ECC 203 : Electromagnetics and Radiating Systems

Antenna Array 2 : Uniform N Element Linear Array

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Geometry of N-Element Linear Array

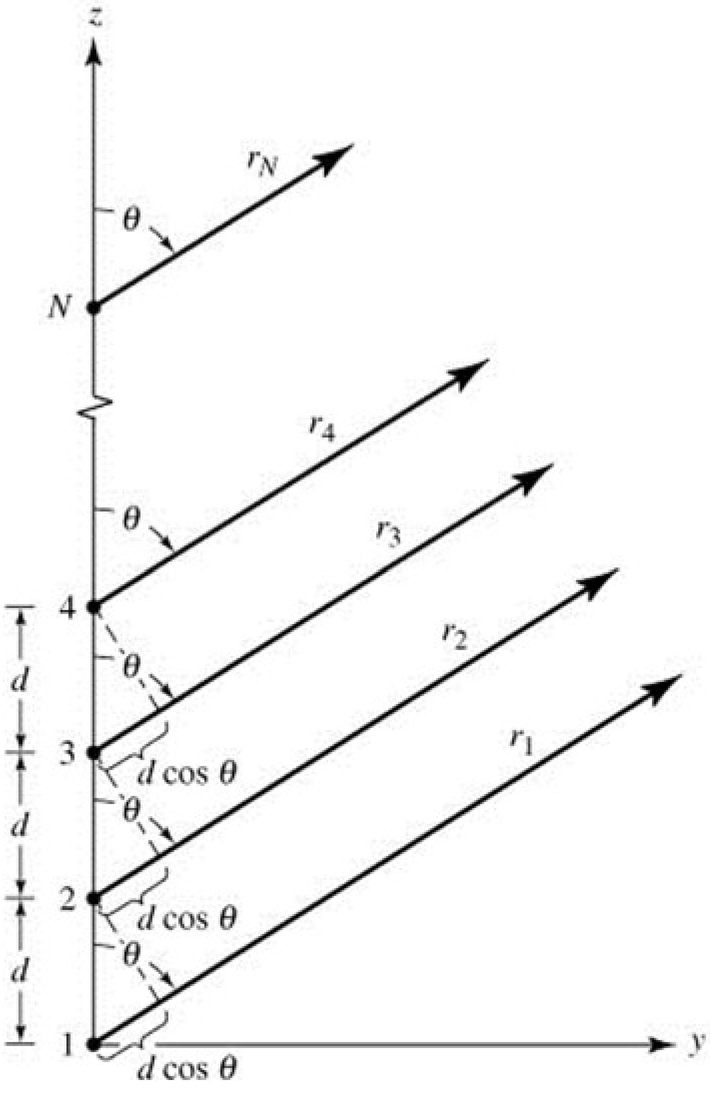


Fig. 6.5a

Array Pattern Multiplication for Identical Elements

$$\underbrace{E(\text{total})}_{\text{ET}} = \underbrace{E(\text{single element})}_{\text{Element Factor (EF)}} \cdot \underbrace{\text{Array Factor}}_{\text{AF}} \qquad (6\text{-}5)$$

N-Elements

$$\mathrm{AF} = \underbrace{1e^{j0}}_{\#1} + \underbrace{1e^{j\beta}e}_{\#2}{}^{jkd\cos\theta} + \underbrace{1e^{j2\beta}e}_{\#3}{}^{j2kd\cos\theta}$$

$$+ \cdots + \underbrace{1e^{j(N-1)\beta}e}_{\#N}{}^{j(N-1)kd\cos\theta}$$

$$\mathrm{AF} = \sum_{n=1}^{N} (1) e^{j(n-1)\beta} e^{j(n-1)kd\cos\theta}$$

N-Elements

$$\mathrm{AF} = \sum_{n=1}^{N} e^{j(n-1)\left(\underbrace{kd\cos\theta+\beta}_{\psi}\right)} \quad (6\text{-}6)$$

$$\boxed{\begin{aligned} \mathrm{AF} &= \sum_{n=1}^{N} e^{j(n-1)\psi} && (6\text{-}7) \\ \psi &= kd\cos\theta + \beta && (6\text{-}7a) \end{aligned}}$$

Another Form of AF

$\boxed{1}$: $$\text{AF} = 1 + e^{j\psi} + e^{j2\psi} + \ldots$$

$$\ldots + e^{j(N-2)\psi} + e^{j(N-1)\psi}$$

$\boxed{2}$: $$e^{j\psi}\text{AF} = e^{j\psi} + e^{j2\psi} + \ldots$$

$$\ldots + e^{j(N-2)\psi} + e^{j(N-1)\psi} + e^{jN\psi}$$

$\boxed{2} - \boxed{1}$: (6-8)

$$\text{AF}\left(-1 + e^{j\psi}\right) = -1 + e^{jN\psi} \qquad \text{(6-9)}$$

Another Form of AF

$$\text{AF}\left(-1+e^{j\psi}\right)=-1+e^{jN\psi} \qquad (6\text{-}9)$$

$$\text{AF}=\frac{e^{jN\psi}-1}{e^{j\psi}-1}=\frac{e^{j\frac{N}{2}\psi}\left(e^{j\frac{N\psi}{2}}-e^{-j\frac{N\psi}{2}}\right)}{e^{j\frac{\psi}{2}}\left(e^{j\frac{\psi}{2}}-e^{-j\frac{\psi}{2}}\right)}$$

