Assignment #6

(Jul xx, 2025)

Modern Physics: Beiser Chaps 2, 3, and 9

The bond length is 1.54 Å.

1. Calculate the average energy of an oscillator of frequency 0.6×10^{14} sec^{-1} at 1800 K treating it as (i) classical oscillator (ii) Planck's oscillator. Repeat the calculation for frequency 1.5×10^{18} sec^{-1}. Analyze your answer.
[Ans. 2.484×10^{-20} J, 1.01×10^{-20} J, 2.484×10^{-20} J, ~ 0 J]

2. Calculate the number of modes of vibration in a 100 c.c. cavity in the wavelength range (i) 5000 to 5002 Å (ii) 8000 to 8005 Å . [Ans. 8.03×10^{12}, 3.066×10^{12}]

3. From Planck's law deduce the value of ν corresponding to peak of $E(\nu)$ vs ν curve at 1000 K. In what spectral range does this frequency lie? [Ans. 6.25×10^{13} sec^{-1}]

4. Sunlight arrives at the earth at the rate of about 1.4 kW/m^2 when the sun is directly overhead. The Average radius of the Earth's orbit is 1.5×10^{11} m and the radius of the Sun is 7.0×10^{8} m. From these values find the surface temperature of the Sun, assuming that it radiates like a blackbody. [Ans. 5.8×10^{3} K]

5. A thermograph measures the rate at which each small portion of a person skin emits infrared radiation. To verify that a small difference in skin temperature means a significant difference in radiation rate, find the percentage difference between the total radiation from skin at 34° and at 35° C. [Ans. 1.3%]

6. Prove that the wavelength corresponding to maximum energy of emission is given by

$$\lambda_m = h\,c/4.965\,k_B T \ .$$

7. Considering the sun as a black body at 6000 K, estimate the portion of its total radiation that consists of yellow light between 570 and 590 nm. [Ans. 2.6%]

8. An X-ray of wavelength 0.05 nm scatters from a gold target. Can the X-ray be Compton-scattered from an electron bound by as much as 62,000 eV? What is the kinetic energy of the most energetic recoil electron and at what angle does it occur. [Ans. 2.3 keV]

9. Prove that the kinetic energy of the recoiling electron in Compton scattering is given by $K = h\nu[2\beta\sin^2(\theta/2)]/[1 + 2\beta\sin^2(\theta/2)]$, where $\beta = h\nu/m_0c^2$ and θ is the scattering angle of incident photon . Also estimate the maximum value of kinetic energy.

10. Photons of the wavelength 0.012 Å are scattered by free electrons at angle 90° and 180°. Find the wavelengths of the scattered photons and the energies transferred to the free electron in both the cases. [Ans. 0.0363 Å, 0.0506 Å, 0.66 MeV, 0.80 MeV]

11. A photon whose energy equals the rest energy of the electron undergoes a Compton collision with an electron. If the electron moves off at an angle of 40° with the original photon direction, what is the energy of the scattered photon? [Ans. 335 KeV]

12. 1.5 mW of 400 nm light is directed at a photo electric cell. If 0.10 % of the incident photons produce photoelectron, find the current in the cell. [Ans. 0.48 μA]

13. The stopping potential for electrons emitted from a surface illuminated by light of wavelength 491 nm is 0.710 V. When the incident wavelength is changed to a new value, the stopping potential is 1.43 V. (a) What is this new wavelength? (b) What is the work function for the surface? [Ans. (a) 382 nm (b) 1.82 eV]

14. What is the wavelength of an electron that has been accelerated through a potential difference of 200 V? Would this electron exhibit particle-like or wave-like characteristic on meeting an obstacle of opening 1 mm in diameter? [Ans: 8.68×10^{-11} m, particle-like]

15. Show that if the total energy of a moving particle greatly exceeds its rest energy, its de Broglie wavelength is nearly the same as the wavelength of a photon with the same total energy.

