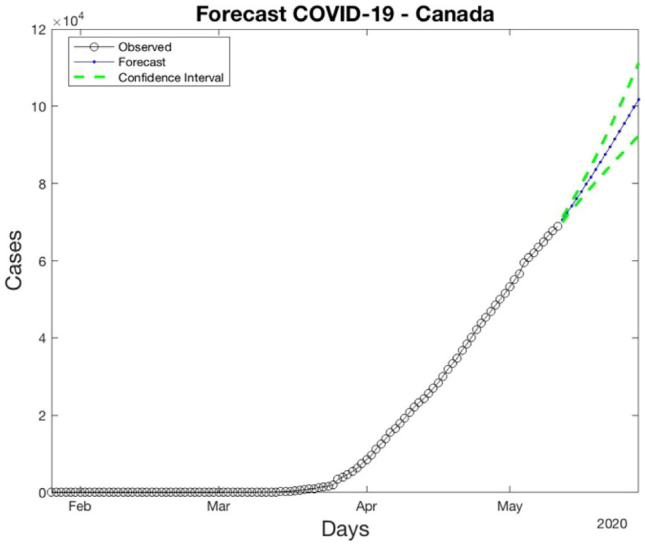


Fig. 9. Forecasting of Canada.

United Kingdom presents an RMSE of 728.13 with a 467.53 ppMp, Germany has 83.51 ppMp and presents an RMSE of 1075.02, and Russia presents an RMSE of 958.44 with a 145.87 ppMp. The RMSE average of this is 218.59.

Brazil presents a RMSE of 591, this country has the largest number of population in the region (211.04 ppMp). In contrast, Paraguay presents the lowest RMSE. Brazil has almost 30 times the population of Paraguay (7.05 ppMp). These countries belong to South America region, the RMSE average of this region is 104.78.

Asia region consists of 40 countries. In this region, Turkey presents the most bigger RMSE (696.35) with a population (83.43 ppMp). China and India have a ppMp major to one thousand, but the RMSE are 117.88 and 250.83, respectively. On the other hand, Yemen with a population less than 30 ppMp has a RMSE close to zero.

Egypt presents a RMSE of 84.08, this country has 110.38 ppMp. In contrast, Namibia presents a RMSE close to zero. Egypt has almost 41 times the population of Namibia (2.49 ppMp). These countries belong to Africa region, the RMSE average of this region is 13.8.

Oceania region consists of 4 countries. In this region, Australia presents the most bigger RMSE (24.76) with a population (25.20 ppMp). The lower RMSE is presented by Fiji with a population less than 1 ppMp.

For the regions North America, South America, Oceania, and Europe, there is a relation between the major ppMp and the error on the prediction. The RMSE of Africa is minor to 15, even though it has countries with 200 ppMp. This fact could be means a relation between the virus spread and the climate for example. For the region Asia the average RMSE is minor to 90, this area was the first area infected by COVID-19 so there are more available data in this area. Thus, we have more data to calculate the Forecast.

Table 3
Comparison between [10] and this work.

Country	RMSE	
	[10]	This work
Italy	1150.31	566.88
Turkey	138.35	1892.33
Spain	379.89	696.35
Average	[a]556.183	[b]144.81

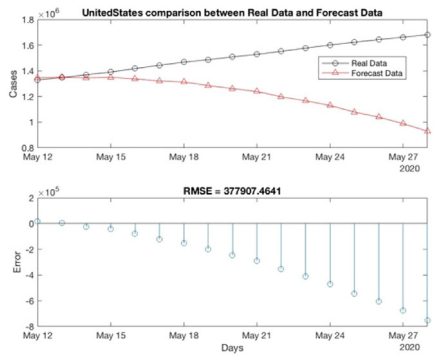
[a]Using 3 countries.
[b]Using 145 countries.

Table 3 shows a comparison between [10] and this work before to create the geographic models. As shown Table 3 this approach has better RMSE to forecast the virus in Italy, on the contrary [10] has better RMSE to predict the virus in Turkey and Spain. At first, it seems that their proposal is better than ours, but when the RMSE averages are compared, we can see that our proposal has a lower RMSE than them, besides we are analyzing 145 countries while they only analyze 3.

