ECC 203 : Electromagnetics and Radiating Systems

Antenna Array 1 : Uniform 2 Element Array

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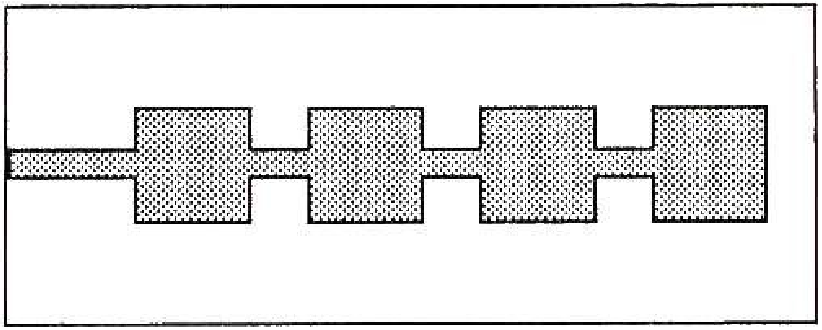
- A single element produces a wide radiation pattern and low directivity.
- There is a demand for very directive antennas (for long distance communications, for example).
- One way is to increase the size of the antenna.
- Having an array is the other way.
- In an array, fields from individual elements interfere constructively in some directions and cancel in some others

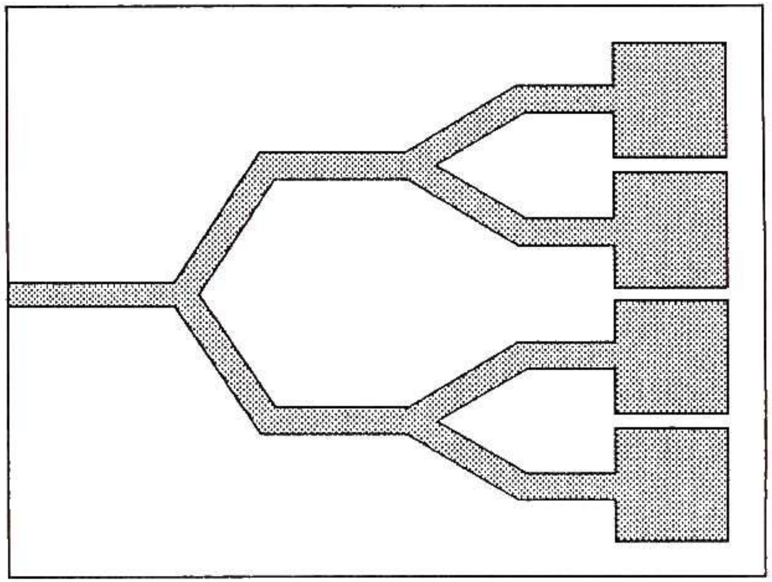
VLA (Very Large Array Antenna)

Individual element

<u>There are 5 controls that affect the overall pattern of an antenna array:</u>

1. **Geometry (linear, circular, rectangular array).**
2. **Distance of separation between elements.**
3. **Amplitude of current in each element.**
4. **Phase of individual elements.**
5. **Pattern of individual element.**





