Lecture-29

26/10/2023.

$$\boxed{-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x,t)}{\partial x^2} + V(x,t)\,\psi(x,t) = i\hbar\frac{\partial\psi(x,t)}{\partial t}}$$

⇒ Consistant with $\lambda = \frac{h}{p}$ & $E = h\nu$.

⇒ " " $E = \frac{p^2}{2m} + V$.

⇒ Must be linear in ψ.

⇒ For free particle, $V = V_0$.

$$\psi = \cos(kx - \omega t) + i\sin(kx - \omega t)$$

* Schrodinger eq^n relates I^{st} time derivative to second derivative.

* $\psi \longrightarrow$ complex ⇒ We should not attemp to give ψ a physical existance ⇒ The reason is that complex quantity cannot be measured by any actual physical instrument.

Probability density ⇒

$$\boxed{P(x,t) = \psi^*(x,t)\,\psi(x,t)}$$

Always real & non negative.

Normalization ⇒

$$\boxed{\int_{-\infty}^{\infty} \psi^*\psi\, d\tau = 1}$$

Expectation Value ⇒

$$\boxed{\langle f(x,t)\rangle = \int_{-\infty}^{\infty} \psi^*(x,t)\, f\, \psi(x,t)\, dx}$$

Expectation value of Momentum $\bar{p}$ & total energy E ⇒

$$\bar{p} = \int_{-\infty}^{\infty} \psi^*(x,t)\, p\, \psi(x,t)\, dx$$

To evaluate this integral $\bar{p}$ must be expressed as a fun^n of variable x & t.

⇒ Uncertainty principle tells us that it is not possible to write $\bar{p}$ as a fun^n of x because $\bar{p}$ & x can not be simultaneously known.

* Let us look at free particle wave f^n ⇒

$$\psi(x,t) = \cos(kx - \omega t) + i \sin(kx - \omega t) \quad \text{———} \quad (1)$$

$$\frac{\partial \psi}{\partial x} = -k \sin(kx - \omega t) + ik \cos(kx - \omega t)$$

$$= ik\left[\cos(kx - \omega t) + i \sin(kx - \omega t)\right]$$

$$p = \hbar k$$

$$\frac{\partial \psi}{\partial x} = \frac{ip}{\hbar} \psi$$

$$p\psi = -i\hbar \frac{\partial}{\partial x} \psi$$

In one dimensional space $\hat{p} = -i\hbar \frac{\partial}{\partial x}$

" Three " " $\boxed{\hat{p} = -i\hbar \nabla}$

$$\therefore \bar{p} = \int_{-\infty}^{\infty} \psi^*(x,t) \left(-i\hbar \frac{\partial}{\partial x}\right) \psi(x,t)\, dx$$

$$= -i\hbar \int_{-\infty}^{\infty} \psi^*(x,t) \frac{\partial \psi}{\partial x}(x,t) \cdot dx$$

$$\bar{E} = \int_{-\infty}^{\infty} \psi^*(x,t)\, E\, \psi(x,t)\, dx$$

$$\psi = \cos(kx - \omega t) + i \sin(kx - \omega t)$$

$$\frac{\partial \psi}{\partial t} = \omega \sin(kx - \omega t) - i\omega \cos(kx - \omega t)$$

$$= -i\omega \left[\cos(kx - \omega t) + i \sin(kx - \omega t) \right]$$

$$E = \hbar \omega$$

$$\frac{\partial \psi}{\partial t} = -\frac{iE}{\hbar} \psi$$

$$E\, \psi(x,t) = i\hbar \frac{\partial}{\partial t} \psi(x,t).$$

$$\boxed{\hat{E} = i\hbar \frac{\partial}{\partial t}}$$

$$\bar{E} = \int_{-\infty}^{\infty} \psi^*(x,t)\, (i\hbar) \frac{\partial}{\partial t} \psi(x,t)\, dx$$

$$\bar{E} = i\hbar \int_{-\infty}^{\infty} \psi^* \frac{\partial \psi}{\partial t}\, dx. \checkmark$$

Are these relations restricted to the case of free particle only? = No.

$$\frac{p^2}{2m} + V(x,t) = E$$

$$\Rightarrow \frac{1}{2m}\left(-i\hbar \frac{\partial}{\partial x}\right)^2 + V(x,t) = i\hbar \frac{\partial}{\partial t}$$

$$\Rightarrow -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V(x,t) = i\hbar \frac{\partial}{\partial t}$$

This is operator eq^n. It has significance when applied on wave f^n.

$$\Rightarrow \boxed{-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \psi + V\psi = i\hbar \frac{\partial \psi}{\partial t}}$$ Schrodinger eq^n.

Operator representing the classical orbital angular Momentum:

$$\bar{L} = \bar{r} \times \bar{p}$$

$$\bar{L} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ p_x & p_y & p_z \end{vmatrix}$$

$$\hat{L}_x = \hat{y}\hat{p}_z - \hat{z}\hat{p}_y = -i\hbar\left(\hat{y}\frac{\partial}{\partial z} - \hat{z}\frac{\partial}{\partial y}\right)$$

$$\hat{L}_y = \hat{z}\hat{p}_x - \hat{x}\hat{p}_z = -i\hbar\left(\hat{z}\frac{\partial}{\partial x} - \hat{x}\frac{\partial}{\partial z}\right)$$

$$\hat{L}_z = \hat{x}\hat{p}_y - \hat{y}\hat{p}_x = -i\hbar\left(\hat{x}\frac{\partial}{\partial y} - \hat{y}\frac{\partial}{\partial x}\right).$$

Q:→ Consider a particle of mass m which is confined in a one dimensional box of length a. Let say it's wave fu^n for lowest energy state of the particle is

$$\psi(x,t) = \begin{cases} A\cos\frac{\pi x}{a}\, e^{-\frac{iEt}{\hbar}} & -a/2 \; x < a/2 \\ = 0 & x \leq -a/2 \text{ or } x \geq a/2. \end{cases}$$

Evaluate the expectation value of x, p, x^2 & p^2.