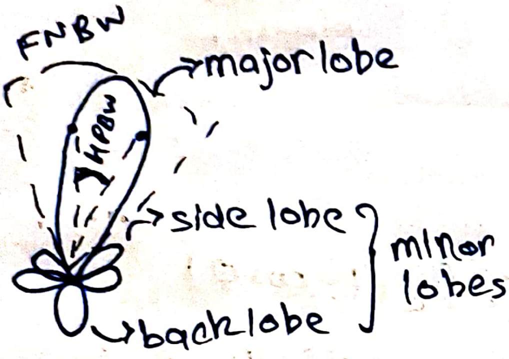
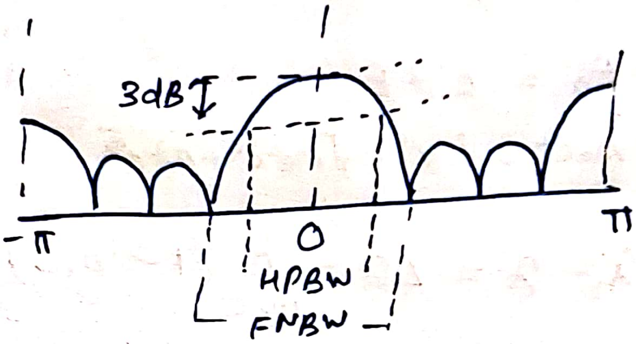
ECC-201
Nov 20, 2025 (self)

Amplitude Radiation Pattern

- Field Pattern: Plot of $|E|/H$ on linear scale
- Power Pattern: Plot of $|E|^2/|H|^2$ on linear or log scale

Radiation Pattern is usually normalized.

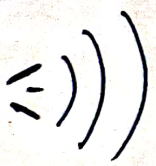




Isotropic ⇒ spherical radiation

Directional ⇒ Radiation in one direction: $U = \begin{cases} \cos^n\theta & \theta \in (0, \frac{\pi}{2}) \\ 0 & \theta \in (\frac{\pi}{2}, \pi) \end{cases}$ $\quad \phi \in (0, 2\pi)$

Omnidirectional ⇒ Radiation in a plane
$$U_0 = |\sin^n\theta| \quad \theta \in (0, \pi)$$



Reactive near field	Radiating near field (Fresnal)	Far-Field (Fraunhofer)

$$R_1 = 0.62\sqrt{\frac{D^3}{\lambda}} \qquad R_2 = \frac{2D^2}{\lambda}$$

$$\mathcal{E} = Re\{Ee^{j\omega t}\} = (Ee^{j\omega t} + E^* e^{-j\omega t})/2$$

$$\mathcal{H} = Re\{He^{j\omega t}\} = (He^{j\omega t} + H^* e^{-j\omega t})/2$$

$$W = \mathcal{E} \times \mathcal{H} = \frac{Re\{E \times H^*\}}{2} + \frac{Re\{E \times H e^{2j\omega t}\}}{2}$$

$$\langle W \rangle = \frac{1}{2} Re[E \times H^*]$$

$$P_{rad} = \oint_S W_{rad}\, ds \qquad ds = r^2 \sin\theta\, d\theta\, d\phi\, \hat{r}$$

for an isotropic antenna, $W_o = \dfrac{P_{rad}}{4\pi r^2}$

$$U = \frac{\text{Power}}{\text{Solid Angle}} = r^2 W_{rad}$$

$$\boxed{U = r^2 W_{rad}}$$

$U \rightarrow \frac{W}{Sr}$, $W_{rad} \rightarrow \frac{W}{m^2}$

$$P_{rad} = \int_0^{2\pi}\int_0^{\pi} U \sin\theta\, d\theta\, d\phi$$

$$d\Omega = \sin\theta\, d\theta\, d\Omega$$

$$U(\theta_h) = \frac{U_{max}}{2} \Rightarrow 2\theta_h : HPBW$$

Directivity : $D = \dfrac{U(\theta,\phi)}{U_o} = \dfrac{4\pi U(\theta,\phi)}{P_{rad}}$

