DEPARTMENT OF MATHEMATICS, IIT ROORKEE
MAB-103: Numerical Methods

Assignment-2 System of Linear Equations Session: 2025-26

1. Solve the following linear system of equations by Gauss elimination method, with partial pivoting. Give your answers in 3 decimal places.
   (i) $3y + 5z = 1.20736, \quad 3x - 4y = -2.34066, \quad 5x + 6z = -0.329193$
   (ii) $x + 10y - z = 3, \quad 10x - y + 2z = 4, \quad 2x + 3y + 20z = 7$
   (iii) $8y + 2z = -7, \quad 3x + 5y + 2z = 8, \quad 6x + 2y + 8z = 26$
2. Solve the linear system $3x + 2y + 100z = 105, \quad -x + 3y + 100z = 102, \quad x + 2y - z = 2$ by Gauss elimination, with scaling.
3. Apply Crout's and Doolittle's method to solve the following system of equations:
   (i) $x + y + z = 1, \quad 4x + 3y - z = 6, \quad 3x + 5y + 3z = 4.$
   (ii) $2x_1 + x_2 + x_3 - 2x_4 = -10,\ 4x_1 + 2x_3 + x_4 = 8,\ 3x_1 + 2x_2 + 2x_3 = 7,\ x_1 + 3x_2 + 2x_3 - x_4 = -5.$
   (iii) $4x + y + z = 4, \quad x + 4y - 2z = 4, \quad 3x + 2y - 4z = 6.$
4. Apply Jacobi's Method to solve the following system of equation (correct up to 2-D)
   (i) $10x + 2y + z = 9, \quad 2x + 20y - 2z = -44, \quad -2x + 3y + 10z = 22.$
   (ii) $83x + 11y - 4z = 95, \quad 7x + 52y + 13z = 104, \quad 3x + 8y + 29z = 71$
5. Apply Gauss-Seidel Method to solve the following system of equation (correct up to 3D)
   (i) $\begin{bmatrix} 20 & 1 & -2 \\ 3 & 20 & -1 \\ 2 & -3 & 20 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 17 \\ -18 \\ 25 \end{bmatrix}$
   (ii) $10x_1 - 2x_2 - x_3 - x_4 = 3$
   $-2x_1 + 10x_2 - x_3 - x_4 = 15$
   $-x_1 - x_2 + 10x_3 - 2x_4 = 27$
   $-x_1 - x_2 - 2x_3 + 10x_4 = -9$
   (iii) $1.4x_1 + 2.3x_2 + 3.7x_3 = 6.5$
   $3.3x_1 + 1.6x_2 + 1.3x_3 = 10.3$
   $1.5x_1 + 2.9x_2 + 1.1x_3 = 8.8$
6. Solve the linear system Ax=b given by $4x_1 + 3x_2 = 24, \quad 3x_1 + 4x_2 - x_3 = 30, \quad -x_2 + 4x_3 = 24,$ taking $\omega = 1.25$, $X_0 = (1, 1, 1)^T$ correct up to 3D, using SOR method with Gauss- Seidel scheme.
7. Find the solution of the following system of equations using SOR method (correct up to 3D) taking $\omega = 1.1$, $X(0) = 0$;
   (i) $3x_1 - x_2 + x_3 = 1, \quad 3x_1 + 6x_2 + 2x_3 = 0, \quad 3x_1 + 3x_2 + 7x_3 = 4,$ using with Jacobi's scheme.

(ii) $10x_1 - x_2 = 9, \quad -x_1 + 10x_2 - 2x_3 = 7, \quad -2x_2 + 10x_3 = 6$, using with Gauss-Seidel scheme.

8. Assume the LU decomposition of a square matrix A of size 500×500 takes 5 seconds. How many systems $A\mathbf{x} = \mathbf{b}_1, \ldots, A\mathbf{x} = \mathbf{b}_k$ can be solved in the next 6 seconds after the decomposition is complete?

9. Prove the following theorems:

   (i) If A is a strictly diagonally dominant matrix, then the Gauss Jacobi iteration scheme converges for any initial starting vector.

   (ii) If A is a strictly diagonally dominant matrix, then the Gauss Seidel iteration scheme converges for any initial starting vector.

Answers:

(1) (i) (0.143, 0.692, -0.174)

(ii) (0.375, 0.289, 0.269)

(iii) (4.000, -1.000, 0.500)

(2) (1, 1, 1)

(3) (i) (1.00, 0.50, -0.50),

(ii) (5, 6, -10, 8)

(iii) (1.00, 0.500, -0.500)

(4) (i) (1.00, -2.00, 3.00)

(ii) (1.06, 1.37, 1.96)

(5) (i) (1.00, -1.00, 1.00)

(ii) (1.000, 2.000, 3.000, 0.000)

(iii) (2.298, 1.975, -0.340)

(6) SOR: (3.000, 4.000, -5.000)

(7) (i) (0.035, -0.237, 0.658)

(ii) (0.996, 0.958, 0.791).

(8) 200