Assignment - 6

Q1. $v = 0.6 \times 10^{14}\ s^{-1}$

$T = 1800K$

(i) As per classical oscillator, avg energy = E

$$E = \frac{8\pi v^2 KT\,dv}{c^3}$$

$K = 1.3806 \times 10^{-23}\ \frac{m^2 kg}{s^2 K}$

$$= \frac{832.33 \times 10^{28} \times 10^{-23}}{10^{24}}$$

$= 832.33 \times 10^{-19}\ J$

$= 8.3233 \times 10^{-17}\ J$

For a harmonic oscillator,

Avg energy = KT

where K = Boltzman constant

$= 1.3806 \times 10^{-23}$

T = Temp.

$= 1.3806 \times 10^{-23} \times 1800$

$= 2485.08 \times 10^{-23}$

$= \boxed{2.485 \times 10^{-20}\ J}$

(ii) As a plank's oscillator, avg energy = E

~~$E = hv$~~

~~$= 6.626 \times 10^{-34} \times 0.6 \times 10^{14}$~~

$$= E = \frac{hv}{e^{\frac{hv}{KT}} - 1}$$

$hv = 3.9756 \times 10^{-20}$

$$E = \frac{3.9756 \times 10^{-20}}{3.9520}$$

$= \boxed{1.01 \times 10^{-20}\ J}$

If new $v = 1.5 \times 10^{18}\ s^{-1}$

Energy as classical oscillator $= KT$

$= \boxed{2.484 \times 10^{-20}\ J}$

Energy as plank's oscillator $= \dfrac{hv}{e^{\frac{hv}{KT}} - 1}$

$hv = 9.939 \times 10^{-16}$

$E = \dfrac{9.939 \times 10^{-16}}{}$

$\approx \boxed{0\ J}$

Q2. As per classical theory, it followed law of equipartition of energies so, $E = KT$ where K = Boltzman constant.
So, it is irrespective of frequency.
As per planck's theory, $E = \dfrac{hv}{e^{hv/KT} - 1}$ which is accurate.

Q2. Vol. $= 100\ cm^3$ $\quad v = \lambda c \quad c = \lambda v$

No. of modes = ? $\quad dv = c d$

(i) λ range $= 5000$ to $5002\ \AA$

No. of nodes $= \dfrac{8\pi v^2\, dv}{c^3\ \cancel{d\lambda}}$ $\quad \cancel{v = \lambda c}$ $\quad \cancel{dv = c\, d\lambda}$

$= \cancel{8\pi \lambda^2 c^2}$

$c = \lambda v$

$c/\lambda = v$

$dv = \dfrac{-c\ d\lambda}{\lambda^2}$

No. of nodes $= \dfrac{8\pi c^2}{\lambda^2 c^3} \left(\dfrac{-c\ d\lambda}{\lambda^2}\right) \times$ vol.

$= \dfrac{8\pi\ d\lambda}{\lambda^4} \times \dfrac{100}{100 \times 100 \times 100}$

$$= \frac{4.017 \times 10^{-14}}{10^{-40}} \times 2 \times 10^{-10} \qquad -24+40$$

$$= \cancel{4.017} \; 8.035 \times 10^{16} \times \cancel{vol.} \; 10^{-4}$$

$$= \boxed{8.035 \times 10^{12}}$$

(ii) λ range $=$ 8000 to 8005 $\mathring{A}$

$\lambda = 8002.5 \times 10^{-10}$ m

No. of nodes $= \frac{25.132 \times 5 \times 10^{-10} \times 10^{-4}}{10^{-40} \times 4.1011 \times 10^{15}}$

$= 30.6405 \times 10^{11}$

$= \boxed{3.06 \times 10^{12}}$

Q3. $T = 1000$ K

As per planck's law $\rightarrow$

$$E = \frac{8\pi v^2 dv}{c^3}\left[\frac{hv}{e^{\frac{hv}{KT}} - 1}\right]$$

For peak of the curve.

$$\frac{d}{dv}\left(\frac{dE}{dv}\right) = 0 \qquad \frac{8\pi h}{c^3}\frac{d}{dv}\left[\frac{v^3}{e^{\frac{hv}{KT}} - 1}\right]$$

$$= \frac{8\pi h}{c^3}\left[\frac{\left(e^{\frac{hv}{KT}} - 1\right)(3v^2) - (v^3)\left(e^{\frac{hv}{KT}}\right)\left(\frac{h}{KT}\right)}{\left(e^{\frac{hv}{KT}} - 1\right)^2}\right] = 0$$

$$3\left(e^{\frac{hv}{KT}} - 1\right) - \left(\frac{hv}{KT} e^{\frac{hv}{KT}}\right) = 0.$$

