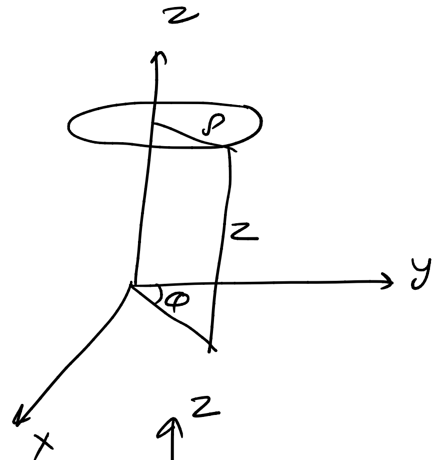
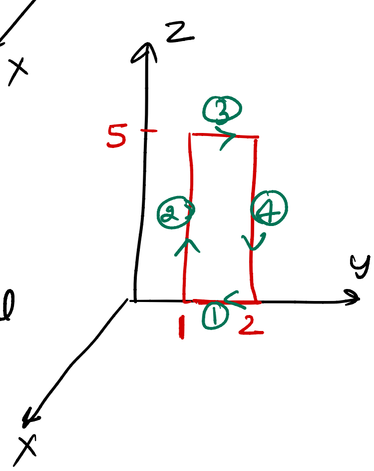
Lecture- 15

8/9/23.

Q.2 ⇒ Given the magnetic vector potential $\bar{A} = -\frac{\rho^2}{4}\hat{a}_z$ Wb/m, Calculate the total magnetic flux crossing the surface
$\varphi = \pi/2, \quad 1 \leq \rho \leq 2\text{ m}, \quad 0 \leq z \leq 5\text{ m}$

Ans ⇒
$$B = \nabla \times A$$
$$= -\frac{\partial}{\partial \rho} A_z \,\hat{a}_\varphi$$
$$= \frac{\rho}{2}\hat{a}_\varphi$$
$$\therefore \int B \cdot ds = \iint \frac{\rho}{2}\, d\rho\, dz$$
$$= \frac{\rho^2}{4} z \Big/_1^2 \Big/_0^5$$
$$= \frac{5}{4}\rho^2 \Big/_1^2 = \frac{15}{4}\text{ Wb}$$





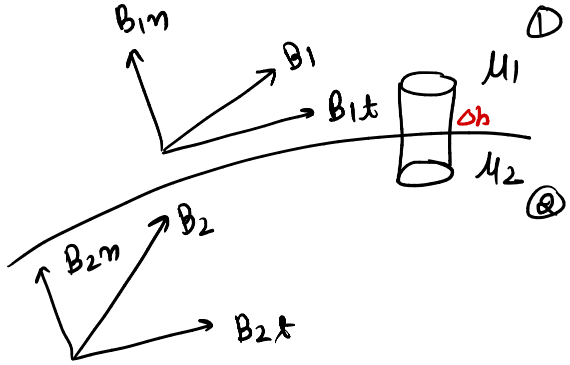
* $\varphi = \oint A \cdot dl$
$$= \cancel{\int_1 A \cdot dl} + \int_2 A \cdot dl + \cancel{\int_3 A \cdot dl} + \int_4 A \cdot dl$$
$$= \underbrace{\int_2 A \cdot dl}_{\rho = 1} + \underbrace{\int_4 A \cdot dl}_{\rho = 2}$$
$$= -\frac{1}{4} \times 5 - 1(-5) = \frac{15}{4}\text{ Wb}$$

Homework

Q ⇒ The magnetic vector potential of a current distribution in free space is given by.
$$\bar{A} = 15 e^{-\rho} \sin\varphi \,\hat{a}_z \text{ Wb/m}$$
Find $\bar{H}$ at $(3, \pi/4, -10)$. Calculate the flux through
$\rho = 5, \quad 0 \leq \varphi \leq \pi/2, \quad 0 \leq z \leq 10.$

Ans ⇒ 1.011 Wb.