Two Infinitesimal Dipoles

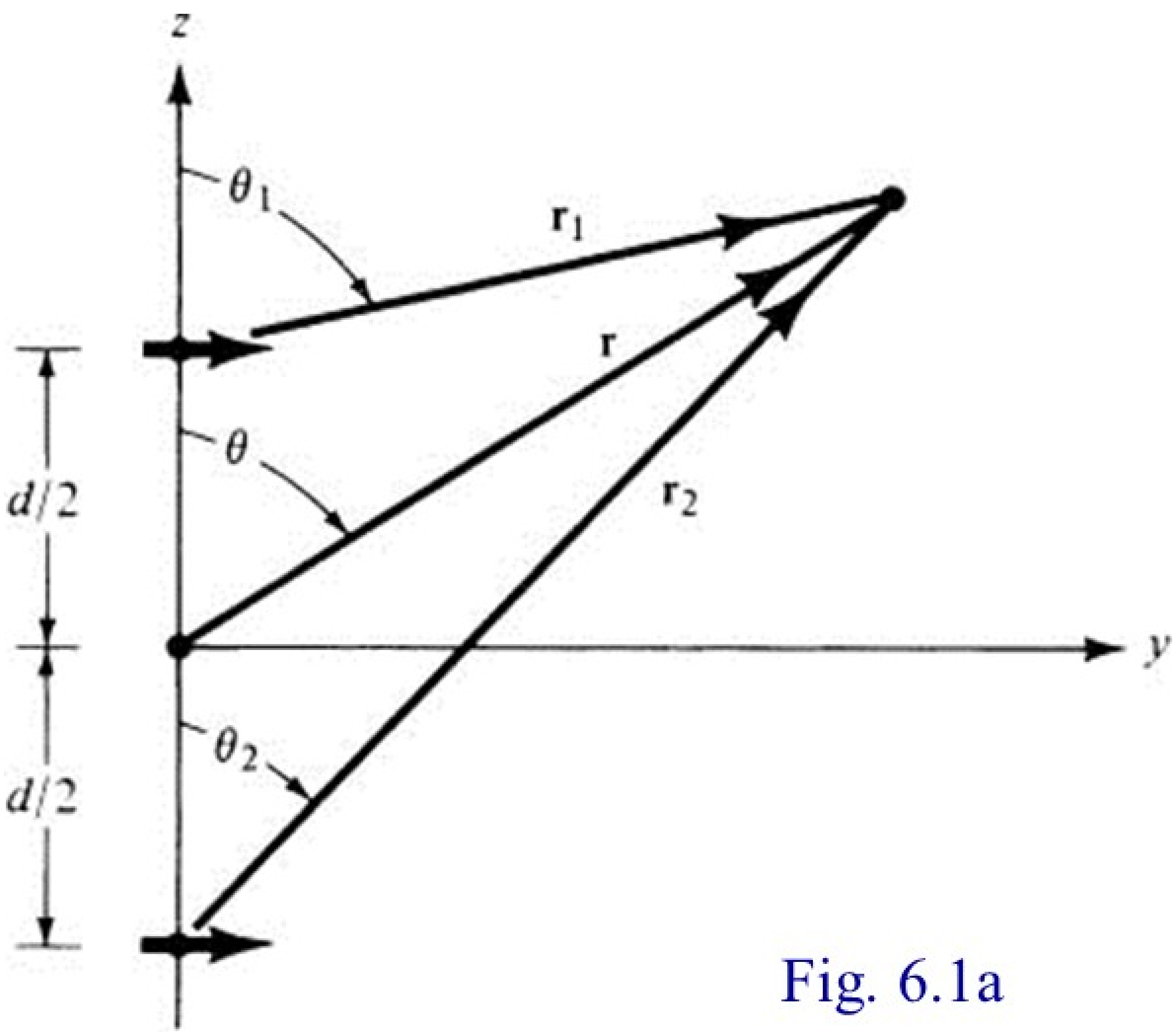
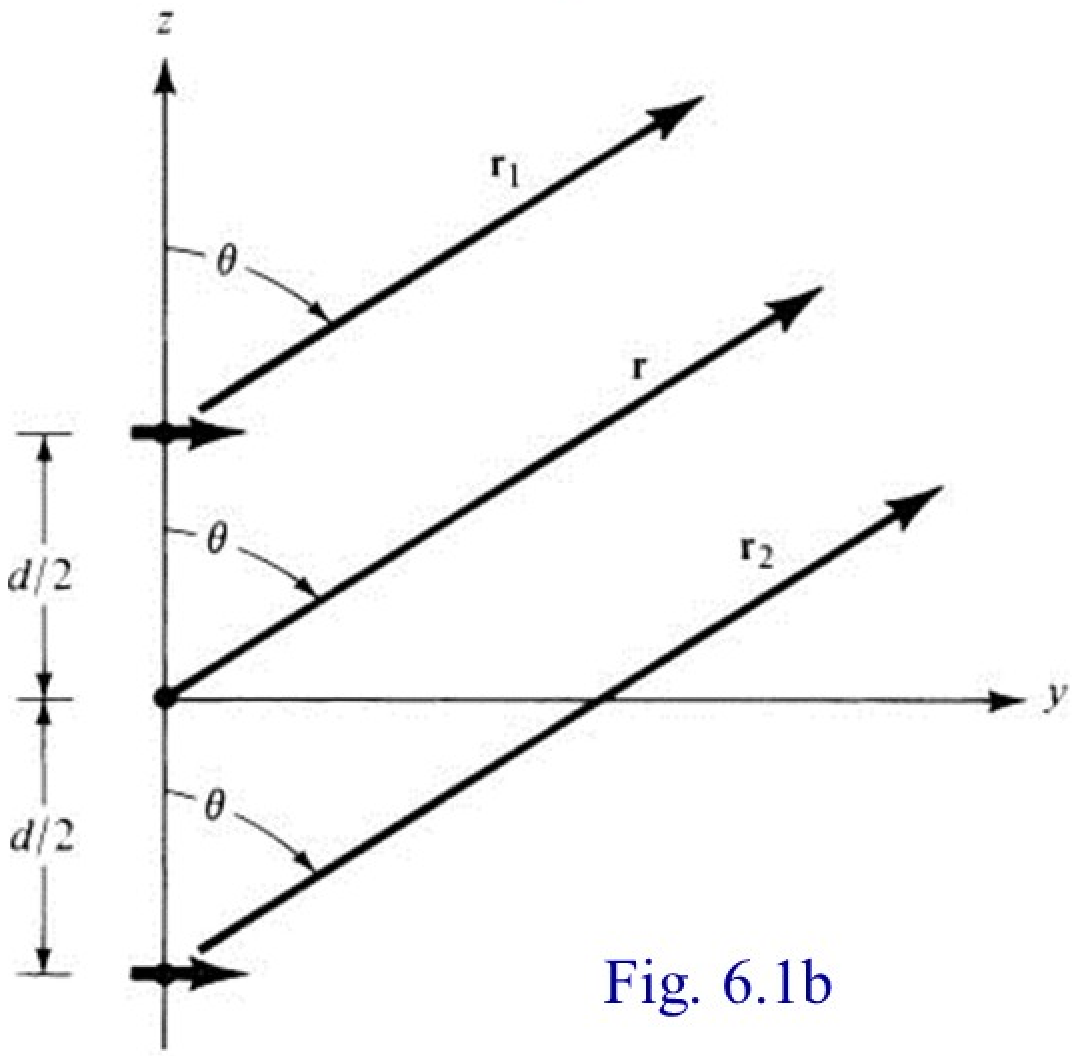


