ECC 203 : Electromagnetics and Radiating Systems

Smith Chart

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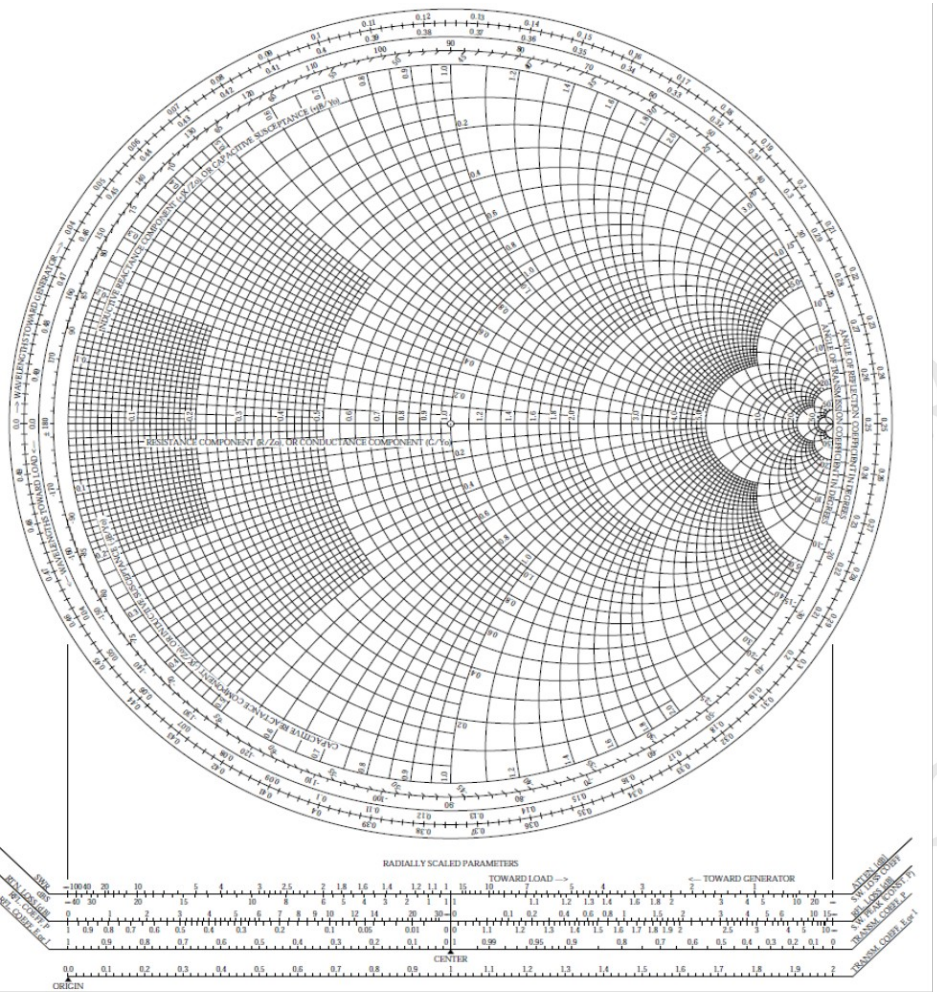
Smith Chart

- The Smith Chart was invented by Phillip Smith in 1939 in order to provide an easily usable graphical representation of the complex reflection coefficient Γ and reading of the associated complex terminating impedance.

- Important Parameters of a Circuit
  - Impedance / Admittance
  - Γ (reflection co-efficient)

- Is there a way to represent both together in a graphical tool ?

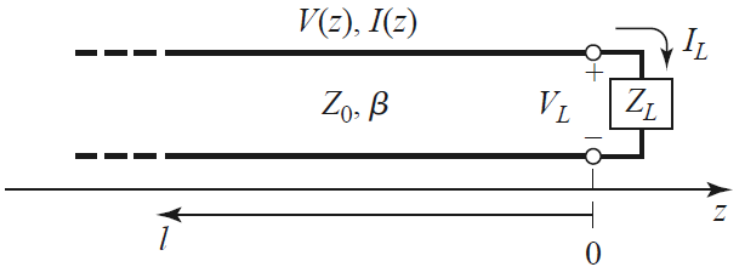


Graphical Aid for solving Transmission Line Problems involving Complex Impedances

Can be used to design Impedance Matching Networks graphically -using both lumped and distributed elements

All Microwave CAD Software and Measurement Instruments have this as the graphical display

Smith Chart – Γ plot



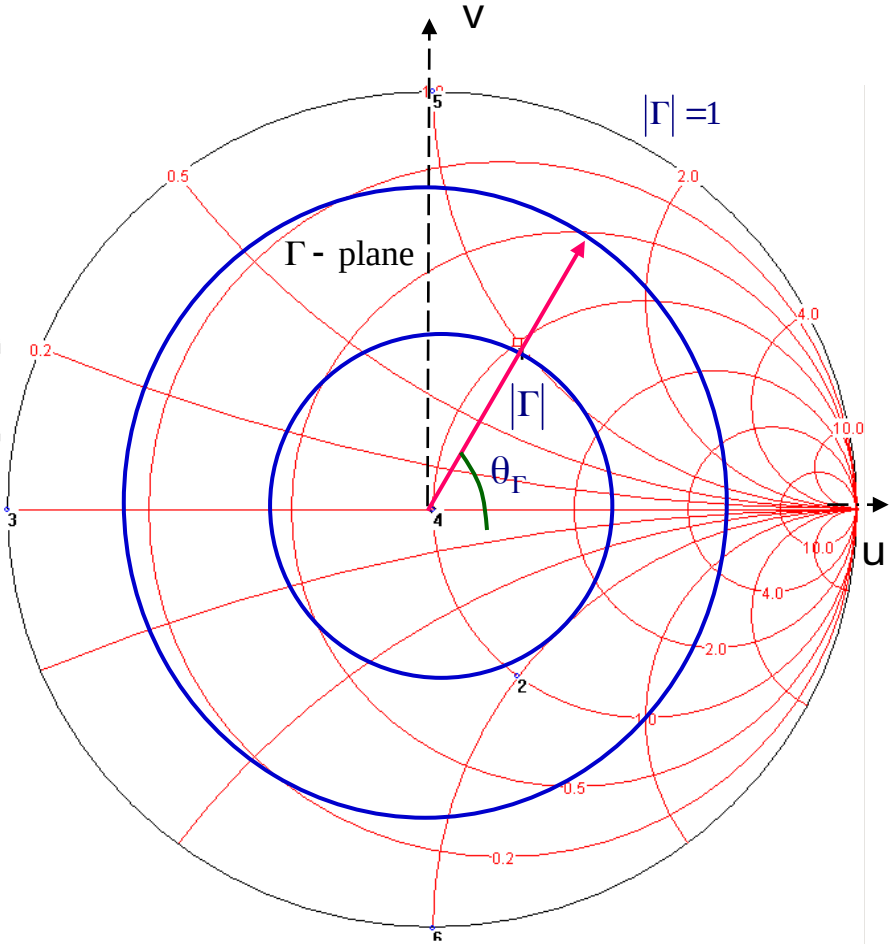
$$Z_L = R + jX$$

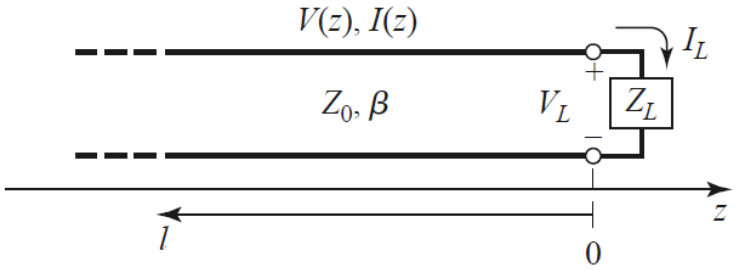
Normalization of Impedance

$$z_L = \frac{Z_L}{Z_0} = r + jx$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{\frac{Z_L}{Z_0} - 1}{\frac{Z_L}{Z_0} + 1}$$

$$\Gamma = \frac{z_L - 1}{z_L + 1} = |\Gamma| e^{j\theta_\Gamma} = u + jv$$

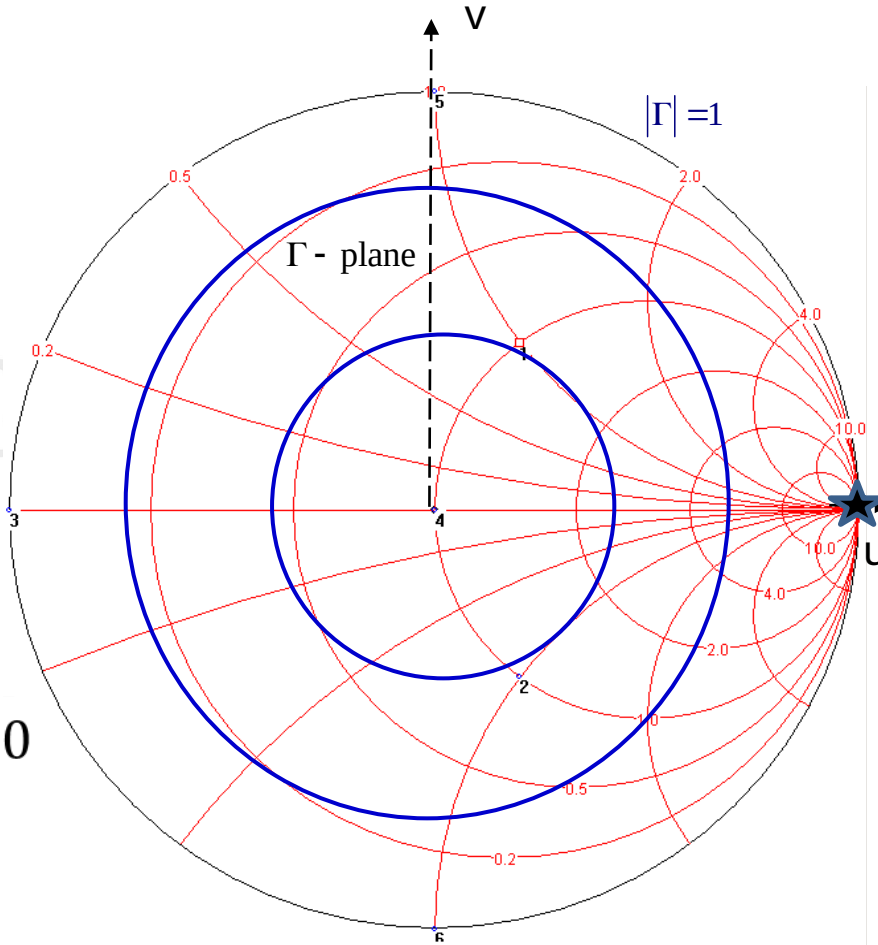




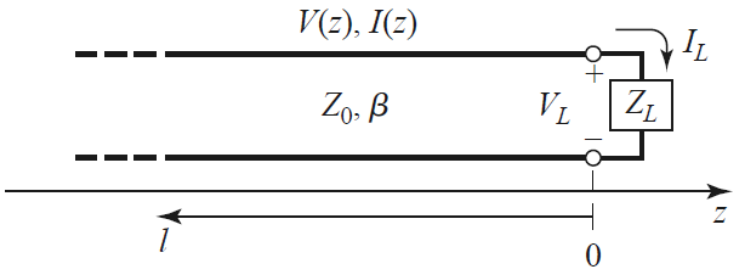
In a 50 Ω system, identify the point on Smith Chart corresponding to Open Circuit Load.

$$z_L = \frac{Z_L}{Z_0} = \frac{\infty}{50} = \infty$$

$$\Gamma = \frac{z_L - 1}{z_L + 1} = \frac{\infty - 1}{\infty + 1} = 1 * e^{j0} = 1 + j0$$



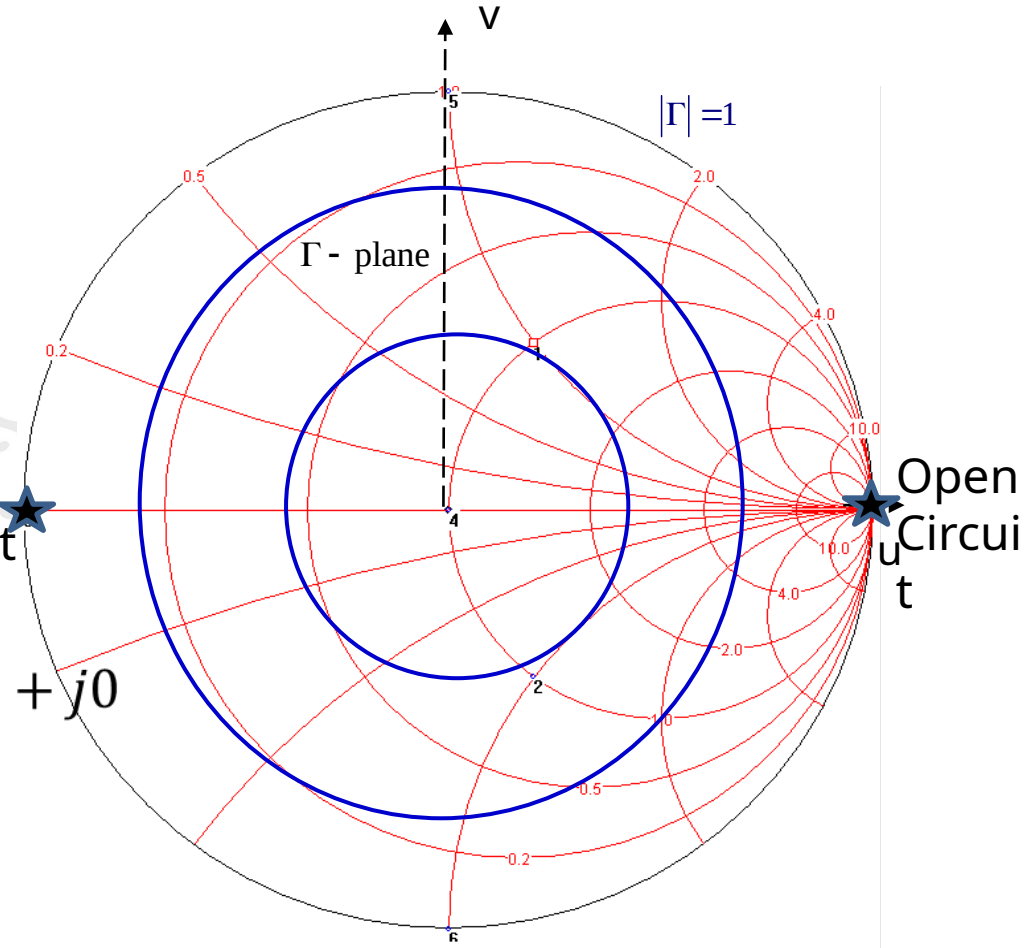
Open Circuit

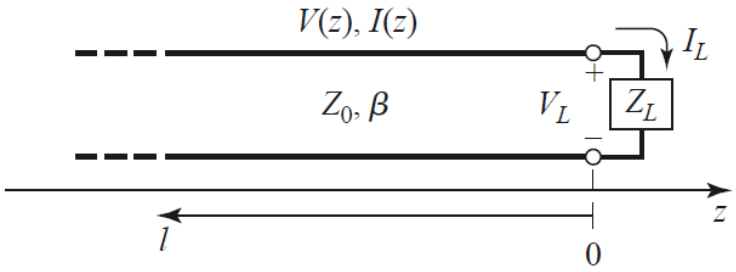


In a 50 Ω system, identify the point on Smith Chart corresponding to Short Circuit Load.

$$z_L = \frac{Z_L}{Z_0} = \frac{0}{50} = 0$$

$$\Gamma = \frac{z_L - 1}{z_L + 1} = \frac{0 - 1}{0 + 1} = 1 * e^{j180} = -1 + j0$$

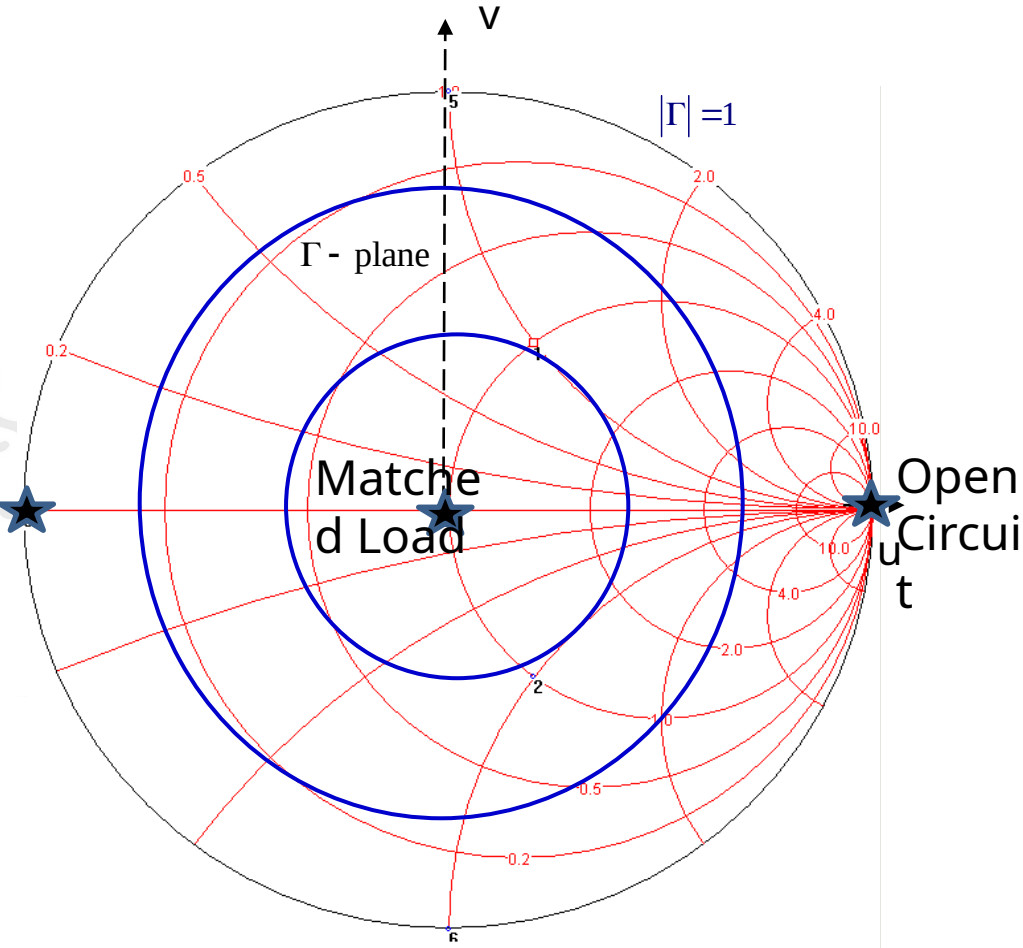




In a 50 Ω system, identify the point on Smith Chart corresponding to Matched Load (i.e. 50 Ω Load).