Magnetic Boundary Condition :→



① Gauss law of magnetic field

$$\oint B \cdot ds = 0 \checkmark$$

② Ampere's law

$$\oint B \cdot dl = \mu I$$

or

$$\oint H \cdot dl = I \checkmark$$

① $\oint B \cdot ds = 0$

$$B_{1n} ds - B_{2n} ds = 0$$

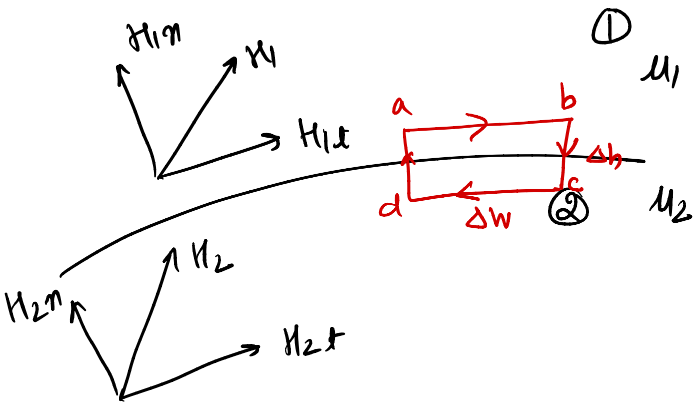
$$\boxed{B_{1n} = B_{2n}}$$

Normal component of B is continious at the boundary

→ $\mu_1 H_{1n} = \mu_2 H_{2n}$

$$H_{1n} = \frac{\mu_2}{\mu_1} H_{2n}$$

⇒ Normal component of H is discontinious.



* Let say there is a surface current K which is normal to the path (latter we will put it to zero).

$$\oint H \cdot dl = I$$

$$H_{1t} \Delta W - \cancel{H_{1n} \frac{\Delta h}{2}} - \cancel{H_{2n} \frac{\Delta h}{2}} - H_{2t} \Delta W + \cancel{H_{2n} \frac{\Delta h}{2}} + \cancel{H_{1n} \frac{\Delta h}{2}} = K \Delta W$$

⇒ $H_{1t} - H_{2t} = K$

* If the boundary is free of current or the media are not conductor $(K = 0)$ · then

$$\boxed{H_{1t} = H_{2t}}$$

Tangencial component of H is continious.

$$\Rightarrow \quad \frac{B_{1t}}{\mu_1} = \frac{B_{2t}}{\mu_2}$$

$$B_{1t} = \frac{\mu_1}{\mu_2} B_{2t}$$

So far in this course we have discussed Electrostatic & Magnetostatic :→

$$\oint E \cdot ds = \frac{q_{enc}}{\epsilon_0} \qquad \nabla \cdot E = \rho/\epsilon_0$$

$$\oint E \cdot dl = 0 \qquad \nabla \times E = 0$$

$$\oint B \cdot ds = 0 \qquad \nabla \cdot B = 0$$

$$\oint B \cdot dl = \mu_0 I_{enc.} \qquad \nabla \times B = \mu_0 J$$

$$f = q[E + v \times B]$$

Dynamics :→ Things change with time.

§ Electromotive force (emf) :→

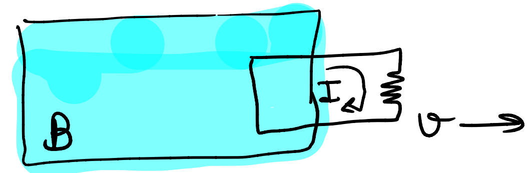
emf of a circuit $\boxed{\mathcal{E} = \oint f \cdot dl}$

Emf in a conducting circuit is total accumulated force on the charge throughout the length of the loop.

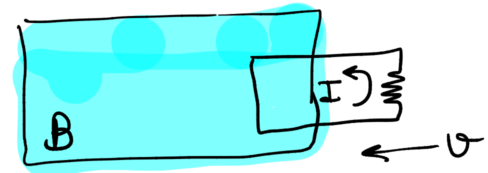
⇒ work done per unit charge by the source.

* Faraday's Law ⇒ (Based on experiments) ⇒

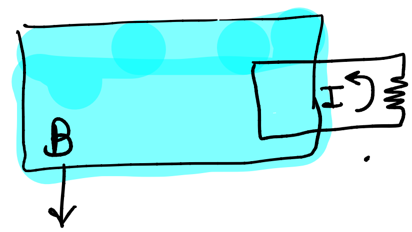
Exp 1 ⇒ Pull a loop of wire to the right through a magnetic field. A current will flow in the loop.