Fig. 6.1a

(a) Two infinitesimal dipoles

Geometry of a Two-Element Array Positioned Along the Z-Axis



(b) Far–field observations

Total Field of 2-Element Array

$$\underline{E}_t = \underline{E}_1 + \underline{E}_2 = \hat{a}_\theta j\eta \frac{kI_o \ell}{4\pi}$$

$$\cdot \left\{ \frac{e^{-j(kr_1 - \beta/2)}}{r_1} \left| \cos\theta_1 \right| + \frac{e^{-j(kr_2 + \beta/2)}}{r_2} \left| \cos\theta_2 \right| \right\} \quad (6\text{-}1)$$

$$\left. \begin{aligned} r_1 &\cong r - \frac{d}{2}\cos\theta \\ r_2 &\cong r + \frac{d}{2}\cos\theta \end{aligned} \right\} \text{ for phase variations} \quad (6\text{-}2b)$$

$$r_1 \cong r_2 \cong r \quad \text{for amplitude variations} \quad (6\text{-}2c)$$

$$\theta_1 \cong \theta_2 \cong \theta \quad (6\text{-}2a)$$

$$\underline{E}_t = \hat{a}_\theta j\eta \frac{kI_0 l e^{-jkr}}{4\pi r}\left|\cos\theta\right|\left\{e^{+j(kd\cos\theta+\beta)/2} + e^{-j(kd\cos\theta+\beta)/2}\right\}$$

