PHI-101 (Physics) Autumn 2025-26

Assignment #4

(Aug 28, 2025)

Electrostatic fields: Sadiku Chaps 7-8, Griffiths Chap 5

1. Consider a volume distribution of current with a current density $\mathbf{J}(\mathbf{r})$. For static fields and a solenoidal $\mathbf{J}(\mathbf{r})$ (steady current), using the appropriate Maxwell's equation, show that $\nabla^2\mathbf{A}(\mathbf{r}) = -\mu_0\mathbf{J}(\mathbf{r})$ in vacuum, where $\mathbf{A}(\mathbf{r})$ is the vector potential. Verify the same by using the expression for $\mathbf{A}(\mathbf{r})$ in terms of $\mathbf{J}(\mathbf{r})$.

2. A long hollow right circular cylindrical shell of inner and outer radii a and b respectively and made of iron of permeability μ (assumed to be a constant in the range of the field involved) is placed with its axis perpendicular to an initially uniform magnetic field B_0. Writing the magnetic field in terms of a magnetic scalar potential as: $\mathbf{B} = -\nabla\phi$, write out the boundary conditions to be satisfied by (derivatives of) ϕ.

3. Consider an infinitely long cylinder of radius $3a$ with an infinitely long cylindrical hole of radius a displaced so that the center of the hole is at a distance a from the center of the big cylinder as shown in the Fig. 1. The solid part of the cylinder carries a current I, distributed uniformly over the cross section and out of the plane of the paper. Find the cartesian components of the magnetic field along a plane containing the axis.

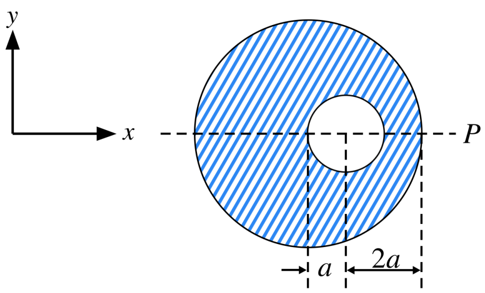

Fig. 1

4. A current distribution gives rise to the magnetic vector potential $\mathbf{A} = x^2y\,\mathbf{a}_x + y^2x\,\mathbf{a}_y - 4xyz\,\mathbf{a}_z$ Wb/m. Calculate the following:
   (a) $\mathbf{B}$ at $(-1, 2, 5)$
   (b) The flux through the surface defined by $z = 1$, $0 \le x \le 1$, $-1 \le y \le 4$.

5. If magnetic vector potential is given by $\mathbf{A} = 10\rho^{3/2}\,\mathbf{a}_z$ Wb/m in free space in cylindrical coordinates. Find
   (a) $\mathbf{H}$,
   (b) $\mathbf{J}$, and
   (c) show that $\oint \mathbf{H}\cdot d\mathbf{l} = I_{enc}$

6. A current sheet $\mathbf{K}_1 = 5\,\mathbf{a}_x$ A/m flows on $y = 10$, while $\mathbf{K}_2 = -10\,\mathbf{a}_x$ A/m flows on $y = -4$. Find $\mathbf{H}$ at the origin.

7. Let $\mathbf{H} = k_0\left(\dfrac{\rho}{a}\right)\mathbf{a}_\phi$, $\rho < a$, where k_0 is a constant. (i) Find $\mathbf{J}$ for $\rho < a$. (ii) Find $\mathbf{H}$ for $\rho > a$.

8. In free space, the magnetic field is

$$\mathbf{B} = y^2\,\mathbf{a}_x + z^2\,\mathbf{a}_y + x^2\,\mathbf{a}_z \text{ Wb/m}^2$$

(a) Show that $\mathbf{B}$ is a magnetic field.

(b) Find the magnetic flux through $x = 1, 0 < y < 1, 1 < z < 4$

(c) Calculate $\mathbf{J}$.

(d) Determine the total magnetic flux through the surface of a cube defined by $0 < x < 2$, $0 < y < 2$, $0 < z < 2$.

9. Consider the following arbitrary fields. Find out which of them can possibly represent electrostatic or magnetostatic field in free space.

(a) $\mathbf{D} = y^2 z\,\mathbf{a}_x + 2(x+1)yz\,\mathbf{a}_y - (x+1)z^2\,\mathbf{a}_z$

(b) $\mathbf{E} = \dfrac{(z+1)}{\rho}\,\mathbf{a}_\rho + \dfrac{\sin\phi}{\rho}\,\mathbf{a}_z$

(c) $\mathbf{F} = \dfrac{1}{r^2}(2\cos\theta\,\mathbf{a}_r + \sin\theta\,\mathbf{a}_\theta)$

10. Find the current density $\mathbf{J}$ due to

$$\mathbf{A} = \frac{10}{\rho^2}\,\mathbf{a}_z \text{ Wb/m}$$

in free space.

11. Let $\mathbf{A} = 10\rho^2\,\mathbf{a}_z\ \mu$Wb/m.

(a) Find $\mathbf{H}$ and $\mathbf{J}$.

(b) Determine the total current crossing the surface $z = 1, 0 \le \rho \le 2, 0 \le \phi \le 2\pi$.

12. A spherical shell of radius R and uniform surface charge density σ, is spinning with a constant angular velocity ω? Find $\mathbf{A}(\mathbf{r})$, inside and outside of the sphere, and consequently $\mathbf{B}(\mathbf{r})$.

13. An infinite cylinder with radius R and surface charge density σ spins around its symmetry axis with angular frequency ω. Find the magnetic field inside the cylinder.

14. A unit normal vector from region 2 ($\mu = 2\mu_0$) to region 1 ($\mu = \mu_0$) is $\mathbf{a}_{n21} = \frac{1}{7}(6\,\mathbf{a}_x + 2\,\mathbf{a}_y - 3\,\mathbf{a}_z)$. If $\mathbf{H}_1 = 10\,\mathbf{a}_x + \mathbf{a}_y + 12\,\mathbf{a}_z$ A/m and $\mathbf{H}_2 = H_{2x}\,\mathbf{a}_x - 5\,\mathbf{a}_y + 4\,\mathbf{a}_z$ A/m, determine

(a) H_{2x}

(b) The surface current density $\mathbf{K}$ on the interface

(c) The angles $\mathbf{B}_1$ and $\mathbf{B}_2$ make with the normal to the interface

15. Region 1, for which $\mu_1 = 2.5\,\mu_0$, is defined by $z < 0$, while region 2, for which $\mu_2 = 4\,\mu_0$, is defined by $z > 0$. If $\mathbf{B}_1 = 6\,\mathbf{a}_x - 4.2\,\mathbf{a}_y + 1.8\,\mathbf{a}_z$ mWb/m^2, find $\mathbf{H}_2$ and the angle $\mathbf{H}_2$ makes with the interface.

16. A current sheet with $\mathbf{K} = 12\,\mathbf{a}_y$ A/m is placed at $x = 0$, which separates region 1, $x < 0$, $\mu = 2\mu_0$ and region 2, $x > 0$, $\mu = 4\mu_0$. If $\mathbf{H}_1 = 10\,\mathbf{a}_x + 6\,\mathbf{a}_z$ A/m, find $\mathbf{H}_2$.