Assignment #1

(Jul 28, 2025)

Coordinate Systems and Transformation: Sadiku Chaps 2 & 3, Griffiths Chap 1

1. Express the following points in cylindrical and spherical coordinates:

(a) $P(2, 5, -1)$

(b) $Q(-3, 4, 0)$

(c) $R(6, 2, -4)$

2. Express the following points in Cartesian coordinates:

(a) $P_1(2, 30°, 5)$

(b) $P_2(1, 90°, -3)$

(c) $P_3(10, \pi/4, \pi/3)$

(d) $P_4(4, 30°, 60°)$

3. Convert the following vectors to cylindrical and spherical systems:

(a) $\mathbf{F} = \dfrac{x\,\mathbf{a}_x + y\,\mathbf{a}_y + 4\mathbf{a}_z}{\sqrt{x^2 + y^2 + z^2}}$

(b) $\mathbf{G} = (x^2 + y^2)\left[\dfrac{x\,\mathbf{a}_x}{\sqrt{x^2 + y^2 + z^2}} + \dfrac{y\,\mathbf{a}_y}{\sqrt{x^2 + y^2 + z^2}} + \dfrac{z\,\mathbf{a}_z}{\sqrt{x^2 + y^2 + z^2}}\right]$

4. Express the following vectors in rectangular coordinates:

(a) $\mathbf{A} = \rho \sin\phi\,\mathbf{a}_\rho + \rho\cos\phi\,\mathbf{a}_\phi - 2z\,\mathbf{a}_z$

(b) $\mathbf{B} = 4r\cos\phi\,\mathbf{a}_r + r\,\mathbf{a}_\phi$

5. Given a vector field $\mathbf{H} = xy^2z\,\mathbf{a}_x + x^2yz\,\mathbf{a}_y + xyz^2\,\mathbf{a}_z$

(a) Express this vector field in cylindrical and spherical coordinates.

(b) In both cylindrical and spherical coordinates, determine $\mathbf{H}$ at $(3, -4, 5)$.

6. Let $\mathbf{A} = \rho\cos\phi\,\mathbf{a}_\rho + \rho z^2 \sin\phi\,\mathbf{a}_z$

(a) Transform $\mathbf{A}$ into rectangular coordinates and calculate its magnitude at point $(3, -4, 0)$.

(b) Transform $\mathbf{A}$ into spherical system and calculate its magnitude at point $(3, -4, 0)$.

7. Let Let $\mathbf{H} = 5\rho\sin\phi\,\mathbf{a}_\rho - \rho z\cos\phi\,\mathbf{a}_\phi + 2\rho\,\mathbf{a}_z$. At point $P(2, 30°, -1)$, find:

(a) a unit vector along $\mathbf{H}$

(b) the component of $\mathbf{H}$ parallel to $\mathbf{a}_x$

(c) the component of $\mathbf{H}$ normal to $\rho = 2$

(d) the component of $\mathbf{H}$ tangential to $\phi = 30°$

8. Let $\mathbf{A} = (2z - \sin\phi)\mathbf{a}_\rho + (4\rho + 2\cos\phi)\mathbf{a}_\phi - 3\rho z\mathbf{a}_z$ and $\mathbf{B} = \rho\cos\phi\mathbf{a}_\rho + \sin\phi\mathbf{a}_\phi + \mathbf{a}_z$.

(a) Find the minimum angle between **A** and **B** at $(1, 60°, -1)$.

(b) Determine a unit vector normal to both **A** and **B** at $(1, 90°, 0)$.

9. If $\mathbf{J} = r\sin\theta\cos\phi\,\mathbf{a}_r - \cos 2\theta\sin\phi\,\mathbf{a}_\theta + \tan\frac{\theta}{2}\ln r\,\mathbf{a}_\phi$ at $T(2, \pi/2, 3\pi/2)$, determine the vector component of **J** that is:

(a) Parallel to $\mathbf{a}_z$

(b) Normal to surface $\phi = 3\pi/2$

(c) Tangential to the spherical surface $r = 2$

(d) Parallel to the line $y = -2, z = 0$

10. Given the vector field $\mathbf{H} = \rho z\cos\phi\,\mathbf{a}_\rho + e^{-2}\sin\frac{\phi}{2}\,\mathbf{a}_\phi + \rho^2\,\mathbf{a}_z$. At a point $(1, \frac{\pi}{3}, 0)$, find

(a) $\mathbf{H}\cdot\mathbf{a}_x$

(b) $\mathbf{H}\times\mathbf{a}_\theta$

(c) The vector component of **H** normal to surface $\rho = 1$

(d) The scalar component of **H** tangential to the plane $z = 0$

11. If $\mathbf{A} = 3\,\mathbf{a}_r + 2\,\mathbf{a}_\theta - 6\,\mathbf{a}_\phi$ and $\mathbf{B} = 4\,\mathbf{a}_r + 3\,\mathbf{a}_\phi$, determine

(a) $\mathbf{A}\cdot\mathbf{B}$

(b) $|\mathbf{A}\times\mathbf{B}|$

(c) The vector component of **A** along $\mathbf{a}_z$ at $(1, \pi/3, 5\pi/4)$

12. Using the differential length dl, find the length of each of the following curves:

(a) $\rho = 3,\ \ \pi/4 < \phi < \pi/2,\ \ z = \text{constant}$

(b) $r = 1,\ \ \theta = 30°,\ \ 0 < \phi < 60°$

(c) $r = 4,\ 30° < \theta < 90°,\ \phi = \text{constant}$

13. Calculate the areas of the following surfaces using the differential surface area dS:

(a) $\rho = 2,\ \ 0 < z < 5,\ \ \pi/3 < \phi < \pi/2$

(b) $z = 1,\ \ 1 < \rho < 3,\ \ 0 < \phi < \pi/4$

(c) $r = 10,\ \ \pi/4 < \theta < 2\pi/3,\ \ 0 < \phi < 2\pi$

(d) $0 < r < 4,\ \ 60° < \theta < 90°,\ \ \phi = \text{constant}$

14. Use the differential volume dv to determine the volumes of the following regions:

(a) $0 < x < 1,\ \ 1 < y < 2,\ \ -3 < z < 3$

(b) $2 < \rho < 5,\ \ \pi/3 < \phi < \pi,\ \ -1 < z < 4$

(c) $1 < r < 3,\ \ \pi/2 < \theta < 2\pi/3,\ \ \pi/6 < \phi < \pi/2$