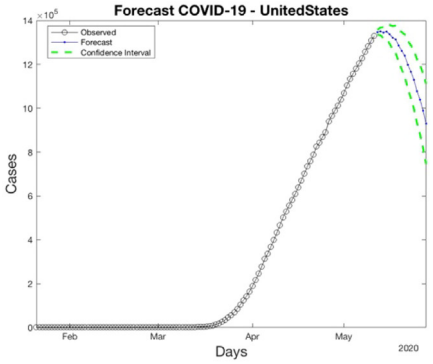
Fig. 8 presents the results per one country in each region. In Fig. 8(a–f), we can see an upward trend in the number of cases, with the exception of the American States, which marks a decrease in the number of cases. Let us remember that from the beginning, the American States had the highest RMSE among all countries. When the geographic models are created, these models are used to predict new cases in a country. The results are shown in Table 2. The forecast is made 17 days after the models are calculated, we take this decision to have a real difference between the cases on April 25 and May 11 as shown the tables in Appendix A. As expected, the RMSE error grew because, the prediction is making 17 days after the models were created and we calculate 15 days of prediction cases. In these time interval, the actions as quarantine control, stay at home campaign, social distance taken by governments significantly affect the prediction. If the lector wants current predictions, the information needs to be updated and repeat the building the model stage.

6. Conclusion

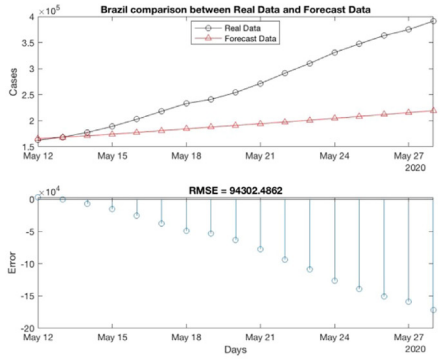
We can conclude that the algorithm to model and evaluate the ARIMA models is able to develop models, which have low RMSE. On the other hand, this work shows a way to model the COVID spread started in particular cases to generate a general case. We can conclude, this work contributes to researchers working in COVID-19 prediction. It shows there is a relation between the virus spread and the different variables present in the countries,



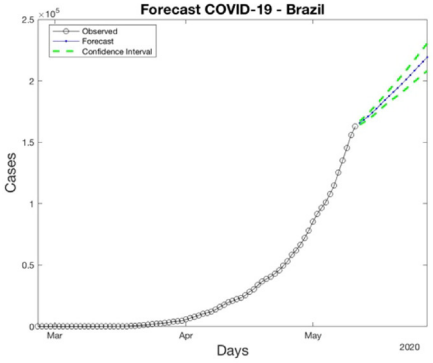
(a.1) United States comparison.



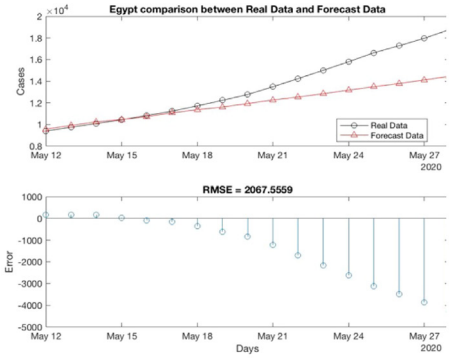
(a.2) United States Forecast.



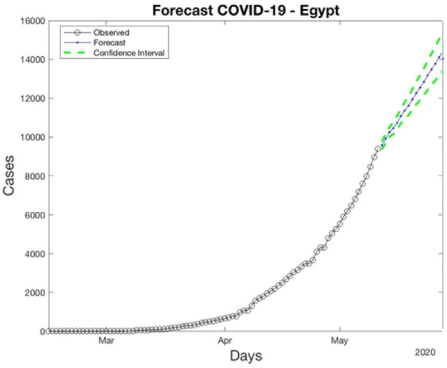
(b.1) Brazil comparison.



