Plane Wave – Incidence

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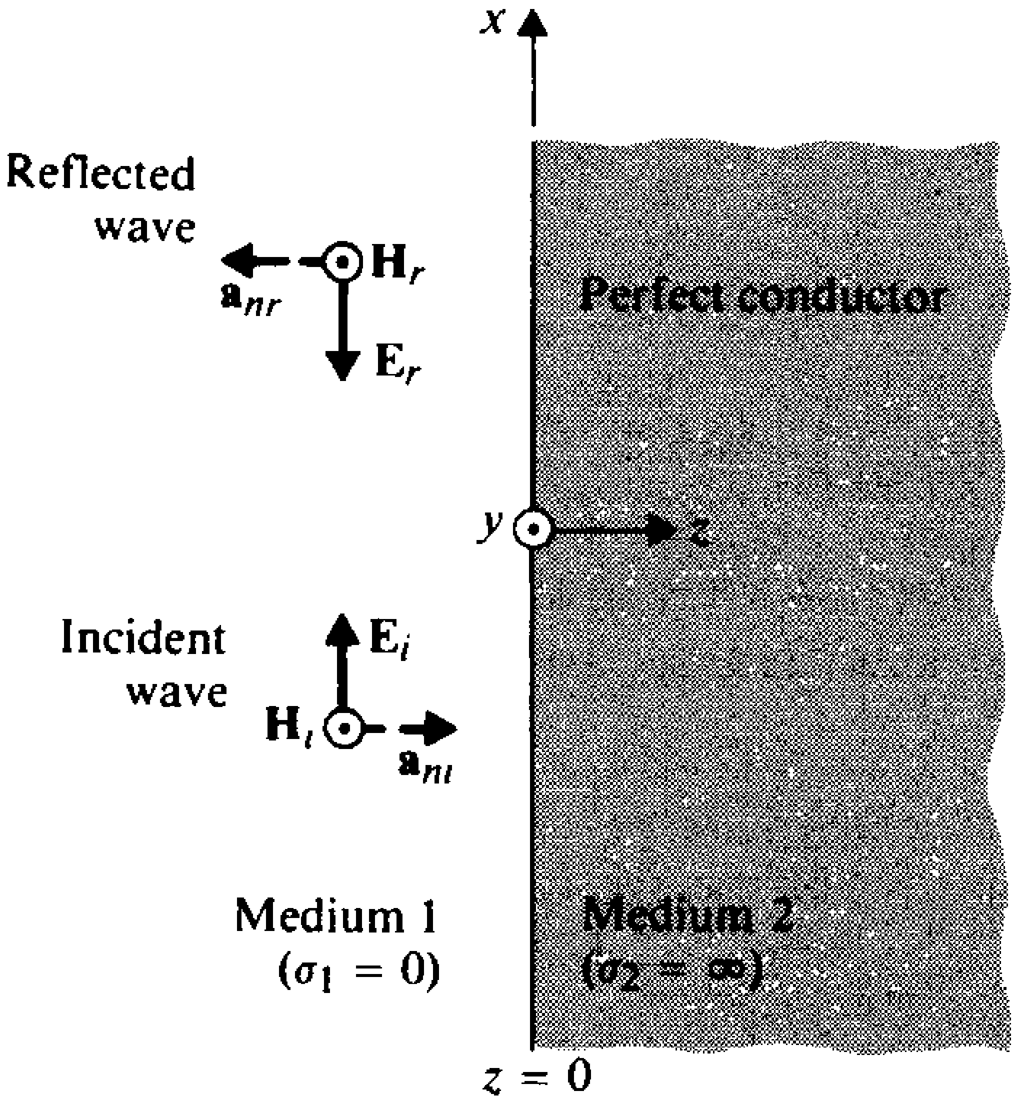
Normal Incidence on Conducting Boundary

For simplicity we shall assume that the incident wave ($\mathbf{E}_i$, $\mathbf{H}_i$) travels in a lossless medium (medium 1: $\sigma_1 = 0$) and that the boundary is an interface with a perfect conductor (medium 2: $\sigma_2 = \infty$).



<u>Incident Wave</u>

E_i → x

Prop_i → z

H_i → y

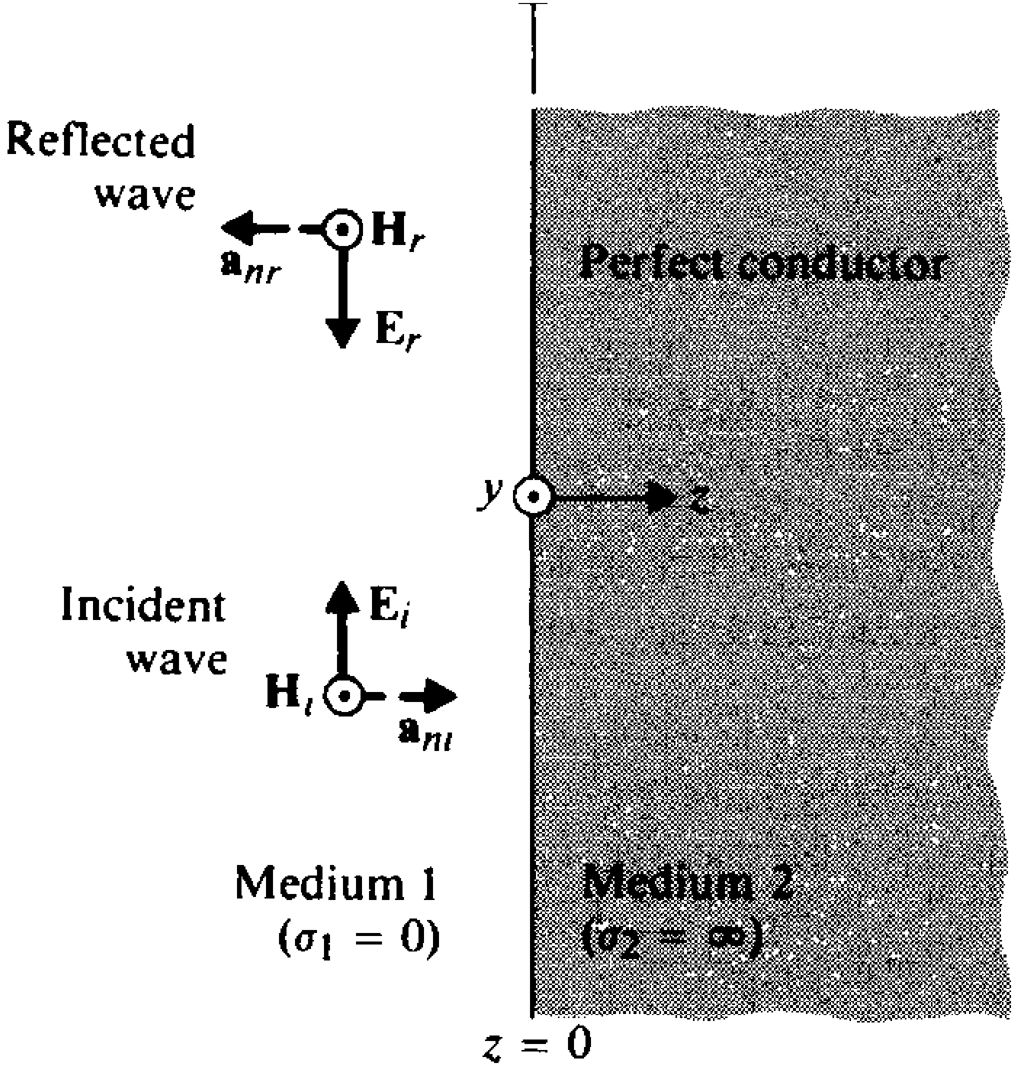
<u>Reflected Wave</u>

E_r → x or –x (yet to be determined)

