Lecture-21 5/10/2023.

Planck's distribution :⇒

$$\boxed{u(\nu, T) = \frac{8\pi \nu^2}{c^3} \cdot \frac{h\nu}{e^{h\nu/kT} - 1}}$$

Total no. of modes b/w 0 to ν is

$$n(\nu) = \frac{1}{8} \times \frac{4}{3}\pi \left(\frac{2a\nu}{c}\right)^3 \times 2$$

$$n(\nu) = \frac{8\pi a^3 \nu^3}{3c^3}$$

$$n(\nu)\, d\nu = \frac{8\pi a^3 \cdot 3\nu^2}{3c^3}\, d\nu$$

No. of modes b/w ν & $\nu + d\nu$

$$N(\nu) = \frac{8\pi \nu^2}{c^3}. \quad \text{——— (a)}$$

$$\langle E \rangle = \frac{h\nu}{e^{h\nu/kT} - 1} \quad \text{——— (b)}$$

$$n(\lambda) = \frac{1}{8} \times \frac{4}{3}\pi \left(\frac{2a}{\lambda}\right)^3 \cdot 2 \qquad \left[\nu = \frac{c}{\lambda}\right]$$

$$n(\lambda) = \frac{8\pi a^3}{3\lambda^3}.$$

$$n(\lambda)\, d\lambda = \frac{8\pi a^3}{\cancel{3}\, \lambda^4} (-\cancel{3})\, d\lambda$$

$$n(\lambda)\, d\lambda = \frac{8\pi a^3}{\lambda^4}\, d\lambda$$

$$N(\lambda) = \frac{8\pi}{\lambda^4}. \quad \text{——— } (a_1)$$

$$\langle E \rangle = \frac{hc}{\lambda \cdot \left(e^{\frac{hc}{\lambda kT}} - 1\right)} \quad \text{——— } (b_1)$$

$$u(\lambda, T) = \frac{8\pi hc}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1}$$

$\lambda_{max} = ?$

$$u(\lambda, T) = \underbrace{\frac{8\pi hc}{\lambda^5}}_{\text{I}} \underbrace{\frac{1}{e^{hc/\lambda kT} - 1}}_{\text{II}}$$

$$\frac{du}{d\lambda} = 0$$

$$\Rightarrow 8\pi hc \left[\frac{1}{\lambda^5} \left(-\frac{1}{(e^{hc/\lambda kT} - 1)^2} e^{hc/\lambda kT} \left(-\frac{hc}{kT} \cdot \frac{1}{\lambda^2}\right)\right) + \frac{1}{e^{hc/\lambda kT} - 1} \left(\frac{-5}{\lambda^6}\right)\right] = 0$$

$$\Rightarrow \cancel{\frac{8\pi hc}{(e^{hc/\lambda kT} - 1)^2}} \cdot \frac{1}{\lambda^6} \left[\frac{e^{hc/\lambda kT}}{\lambda} \cdot \frac{hc}{kT} - 5\left(e^{hc/\lambda kT} - 1\right)\right] = 0$$

$$\Rightarrow \cancel{e^{hc/\lambda kT}} \left[\frac{hc}{\lambda kT} - 5\left[1 - e^{-hc/\lambda kT}\right]\right] = 0$$

$$\Rightarrow \frac{hc}{\lambda k_B T} - 5\left[1 - e^{-hc/\lambda k_B T}\right] = 0$$

Let say $\frac{hc}{\lambda k_B T} = x$

$$\Rightarrow x - 5\left[1 - e^{-x}\right] = 0$$

$$x = 5\left(1 - e^{-x}\right)$$

This is Transcendental eqⁿ & it can be solve graphically.

Solⁿ, $x = 4.965$

$$\frac{hc}{\lambda k_B T} = 4.965$$

$$\lambda_{max} = \frac{hc}{4.965} \times \frac{1}{T}$$

$$\lambda_{max} = \frac{2.9 \times 10^{-3}\ mK}{T}$$

Wien's displacement law.