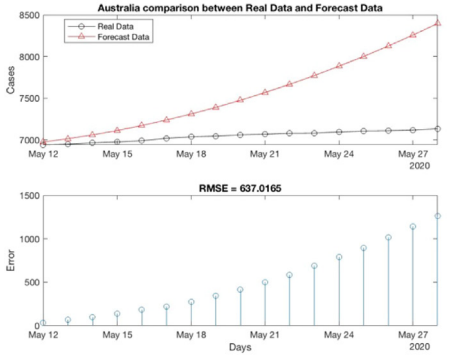
(b.2) Brazil Forecast.



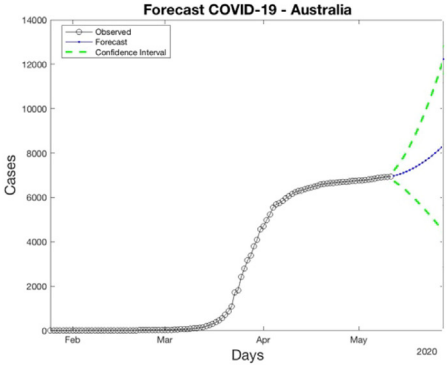
(c.1) Egypt comparison.



(c.2) Egypt Forecast.



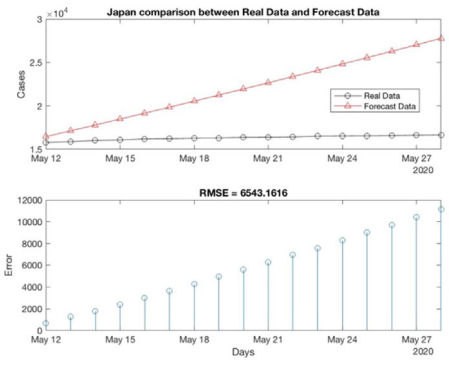
(d.1) Australia comparison.



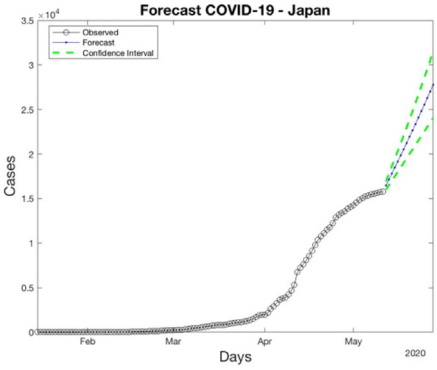
(d.2) Australia Forecast.

Fig. 10. An example of results per region. RMSE and Forecast (a) North America, (B) South America, (c) Africa, (d) Oceania, (e) Asia and (f) Europe.

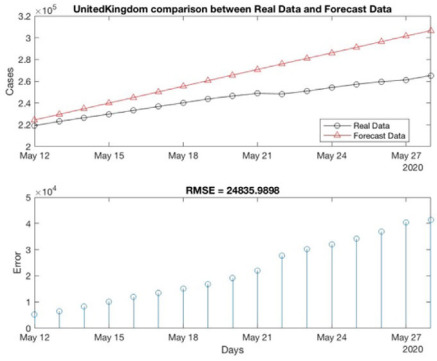
which belong to the same geographic region. Interestingly, we can find a show relation between the population in a country and RMSE error in a prediction. In future challenges of the proposed work different variables could be analyzed, for example, the date when the first coronavirus case is detected in the country, humidity, temperature, among other variables. Other kinds of clusters



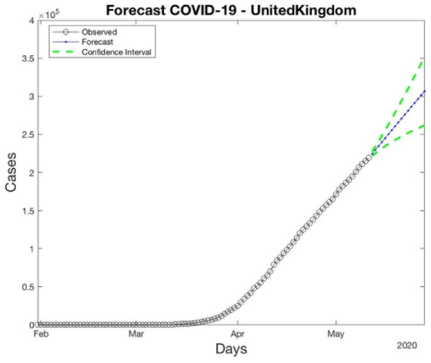
(e.1) Japan comparison.