Prop_r → -z (known)

H_r → y or –y (yet to be determined)

Inside medium 2 (a perfect conductor), both electric and magnetic fields vanish, $\mathbf{E}_2 = 0$, $\mathbf{H}_2 = 0$; hence no wave is transmitted across the boundary into the $z > 0$ region. The incident wave is reflected, giving rise to a reflected wave ($\mathbf{E}_r$, $\mathbf{H}_r$). The reflected electric field intensity can be written as



$$\mathbf{E}_i(z) = \mathbf{a}_x E_{i0} e^{-j\beta_1 z},$$

$$\mathbf{H}_i(z) = \mathbf{a}_y \frac{E_{i0}}{\eta_1} e^{-j\beta_1 z},$$

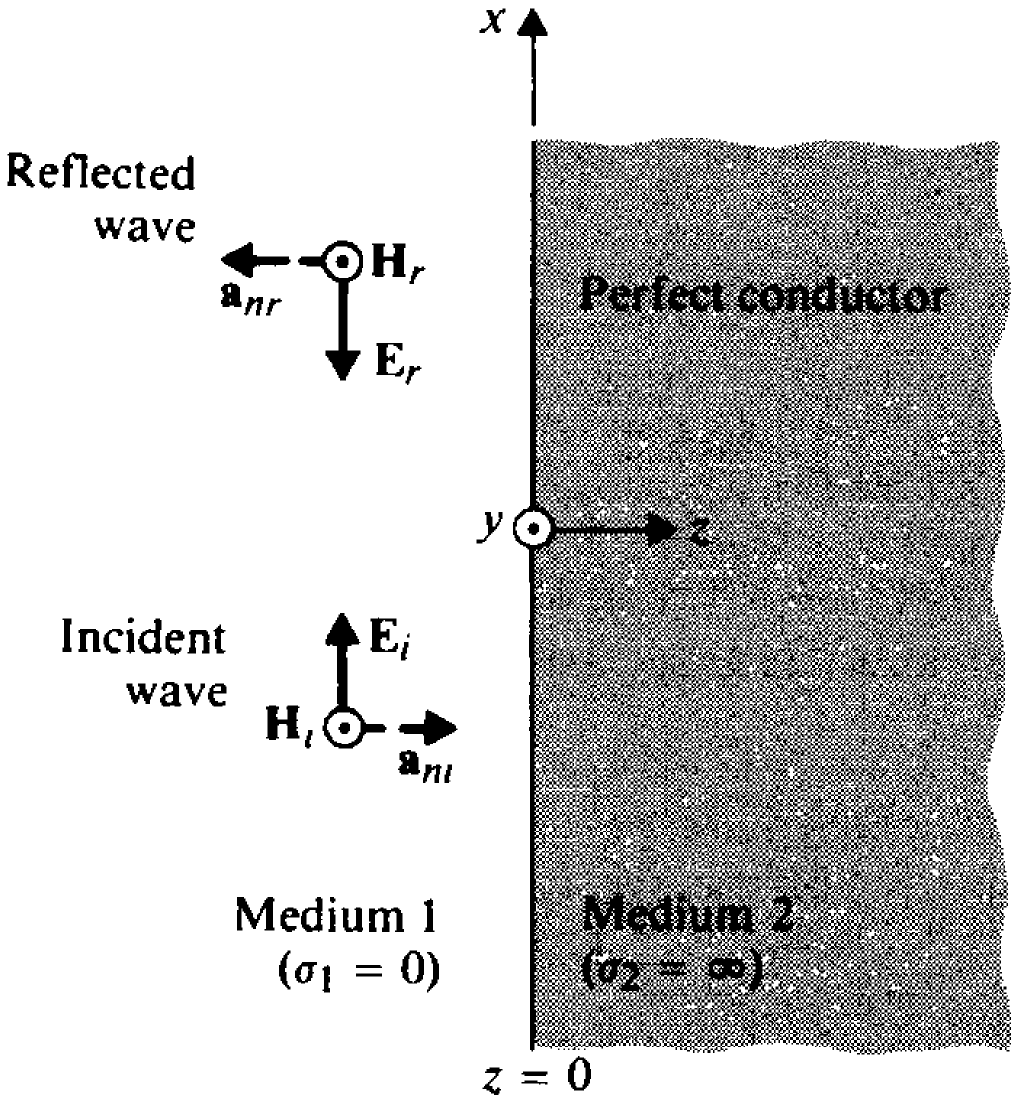
$$\mathbf{E}_r(\mathrm{z}) = \mathbf{a}_x E_{r0} e^{+j\beta_1 z},$$

$$\mathbf{E}_1(z) = \mathbf{E}_i(z) + \mathbf{E}_r(z) = \mathbf{a}_x(E_{i0}e^{-j\beta_1 z} + E_{r0}e^{+j\beta_1 z}).$$

Number of unknowns : 1 (E_r)

Boundary Conditions : 2

Continuity of the tangential component of the **E**-field at the boundary $z = 0$ demands that



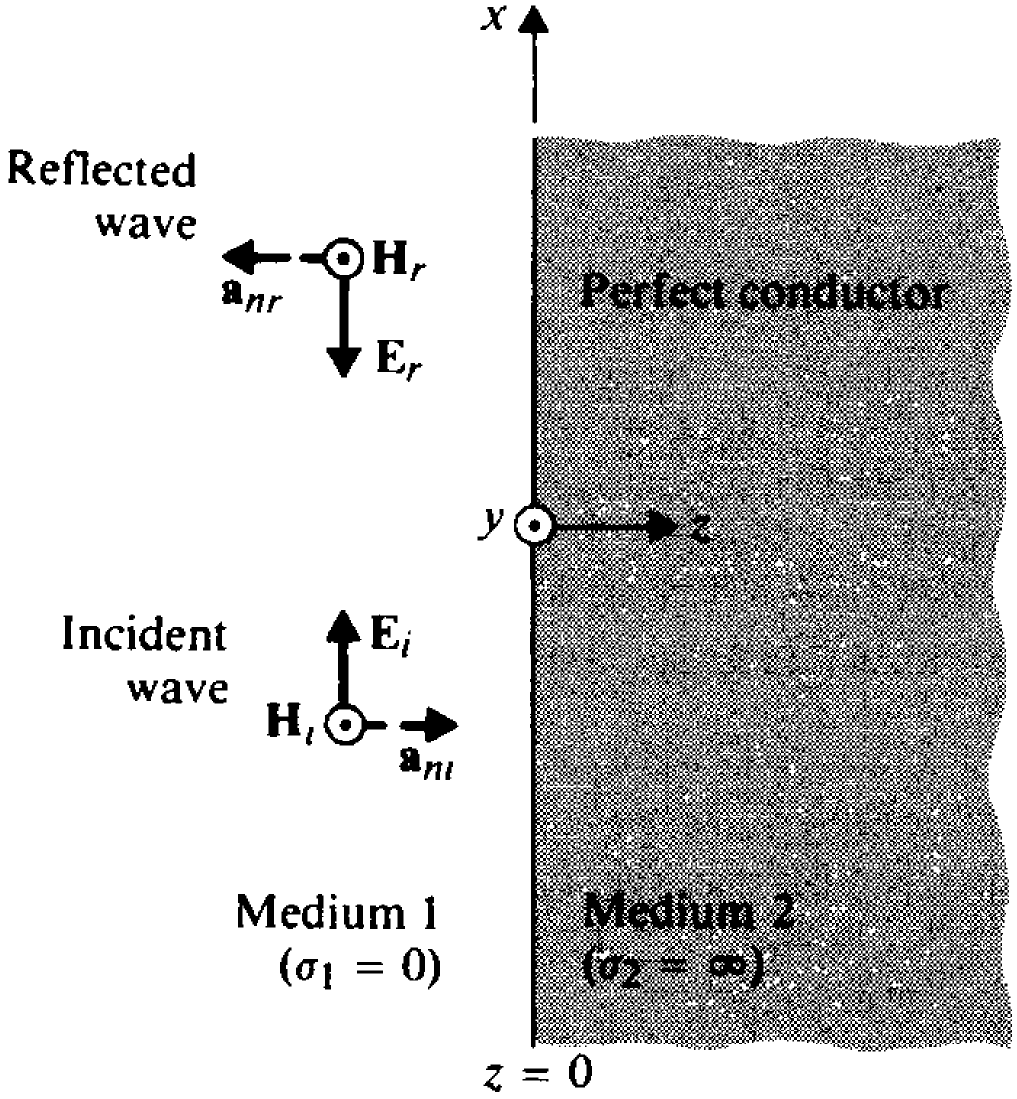
$$\mathbf{E}_1(0) = \mathbf{a}_x(E_{i0} + E_{r0}) = \mathbf{E}_2(0) = 0,$$