For finding peak we will differentiate E wrt v

$$E = \frac{hv}{e^{\frac{hv}{KT}} - 1}$$

$$\frac{dE}{dv} = \frac{h\left[\left(e^{\frac{hv}{KT}} - 1\right) - v\left(e^{\frac{hv}{KT}}\right)\left(\frac{h}{KT}\right)\right]}{\left(e^{\frac{hv}{KT}} - 1\right)^2} = 0$$

$$e^{\frac{hv}{KT}} - 1 - \frac{hv}{KT} e^{\frac{hv}{KT}} = 0.$$

$$e^{\frac{hv}{KT}} \left[1 - \frac{hv}{KT}\right] = 1$$

$$1 - \frac{hv}{KT} = e^{-\frac{hv}{KT}}$$

$$e^{\frac{hv}{KT}} \left[3 - \frac{hv}{KT}\right] = 3.$$

solving this equation, we get,

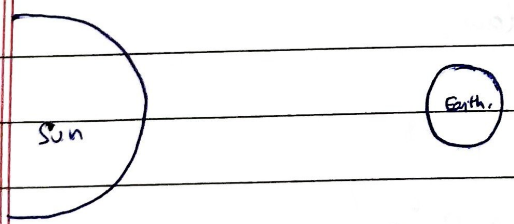
$$v = \boxed{6.25 \times 10^{13}\ Hz}$$

Q4. Rate $= 1.4\ kW/m^2 = 1400\ \frac{Watt}{m^2}$

r_o = radius of orbit $= 1.5 \times 10^{11} m$

r_s = " " sun $= 7 \times 10^8 m$

Temp. of Sun = ?



Power emitted by sun $= A \sigma T^4 = P$

$= (4\pi r_s^2)(\sigma T^4) = P$

Power recieved by Earth $= \frac{P}{4\pi r_o^2} = 1400\ \frac{J}{m^2 s}$

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$$\frac{r_s^2 (\sigma T^4)}{r_0^2} = 1400$$

$$T^4 = \frac{1400}{\sigma}\left(\frac{r_0^2}{r_s^2}\right)$$

$$= \frac{1400}{5.67 \times 10^{-8}} \times \left(\frac{1500}{7}\right)^2$$

$$= 246.913 \times 10^8 \times 45918.367$$

$$= 1133.7841.821 \times 10^8$$

$$= 1133.7841821 \times 10^{12}$$

$$T = \boxed{5.802 \times 10^3 \ K}$$

Q5. At $T_1 = 34^\circ C = 307 K$

273

$T_2 = 35^\circ C = 308 K$

615

$R_1 = A\sigma T_1^4$

$R_2 = A\sigma T_2^4$

$$\%\ \text{difference} = \frac{A\sigma T_2^4 - \sigma A T_1^4}{\sigma A T_1^4} \times 100$$

$$= \frac{T_2^4 - T_1^4}{T_1^4} \times 100$$

$$= \frac{(T_2^2 - T_1^2)(T_2^2 + T_1^2)}{T_1^4} \times 100$$

$$= \frac{615 \times (189113)}{94249 \times 94249} \times 100$$

$$= \boxed{1.3\ \%}$$

Q6. To prove :- $\lambda_{max} = \dfrac{hc}{4.965\ K_B T}$

Emitted energy is given by → $E = \dfrac{hc^2}{\lambda^5} \dfrac{1}{(e^{\frac{hc}{\lambda K T}} - 1)}$

We have to find λ corresponding to max. energy

So, $\dfrac{dE}{d\lambda} = 0.$

let $x = \frac{hc}{\lambda KT}$ $\lambda = \frac{hc}{KTx}$

So, substituting, we get a function →

$g(x) \propto \frac{x^5}{e^x - 1}$

$g'(x) = \frac{(e^x - 1)\, 5x^4 - x^5 (e^x)}{(e^x - 1)^2} = 0$

$e^x (5 - x) - 5 = 0.$

Solving this eq^n

∴ we get $x \approx 4.965$

$\frac{hc}{\lambda K_B T} = 4.965$

$$\boxed{\lambda = \frac{hc}{4.965\, K_B T}}$$

∴ proved.

Q7. Temp. of sun = 6000 K 580×10^{-9} (58×10^{-8})

λ from = 570 nm to 590 nm.

Portion of total radiation comprising of yellow light = ?

~~$E = \frac{8\pi hc \; d\lambda}{\lambda^5 (e^{\frac{hc}{\lambda KT}} - 1)}$~~

Total radiancy = $\sigma A T^4$

spectral Radiancy = $\frac{8\pi v^2 dv}{c^3} \left[\frac{hv}{e^{\frac{hv}{KT}} - 1}\right] = \frac{8\pi hc}{\lambda^5} \frac{d\lambda}{(e^{\frac{hc}{\lambda KT}} - 1)}$

Total radiancy per unit area = σT^4

$$\% \text{ Portion} = \frac{\left(\frac{8\pi hc}{\lambda^5}\frac{d\lambda}{(e^{\frac{hc}{\lambda kT}}-1)}\right)}{\sigma T^4} \times 100$$

$$= \frac{499.588 \times 10^{-26} \times 20\times10^{-9} \times 100}{6.56356768 \times 10^{-32} \times 61.6146}$$

$$= \frac{999.176 \times 10^{-32}}{404.4111 \times 10^{-32} \times 5.67\times10^{-8} \times 1296\times10^{12}}$$

$$= \boxed{2.47\ \%}$$

Q8. λ of x ray $= 0.05\,nm$

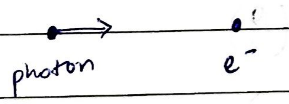
If Binding energy of e^- $= 62{,}000\,eV$

Energy of x ray $= \frac{hc}{\lambda} = 24{,}800\,eV$

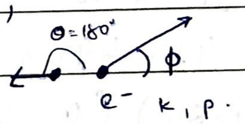
Compton scattering will not take place, because energy of photons is far less than energy needed to free the electron, so the collision of photon will not be with the e^- but with the whole ionic core so, in the equation $\Delta\lambda = \frac{h(1-\cos\theta)}{m_0c}$,

M_0 = mass of whole ionic core instead of e^-

So, $\Delta\lambda$ will become negligible so no scattering will be visible.