(e.2) Japan Forecast.



(f.1) United Kingdom comparison.



(f.2) United Kingdom comparison.

Fig. 10. (*continued*).

Table A.1
Data of the countries separated by geographical regions.

North America							
Country	Population per million people (ppMp)	Total confirmed COVID-19 cases per million people		ARIMA parameters			RMSE average 640.61
		April 25	May-11	p	D	q	
Belize	0.390351	45.269	45.269	0	2	0	0.000000283
Canada	37.411038	1162.546	1824.167	3	2	3	261.0992307
Cuba	11.333484	113.45	155.916	5	1	3	21.28255446
Dominican Republic	10.738957	529.964	953.825	5	0	3	68.51095035
Grenada	0.112002	133.311	186.635	5	1	3	0.245202091
Guatemala	17.581476	24.001	58.72	2	2	5	16.71287027
Haiti	11.263079	6.314	15.961	3	3	1	2.392474128
Honduras	9.746115	59.669	199.099	4	3	1	17.1948204
Jamaica	2.948277	97.259	169.528	5	3	3	7.44624079
Mexico	127.575529	99.835	271.63	2	3	0	161.3544175
Nicaragua	6.545503	1.66	2.415	5	5	3	0.385861038
Panama	4.24644	1237.146	1957.927	3	1	5	21.41200047
United States	329.064917	2690.383	4017.488	4	1	3	7749.995645
South America							
Country	Population per million people (ppMp)	Total confirmed COVID-19 cases per million people		ARIMA parameters			RMSE average 104.78
		April 25	May-11	p	D	q	
Suriname	0.581363	17.046	17.046	2	2	0	2.42E−09
Uruguay	3.461731	162.074	203.528	1	2	4	0.536253127
Guyana	0.782775	92.809	132.221	5	0	1	0.945527112
Paraguay	7.044639	31.265	99.965	4	3	1	3.977159335
Venezuela	28.515829	11.183	14.559	4	2	4	7.768192986
Bolivia	11.513102	69.134	218.966	5	5	4	13.19241251
Chile	18.952035	643.747	1510.027	5	3	4	31.85255857
Argentina	44.780675	75.737	127.8	2	1	2	34.307185
Colombia	50.339443	95.926	217.421	1	2	0	42.19486859
Ecuador	17.373657	633.847	1675.39	3	4	4	206.1259375
Peru	32.510462	656.56	2041.348	4	2	1	325.2189929
Brazil	211.049519	249.319	765.428	4	2	1	591.2483357

(*continued on next page*)

could be applied like cultural behavior, religious behavior, hygiene habits, feeding habits, among others. The approach is able to make current predictions, just the information needs to be updated.

Table A.1 (*continued*).