Q.1 ⇒ Estimate the surface tem. of a star if the radiation it emit has a maximum wavelength of 446 nm. What is the intensity radiated by the star?

A ⇒

$$T = \frac{2.9 \times 10^{-3}\ mK}{\lambda_{max}} \quad [\text{Wien's law}]$$

$$T = \frac{2.9 \times 10^{-3}\ mK}{446 \times 10^{-9}} = 6500\ K$$ A

Stefan's law $E = \sigma T^4$

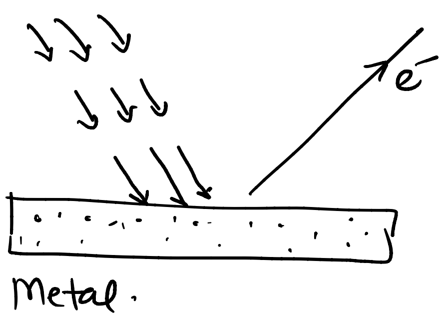
$$E = 5.67 \times 10^{-8}\ \frac{W}{m^2} K^{-4} \times (6500\ K)^4$$

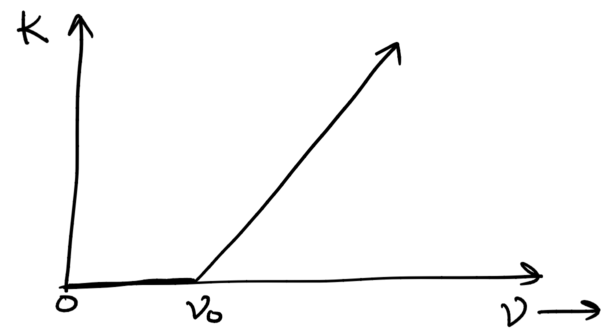
$$= 101.2\ W/m^2.$$ A

Photoelectric effect ⇒

In 1887 Hetz discovered the photoelectric effect. When irradiated with light, metal ejects $\bar{e}$.

⇒ It provides a direct confirmation for the energy quantization of light.





Experimental observation ⇒

① If the frequency of incident radiation is smaller than a threshold

⟹ No $\bar{e}$ can be emitted regardless of the intensity of the radiation.

Homework

② No matter how low the intensity is, the electrons will be ejected instantly.

③ If fixed frequency, No. of ejected e^- increases with intensity but independent of frequency.

④ K.E. of $\bar{e}$ depends on frequency but not on intensity.

⇒ Classical wave theory of light

As intensity increases → $\vec{E}$ of light wave increases → Force applied on e should increase

$F = eE$

→ K.E. of e^- should also increase → XX

§ Inspired by Planck's quantization of EM radiation. Einstein proposed that radiant energy is quantized into concentrated bundles (Photons), each carrying an energy $h\nu$.

$$\boxed{h\nu = W + K}$$

W is work fun^n.

$$h\nu = h\nu_0 + K$$

$$K = h(\nu - \nu_0)$$

Q.1 ⇒ When an ultraviolet beam of wavelength $\lambda = 280$ nm falls on a metal surface, it produce photoelectrons with max energy of 8.57eV. Calculate the work function & the cutoff frequency of metal.

A ⇒ $h\nu = W + K.E. \Rightarrow \frac{hc}{\lambda}$

$$W = \frac{hc}{\lambda} - K.E.$$

$$= \frac{6.62 \times 10^{-34} \times 3 \times 10^{8}}{280 \times 10^{-9}} - 8.57 \times 1.6 \times 10^{-19} \text{ J}$$

$$= -\underline{6.62 \times 10^{-19} \text{J}} = -4.14 \text{ eV}$$

cutoff frequency

$$W = h\nu_0$$

$$\nu_0 = \frac{W}{h} = \frac{6.62 \times 10^{-19}}{6.62 \times 10^{-34}} = 10^{15} \text{ Hz}$$

Compton effect ⇒

It provide the confirmation of the particle aspect of radiation.