Lecture-27

19/10/2023.

Wave function ⇒

* The quantity with which Q.M. is concerned is wave fun^n ψ.

* ψ itself has no physical interpretation.

* $|\psi|^2 = \psi^* \psi$

evaluated at a particular place & time is proportional to the probability of finding the particle there at that time.

* Wave fun^n ψ are usually complex.

$$\psi = A + iB$$
$$\psi^* = A - iB$$
$$|\psi|^2 = \psi^* \psi = (A - iB)(A + iB) = A^2 + B^2.$$

always a positive, real quantity.

Normalization ⇒

$$\boxed{\int_{-\infty}^{\infty} |\psi|^2 \, dV = 1}$$

The particle exist somewhere at all time.

Well-behaved wave f^n ⇒

* Normalized ⇒ $x \to \pm\infty$
  $\psi \to 0$

* ψ must be single-valued & continuous.

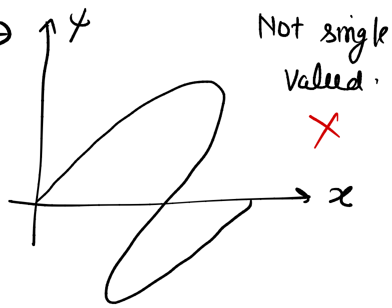
  ↳ probability can have only one value at a particular place & time.

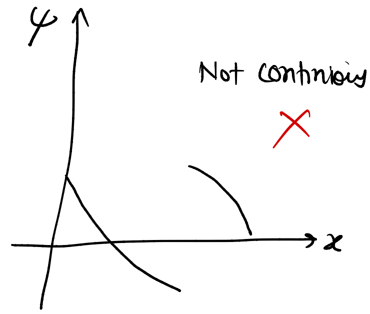
* $\frac{\partial \psi}{\partial x}, \frac{\partial \psi}{\partial y}, \frac{\partial \psi}{\partial z}$ must be finite, continuous & single valued.

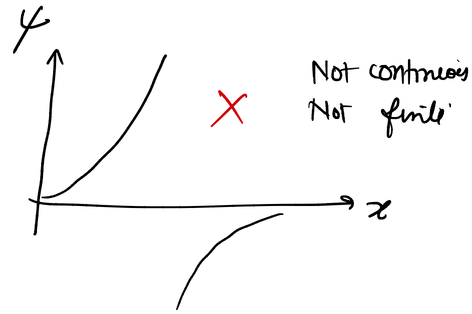
* If ψ is normalized, then the probability of finding the particle b/w x_1 & x_2

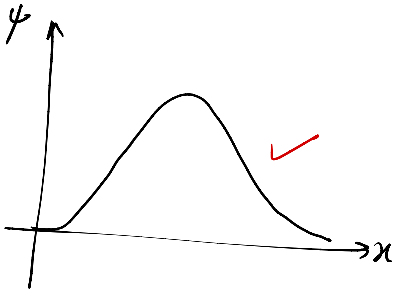
$$P_{x_1,x_2} = \int_{x_1}^{x_2} |\psi|^2 \, dx .$$

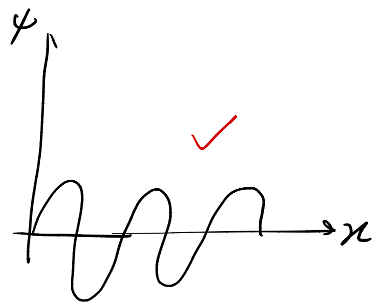
Ex ⇒

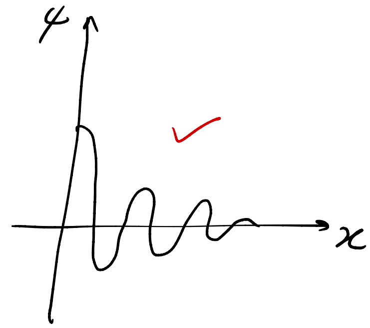


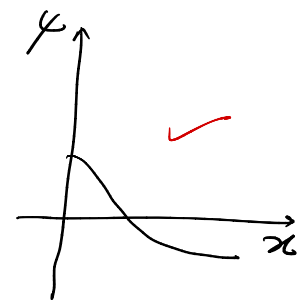












Q ⇒ A wave funⁿ $\psi(x)$ given by $\psi(x) = A_n \sin \frac{2n\pi x}{L}$ in the region $0 \leq x \leq L$. Find the value of A_n using normalisation condⁿ.