U_o → radiation intensity of isotropic antenna

$$D(dB) = 10 \log_{10}(D)$$

$$D_o = \frac{4\pi U_{max}}{P_{rad}} = \frac{4\pi}{\Omega_A}$$

Ω_A ↳ beam solid angle

Directional Antenna:

$$D_0 \cong \frac{4\pi}{\Theta_{1r}\Theta_{2r}} = \frac{4\pi(180/\pi)^2}{\Theta_{1d}\Theta_{2d}} \quad \text{(Kraus)}$$

$$D_0 \cong \frac{32\ln(2)}{\Theta_{1r}^2 + \Theta_{2r}^2} = \frac{32\ln 2\,(180/\pi)^2}{\Theta_{1d}^2 + \Theta_{2d}^2} \quad \text{(Tai \& Pereira)}$$

$$= \frac{2D_1 D_2}{D_1 + D_2}$$

Omnidirectional Antenna:

$$D_0 \cong \frac{101}{\Theta_d - 0.0027\,\Theta_d^2} \quad \text{(McDonald)}$$

$$D_0 \cong -172.4 + 191\sqrt{0.818 + \frac{1}{\Theta_d}} \quad \text{(Pozar)}$$

Antenna Efficiency: $e_0 = e_r \underbrace{e_c e_d}_{e_{cd}} = \underbrace{(1-|\Gamma_{in}|^2)}_{e_r} e_{cd}$

$$\Gamma_{in} = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0}$$

$$|\Gamma_{in}| = \frac{VSWR - 1}{VSWR + 1}$$

Gain: $G = e_{cd} D$

Realized Gain: $G_{re} = e_r e_{cd} D$

$$G_0(dB) = 10\log_{10}(e_{cd} D_0) = 10\log_{10}(e_{cd}) + D_0(dB)$$

For broadband antenna, the bandwidth is the ratio of upper-to-lower frequencies of acceptable operation.

For narrowband antenna, the bandwidth is the percentage of frequency difference, upper-lower over center frequency of bandwidth.

Linearly Polarized Wave :

$$E(z) = E_x e^{-\gamma z} \hat{x} + E_y e^{-\gamma z} \hat{y}$$

$$E(z,t) = E_x e^{-\alpha z} \cos(\omega t - \beta z) \hat{x} + E_y e^{-\alpha z} \cos(\omega t - \beta z) \hat{y}$$

Elliptically $ circularly polarized :

$$E(z) = E_x e^{-j\beta z} \hat{x} - jE_y e^{-j\beta z} \hat{y}$$ → Right hand positive circular

$$E(z,t) = E_x \cos(\omega t - Kz) \hat{x} + E_y \cos(\omega t - Kz - \pi/2) \hat{y}$$

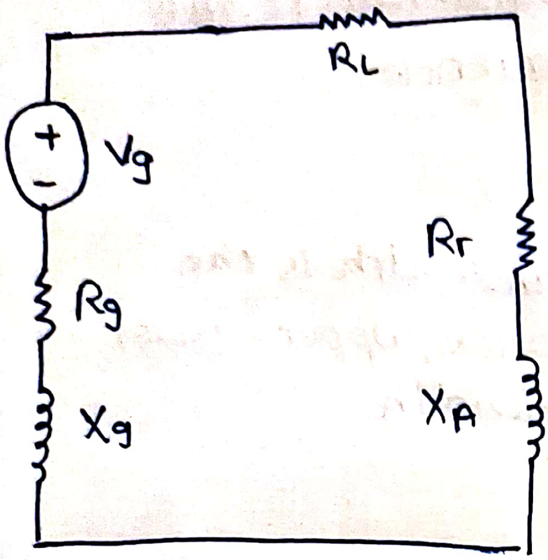
$$E(z) = E_x e^{-j\beta z} \hat{x} + jE_y e^{-j\beta z} \hat{y}$$ → left hand negatively circular

