DEPARTMENT OF MATHEMATICS, IIT ROORKEE
MAB-103: Numerical Methods

Assignment-1 Error Analysis Session-2025-26

1. Round off the following numbers to 4 significant digits:
2.34567, 2.3455, 2.34449, 1.47383, 1473.27, 0.00276, 0.0027657.

2. If the numbers are correct to the last digit, find the correct significant figures for each problem:
$2.33 \times 6.085 \times 2.1$, $(4.52 \times 10^{-4}) \div (3.980 \times 10^{-6})$, 3.10×4.520.

3. Calculate the value of $\sqrt{626} - \sqrt{625}$ correct to 4 significant figures.

4. Round off the numbers 865250 and 37.46235 to four significant figures and compute absolute, relative and percentage error in each case.

5. Find $0.348 + 0.1834 + 435.4 + 235.2 + 11.75 + 9.27 + 0.0849 + 0.0214 + 0.000354$, assume that all their digits being correct. Find the maximum absolute error and maximum relative error in the sum.

6. If $S = \sqrt{6} + \sqrt{7} + \sqrt{8}$, find, correct to 4 significant digits,
(i) the absolute error and the relative error in S,
(ii) the maximum absolute error and the maximum relative error in S.

7. If $x = 5.675, y = 4.737$ and $z = 4.373$, calculate $x(y - z)$ and $xy - xz$, to four significant figures, which one is more accurate.

8. Compute $y = x^3 \sin(x)$ for $x = \sqrt{2} (\approx 1.414)$. Determine the maximum absolute error and maximum relative error in y ($\sin(x)$ to be calculated in radians)).

9. If $u = \frac{4xy^2}{z^2}$ and error in x, y, z be 0.001, compute
(i) the absolute error and the relative error in u, when $x = y = z = 1$,
(ii) the maximum absolute error and the maximum relative error in u, when $x = y = z = 1$.

10. Find $u = log_e(x_1 + x_2^2), x_1 = 0.97, x_2 = 1.132$. Obtain the maximum absolute error and maximum relative error in u.

11. The function $y = k_1 \cos(x) + k_2 \ln(x)$ is said to be evaluated for $x = 1.36$. The values $k_1 = 2.0$ and $k_2 = 3.0 \times 10^{-3}$ are correct only to the number of significant digits shown. Find the maximum absolute error and the maximum relative error in y.

12. The derivative of a function $f(x)$ at a particular value of x can be approximately calculated by $f'(x) \approx \frac{f(x+h)-f(x)}{h}$. For $f(x) = 7e^{0.5x}$, find the percentage error in calculating $f'(2)$ using values from $h = 0.3$ and $h = 0.15$.

13. If one chooses 6 terms of the Maclaurin series for e^x to calculate $e^{0.7}$, how many significant digits can you trust in the solution? Find your answer without knowing or using the exact answer of $e^{0.7}$.

14. In the Taylor's series expansion of e^x, find the minimum number of terms it would require to get an approximation of e^1 within a magnitude of true error less than 10^{-6}.

15. Find the condition number of the following functions:

    (a) $f(x) = \sqrt{x}$, for all $x \in [0, \infty)$

    (b) $f(x) = 10/(1 - x^2)$, for all $x \in \mathbb{R}$.

Answers:

(1) 2.346, 2.346, 2.344, 1.474, 1473, 0.002760, 0.002766.

(2) 2 (30),3 (114),3 (14.0).

(3) 0.01999.

(4) 8.652×10^5, 50, 0.5779×10^{-4}, 0.5779×10^{-2}.
37.46, 0.2350×10^{-2}, 0.6273×10^{-4}, 0.6273×10^{-2}.

(5) 692.3, 0.1, 0.2×10^{-3}.

(6) (i) 0.3300×10^{-3}, 0.4165×10^{-4}
(ii) 0.1170×10^{-2}, 0.1477×10^{-3}.

(7) 2.066, 2.06, former is more accurate.

(8) 0.3183×10^{-3}, 1.140×10^{-3}.

(9) (i) 0.004, 0.001 (ii) 0.020, 0.005.

(10) 0.27×10^{-2}, 0.34×10^{-2}.

(11) 0.020, 0.010.

(12) Percentage error = 4.3 %.

(13) 3 significant digits.

(14) Minimum 11 terms.

(15) (a)1/2
(b) The function is ill-conditioned near x = 1 and x = -1, otherwise it is well-conditioned for all $x \in \mathbb{R}$.