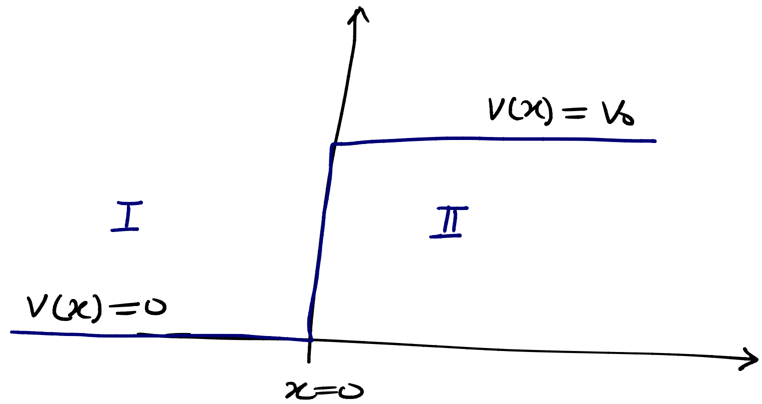
Lecture-34 3/11/2023.

Potential Step :→

* Let say e^- of energy $E > V_0$ moves from left to right & face a sudden shift in the pot. at $x=0$.



$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2} E\psi = 0 \qquad x<0 \quad \text{——— (1)}$$

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E-V)\psi = 0 \quad \text{——— (2)} \qquad x \geq 0$$

Let say $k_1 = \frac{\sqrt{2mE}}{\hbar}$, $k_2 = \frac{\sqrt{2m(E-V)}}{\hbar}$

$$\left(\frac{d^2}{dx^2} + k_1^2\right)\psi_I(x) = 0 \qquad x<0$$

$$\left(\frac{d^2}{dx^2} + k_2^2\right)\psi_{II}(x) = 0 \qquad x \geq 0$$

Sol^n :

$$\psi_1(x) = Ae^{ik_1x} + Be^{-ik_1x} \quad \text{——— (3)}$$

$$\psi_2(x) = ce^{ik_2x} + \cancel{De^{-ik_2x}} \quad \text{——— (4)}$$

* Ae^{ik_1x} & ce^{ik_2x} represent waves moving in the +ve X direction
Be^{-ik_1x} & De^{-ik_2x} " " " " -ve " ".

Since No wave is reflected from the region $x>0$ $\therefore$ $D=0$.

§ Reflection & Transmission coefficient :→

$$R = \left|\frac{\text{Reflected current density}}{\text{Incident " "}}\right| = \left|\frac{J_{reflected}}{J_{incident}}\right|$$

$$T = \left|\frac{\text{Transmitted " "}}{\text{Incident " "}}\right| = \left|\frac{J_{Transmitted}}{J_{incident}}\right|.$$

$$\boxed{J = \frac{\hbar}{2mi}\left(\psi^* \nabla \psi - \psi \nabla \psi^*\right)}$$

Lecture-31

$J_{incident} \Rightarrow \frac{\hbar}{2mi}\left[A^* \cancel{e^{-ik_1x}} A ik_1 \cancel{e^{ik_1x}} - A \cancel{e^{ik_1x}}\left(-A^* ik_1 \cancel{e^{-ik_1x}}\right)\right]$

$\psi_i = Ae^{ik_1x}$

$\frac{d\psi}{dx} = Aik_1 e^{ik_1x}$

$= \frac{\hbar}{2mi}\left[|A|^2 ik_1 + |A|^2 ik_1\right]$

$= \frac{\hbar}{\cancel{2}m\cancel{i}} \cancel{2}\cancel{i}k_1 |A|^2$

$= \frac{\hbar k_1}{m} |A|^2$ ——— (5)

Similarly, $J_{reflected} = -\frac{\hbar k_1}{m}|B|^2$

$J_{transmitted} = \frac{\hbar k_2}{m}|C|^2.$

$R = \frac{|B|^2}{|A|^2}$ & $T = \frac{k_2}{k_1}\frac{|C|^2}{|A|^2}$ ——— (6)

$\psi_1(x) = Ae^{ik_1x} + Be^{-ik_1x}$ $\quad x<0$

$\psi_{II}(x) = Ce^{ik_2x}$ $\quad x \geq 0$

B.C. $\Rightarrow$ $\psi_1(0) = \psi_{II}(0).$

$A + B = C$ ——— (7)

$\frac{\partial \psi_1(0)}{\partial x} = \frac{\partial \psi_2(0)}{\partial x}.$

$A\cancel{i}k_1 \cancel{e^{ik_1x}} - B\cancel{i}k_1 \cancel{e^{-ik_1x}} = C\cancel{i}k_2 \cancel{e^{ik_2x}}$ (put $x=0$)

$k_1[A - B] = k_2 C$ ——— (8)

$$A + B = C \qquad \text{--- (7)} \quad \times k_2$$
$$k_1[A-B] = k_2 C \qquad \text{--- 8}$$

substract.

$$k_2 A + k_2 B = k_1 A - k_1 B$$
$$k_1 B + k_2 B = k_1 A - k_2 A$$
$$B[k_1 + k_2] = A[k_1 - k_2]$$
$$\frac{B}{A} = \frac{k_1 - k_2}{k_1 + k_2}$$
$$\therefore \boxed{R = \frac{(k_1 - k_2)^2}{(k_1 + k_2)^2}}$$

$$R = \frac{|B|^2}{|A|^2}$$
$$T = \frac{k_2}{k_1}\frac{|C|^2}{|A|^2}$$

$$A + B = C \qquad \text{--- (7)} \quad \times k_1$$
$$k_1[A - B) = k_2 C \qquad \text{--- (8)}$$

add.

$$k_1 A + \cancel{k_1 B} + k_1 A - \cancel{k_1 B} = k_1 C + k_2 C$$
$$2k_1 A = (k_1 + k_2) C$$
$$\frac{C}{A} = \frac{2k_1}{k_1 + k_2}$$
$$T = \frac{k_2}{k_1} \times \frac{(2k_1)^2}{(k_1 + k_2)^2}$$
$$\boxed{T = \frac{4 k_1 k_2}{(k_1 + k_2)^2}}$$

$R + T = 1$.

§ As E gets smallar

k_2 " "

T " "

At $E = V$ $\quad k_2 \Rightarrow 0 \quad \therefore T = 0$

& $R = 1$

$$k_2 = \frac{\sqrt{2m(E-V)}}{\cancel{h}}$$

$E \gg V$

$T \Rightarrow 1$

$R = 0$