If $E_x = E_y \Rightarrow$ circular polarized

$E_x \neq E_y \Rightarrow$ elliptical polarized

Polarization Loss Factor : $PLF = |\hat{\rho}_w \cdot \hat{\rho}_a|^2 = |\cos\psi_p|^2$

$$-\infty < PLF(dB) = 10 \log_{10}(PLF) < 0$$



$R_A = R_r + R_L$

$R_r \downarrow$ Radiation Resistance

$R_L \searrow$ Loss Resistance

For max power transfer:

$$\left.\begin{aligned} R_A &= R_g \\ X_A &= -X_g \end{aligned}\right\} Z_g = Z_A^*$$

$$e_{cd} = \frac{R_r}{R_r + R_L}$$

Effective Aperture: $A_e = \frac{P_T}{W} = \frac{\lambda^2}{4\pi} D$

$$A_{em} = \frac{\lambda^2}{4\pi} D_0 \rightarrow \text{assuming no loss}$$

Friss Transmission:

$$\frac{P_r}{P_t} = G_{ot} G_{or} \underbrace{\left(\frac{\lambda}{4\pi R}\right)^2}_{\text{Free space loss}} \underbrace{|\hat{\rho}_t \cdot \hat{\rho}_r|^2}_{\text{polarization loss}}$$

$G_{ot} = e_{cdt} (1 - |\Gamma_t|^2) D_{ot}$

$G_{rr} = e_{cdr} (1 - |\Gamma_r|^2) D_{or}$

$R \rightarrow$ distance b/w antenna

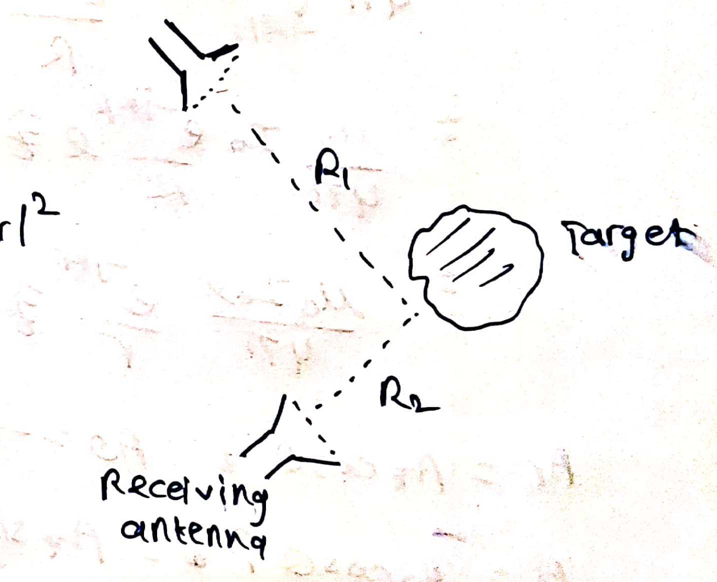
$$\text{Radar cross section } (\sigma) = \lim_{R\to\infty}\left[4\pi R^2 \frac{W_s}{W_i}\right]$$

$$\Rightarrow W_s = \frac{\sigma W_i}{4\pi R^2}$$

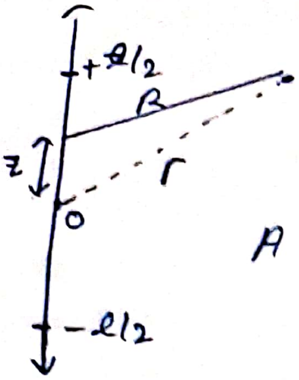
RCS is a function of:

- shape of target
- Material of target
- Polarization of incident wave
- Angle of incidence
- Observation angle

Radar range equation:



$$\frac{P_r}{P_t} = \frac{\sigma}{4\pi} G_{ot} G_{rt} \left(\frac{\lambda}{4\pi R_1 R_2}\right)^2 |\hat{\rho}_t \cdot \hat{\rho}_r|^2$$



$$A = \frac{\mu}{4\pi}\int_{-l/2}^{l/2} I_0 \hat{z} \cdot \frac{e^{-jKR}}{R}\, dz = \frac{\mu l I_0}{4\pi r} e^{-jKr}\,\hat{z}$$

$$A = A_z \cos\theta\, \hat{r} - A_z \sin\theta\, \hat{\theta}$$

$$K = \omega\sqrt{\mu\epsilon}$$

$$\eta = \sqrt{\frac{\mu}{\epsilon}}$$

$$H = \frac{\nabla \times A}{\mu} = j\,\frac{K l I_0}{4\pi r} e^{-jKr} \sin\theta \left(1 + \frac{1}{jKr}\right)\hat{\phi}$$

$$E = \frac{\nabla \times H}{j\omega\epsilon} = \frac{\eta l I_0}{2\pi r^2} e^{-jKr} \cos\theta \left(1 + \frac{1}{jKr}\right)\hat{r}$$

$$+ j\,\frac{\eta K I_0}{4\pi r} e^{-jKr} \sin\theta \left(1 + \frac{1}{jKr} - \frac{1}{K^2 r^2}\right)\hat{\theta}$$