$$\text{which yields } E_{r0} = -E_{i0}$$

$$\mathbf{E}_1(z) = \mathbf{a}_x E_{i0}(e^{-j\beta_1 z} - e^{+j\beta_1 z})$$
$$= -\mathbf{a}_x j2E_{i0} \sin \beta_1 z.$$

$$\mathbf{H}_r(z) = \frac{1}{\eta_1}\mathbf{a}_{nr} \times \mathbf{E}_r(z) = \frac{1}{\eta_1}(-\mathbf{a}_z) \times \mathbf{E}_r(z)$$
$$= -\mathbf{a}_y \frac{1}{\eta_1} E_{r0} e^{+j\beta_1 z} = \mathbf{a}_y \frac{E_{i0}}{\eta_1} e^{+j\beta_1 z}.$$

$$\mathbf{H}_1(z) = \mathbf{H}_i(z) + \mathbf{H}_r(z) = \mathbf{a}_y 2\frac{E_{i0}}{\eta_1} \cos \beta_1 z.$$



$$\mathbf{E}_1(z) = \mathbf{a}_x E_{i0}(e^{-j\beta_1 z} - e^{+j\beta_1 z})$$
$$= -\mathbf{a}_x j2E_{i0} \sin \beta_1 z.$$

$$\mathbf{H}_1(z) = \mathbf{H}_i(z) + \mathbf{H}_r(z) = \mathbf{a}_y 2\frac{E_{i0}}{\eta_1} \cos \beta_1 z.$$

$$\mathscr{P}_{av} = \frac{1}{2}\mathscr{R}e(\mathbf{E} \times \mathbf{H}^*)$$

P_av = 0

$$\mathbf{E}_1(z, t) = \mathscr{R}e[\mathbf{E}_1(z)e^{j\omega t}] = \mathbf{a}_x 2E_{i0} \sin \beta_1 z \sin \omega t,$$

$$\mathbf{H}_1(z, t) = \mathscr{R}e[\mathbf{H}_1(z)e^{j\omega t}] = \mathbf{a}_y 2\frac{E_{i0}}{\eta_1} \cos \beta_1 z \cos \omega t.$$

$$\mathbf{E}_1(z, t) = \mathscr{R}e[\mathbf{E}_1(z)e^{j\omega t}] = \mathbf{a}_x 2E_{i0} \sin \beta_1 z \sin \omega t,$$

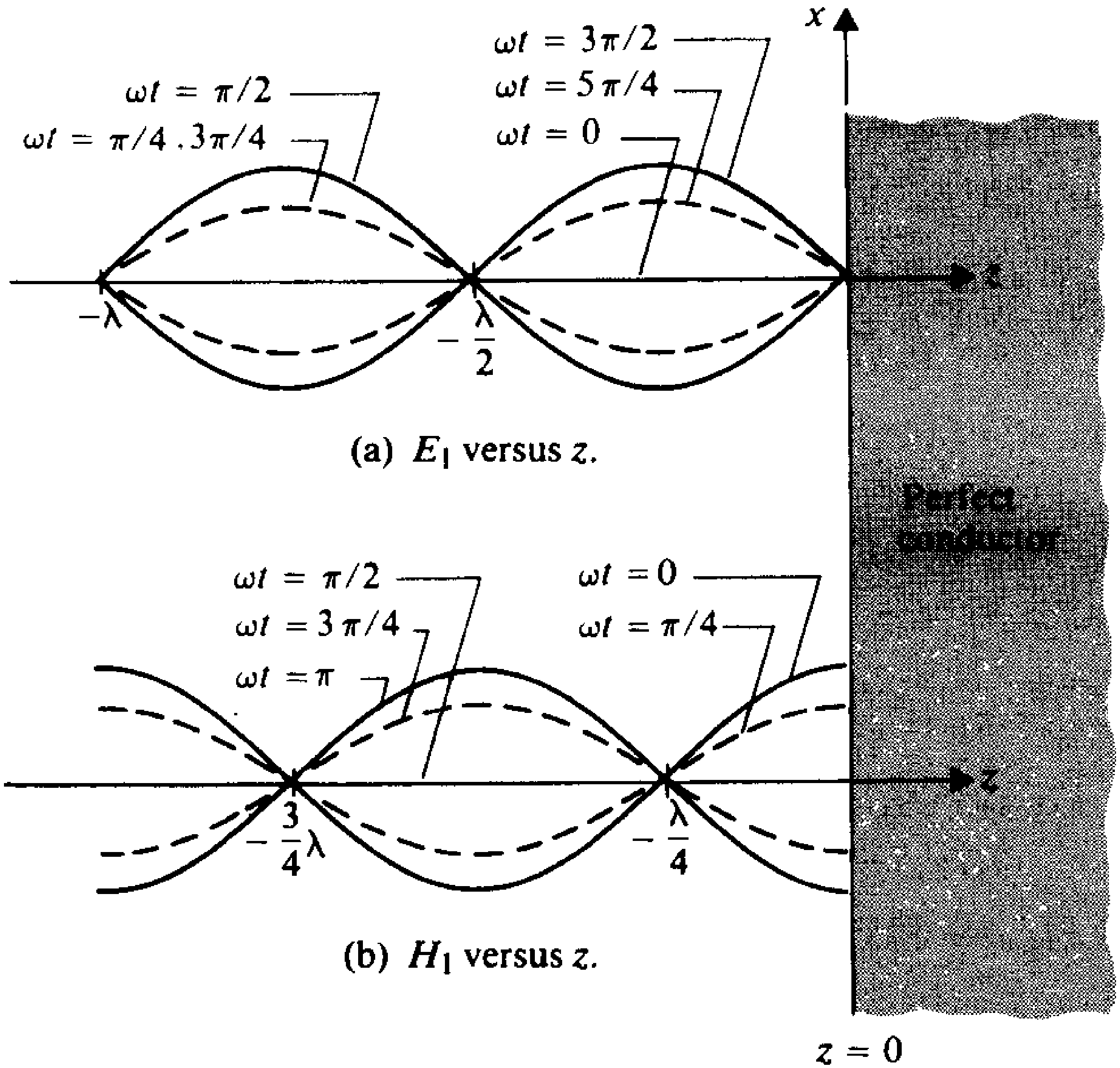
$$\mathbf{H}_1(z, t) = \mathscr{R}e[\mathbf{H}_1(z)e^{j\omega t}] = \mathbf{a}_y 2 \frac{E_{i0}}{\eta_1} \cos \beta_1 z \cos \omega t.$$