16. Find the kinetic energy of (a) photons (b) electrons (c) neutrons and (d) α particles that have a de Broglie wavelength of 0.15 nm.
[Ans. (a) 8.27 KeV, (b) 66.9 eV, (c) 0.036 eV, (d) 9.17×10^{-3} eV]

17. Calculate the phase and group velocities of an electron whose de Broglie wavelength is exactly 1×10^{-13} m.

18. Calculate the de Broglie wavelength of (a) a tennis ball of mass 70 gm traveling 25 m/s and (b) an electron of energy 50 eV. [Ans. 3.8×10^{-34} nm, 0.17 nm]

19. Newton showed that deep-water waves have a phase velocity of $\sqrt{g\lambda}/2\pi$. Find the group velocity of such waves. [Ans. $v_{gr} = \frac{1}{2} v_{ph}$]

20. Calculate the minimum kinetic energy of an electron that is localized within a typical nuclear radius of 6×10^{-15} m. [Ans. 15.9 MeV]

21. A beam of 2 keV electron is transmitted through a thin crystal and diffracted beams are recorded on a screen 35 cm away from the crystal. The radius of three concentric rings on the screen, all corresponding to first order diffraction, are 2.1 cm, 2.3 cm, and 3.2 cm. What is the lattice plane spacing corresponding to each of the three rings ?
[Ans. 0.457 nm, 0.412 nm, 0.301 nm]

22. Two waves are traveling simultaneously in a medium. They can be represented by $\psi_1(x,t) = 0.003 \sin(6x - 300t)$ and $\psi_2(x,t) = 0.003 \sin(7.0x - 250t)$, where x is in meters and t is in seconds.

(a) Write the expression for the resulting wave ψ.

(b) What are the phase and group velocities?

(c) What is x between the adjacent zeroes of ψ?

(d) What is $\Delta k \Delta x$?

[Ans. (a) $\psi = 0.006 \sin(6.5x - 275t) \cos(0.5x + 25t)$, (b) 42.3 m/s, 50 m/s, $(c)\pi$ m, (d) $2\,\pi$]

23. A 0.083 eV neutron beam scatters from an unknown sample and a Brags reflection peak is observed centered at 22°. What is the Brags Plane spacing? (Consider first order diffraction) [Ans. d = 1.33]

24. Suppose that the momentum of a certain particle can be measured to an accuracy of one part in a thousand. Determine the minimum uncertainty in the position of the particle if the particle is (a) a 5×10^{-3} kg mass moving with a speed of 2 m/s, (b) an electron moving with a speed of 1.8×10^{8} m/s. [Ans. (a) 5.28×10^{-20} Å, (b) 2.57 Å]

25. What is the uncertainty in the location of a photon of wavelength 3000 Å if this wavelength is known to an accuracy of one part in a million? [Ans. 23.9 mm]

26. What is the minimum uncertainty in the energy state of an atom if an electron remains in this state for 10^{-8} sec. [Ans. 0.329×10^{-7} eV]

27. Consider an element having excited states at 3.6 eV and 4.6 eV used as a gas in the Franck-Hertz experiment. Assume the work function of the materials involved cancel out. List all the possible peaks that might be observed with electron scattering up to an accelerating voltage of 18 V.

28. Consider an electron of momentum **p** in the Coulomb field of a proton. The total energy is

$$E = \frac{p^2}{2m} - \frac{e^2}{4\pi\varepsilon_0 r},$$

where, r is the distance of the electron from proton. Assuming that the uncertainty $\Delta r = r$ and $\Delta p = p$. Use the Heisenberg's uncertainty principle to obtain an estimate of the size and the energy of the hydrogen atom in the ground state. Compare these results with those obtained by Bohr's theory of hydrogen atom.

29. Use the Heisenberg's uncertainty relation to estimate the ground state energy of a linear harmonic oscillator. [Ans. $\hbar\omega/4\pi$]

30. Write down the Rayleigh-Jeans law for blackbody radiation and plot the spectral energy density as a function of wavelength. Explain how Plank's treatment of blackbody radiation fixed the ultraviolet catastrophe.