For far-field, $E_r \cong 0$, $E_\theta \cong \eta H_\phi$

$$W_r = \frac{|E_\phi|^2}{2\eta} = \frac{\eta}{2}|H_\phi|^2$$

$$W = \frac{1}{2}(E \times H^*) = \underbrace{\frac{1}{2} E_r H_\phi^*\, \hat{\theta}}_{W_\theta} + \underbrace{\frac{1}{2} E_\theta H_\phi\, \hat{r}}_{W_r}$$

$$P_{rad} = \int_0^{2\pi}\int_0^{\pi} W_r \cdot r^2 \sin\theta\, d\theta\, d\Omega = \eta\left(\frac{\pi}{3}\right)\left|\frac{I_0 l}{\lambda}\right|^2 \left[1 - \frac{j}{(Kr)^3}\right]$$

$$P = P_{rad} + j2\omega(\tilde{W}_m - \tilde{W}_e)$$

↳ decreases rapidly with r
almost 0 for far-field

$$P_{rad} = \eta\left(\frac{\pi}{3}\right)\left|\frac{I_0 l}{\lambda}\right|^2$$

$$W_{rad} = \frac{\eta}{8}\left|\frac{I_0 l}{\lambda}\right|^2 \frac{\sin^2\theta}{r^2} = B_0 \frac{\sin^2\theta}{r^2}$$

$$U = r^2 W_{rad} = B_0 \sin^2\theta$$

$$D_0 = \frac{4\pi U_{max}}{P_{rad}} = \frac{4\pi \frac{\eta}{8}\left|\frac{I_0 l}{\lambda}\right|^2}{\eta\left(\frac{\pi}{3}\right)\left|\frac{I_0 l}{\lambda}\right|^2} = 1.5$$

$\Rightarrow \eta_0 = 120\pi$

$$P_{rad} = \frac{1}{2}|I_0|^2 R_r \Rightarrow R_r = \frac{2P_{rad}}{|I_0|^2} = 80\pi^2\left(\frac{l}{\lambda}\right)^2$$

$$A_{em} = \frac{\lambda^2}{4\pi} D_0 = \frac{3}{8\pi}\lambda^2$$

Finite length Dipole :

$$I(z) = I_0 \sin\left[K\left(\frac{l}{2} - z\right)\right]$$

$$I(z) = I_0 \sin\left[K\left(\frac{l}{2} + z\right)\right]$$

$$R \cong \underbrace{r - z\cos\theta}_{\text{phase}}$$
$$\cong \underbrace{r}_{\text{amplitude}}$$

$$A = \frac{\mu}{4\pi} \int_{-l/2}^{l/2} I(z) \frac{e^{-jKR}}{R} \, dz$$

$$= \frac{\mu}{4\pi r} e^{-jKr} \int_{-l/2}^{l/2} I(z) \, e^{jKz\cos\theta} \, dz \, \hat{z}$$

for far-field, $E_\theta \cong j\omega \sin\theta \, A_z$

$$E_\theta = \frac{j\eta K}{4\pi r} e^{-jKr} \sin\theta \int_{-l/2}^{l/2} I(z) \, e^{jKz\cos\theta} \, dz$$

$$E_\theta = j \frac{\eta I_0}{2\pi r} e^{-jKr} \left[\frac{\cos\left(\frac{Kl}{2}\cos\theta\right) - \cos\left(\frac{Kl}{2}\right)}{\sin\theta} \right]$$

$$H_\phi \cong \frac{E_\theta}{\eta}$$

$$W_{rad} = \underbrace{\frac{\eta |I_0|^2}{8\pi^2 r^2}}_{\frac{B_0}{r^2}} \left[\frac{\cos\left(\frac{Kl}{2}\cos\theta\right) - \cos\left(\frac{Kl}{2}\right)}{\sin\theta} \right]^2$$

$$U = B_0 \left[\frac{\cos\left(\frac{Kl}{2}\cos\theta\right) - \cos\left(\frac{Kl}{2}\right)}{\sin\theta} \right]^2$$

