DEPARTMENT OF MATHEMATICS, IIT ROORKEE
MAB-103: Numerical Methods

Assignment-3 Non-Linear Equations Session: 2025-26

1. Solve by bisection method to obtain a root of the equation $3x^3 + 5x - 40 = 0$, correct to 3D.

2. Use the method of bisection to obtain the smallest positive root of the equations
(i) $x^3 - 5x + 1 = 0$,
(ii) $\cos x = 3x - 1$, correct to 3D.

3. Use Regula Falsi (Method of False Position),
(i) to find both the roots of the equation $3x^2 - 5x - 7 = 0$, correct to 3D.
(ii) to find a positive root of the equation $\cos x - xe^x = 0$, correct to 3D.

4. Apply the method of Regula Falsi to find the smallest positive root of $x^3 - 3x + 1 = 0$, correct to 3D.

5. Verifying the condition for convergence, find a positive root of the following equations, correct to 4 significant digits, using fixed point iteration method:
(i) $2x = \cos x + 3$,
(ii) $x^3 + x^2 - 100 = 0$,
(iii) $8e^{-x}\sin x - 1 = 0$,
(iv) $\sin x + \cos(1 + x^2) - 1 = 0$.

6. Use the fixed point iteration method to find the root of the equation $x^3 + 3x^2 + 2 = 0$ near $x = -3$, correct to 5D.

7. (i) Find a real root of the equation $-4x + \cos x + 2 = 0$, by Newton Raphson method, correct to 5D.
(ii) Find the smallest positive root of (a) $e^x \sin x = 1$ and (b) $e^x = 4x$, by Newton-Raphson method, correct to 5D.

8. (i) Use Newton-Raphson method to derive the algorithm for computing
(a) $\sqrt{q}$, (b) $\sqrt[4]{q}$, $q \in \mathbb{R}_+$.
(ii) Use the algorithm to compute
(a) $\sqrt{10}$, correct to 3D.
(b) $\sqrt{21}$, correct to 5D.
(c) the fourth-root of 32, correct to 4D.

9. Solve the following system of nonlinear equations correct to 4D, by iteration method
(i) $x - 3log_{10}x - y^2 = 0,\ 2x^2 - xy - 5x = 0$, when $x_0 = 3.4,\ y_0 = 2.2$.
(ii) $x^3 + 3y^2 - 20.92 = 0,\ x^2 + 2y + 1.958 = 0$, when $x + 0 = 1.30,\ y_0 = -2.00$.

10. Solve the following system of equations by Newton-Raphson method, correct to 3D:
(i) $x^2 + y^2 = 1.5,\ xy = 0.4$ when $x_0 = 1.5,\ y_0 = 0.5$.
(ii) $log_e(x^2 + y) - 2 + y = 0,\ \sqrt{x} + xy = 0.4$ when $x_0 = 3,\ y_0 = 0$.

11. Determine the formula

$$n \geq \frac{\log(b_0 - a_0) - \log(\epsilon) - \log(a_0)}{\log 2} - 1$$

involving a_0, b_0 and ϵ for the number of steps that should be taken in the bisection algorithm to ensure that the root is determined with relative accuracy $\leq \epsilon$. Assume $a_0 > 0$.

12. (a) Obtain the condition of convergence and the rate of convergence of the Fixed-Point iteration method.

    (b) Obtain the condition of convergence and the rate of convergence of Newton Raphson's method.

13. Find the double root of the equation

$$x^4 - 6.75x^2 + 6.25x - 1.5 = 0,$$

correct to 4-decimal places, when
(i) multiplipity is known;
(ii) multiplicity is unknown.

14. If f(x) $=$ 0 has a root α of multiplicity 2, then show that the Newton-Raphson iterative scheme is given by

$$x_{n+1} = x_n - 2\frac{f(x_n)}{f'(x_n)},$$

and has a quadratic rate of convergence.

15. If f(x) $=$ 0 has a double root α, then show that the Newton-Raphson iterative scheme given by

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)},$$

has a linear rate of convergence.

Answers:

(1) 2.138

(2) (i) 0.202 (ii) 0.607

(3) (i) 2.57, -0.907 (ii) 0.518

(4) 0.347

(5) (i) 1.524, (ii) 4.331, (iii) 1.989 (iv) 1.945

(6) -3.1958

(7) (i) 0.69243, (ii) (a) 0.58853, (b) 0.35740

(8) (i) (a) $x_{n+1} = \frac{1}{2}\left(x_n + \frac{q}{x_n}\right)$, (b) $x_{n+1} = \frac{1}{4}\left(3x_n + \frac{q}{x_n^3}\right)$
(ii) (a) 3.162, (b) 4.58258, (c) 2.3784

(9) (i) (3.1424, 1.2848) (ii) (1.6499, -2.3401)

(10) (i) (1.177, 0.340) (ii) (3.422, -0.424)

(13) 0.5000