Divide Numerator and Denominator by 2j

$$\text{AF} = e^{j\frac{\psi}{2}(N-1)} \frac{\sin\left(\frac{N}{2}\psi\right)}{\sin\left(\frac{\psi}{2}\right)} \tag{6-10}$$

$$|\text{AF}| = \left|e^{j\frac{\psi}{2}(N-1)}\right| \left|\frac{\sin\left(\frac{N}{2}\psi\right)}{\sin\left(\frac{\psi}{2}\right)}\right|$$

$$|\text{AF}| = \left|\frac{\sin\left(\frac{N}{2}\psi\right)}{\sin\left(\frac{\psi}{2}\right)}\right| \tag{6-10a}$$

$$\mathrm{AF} = \frac{\sin\left(\frac{N}{2}\psi\right)}{\sin\left(\frac{1}{2}\psi\right)} \overset{\psi \to 0}{\cong} \frac{\sin\left(\frac{N}{2}\psi\right)}{\frac{\psi}{2}} \qquad \text{(6-10b)}$$

Normalize by N:

$$(\mathrm{AF})_n = \frac{1}{N}\frac{\sin\left(\frac{N}{2}\psi\right)}{\sin\left(\frac{1}{2}\psi\right)} \overset{\psi \to 0}{\cong} \frac{\sin\left(\frac{N}{2}\psi\right)}{\frac{N}{2}\psi} \qquad \text{(6-10c,d)}$$

$$\psi = kd\cos\theta + \beta$$

Normalized Array Factor

$$|\mathrm{AF}|_n = \begin{cases} \dfrac{1}{N}\left|\displaystyle\sum_{n=1}^{N} e^{j(n-1)\psi}\right| \\[2ex] \dfrac{1}{N}\left|\dfrac{\sin\left(\frac{N}{2}\psi\right)}{\sin\left(\frac{\psi}{2}\right)}\right| \underbrace{\approx}_{\psi\to 0} \left|\dfrac{\sin\left(\frac{N}{2}\psi\right)}{\left(\frac{N}{2}\psi\right)}\right| \end{cases}$$

$$\psi = kd\cos\theta + \beta$$

Nulls

$$(\mathrm{AF})_n = \frac{\sin\left(\frac{N\psi}{2}\right)}{N\sin\left(\frac{\psi}{2}\right)} \simeq \frac{\sin\left(\frac{N\psi}{2}\right)}{\frac{N\psi}{2}} = 0$$

$$\sin\left(\frac{N\psi}{2}\right) = 0$$

$$\frac{N\psi}{2} = \sin^{-1}(0) = \pm n\pi, \quad n = \cancel{0},1,2,\cdots$$

$$n \neq 0, N, 2N, \cdots$$

$$\frac{N}{2}\left(kd\cos\theta_n + \beta\right) = \pm n\pi$$

$$\theta_n = \cos^{-1}\left[\frac{\lambda}{2\pi d}\left(-\beta \pm \frac{2n}{N}\pi\right)\right] \qquad (6\text{-}11)$$

Maxima (Principal)

$$\mathrm{AF} = \frac{\sin\left(\frac{N}{2}\psi\right)}{N\sin\left(\frac{\psi}{2}\right)} \approx \frac{\sin\left(\frac{N}{2}\psi\right)}{\frac{N}{2}\psi} = 1$$

$$\sin\left(\frac{\psi}{2}\right) = 0 \Rightarrow \frac{\psi}{2} = \sin^{-1}(0) = \pm m\pi$$

$$m = 0, 1, 2, \cdots$$

$$\psi = \pm 2m\pi = kd\cos\theta_m + \beta$$

$$\theta_m = \cos^{-1}\left[\frac{\lambda}{2\pi d}(-\beta \pm 2m\pi)\right], \quad m = 0, 1, 2 \qquad (6\text{-}12)$$

$$\underline{m = 0}: \quad \theta_m = \cos^{-1}\left(-\frac{\lambda\beta}{2\pi d}\right) \qquad (6\text{-}13)$$

Half-Power (3-dB)

$$\mathrm{AF} \approx \frac{\sin\left(\frac{N}{2}\psi\right)}{\frac{N}{2}\psi} = 0.707 \Rightarrow \frac{N\psi}{2} = \pm 1.391$$

$$\frac{N}{2}\psi = \frac{N}{2}\left(kd\cos\theta_h + \beta\right) = \pm 1.391$$

$$\theta_h \approx \cos^{-1}\left[\frac{\lambda}{2\pi d}\left(-\beta \pm \frac{2.782}{N}\right)\right] \qquad (6\text{-}14)$$

$$\theta_h \approx \frac{\pi}{2} - \sin^{-1}\left[\frac{\lambda}{2\pi d}\left(-\beta \pm \frac{2.782}{N}\right)\right] \qquad (6\text{-}14a)$$