Exp 2 ⇒ Move the magnet to the left. A current will flow in the loop.



Exp 3 ⇒ If the strength of the magnetic field changes, The current flows.



Direction of current ⇒

* Given by the Lentz's Law

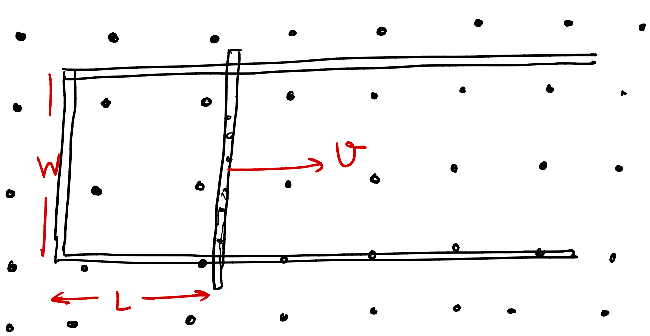
* The direction is such that it opposes the cause of change.

$$\boxed{emf = -\frac{d\phi}{dt}} = -\frac{d}{dt}\int B \cdot ds$$

B can change in time

ds can change in time

#



$$emf = \frac{d}{dt}\int B \cdot ds$$
$$= B\frac{d}{dt}\int ds$$
$$= B\, w\, v \frac{d}{dt}\int dt$$
$$= B w v$$

§ Now Let us understand this result from the magnetic force ⇒

$$f = v \times B$$

$$\therefore \ \mathcal{E} = vBw$$

$$emf = \oint f \cdot dl$$

force per unit charge.

The case where the loop is stationary & the magnetic field is changed.

$v = 0$
$f = 0$
$\varepsilon = 0$

[Now we cannot use the same argument]

<u>Farday's discovery</u> ⇒ * Flux rule is still correct.

* The force on electric charges is given in complete generality by

$$F = q[\bar{E} + v \times \bar{B}]$$

* Any force on charges at rest in a stationary wire come from the $\bar{E}$ term. ⇒ Electric & magnetic field are related.

* In a region where the magnetic field is changing in time, electric fields are generated / induced.

* The general law for the electric field associated with changing magnetic field is

$$\varepsilon = \boxed{\oint E \cdot dl = -\frac{d\phi}{dt}}$$

$$\Rightarrow \oint E \cdot dl = -\int \frac{\partial}{\partial t} B \cdot ds$$

Stoke's theorem

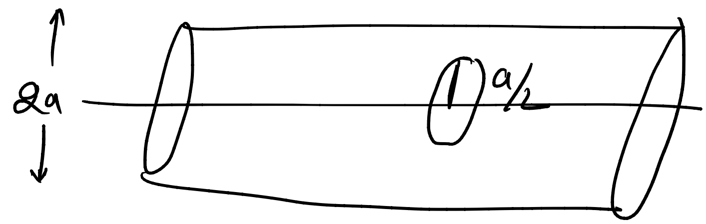
$$\int (\nabla \times E) \cdot ds = -\int \frac{\partial B}{\partial t} \cdot ds$$

$$\boxed{\nabla \times E = -\frac{\partial B}{\partial t}}$$

If B is const
$\nabla \times E = 0$
or
$\oint E \cdot dl = 0$
old Rule!

Q1:⇒ A long solenoid of radius a is driven by an AC so that the field inside is sinusoidal : $B(t) = B_0 \cos(\omega t)\hat{z}$. A circular loop of wire of radius $a/2$ & resistance R is placed inside the solenoid & coaxial with it. Find the current in the loop as fu^n of time.

Ans:⇒ $B(t) = B_0 \cos(\omega t)\hat{z}$



$$\varphi = B\,\pi\left(\frac{a}{2}\right)^2$$

$$\varphi = B_0 \cos(\omega t)\,\pi \frac{a^2}{4}$$

$$\mathcal{E} = -\frac{d\varphi}{dt}$$

$$= \frac{\pi a^2}{4} B_0\, \omega \sin \omega t$$

$$I = \frac{\mathcal{E}}{R} = \frac{\pi a^2 \omega B_0 \sin \omega t}{4R}$$