For max. energy transfer to happen, photon should completely reflect back,



So, $\theta = 180°$, $\Delta\lambda = 2\lambda_c$

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Initial energy $= \dfrac{hc}{\lambda}$

Final energy $= \dfrac{hc}{(\lambda + 2\lambda_c)}$ + energy of e^-

$$E \text{ of } e^- = \frac{hc}{\lambda} - \frac{hc}{\lambda + 2\lambda_c} =$$

$$= hc\left[\frac{2\lambda_c}{\lambda(\lambda + 2\lambda_c)}\right]$$

$$= \frac{2(1240)(0.00243) \times 20}{\cancel{0.05}\ (0.05243)}$$

$$= \frac{496 \times 0.243}{0.05243.}$$

$$= 2298.83 \text{ eV}$$

$$= \boxed{2.3 \text{ keV}}$$

2480.00
− 2298.83
22501.17

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Q9. To prove : K.E. of recoil e^- = K

$$K = \frac{h\nu\,[2\beta \sin^2(\theta/2)]}{1 + 2\beta \sin^2(\theta/2)} \qquad \beta = \frac{h\nu}{M_0 c^2}$$

$$E_0 = E_1 + K$$

$$K = E_0 - E_1$$
$$= c\,(P_0 - P_1)$$
$$= c\left[\frac{h}{\lambda_0} - \frac{h}{(\lambda_0 + \Delta\lambda)}\right]$$
$$= hc\left[\frac{\Delta\lambda}{\lambda_0(\lambda_0 + \Delta\lambda)}\right]$$
$$= hc\left[\frac{\lambda_c(1-\cos\theta)}{\lambda_0(\lambda_0 + \lambda_c(1-\cos\theta))}\right]$$
$$= h\nu\left[\frac{\lambda_c(1-\cos\theta)}{\lambda_0 + \lambda_c(1-\cos\theta)}\right]$$
$$= h\nu\left[\frac{\lambda_c(2\sin^2\theta/2)}{\lambda_0 + \lambda_c(2\sin^2\theta/2)}\right]$$

$$\frac{x \sin^2\theta/2}{1 + x\sin^2\theta/2} \qquad \lambda_0\nu = c \qquad \lambda_0 = c/\nu$$

$$K = h\nu\left[\frac{\nu\lambda_c(2\sin^2\theta/2)}{c + \nu\lambda_c(2\sin^2\theta/2)}\right]$$

$$\lambda_c = \frac{h}{M_0 c}$$

So,

$$K = h\nu\left[\frac{\frac{h\nu}{M_0c}(2\sin^2\theta/2)}{c + \frac{h\nu}{M_0c}(2\sin^2\theta/2)}\right]$$

$$= \boxed{h\nu\left[\frac{2\beta\sin^2\theta/2}{1 + 2\beta\sin^2\theta/2}\right]}$$

∴ Hence proved.

$$\boxed{K_{max} = 2\beta h\nu = \frac{2h^2\nu^2}{m_0c^2}}$$

$$K = c\,(P_0 - P_1) \qquad \cancel{P^2} = P_0^2 + P_1^2 - 2P_0P_1\cos\theta$$

$$p^2 = 2Km_0 + K^2/c^2$$

$$p^2 = 2m_0c\,(P_0 - P_1) + P_0^2 + P_1^2 - 2P_0P_1$$

$$P_0^2 + P_1^2 - 2P_0P_1\cos\theta = 2m_0c\,(P_0 - P_1) + \cancel{P_0^2} + \cancel{P_1^2} - 2P_0P_1$$

$$\cancel{2}P_0P_1\,(1-\cos\theta) = \cancel{2}m_0c\,(P_0 - P_1)$$

$$\boxed{\frac{2\sin^2\theta/2}{m_0c} = \frac{1}{P_1} - \frac{1}{P_0}}$$

Q10. $\lambda = 0.012\,\text{Å}$

0.0486

$\theta_1 = 90^\circ \qquad \Delta\lambda_1 = ?$

$\theta_2 = 180^\circ \qquad \Delta\lambda_2 = ?$

As per compton effect, change in wavelength is given by →

$$\Delta\lambda = \lambda_c(1-\cos\theta)$$

$$\Delta\lambda_1 = \lambda_c(1-\cos 90^\circ) = \lambda_c = 0.0243\,\text{Å}$$

$$\Delta\lambda_2 = \lambda_c(1-\cos 180^\circ) = 2\lambda_c = 0.0486\,\text{Å}$$