Broadside Linear Array

Geometry of N-Element Linear Array

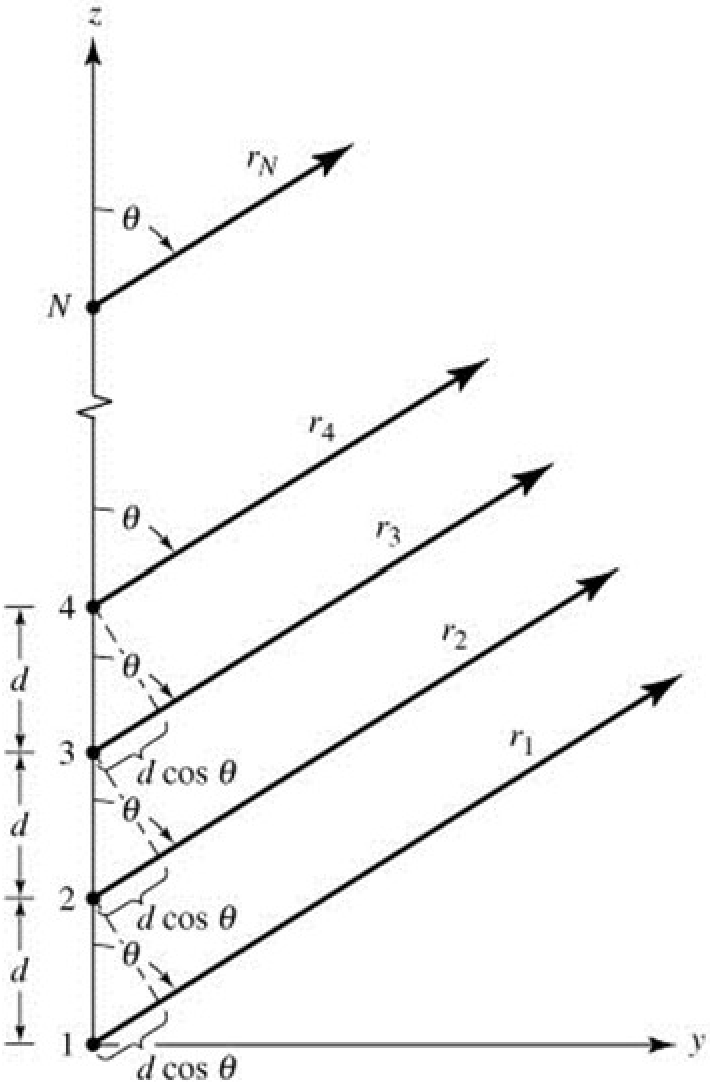


Fig. 6.5a

Broadside Array

$$\psi\Big|_{\theta=90^\circ} = (kd\cos\theta + \beta)\Big|_{\theta=90^\circ} = 0 + \beta = 0$$

(6-18a)