Both $\mathbf{E}_1(z, t)$ and $\mathbf{H}_1(z, t)$ possess zeros and maxima at fixed distances from the conducting boundary for all t, as follows:

$$\left.\begin{array}{r}\text{Zeros of } \mathbf{E}_1(z, t) \\ \text{Maxima of } \mathbf{H}_1(z, t)\end{array}\right\} \text{occur at } \beta_1 z = -n\pi, \quad \text{or } z = -n\frac{\lambda}{2}, \qquad n = 0, 1, 2, \ldots$$

$$\left.\begin{array}{r}\text{Maxima of } \mathbf{E}_1(z, t) \\ \text{Zeros of } \mathbf{H}_1(z, t)\end{array}\right\} \text{occur at } \beta_1 z = -(2n+1)\frac{\pi}{2}, \quad \text{or } z = -(2n+1)\frac{\lambda}{4},$$
$$n = 0, 1, 2, \ldots$$

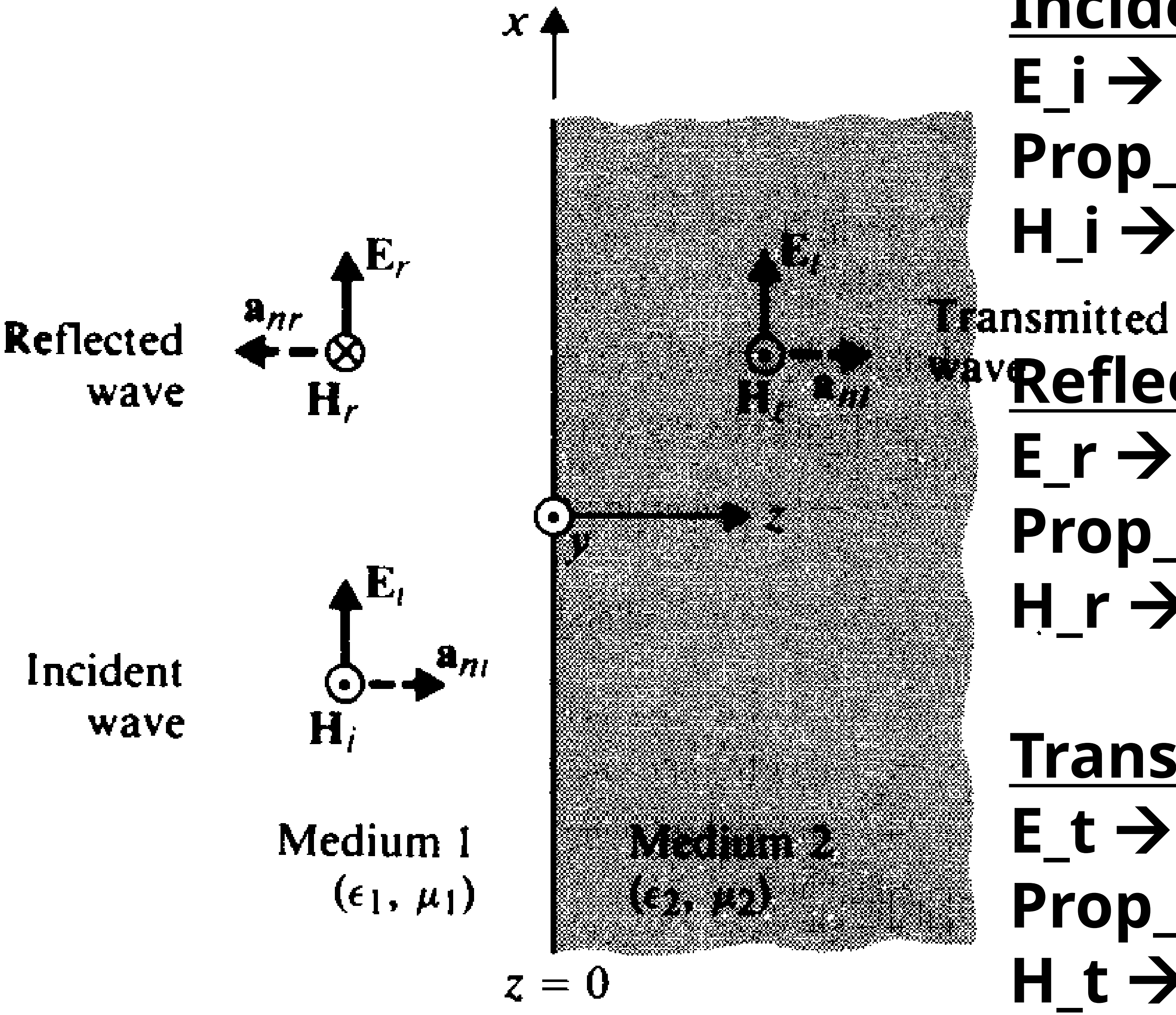
The total wave in medium 1 is not a traveling wave. It is a ***standing wave***,



(a) E_1 versus z.

(b) H_1 versus z.