For $\theta = 180^\circ$

New wavelength $= 0.0606\,Å$

$$K = \frac{hc}{\lambda}\left[\frac{\Delta\lambda}{\lambda'}\right]$$

$$= 1033333.33\left(\frac{0.0486}{0.0606}\right)$$

$$= 828712.87\ eV$$

$$= \boxed{0.828\ MeV}$$

For $\theta = 90^\circ$

New wavelength $= 0.0363\,Å$

$$K = h\nu\left[\frac{2\beta\sin^2\theta/2}{1 + 2\beta\sin^2\theta/2}\right]$$

$$= c\left[\frac{h}{\lambda} - \frac{h}{\lambda'}\right]$$

$$= hc\left[\frac{\Delta\lambda}{\lambda(\lambda')}\right]$$

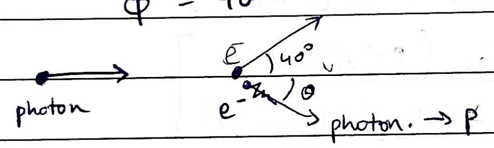
$$= 1033333.33\left(\frac{0.0243}{0.0363}\right)$$

$$= 691135.53\ eV$$

$$= \boxed{0.691\ MeV}$$

Q11. Energy of photon $= m_e c^2 = E$

$\phi = 40^\circ$



Applying momentum conservation.

$$\frac{E}{c} = P_e\cos 40^\circ + p\cos\theta$$

$$E = c(P_e\cos 40^\circ) + cP\cos\theta$$

$$P_e\sin 40^\circ = p\sin\theta$$

$$\left(\frac{P_e\sin 40^\circ}{p}\right)^2 + \frac{1}{p^2}\left(\frac{E}{c} - P_e\cos 40^\circ\right)^2 = 1$$

$$(P_e\sin 40^\circ)^2 + \left(E/c - P_e\cos 40^\circ\right)^2 = p^2$$

Applying energy conservation

$$M_e c^2 + E = \sqrt{P_e^2c^2 + (m_ec^2)^2} + pc$$

$$(2E - pc)^2 = P_e^2c^2 + E^2$$

$$4E^2 + c^2p^2 - 4Epc = P_e^2c^2 + E^2$$

$$3E^2 - 4Epc + c^2(p^2 - P_e^2) = 0$$

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$\frac{\cancel{hc}}{\cancel{hc}}$

$\frac{h}{\lambda} = \frac{E}{c}$ $\lambda = \frac{hc}{E}$

$E = (P_e) c \cos 40^\circ + p \cos\theta$

$= c \left(\frac{P_e}{\cancel{\ }}\right) \cos 40^\circ + \frac{h \cos\theta}{\lambda + \Delta\lambda}$

$= c$

$\theta = ?$

$\phi = 40^\circ$

The relation is given by $\rightarrow$

$$\cot\phi = \left(1 + \frac{h\nu}{m_e c^2}\right) \tan\left(\frac{\theta}{2}\right)$$

$$\tan(\theta/2) = \frac{\cot 40^\circ}{2 \cancel{1}} = \cancel{0.2329} \quad 1.1367.$$

$$\theta = 2 \tan^{-1}\left(\cancel{0.2329}\ 1.1367\right)$$

$$= \boxed{\cancel{26.228^\circ}}$$

$$= \boxed{\cancel{91.2^\circ}} \qquad \boxed{\theta \simeq 61.56^\circ}$$

Energy of scattered photon $= \frac{hc}{\lambda'} = \frac{\cancel{1240}}{}$

$$E' = \frac{E}{1 + \beta(1 - \cos\theta)} \qquad \left[\beta = \frac{h\nu}{m_0 c^2}\right]$$

For us $\beta = 1$

$$E' = \frac{E}{2 - \cos\theta} = \frac{511\ \text{keV}}{1.5235} \simeq \boxed{335\ \text{keV}}$$

Q12. Power = 1.5 mW = 1.5×10^{-3} W

λ = 400 nm

0.1 % → produce photoelectrons

current = ? = $\frac{\text{charge}}{\text{time}}$.

For unit time,

Energy incident = 1.5×10^{-3} J

Energy of unit photon = $\frac{\cancel{1240}^{31}}{\cancel{400}_{10}}$ eV = 3.1 eV

No. of photons = $1.5 \times 10^{-3} / 3.1 \times 1.6 \times 10^{-19}$ J

$= \frac{15 \times 10^{16}}{3.1 \times 16}$

Effective e^- produced = $\frac{15 \times 10^{16}}{31 \times 1.6} \times \frac{0.1}{100}$

current = $\frac{15 \times 10^{15}}{31 \times \cancel{1.6}} \times \frac{\cancel{0.1}}{\cancel{100}} \times \cancel{1.6} \times 10^{-19}$

$= 0.483 \times 10^{-6}$ A

$= \boxed{0.48\ \mu A}$

Q13. stopping potential V_{01} = 0.71 V → $(KE)_{max}$ = 0.71 eV

λ_1 = 491 nm

λ_2 = ?

V_{02} = 1.43 V

work function (ϕ) = ?

As per photoelectric equation,

$K_{max} = hc/\lambda - \phi$

$0.71 = \frac{1240}{491} - \phi$

$0.71 = 2.5254 - \phi$

$\boxed{\phi = 1.815 \text{ eV}}$

$1.43 = \frac{1240}{\lambda} - 1.815$

$\frac{1240}{\lambda} = 3.245$

$\lambda = \boxed{382.126 \text{ nm}}$

Q14. Kinetic energy $= 200 eV$

$p = \sqrt{2mK} = \frac{h}{\lambda}$

$\lambda = \frac{h}{\sqrt{2mK}} = \frac{6.6 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 200 \times 1.6 \times 10^{-19}}}$

$= \frac{6.626 \times 10^{-34}}{76.315 \times 10^{-25}}$

$= 0.0868 \times 10^{-9} m = \boxed{8.68 \times 10^{-11} m}$

- It would exhibit particle-like behaviour because wave like behaviour like interference, diffraction is only visible when it encounters obstacles whose dimensions are nearly same as wavelength of e^-.