$$z_L = \frac{Z_L}{Z_0} = \frac{50}{50} = 1$$

$$\Gamma = \frac{z_L - 1}{z_L + 1} = \frac{1 - 1}{1 + 1} = 0 = 0 + j0$$



Smith Chart – Z plot

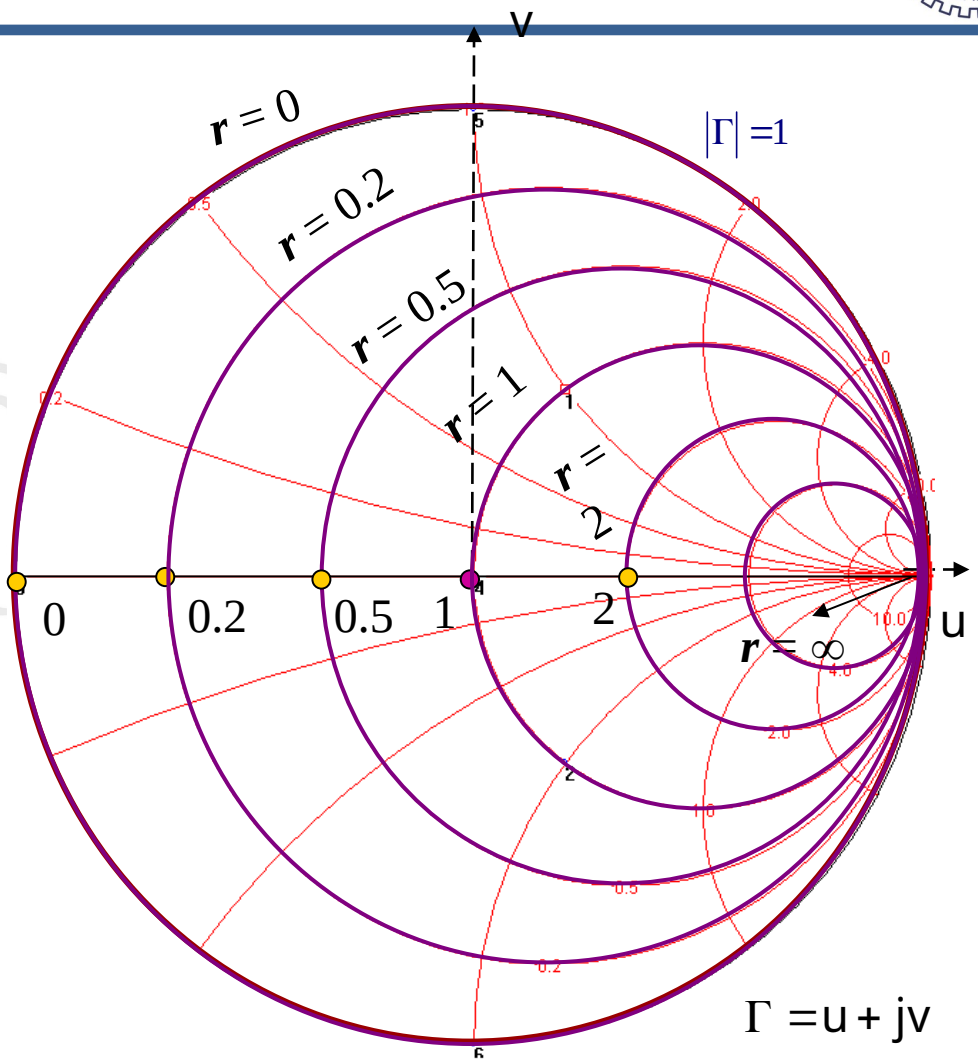
Constant Resistance Circle

$$r = \frac{R}{Z_0}$$

each circle

$r \rightarrow$ *is fixed*

$x \rightarrow$ *varies*



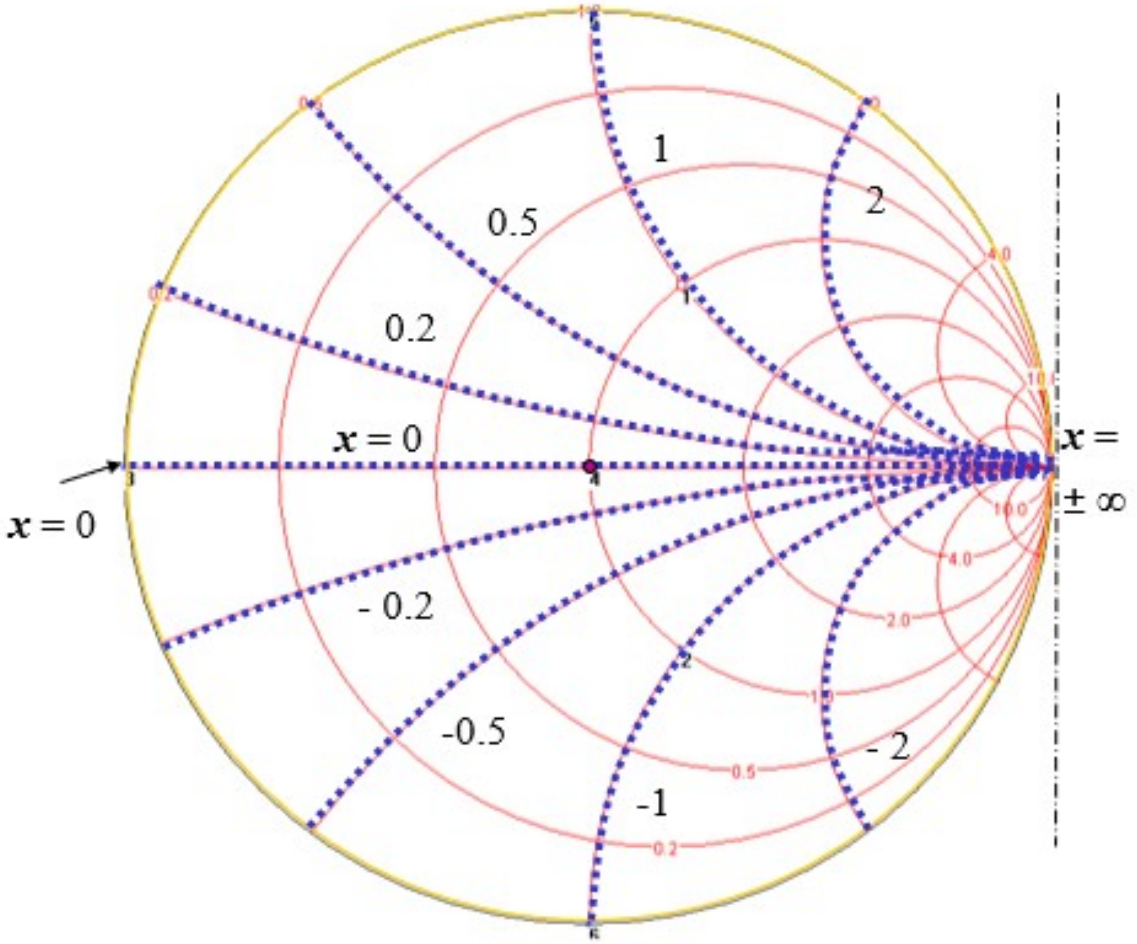
Constant Reactance Circle

$$x = \frac{X}{Z_0}$$

each circle

$r \rightarrow varies$

$x \rightarrow fixed$



Constant Reactance Circle

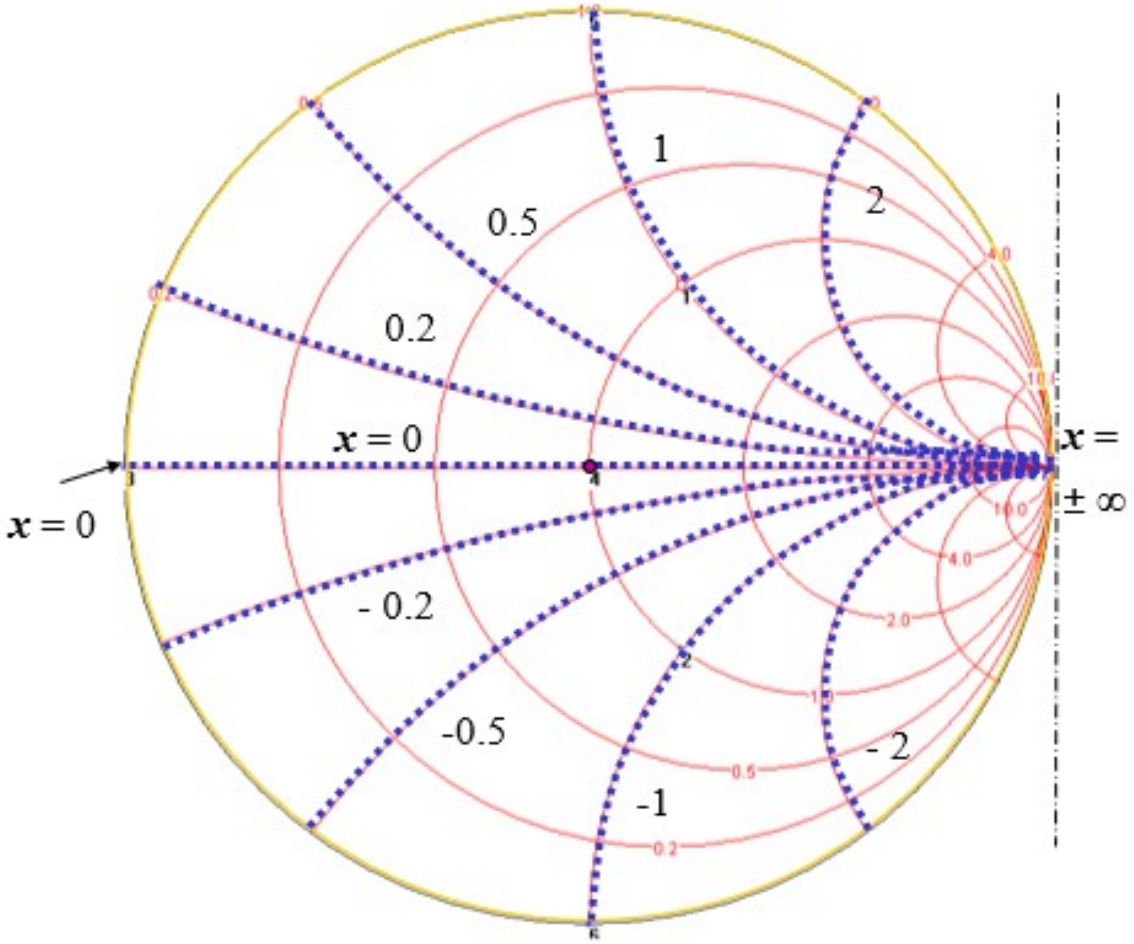
$$x = \frac{X}{Z_0}$$

each circle

$r \rightarrow varies$

$x \rightarrow fixed$

Identify the Inductance Region of the Smith Chart



Constant Reactance Circle

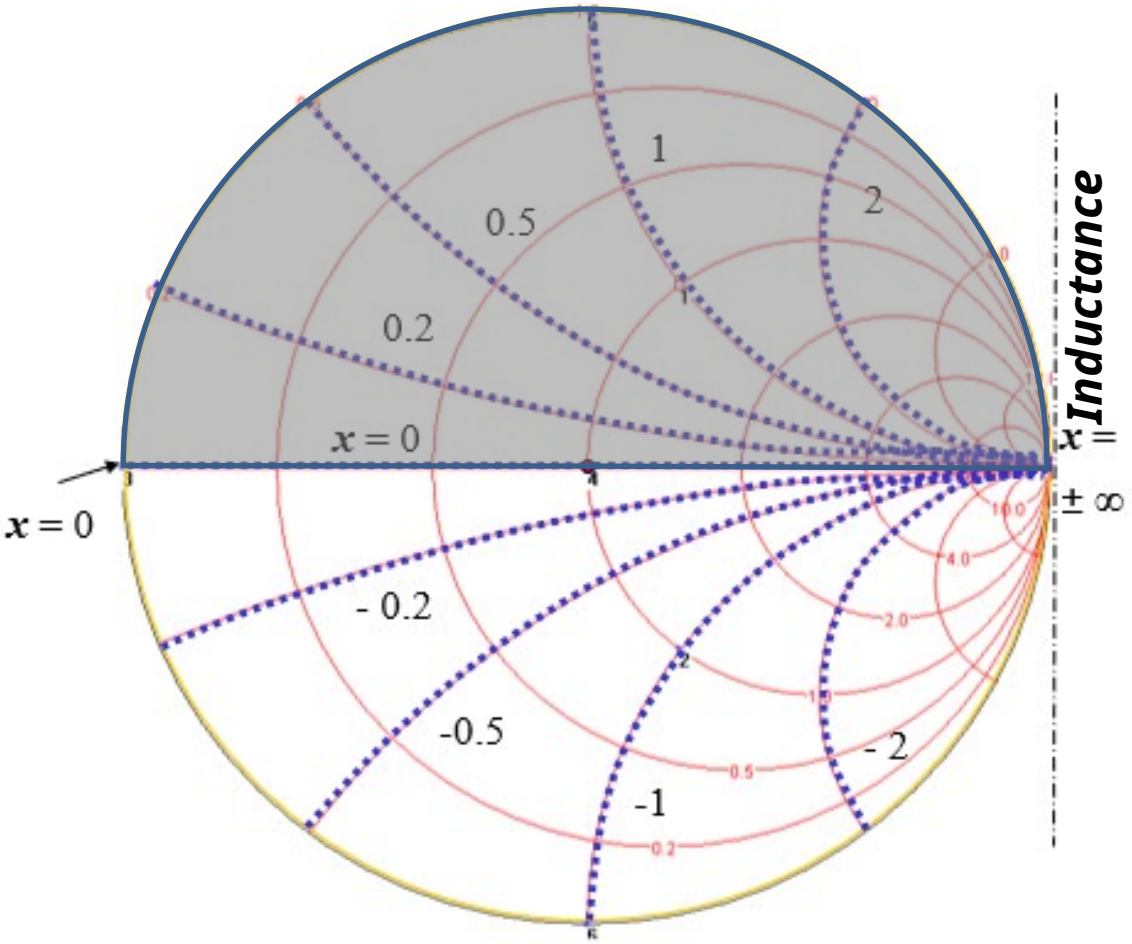
$$x = \frac{X}{Z_0}$$

each circle

$r \rightarrow varies$

$x \rightarrow fixed$

Identify the Inductance Region of the Smith Chart



Constant Reactance Circle

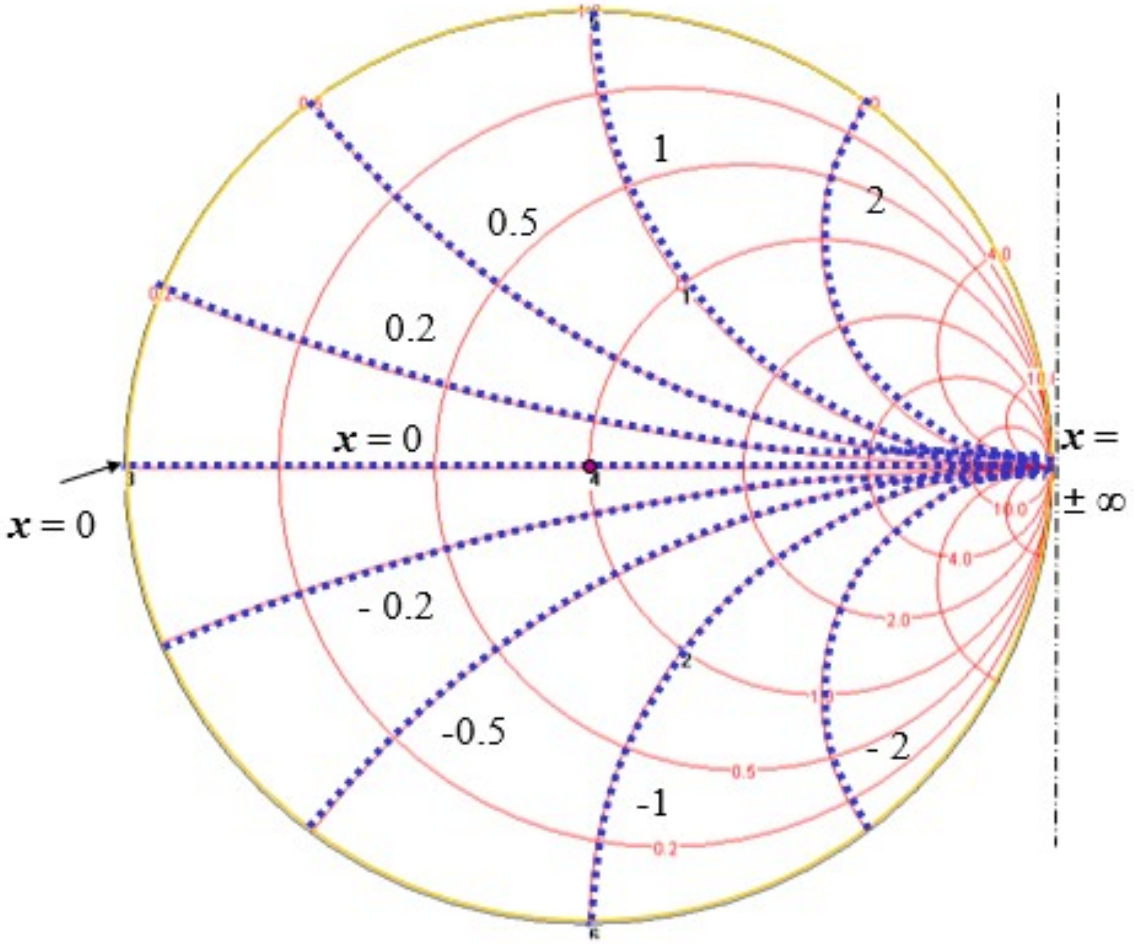
$$x = \frac{X}{Z_0}$$

each circle

$r \rightarrow varies$

$x \rightarrow fixed$

Identify the Capacitance Region of the Smith Chart



Constant Reactance Circle

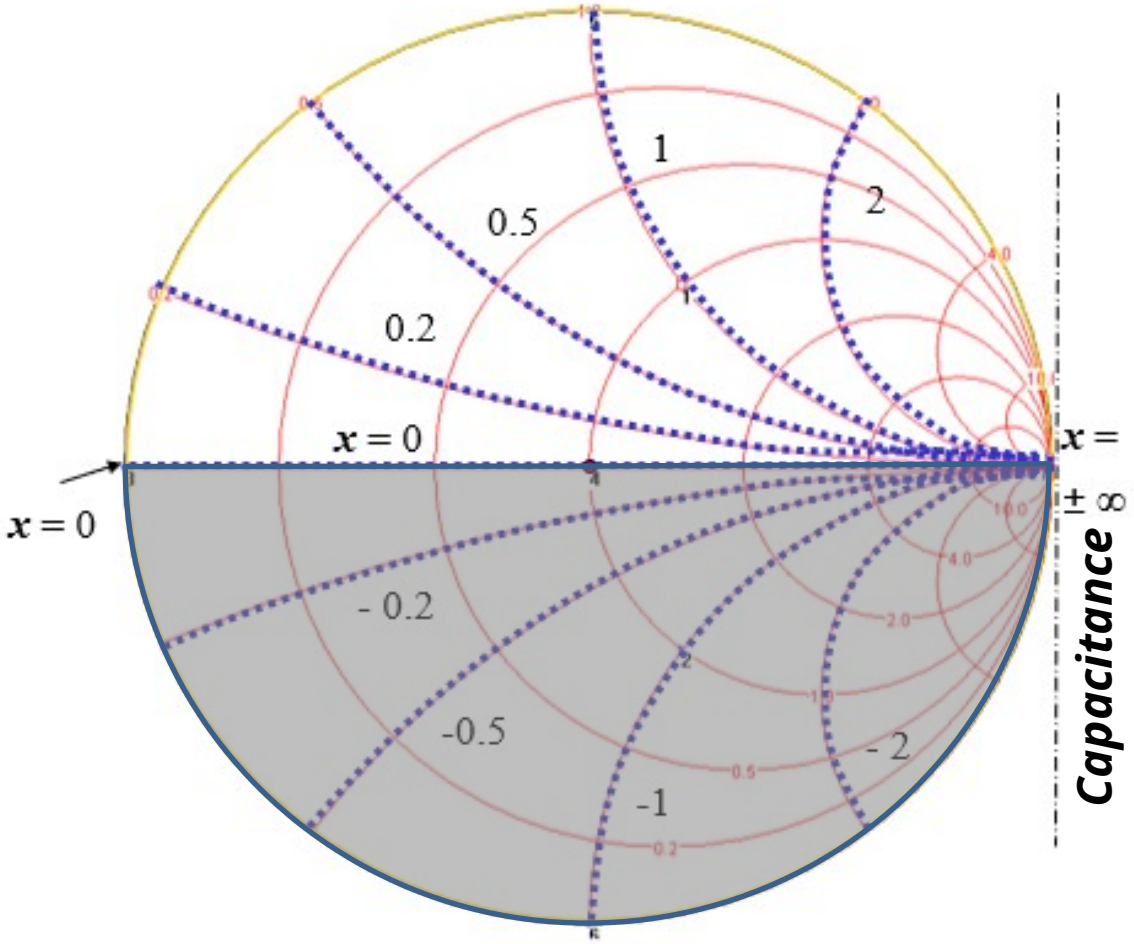
$$x = \frac{X}{Z_0}$$

each circle

$r \rightarrow varies$

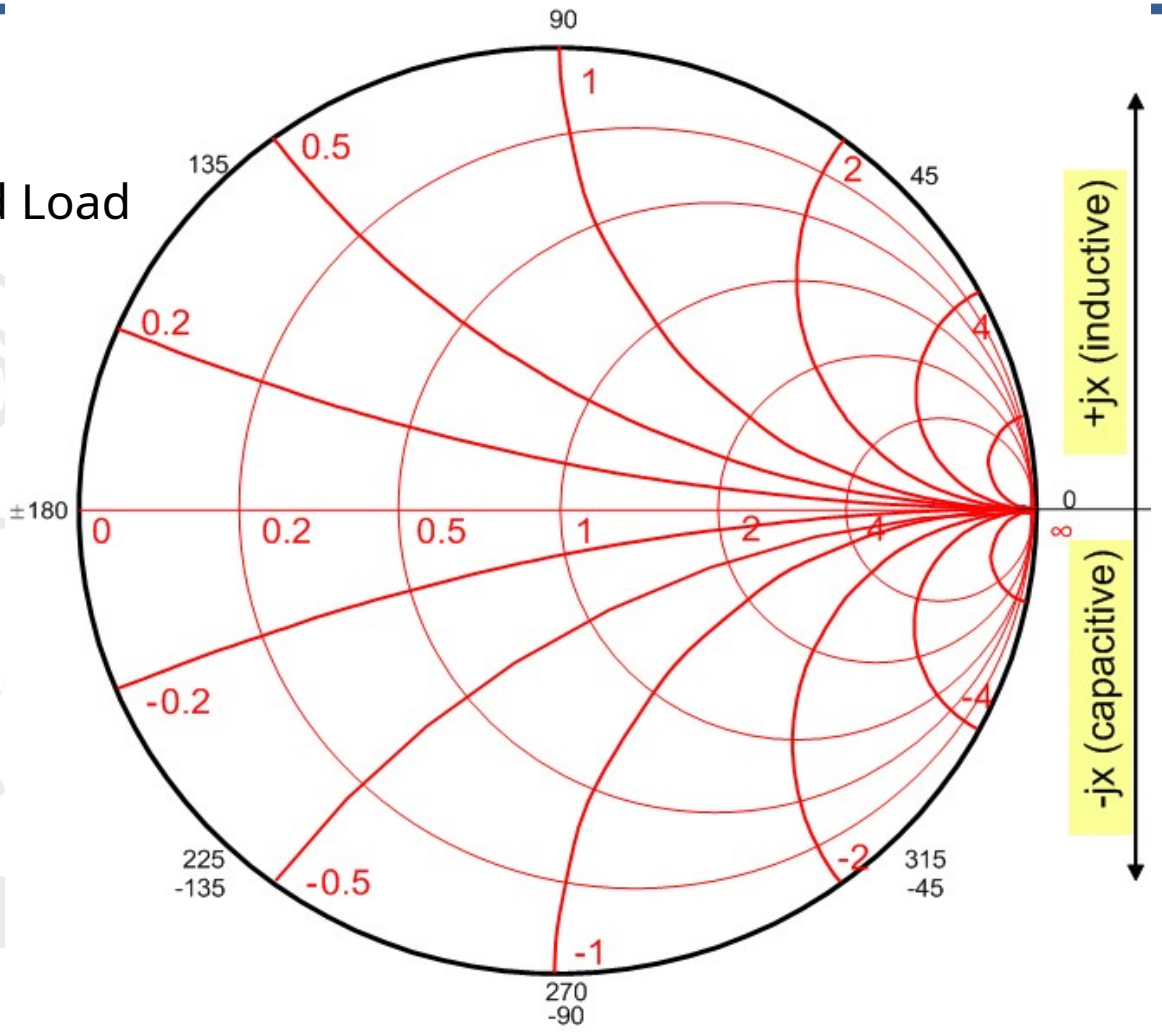
$x \rightarrow fixed$

Identify the Capacitance Region of the Smith Chart



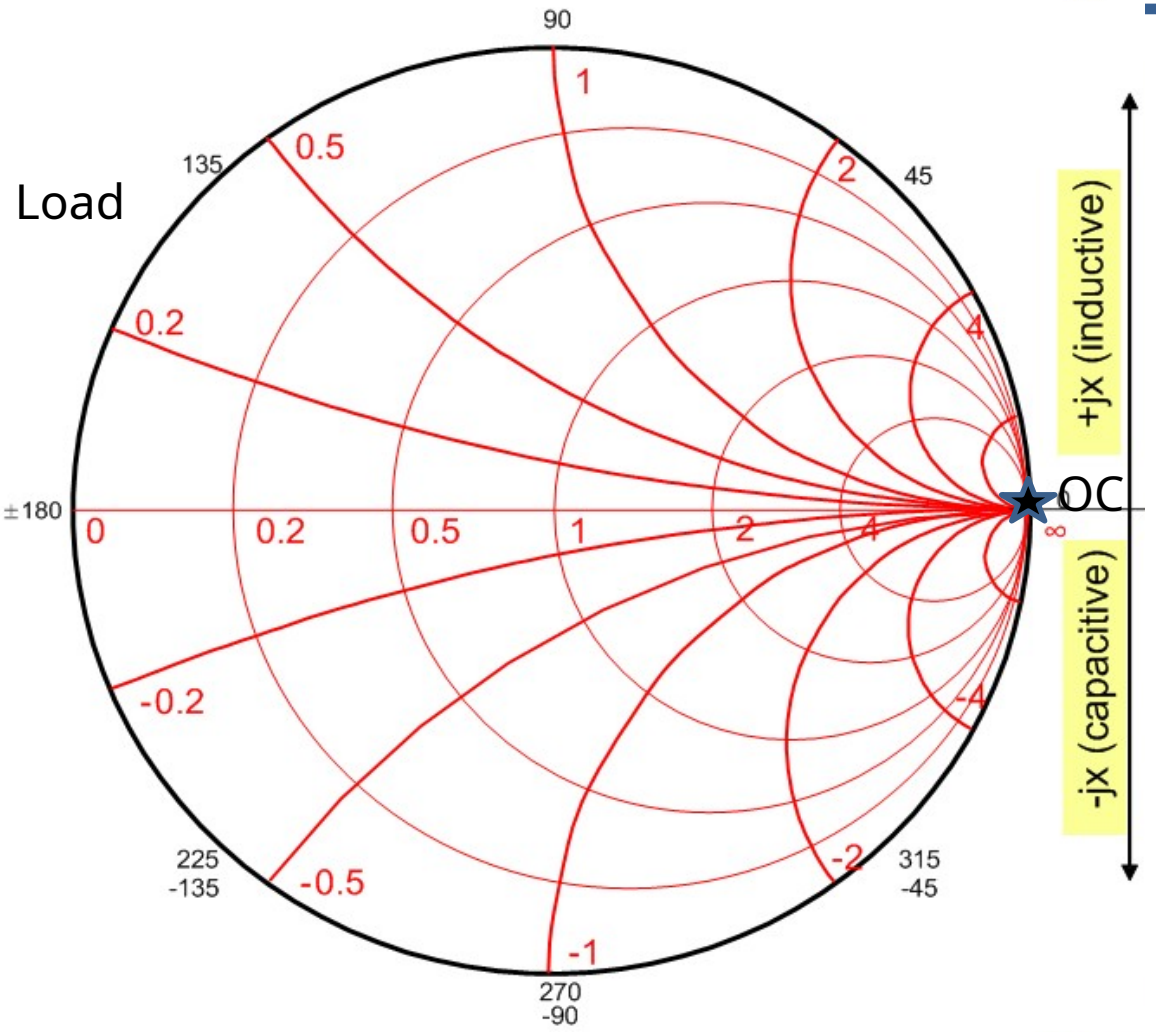
Impedance Plane

Identify O.C , S.C and Matched Load points on the Smith Chart.



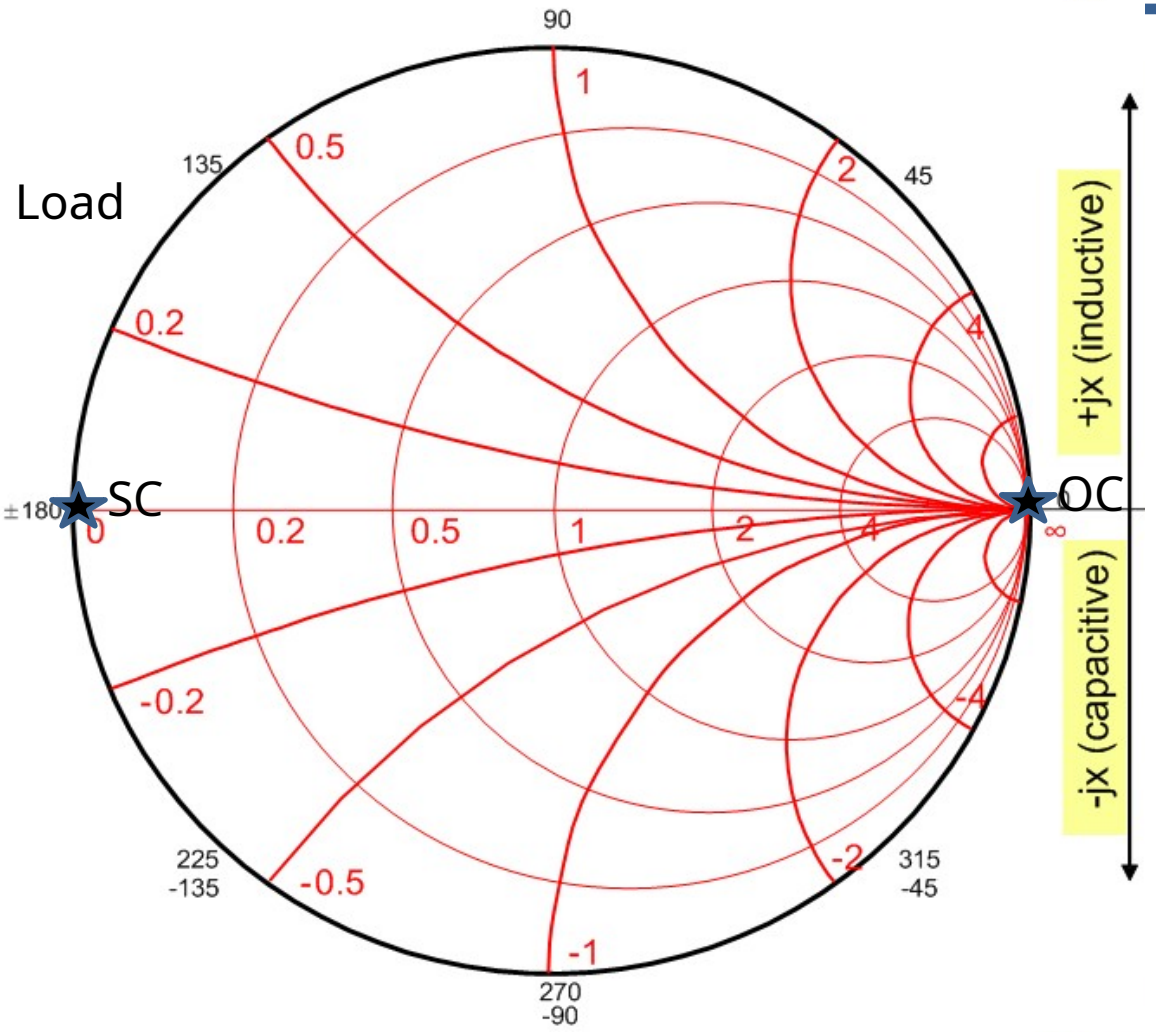
Impedance Plane

Identify O.C , S.C and Matched Load points on the Smith Chart._



Impedance Plane

Identify O.C , S.C and Matched Load points on the Smith Chart.



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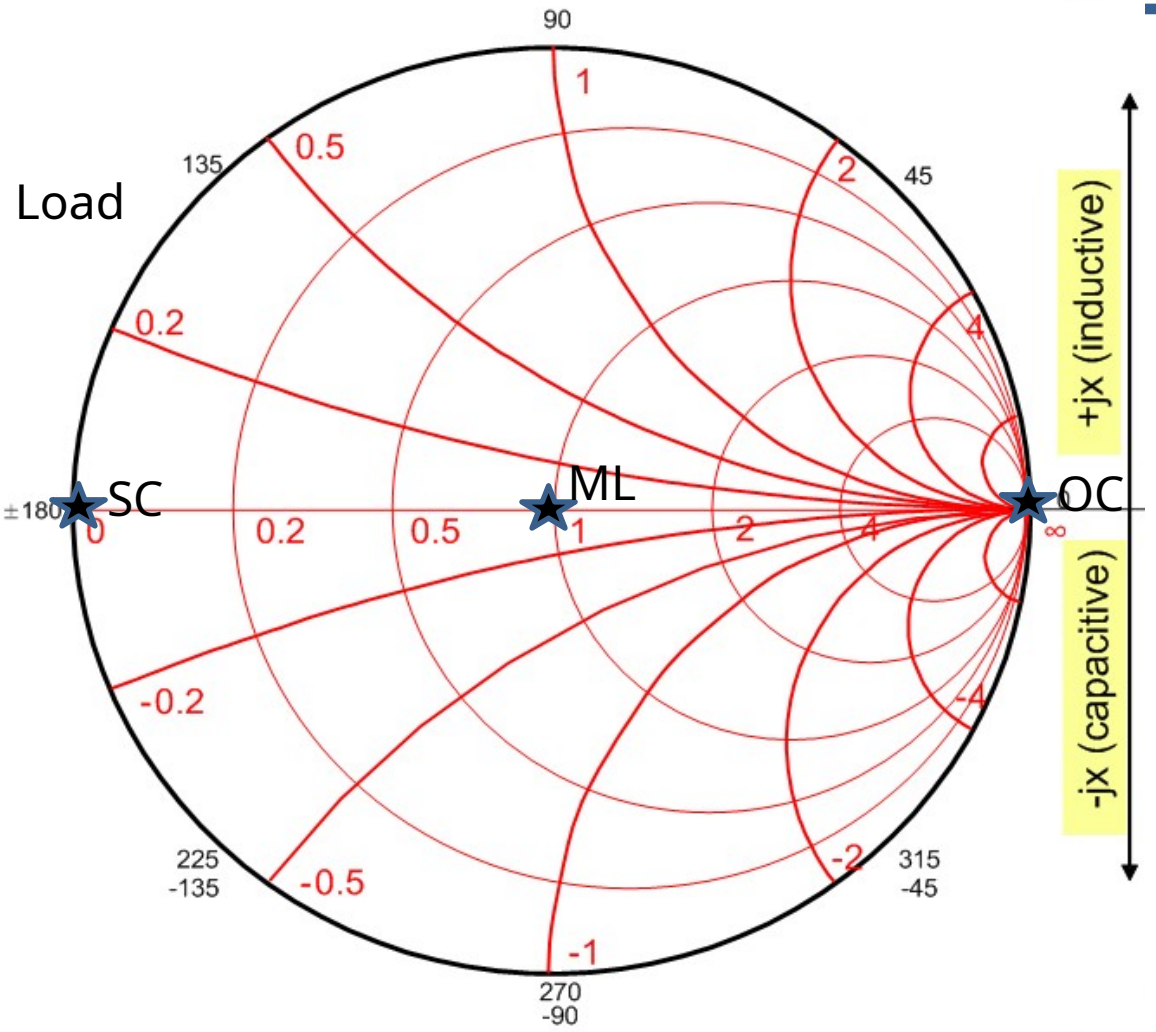
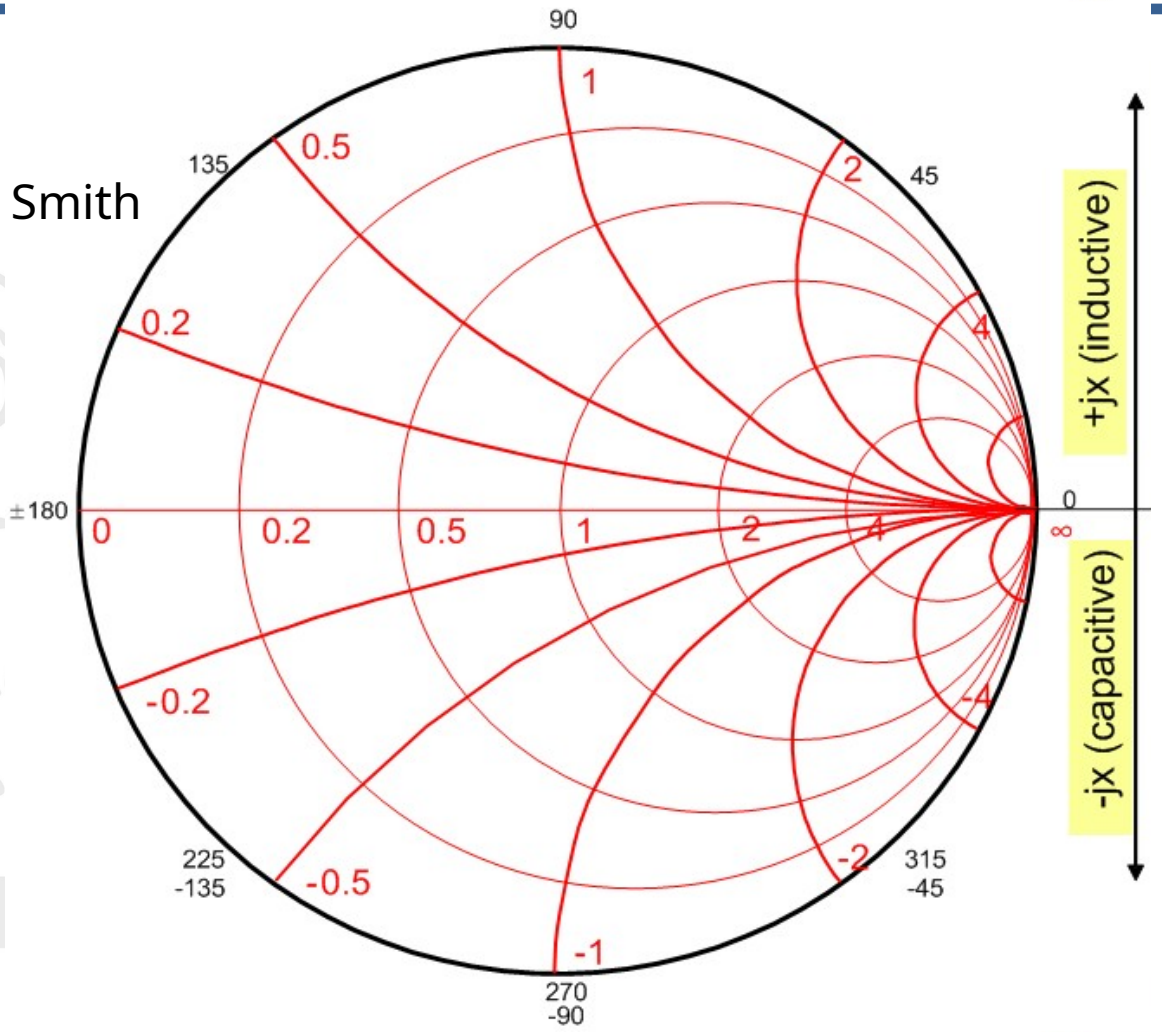