Africa							
Country	Population per million people (ppMp)	Total confirmed COVID-19 cases per million people		ARIMA parameters			RMSE average 13.80
		April 25	May-11	p	D	q	
Namibia	2.494524	6.297	6.297	4	2	0	3.38E−09
Mauritania	4.525698	1.505	1.721	1	2	0	1.88E−08
Seychelles	0.097741	111.857	111.857	0	3	0	0.00000302
South Sudan	11.062114	0.447	13.936	3	4	0	0.231695815
Angola	31.825299	0.761	1.369	2	0	1	0.272840126
Madagascar	26.969306	4.406	6.175	0	0	0	0.5
Botswana	2.303703	9.355	9.78	3	0	3	0.561333394
Mauritius	1.26967	260.268	261.054	5	2	0	0.577046983
Togo	8.082359	10.871	21.018	2	1	1	0.597788103
Central African Republic	4.745179	3.934	37.062	2	3	4	0.690491146
Zimbabwe	14.645473	1.951	2.422	4	1	4	0.920238223
Eritrea	3.497117	10.997	10.997	4	0	3	1.098426352
Benin	11.801151	4.784	26.313	1	1	5	1.123599466
Chad	15.946882	2.435	19.603	2	2	3	1.486804327
Libya	6.777453	8.878	9.314	5	1	0	1.802690408
Zambia	17.861034	4.569	14.524	4	1	5	2.264558145
Ethiopia	112.078727	1.018	2.079	3	0	3	2.448214102
Burundi	11.530577	1.009	1.598	1	4	4	2.535566664
Cape Verde	0.549936	158.277	442.456	4	2	2	2.574723428
Burkina Faso	20.321383	29.947	35.927	3	0	5	2.824155601
Uganda	44.269587	1.64	2.645	5	2	5	2.97488775
Sierra Leone	7.813207	10.28	38.486	3	3	1	3.214739643
Liberia	4.937374	23.133	39.346	4	1	5	3.84699105
Niger	23.310719	28.133	33.916	4	2	2	3.917381572
Sudan	42.813237	3.968	31.084	2	2	4	4.558386235
Mali	19.658023	16.049	34.764	3	2	2	5.436959696
Kenya	52.573967	6.249	12.497	5	2	2	5.646461767
Mozambique	30.366043	2.08	2.911	5	3	5	6.014669312
Malawi	18.628749	1.725	2.927	1	4	3	6.1920973
Tunisia	11.694721	78.013	87.32	1	0	0	8.775869024
Gabon	2.172578	77.278	296.981	4	3	2	11.10807476
Nigeria	200.963603	5.312	21.34	5	4	0	12.60385557
Somalia	15.442906	20.638	66.318	2	1	4	14.76084313
Senegal	16.296362	32.549	102.067	5	3	3	26.08610014
Djibouti	0.973557	1011.132	1224.694	1	1	3	26.43208425
Cameroon	25.876387	52.852	97.153	1	2	2	30.35431279
Algeria	43.053054	71.31	130.51	2	0	3	30.43462472
Equatorial Guinea	1.355982	151.106	312.904	1	2	1	41.34353663
Ghana	30.417858	41.161	137.193	5	2	2	47.53364114
Guinea	12.771246	72.643	163.408	3	5	0	59.82419028
South Africa	58.558267	71.153	168.862	3	3	5	65.82375285
Morocco	36.471766	101.814	164.262	1	1	0	70.07161038
Egypt	100.388076	39.987	91.856	5	1	3	84.08557986
Oceania							
Country	Population per million people (ppMp)	Total confirmed COVID-19 cases per million people		ARIMA parameters			RMSE average 6.79
		April 25	May-11	p	D	q	
Fiji	0.889955	20.079	20.079	0	2	0	0.000000105
Papua New Guinea	8.776119	0.894	0.894	3	2	2	0.321552716
New Zealand	4.783062	231.635	237.857	3	3	0	2.092622312
Australia	25.2032	262.237	272.197	1	3	1	24.7684965
Asia							
Country	Population per million people (ppMp)	Total confirmed COVID-19 cases per million people		ARIMA parameters			RMSE average 89.46
		April 25	May-11	p	D	q	
Yemen	29.161922	0.034	1.71	1	2	1	8.46E−16
Laos	7.169456	2.611	2.611	0	2	0	0.000000474
Brunei	0.433296	315.441	322.298	5	1	1	0.201374663
Mongolia	3.225166	10.981	12.812	5	3	2	0.42737412
Bhutan	0.763094	9.072	11.664	0	5	3	0.854706352
Vietnam	96.462108	2.774	2.959	0	0	0	0.894427191
Syria	17.070132	2.4	2.686	1	2	3	1.2290986
Kyrgyzstan	6.415851	101.928	155.728	4	3	1	1.764150638
Myanmar	54.045422	2.647	3.308	4	1	3	1.810985483
Jordan	10.101697	43.222	52.925	5	2	0	1.976423483
Lebanon	6.855709	101.971	123.802	3	0	2	3.248815136
Nepal	28.608715	1.682	4.119	3	2	4	4.05357834
Georgia	3.996762	111.301	159.181	0	1	4	4.895198267
Maldives	0.530957	236.799	1544.746	5	4	1	5.924589657
Cyprus	1.198574	917.914	1025.232	4	0	4	6.066194022
Azerbaijan	10.047719	157.015	248.442	4	0	5	11.83730777

(*continued on next page*)

Table A.1 (*continued*).