(6-3)

$$\underbrace{\underline{E}_t = \underbrace{\hat{a}_\theta j\eta \frac{kI_0 l e^{-jkr}}{4\pi r}\left|\cos\theta\right|}_{\text{Single Element}}\underbrace{\left\{2\cos\left[\frac{1}{2}(kd\cos\theta+\beta)\right]\right\}}_{\text{Array Factor}}}_{\text{Total Field}}$$

Array Pattern Multiplication for Identical Elements

$$\underbrace{E(\text{total})}_{\text{ET}} = \underbrace{E(\text{single element})}_{\text{Element Factor (EF)}} \cdot \underbrace{\text{Array Factor}}_{\text{AF}} \tag{6-5}$$

$$\mathrm{AF} = 2\cos\left[\frac{1}{2}(kd\cos\theta + \beta)\right] \qquad (6\text{-}4)$$

$$(\mathrm{AF})_n = \cos\left[\frac{1}{2}(kd\cos\theta + \beta)\right] \qquad (6\text{-}4a)$$

$$\boxed{E_{tn} = C\left|\cos\theta\right|\cos\left[\frac{1}{2}(kd\cos\theta + \beta)\right]}$$

a. $d = \lambda/4,\ \beta = 0$

b. $d = \lambda/4,\ \beta = +\dfrac{\pi}{2}$

c. $d = \lambda/4,\ \beta = -\dfrac{\pi}{2}$

For $d = \lambda / 4$

$$(AF)_n = \cos\left[\frac{1}{2}(kd\cos\theta + \beta)\right]$$

$$= \cos\left[\frac{1}{2}\left(\frac{2\pi}{\lambda}\frac{\lambda}{4}\cos\theta + \beta\right)\right]$$

$$= \cos\left[\frac{1}{2}\left(\frac{\pi}{2}\cos\theta + \beta\right)\right]$$

$$(AF)_n = \cos\left[\frac{\pi}{4}\cos\theta + \frac{\beta}{2}\right]$$

a. $d = \lambda/4, \beta = 0$:

$$E_{tn} = |\cos\theta| \left| \cos\left(\frac{\pi}{4}\cos\theta\right) \right|_{\theta=\theta_n} = 0$$

$$1.\ |\cos(\theta_n)| = 0 \quad \Rightarrow \theta_n = \cos^{-1}(0) = \frac{\pi}{2}, \cancel{\frac{3\pi}{2}}, \cdots$$

