ECC-201

Frequency Modulation

PM : $s(t) = A_c \cos[2\pi f_c t + K_p m(t)]$

FM : $s(t) = A_c \cos[2\pi f_c t + 2\pi K_f \int_0^t m(\tau)d\tau]$

$f_i = \frac{1}{2\pi}\frac{d\theta_i(t)}{dt}$ $\quad \theta_i(t) = \int_0^t f_i(t)\,dt \times 2\pi$

For singletone modulation: $m(t) = A_m \cos(2\pi f_m t)$

$\theta_i(t) = 2\pi f_c t + \underbrace{\frac{K_f A_m}{f_m}}_{\beta_f} \sin(2\pi f_m t)$ $\quad \boxed{\Delta f = K_f A_m}$

$f_i(t) = f_c + \underbrace{K_f A_m}_{\Delta f} \cos(2\pi f_m t)$ $\quad \boxed{\beta_f = \frac{\Delta f}{f_m}}$

$P = \frac{A_c^2}{2}$ constant , AM is non-linear

Bandwidth - Noise Tradeoff

$\beta_f << 1$: Narrow Band FM

$\beta_f >> 1$: Wide Band FM

Carson's Rule : $B_T \cong 2(\Delta f + f_m)$

Generation : Voltage Controlled Oscillator (VCO)

Demodulation : Balanced Frequency Discriminator

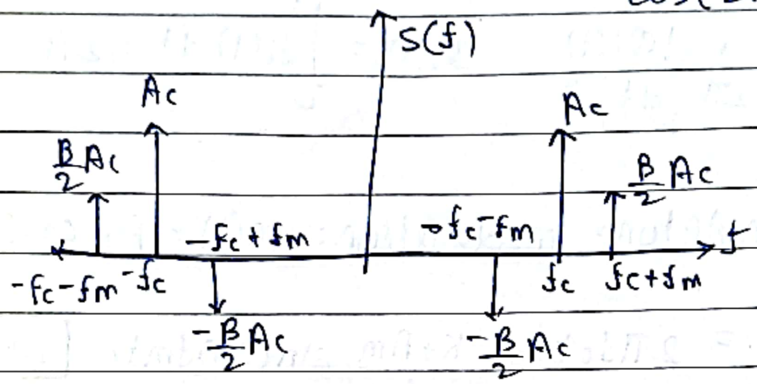
NBFM

$\beta << 1$

$$s(t) = A_c \cos\left[2\pi f_c t + \beta \sin(2\pi f_m t)\right]$$

$$\cong A_c \cos(2\pi f_c t) - \beta A_c \sin(2\pi f_c t)\sin(2\pi f_m t)$$

$$\cong A_c \cos(2\pi f_c t) + \frac{\beta A_c}{2}\left[\cos(2\pi(f_c+f_m)t) - \cos(2\pi(f_c-f_m)t)\right]$$



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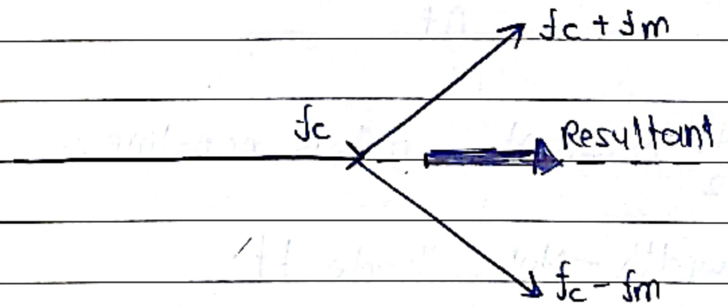
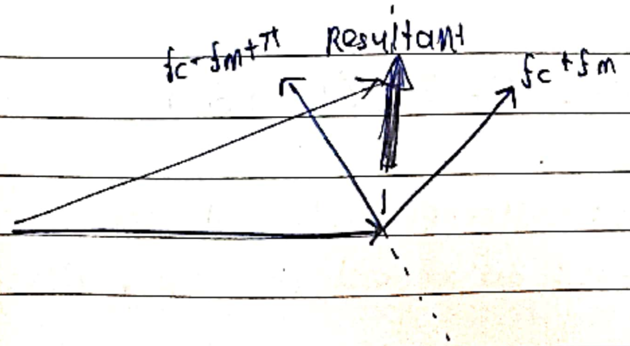
Amplitude Modulation:



Frequency Modulation:



WBFM

$$s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int m(\tau)d\tau\right]$$
$$= A_c \, Re\left\{ e^{2\pi f_c t + 2\pi k_f \int m(\tau)d\tau} \right\}$$
$$= A_c \, Re\left\{ e^{2\pi f_c t} * \underbrace{e^{2\pi k_f \int m(\tau)d\tau}}_{\tilde{s}(t)} \right\}$$

$$\tilde{s}(t) = A e^{2\pi k_f \int m(\tau)d\tau}$$

For singletone, $\tilde{s}(t) = A e^{j\beta \sin(2\pi f_m t)}$

↳ Periodic

Taking Fourier series: $\tilde{s}(t) = \sum_{n=-\infty}^{\infty} C_n e^{j2\pi f_m t n}$

$$C_n = f_m \int_{-1/2f_m}^{1/2f_m} \tilde{s}(t) e^{-j2\pi f_m t n} dt$$

$$= f_m \int_{-1/2f_m}^{1/2f_m} A_c e^{j\beta \sin(2\pi f_m t) - j2\pi f_m t n} dt$$

$x = 2\pi f_m t \quad \therefore \quad dx = 2\pi f_m dt$

$$= \frac{A_c}{2\pi} \int_{-\pi}^{\pi} e^{j\beta \sin(x) - nx} dx$$

$$= A_c J_n(\beta) \qquad J_n(\beta) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{j\beta \sin x - jnx} dx$$

$$\tilde{s}(t) = \sum_{n=-\infty}^{\infty} A_c J_n(\beta) e^{j2\pi n f_m t}$$

$$s(t) = \mathrm{Re}\{ e^{j2\pi f_c t} \cdot \tilde{s}(t) \}$$

$$= \mathrm{Re}\left\{ e^{j2\pi f_c t} \cdot \sum_{n=-\infty}^{n=\infty} A_c J_n(\beta) e^{j2\pi n f_m t} \right\}$$

$$= \sum_{n=-\infty}^{\infty} A_c J_n(\beta) \mathrm{Re}\left\{ e^{j2\pi f_c t + j2\pi n f_m t} \right\}$$

$$= \sum_{n=-\infty}^{\infty} A_c J_n(\beta) \cos(2\pi f_c t + 2\pi n f_m t)$$

$$S(f) = \frac{A_c}{2} \sum_{n=-\infty}^{\infty} J_n(\beta) \left[\delta(f - f_c - n f_m) + \delta(f + f_c + n f_m) \right]$$

$J_n(\beta)$ is a real number \$ $J_n(\beta) = (-1)^n J_{-n}(\beta)$

For small β, $J_0(\beta) \cong 1$

$J_1(\beta) \cong \beta/2$

$J_n(\beta) \cong 0 \;\; \forall\, n \geq 2$

Power of one sideband : $P_s = \frac{1}{2} A_c^2 J_n^2(\beta)$

$$P_{total} = \sum P_s = \frac{1}{2} A_c^2 \sum_{-\infty}^{\infty} J_n^2(\beta) = \frac{A_c^2}{2}$$

For singletone modulation:

$$\boxed{B_T = 2(\Delta f + f_m)}$$ Carson's Rule

$\Delta f = \beta f_m$ $\quad$ f_m is the highest frequency in $m(t)$

$$B_T = 2\Delta f\left(1 + \frac{1}{\beta}\right) = 2 f_m (1+\beta)$$

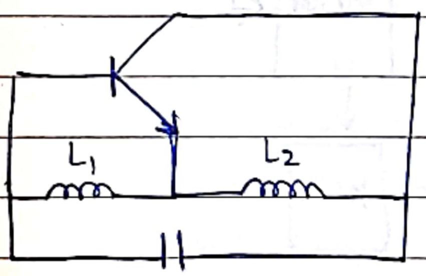
for $\beta \ll 1$: $B_T \rightarrow 2 f_m$

$\beta \gg 1$: $B_T \rightarrow 2\Delta f$

A more practical Universal Curve approach is the 1% bandwitd rule: $B_T = 2 n_{max} f_m$
Where n_{max} is the maximum integer n for which $J_n(\beta) \geq 0.01$

FM Modulation

Direct Method: Hartley Oscillator (VCO)



$$f_i(t) = \frac{1}{2\pi\sqrt{(L_1+L_2)C(t)}}$$

$C(t) \Rightarrow$ Varactor Diode

$$C(t) = C_0 + \Delta C \cos(2\pi f_m t)$$

$$f_i(t) = f_0 \left(1 + \frac{\Delta C}{C_0}\cos(2\pi f_m t)\right)^{-1/2}$$

$$f_0 = \frac{1}{2\pi\sqrt{C_0(L_1+L_2)}}$$

$$f_i(t) \cong f_0\left(1 - \frac{\Delta C}{2C_0}\cos(2\pi f_m t)\right)$$

$$= f_0 - \underbrace{\frac{\Delta C\, f_0}{2C_0}}_{\Delta f}\cos(2\pi f_m t)$$

$$= f_0 + \Delta f \cos(2\pi f_m t)$$

A big problem with this direct method is the instability of frequency of the varactor diode, hence we use the indirect method