Impedance Plane

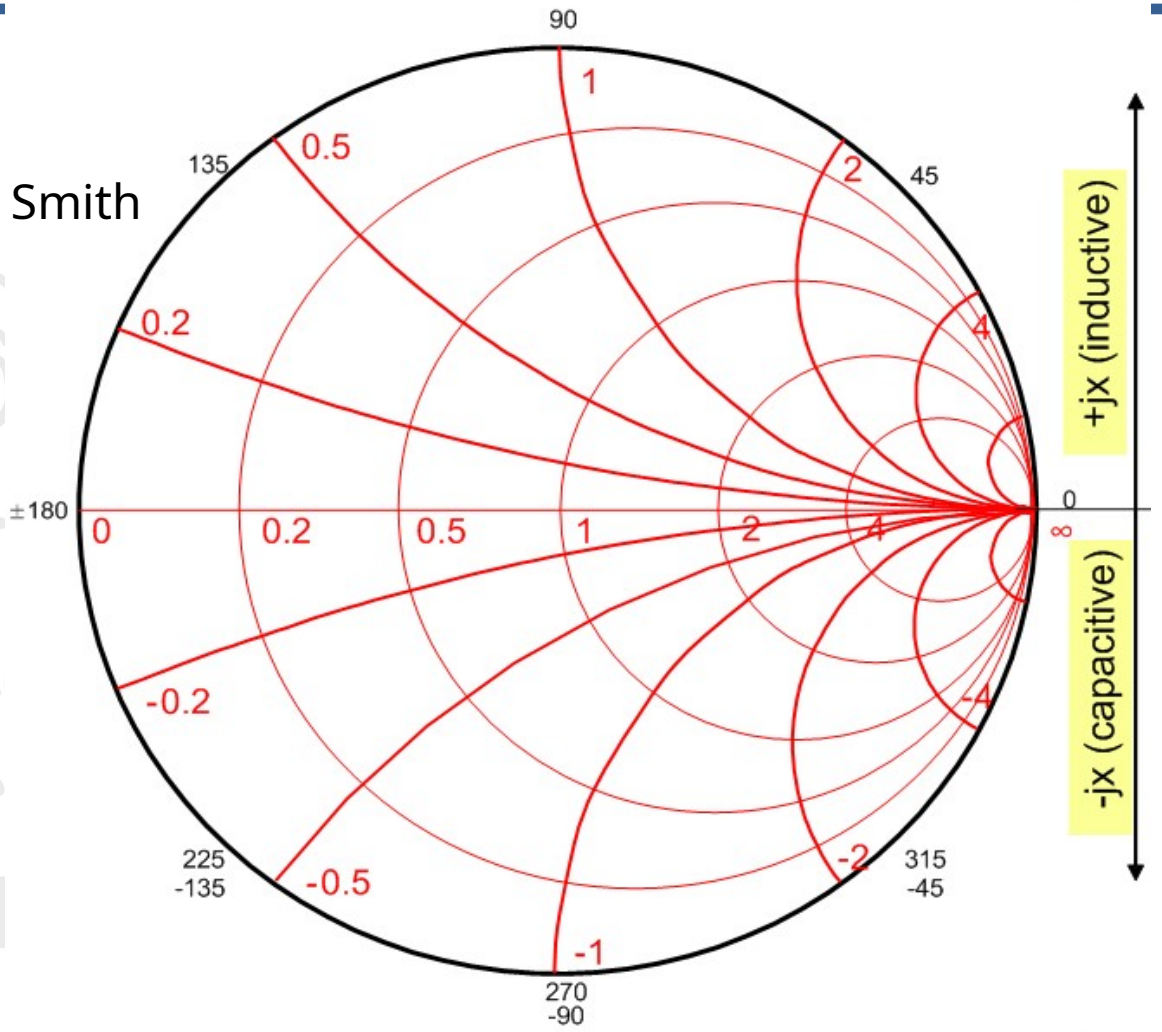
Identify a load = 100 Ω on the Smith Chart



Impedance Plane

Identify a load = 100 Ω on the Smith Chart
(assume a 50 Ω system)

$$r = \frac{R}{Z_0} = \frac{100}{50} = 2$$

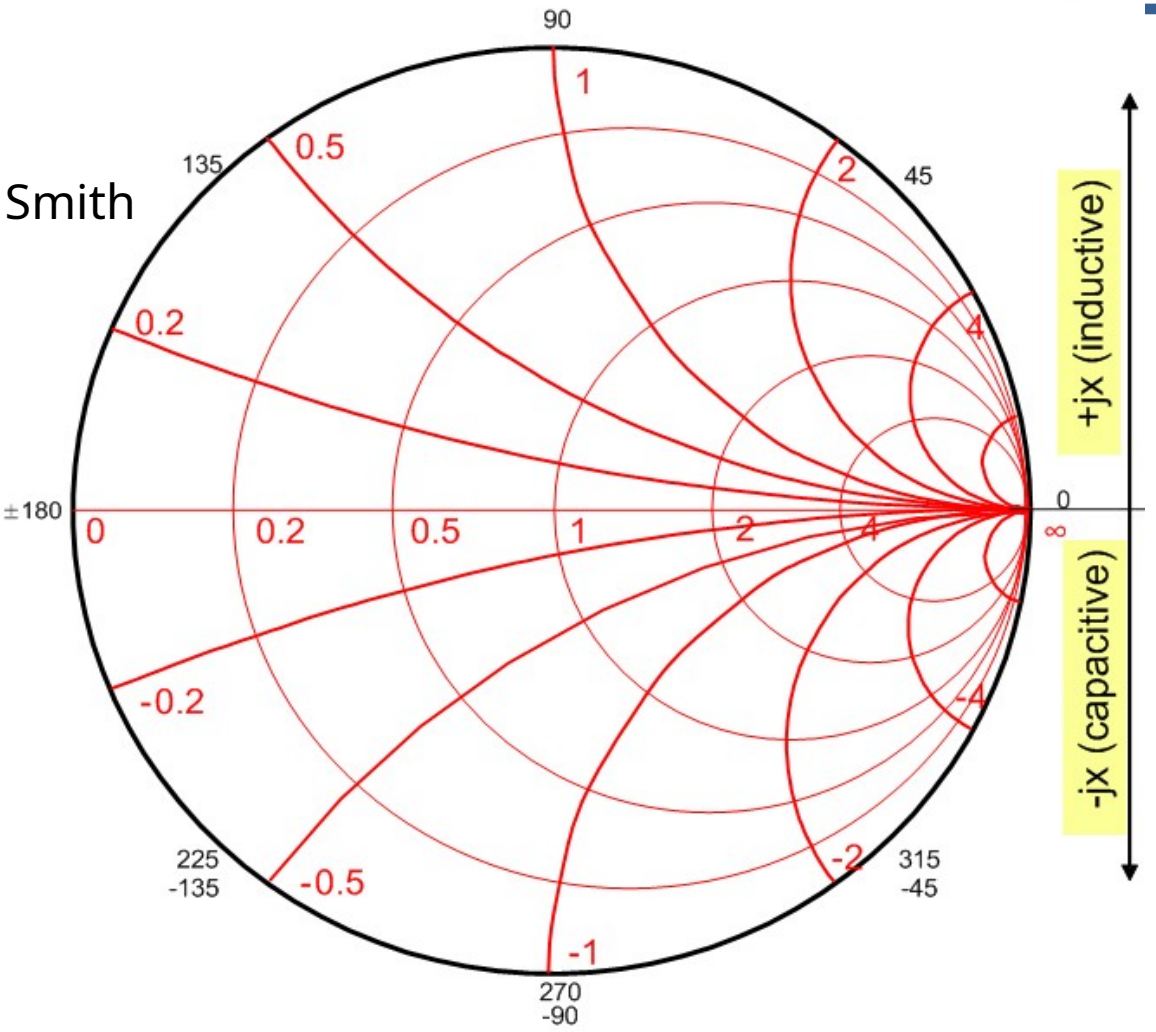


Impedance Plane

Identify a load = 100 Ω on the Smith Chart
(assume a 50 Ω system)

$$r = \frac{R}{Z_0} = \frac{100}{50} = 2$$

$$x = \frac{X}{Z_0} = \frac{0}{50} = 0$$



Impedance Plane

Identify a load = 100 Ω on the Smith Chart
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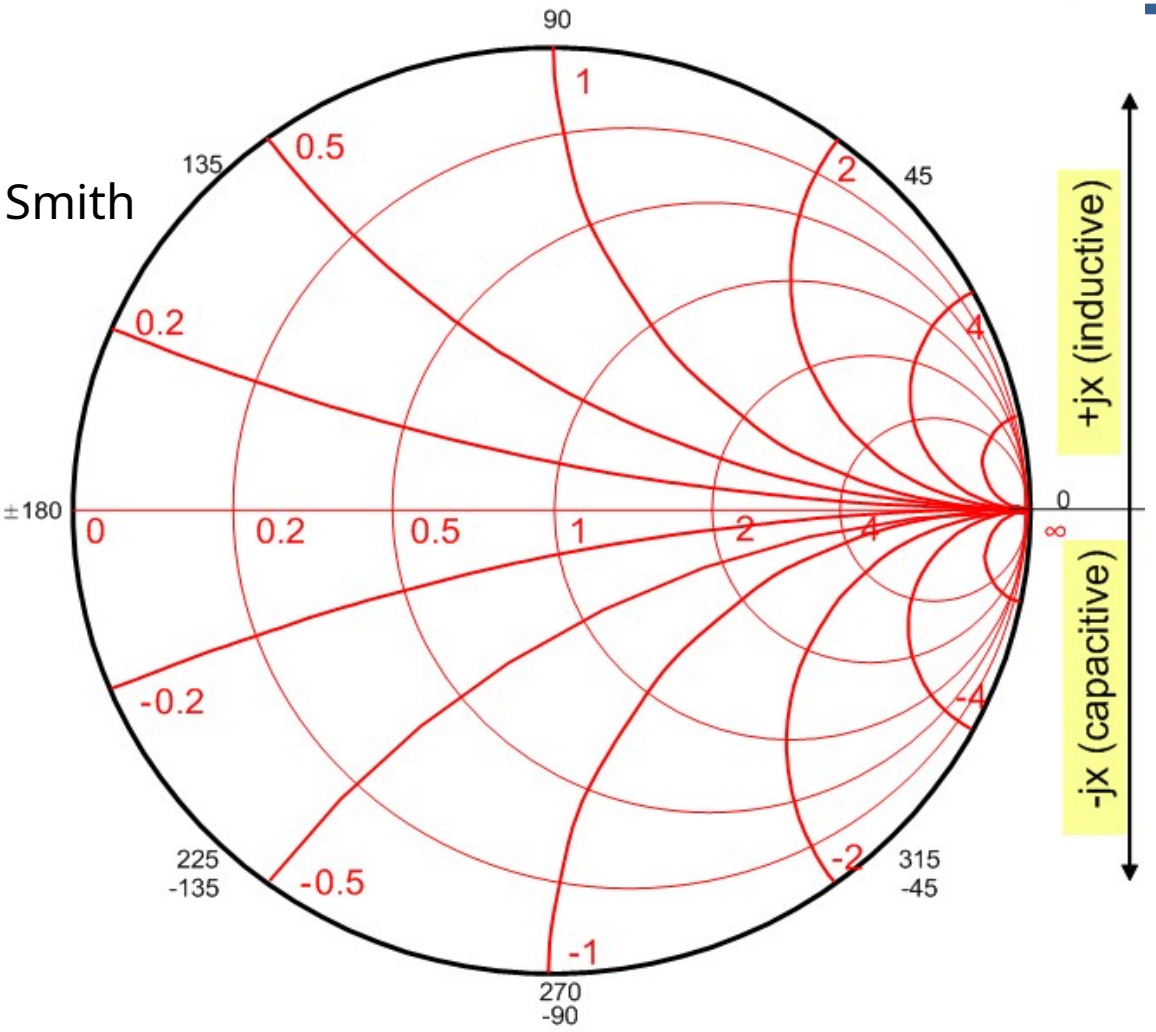
$$r = \frac{R}{Z_0} = \frac{100}{50} = 2$$

$$x = \frac{X}{Z_0} = \frac{0}{50} = 0$$

$$z_L = r + jx = 2 + j0$$



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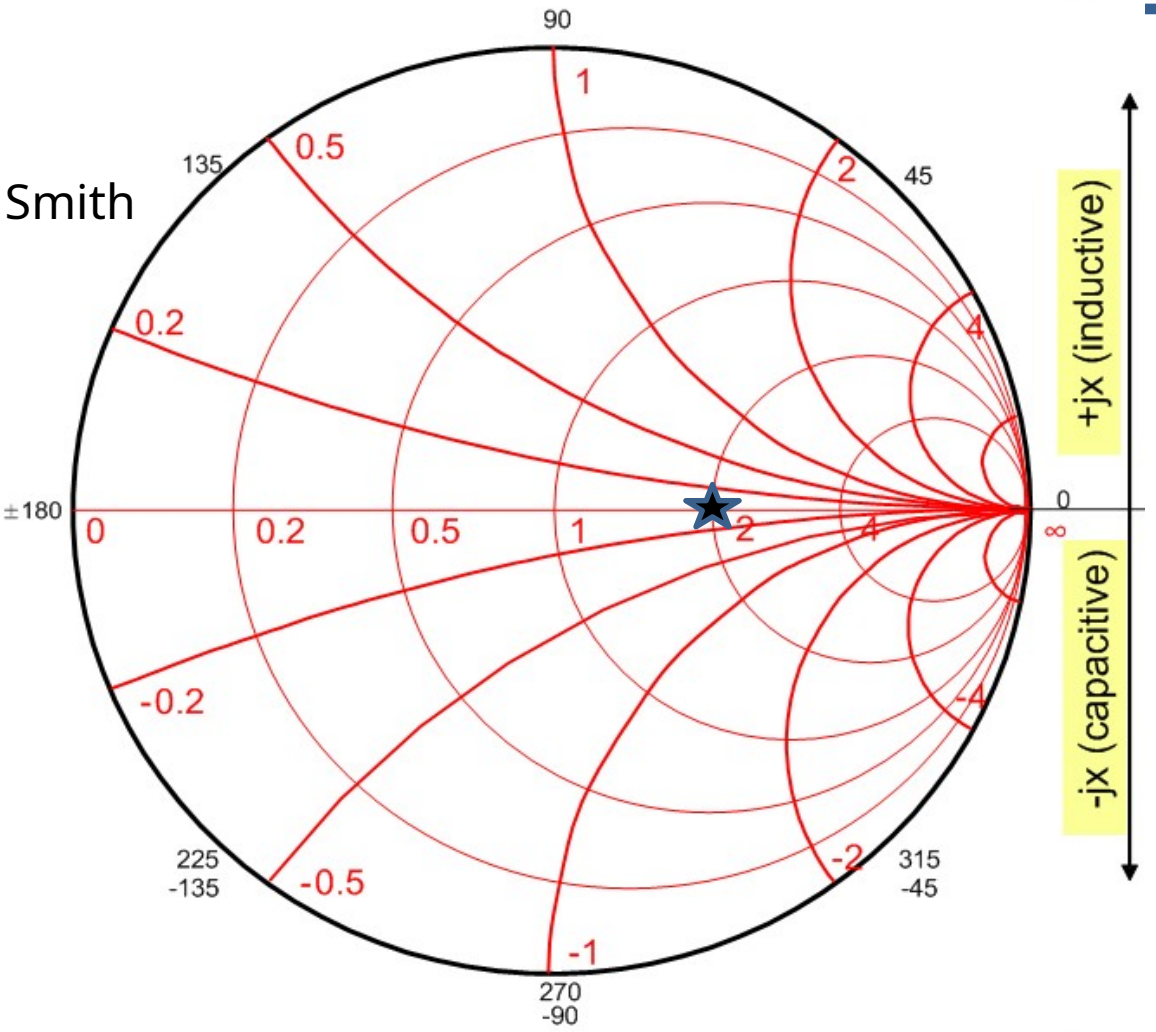
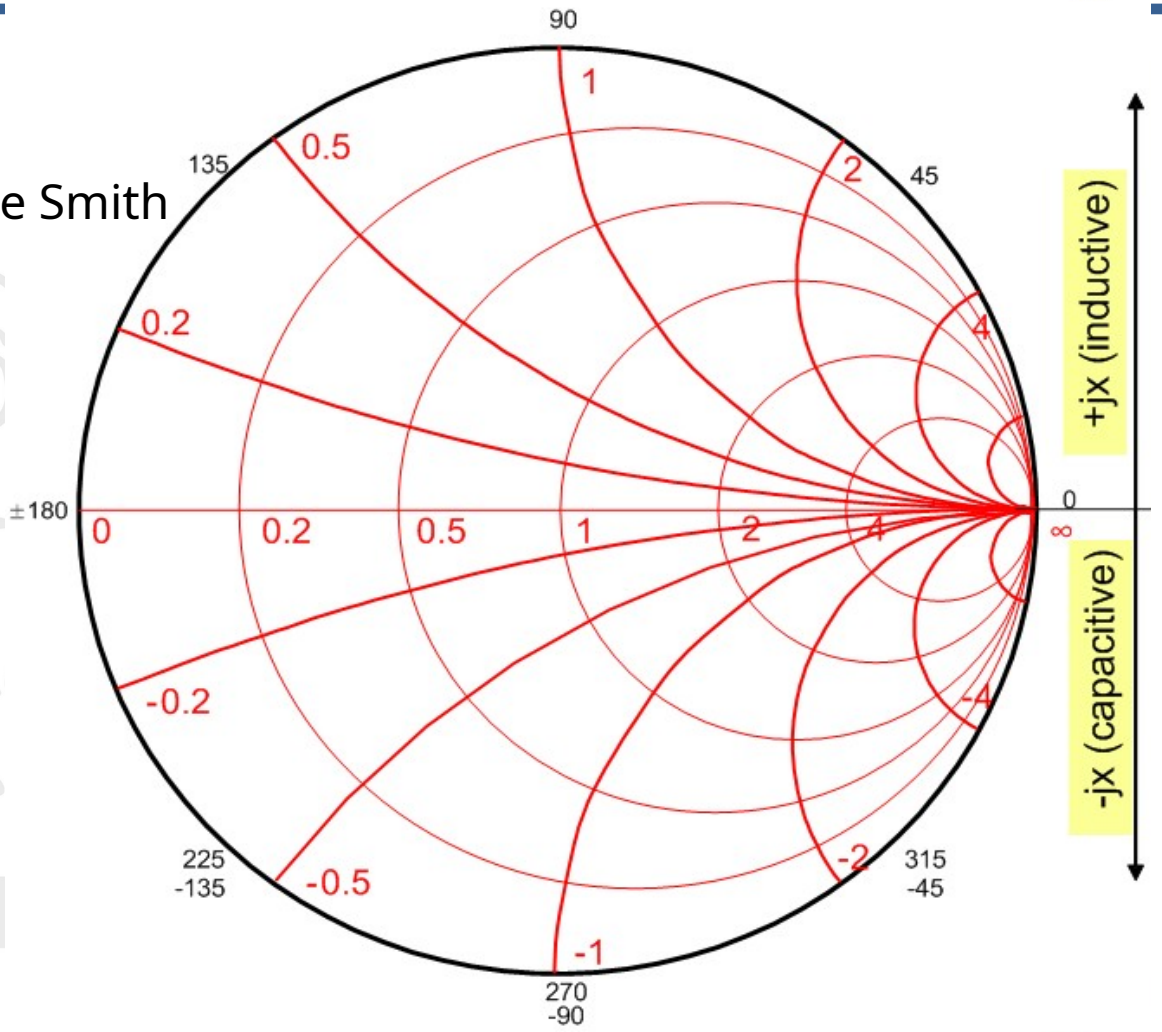


Impedance Plane

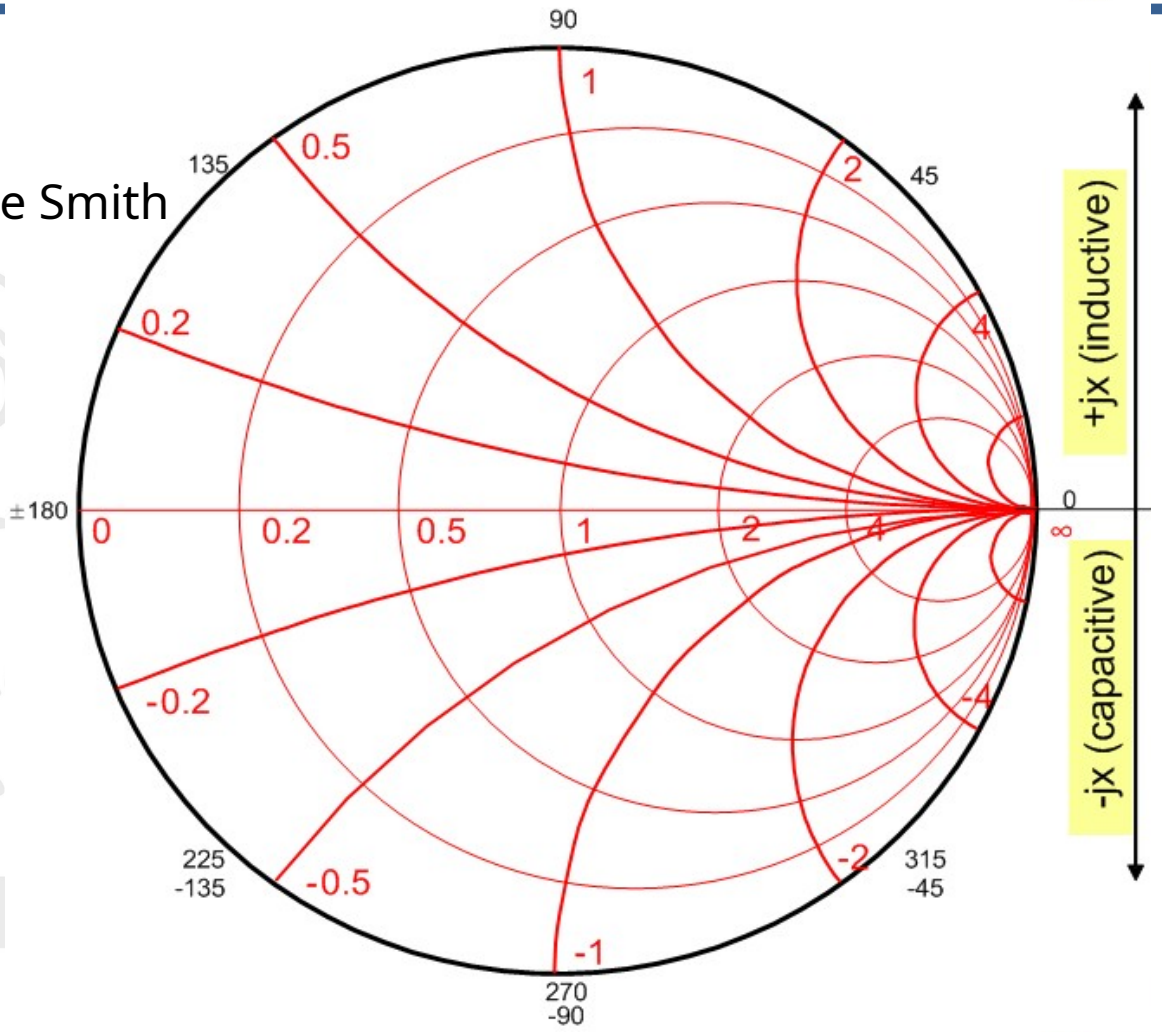
Identify a load = -100j Ω on the Smith Chart
(assume a 50 Ω system)