$l = \lambda/50 \Rightarrow HPBW \cong 90^\circ$

$l = \lambda/4 \Rightarrow HPBW = 87^\circ$

$l = \lambda/2 \Rightarrow HPBW = 78^\circ$

$l = \lambda \Rightarrow HPBW = 47.8^\circ$

As we increase the length of the dipole, the directivity increases.

<u>Half-wavelength dipole</u> ($l = \lambda/2$)

$$E_\theta = j\frac{\eta I_0}{2\pi r} e^{-jkr} \left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} \right] = \eta H_\phi$$

$$W_{rad} = \frac{|E_\theta|^2}{2\eta} \cong \frac{\eta |I_0|^2}{8\pi^2 r^2} \sin^3\theta$$

$$U = \frac{\eta |I_0|^2}{8\pi^2} \sin^3\theta$$

$$P_{rad} = \frac{\eta |I_0|^2}{8\pi^2} \underbrace{\int_0^{2\pi}\!\!\int \frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} \, d\theta \, d\Omega}_{2.435 = C_{in}(2\pi)} = \frac{\eta |I_0|^2}{8\pi} C_{in}(2\pi)$$

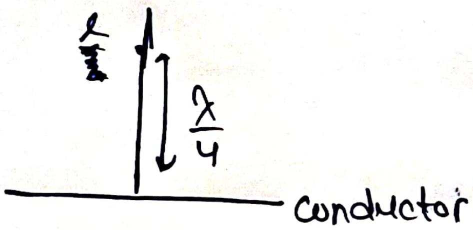
$$D_0 = \frac{4\pi U_{max}}{P_{rad}} = 4\pi \frac{\eta |I_0|^2}{8\pi^2} \cdot \frac{8\pi}{\eta |I_0|^2 \, C_{in}(2\pi)} = 1.643$$

$$A_{em} = \frac{\lambda^2}{4\pi} D_0 = 0.13\lambda^2$$

$$R_r = \frac{2P_{rad}}{|I_0|^2} = \frac{\eta}{4\pi} C_{in}(2\pi) = 73.05$$

to get this resistance, we typically get an impedance like $Z_{in} = 73.1 + j42.5\,\Omega$, to eliminate the imaginary part, we reduce antenna length to $0.47\lambda - 0.48\lambda$ so resonance occurs.

<u>Monopole</u> ($l = \lambda/4$)



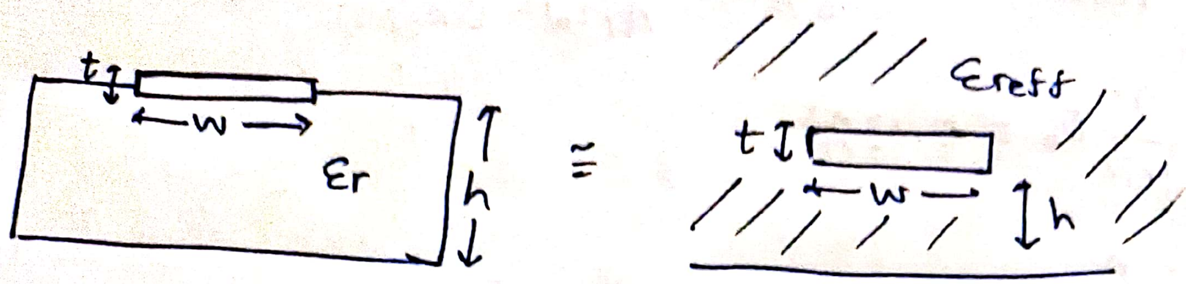
Comparing to half-wavelength dipole :