Normal Incidence on Dielectric Boundary



Incident Wave

E_i → x
Prop_i → z
H_i → y

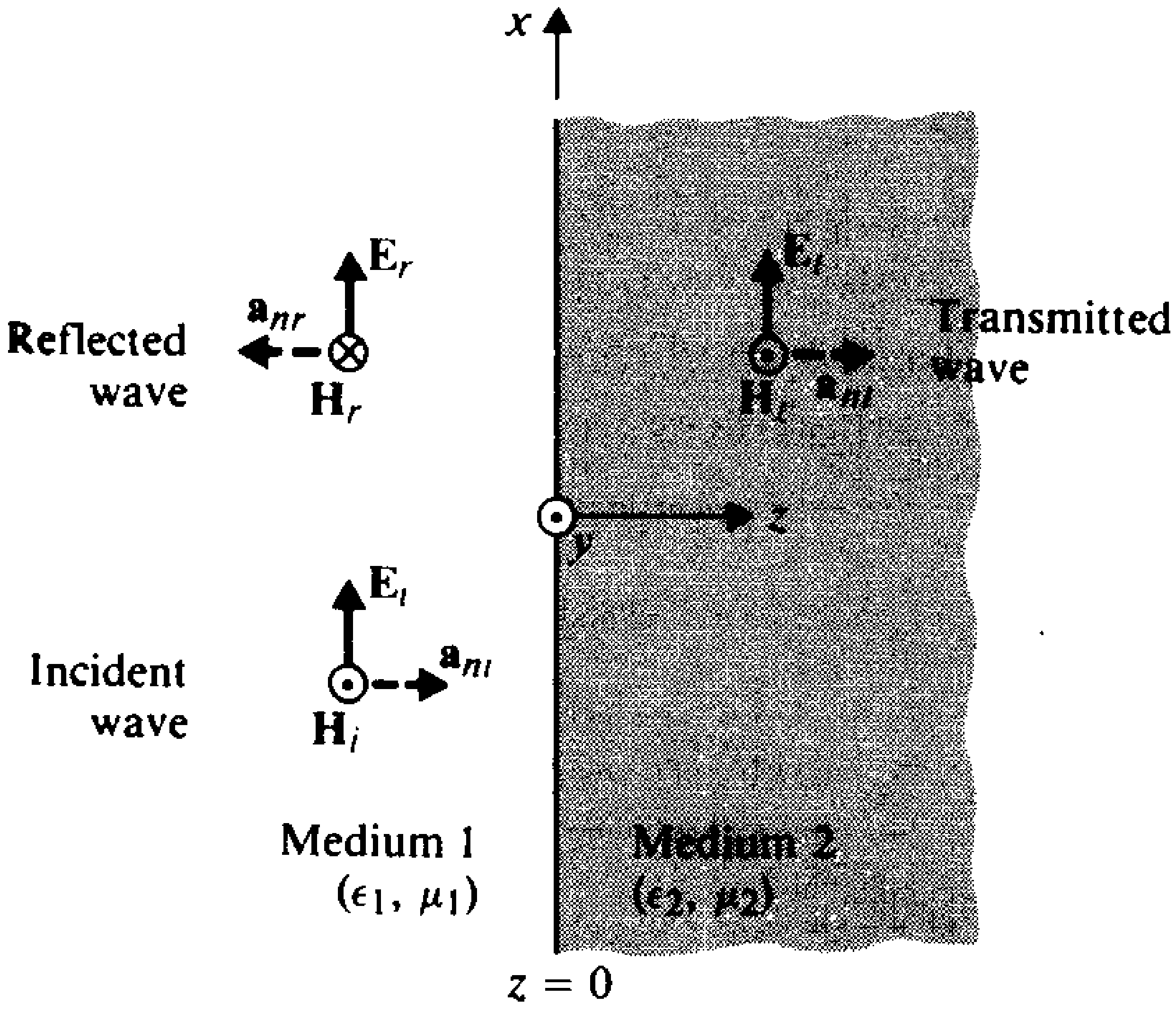
Reflected Wave

E_r → x or –x (yet to be determined)
Prop_r → -z (known)
H_r → y or –y (yet to be determined

Transmitted Wave

E_t → x or –x (yet to be determined)
Prop_i → z
H_t → y or –y (yet to be determined

Because of the medium discontinuity at $z = 0$, the incident wave is partly reflected back into medium 1 and partly transmitted into medium 2.



$$\mathbf{E}_i(z) = \mathbf{a}_x E_{i0} e^{-j\beta_1 z},$$

$$\mathbf{H}_i(z) = \mathbf{a}_y \frac{E_{i0}}{\eta_1} e^{-j\beta_1 z}.$$

$$\mathbf{E}_r(z) = \mathbf{a}_x E_{r0} e^{j\beta_1 z},$$

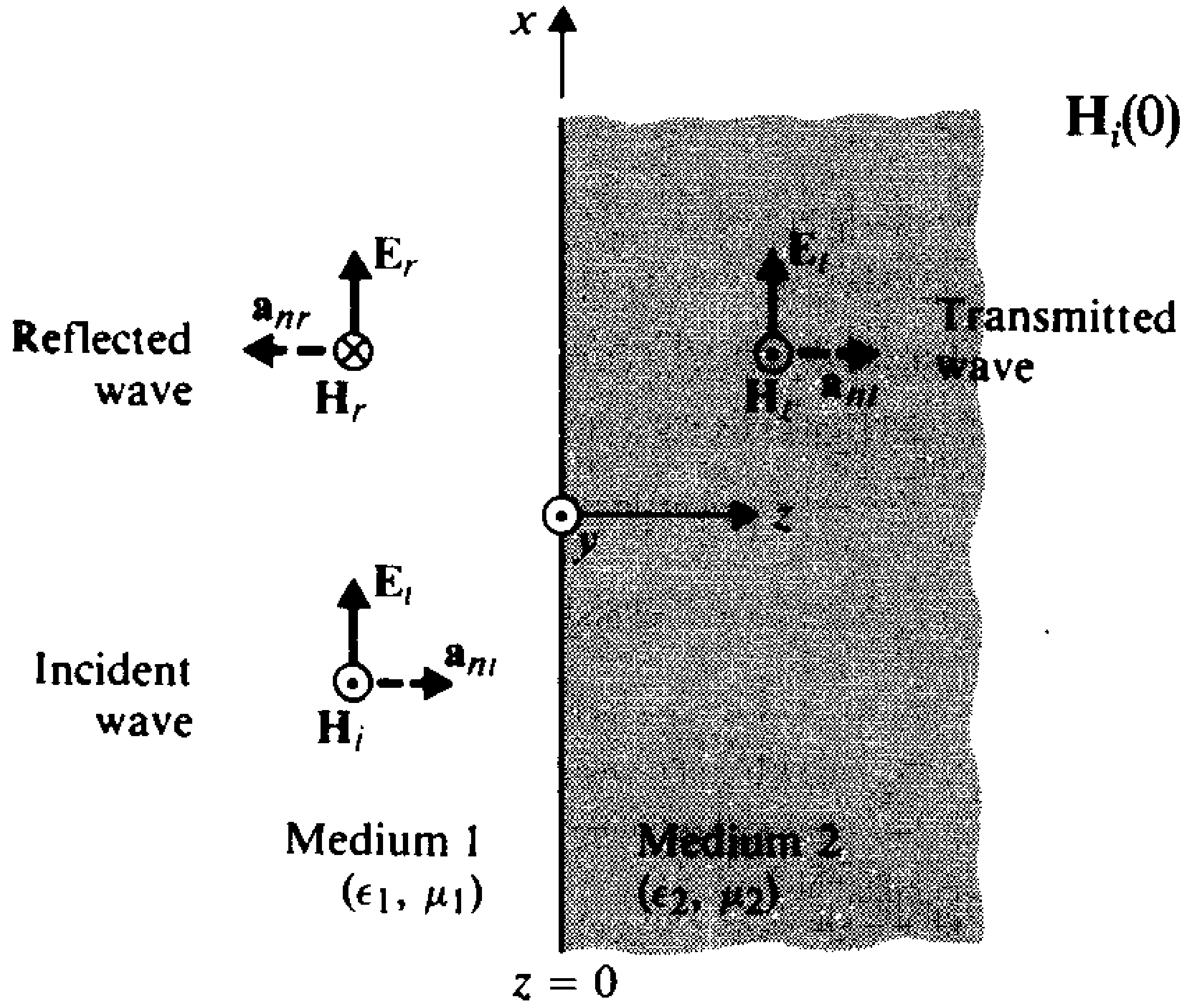
$$\mathbf{H}_r(z) = (-\mathbf{a}_z) \times \frac{1}{\eta_1} \mathbf{E}_r(z) = -\mathbf{a}_y \frac{E_{r0}}{\eta_1} e^{j\beta_1 z}.$$

$$\mathbf{E}_t(z) = \mathbf{a}_x E_{t0} e^{-j\beta_2 z},$$

$$\mathbf{H}_t(z) = \mathbf{a}_z \times \frac{1}{\eta_2} \mathbf{E}_t(z) = \mathbf{a}_y \frac{E_{t0}}{\eta_2} e^{-j\beta_2 z},$$

Number of unknowns : 2 (E_r, E_t)

