DEPARTMENT OF MATHEMATICS, IIT ROORKEE
MAB-103: Numerical Methods

Assignment-8 Numerical solutions of Ordinary Differential equations Session: 2025-26

1. Using Picard's process of successive approximations, obtain a solution up to the fifth approximation of the equation
$$\frac{dy}{dx} = y + x, \; y(0) = 1.$$
Check your answer by finding the exact solution.

2. Given the initial value problem
$$\frac{dy}{dx} = x^2 y - 1, \; y(0) = 1,$$
determine the first four non-zero terms in the Taylor series for y(x) and hence obtain the values for $y(0.1)$ and $y(0.2)$, to five places of decimals.

3. Solve, by Taylor's series method, by determining the first four non-zero terms, the differential equation
$$\frac{dy}{dx} = \ln(xy), \; y(1) = 2,$$
for y(1.1) and y(1.2), correct to 4 significant figures.

4. Using Euler's method, solve the following differential equations:
(a) $y'(x) + 2y = 0, \; y(0) = 1.$
(b) $y'(x) - 1 = y^2, \; y(0) = 0.$
In each case, take $h = 0.1$ and obtain $y(0.1), y(0.2)$ and $y(0.3)$, correct to 3 decimal places.

5. Using the modified Euler's method, find y(0.2) and y(0.4) given
$$\frac{dy}{dx} = y + e^x, \; y(0) = 0,$$
correct to 4 decimal places, with $h = 0.1$.

6. Given that
$$\frac{dy}{dx} = x^2 + y, \; y(0) = 1.$$
Determine $y(0.2), y(0.4)$ and $y(0.6)$ using Euler's modified method, correct to 5 significant figures, with $h = 0.2$.

7. Use Runge-Kutta method of fourth order to find the value of y when $x = 1$, correct to 4 decimal places, given that
$$\frac{dy}{dx} = \frac{y - x}{y + x}, \;\; y(0) = 1, \text{ with } h = 0.1.$$

8. Using the Runge-Kutta method of fourth order, solve

$$\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}, \quad y(0) = 1,$$

for the interval $0 < x \leq 0.4$ with $h = 0.1$, correct to 4 decimal places.

9. Using Milne's predictor-corrector method, evaluate $y(4.5)$, if y satisfies

$$5x\frac{dy}{dx} + y^2 - 2 = 0,$$

and $y(4) = 1$, $y(4.1) = 1.0049$, $y(4.2) = 1.0097$, $y(4.3) = 1.0143$, $y(4.4) = 1.0187$.

10. Given

$$\frac{dy}{dx} = x(x^2 + y^2)e^{-x}, \; y(0) = 1,$$

find y at x = 0.1, 0.2, and 0.3, by Taylor's series method (by determining the first four non-zero terms) and compute y(0.4) by Milne's predictor-corrector method, correct to 3 decimal places.

11. Using Picard's process of successive approximations, obtain a solution, up to the third approximation (correct to 4D), of y and z corresponding to $x = 0.1$, given that y(0) = 2, z(0)= 1 and

$$\frac{dy}{dx} = x + z, \quad \frac{dz}{dx} = x - y^2.$$

12. Using Taylor series method, solve

$$\frac{d^2y}{dx^2} + y^2\frac{dy}{dx} = x^3, \; y(1) = 1, y'(1) = 1,$$

for $x = 1.1$, and $x = 1.2$, correct to 4 decimal places.

Answers:

(1) $1 + x + x^2 + \frac{x^3}{3} + \frac{x^4}{12} + \frac{x^5}{120} + \frac{x^6}{720}$

(2) 0.90033, 0.80227

(3) 2.081

(4) (a) 0.800, 0.640, 0.512; (b) 0.100, 0.201, 0.305

(5) 0.2468, 0.6031

(6) 1.0202, 1.0408, 1.0619

(7) 1.4983

(8) 1.0101, 1.0207, 1.0318, 1.0438

(9) 1.0230

(10) 1.071

(11) 2.0845, 0.5867

(12) 1.1002, 1.0055