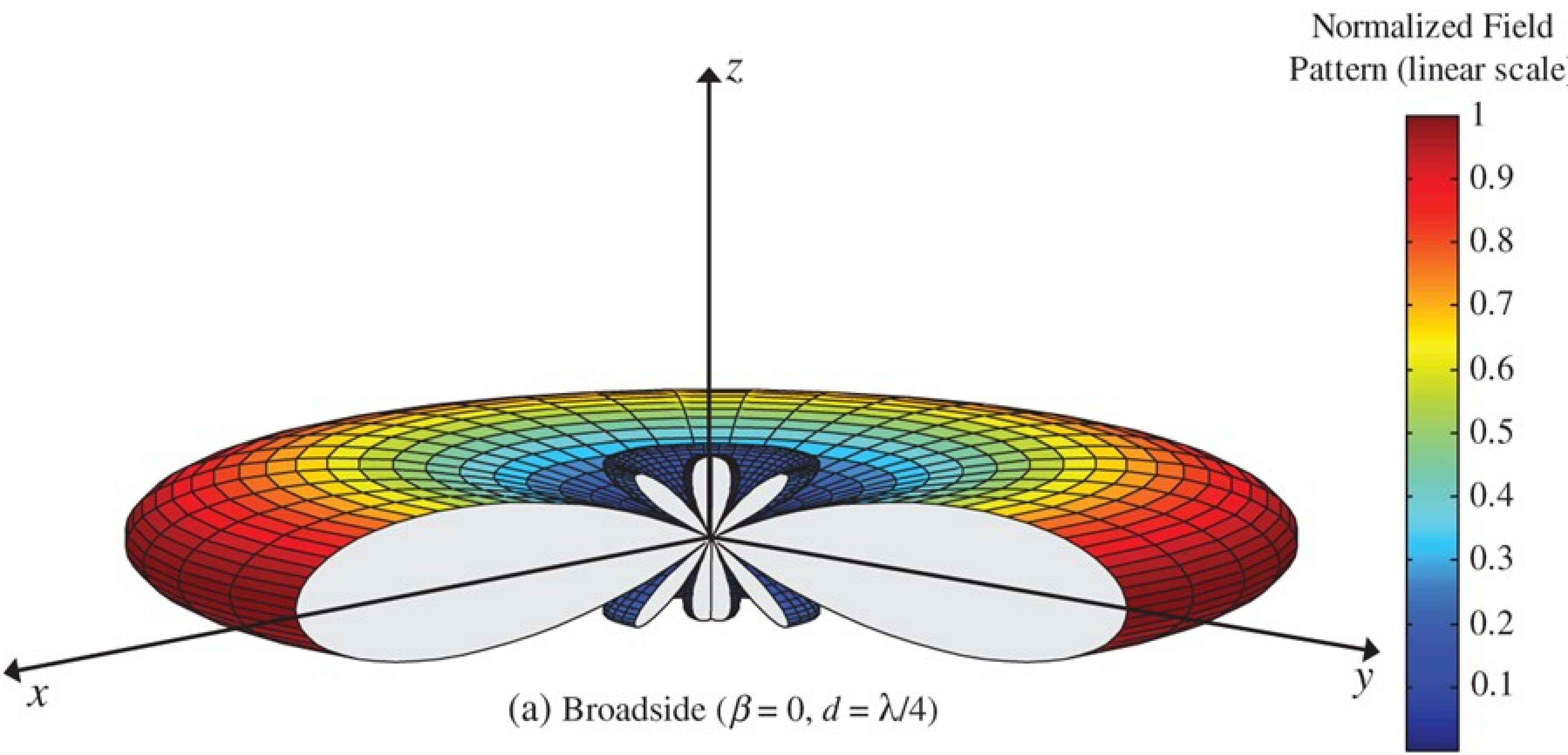
$$\boxed{\beta = 0}$$

Avoid $\underline{d = n\lambda}$ because

$$\psi\Big|_{\substack{\beta=0 \\ d=n\lambda}} = (kd\cos\theta + \beta)_{\substack{\beta=0 \\ d=n\lambda}} = 2\pi n\cos\theta$$

$$\psi = 2\pi n\cos\theta\Big|_{\theta=0^\circ,180} = \pm\, 2\pi n \qquad \text{(6-19)}$$