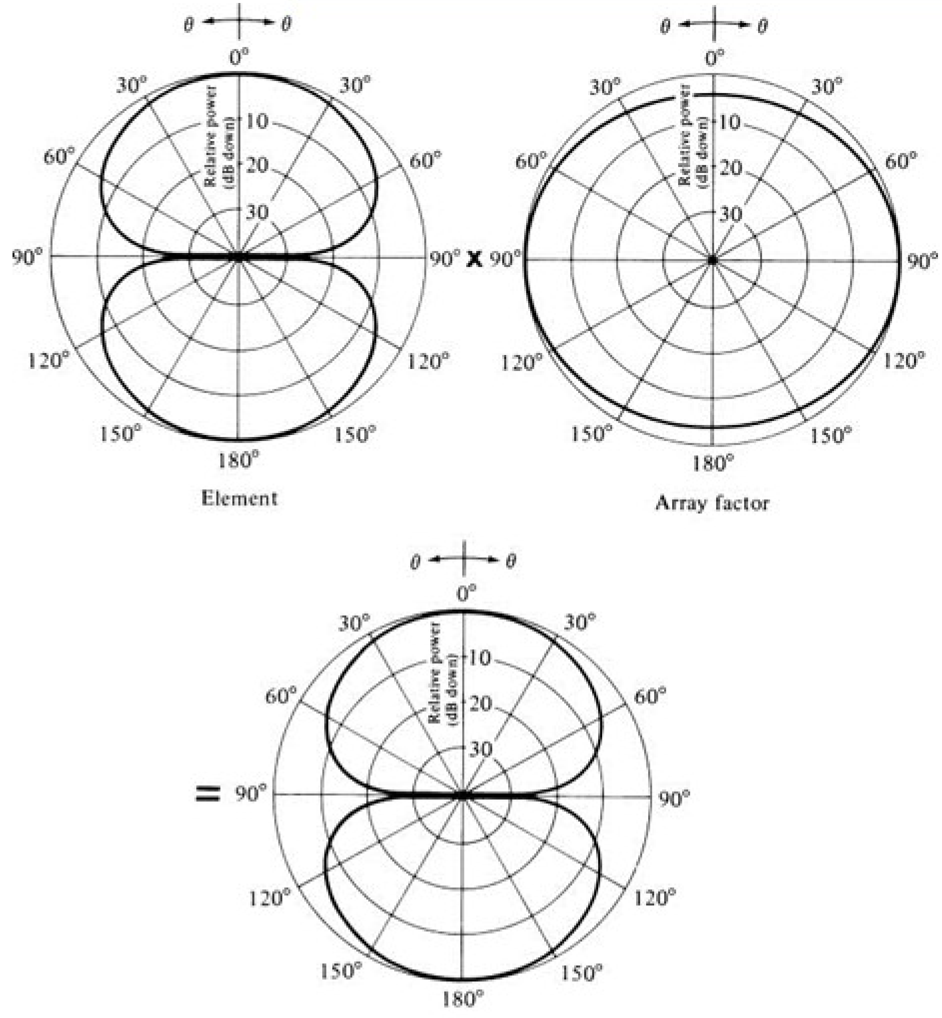
$$2.\ \left|\cos\left(\frac{\pi}{4}\cos\theta_n\right)\right| = 0 \quad \Rightarrow \frac{\pi}{4}\cos\theta_n = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \cdots$$

$$\frac{\pi}{4}\cos\theta_n = +\frac{\pi}{2} \quad \Rightarrow \theta_n = \cos^{-1}(2) = \text{does not exist}$$

$$\frac{\pi}{4}\cos\theta_n = -\frac{\pi}{2} \quad \Rightarrow \theta_n = \cos^{-1}(-2) = \text{does not exist}$$

$$\therefore \quad \theta_n = \frac{\pi}{2} = 90^\circ$$

Element, Array Factor, and Total Field Patterns of a 2-element Array of Infinitesimal Horizontal Dipoles With Identical Phase Excitation



$$\beta = 0,\ d = \lambda / 4$$

$$\Rightarrow \theta_n = 90^o$$

Fig. 6.3

b. $d = \lambda/4, \beta = +\pi/2:$

$$E_{tn} = |\cos(\theta)| \left| \cos\left(\frac{\pi}{4}\cos\theta + \frac{\pi}{4} \right) \right|_{\theta=\theta_n} = 0$$

1. $|\cos(\theta_n)| = 0 \quad \Rightarrow \theta_n = \frac{\pi}{2}, \cancel{\frac{3\pi}{2}}, \cdots$

2. $\left| \cos\left(\frac{\pi}{4}\cos\theta_n + \frac{\pi}{4} \right) \right| = 0 \quad \Rightarrow \frac{\pi}{4}\cos\theta_n + \frac{\pi}{4} = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \cdots$