A ⇒

$$\int_{-\infty}^{\infty} \psi^*(x)\, \psi(x)\, dx = 1$$

$$\Rightarrow \int_0^L \left(A_n^* \sin \frac{2n\pi x}{L}\right) \cdot \left(A_n \sin \frac{2n\pi x}{L}\right) dx = 1$$

$$\Rightarrow |A_n|^2 \int_0^L \sin^2 \frac{2n\pi x}{L} \, dx = 1$$

$$\Rightarrow \frac{|A_n|^2}{2} \int_0^L \left(1 - \cos \frac{4n\pi x}{L}\right) dx = 1$$

$$\Rightarrow \frac{|A_n|^2}{2}\left[x - \frac{\sin\frac{4n\pi x}{L}}{\left(\frac{4n\pi}{L}\right)} \right]_0^L = 1$$

$$\Rightarrow \frac{|A_n|^2}{2} \cdot L = 1$$

$$\Rightarrow \quad |A_n| = \sqrt{\frac{2}{L}}$$

$$\Rightarrow \quad \psi(x) = \sqrt{\frac{2}{L}} \sin \frac{2n\pi x}{L} \cdot$$

Q :→ Normalize a wavefn

$$\psi = A\, e^{-\left(\frac{\sqrt{cm}}{2\hbar}\right)x^2} \cdot e^{-\frac{i}{2}\sqrt{\frac{c}{m}}\,t}$$

by determing A.

Ans :→

$$\int_{-\infty}^{\infty} \psi^* \psi \, dx = 1$$

$$\int_{-\infty}^{\infty} \left(A^* e^{-\frac{\sqrt{cm}}{2\hbar}x^2} \cancel{e^{\frac{i}{2}\sqrt{\frac{c}{m}}\,t}}\right) \cdot \left(A\, e^{-\left(\frac{\sqrt{cm}}{2\hbar}\right)x^2} \cdot \cancel{e^{-\frac{i}{2}\sqrt{\frac{c}{m}}\,t}}\right) dx = 1$$

$$A^2 \int_{-\infty}^{\infty} e^{-\frac{\sqrt{cm}}{\hbar}x^2} \, dx = 1$$

$$A^2 \int_{-\infty}^{\infty} e^{-\left[\left(\frac{\sqrt{cm}}{\hbar}\right)^{1/2} x\right]^2} dx = 1$$

$$\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$$

$$A^2 \cdot \frac{\sqrt{\pi}}{\left(\frac{\sqrt{cm}}{\hbar}\right)^{1/2}} = 1$$

$$A = \frac{(cm)^{1/8}}{(\pi\hbar)^{1/4}}$$

$$\therefore \quad \psi(x,t) = \frac{(cm)^{1/8}}{(\pi\hbar)^{1/4}}\, e^{-\left(\sqrt{cm}/2\hbar\right)x^2} \cdot e^{-\frac{i}{2}\sqrt{c/m}\,t} \cdot \quad \text{Ans.}$$

Homework.

Q ⇒ Normalize the wave fn. $\varphi(x) = e^{-|x|} \sin\alpha x$.

A ⇒ $$\psi(x) = A e^{-|x|} \sin\alpha x$$

$$A = \left\{ 2 \frac{(1+\alpha^2)}{\alpha^2} \right\}^{1/2}$$

Q ⇒ A particle limited to the x axis has the wave fn $\psi = ax$ b/w $x=0$ & $x=1$ & $\psi=0$ elsewhere. Find the probability that the particle can be found b/w $x = 0.45$ & $x = 0.55$.

A ⇒ $$\int_{x_1}^{x_2} |\psi|^2 dx = \int_{0.45}^{0.55} a^2 x^2 \, dx = a^2 \frac{x^3}{3} \Big|_{0.45}^{0.55} = .0251 a^2$$

Expectation value ⇒

Average value.

In Q.M. every observable is represented by a Quantum mechanical operator $(\hat{O})$.

The expectation value of $\hat{O}$ is

$$\boxed{\langle O(x,p) \rangle = \int_{-\infty}^{\infty} \psi^* \hat{O} \psi \, dx.}$$

$$\langle O(x,p) \rangle = \frac{\int_{-\infty}^{\infty} \psi^* \hat{O} \psi \, dx}{\int \psi^* \psi \, dx}$$

Homework.

Q ⇒ In the previous question, Find the expectation value of $\langle x \rangle$.

A $$\langle x \rangle = \int_0^1 \psi^* x \psi \, dx$$

$$= \frac{a^2}{4}$$