

DEPARTMENT OF MATHEMATICS, IIT ROORKEE
MAB-103: Numerical Methods

Assignment-7

Numerical Integration

Session: 2025-26

1. Evaluate the following integrals by (a) Trapezoidal rule (b) Simpson's 1/3 rule, and (c) Simpson's 3/8 rule, for the given number of sub-intervals:

(i) $\int_0^{1.2} \log(1+x^2)dx, n = 6.$ (ii) $\int_0^1 \cos(x^2)dx, n = 12.$
(iii) $\int_0^{0.5} (\tan^{-1} x)^2, n = 18.$ (iv) $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\sin x}{x} dx, n = 6.$
(v) $\int_{0.4}^{1.6} \frac{x}{\sin x} dx, n = 12.$ (vi) $\int_{0.125}^{0.875} \frac{\cos x}{\sqrt{x}} dx, n = 18.$

2. Use Trapezoidal rule and Simpson's 1/3 rule, to evaluate the integral

$$\int_0^1 \frac{1}{1+x^2} dx,$$

by taking 10 subintervals and hence find an approximate value of π .

3. Evaluate the integral

$$\int_0^{1.2} e^{-x^2} dx$$

by Trapezoidal rule, taking $h = 0.2$. Also, estimate the error.

4. Consider evaluating the integral

$$\int_0^1 \frac{1}{1+x^2} dx$$

using Simpson's 1/3 rule. How large should n be chosen in order to ensure that

$$|\text{Error}| \leq 5 \times 10^{-6} ?$$

5. Using Simpson's $(3/8)^{th}$ rule, evaluate

$$\int_0^{1.2} (\sin x - \ln(1+x) + e^x) dx,$$

correct to 5-significant places, where $h = 0.1$, and compare it with the exact solution.

6. Evaluate the following integrals using Gauss 2-point and 3-point quadrature formulae:

(a) $\int_{0.2}^{1.5} e^{-x^2} dx$ (b) $\int_0^\pi \sin(x^2) dx$
(c) $\int_{-4}^4 \frac{dx}{1+x^2}$ (d) $\int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{(1-0.25 \sin^2 x)}} dx$

7. Prove that

$$S(2m) = \frac{4}{3}T(2m) - \frac{1}{3}T(m),$$

where $T(m)$ and $S(2m)$ are composite trapezoidal rule and composite Simpson's rule, respectively.

8. Determine the values of $c_j, j = -1, 0, 1, 2$, such that the quadrature rule

$$Q(f) = c_{-1}f(-1) + c_0f(0) + c_1f(1) + c_2f(2)$$

gives the correct value for the integral

$$\int_0^1 f(x)dx$$

when f is any polynomial of degree 3. Show that, with these values of the weights c_j , and under appropriate conditions on the function f ,

$$\left| \int_0^1 f(x)dx - Q(f) \right| \leq \frac{11}{720}M_4.$$

Give suitable conditions for the validity of this bound, and a definition of the quantity M_4 .

9. Suppose that f is a real-valued function, defined and continuous on the interval $[a, b]$, and that $f^{iv} = f^4$, the fourth derivative of f , is continuous on $[a, b]$. Then,

$$\int_a^b f(x)dx - \frac{b-a}{6}(f(a) + 4f((a+b)/2) + f(b)) = -\frac{(b-a)^5}{2880}f^{iv}(\xi)$$

for some ξ in (a, b) .

Answers:

- (1) (i) 0.421603, 0.422509, 0.422506.
(ii) 0.904670, 0.904524, 0.904524.
(iii) 0.037968, 0.037993, 0.037993.
(iv) 0.611924, 0.611786, 0.611787.
(v) 1.47445, 0.992719, 0.992719.
(vi) 1.0677, 1.02824, 1.02824.

(2) 0.2760

(3) 0.8048, 0.008

(4) 32

(5) 1.7916

(6) 0.8881, 1.3375, 0.9150, 1.3557.