DEPARTMENT OF MATHEMATICS, IIT ROORKEE
MAB-103: Numerical Methods

Assignment-5 Finite Differences and Interpolation Session: 2025-26

1. Prove the following operator relations:

   (a) E=1+Δ=$(1-\nabla)^{-1}$, (b) $\mu^2 = 1 + \left(\frac{\delta^2}{4}\right)$, (c) $\delta y_{n+\frac{1}{2}} + y_{n-\frac{1}{2}} = 2\mu\delta y_n$,

   (d) $\delta^2(1+\Delta) = \Delta^2$, (e) $\nabla\Delta = \Delta - \nabla = \delta^2$, (f) $\Delta = \mu\delta + \left(\frac{\delta^2}{2}\right)$,

   (g) $\mu\delta = \sinh(hD)$, (h) $e^{-hD} = 1 - \nabla$, (i) $\Delta = e^{hD} - 1$,

2. For the quadratic polynomial $f(x) = ax^2 + bx + c$, compute $\Delta^r f(x)$, $r = 1, 2, 3, ...$

3. For the polynomial $P_n(x) = a_0x^n + a_1x^{n-1} + ... + a_{n-1}x + a_n$. Show that $\Delta^n P_n(x) = a_0 n! h^n$. Verify the result by preparing the finite difference table of $P_3(x) = 2x^3+3x-1$. By tabulating it for $x = -2(1)3$. Extend the table using finite difference to compute $P_3(-3)$ and $P_3(4)$.

4. Determine the lowest degree polynomial which pass through the points:

X:	0	1	2	3	4	5
Y:	3	1	-1	3	19	53

   Hence, compute $y(1.2)$.

5. Find the missing term in the following table:

X:	2	3	4	5	6
Y:	45.0	49.2	54.1	?	67.4

6. Find the missing values in the following table:

x:	45	50	55	60	65
f(x):	3.0	?	2.0	?	-2.4

7. Find the value of y for $x = 23°$ and $x = 75°$, from the table by using Newton's forward and backward formulae:

x:	10°	20°	30°	40°	50°	60°	70°	80°
$\cos x$:	0.9848	0.9397	0.8660	0.7660	0.6428	0.5000	0.3420	0.1737

8. From the given table, find the values of y, for $x = 0.05$ and $x = 0.37$, by using suitable Newton's formulae:

x:	0.00	0.10	0.20	0.30	0.40
y:	1.0000	1.2214	1.4918	1.8221	2.2255

9. Use Stirling's formula to evaluate $f(1.315)$ from the following table:

x(deg):	1.0	1.1	1.2	1.3	1.4	1.5	1.6
cos x:	1.54308	1.66852	1.81066	1.97091	2.15090	2.35241	2.57746

10. Use Bessel's formula to evaluate $f(15°)$ from the following table:

x:	10°	12°	14°	16°	18°	20°
$\cos x$:	0.176327	0.212556	0.249328	0.286745	0.324920	0.363970

11. Find by Lagrange's formula, the interpolation polynomial, which corresponds to the following table:

x:	-1	0	2	5
f(x):	9	5	3	15

Hence, compute $f(1)$.

12. Compute $f(11.7)$ by Lagrange's formula from the following table:

x:	11.5	11.6	11.8	11.9	12.1	12.4
f(x):	0.26969	0.33839	0.39544	0.40022	0.38332	0.32257

13. Let $f \in C^{n+1}[a,b]$, and let the interpolation nodes $x_0, x_1, \ldots, x_n$ be distinct points in $[a,b]$. The interpolation error for the degree-n Lagrange polynomial $P_n(x)$ is known to satisfy

$$E_n(x) = f(x) - P_n(x) = \frac{f^{(n+1)}(\xi_x)}{(n+1)!} \prod_{i=0}^{n} (x - x_i),$$

for some $\xi_x \in (a,b)$, provided $x \neq x_i$.

Answers:

(5) 60.05

(6) 2.925, 0.225

(8) 1.1052, 2.0959

(9) 1.99661

(10) 0.267949

(11) $x^2 - 3x + 5$, $f(1) = 3$