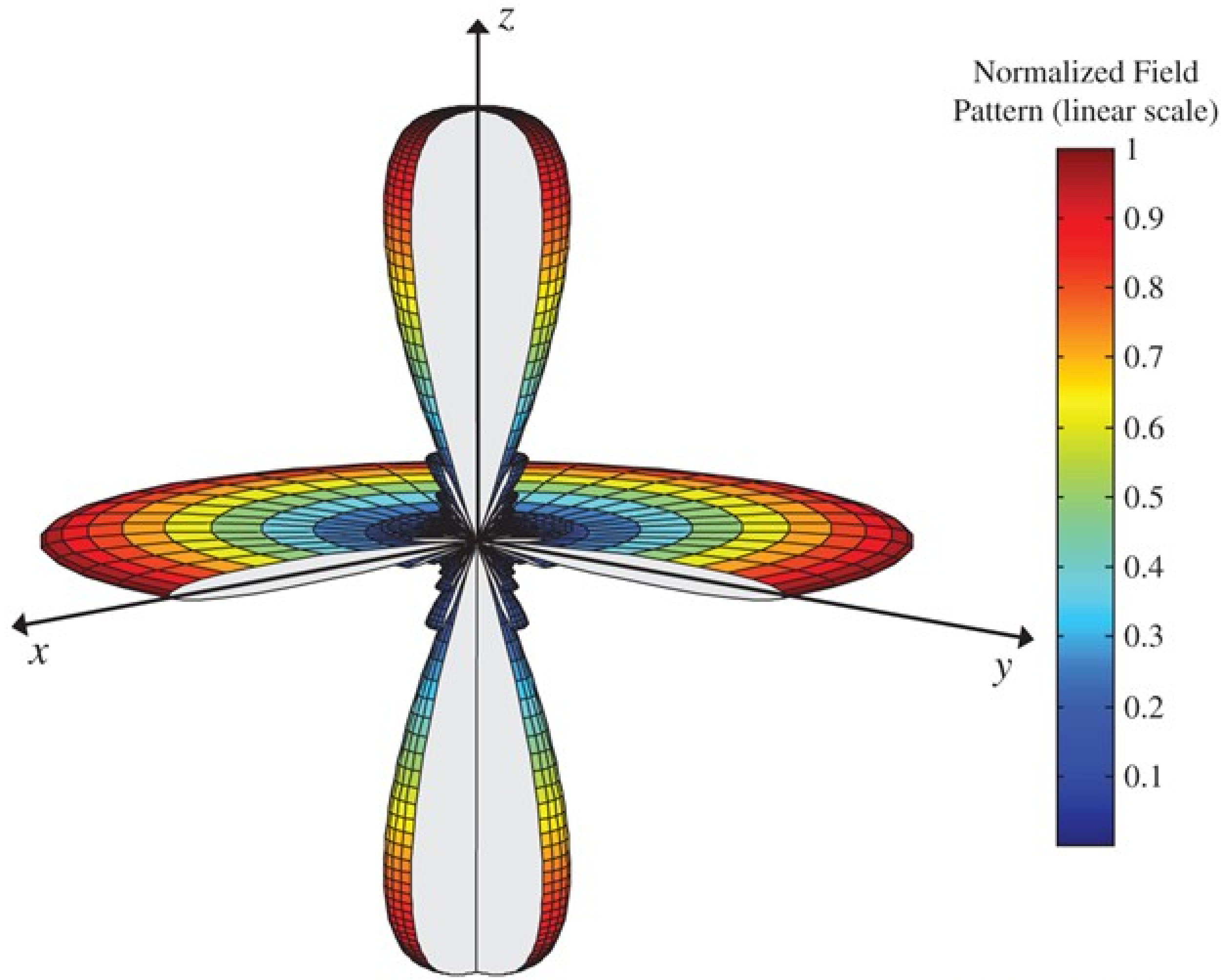
Broadside $(d = \lambda / 4)$ $N = 10$



(a) Broadside ($\beta = 0$, $d = \lambda/4$)

Fig. 6.6a

Broadside/End-Fire ($d = \lambda, N = 10$)



(b) Broadside/end-fire ($\beta = 0$, $d = \lambda$)

Fig. 6.6b

No Grating Lobes

$$\boxed{d < \lambda}$$

Grating Lobes

$$\boxed{d \geq \lambda}$$

Ordinary End-Fire Linear Array

Geometry of N-Element Linear Array

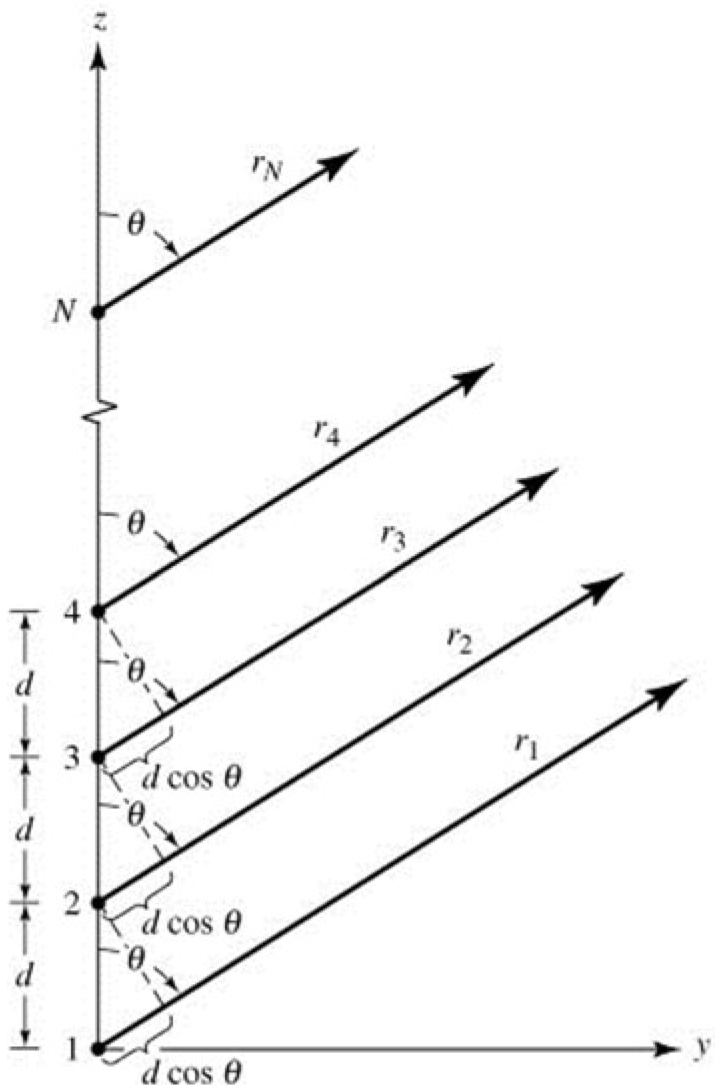


Fig. 6.5a

Ordinary End-Fire

A. Maximum Toward θ=0°

B. Maximum Toward θ=180°

A. Toward θ=0°

$$\psi\Big|_{\theta=0^\circ} = (kd\cos\theta + \beta)\Big|_{\theta=0^\circ} = kd + \beta = 0$$