Q15. If E = Total energy

$E >>> m_0c^2$, then de broglie wavelength $\simeq$ wavelength of ~~light~~ photon of same energy.

If energy of photon $= E$

then wavelength : $E = \frac{hc}{\lambda}$

$\boxed{\lambda = \frac{hc}{E}}$ = RHS

For a moving particle,

$E = K + m_0c^2$

$K = E - m_0c^2 \simeq E$ (approx)

De Broglie wavelength $\lambda = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2mE}} = \frac{h\sqrt{\sqrt{1-\frac{v^2}{c^2}}}}{\sqrt{2 m_0 E}}$

$E = m_0c^2 + pc$

$= cp$

$\lambda = \frac{h}{p} = \boxed{\frac{hc}{E}} = LHS$

$\therefore LHS = RHS$

Q16. (a) $\lambda = 0.15\ nm$

$KE = \frac{hc}{\lambda} = \frac{1240}{0.15} = 8266.66\ eV \neq \boxed{8.3\ KeV}$

(b) $K = \frac{h^2}{2m\lambda^2} = \frac{1951.283 \times 10^{-50}}{2 \times 9.1 \times 10^{-31}} = 107.21 \times 10^{-19}\ J \simeq \boxed{67\ eV}$

$\lambda = \frac{h}{\sqrt{2mK}}$

(c) $K = \frac{h^2}{2m\lambda^2} = \frac{1957.283 \times 10^{-50}}{2 \times 1.67 \times 10^{-27} \times 1.6 \times 10^{-19}} = \boxed{0.036\ eV}$

(d) $K = \frac{h^2}{2m\lambda^2} = \frac{1951.283 \times 10^{-50}}{2 \times 1.6 \times 10^{-19} \times 4 \times 1.67 \times 10^{-27}} = \boxed{9.17 \times 10^{-3}\ eV}$

Q17. $\lambda = 10^{-13}\ m = 10^{-9} \times 10^{4}\ nm$

$V_{ph} = ?$

$V_g = ?$

$V_g V_p = c^2$

$V_g = V$

$V_p = c\sqrt{1 - \frac{E_0^2}{E^2}}$

$K = \frac{h^2}{2m\lambda^2} = \frac{(6.626 \times 10^{-34})^2}{10^{-26} \times 2 \times 9.1 \times 10^{-31}}$

$= 2.41 \times 10^{-11}\ J$

$\frac{0.819}{3.229} \times 10^{-11}\ J$

$E_0 = m_0c^2 = 81.9 \times 10^{-15}\ J = 51.1875 \times 10^{4}\ eV$

$E = hc/\lambda = 0.1240\ eV$

$\left(\frac{E_0}{E}\right)^2 = \left(\frac{m_0c^2\lambda}{hc}\right)$

$V_g V_p = c^2$

$V_g = V = c\sqrt{1 - \frac{E_0^2}{E^2}} =$

$V_p = \frac{c^2}{V}$

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$$v = c\sqrt{1 - \left(\frac{0.819}{3.229}\right)^2}$$

$$= c\sqrt{0.9356}$$

$$= \boxed{2.9017 \times 10^8 \text{ m/s}} = \text{Group velocity} = v_g.$$

$$v_p = \frac{9 \times 10^{16}}{2.9017 \times 10^8} = \boxed{3.101 \times 10^8 \text{ m/s}} = v_p$$

Q18.

$$-34+2$$

$$\lambda_b = \frac{h}{\sqrt{2mK}} = \frac{h}{mv} = \frac{6.626 \times 10^{-34}}{(70 \times 10^{-3})(25)} = \boxed{0.0378 \times 10^{-32} \text{ m}}$$

$$\lambda_e = \frac{h}{\sqrt{2mK}} = \frac{6.626 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 50 \times 1.6 \times 10^{-19}}} = \frac{6.626 \times 10^{-34}}{3.815 \times 10^{-24}}$$

$$10^{-50+2}$$

$$= \boxed{1.736 \times 10^{-10} \text{ m}}$$

$$= \boxed{0.17 \text{ nm}}$$

$$v_p = \frac{\sqrt{g\lambda}}{2\pi} = \frac{d\omega}{dK}$$

$$\int d\omega = \int \frac{\sqrt{g\lambda}}{2\pi}\, dK$$

$$K = \frac{2\pi}{\lambda}$$

$$\lambda = \frac{2\pi}{K}$$

$$\int d\omega = \frac{1}{2\pi}\int \sqrt{\frac{2\pi g}{K}}\, dK$$

$$\omega = \frac{\cancel{2}\sqrt{2\pi g}\sqrt{K}}{\cancel{2}\pi}$$

$$\omega = \sqrt{\frac{2gK}{\pi}}$$

$$v_g = \frac{\omega}{K}$$

$$= \frac{1}{K}\sqrt{\frac{2gK}{\pi}}$$

$$= \sqrt{\frac{2g}{\pi K}}$$

Substituting $K = \frac{2\pi}{\lambda}$

$$v_g = \sqrt{\frac{2g\lambda}{\pi(2\pi)}}$$

$$= \frac{\sqrt{g\lambda}}{\pi}$$

$$\frac{\omega}{\sqrt{K}} = \sqrt{\frac{2\pi g}{K}}$$

$$d\omega = \frac{\sqrt{2\pi g}}{2\sqrt{K}}\, dK$$

$$\frac{d\omega}{dK} = \sqrt{\frac{g\pi\lambda}{2 \cdot 2\pi}} = \frac{1}{2}\sqrt{g\lambda}$$

