Lecture - 32 31/10/2023.

Free particle ⇒ $V(x) = 0$

$$\frac{-\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} = E\psi(x)$$

$$\Rightarrow \left(\frac{d^2}{dx^2} + \frac{2mE}{\hbar^2}\right)\psi(x) = 0$$

$$\boxed{\frac{2mE}{\hbar^2} = k^2}$$

k ↓ wave no.

$$\Rightarrow \left(\frac{d^2}{dx^2} + k^2\right)\psi = 0$$

Sol^n ⇒

$$\psi(x) = A\, e^{ikx}$$

$$\psi_+(x,t) = A_+ e^{i(kx-\omega t)}$$

$$\psi_-(x,t) = A_- e^{-i(kx-\omega t)}.$$

well define momentum $p = \hbar k$

" " energy $E = \frac{\hbar^2 k^2}{2m}$

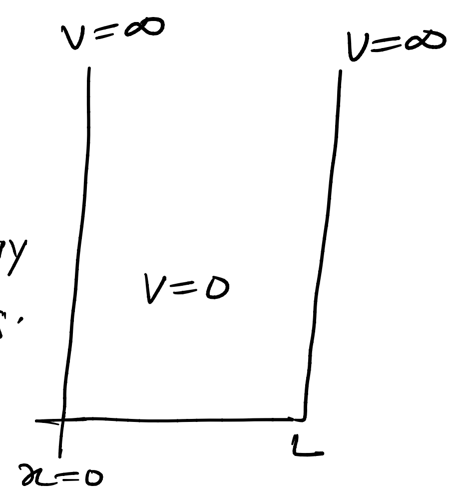
$\Delta x \to \infty$

$\Delta t \to \infty$

ψ_+ & ψ_- are not normalizable.

⇒ Wave packet $\psi(x,t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} \phi(k)\, e^{i(kx-\omega t)}\, dk.$

Particle in box ⇒



* A particle is trapped in a box with infinitely hard walls.
* hard wall :- The particle does not lose energy when it collides with the walls.

$$V(x) = \infty \quad \text{for } x<0 \ \& \ x>a$$
$$= 0 \quad \text{for } 0 \le x \le a$$

Sc eq^n ⇒

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + \cancel{V\psi} = E\psi$$

$$\Rightarrow \quad \frac{d^2\psi}{dx^2} + \frac{2m}{\hbar}E\psi = 0$$

$$\boxed{\frac{2m}{\hbar^2}E = k}$$

$$\Rightarrow \quad \left(\frac{d^2}{dx^2} + k^2\right)\psi = 0.$$

Sol^n ⇒

$$\psi(x) = A\sin kx + B\cos kx.$$

$$\psi(x) = A\sin\left(\frac{\sqrt{2mE}}{\hbar}x\right) + B\cos\left(\frac{\sqrt{2mE}}{\hbar}x\right) \quad \text{—— (1)}$$

B.C ⇒ at $x=0$, $\psi=0$

Therefore, $B=0$

$$\psi(x) = A\sin\frac{\sqrt{2mE}}{\hbar}x \quad \text{—— (1)}$$

B.C ⇒ at $x=L$, $\psi=0$.

$$A\sin\left(\frac{\sqrt{2mE}}{\hbar}L\right) = 0$$

$$\therefore \quad \frac{\sqrt{2mE}}{\hbar}L = n\pi$$

$n = 1, 2, 3 \ldots\ldots$

$n \neq 0$ because if $n=0$ then $\psi=0$ everywhere which indicates that the particle does not exist in the well anywhere.

$$2mE = \frac{n^2\pi^2\hbar^2}{L^2}$$

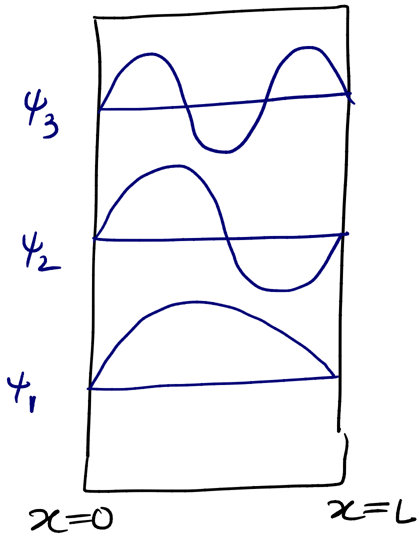
$$\boxed{E = \frac{n^2h^2}{8mL^2}}$$

The energy spectrum in an infinite step pot is discrete.

