Maxwell’s Equations

Electrostatics

Question : What are the fundamental postulates of Electrostatic Model ?

In constructing the electrostatic model we defined an electric field intensity vector, $\mathbf{E}$, and an electric flux density (electric displacement) vector, $\mathbf{D}$. The fundamental governing differential equations are

$$\nabla \times \mathbf{E} = 0,$$

$$\nabla \cdot \mathbf{D} = \rho.$$

For linear and isotropic (not necessarily homogeneous) media, $\mathbf{E}$ and $\mathbf{D}$ are related by the constitutive relation

$$\mathbf{D} = \epsilon \mathbf{E}.$$

Magnetostatics

Question : What are the fundamental postulates of Magnetostatics Mode

For the magnetostatic model we defined a magnetic flux density vector, $\mathbf{B}$, and a magnetic field intensity vector, $\mathbf{H}$. The fundamental governing differential equations are

$$\nabla \cdot \mathbf{B} = 0,$$

$$\nabla \times \mathbf{H} = \mathbf{J}.$$

The constitutive relation for $\mathbf{B}$ and $\mathbf{H}$ in linear and isotropic media is

$$\mathbf{H} = \frac{1}{\mu}\mathbf{B}.$$

Maxwell’s Equations

Maxwell’s Equations

Differential Form	Integral Form	Significance
$\nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t}$	$\oint_C \mathbf{E} \cdot d\boldsymbol{\ell} = -\dfrac{d\Phi}{dt}$	Faraday’s law
$\nabla \times \mathbf{H} = \mathbf{J} + \dfrac{\partial \mathbf{D}}{\partial t}$	$\oint_C \mathbf{H} \cdot d\boldsymbol{\ell} = I + \int_S \dfrac{\partial \mathbf{D}}{\partial t} \cdot d\mathbf{s}$	Ampère’s circuital law
$\nabla \cdot \mathbf{D} = \rho$	$\oint_S \mathbf{D} \cdot d\mathbf{s} = Q$	Gauss’s law
$\nabla \cdot \mathbf{B} = 0$	$\oint_S \mathbf{B} \cdot d\mathbf{s} = 0$	No isolated magnetic charge

Question : Are these equations sufficient ?

Maxwell's Equations

Question : Are the four Maxwell's equations independent ?
(Hint : Conservation of Charge → Continuity Equation)

$$\nabla \cdot (\nabla \times \mathbf{H}) = \nabla \cdot \mathbf{J} + \nabla \cdot \frac{\partial \mathbf{D}}{\partial t}$$

$$0 = \nabla \cdot \mathbf{J} + \frac{\partial}{\partial t}(\nabla \cdot \mathbf{D})$$

$$\nabla \cdot \mathbf{J} = -\frac{\partial \rho_v}{\partial t} \rightarrow 0 = -\frac{\partial \rho_v}{\partial t} + \frac{\partial}{\partial t}(\nabla \cdot \mathbf{D})$$

$$\nabla \cdot \mathbf{D} = \rho_v$$

ilarly, we can obtain last of the Maxwell's equations by taking the diverger
of the first Maxwell's equations.

Maxwell's Equations

Number of unknowns ? E, D, H, B (4 vectors, 12 scalar components)

Number of equations ? 2 vector equations, 6 scalar equations

Other two equations ? Constitutive vector equations ($D = \varepsilon E$, $B = \mu H$)
2 vector equations, 6 scalar equations

What about Lorentz force equation ? Is it possible to obtain the Lorentz force equation from Maxwell's Equations?

Complete EM Model

Maxwell's equations	Differential form	Integral form
(Faraday's law)	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$ (11.24)	$\oint_C \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi}{dt}$
(Ampere's law)	$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$ (11.25)	$\oint_C \mathbf{H} \cdot d\mathbf{l} = \int_s \left(\mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}\right) \cdot d\mathbf{s}$
(Gauss' law)	$\nabla \cdot \mathbf{D} = \rho$ (11.26)	$\oint_s \mathbf{D} \cdot d\mathbf{s} = Q$
(No monopoles)	$\nabla \cdot \mathbf{B} = 0$ (11.27)	$\oint_s \mathbf{B} \cdot d\mathbf{s} = 0$
Constitutive relations	$\mathbf{B} = \mu \mathbf{H}$	
	$\mathbf{D} = \varepsilon \mathbf{E}$	
The Lorentz force equation	$\mathbf{F} = Q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$	

Boundary Conditions

	Electric field	Magnetic field
Tangential components:	$E_{1t} = E_{2t}$	$H_{1t} - H_{2t} = \mathcal{J}$
	$\dfrac{D_{1t}}{\varepsilon_1} = \dfrac{D_{2t}}{\varepsilon_2}$	$\dfrac{B_{1t}}{\mu_1} = \dfrac{B_{2t}}{\mu_2} = \mathcal{J}$
Normal components:	$D_{1n} - D_{2n} = \rho_s$	$B_{1n} = B_{2n}$
	$\varepsilon_1 E_{1n} - \varepsilon_2 E_{2n} = \rho_s$	$\mu_1 H_{1n} = \mu_2 H_{2n}$

Time Harmonic Maxwell's Equations

Phasors $$\mathbf{E}(x, y, z, t) = \mathcal{R}e[\mathbf{E}(x, y, z)e^{j\omega t}],$$

Maxwell's equations	Differential form		Integral Form
	$\nabla \times \mathbf{E} = -j\omega\mathbf{B}$	(11.68)	$\oint_C \mathbf{E} \cdot d\mathbf{l} = -j\omega \int_s \mathbf{B} \cdot d\mathbf{s}$
	$\nabla \times \mathbf{H} = \mathbf{J} + j\omega\mathbf{D}$	(11.69)	$\oint_C \mathbf{H} \cdot d\mathbf{l} = \int_s (\mathbf{J} + j\omega\mathbf{D}) \cdot d\mathbf{s}$
	$\nabla \cdot \mathbf{D} = \rho$	(11.70)	$\int_s \mathbf{D} \cdot d\mathbf{s} = Q$
	$\nabla \cdot \mathbf{B} = 0$	(11.71)	$\int_s \mathbf{B} \cdot d\mathbf{s} = 0$
Constitutive relations	$\mathbf{B} = \mu\mathbf{H}$		
	$\mathbf{D} = \varepsilon\mathbf{E}$		
The Lorentz force equation	$\mathbf{F} = Q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$		

Thank You