For water waves,

$$\omega^2 = gK$$

$$\omega = \sqrt{gK}$$

$$v_g = \frac{d\omega}{dK} = \frac{1}{2}\sqrt{\frac{g}{K}}$$

$$v_p = \frac{\omega}{K} = \sqrt{\frac{g}{K}}$$

$$\boxed{v_g = \frac{1}{2} v_p}$$

Q20. $(KE)_{min} = ?$ $\quad r = 6\times10^{-15}\,m = \Delta x$

$$K = \tfrac{1}{2}mv^2$$

$$\Delta x \cdot \Delta v \geq \frac{\hbar}{2m}$$

$$\Delta K = \frac{m}{\cancel{2}}(2v)\Delta v = mv(\Delta v)$$

$$= \cancel{m}v\left(\frac{\hbar}{2\cancel{m}\,\Delta x}\right)$$

$-34+15$

$$= v\,(0.087\times10^{-19})$$

Q20. As per uncertainity principle, $r = 6 \times 10^{-15}$ m

$$\Delta x \cdot \Delta p \geq \frac{\hbar}{2}$$

$$\Delta x \approx r$$

$$\Delta p \approx \frac{\hbar}{2\Delta x}$$

$$\boxed{p = \frac{h}{4\pi r}} = \frac{0.5272 \times 10^{-34}}{6 \times 10^{-15}} = 0.0878 \times 10^{-19} = 8.78 \times 10^{-17+8}$$

$$pc = \cancel{26.34}\ 2.63 \times 10^{-12} = \underline{16.5\ MeV}$$

$$E^2 = p^2c^2 + (m_e c^2)^2$$

$$E = \sqrt{[693,795 \times 10^{-24}] + [6707.61 \times 10^{-30}]}$$

$$= 10^{-9}\sqrt{693.795 + (6707.61)(10^{-12})}$$

$$\simeq 26.33 \times 10^{-9}$$

$$K = E - m_e c^2$$

$$= (26.33 \times 10^{-9}) - (81.9 \times 10^{-15})$$

$$= \frac{26.33 \times 10^{-9}\ J}{1.6 \times 10^{-19}} \simeq \cancel{16.462 \times 10^{10}\ eV} \quad \boxed{16\ MeV}$$

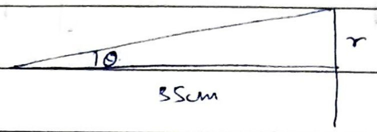
30×16, 12, $9.1 \times 10^{-31} \times 9 \times 10^{16}$, 81.9×10^{-15}

Q21. $E = 2000\ eV = \frac{hc}{\lambda}$ $\lambda = \frac{2000}{1240} = \boxed{\cancel{1.6129}\ nm}$

Screen distance $= 35cm$ $= 2.7423 \times 10^{-11}$

$r_1 = 2.1\ cm$ $= \boxed{0.0274\ nm}$

$r_2 = 2.3\ cm$



$r_3 = 3.2\ cm$

Lattice spacing $= d = ?$

As per Davisson - Germer experiment,

$n\lambda = 2d\sin\phi$ ϕ = scattering angle $= 2\theta$

For first order diffraction, $n = 1$

$\lambda = 2d\sin\phi$

$$d = \frac{\lambda}{2\sin\phi} = \frac{\overset{0.0274}{\cancel{1.6129}}\ (35)}{2 \times r \times 2}$$

$$d_1 = \frac{\overset{0.4795}{\cancel{28.2257}}}{2.1} = (0.2289\ nm) \times 2 = \boxed{0.4578\ nm}$$

$$d_2 = \frac{\overset{0.4795}{\cancel{28.2257}}}{2.3} = 0.2091\ nm = \boxed{0.4182\ nm}$$

$$d_3 = \frac{\overset{0.4795}{\cancel{28.2257}}}{3.2} = 0.1506\ nm = \boxed{0.3012\ nm}$$

Q22. $\psi_1(x,t) = 0.003\sin(6x - 300t)$ C

$\psi_2(x,t) = 0.003\sin(7x - 250t)$ D

(a) $\psi = \psi_1 + \psi_2 = ? = 0.003\left[2\sin\left[\frac{13x - 550t}{2}\right]\cos\left[\frac{x + 50t}{2}\right]\right]$

$= 0.006\sin\left(\frac{13x - 550t}{2}\right)\cos\left(\frac{x + 50t}{2}\right)$

(b) $v_g = \frac{d\omega}{dk} = \frac{\Delta\omega}{\Delta k}$ $k_{avg} = \frac{k_1 + k_2}{2}$ $\omega_{avg} = \frac{\omega_1 + \omega_2}{2}$

$\Delta k = \frac{k_1 - k_2}{2}$ $\Delta\omega = \frac{\omega_1 - \omega_2}{2}$

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$K_{avg} = \frac{6+7}{2} = 6.5$ $\Delta K = \frac{7-6}{2} = 0.5$