Impedance Plane

Identify a load = -100j Ω on the Smith Chart
(assume a 50 Ω system)

$$r = \frac{R}{Z_0} = \frac{0}{50} = 0$$

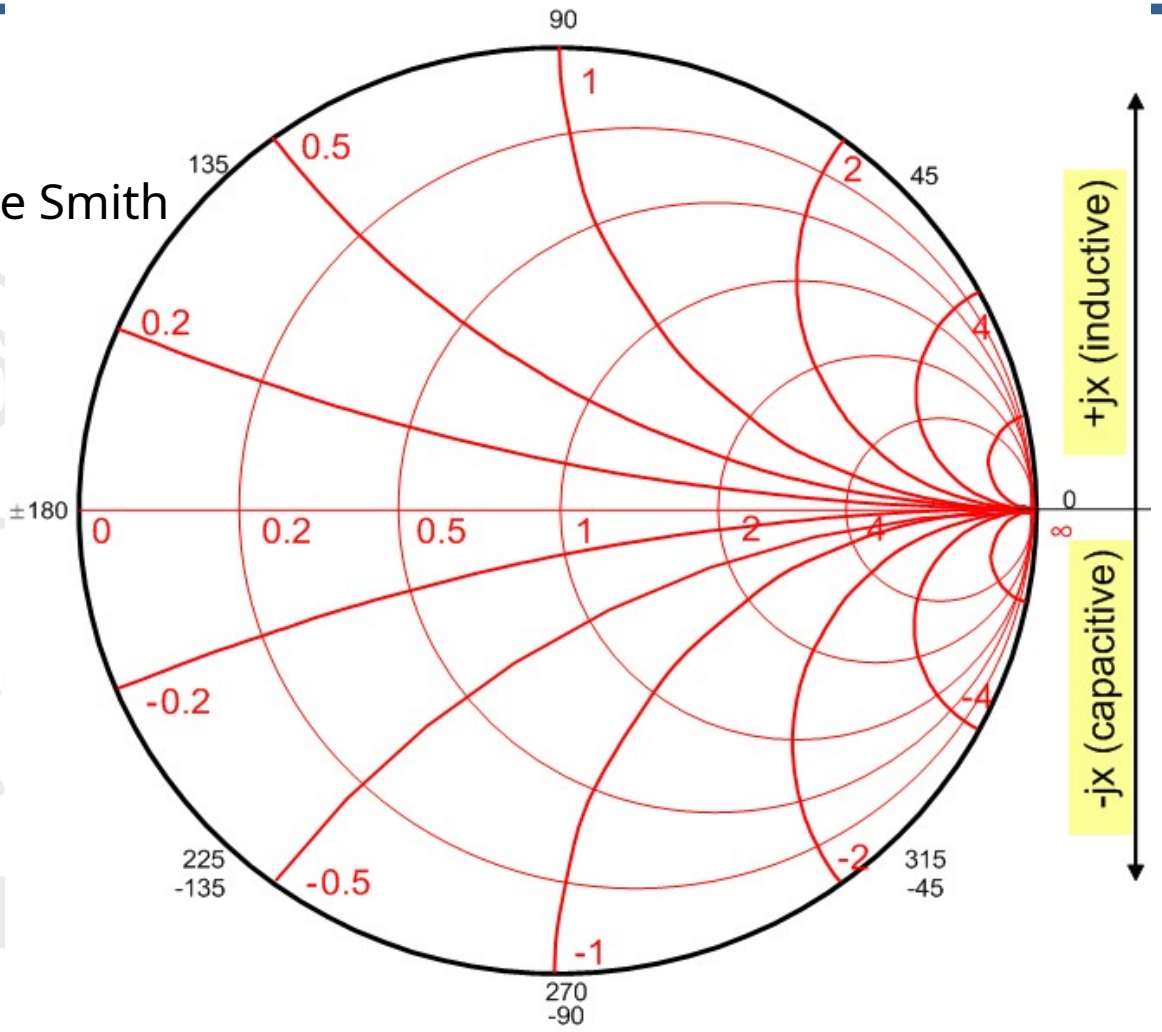


Impedance Plane

Identify a load = -100j Ω on the Smith Chart
(assume a 50 Ω system)

$$r = \frac{R}{Z_0} = \frac{0}{50} = 0$$

$$x = \frac{X}{Z_0} = \frac{-100}{50} = -2$$



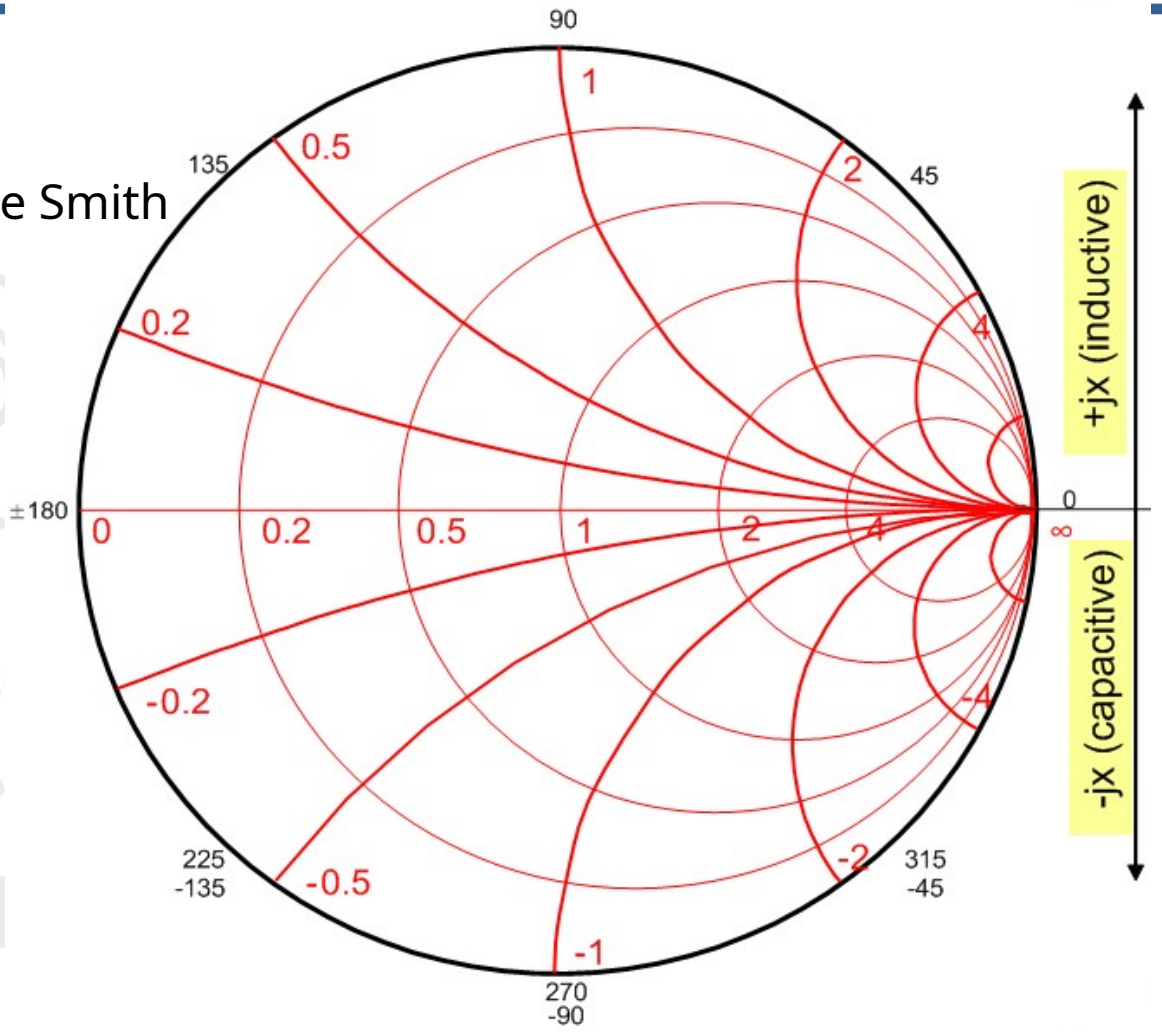
Impedance Plane

Identify a load = -100j Ω on the Smith Chart
(assume a 50 Ω system)

$$r = \frac{R}{Z_0} = \frac{0}{50} = 0$$

$$x = \frac{X}{Z_0} = \frac{-100}{50} = -2$$

$$z_L = r + jx = 0 - j2$$



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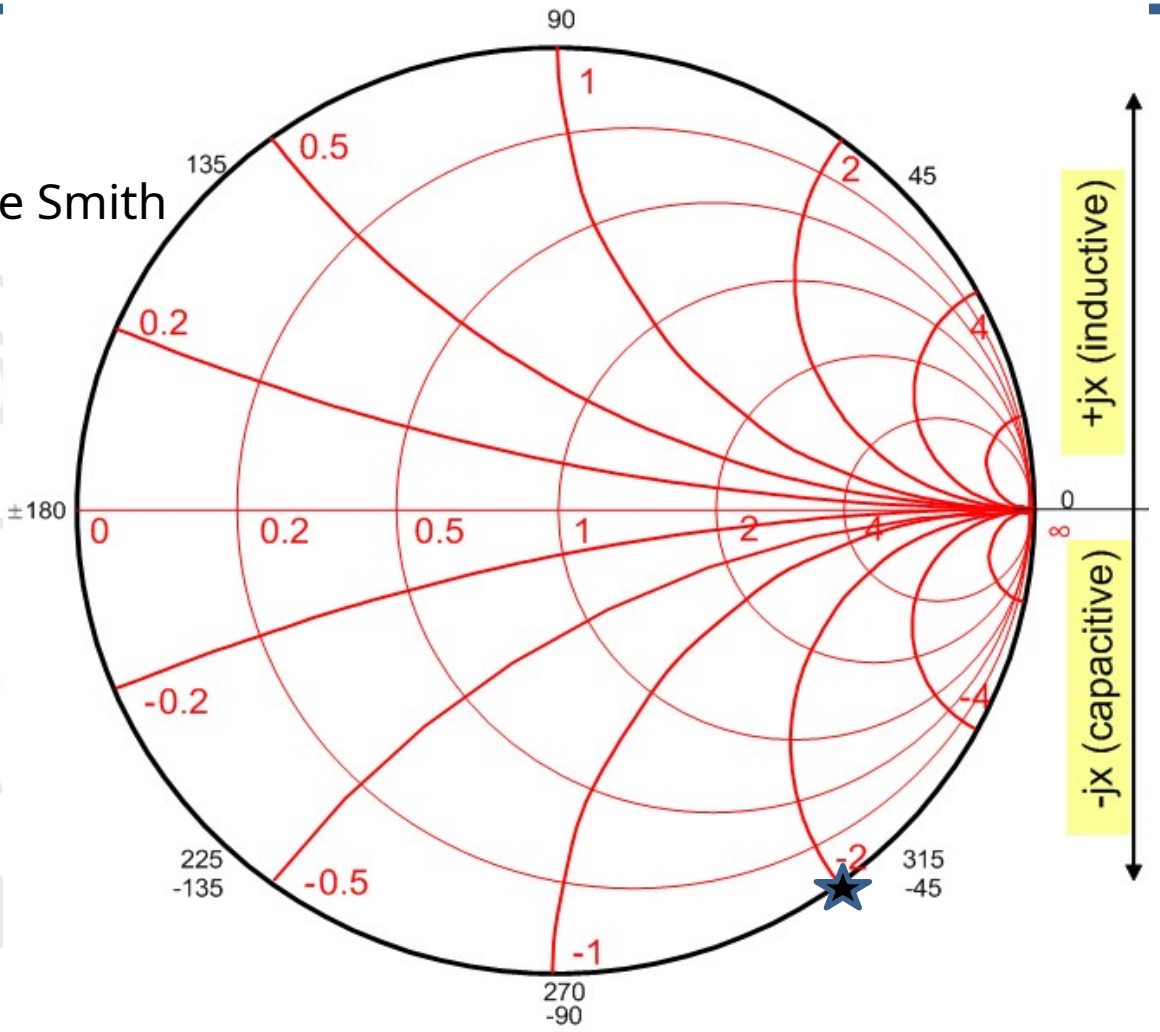
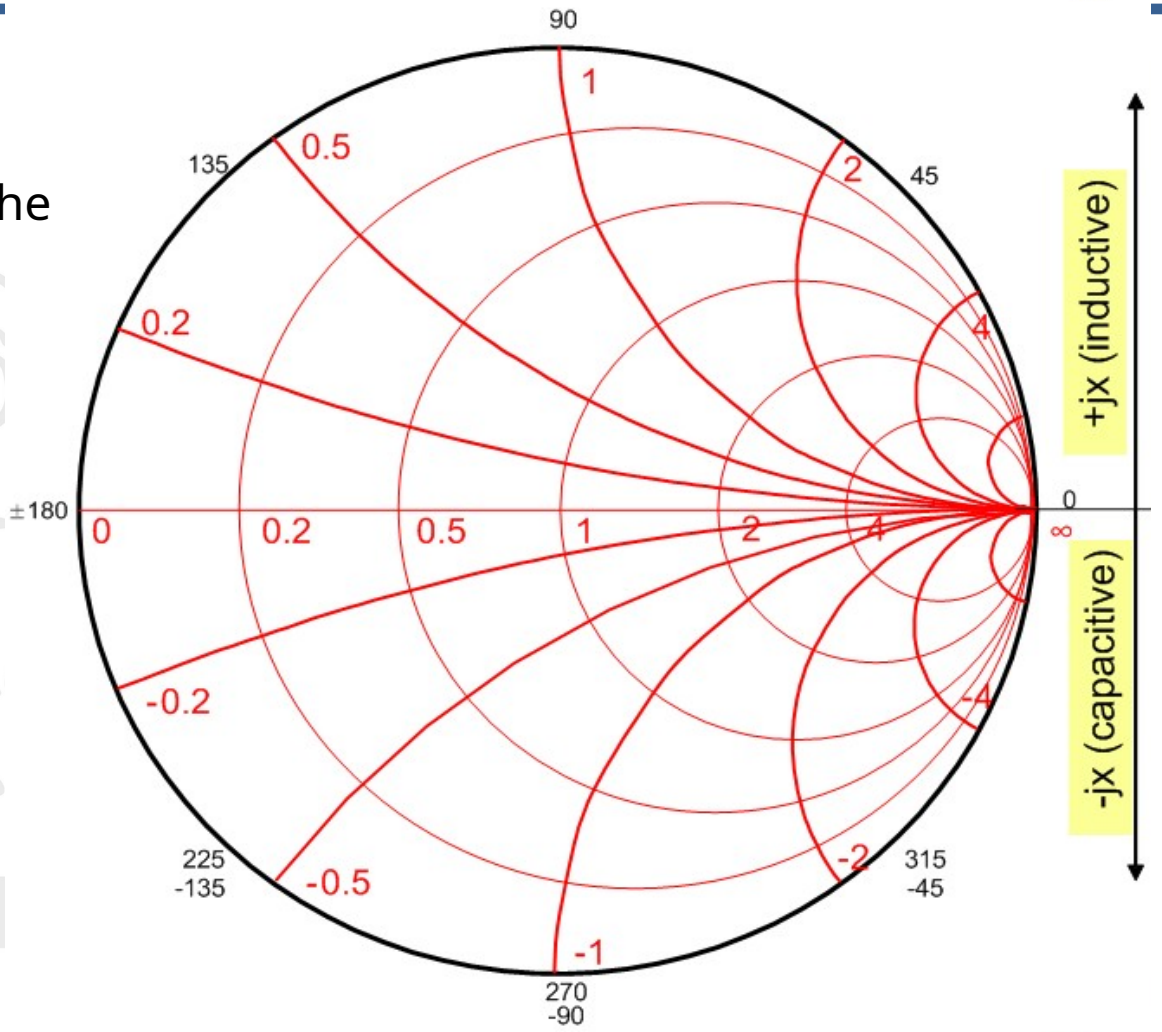
$$x = \frac{X}{Z_0} = \frac{-100}{50} = -2$$

$$z_L = r + jx = 0 - j2$$



Impedance Plane

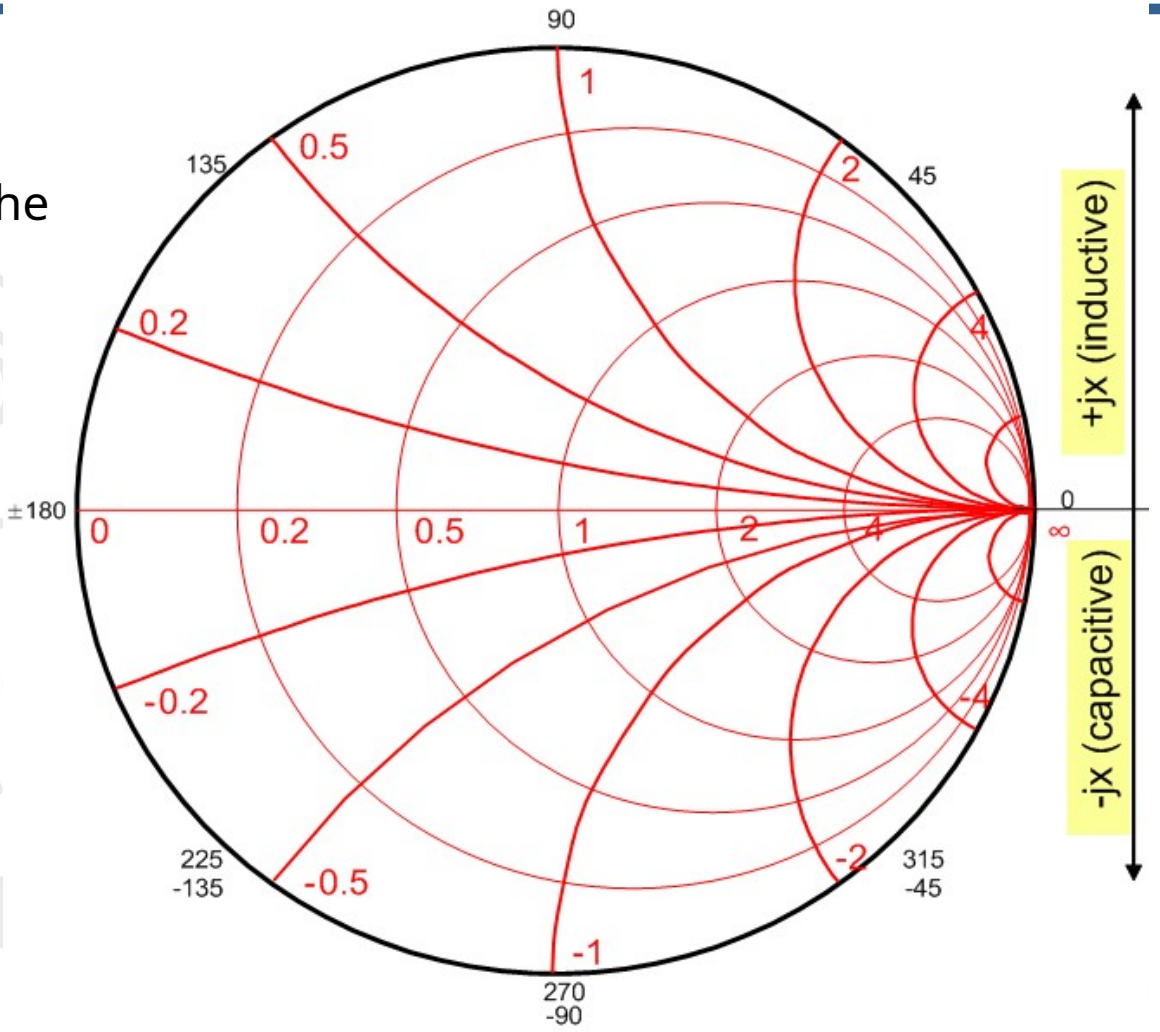
Identify a load = 50+100j Ω on the Smith Chart
(assume a 50 Ω system)



Impedance Plane

Identify a load = 50+100j Ω on the Smith Chart
(assume a 50 Ω system)

$$r = \frac{R}{Z_0} = \frac{50}{50} = 1$$

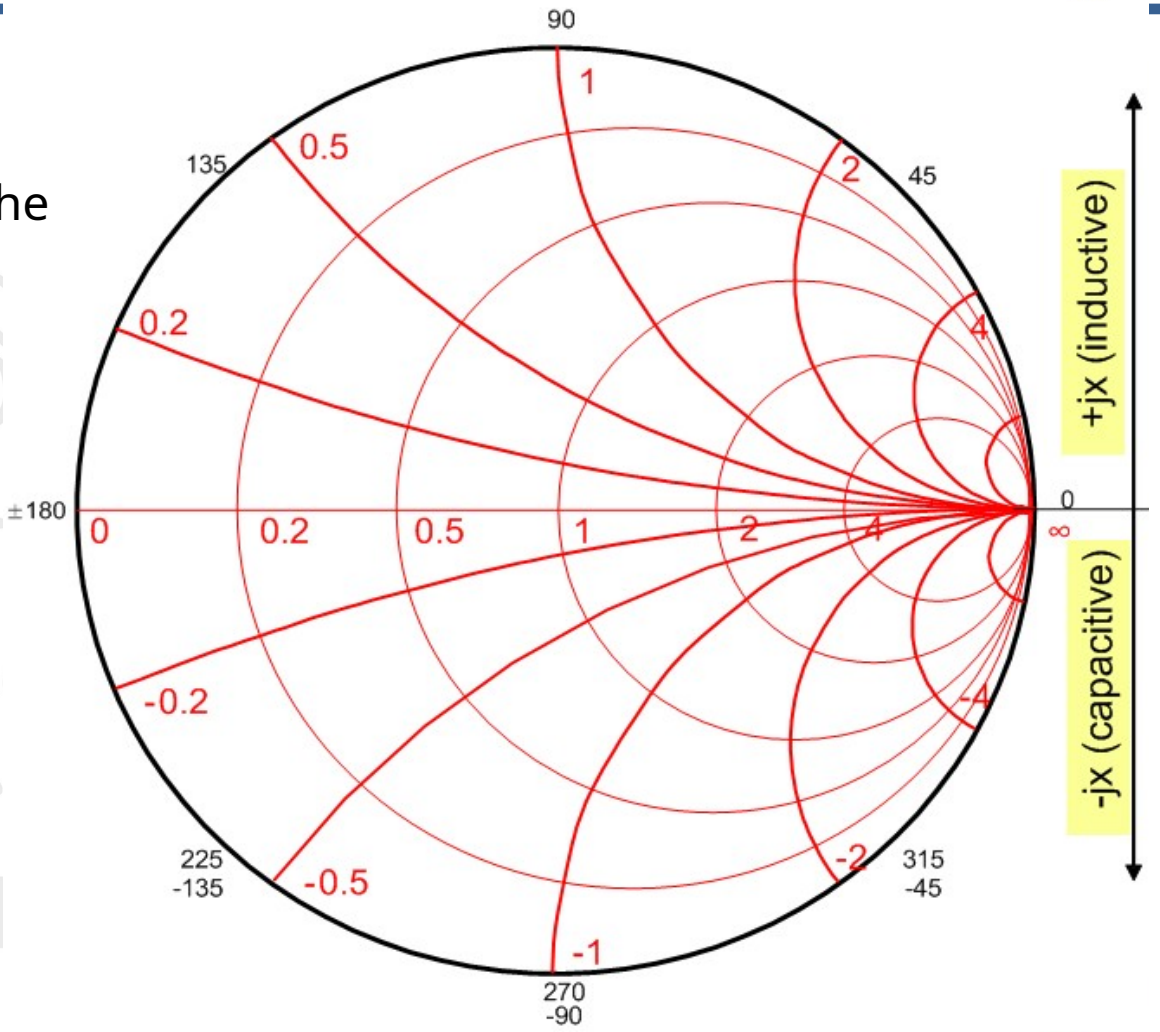


Impedance Plane

Identify a load = 50+100j Ω on the Smith Chart
(assume a 50 Ω system)

$$r = \frac{R}{Z_0} = \frac{50}{50} = 1$$

$$x = \frac{X}{Z_0} = \frac{100}{50} = 2$$



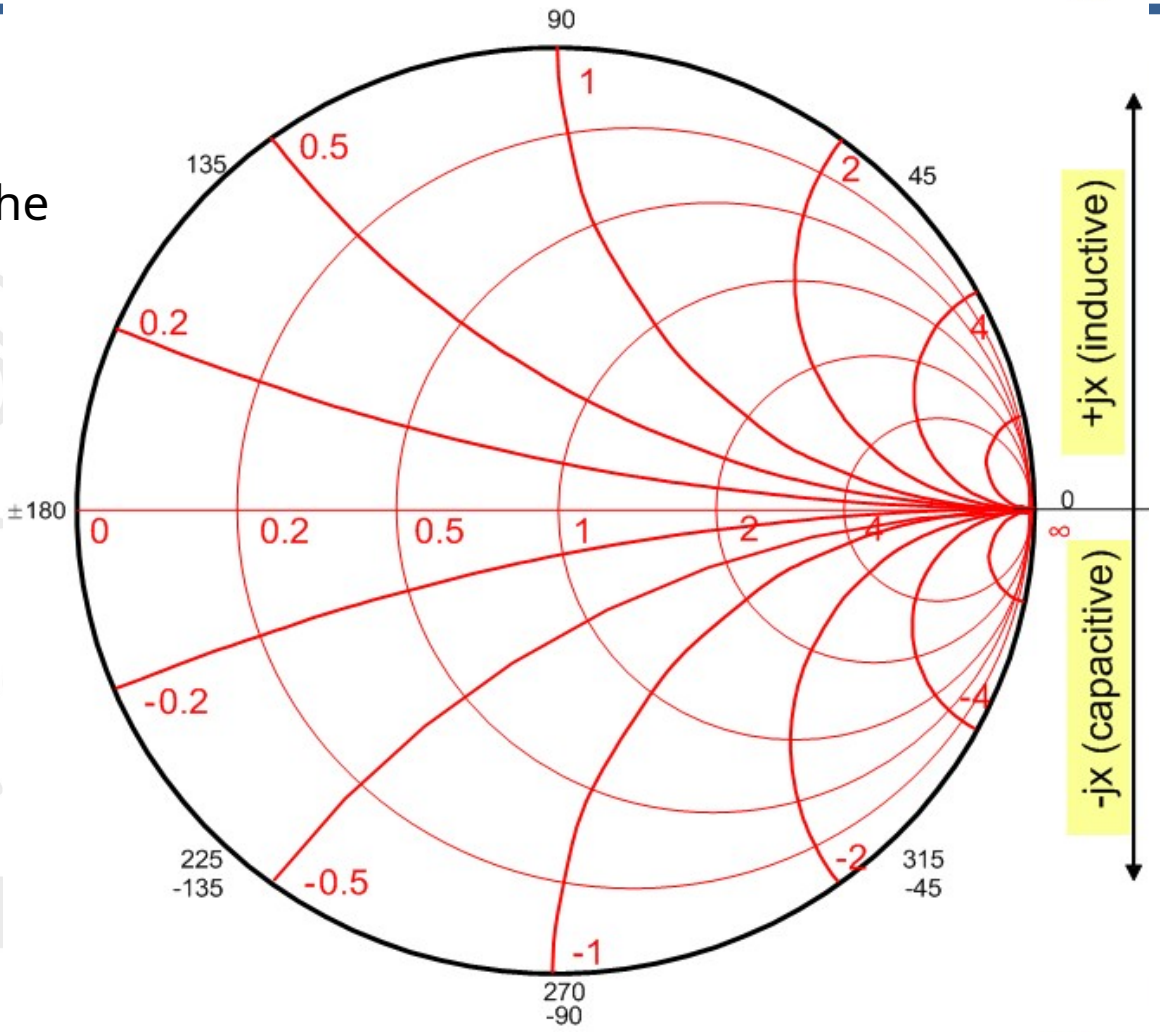
Impedance Plane

Identify a load = 50+100j Ω on the Smith Chart
(assume a 50 Ω system)