$$\boxed{\beta = -kd}$$

B. Toward θ=180°

$$\psi = (kd\cos\theta + \beta)\Big|_{\theta=180^\circ} = -kd + \beta = 0$$

$$\boxed{\beta = +kd}$$

$N = 10, d = \lambda / 4$

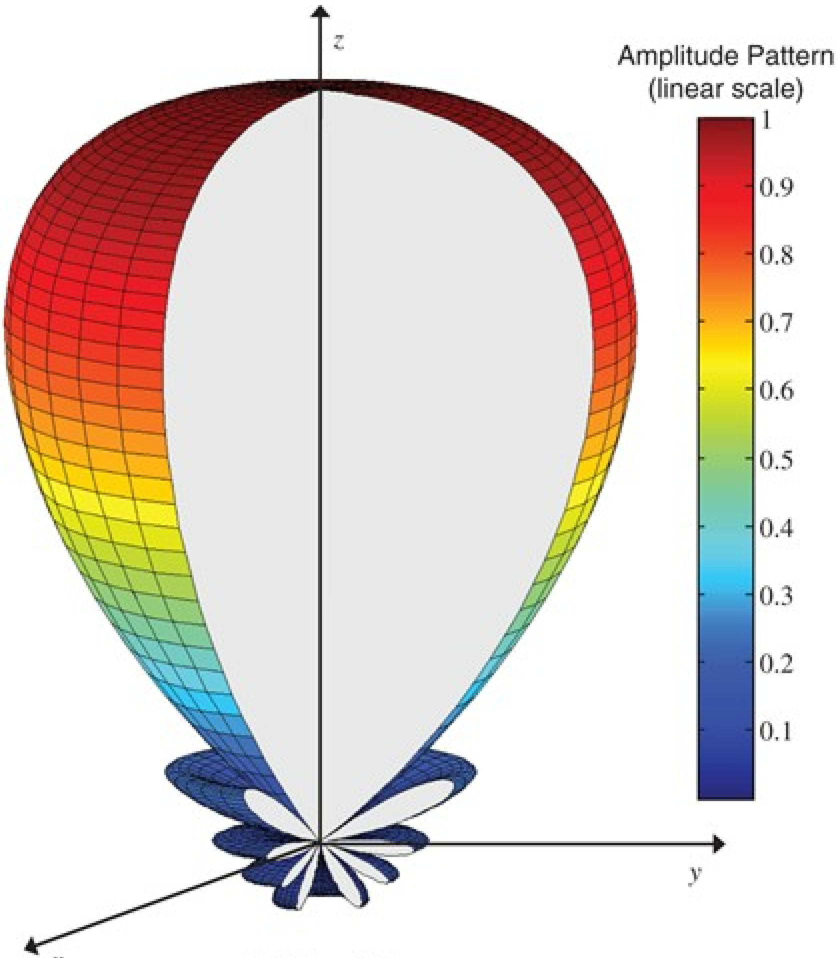
$$1.\quad \beta = -kd = -\frac{2\pi}{\lambda}\left(\frac{\lambda}{4}\right) = -\frac{\pi}{2} \qquad \text{(6-20a)}$$

Max @ $\theta = 0^{\circ}$

$$2.\quad \beta = +kd = +\frac{2\pi}{\lambda}\left(\frac{\lambda}{4}\right) = +\frac{\pi}{2} \qquad \text{(6-20b)}$$

