Assignment #3

(Aug 08, 2025)

Electrostatic fields: Sadiku Chaps 4-5, Griffiths Chap 2

1. Describe the differential and integral forms of Gauss's law. Prove that Gauss's law is an alternative statement of Coulomb's law.

2. Use Gauss's law to find the electric field inside and outside a spherical shell of radius R, which carries a uniform surface charge density σ.

3. Find the electric field at a distance r from an infinitely long straight wire, which carries a uniform line charge λ.

4. Find the electric field inside a sphere which carries a charge density proportional to the distance from the origin, $\rho = kr$, for some constant k.

5. A hollow spherical shell carries charge density $\rho = k/r^2$ in the region $a \leq r \leq b$ (Fig. 1).

(a) Find the electric field in the three regions:

(i) $r < a$,

(ii) $a < r < b$,

(iii) $r > b$.

(b) Plot $|\mathbf{E}|$ as a function of r, for the case $b = 2a$.

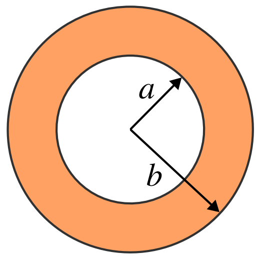

Fig. 1

6. If $\mathbf{D} = (2y^2 + z)\,\mathbf{a}_x + 4xy\,\mathbf{a}_y + x\,\mathbf{a}_z$ C/m^2, find

(a) the volume charge density at $(-1, 0, 3)$,

(b) the flux through the cube defined by $0 \leq x \leq 1, 0 \leq y \leq 1, 0 \leq z \leq 1$,

(c) the total charge enclosed by the cube.

7. A charge distribution in free space has $\rho_v = 2r$ nC/m^3 for $0 \leq r \leq 10$ m and zero otherwise. Determine $\mathbf{E}$ at $r = 2$ m and $r = 12\,m$.

8. In a certain region, the electric field is given by

$$\mathbf{D} = 2\rho(z+1)\cos\phi\,\mathbf{a}_\rho - \rho(z+1)\sin\phi\,\mathbf{a}_\phi + \rho^2\cos\phi\,\mathbf{a}_z\ \mu\text{C/m}^2$$

(a) Find the charge density.

(b) Calculate the total charge enclosed by the volume $0 < \rho < 2, 0 < \phi < \pi/2, 0 < z < 4$.

(c) Confirm Gauss's law by finding the net flux through the surface of the volume defined in (b).

9. $V = x^2 y(z+3)$ V. Find

(a) $\mathbf{E}$ at $(3, 4, -6)$.

(b) the charge within cube $0 < x, y, z < 1$.

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10. In free space, an electric field is given by

$$\mathbf{E} = \begin{cases} E_0 \left(\dfrac{\rho}{a}\right) \mathbf{a}_\rho, & 0 < \rho < a \\ 0, & \text{otherwise} \end{cases}$$

Calculate the volume charge density.

11. The electric field in free space is given by

$$\mathbf{E} = 2xyz\, \mathbf{a}_x + x^2 z\, \mathbf{a}_y + x^2 y\, \mathbf{a}_z \text{ V/m}.$$

Calculate the amount of work necessary to move a $2\,\mu$C charge from $(2, 1, -1)$ to $(5, 1, 2)$.

12. Let $\mathbf{E} = \dfrac{10}{r^2}\mathbf{a}_r$ V/m. Find V_{AB}, where A is $(1, \pi/4, \pi/2)$ and B is $(5, \pi, 0)$.

13. Let $V = \rho e^{-z} \sin\phi$. (a) Find $\mathbf{E}$. (b) Show that $\mathbf{E}$ is conservative.

14. In free space, $V = \dfrac{1}{r^3} \sin\theta \cos\phi$. Find $\mathbf{D}$ at $(1,\ 30°,\ 60°)$.

15. A uniform surface charge with density ρ_s exists on a hemispherical surface with $r = a$ and $\theta \leq \pi/2$. Calculate the electric potential at the center.

16. Determine the work done in carrying a $-2\,\mu$C charge from $P_1(2, 1, -1)$ to $P_2(8, 2, -1)$ in the field $\mathbf{E} = y\, \mathbf{a}_x + x\, \mathbf{a}_y$ V/m

(a) along the parabola $x = 2y^2$.

(b) along the straight line joining P_1 and P_2.

17. Region 1 ($z < 0$) contains a dielectric for which $\varepsilon_r = 2.5$, while region 2 ($z > 0$) is characterized by $\varepsilon_r = 4$. Let $\mathbf{E}_1 = -30\, \mathbf{a}_x + 50\, \mathbf{a}_y + 70\, \mathbf{a}_z$ V/m. Find $\mathbf{D}_2$ and the angle between $\mathbf{E}_1$ and the normal to the surface.

18. Two homogeneous dielectric regions 1 ($\rho \leq 4$ cm) and 2 ($\rho \geq 4$ cm) have dielectric constants 3.5 and 1.5, respectively. If $\mathbf{D}_2 = 12\, \mathbf{a}_\rho - 6\, \mathbf{a}_\phi + 9\, \mathbf{a}_z$ nC/m^2, calculate.

(a) $\mathbf{E}_1$ and $\mathbf{D}_1$,

(b) $\mathbf{P}_2$ and ρ_{pv2}

(c) the energy density for each region.