Thailand	69.625581	40.888	43.195	3	0	0	12.58960875
Iraq	39.309789	42.464	68.792	1	3	3	15.52536699
Uzbekistan	32.981715	54.856	72.246	5	1	3	18.9777337
Armenia	2.957728	565.936	1118.035	0	3	4	19.92595139
SriLanka	21.323734	19.614	40.302	3	2	4	20.83410943
Malaysia	31.949789	175.833	205.648	2	1	5	27.80701783
Kuwait	4.207077	612.097	2034.392	0	2	2	44.83279931
Oman	4.974992	350.525	665.606	2	0	2	48.21943289
Iran	82.913893	1050.017	1281.096	5	3	0	50.19983585
Afghanistan	38.041757	34.705	113.08	5	2	0	75.99268382
Bangladesh	163.046173	28.472	88.998	5	3	0	79.01328851
Qatar	2.832071	2958.98	7816.568	1	2	0	97.6076415
Bahrain	1.641164	1479.799	2903.767	1	2	2	99.73990182
Philippines	108.116622	65.632	98.502	4	0	0	103.1445092
Indonesia	270.625567	30.019	51.301	5	1	5	110.4601355
Israel	8.519373	1739.695	1903.636	0	2	4	113.630396
China	1433.783692	58.291	58.368	5	0	0	117.8792989
United Arab Emirates	9.770526	938.385	1839.966	5	5	0	188.3682723
Saudi Arabia	34.268529	433.793	1121.622	3	3	2	226.4705085
India	1366.417756	17.758	48.661	5	2	3	250.833
Pakistan	216.565317	54.053	140.073	4	3	4	266.1868764
Japan	126.860299	101.932	124.909	1	1	2	399.4484529
Singapore	5.804343	1910.657	3988.826	1	2	2	449.3506577
Turkey	83.429607	1243.931	1644.042	5	3	3	696.3506539
Europe							
Country	Population per million people (ppMp)	Total confirmed COVID-19 cases per million people		ARIMA parameters			RMSE average 218.59
		April 25	May-11	p	D	q	
Montenegro	0.627988	507.912	515.873	2	2	0	0.988109524
Malta	0.440377	1012.368	1123.344	5	3	0	3.539156254
Latvia	1.90674	415.65	497.826	5	1	4	3.594220963
Iceland	0.339037	5242.491	5277.656	3	2	5	5.129808852
Croatia	4.130299	489.371	532.73	1	0	5	7.425098776
Estonia	1.325649	1209.915	1310.93	4	0	5	10.28679943
Hungary	9.68468	252.889	339.946	3	1	5	13.10475646
Norway	5.378859	1366.477	1493.938	5	2	1	14.35761421
Slovakia	5.457012	249.101	266.867	4	2	1	15.43096651
Albania	2.880913	235.597	301.619	5	2	4	15.44541938
Lithuania	2.759631	517.946	543.292	1	0	5	15.54234467
Luxembourg	0.61573	5902.782	6207.906	1	2	4	18.86761887
Bulgaria	7.000117	170.974	282.797	5	3	3	26.9764822
Slovenia	2.078654	660.435	700.841	4	2	5	29.45549614
Czech Republic	10.689213	679.15	758.522	5	1	4	32.36197356
Austria	8.955108	1673.033	1752.865	4	1	4	41.48747638
Serbia	8.772228	1099.698	1486.348	3	2	2	43.17539749
Finland	5.532159	793.218	1076.034	0	3	2	46.3725334
Denmark	5.771877	1417.423	1800.524	2	1	1	50.60316184
Sweden	10.036391	1739.433	2606.327	3	2	5	87.03004062
Romania	19.364558	541.489	798.537	4	2	5	102.4828358
Poland	37.887771	287.793	422.653	5	1	4	105.4066726
Portugal	10.226178	2282.207	2704.893	4	1	5	108.4530628
Ukraine	43.993643	185.783	348.289	2	5	2	159.7185476
Ireland	4.882498	3682.615	4657.139	1	4	2	172.2604087
Belgium	11.539326	3821.783	4580.048	5	2	0	207.3927744
Belarus	9.452409	928.426	2431.18	5	2	5	268.7103234
Netherlands	17.097123	2132.201	2487.734	5	0	0	387.229697
Italy	60.550092	3191.997	3623.278	4	3	3	566.8847446
United Kingdom	67.530161	2113.307	3228.692	2	1	0	728.1254615
Russia	145.87226	470.225	1436.864	5	5	1	958.4432358
Germany	83.517046	1819.418	2023.956	4	1	0	1075.024974
Spain	46.736782	4454.496	4871.587	0	1	0	1892.331932