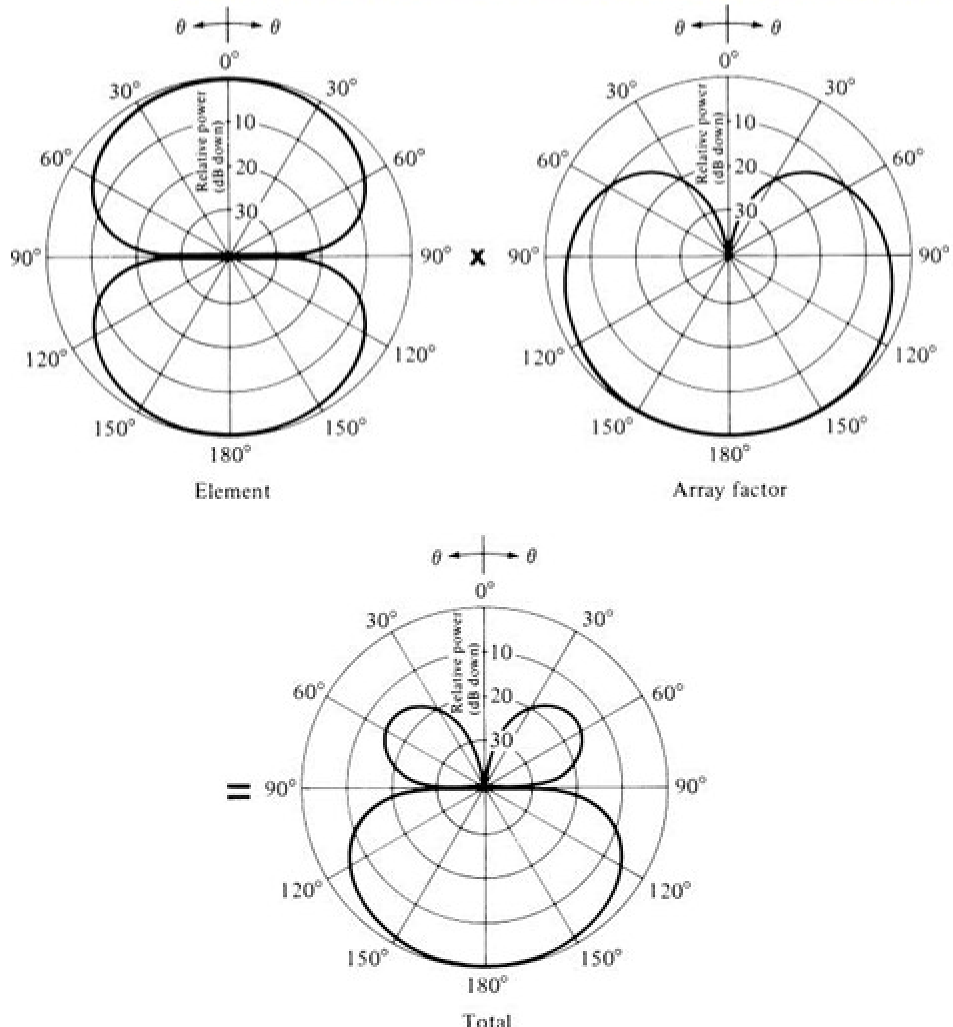
A. $+\pi/2: \quad \theta_n = \cos^{-1}(1) = 0, \cancel{2\pi}, \ldots$

B. $-\pi/2: \quad \theta_n = \cos^{-1}(-3) =$ does not exist

C. $3\pi/2: \quad \theta_n = \cos^{-1}(5) =$ does not exist

$$\therefore \quad \theta_n = 90^\circ, 0^\circ$$

Pattern multiplication of Element, Array Factor, and Total Array Patterns of a 2-element Array of Infinitesimal Horizontal Dipoles



$\beta = +90^o,\ d = \lambda/4$

$\Rightarrow \theta_n = 0^o, 90^0$

Fig. 6.4a

c. $d = \lambda/4, \beta = -\pi/2$:

$$E_{tn} = |\cos\theta| \left| \cos\left(\frac{\pi}{4}\cos\theta - \frac{\pi}{4} \right) \right|_{\theta=\theta_n} = 0$$

1. $|\cos(\theta_n)| = 0 \quad \Rightarrow \theta_n = \frac{\pi}{2}, \cancel{\frac{3\pi}{2}}, \cdots$

