DEPARTMENT OF MATHEMATICS, IIT ROORKEE
MAB-103: Numerical Methods

Assignment-1 Error Analysis Session-2025-26

1. Round off the following numbers to 4 significant digits:
   2.34567, 2.3455, 2.34449, 1.47383, 1473.27, 0.00276, 0.0027657.

2. If the numbers are correct to the last digit, find the correct significant figures for each problem:
   $2.33 \times 6.085 \times 2.1$, $(4.52 \times 10^{-4}) \div (3.980 \times 10^{-6})$, 3.10×4.520.

3. Calculate the value of $\sqrt{626} - \sqrt{625}$ correct to 4 significant figures.

4. Round off the numbers 865250 and 37.46235 to four significant figures and compute absolute, relative and percentage error in each case.

5. Find $0.348 + 0.1834 + 435.4 + 235.2 + 11.75 + 9.27 + 0.0849 + 0.0214 + 0.000354$, assume that all their digits being correct. Find the maximum absolute error and maximum relative error in the sum.

6. Round off the numbers 925150 and 26.36125 to four significant figures and compute absolute, relative and percentage errors.

7. If $x = 5.675, y = 4.737$ and $z = 4.373$, calculate $x(y - z)$ and $xy - xz$, to four significant figures, which one is more accurate.

8. Compute $y = x^3 \sin(x)$ for $x = \sqrt{2}(\approx 1.414)$. Determine the maximum absolute error and maximum relative error in y ($\sin(x)$ to be calculated in radians)).

9. If $u = \frac{4xy^2}{z^3}$ and error in x, y, z be 0.001, compute the maximum relative error in u when $x = y = z = 1$.

10. Find $u = log_e(x_1 + x_2^2), x_1 = 0.97, x_2 = 1.132$. Obtain the maximum absolute error and maximum relative error in u.

11. The function $y = k_1 \cos(x) + k_2 \ln(x)$ is said to be evaluated for $x = 1.36$. The values $k_1 = 2.0$ and $k_2 = 3.0 \times 10^{-3}$ are correct only to the number of significant digits shown. Find
    (i) the absolute error and the relative error in y,
    (ii) the maximum absolute error and the maximum relative error in y.

12. The derivative of a function $f(x)$ at a particular value of x can be approximately calculated by $f'(x) \approx \frac{f(x+h)-f(x)}{h}$. For $f(x) = 7e^{0.5x}$, find the percentage error in calculating $f'(2)$ using values from $h = 0.3$ and $h = 0.15$.

13. If one chooses 6 terms of the Maclaurin series for e^x to calculate $e^{0.7}$, how many significant digits can you trust in the solution? Find your answer without knowing or using the exact answer of $e^{0.7}$.

14. In the Taylor's series expansion of e^x, how many terms it would require to get an approximation of e^1 within a magnitude of true error of less than 10^{-6}.

15. Find the condition number of the following functions:

    (a) $f(x) = \sqrt{x}$, for all $x \in [0, \infty)$

    (b) $f(x) = 10/(1 - x^2)$, for all $x \in \mathbb{R}$.

Answers:

(1) 2.346, 2.346, 2.344, 1.474, 1473, 0.002760, 0.002766.

(2) 2 (30),3 (114),3 (14.0).

(3) 0.01999.

(4) 8.652×10^5, 50, 0.5779×10^{-5}, 0.5779×10^{-3};
37.46, 0.2350×10^{-1}, 0.6273×10^{-3}, 0.6273×10^{-1}.

(5) 692.3, 0.1, 0.2×10^{-3}

(6) 925200, 50, 5.405×10^{-5}, 5.405×10^{-3}
26.36, 0.00125, 4.742×10^{-5}, 4.742×10^{-3}.

(7) 2.066, 2.06, former is more accurate.

(8) 0.3186×10^{-2},0.001141

(9) 0.006.

(10) 0.24×10^{-2}, 0.30×10^{-2}

(11) (i) 0.0022, 0.0011 (ii) 0.022, 0.011.

(12) 4.3 %.

(13) 3.

(14) 11^{th} term or more.

(15) (a)1/2
(b) The function is ill-conditioned near x = 1 and x = -1, otherwise it is well-conditioned for all $x \in \mathbb{R}$.