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgments

This study is supported by JSPS, Japan KAKENHI (Grants-in-Aid for Scientific Research) #JP20K11955.

Appendix A

See Table A.1.

Appendix B

North America

$p_{NAp}(t) = p_1 t^n + p_2 t^{n-1} + p_3 t^{n-2} + \cdots + p_n t + p_{n+1}; n = 14$

$p_{NAd}(t) = p_1 t^n + p_2 t^{n-1} + p_3 t^{n-2} + \cdots + p_n t + p_{n+1}; n = 15$

$p_{NAq}(t) = p_1 t^n + p_2 t^{n-1} + p_3 t^{n-2} + \cdots + p_n t + p_{n+1}; n = 47$

South America

$p_{SAp}(t) = p_1 t^n + p_2 t^{n-1} + p_3 t^{n-2} + \cdots + p_n t + p_{n+1}; n = 23$

$p_{SAd}(t) = p_1 t^n + p_2 t^{n-1} + p_3 t^{n-2} + \cdots + p_n t + p_{n+1}; n = 23$

$p_{SAq}(t) = p_1 t^n + p_2 t^{n-1} + p_3 t^{n-2} + \cdots + p_n t + p_{n+1}; n = 47$

Africa

$p_{Afp}(t) = p_1 t^n + p_2 t^{n-1} + p_3 t^{n-2} + \cdots + p_n t + p_{n+1}; n = 47$

$p_{Afd}(t) = p_1 t^n + p_2 t^{n-1} + p_3 t^{n-2} + \cdots + p_n t + p_{n+1}; n = 47$

$p_{Afq}(t) = p_1 t^n + p_2 t^{n-1} + p_3 t^{n-2} + \cdots + p_n t + p_{n+1}; n = 47$

Oceania

$p_{Op}(t) = p_1 t^n + p_2 t^{n-1} + p_3 t^{n-2} + \cdots + p_n t + p_{n+1}; n = 47$

$p_{Od}(t) = p_1 t^n + p_2 t^{n-1} + p_3 t^{n-2} + \cdots + p_n t + p_{n+1}; n = 47$

$p_{Oq}(t) = p_1 t^n + p_2 t^{n-1} + p_3 t^{n-2} + \cdots + p_n t + p_{n+1}; n = 47$

Asia

$p_{Asp}(t) = p_1 t^n + p_2 t^{n-1} + p_3 t^{n-2} + \cdots + p_n t + p_{n+1}; n = 14$

$p_{Asd}(t) = p_1 t^n + p_2 t^{n-1} + p_3 t^{n-2} + \cdots + p_n t + p_{n+1}; n = 40$

$p_{Asq}(t) = p_1 t^n + p_2 t^{n-1} + p_3 t^{n-2} + \cdots + p_n t + p_{n+1}; n = 50$

Europe

$p_{Ep}(t) = p_1 t^n + p_2 t^{n-1} + p_3 t^{n-2} + \cdots + p_n t + p_{n+1}; n = 12$

$p_{Ed}(t) = p_1 t^n + p_2 t^{n-1} + p_3 t^{n-2} + \cdots + p_n t + p_{n+1}; n = 50$

$p_{Eq}(t) = p_1 t^n + p_2 t^{n-1} + p_3 t^{n-2} + \cdots + p_n t + p_{n+1}; n = 50$

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