$\omega_{avg} = \frac{300+250}{2} = 275$ $\Delta\omega = \frac{300-250}{2} = 25$

Phase velocity $= \frac{\omega}{K} = \frac{\cancel{275}\ 550}{\cancel{6.5}\ 13} = \boxed{42.30\ m/s}$

Group velocity $= \frac{\Delta\omega}{\Delta K} = \frac{25}{0.5} = \boxed{50\ m/s}$

(c) $\psi = 0$ when $\frac{13x - 550t}{2} = n\pi$

OR

$\frac{x + 50t}{\cancel{2}} = \frac{(2n+1)\pi}{\cancel{2}} = ?$

$x + 50t = (2n+1)\pi$

or

$13x - 550t = 2n\pi$

λ for $\sin\left(\frac{\overset{K}{13}x - 550t}{2}\right) = ?$

Fast-moving part

$K = \frac{2\pi}{\lambda}$ $\lambda = \frac{2\pi}{K} = \frac{2\pi \times 2}{13} = \frac{4\pi}{13}$.

λ for $\cos\left(\frac{x + 50t}{2}\right) = ?$ → varies slowly

$K = \frac{2\pi}{\lambda}$ $\lambda = \frac{2\pi}{K} = \frac{2\pi}{1/2} = 4\pi$

So distance between adjacent zeroes $= \lambda/2 = \boxed{2\pi}$ //.

(d) $(\Delta K)(\Delta x) = ?$

$\Delta K = K_1 - K_2 = \boxed{1}$

$\Delta x = \boxed{2\pi}$

$(\Delta K)(\Delta x) = \boxed{2\pi}$

Q23. $E = 0.083\ eV$ $\Rightarrow \lambda = 14939.75\ nm = 1.49 \times 10^{-5}\ m$

$\theta = 22^\circ$

$d = ?$

$n = 1$

$E = \frac{1240}{\lambda (nm)} = ...$

For Bragg's Diffraction $\rightarrow$

$$n\lambda = 2d\sin\theta$$

$$1.49 \times 10^{-5} = d\,(2 \times 0.3746)$$

$$d = \frac{1.49 \times 10^{-5}}{0.7492}$$

$$= 1.988 \times 10^{-5}$$

Q23. $E = 0.083\ eV$

$$\lambda = \frac{h}{\sqrt{2mE}} = \frac{6.626 \times 10^{-34}}{\sqrt{2 \times 1.67 \times 10^{-27} \times 0.083 \times 1.6 \times 10^{-19}}}$$

$$= 9.9489 \times 10^{-11}\ m$$

For Bragg's Diffraction $\rightarrow$

$$n\lambda = 2d\sin\theta$$

$$\frac{9.9489 \times 10^{-11}}{2\sin\theta} = d$$

$$d = 13.279 \times 10^{-11}$$

$$= \boxed{1.33 \times 10^{-10}\ m}$$

Q24. $\Delta p = \left(\frac{1}{1000}\right) p$

$\Delta x_{min} = ?$

$$\Delta p \cdot \Delta x \geq \frac{\hbar}{2}$$

$$\Delta x_{min} = \frac{\hbar}{2\Delta p}$$

(a) $p = mv$

$= 10^{-2}\ \frac{kg\,m}{s}$

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$$\Delta p = \frac{p}{1000} = 10^{-5}$$

$$\Delta x = \frac{6.626 \times 10^{-34}}{4(\pi)(10^{-5})} = 0.5272 \times 10^{-29}$$

$$= 5.272 \times 10^{-30}\ m$$

$$= \boxed{(5.272 \times 10^{-20})\ \mathring{A}}$$

(b) $p = mu$

$$= \cancel{(1.6\times10^{-19})}\ (9.1\times10^{-31})\ (1.8\times10^{8})$$

$$= 16.38 \times 10^{-23} = 1.638 \times 10^{-22}$$

$$\Delta p = 16.38 \times 10^{-26}$$

$$\Delta x = \frac{h}{4\pi \Delta p} \quad 10^{-34+22} = \cancel{0.32185 \times 10^{-12}\ m}$$

$$= \frac{0.5272 \times 10^{-34+23}}{16.38 \times 10^{-23}}$$

$$= 0.0321 \times 10^{-11}\ m$$

(b) $\Delta p = \frac{1}{1000}\left(\frac{m_0}{\sqrt{1 - v^2/c^2}} \times v\right) = \frac{9.1 \times 10^{-31} \times 1.8\times10^{8}}{\sqrt{0.4}} = 25.899 \times 10^{-26}$

$-34+26$

0.1

$1 - \frac{1.8\times10}{3}$

$$\Delta x = \frac{h}{4\pi\Delta p} = 0.0203 \times 10^{-8}$$

$$= \boxed{2.03\ \mathring{A}}$$

Q25. $\lambda = 3000\ \mathring{A}$ As per uncertainity principle $\rightarrow \Delta x \Delta p \geq \frac{\hbar}{2}$

$$\Delta\lambda = \frac{1}{10^7}\lambda$$

$$p = h/\lambda$$

$$\Delta x\left(\frac{h}{\lambda^2}\right)(\Delta\lambda) \geq \frac{h}{4\pi}$$

$\Delta x = ?$ $\quad \Delta p = (h/\lambda^2)(\Delta\lambda)$

$$\Delta x \geq \frac{\lambda^2}{4\pi\Delta\lambda}$$

So, $\Delta x_{min} = \frac{(9 \times 10^{-14})}{4\pi \times 3\times10^{-14}} = 0.239\ m$