$$r = \frac{R}{Z_0} = \frac{50}{50} = 1$$

$$x = \frac{X}{Z_0} = \frac{100}{50} = 2$$

$$z_L = r + jx = 1 + j2$$



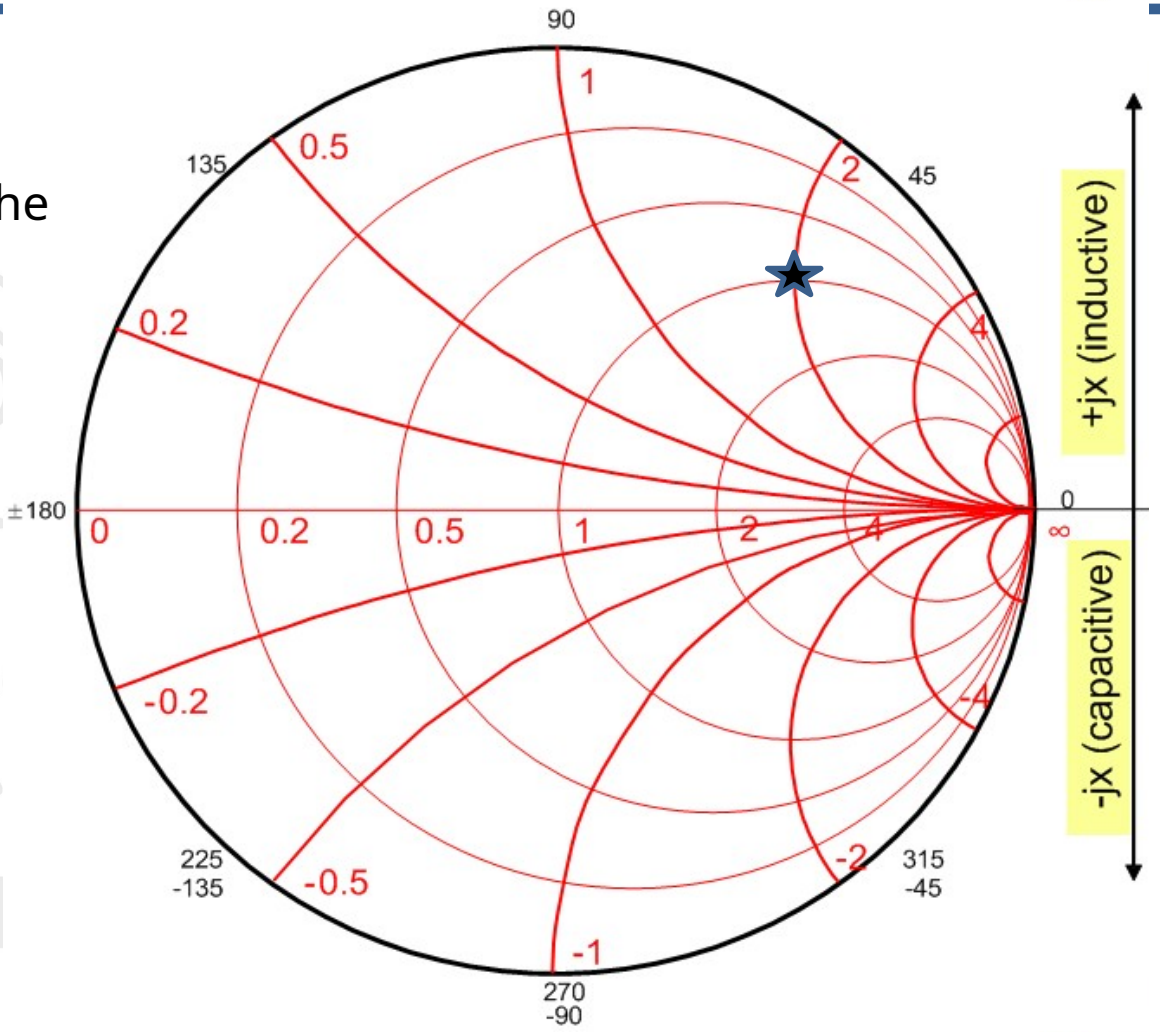
Impedance Plane

Identify a load = 50+100j Ω on the Smith Chart
(assume a 50 Ω system)

$$r = \frac{R}{Z_0} = \frac{50}{50} = 1$$

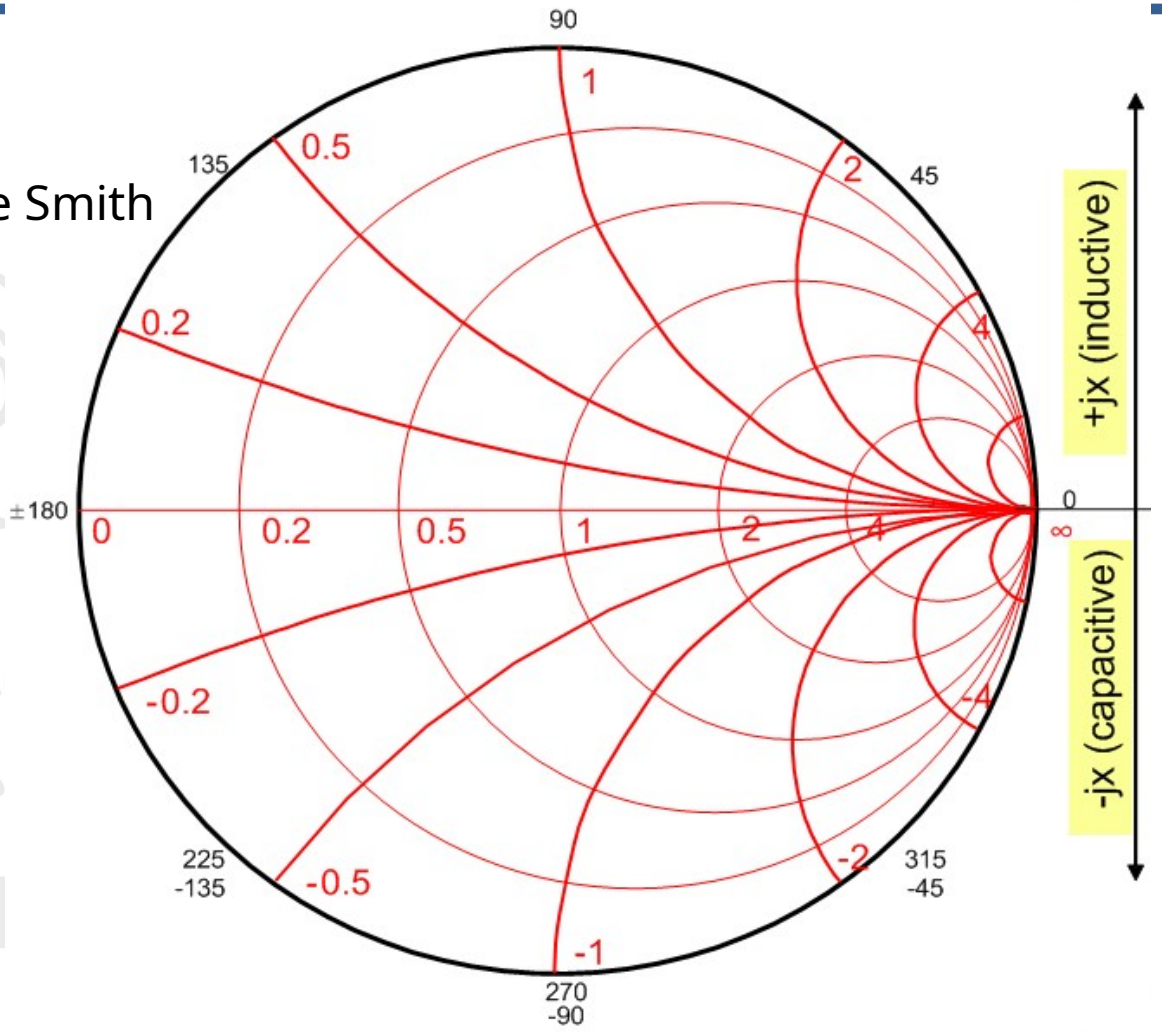
$$x = \frac{X}{Z_0} = \frac{100}{50} = 2$$

$$z_L = r + jx = 1 + j2$$



Impedance Plane

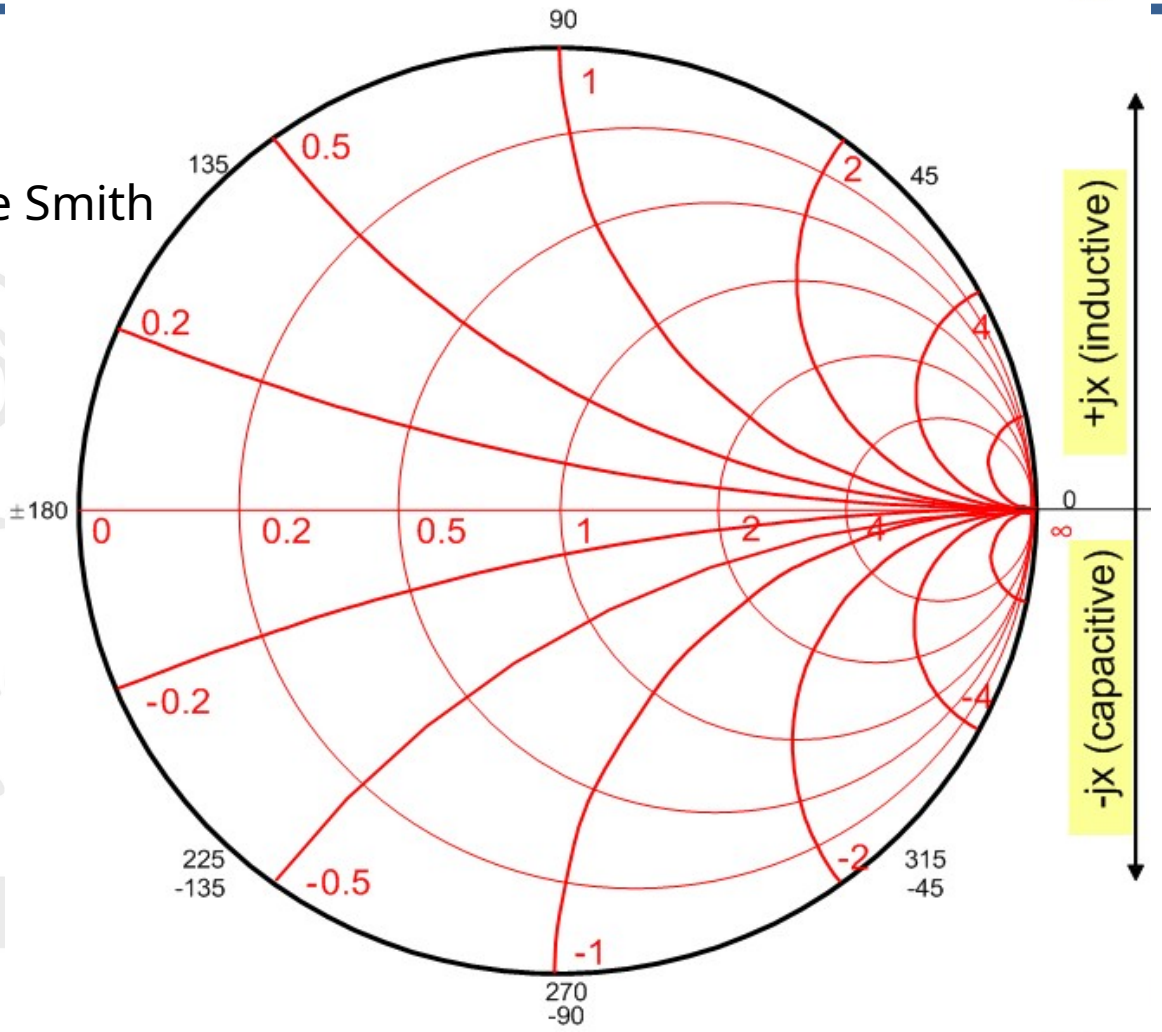
Identify a load = 10-25j Ω on the Smith Chart
(assume a 50 Ω system)



Impedance Plane

Identify a load = 10-25j Ω on the Smith Chart
(assume a 50 Ω system)

$$r = \frac{R}{Z_0} = \frac{10}{50} = 0.2$$

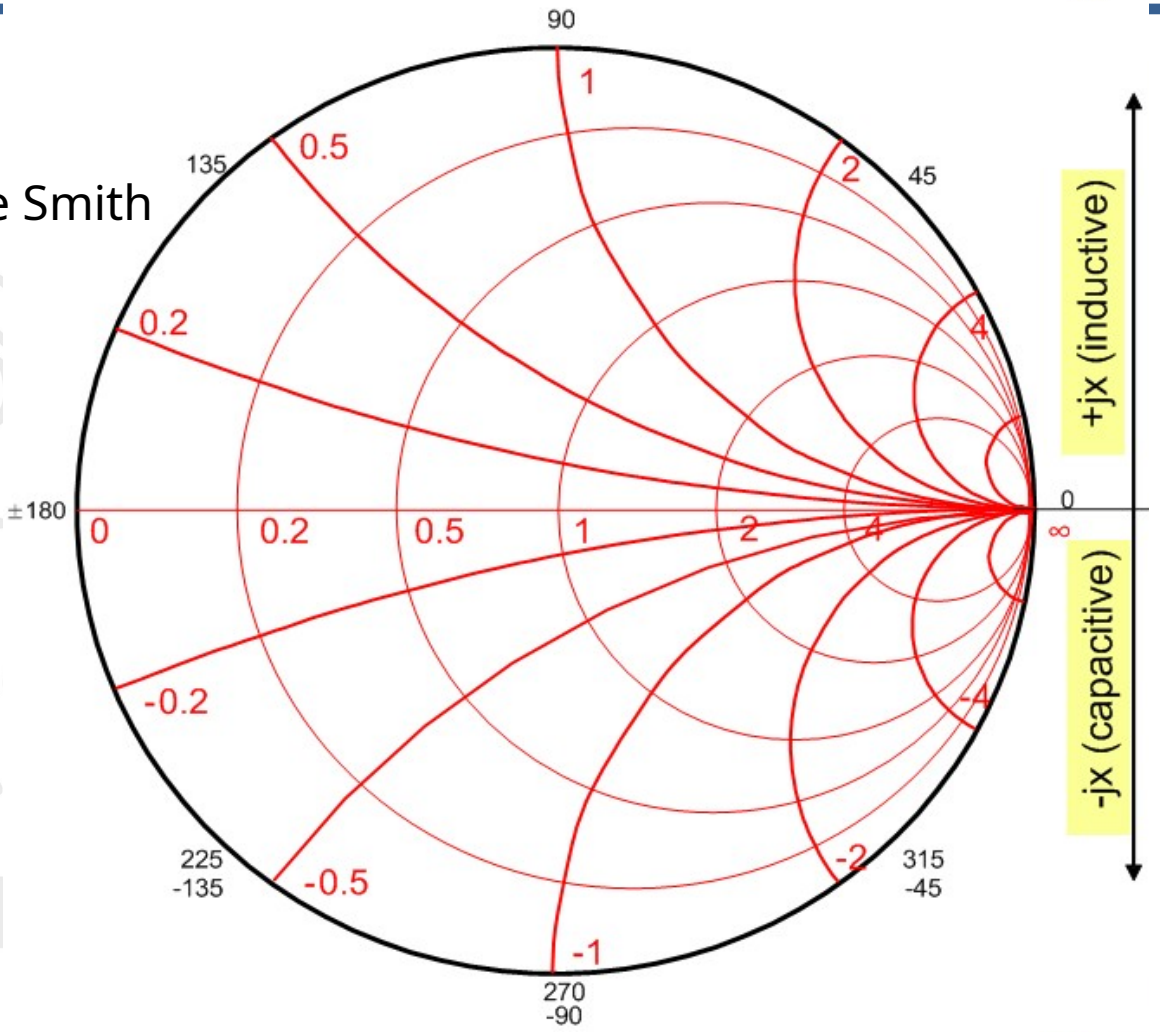


Impedance Plane

Identify a load = 10-25j Ω on the Smith Chart
(assume a 50 Ω system)

$$r = \frac{R}{Z_0} = \frac{10}{50} = 0.2$$

$$x = \frac{X}{Z_0} = \frac{-25}{50} = -0.5$$



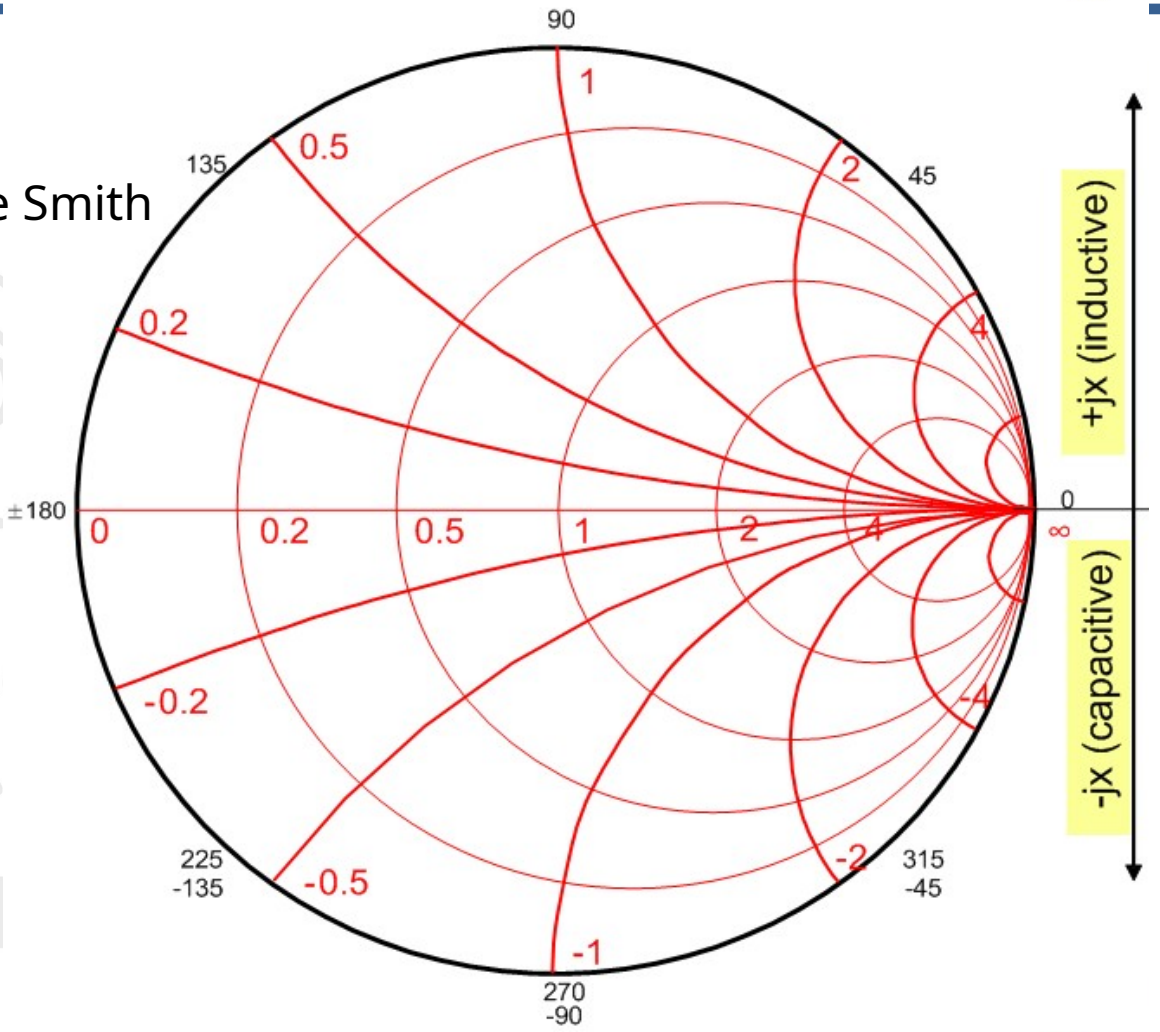
Impedance Plane

Identify a load = 10-25j Ω on the Smith Chart
(assume a 50 Ω system)

$$r = \frac{R}{Z_0} = \frac{10}{50} = 0.2$$

$$x = \frac{X}{Z_0} = \frac{-25}{50} = -0.5$$

$$z_L = r + jx = 0.2 - j0.5$$



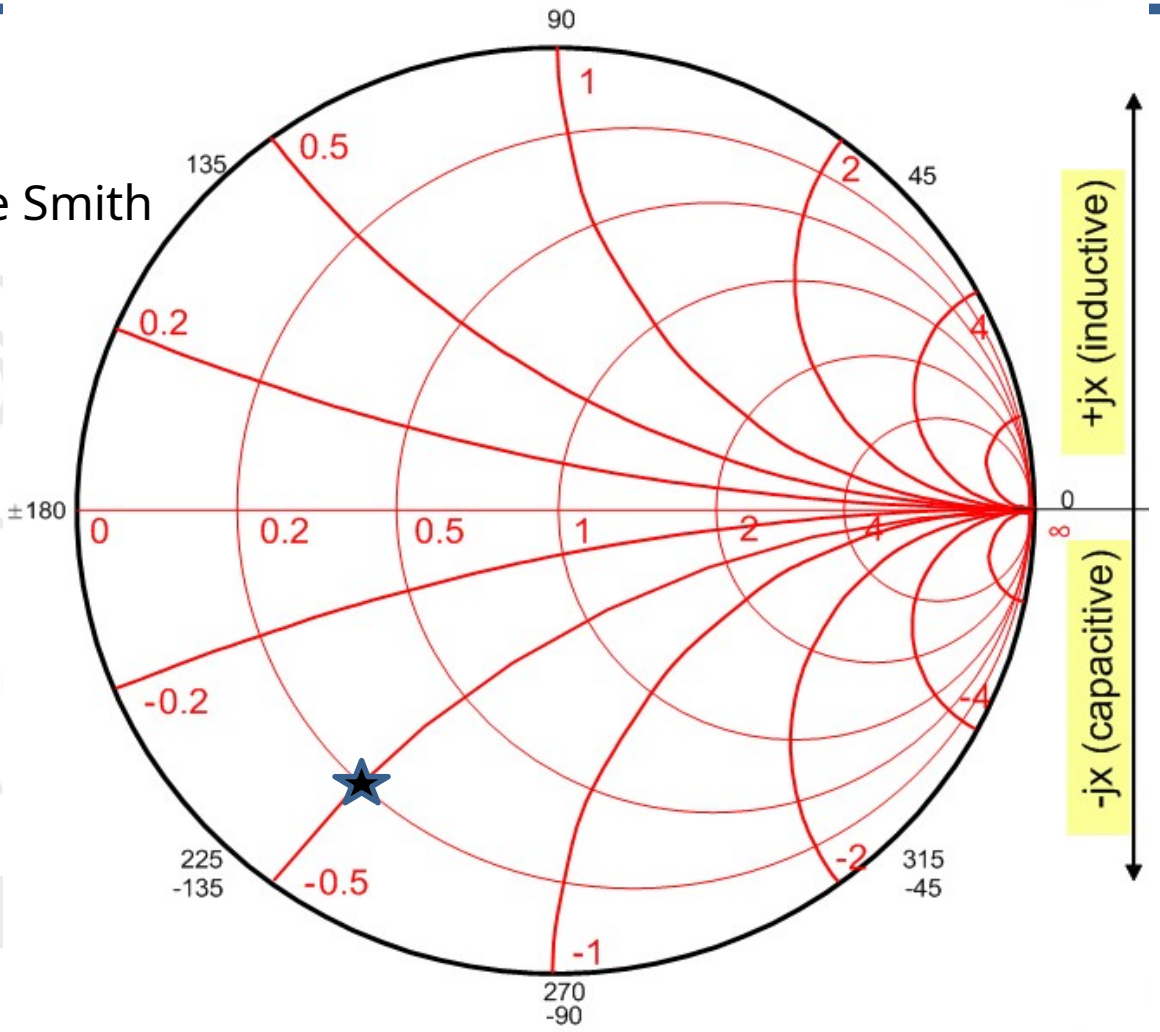
Impedance Plane

Identify a load = 10-25j Ω on the Smith Chart
(assume a 50 Ω system)

$$r = \frac{R}{Z_0} = \frac{10}{50} = 0.2$$

$$x = \frac{X}{Z_0} = \frac{-25}{50} = -0.5$$

$$z_L = r + jx = 0.2 - j0.5$$

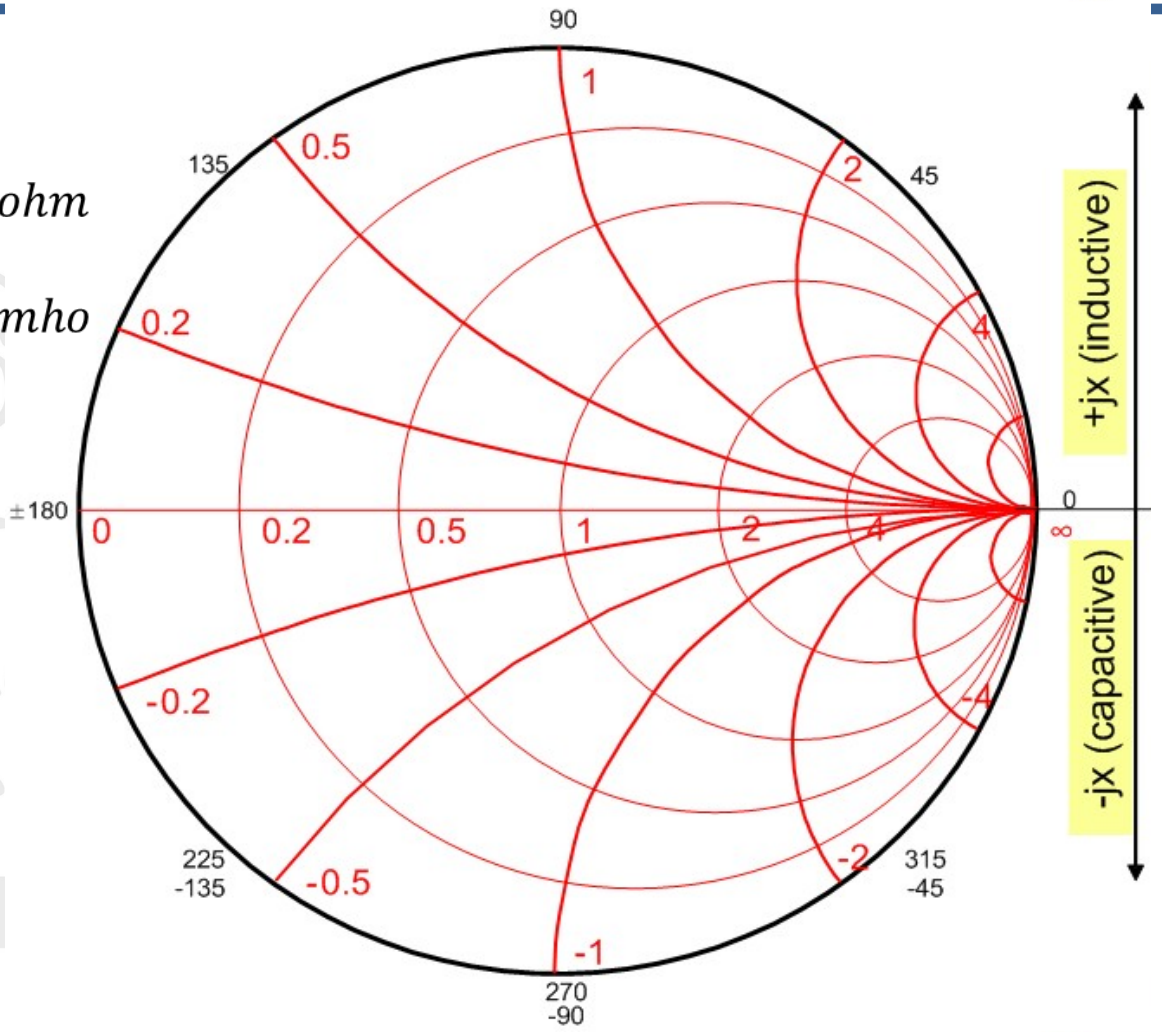


Smith Chart – Y plot (Admittance)