Boundary Conditions : 2



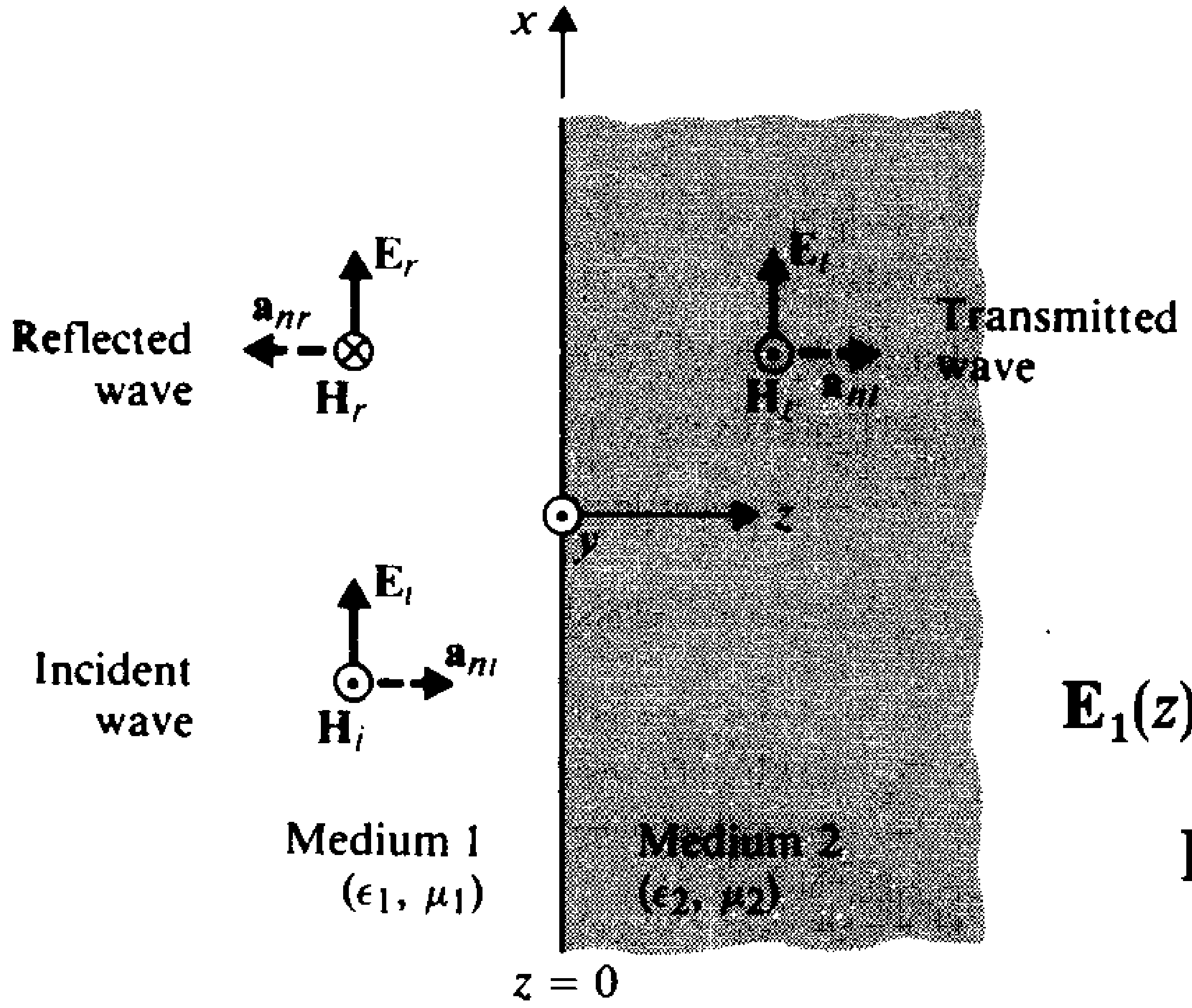
$$\mathbf{E}_i(0) + \mathbf{E}_r(0) = \mathbf{E}_t(0) \qquad \text{or} \qquad E_{i0} + E_{r0} = E_{t0}$$

$$\mathbf{H}_i(0) + \mathbf{H}_r(0) = \mathbf{H}_t(0) \qquad \text{or} \qquad \frac{1}{\eta_1}(E_{i0} - E_{r0}) = \frac{E_{t0}}{\eta_2}.$$

$$E_{r0} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} E_{i0},$$

$$E_{t0} = \frac{2\eta_2}{\eta_2 + \eta_1} E_{i0}.$$

The ratios E_{r0}/E_{i0} and E_{t0}/E_{i0} are called ***reflection coefficient*** and ***transmission coefficient***, respectively. In terms of the intrinsic impedances they are



$$\Gamma = \frac{E_{r0}}{E_{i0}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

$$\tau = \frac{E_{t0}}{E_{i0}} = \frac{2\eta_2}{\eta_2 + \eta_1}$$

$$1 + \Gamma = \tau$$

$$\mathbf{E}_1(z) = \mathbf{E}_i(z) + \mathbf{E}_r(z) = \mathbf{a}_x E_{i0}(e^{-j\beta_1 z} + \Gamma e^{j\beta_1 z})$$

$$\mathbf{E}_1(z) = \mathbf{a}_x E_{i0} e^{-j\beta_1 z}(1 + \Gamma e^{j2\beta_1 z}).$$

The ratio of the maximum value to the minimum value of the electric field intensity of a standing wave is called the ***standing-wave ratio*** **(SWR)**, S.

$$S = \frac{|E|_{\max}}{|E|_{\min}} = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

$$|\Gamma| = \frac{S - 1}{S + 1}$$

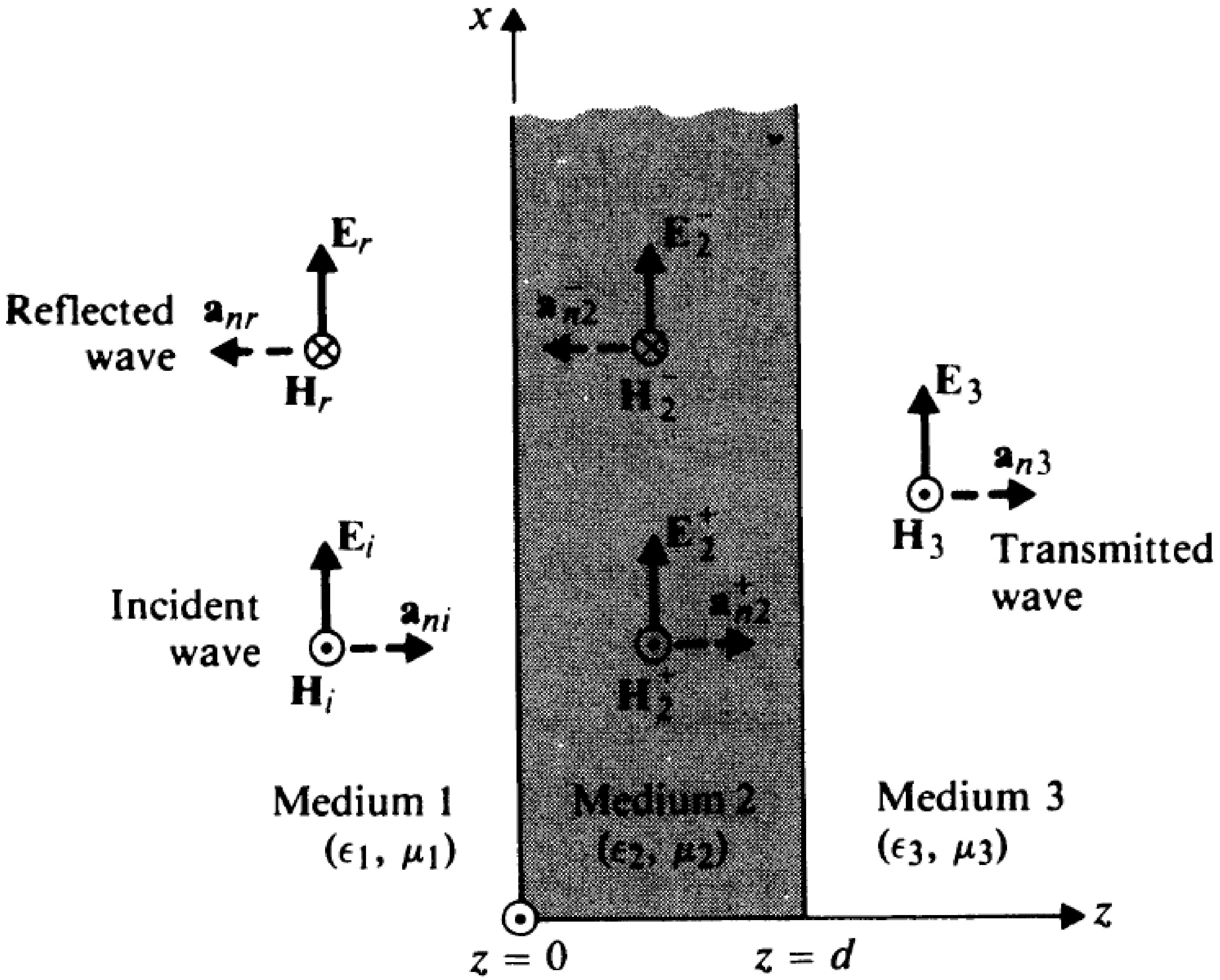
While the value of Γ ranges from -1 to $+1$, the value of S ranges from 1 to ∞