→ Power radiated is half

→ Radiation resistance is half

→ Directivity is doubled

Microstrip Patch Antenna



$$\varepsilon_{reff} = \left(\frac{\varepsilon_r + 1}{2}\right) + \left(\frac{\varepsilon_r - 1}{2}\right)\left(1 + 12\frac{h}{W}\right)^{-1/2}$$

specify : ε_r, f_r, h/λ_0

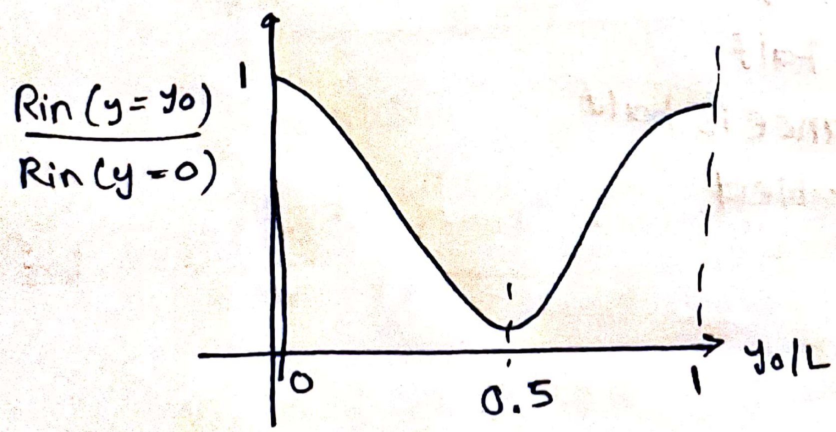
determine : W, L

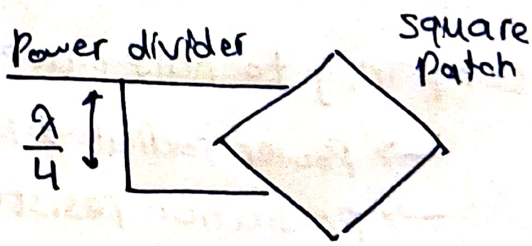
$$W = \frac{v_0}{2 f_r}\sqrt{\frac{2}{\varepsilon_r + 1}}$$

$v_0 = 3 \times 10^8$ m/s

$$L = \frac{v_0}{2 f_r \sqrt{\varepsilon_{reff}}} - 2\Delta L$$

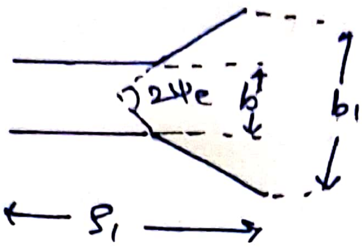
$$\frac{\Delta L}{h} = 0.412 \frac{(\varepsilon_{reff} + 0.3)\left(\frac{W}{h} + 0.264\right)}{(\varepsilon_{reff} - 0.258)\left(\frac{W}{h} + 0.8\right)}$$

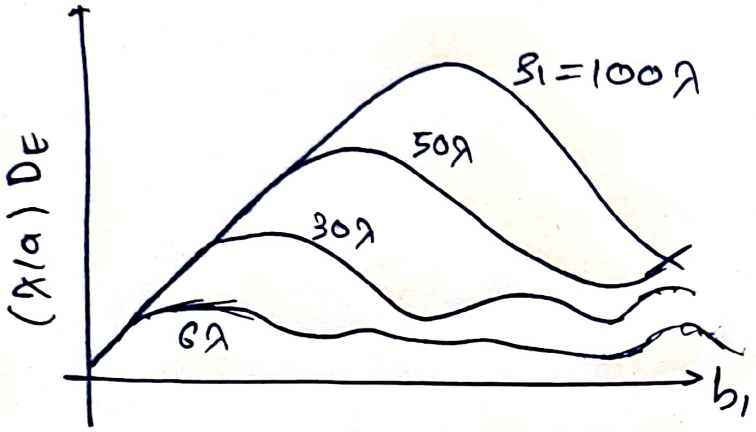




Horn Antenna

E-plane sectoral horn antenna :