Admittance

Terminated Load Impedance = $Z_L\ ohm$

Terminated Load Admittance = $Y_L\ mho$

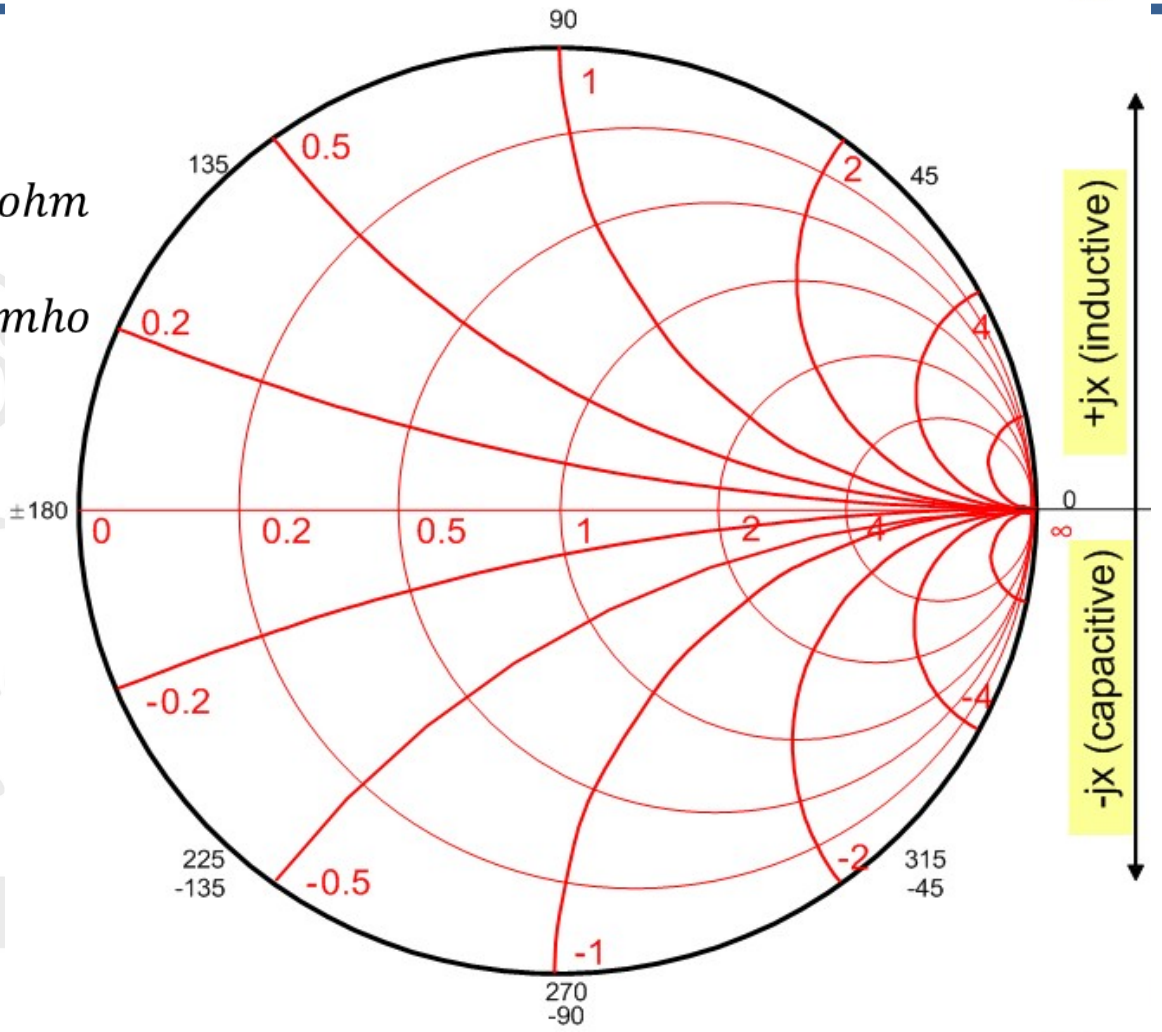


Admittance

Terminated Load Impedance = $Z_L\ ohm$

Terminated Load Admittance = $Y_L\ mho$

$$Y_L = \frac{1}{Z_L} \qquad Y_0 = \frac{1}{Z_0}$$



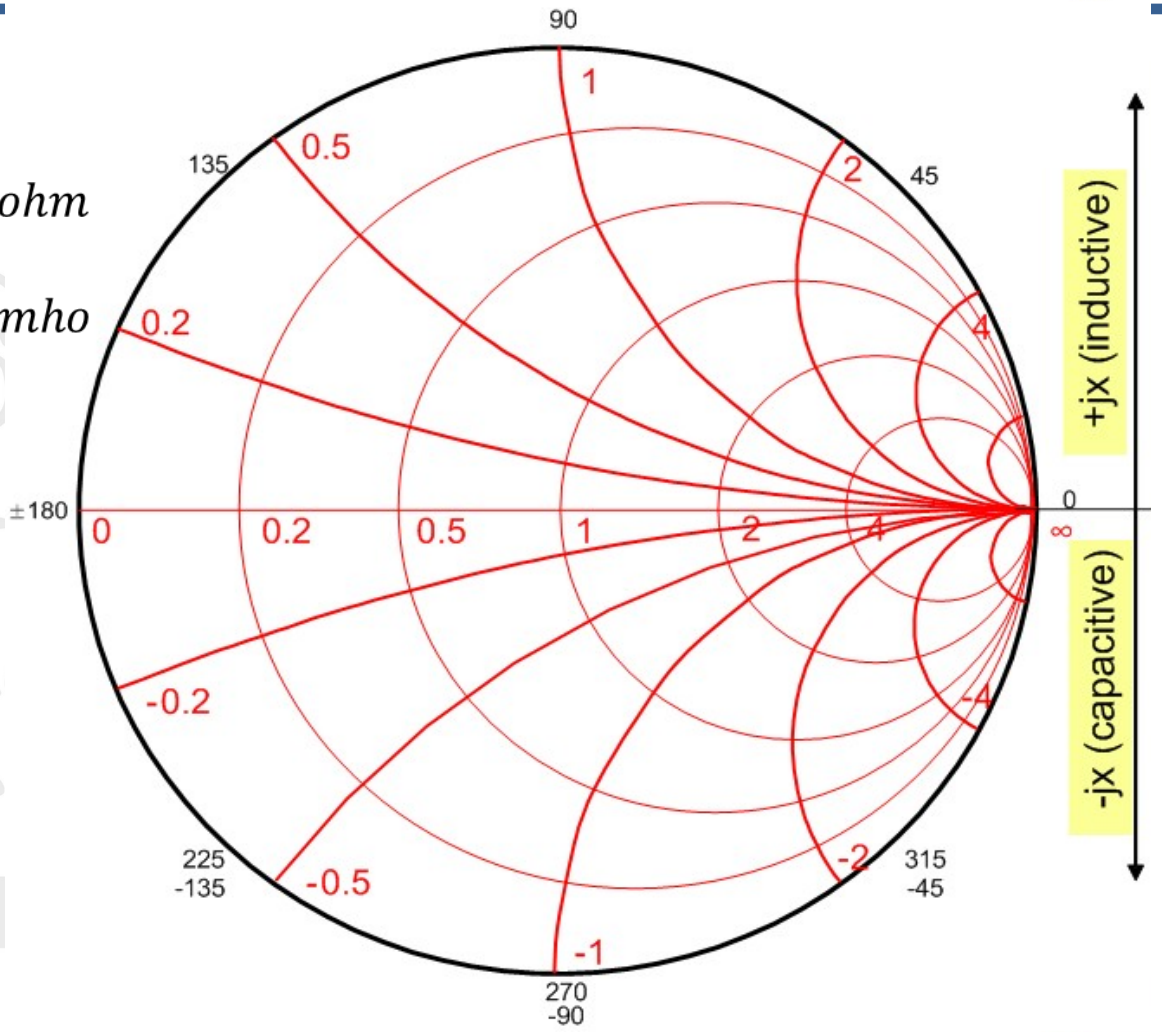
Admittance

Terminated Load Impedance = $Z_L\ ohm$

Terminated Load Admittance = $Y_L\ mho$

$$Y_L = \frac{1}{Z_L} \qquad Y_0 = \frac{1}{Z_0}$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{\frac{1}{Y_L} - \frac{1}{Y_0}}{\frac{1}{Y_L} + \frac{1}{Y_0}}$$



Admittance

Terminated Load Impedance = $Z_L\ ohm$

Terminated Load Admittance = $Y_L\ mho$

$$Y_L = \frac{1}{Z_L} \qquad Y_0 = \frac{1}{Z_0}$$

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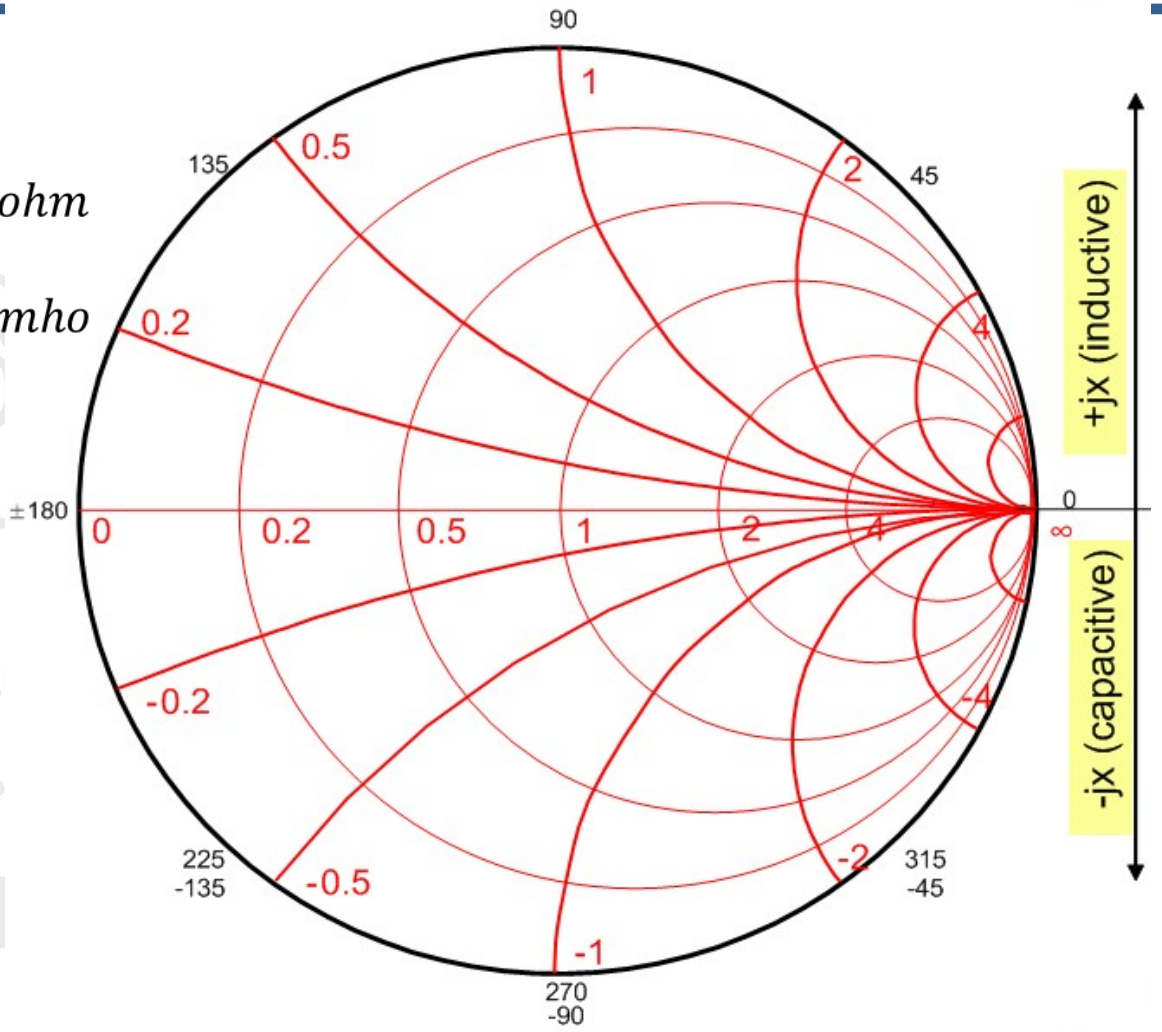
$$\Gamma = -\left(\frac{Y_L - Y_0}{Y_L + Y_0}\right)$$



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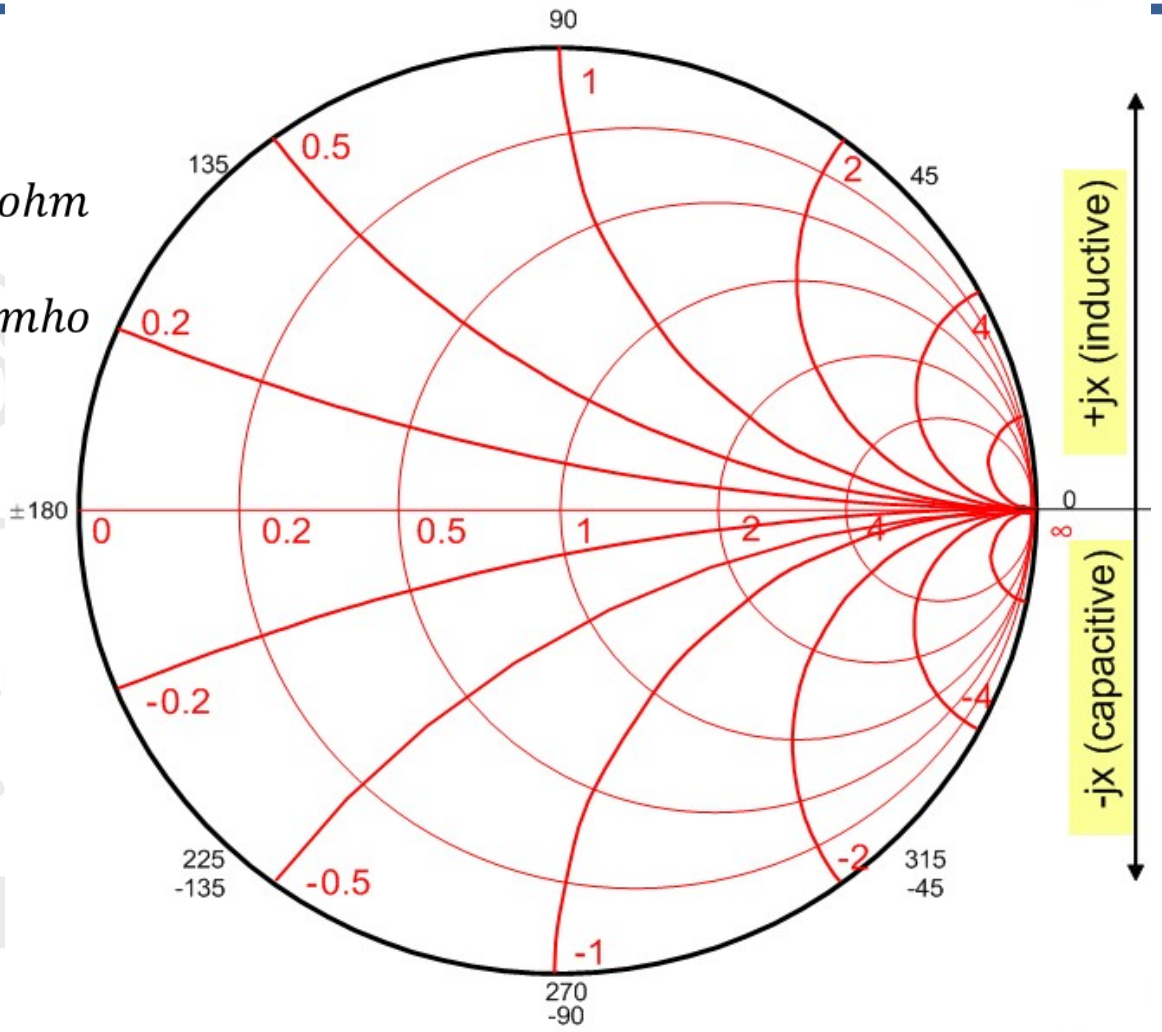

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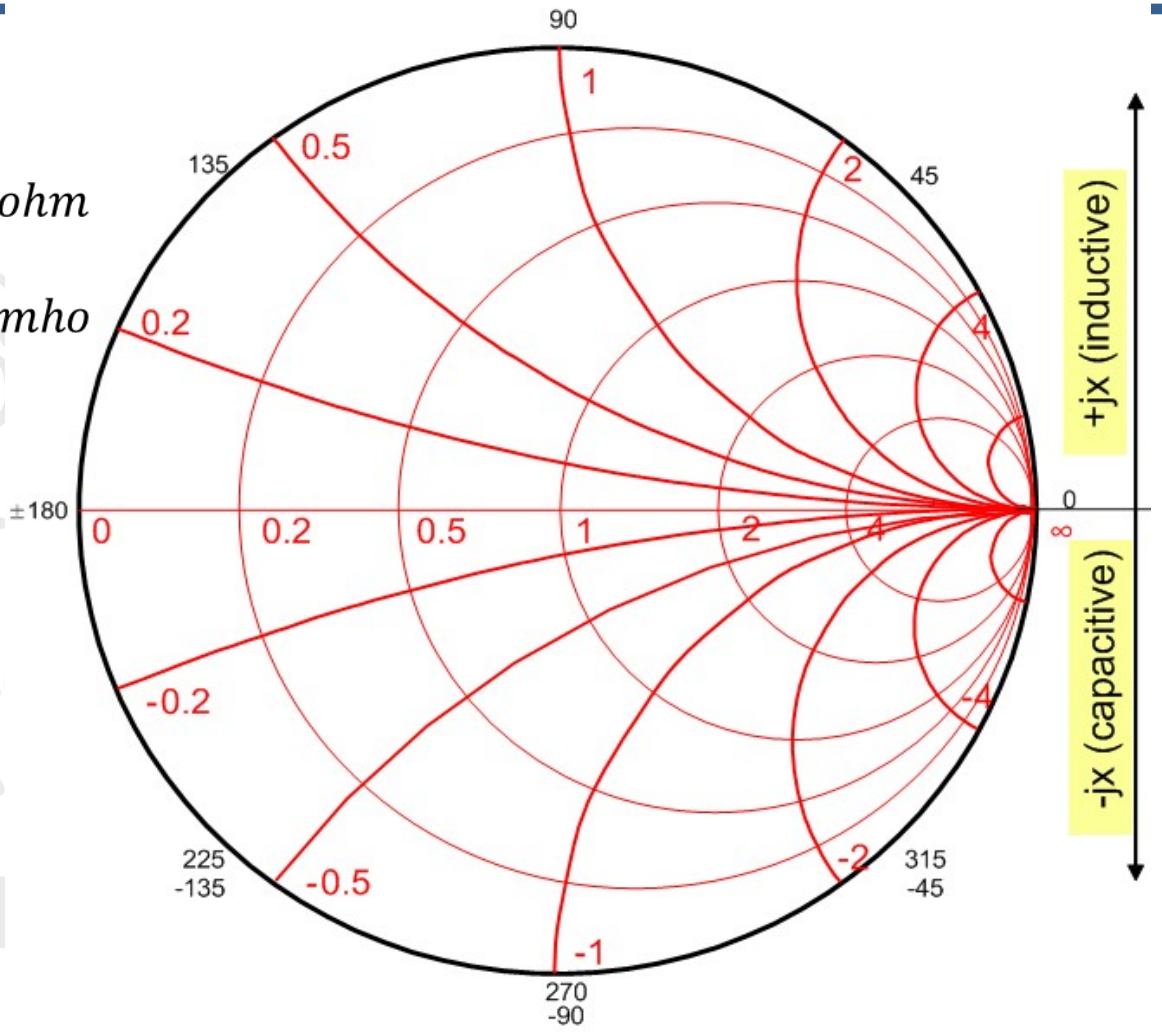

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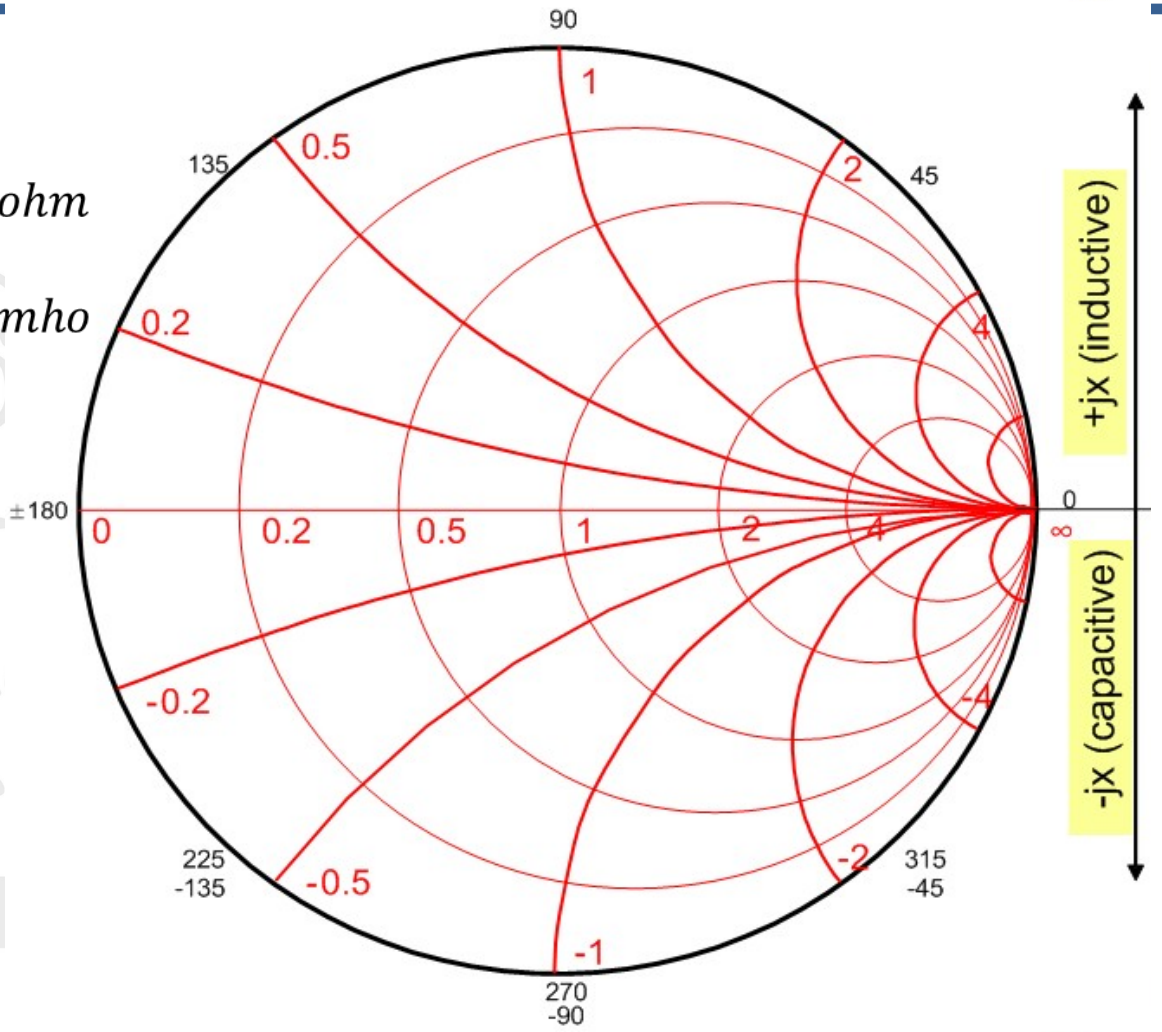

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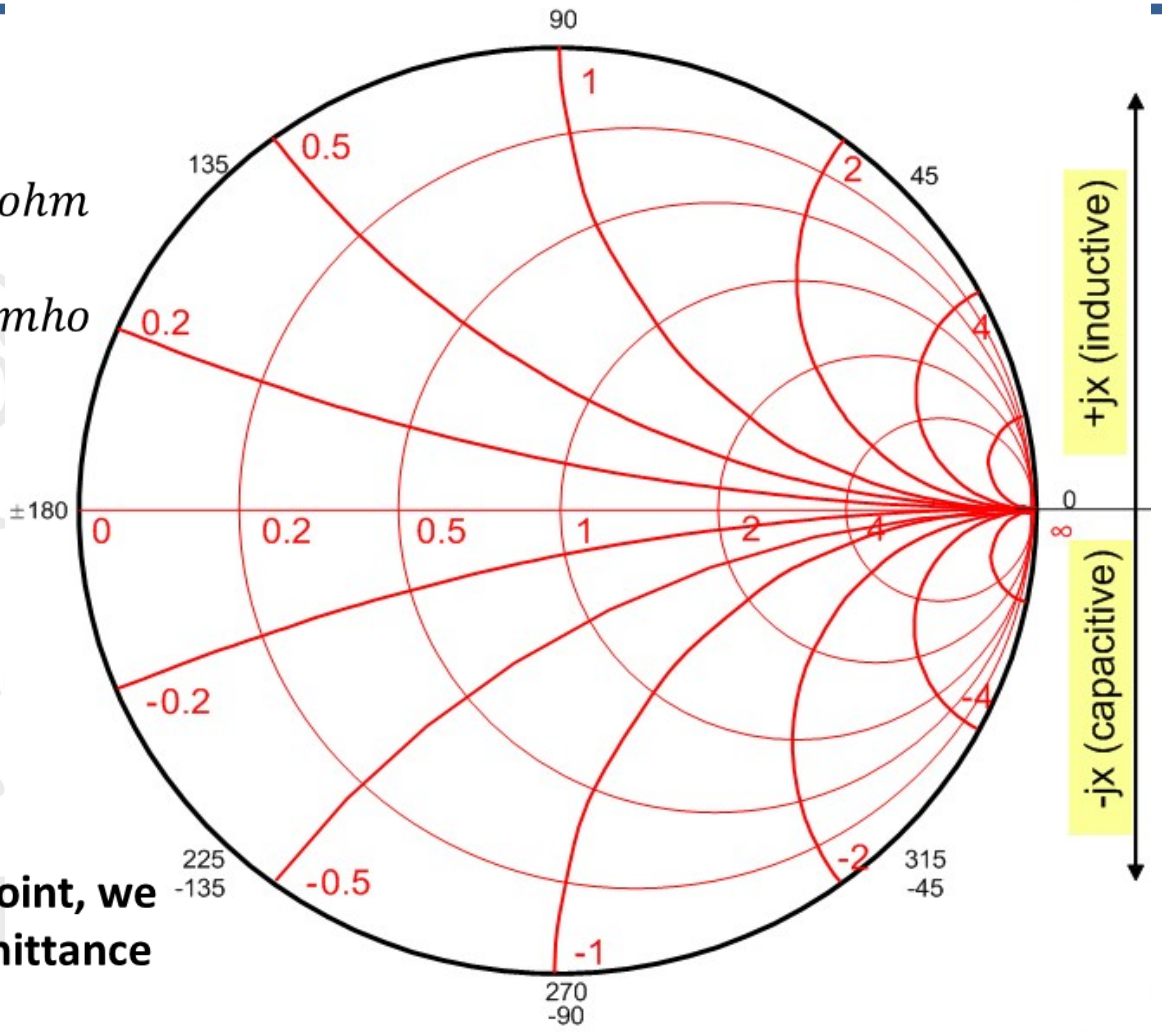
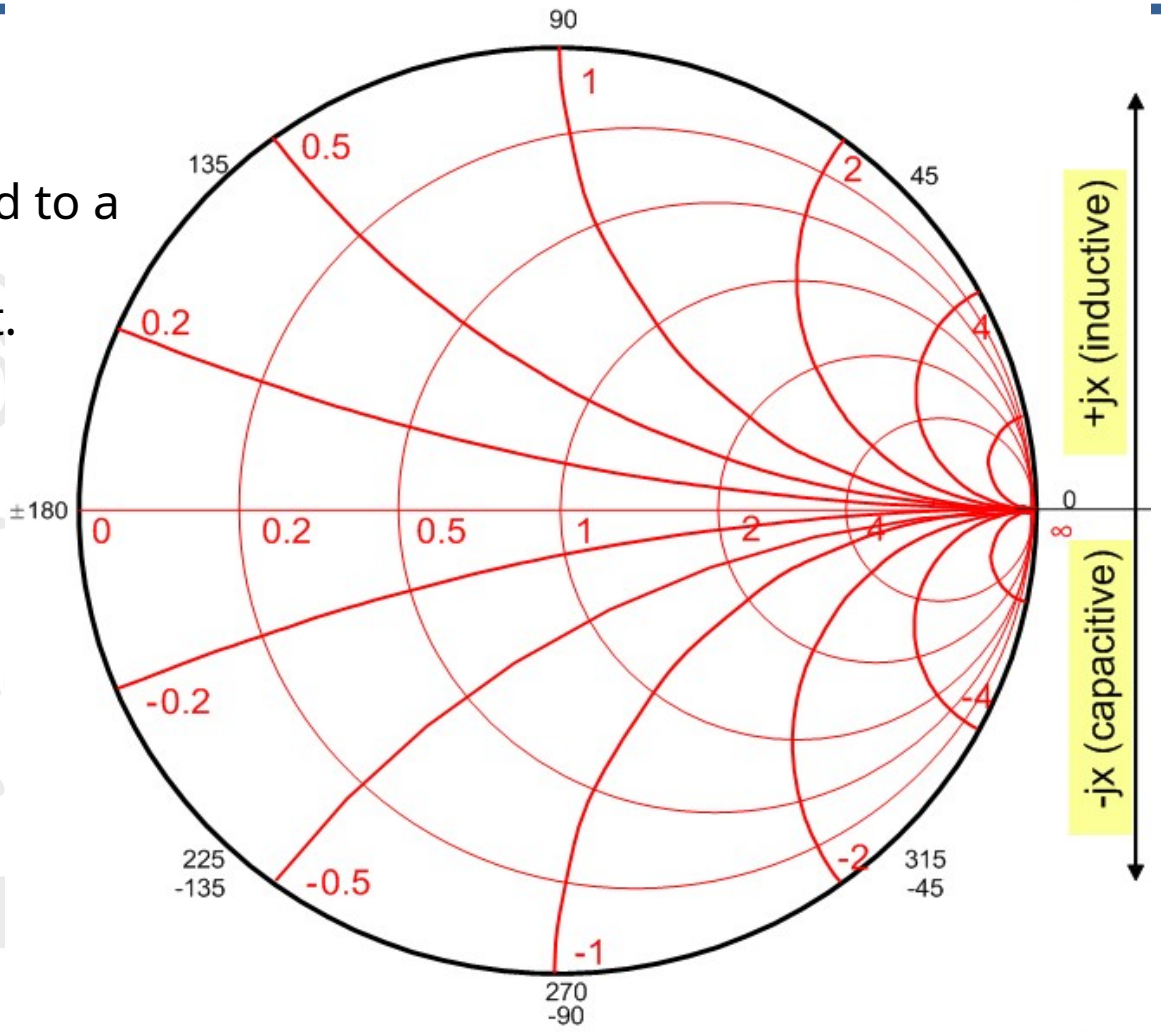
Hence, taking 180 degrees on Γ point, we can read the normalized load admittance as well.