$$\mathbf{E}_1(z) = \mathbf{E}_i(z) + \mathbf{E}_r(z) = \mathbf{a}_x E_{i0}(e^{-j\beta_1 z} + \Gamma e^{j\beta_1 z})$$

$$\mathbf{H}_1(z) = \mathbf{a}_y \frac{E_{i0}}{\eta_1}(e^{-j\beta_1 z} - \Gamma e^{j\beta_1 z})$$

$$\mathbf{E}_t(z) = \mathbf{a}_x \tau E_{i0} e^{-j\beta_2 z}. \qquad \mathbf{H}_t(z) = \mathbf{a}_y \frac{\tau}{\eta_2} E_{i0} e^{-j\beta_2 z}.$$

Normal Incidence on Multiple Dielectric Interfaces


x
$\mathbf{E}_r$
Reflected wave
$\mathbf{a}_{nr}$
$\mathbf{H}_r$
$\mathbf{E}_2^-$
$\mathbf{a}_{n2}^-$
$\mathbf{H}_2^-$
$\mathbf{E}_3$
$\mathbf{a}_{n3}$
$\mathbf{H}_3$
Transmitted wave
$\mathbf{E}_i$
Incident wave
$\mathbf{a}_{ni}$
$\mathbf{H}_i$
$\mathbf{E}_2^+$
$\mathbf{a}_{n2}^+$
$\mathbf{H}_2^+$
Medium 1
(ϵ_1, μ_1)
Medium 2
(ϵ_2, μ_2)
Medium 3
(ϵ_3, μ_3)
z
$z = 0$
$z = d$

A uniform plane wave traveling in the $+z$-direction in medium 1 (ϵ_1, μ_1) impinges normally at a plane boundary with medium 2 (ϵ_2, μ_2), at $z = 0$. Medium 2 has a finite thickness and interfaces with medium 3 (ϵ_3, μ_3) at $z = d$. Reflection occurs at both $z = 0$ and $z = d$. Assuming an x-polarized incident field, the total electric field intensity in medium 1 can always be written as the sum of the incident component $\mathbf{a}_x E_{i0} e^{-j\beta_1 z}$ and a reflected component $\mathbf{a}_x E_{r0} e^{j\beta_1 z}$:

$$\mathbf{E}_1 = \mathbf{a}_x(E_{i0}e^{-j\beta_1 z} + E_{r0}e^{j\beta_1 z}).$$

$$\mathbf{H}_1 = \mathbf{a}_y \frac{1}{\eta_1}(E_{i0}e^{-j\beta_1 z} - E_{r0}e^{j\beta_1 z}).$$

$$\mathbf{E}_2 = \mathbf{a}_x(E_2^+ e^{-j\beta_2 z} + E_2^- e^{j\beta_2 z}),$$

$$\mathbf{H}_2 = \mathbf{a}_y \frac{1}{\eta_2}(E_2^+ e^{-j\beta_2 z} - E_2^- e^{j\beta_2 z}).$$

$$\mathbf{E}_3 = \mathbf{a}_x E_3^+ e^{-j\beta_3 z},$$

$$\mathbf{H}_3 = \mathbf{a}_y \frac{E_3^+}{\eta_3} e^{-j\beta_3 z}.$$

At $z = 0$:

$$\mathbf{E}_1(0) = \mathbf{E}_2(0),$$
$$\mathbf{H}_1(0) = \mathbf{H}_2(0).$$

At $z = d$:

$$\mathbf{E}_2(d) = \mathbf{E}_3(d),$$
$$\mathbf{H}_2(d) = \mathbf{H}_3(d).$$

Number of unknowns : 4 (E_r1, E_2+, E_2-, E_3+)

Boundary Conditions : 4

Purely algebraic exercise

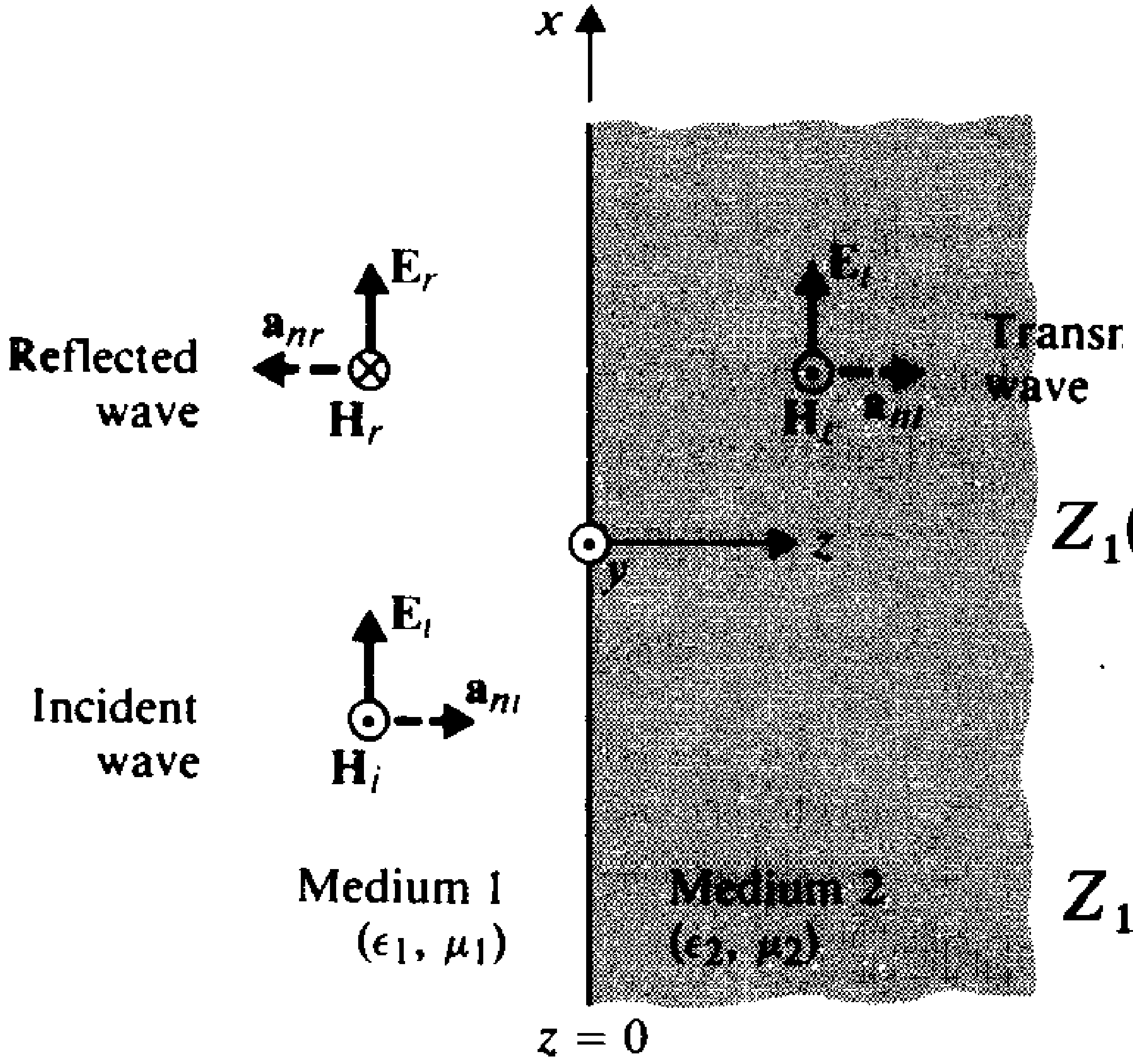
Suppose we are interested in determining only E_r1.