Max @ $\theta = 180^{\circ}$

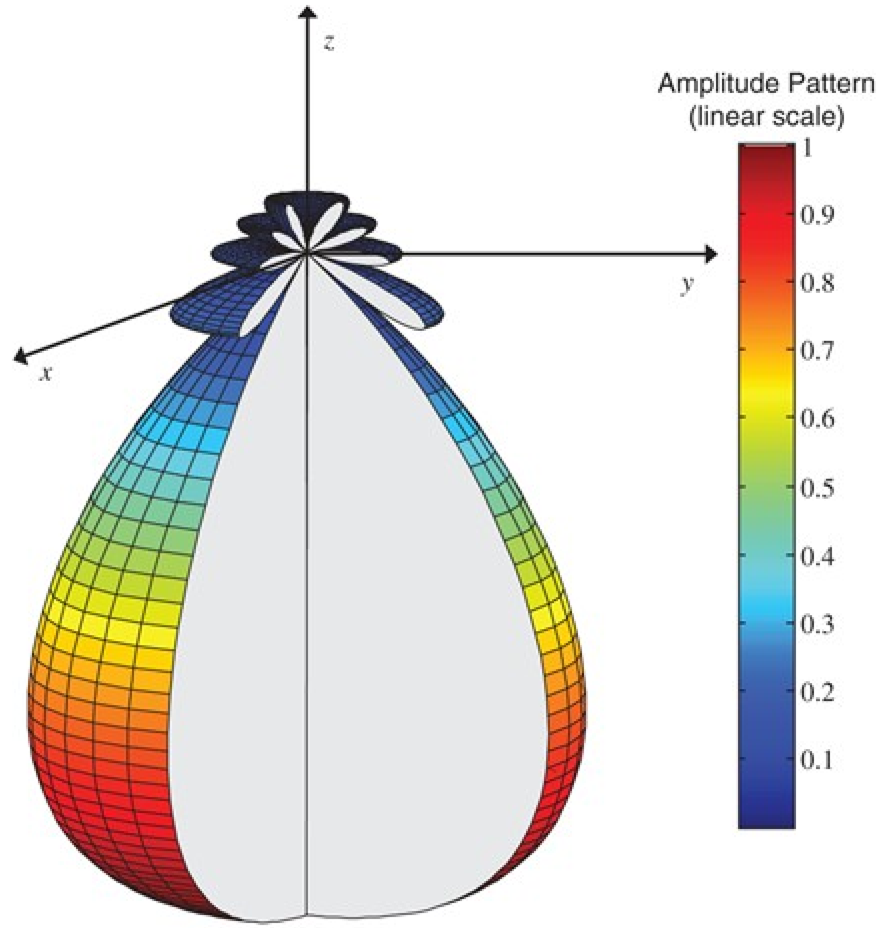
End-Fire $(d = \lambda / 4)$ $N = 10$



(a) $\theta_0 = 0°$

Fig. 6.8(a)

End-Fire ($d = \lambda / 4$) $N = 10$



(b) $\theta_0 = 180°$

Fig. 6.7(b)

Ordinary End-Fire Arrays

1. $d < \lambda/2$: End-fire only in one direction ($\theta = 0^o$ or 180^o)
2. $d = \lambda/2$: End-fire in both directions ($\theta = 0^o$ & 180^o)
3. $\lambda/2 < d < \lambda$: End-fire in one direction ($\theta = 0^o$ or 180^o) and maximum toward two other directions
4. $d = \lambda = n\lambda$: End-fire in both directions ($\theta = 0^o$ & 180^o) and broadside

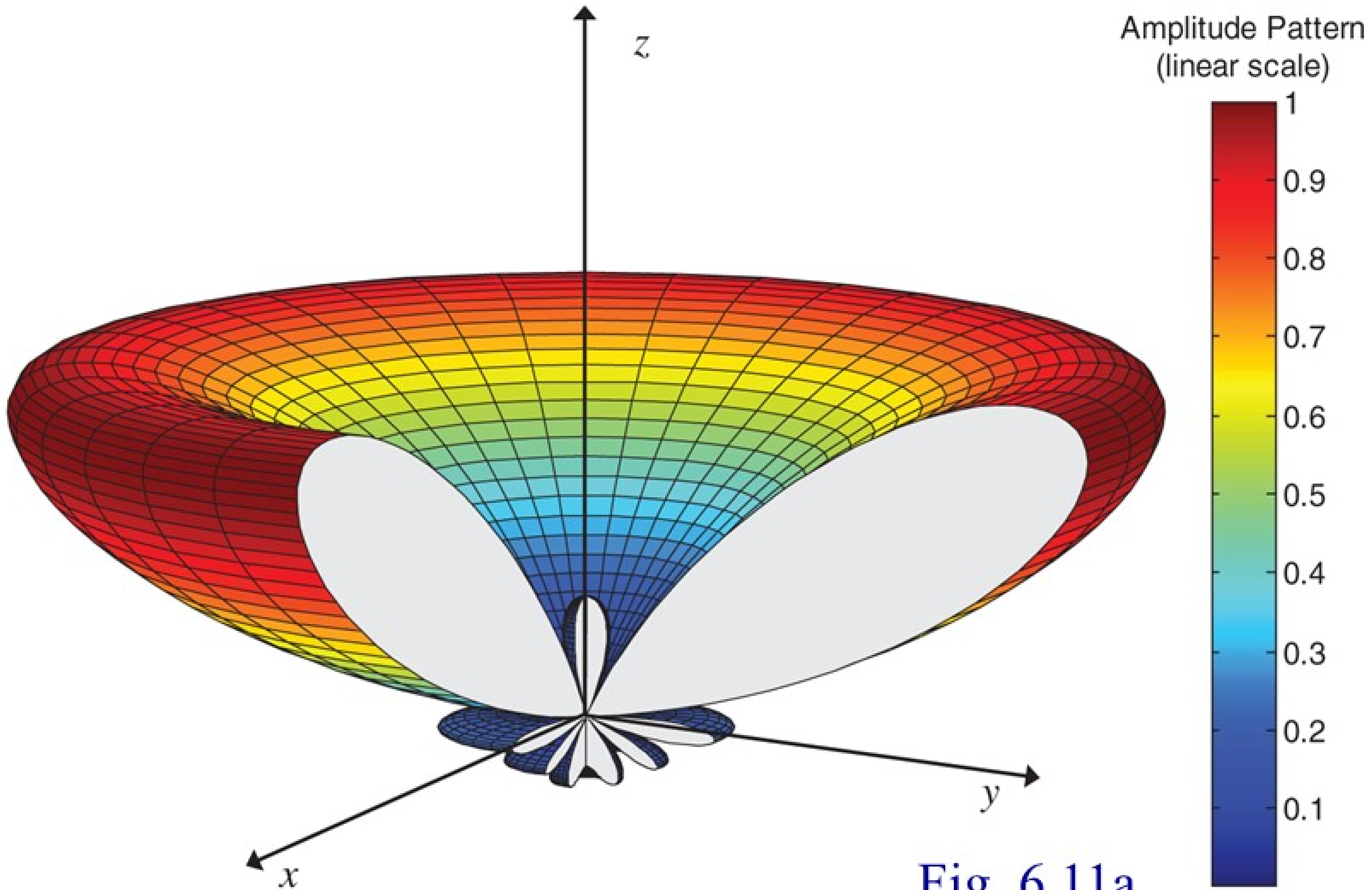
Scanning
Linear Array

Scanning Array ($\theta = \theta_o$)