$$= \boxed{23.9\ cm}$$

Q 26. As per uncertainity principle,

$\Delta E \cdot \Delta t \geq \frac{\hbar}{2}$

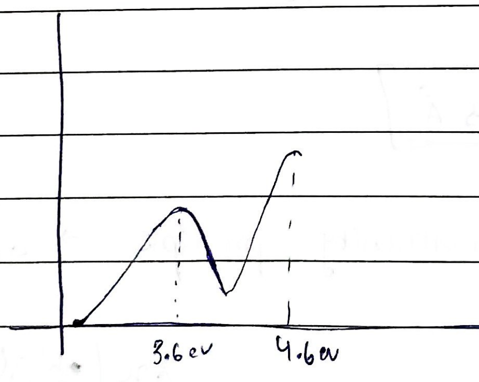
$\Delta E_{min} = \frac{h}{4\pi(\Delta t)} = \frac{0.5272 \times 10^{-34}}{10^{-8}} = \frac{0.5272 \times 10^{-26}}{1.6 \times 10^{-19}}$

$= \boxed{0.3295 \times 10^{-7}\ eV}$

Q 27. State - 1 = 3.6 eV
State - 2 = 4.6 eV

accelerating voltage = 18 V
All possible peaks = ?
For accelerating voltage - V.
If $0 \leq V \leq 3.6\ V$
- e^- will have elastic collisions & current will increase smoothly.
- V = 3.6 V
e^- will completely transfer energy for excitation & current drop

Graphically it would look like →



Peaks → 3.6, 7.2, 10.8, 14.4, 18
4.6, 9.2, 13.8,

8 peaks, 7 valleys.

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Q. 8 $E = \frac{p^2}{2m} - \frac{e^2}{4\pi\epsilon_0 r}$

uncertainity $\rightarrow$ $\Delta r = r$

$\Delta p = p$.

R = size = ?] $\rightarrow$ of hydrogen atom in ground state.

Energy = ?

$\Delta x \Delta p \geq \hbar/2$ $\quad \Delta x = R$ in this case.

$\Delta x \geq \frac{\hbar}{2p}$

$r \geq \frac{\hbar}{2p} = \frac{h}{4\pi p}$

$\boxed{p = \frac{h}{4\pi r}}$

Substituting this in place of energy eq^n.

$E = \frac{h^2}{16\pi^2 r^2 (2m)} - \frac{e^2}{4\pi\epsilon_0 r}$

$= \frac{\hbar^2}{2mr^2} - \frac{e^2}{4\pi\epsilon_0 r}$

In ground state, energy will be minimum, so,

$\frac{dE}{dr} = 0$.

$\frac{-2\hbar^2}{2mr^3} + \frac{e^2}{4\pi\epsilon_0 r^{\cancel{2}}} = 0$.

$\frac{\cancel{2}\hbar^2}{\cancel{2}mr} = \frac{e^2}{4\pi\epsilon_0}$

$r = \frac{4\pi\epsilon_0 \hbar^2}{me^2} = 5.29 \times 10^{-11}$ m $= a_0$

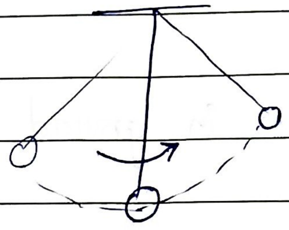
a_0 = Bohr's radius

Substituting this r, to find energy

$E = \frac{\cancel{\hbar^2} m^2 e^4}{2\cancel{m}(16\pi^2\epsilon_0^2\hbar^4)} - \frac{e^4 m}{4\pi\epsilon_0(4\pi\epsilon_0\hbar^2)}$

$= -\frac{me^4}{32\pi^2\epsilon_0^2\hbar^2} = \boxed{-13.6 eV}$ = As predicted by Bohr.

Q29. For a linear harmonic oscillator, ground state energy = ?



$v = A\omega$.

Amplitude = A.

Angular frequency = $\omega = 2\pi f$.

As per uncertainity principle,

$$\Delta E \Delta t \geq \frac{\hbar}{2}$$

$$\Delta E \geq \frac{\hbar}{2\Delta t} = $$

$$\geq \frac{\hbar(\Delta f)}{2}$$

$$\geq \boxed{\frac{\hbar\omega}{4\pi}}$$

Hence proved.

Q30. Rayleigh - Jeans law for blackbody radiation -

As per this law, spectral energy density of blackbody radiation at given temperature t is given by →

$$\boxed{u = \frac{8\pi kT}{\lambda^4}} = \frac{8\pi v^2 kT}{c^3}$$

As per this law, the energy emitted per unit volume per unit wavelength is proportional to square of frequency and it followed the kinetic theory & law of equipartition of energies and assumed that energy of all standing waves will be equal to KT, irrespective of their frequencies. This led to ultraviolet catastrophe.

Planck modified the energy of a single standing wave. He used statistics and defined energy as =

$$E = \left(\frac{h\nu}{e^{\frac{h\nu}{KT}} - 1}\right)$$ where K = Boltzman Constant

As per planck, spectral energy density (u) →

$$u = \frac{8\pi\nu^2}{c^3}\left(\frac{h\nu}{e^{\frac{h\nu}{KT}} - 1}\right)$$

This gave a graph which exactly agreed with the experimental values.