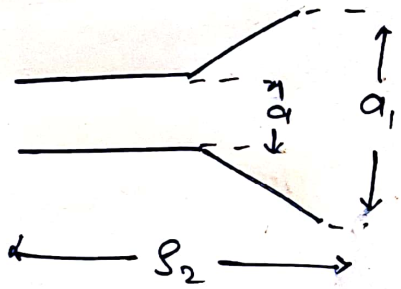




For maximum directivity,

$$b_1 = \sqrt{2\lambda\rho_1}$$

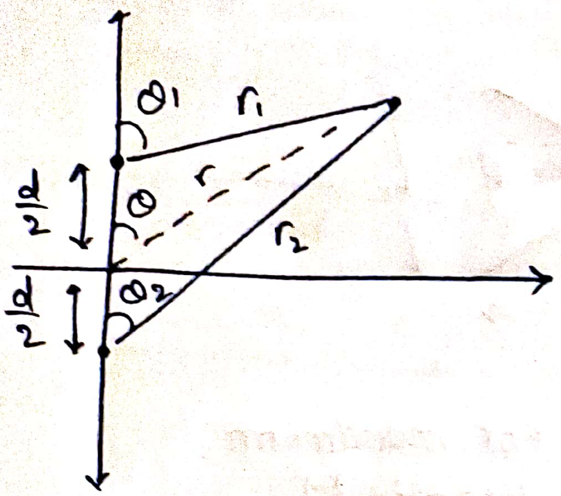
H-plane sectoral horn antenna :



max directivity

$$a_1 = \sqrt{3\lambda\rho_2}$$

Two Element Array



$$E_t = E_1 + E_2$$

$$= \frac{j\eta k I_0 l}{4\pi}\left(\frac{e^{-j(kr_1-\beta/2)}}{r_1}\cos\theta_1 + \frac{e^{-j(kr_2-\beta/2)}}{r_2}\cos\theta_2\right)$$

$$\left.\begin{aligned} r_1 &\cong r - \frac{d}{2}\cos\theta \\ r_2 &\cong r - \frac{d}{2}\cos\theta \end{aligned}\right\} \text{for phase variation}$$

$r_1 \cong r_2 \cong r \rightarrow$ for amplitude variation

$\theta_1 \cong \theta_2 \cong \theta$

$$E_t = \underbrace{j\frac{\eta k I_0 l}{4\pi}|\cos\theta|}_{\text{single element}}\ \underbrace{\left(2\cos\left[\frac{kd\cos\theta + \beta}{2}\right]\right)}_{AF}$$

$$(AF)_n = \cos\left(\frac{kd\cos\theta + \beta}{2}\right)$$

$$E_{tn} = C\,|\cos\theta|\cos\left(\frac{kd\cos\theta + \beta}{2}\right)$$

for $d = \lambda/4 \Rightarrow (AF)_n = \cos\left(\frac{\pi}{4}\cos\theta + \frac{\beta}{2}\right)$

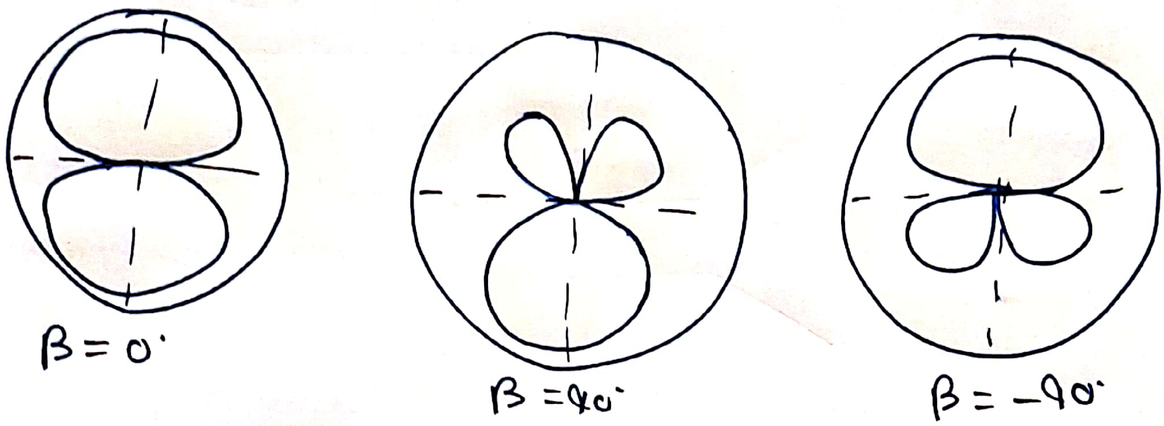
$$E_{tn} = |\cos\theta| \left|\cos\left(\frac{\pi}{4}\cos\theta + \frac{\beta}{2}\right)\right|$$