$$\psi\Big|_{\theta=\theta_o} = (kd\cos\theta + \beta)\Big|_{\theta=\theta_o} = kd\cos\theta_o + \beta = 0$$

$$\boxed{\beta = -kd\cos\theta_o} \qquad (6\text{-}21)$$

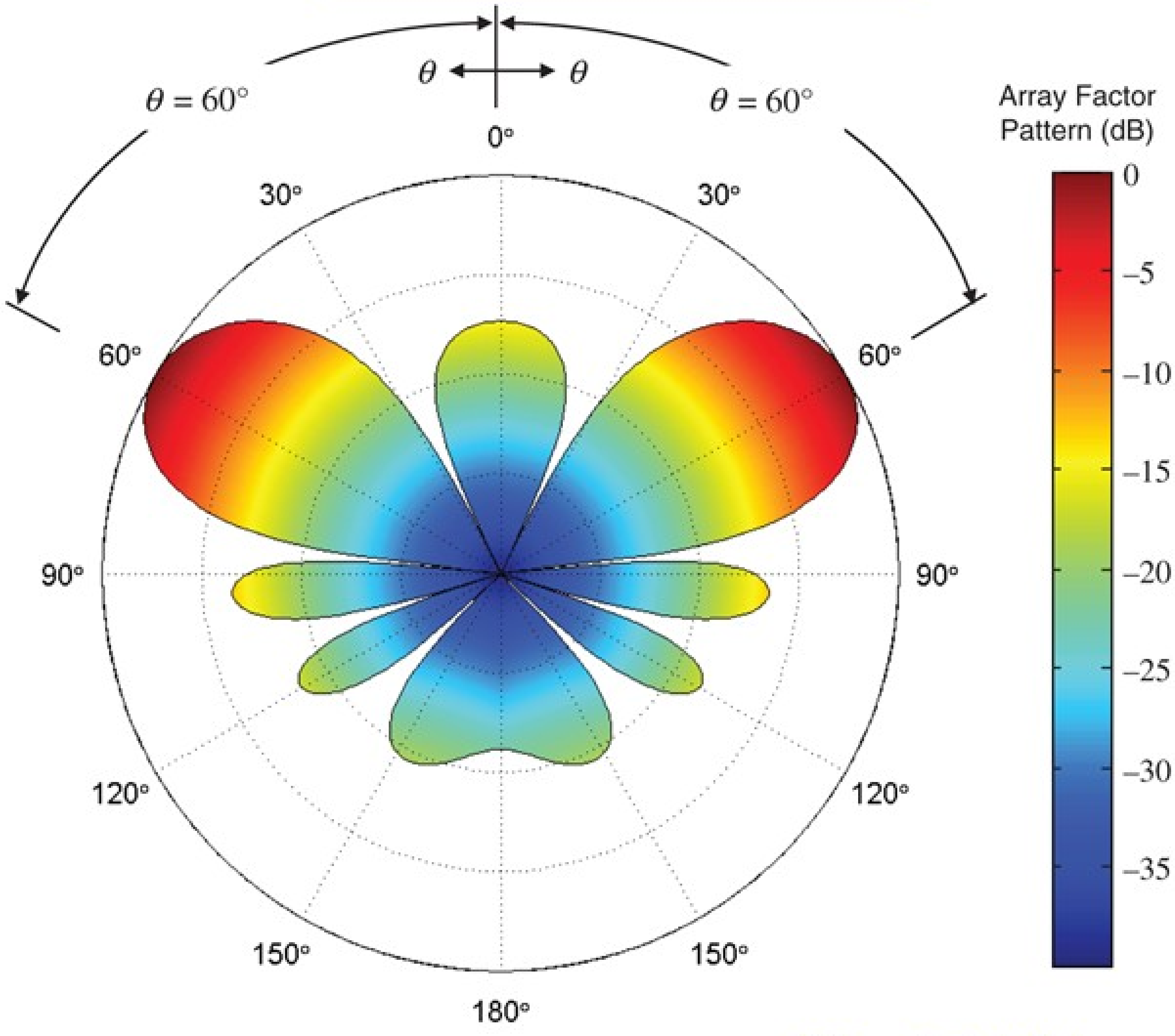
Three-Dimensional



(a) Three-dimensional

Fig. 6.11a

Two-Dimensional



(b) Two-dimensional

Fig. 6.11(b)

Thank You