2. $\left|\cos\left(\frac{\pi}{4}\cos\theta_n - \frac{\pi}{4}\right)\right| = 0 \quad \Rightarrow \frac{\pi}{4}\cos\theta_n - \frac{\pi}{4} = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \cdots$

A. $+\pi/2$: $\theta_n = \cos^{-1}(3) =$ does not exist

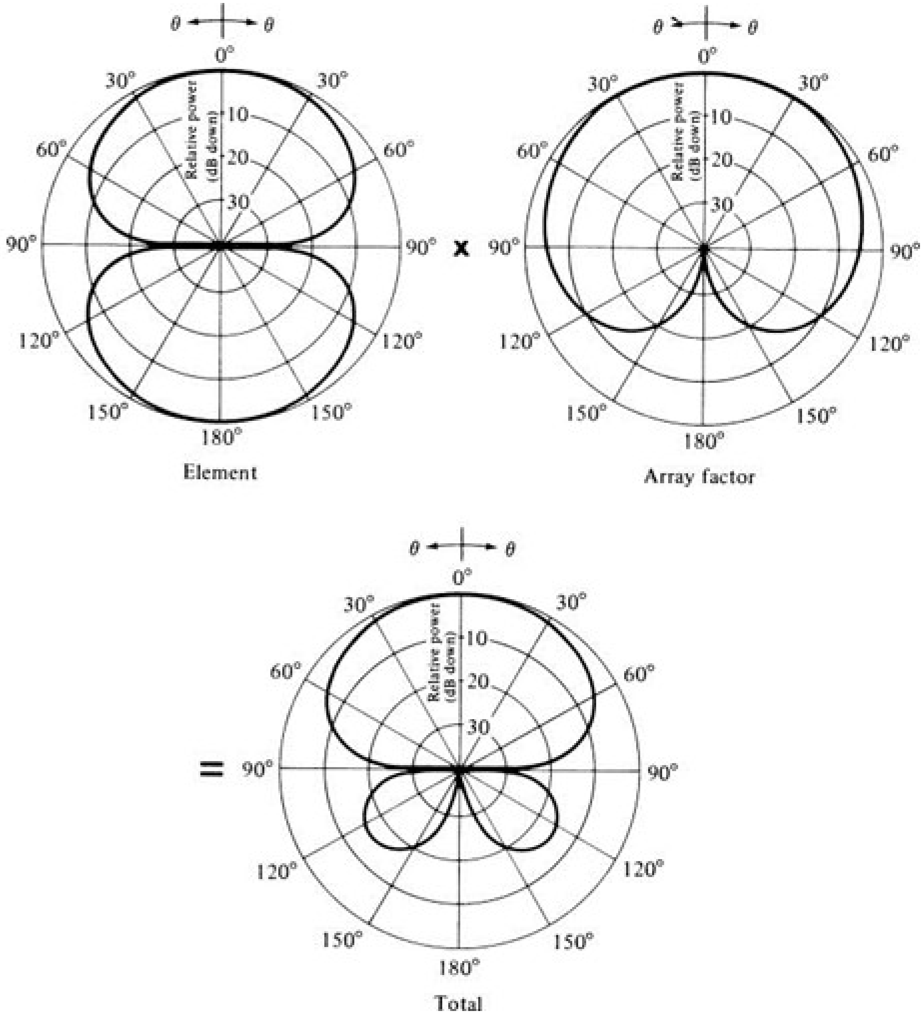
B. $-\pi/2$: $\theta_n = \cos^{-1}(-1) = 180^\circ$

C. $3\pi/2$: $\theta_n = \cos^{-1}(7) =$ does not exist

D. $-3\pi/2$: $\theta_n = \cos^{-1}(-5) =$ does not exist

$$\therefore \quad \theta_n = 90^\circ, 180^\circ$$

Pattern multiplication of Element, Array Factor, and Total Array Patterns of a 2-element Array of Infinitesimal Horizontal Dipoles



$$\underline{\beta = -90^{o},\ d = \lambda / 4}$$

$$\Rightarrow \theta_n = 0^{o}, 180^{0}$$

Fig. 6.4b

Thank You