Can we simply the analysis ?

We define the ***wave impedance of the total field*** at any plane parallel to the plane boundary as the ratio of the total electric field intensity to the total magnetic field intensity.

$$Z(z) = \frac{\text{Total } E_x(z)}{\text{Total } H_y(z)}$$

For a single wave propagating in the $+z$-direction in an unbounded medium, the wave impedance equals the intrinsic impedance, η, of the medium; for a single wave traveling in the $-z$-direction, it is $-\eta$ for all z.



$$E_{1x}(z) = E_{i0}(e^{-j\beta_1 z} + \Gamma e^{j\beta_1 z}),$$

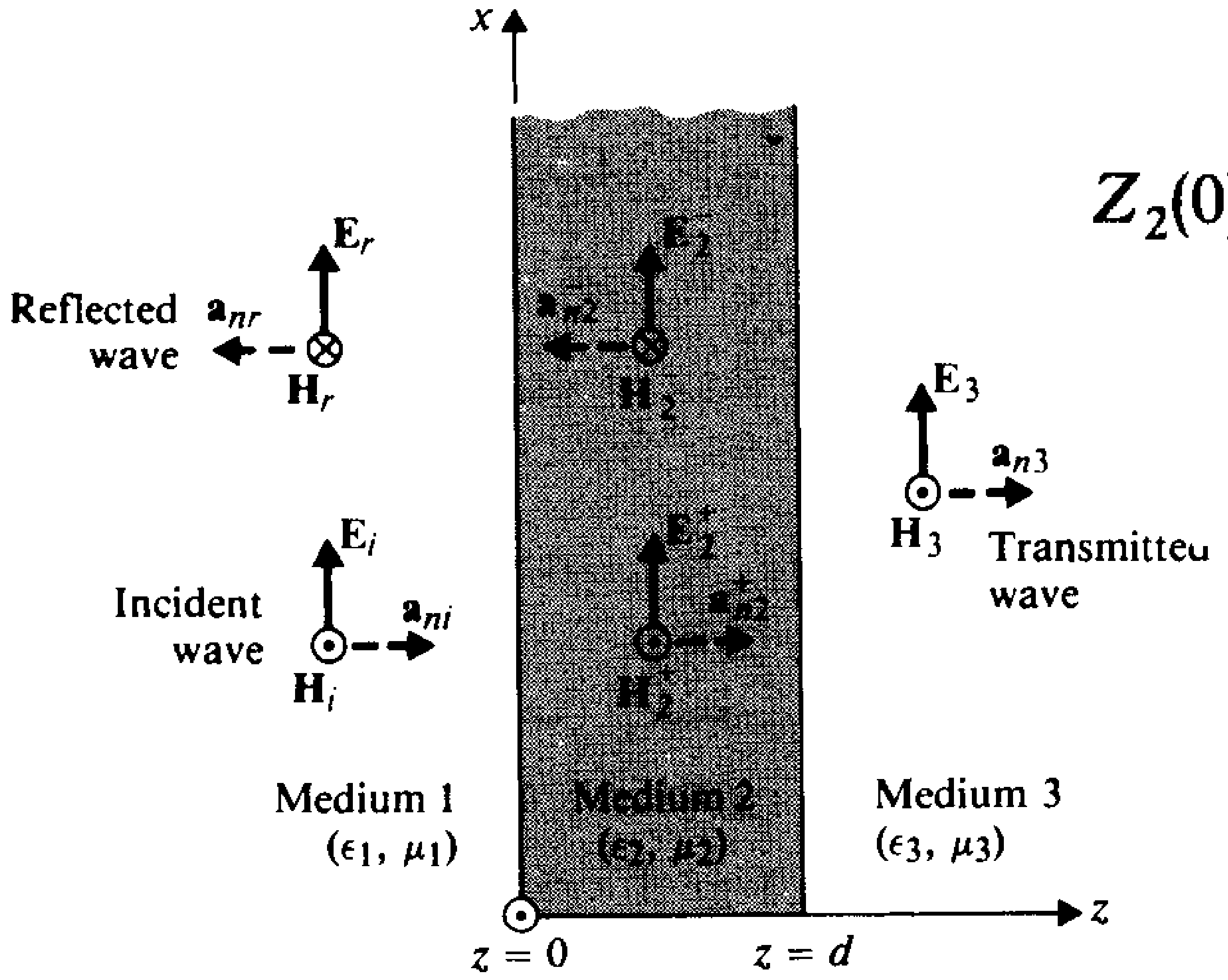
$$H_{1y}(z) = \frac{E_{i0}}{\eta_1}(e^{-j\beta_1 z} - \Gamma e^{j\beta_1 z}).$$

$$Z_1(z) = \frac{E_{1x}(z)}{H_{1y}(z)} = \eta_1 \frac{e^{-j\beta_1 z} + \Gamma e^{j\beta_1 z}}{e^{-j\beta_1 z} - \Gamma e^{j\beta_1 z}},$$

$$Z_1(-\ell) = \frac{E_{1x}(-\ell)}{H_{1y}(-\ell)} = \eta_1 \frac{e^{j\beta_1 \ell} + \Gamma e^{-j\beta_1 \ell}}{e^{j\beta_1 \ell} - \Gamma e^{-j\beta_1 \ell}}.$$

$$\Gamma = (\eta_2 - \eta_1)/(\eta_2 + \eta_1)$$

$$Z_1(-\ell) = \eta_1 \frac{\eta_2 \cos\beta_1 \ell + j\eta_1 \sin\beta_1 \ell}{\eta_1 \cos\beta_1 \ell + j\eta_2 \sin\beta_1 \ell},$$



$$Z_1(-\ell) = \eta_1 \frac{\eta_2 \cos \beta_1 \ell + j\eta_1 \sin \beta_1 \ell}{\eta_1 \cos \beta_1 \ell + j\eta_2 \sin \beta_1 \ell},$$

$$Z_2(0) = \eta_2 \frac{\eta_3 \cos \beta_2 d + j\eta_2 \sin \beta_2 d}{\eta_2 \cos \beta_2 d + j\eta_3 \sin \beta_2 d}.$$

$$\Gamma_0 = \frac{E_{r0}}{E_{i0}} = -\frac{H_{r0}}{H_{i0}} = \frac{Z_2(0) - \eta_1}{Z_2(0) + \eta_1}.$$

Thank You