Admittance

A load = 50+50j Ω is connected to a 50 Ω TL. Determine the load admittance using Smith Chart.



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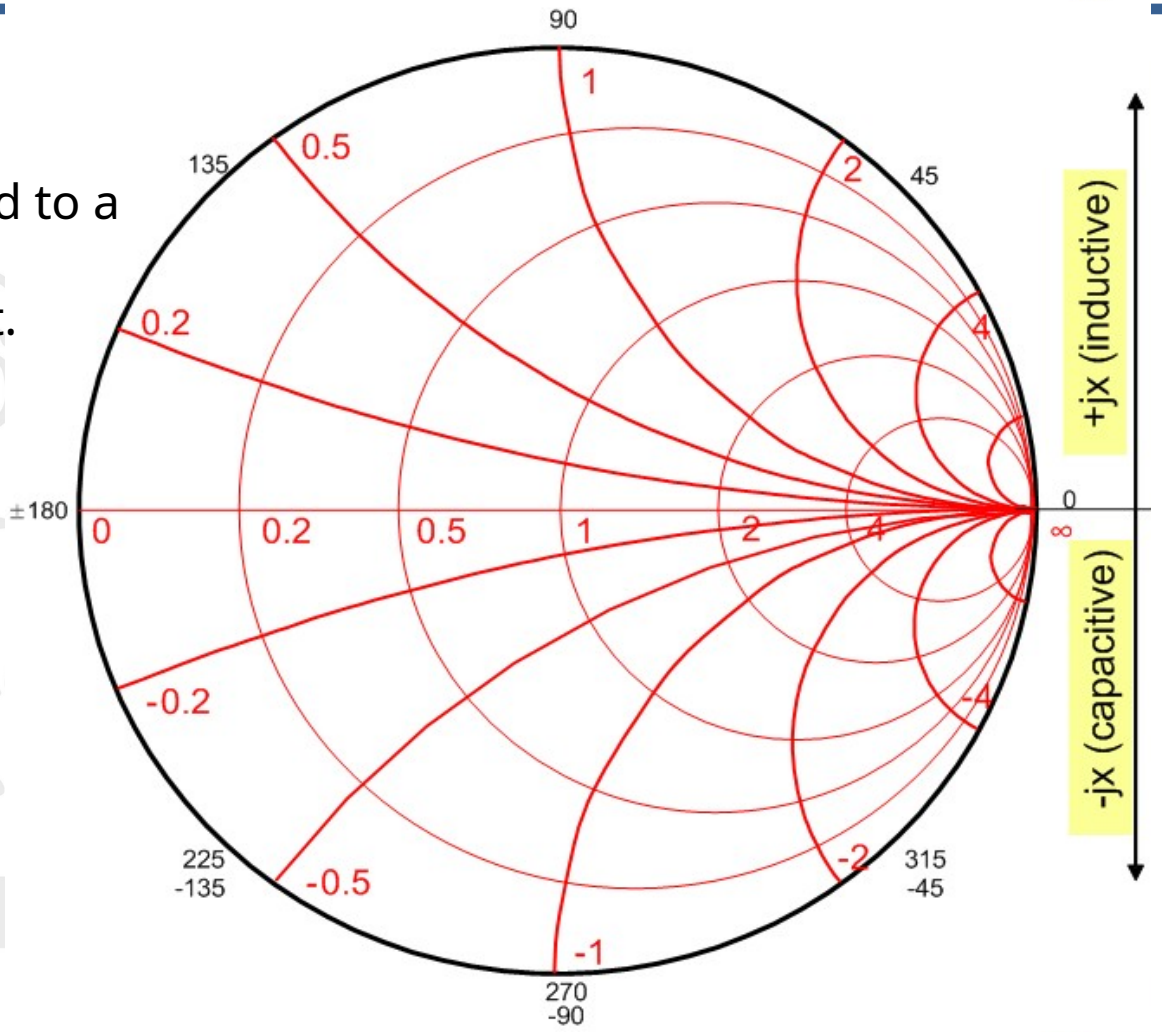
$$Z_L = 50 + 50\,j$$

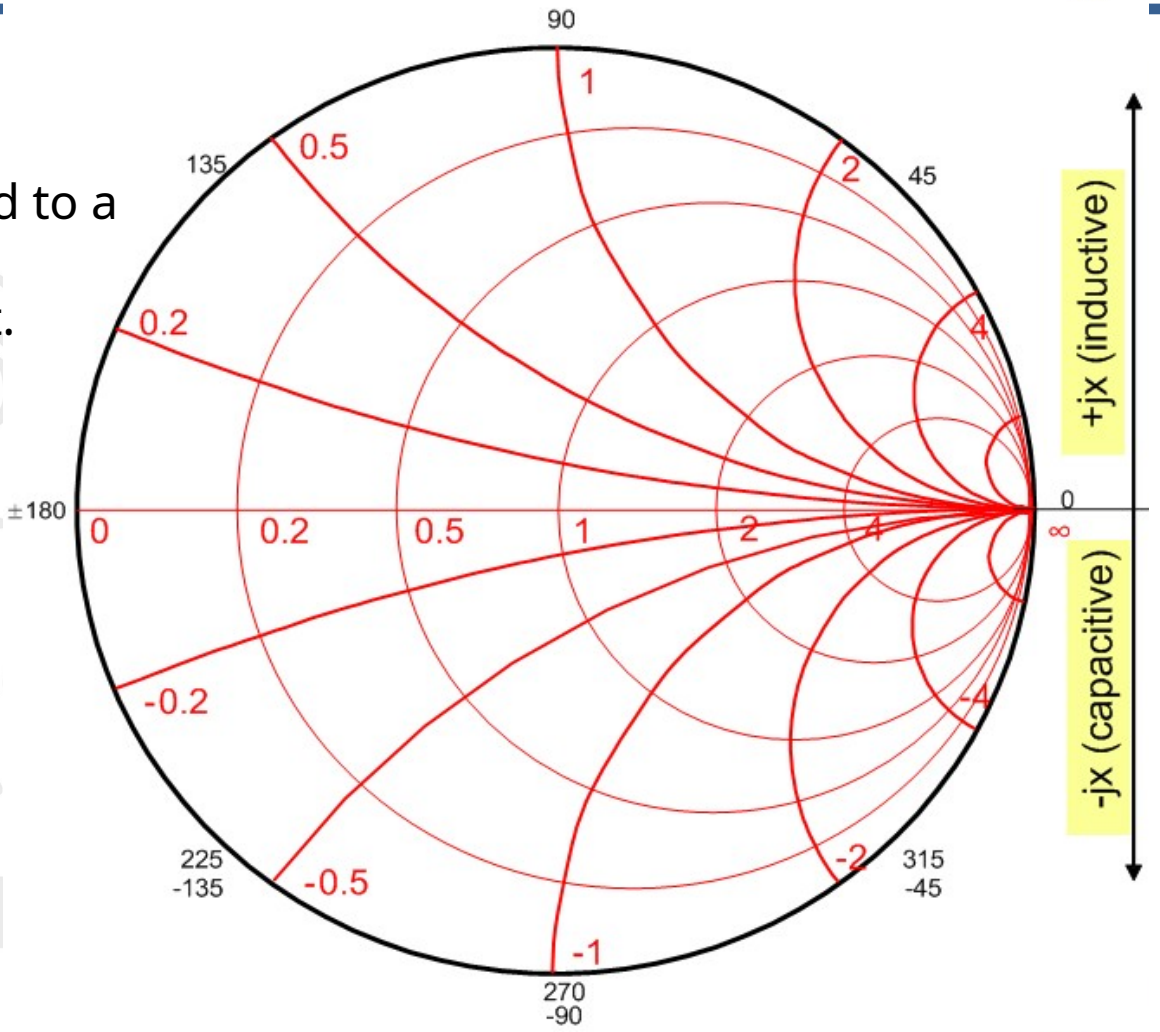


Admittance

A load = 50+50j Ω is connected to a 50 Ω TL. Determine the load admittance using Smith Chart.

$$Z_L = 50 + 50\,j$$

$$z_L = \frac{Z_L}{Z_0} = \frac{50 + 50\,j}{50}$$



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$$Z_L = 50 + 50\,j$$

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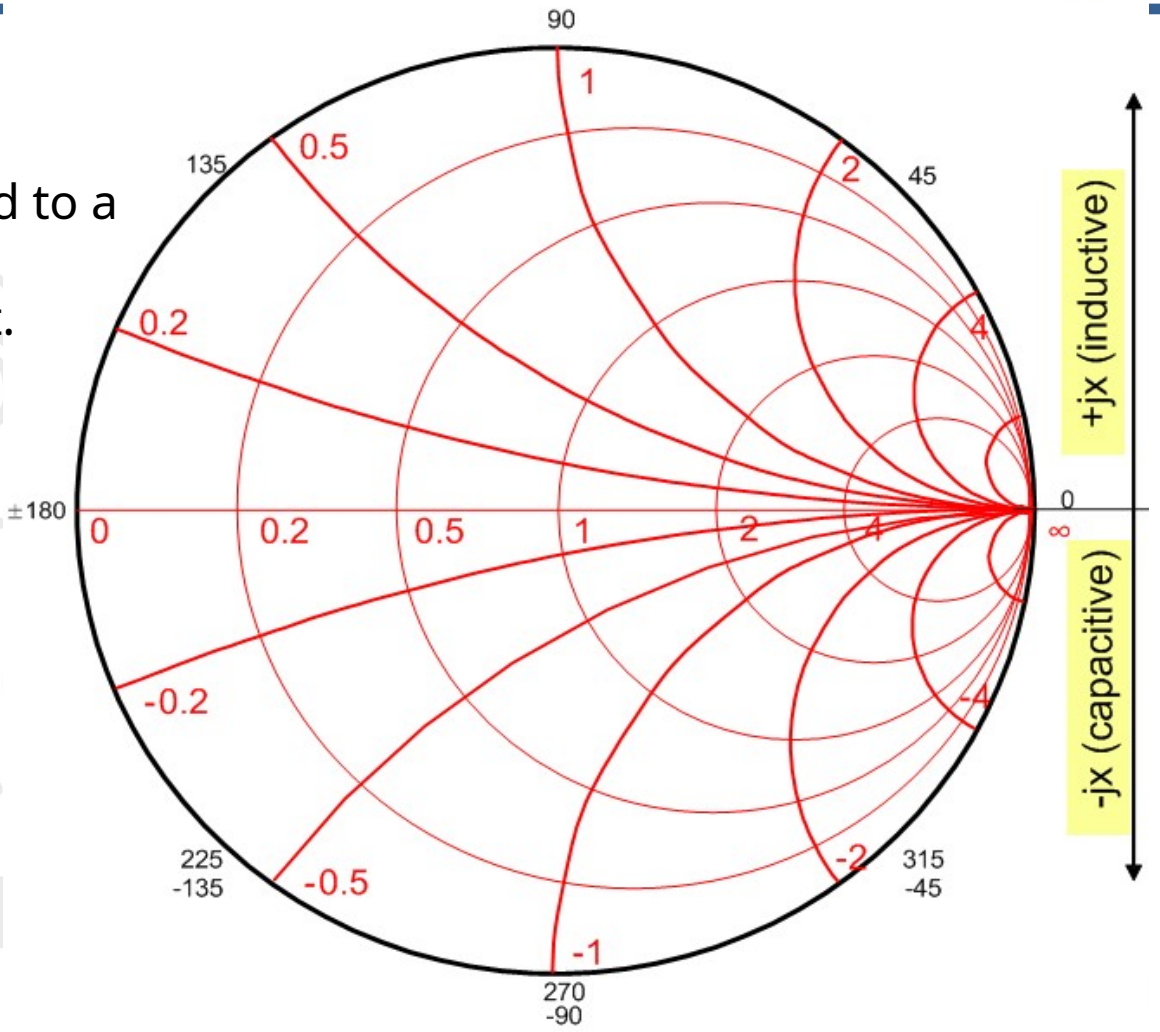
$$z_L = \frac{Z_L}{Z_0} = 1 + j$$



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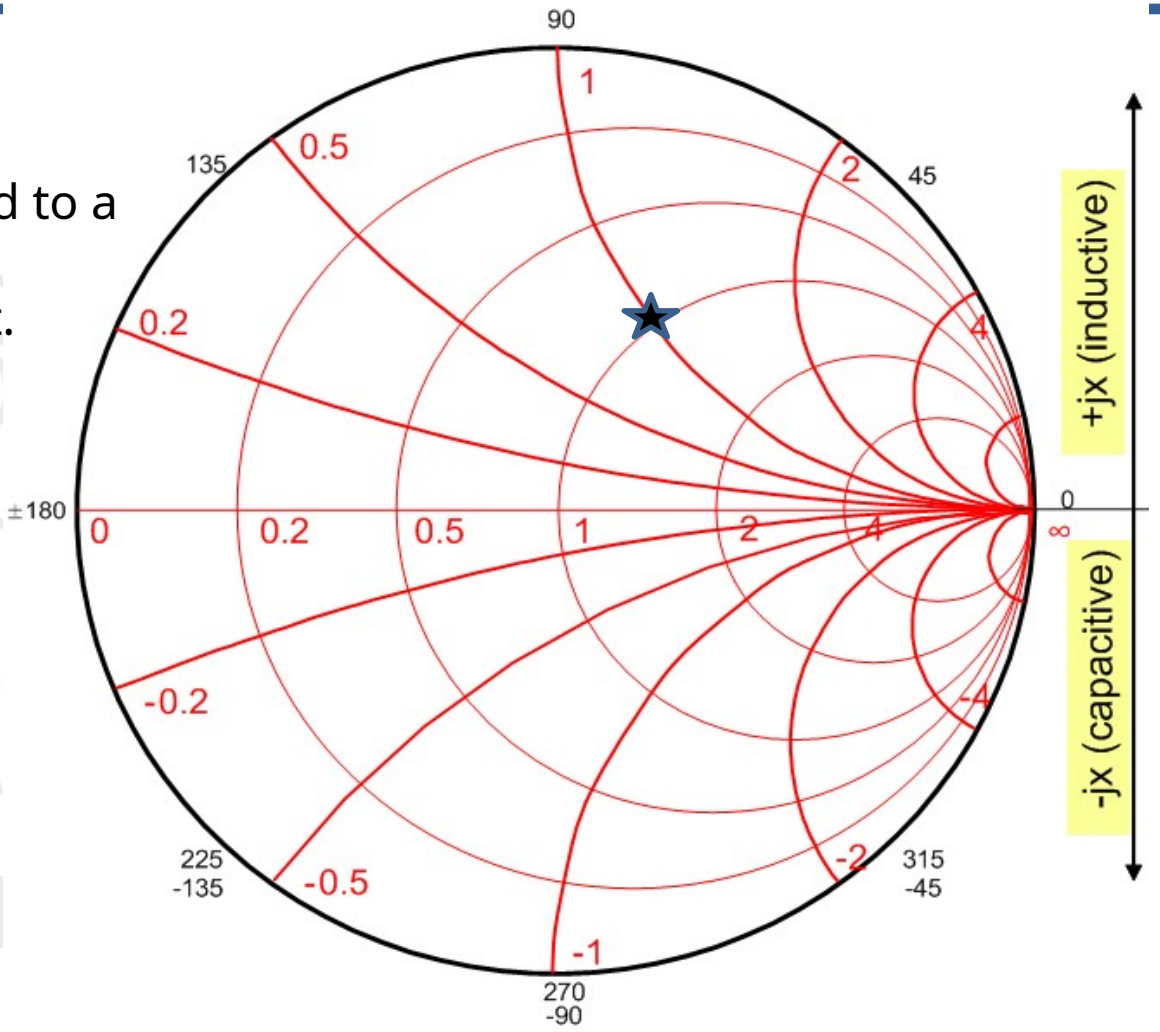

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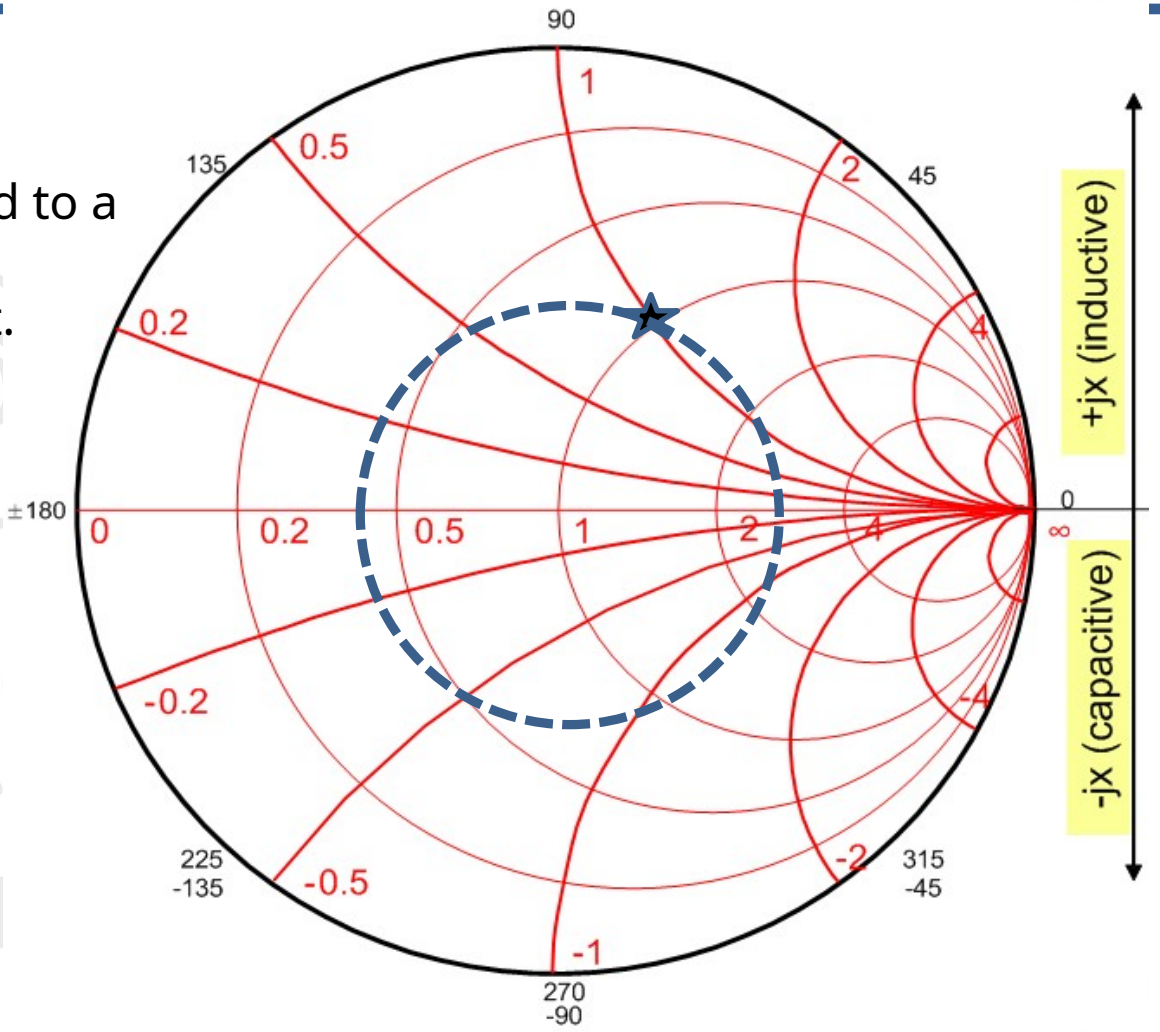

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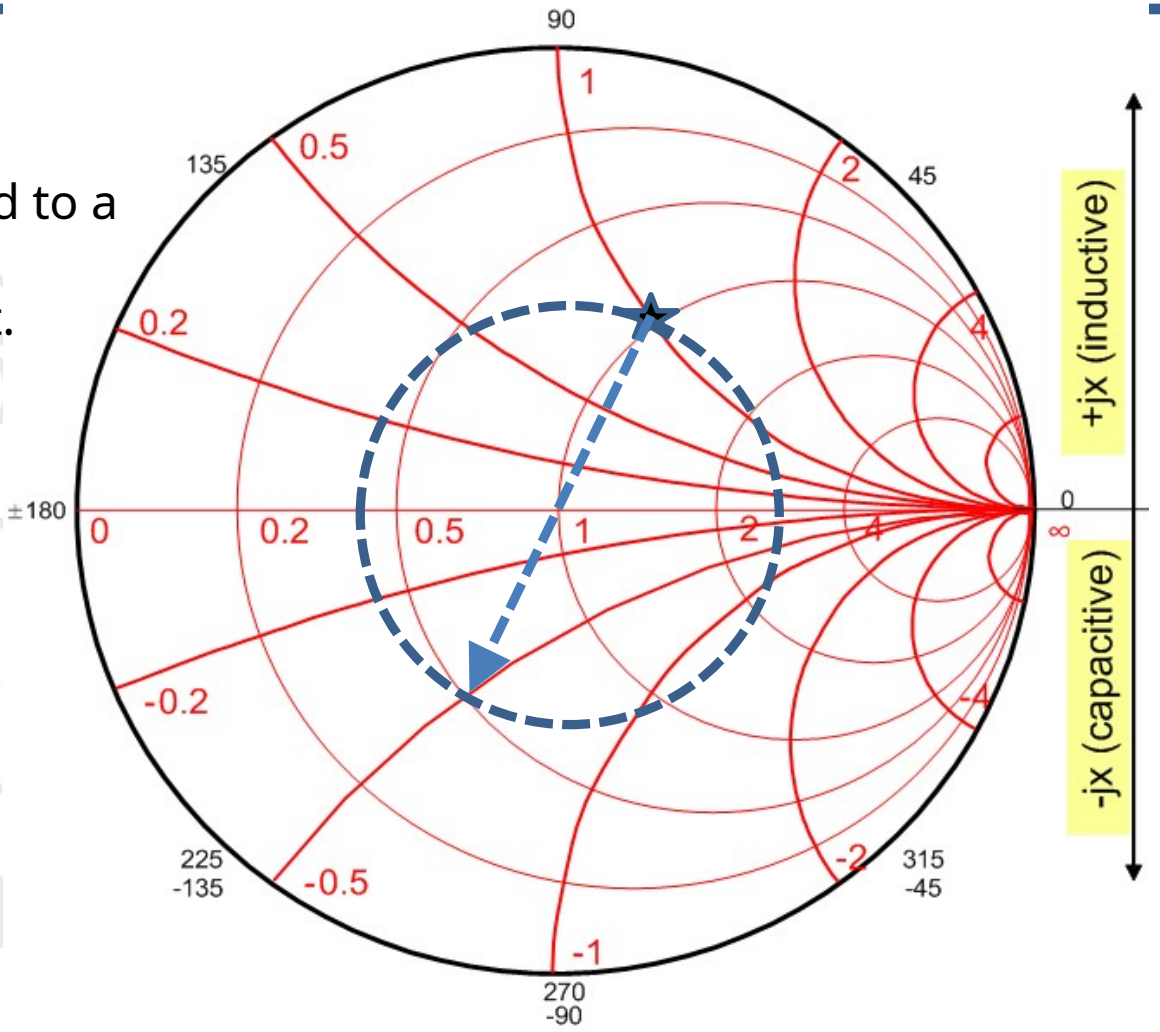

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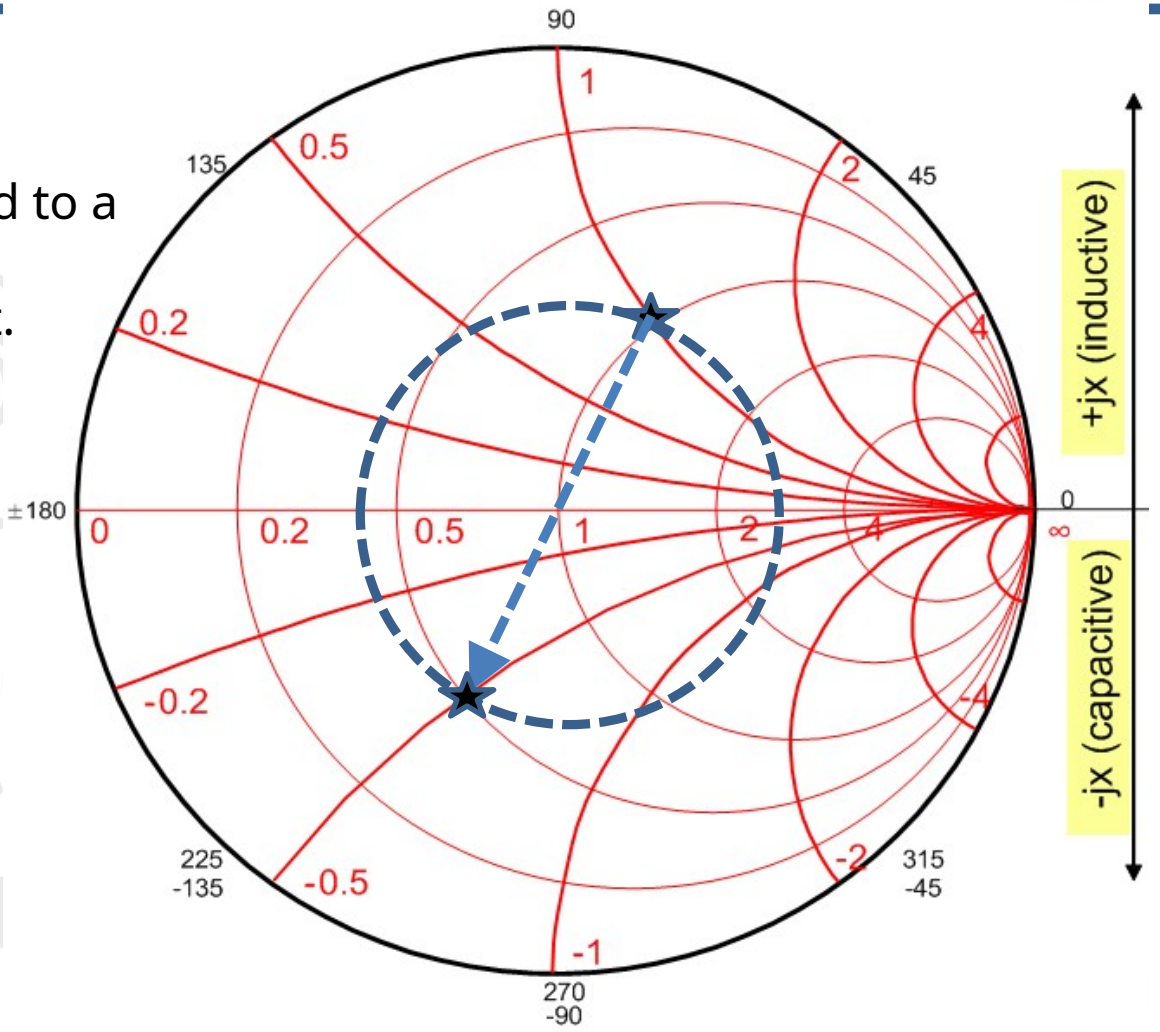