$\beta = 0^\circ \Rightarrow 90^\circ$

$\beta = 90^\circ \Rightarrow 90^\circ, 0^\circ$

$\beta = -90^\circ \Rightarrow 90^\circ, 180^\circ$

Null points



N element Array

$$E_t = E(\text{single}) \cdot AF$$

$$AF = \sum_{n=1}^{N} e^{j(n-1)\psi} = e^{j\frac{\psi}{2}(N-1)} \frac{\sin\left(\frac{N\psi}{2}\right)}{\sin\left(\frac{\psi}{2}\right)}$$

$$\psi = Kd\cos\theta + \beta$$

Nulls $\Rightarrow \frac{N\psi}{2} = n\pi \quad (n \neq 0, N, 2N \cdots)$

$$\theta_n = \cos^{-1}\left[\frac{1}{Kd}\left(-\beta \pm \frac{2n}{N}\pi\right)\right]$$

Maxima $\Rightarrow \frac{\psi}{2} = m\pi$

$$\theta_n = \cos^{-1}\left[\frac{1}{Kd}\left(-\beta \pm 2m\pi\right)\right]$$

$$|AF|_n = \left| \frac{1}{N} \frac{\sin(N\psi/2)}{\sin(\psi/2)} \right|$$

Half Power $\Rightarrow \frac{\sin(N\psi/2)}{N\psi/2} = 0.707 \Rightarrow \frac{N\psi}{2} = \pm 1.391$

$$\theta_h = \cos^{-1}\left[\frac{1}{Kd}\left(-\beta \pm \frac{2.782}{N}\right)\right]$$

for maxima $\Rightarrow \frac{\psi}{2} = m\pi \quad (m=0) \Rightarrow \psi = 0$

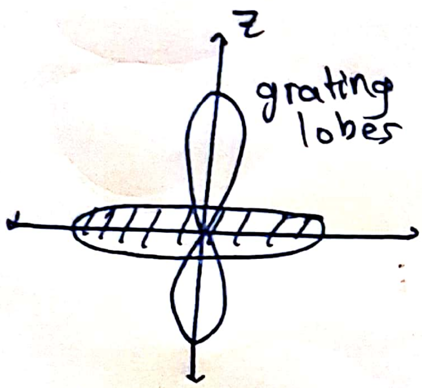
$$\psi \big|_{\theta=\theta_0} = Kd\cos\theta_0 + \beta = 0$$

$$\Rightarrow \quad \underline{\beta = -Kd\cos\theta_0}$$

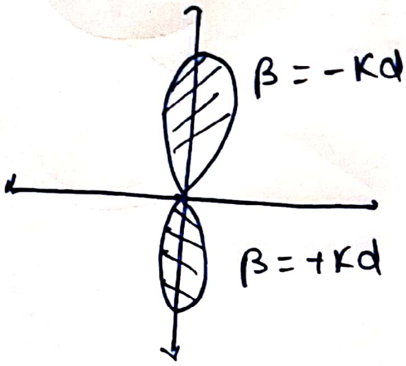
Broadband $\Rightarrow \theta_0 = \pm 90^\circ \quad \Rightarrow \beta = 0$

End-fire $\Rightarrow \theta_0 = 0^\circ, 180^\circ \Rightarrow \beta = \pm Kd$

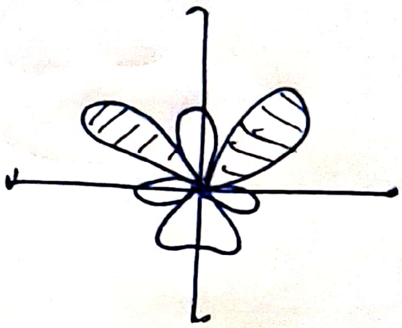
Scanning $\Rightarrow \theta_0 = \theta_0 \quad \Rightarrow \beta = -Kd\cos\theta_0$



Broadband



End-fire



Scanning

$d < \lambda \Rightarrow$ No grating lobes

$d \geq \lambda \Rightarrow$ grating lobes