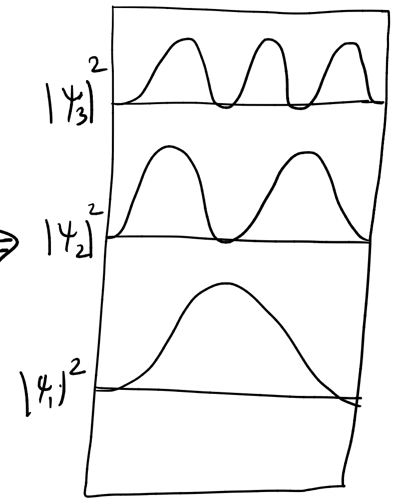
$$\psi(x) = A\sin\left(\frac{\sqrt{2mE}}{\hbar}x\right)$$

$$\psi(x) = A\sin\left(\frac{n\pi x}{L}\right) \qquad A = \sqrt{\frac{2}{L}} \text{ (Lecture 27)}$$

$$\boxed{\psi(x) = \sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L}}$$



$\Rightarrow$



& $\langle x \rangle = L/2$

[lecture - 30].

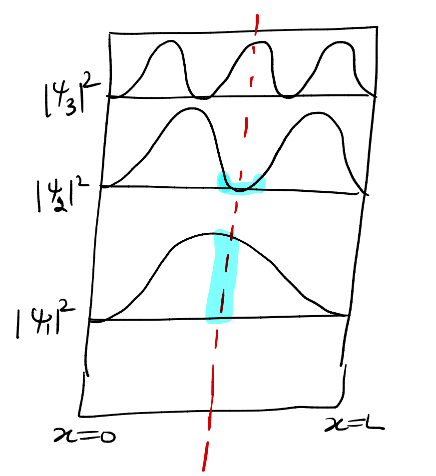
Q:⇒ Find the probability that a particle trapped in a box 'L' wide can be found b/w 0.45L & 0.55L for the ground state & Ist excited state.

A ⇒ $$P_{x_1,x_2 \text{ ground state}} = \int_{x_1}^{x_2} \psi^*_{\text{ground}}\,\psi_{\text{ground}}\,dx$$

$$= \frac{2}{L}\int_{.45}^{.55} \sin^2\frac{\pi x}{L}\,dx$$

$$= .198$$

$$= 19.8\%$$

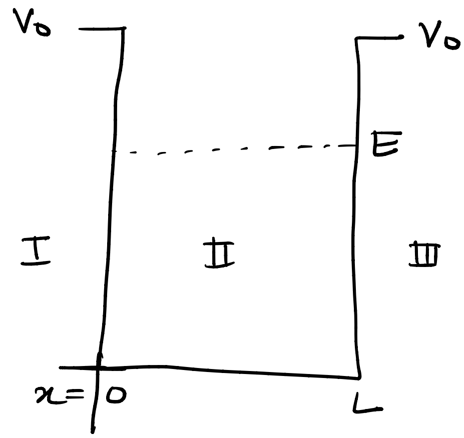


$$P_{x_1 \& x_2 \text{ I}^{st} \text{ excited state}} = \frac{2}{L}\int_{.45}^{.55} \sin^2\frac{2\pi x}{2}\,dx$$

$$= .0065 = .65\%$$

Finite Potential Well:→

Sc. eqⁿ :→ $$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V\psi = E\psi$$



Ist, IIIrd. $$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E-V_0)\psi = 0$$

$$\frac{d^2\psi}{dx^2} - \frac{2m}{\hbar^2}(V_0-E)\psi = 0$$

IInd $$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}E\psi = 0$$

define $$k_1 = \frac{\sqrt{2m(V_0-E)}}{\hbar}$$

$$k_2 = \frac{\sqrt{2mE}}{\hbar}$$

$$\left(\frac{d^2}{dx^2} - k_1^2\right)\psi_1(x) = 0 \qquad x<0$$

$$\left(\frac{d^2}{dx^2} + k_2^2\right)\psi_2(x) = 0 \qquad 0<x<L$$

$$\left(\frac{d^2}{dx^2} - k_1^2\right)\psi_3(x) = 0 \qquad x>L$$

$$\psi_I = A_1 e^{k_1 x} + A_2 e^{-k_1 x} \quad \text{——— ①}$$

$$\psi_2 = B_1 \sin k_2 x + B_2 \cos k_2 x \quad \text{——— ②}$$

$$\psi_3 = C_1 e^{k_1 x} + C_2 e^{-k_1 x} \quad \text{——— ③}$$

BC:→ as $x \to -\infty$, $\psi(x) \to 0$ } $\therefore A_2 \Rightarrow 0$

as $x \to +\infty$, $\psi(x) = 0$ } $\therefore C_1 \Rightarrow 0$

$$\psi_I = A_1 e^{k_1 x} \quad \text{——— ①}$$

$$\psi_2 = B_1 \sin k_2 x + B_2 \cos k_2 x \quad \text{——— ②}$$

$$\psi_3 = C_2 e^{-k_1 x} \quad \text{——— ③}$$