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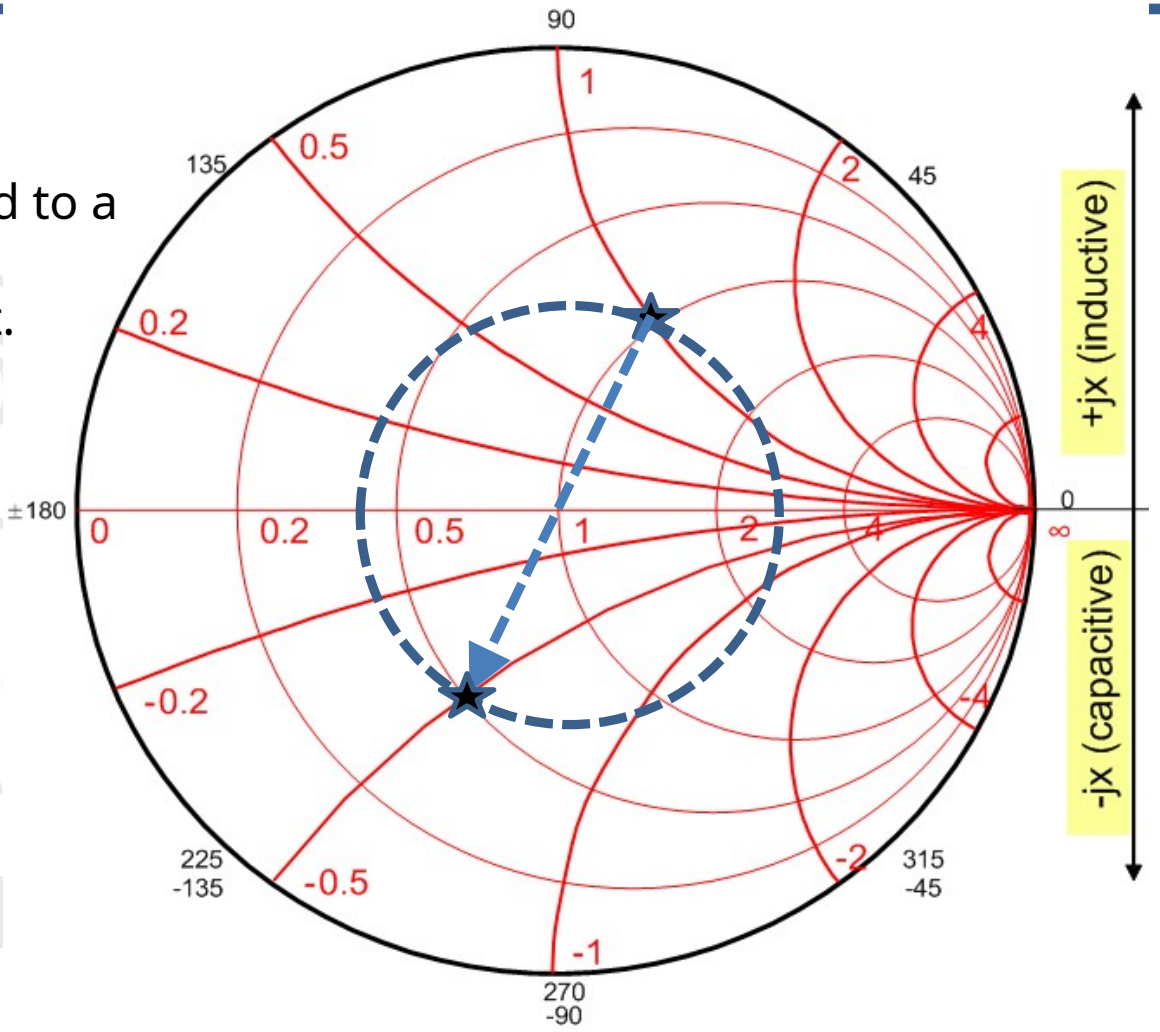
$$y_L = \frac{Y_L}{Y_0} = 0.5 - 0.5j$$



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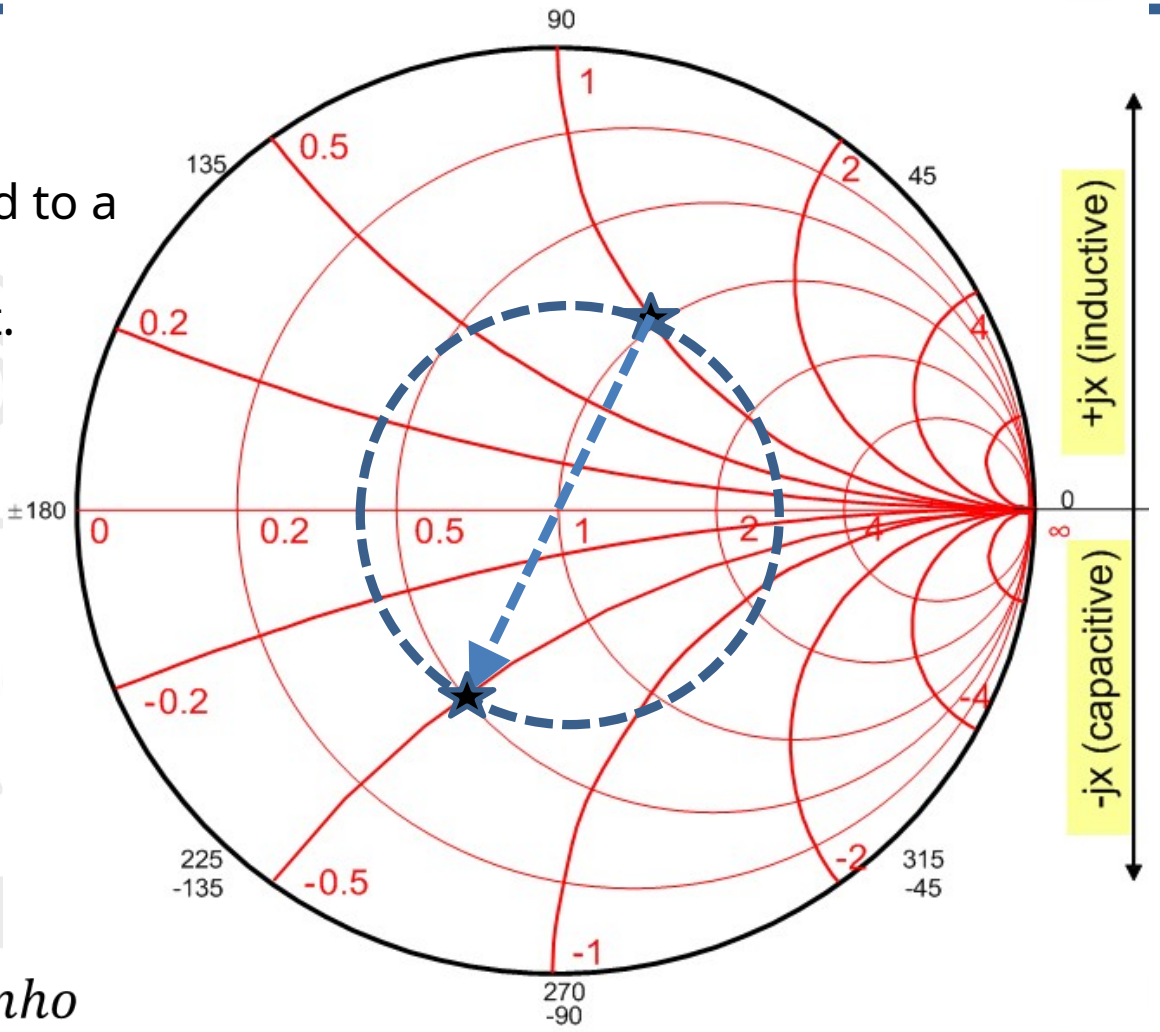
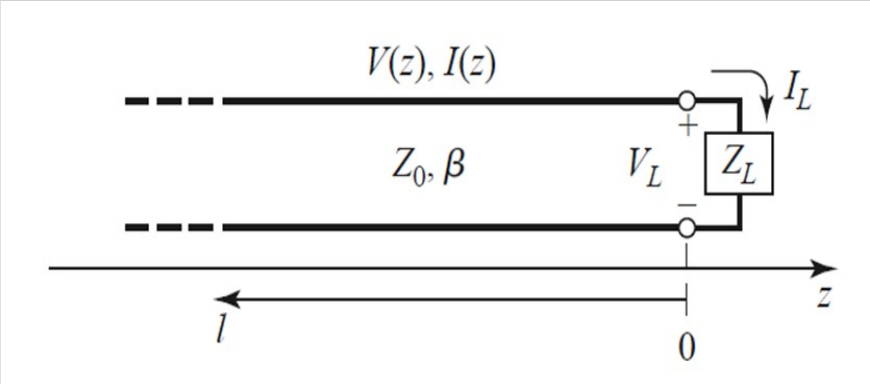
$$y_L = \frac{Y_L}{Y_0} = 0.5 - 0.5j$$

$$Y_L = y_L * Y_0 = \frac{0.5 - 0.5j}{50}\ mho$$



Smith Chart – Impact of TL

- So far, we have understood how to represent load impedance on Smith Chart.
- This is at Load Plane i.e. z = 0



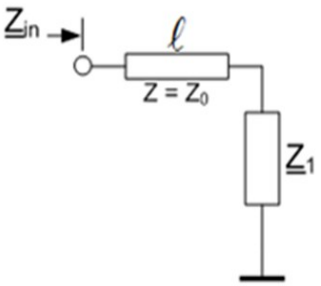
- What is the impact of adding a TL ?

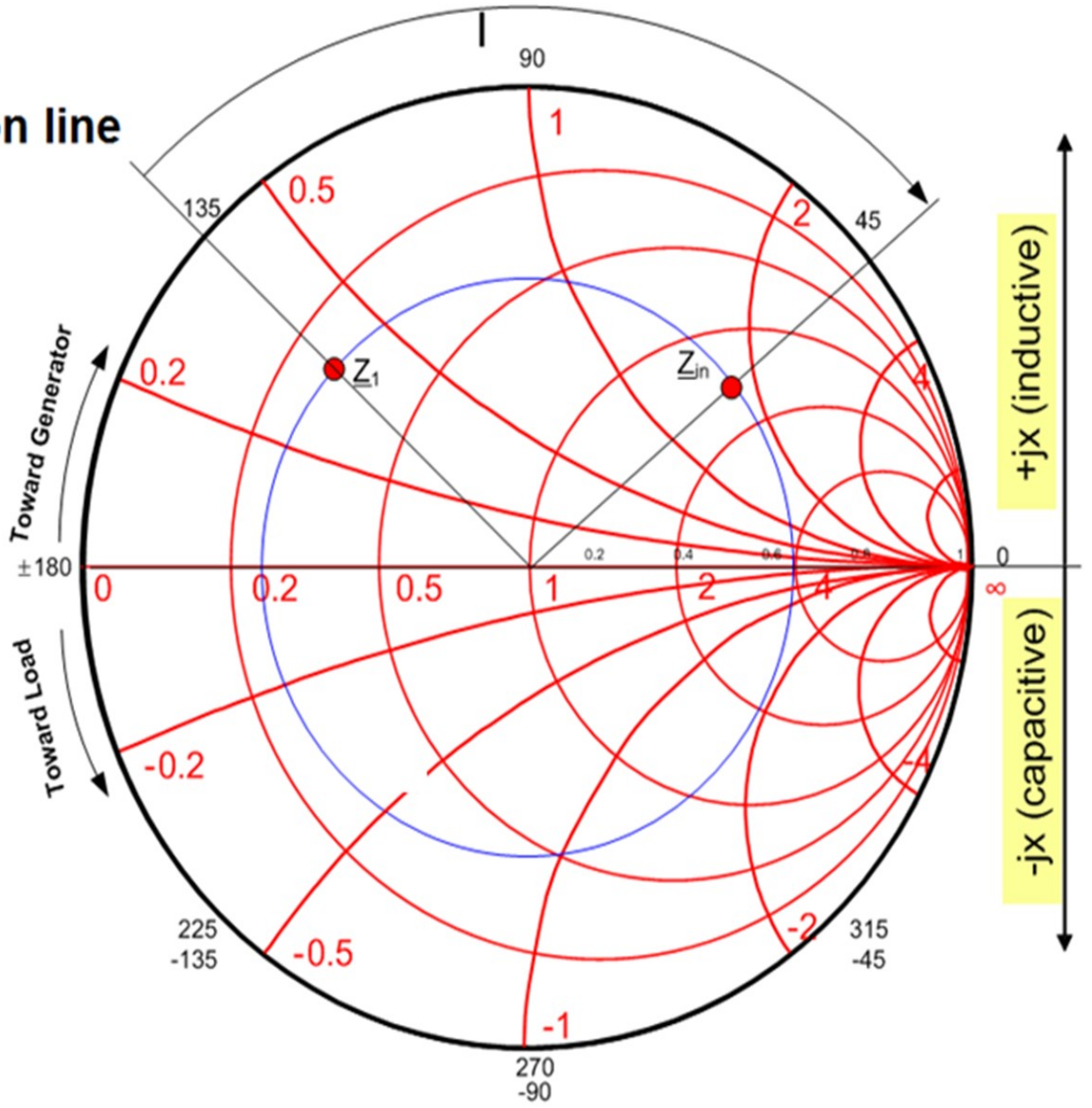
- We know that basically Smith Chart is a polar plot of Γ
- Hence, magnitude of Γ remains same, and only phase changes by $2\beta l$

Impedance plane

Serie transmission line

with $Z = Z_0$

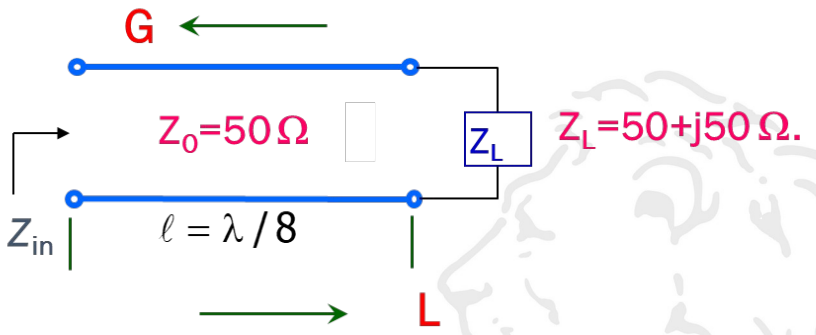




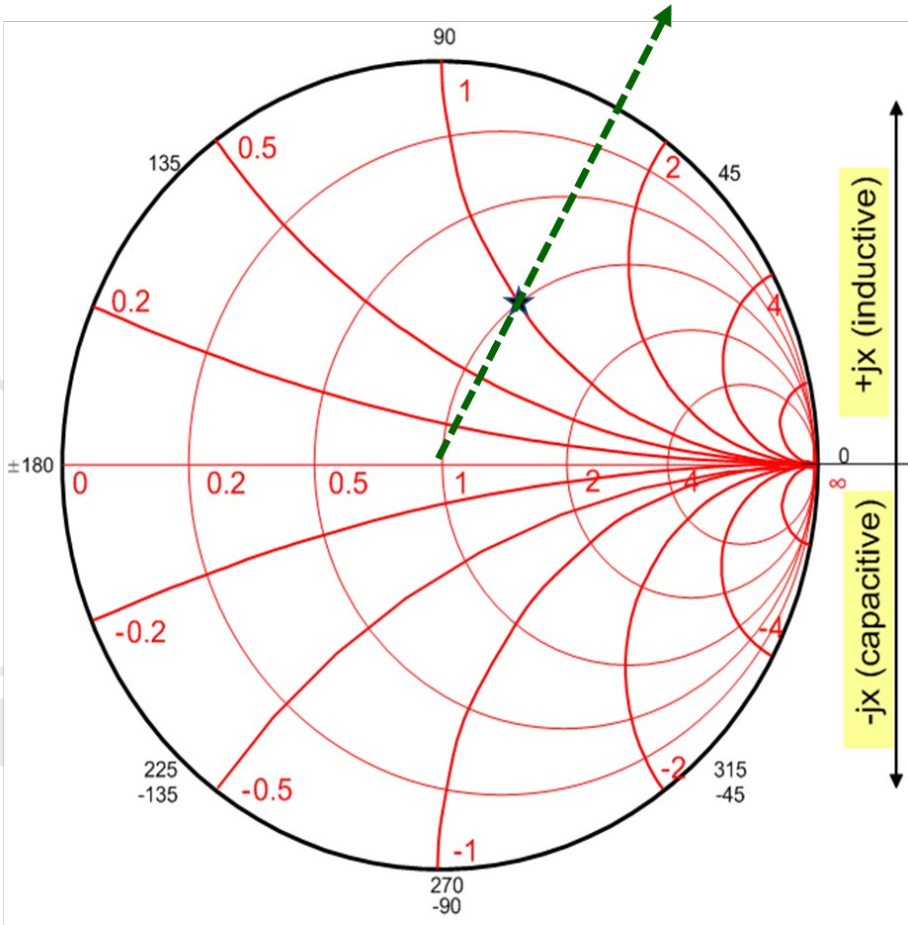
Towards the Generator :
Clockwise

Towards the Load :
Anti-Clockwise

Find the input impedance of a transmission line (Z_0=50Ω) that has a length of λ/8 and is connected to a load impedance Z_L=50+j50 Ω.



1. Locate $z=Z_L/Z_0=1+j1$ on the Smith Chart



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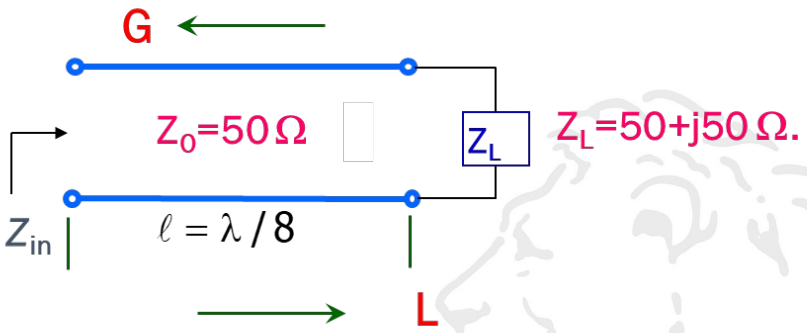
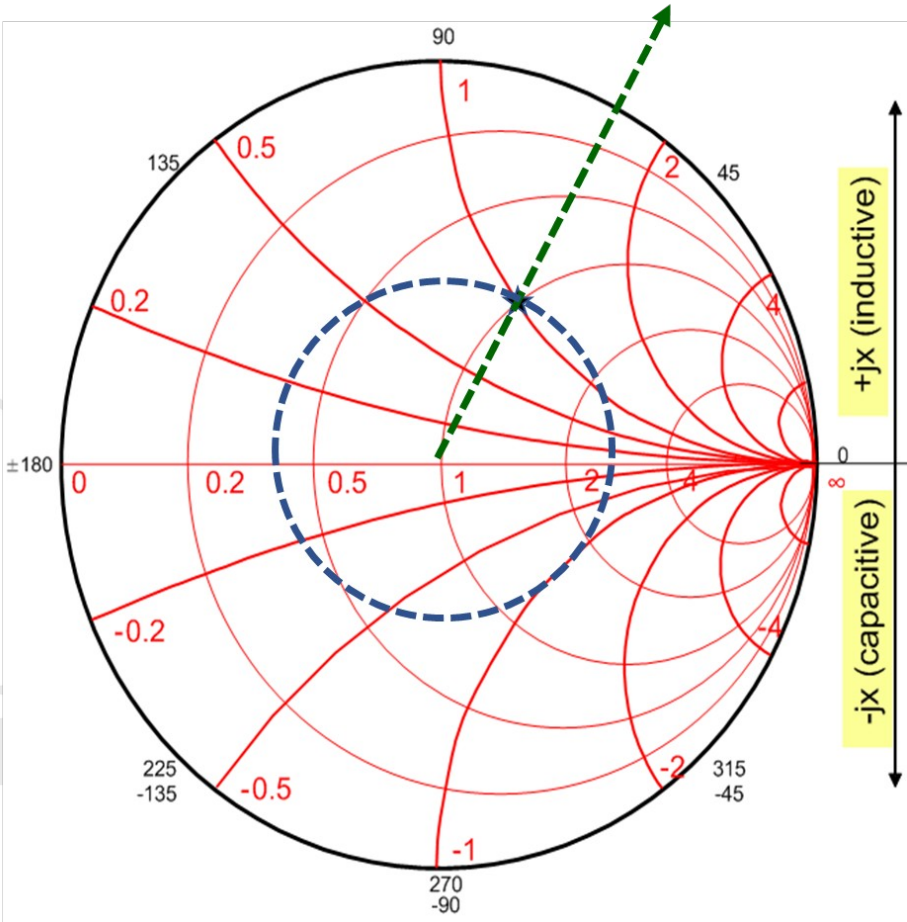


1. Locate $z=Z_L/Z_0=1+j1$ on the Smith Chart
2. Draw the constant Γ circle through the above point



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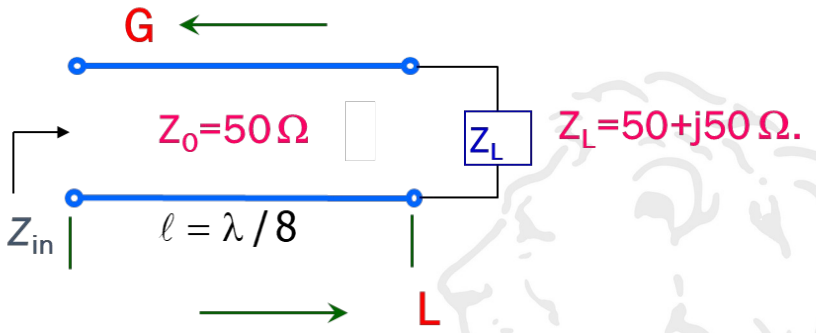
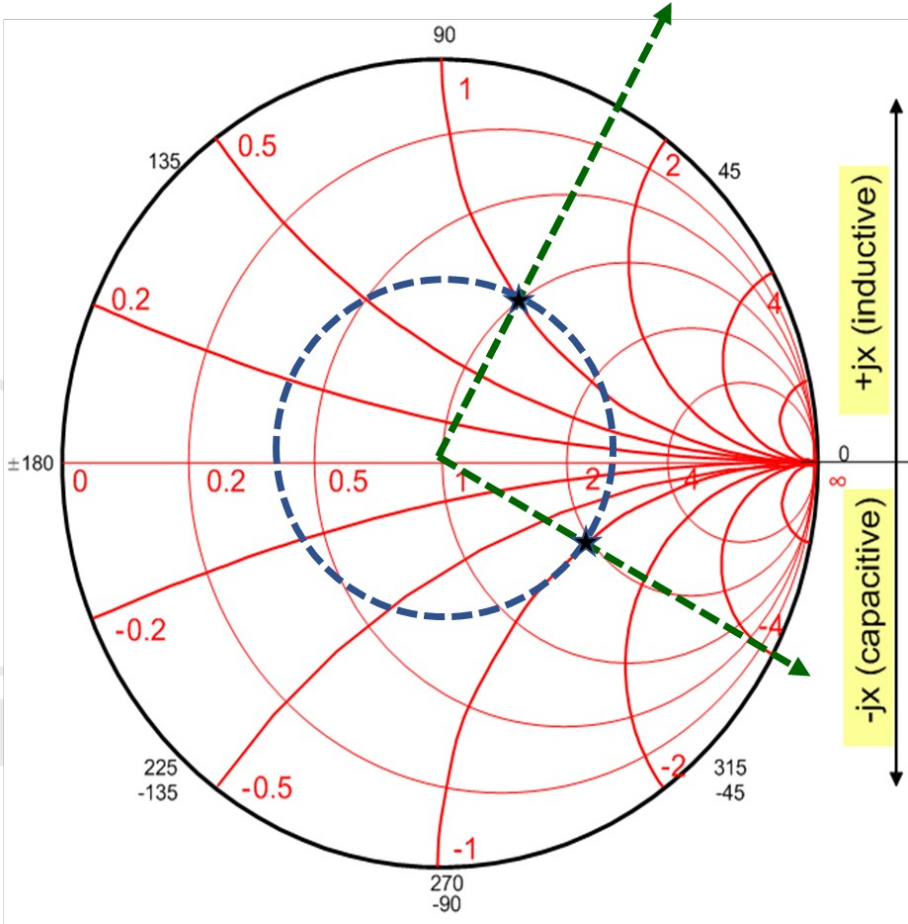


1. Locate $z=Z_L/Z_0=1+j1$ on the Smith Chart
2. Draw the constant Γ circle through the above point
3. Move towards generator (clockwise) on the constant Γ circle a distance of 0.125 λ to obtain new z=2-j1.



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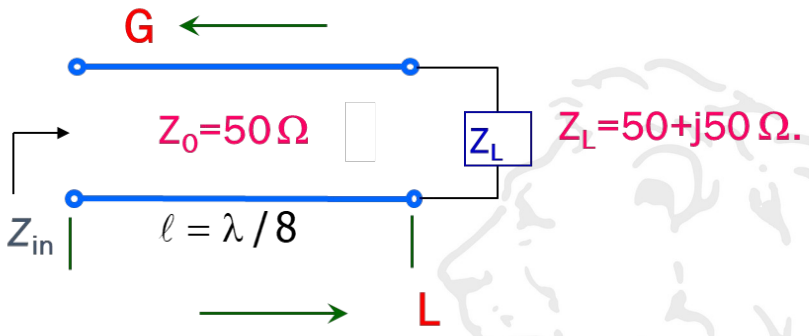

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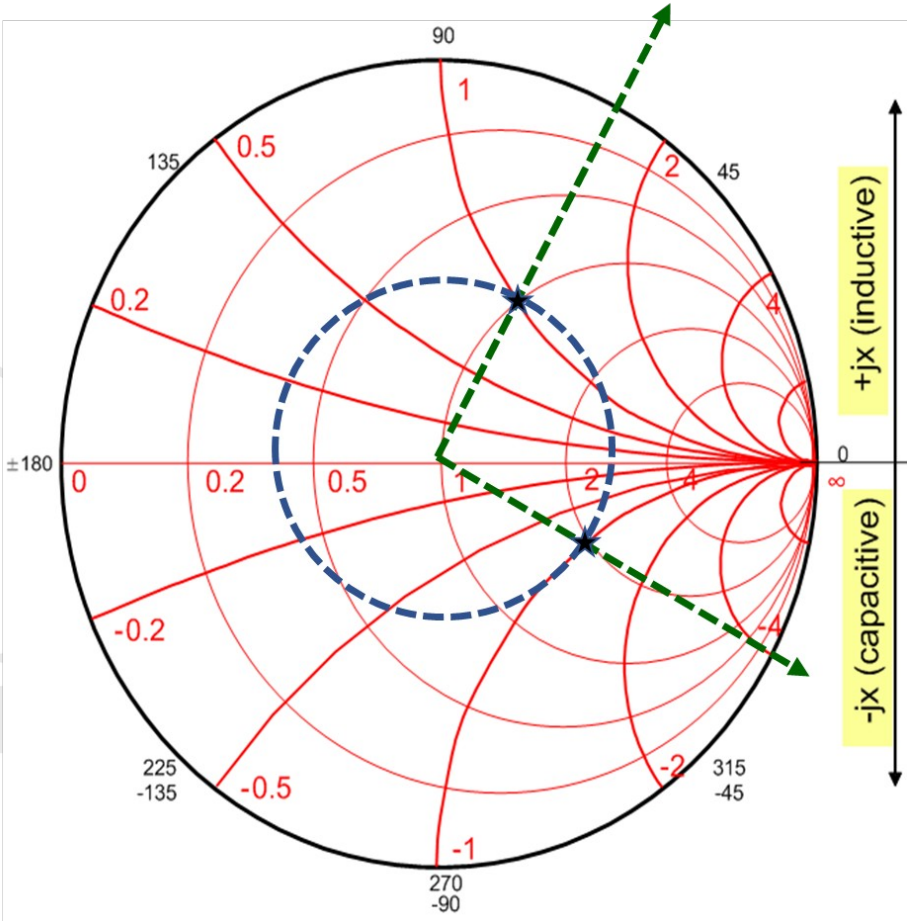
2. Draw the constant Γ circle through the above point
3. Move towards generator (clockwise) on the constant Γ circle a distance of 0.125 λ to obtain new z=2-j1.
4. De-normalize w.r.t 50 ohms to get Z=100-j50 Ω



Thank You

Questions?