Q.1: The material parameters of a certain food item are given by $\sigma = 2.17\ S/m$, $\varepsilon = 47\varepsilon_0$, and $\mu = \mu_0$ at the operating frequency $f = 2.45\ GHz$ of a microwave oven. We wish to find the propagation parameters $\alpha, \beta, \lambda, v_p$, and η.

Solution:

$$\begin{aligned}
\bar{\gamma} &= \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)} \\
&= \sqrt{j\omega\mu \cdot j\omega\varepsilon\left(1 - j\frac{\sigma}{\omega\varepsilon}\right)} \\
&= j\frac{\omega\sqrt{\varepsilon_r}}{c}\sqrt{1 - j\frac{\sigma}{\omega\varepsilon_r\varepsilon_0}} \\
&= j\frac{2\pi \times 2.45 \times 10^9 \times \sqrt{47}}{3 \times 10^8}\sqrt{1 - j\frac{2.17 \times 36\pi}{2\pi \times 2.45 \times 10^9 \times 47 \times 10^{-9}}} \\
&= j351.782\sqrt{1 - j0.3392} \\
&= j351.782\sqrt{1.0560\angle{-18.7369^\circ}} \\
&= 351.782\angle{90^\circ} \times 1.0276\angle{-9.3685^\circ} \\
&= 361.4912\angle{80.6315^\circ} \\
&= 58.85 + j356.67
\end{aligned}$$

$$\begin{aligned}
\alpha &= 58.85\ \text{Np/m} \\
\beta &= 356.67\ \text{rad/m} \\
\lambda &= \frac{2\pi}{\beta} = 0.0176\ \text{m} \\
v_p &= \frac{\omega}{\beta} = 0.4316 \times 10^8\ \text{m/s}
\end{aligned}$$

$$\begin{aligned}
\bar{\eta} &= \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} \\
&= \sqrt{\frac{j\omega\mu}{j\omega\varepsilon[1 - j(\sigma/\omega\varepsilon)]}} \\
&= \frac{\eta_0}{\sqrt{\varepsilon_r}}\frac{1}{\sqrt{1 - j(\sigma/\omega\varepsilon)}} \\
&= \frac{120\pi}{\sqrt{47}}\frac{1}{\sqrt{1 - j0.3392}} \\
&= \frac{54.9898}{1.0276\angle{-9.3685^\circ}} \\
&= 53.51\angle{9.37^\circ}\ \Omega
\end{aligned}$$

Q.2: The constitutive parameters of copper are $\mu = \mu_0 = 4\pi\ x\ 10^{-7}$ (H/m), $\varepsilon = \varepsilon_0 \simeq {}^{1}/_{36\pi}\ x\ 10^{-9}$ (F/m), and $\sigma = 5.8\ x\ 10^7$ (S/m). Assuming that these parameters are frequency independent, over what frequency range of the electromagnetic spectrum is copper a good conductor ?

Solution 2: Good conductor implies that

$$\frac{\varepsilon''}{\varepsilon'} = \frac{\sigma}{\omega\varepsilon} > 100$$

or

$$\omega = 2\pi f < \frac{\sigma}{100\varepsilon}$$

$$f < \frac{\sigma}{200\pi\varepsilon} = \frac{5.8 \times 10^7}{200\pi \times (1/36\pi) \times 10^{-9}} = 1.04 \times 10^{16}\ \text{Hz.}$$

Q.3: A radar installation transmits a wave whose magnetic field intensity is

$$H = \hat{x} H_0 \cos(wt - k_0 z)\ A/m$$

where $H_0 = 25\ A/m$ and $f = 30\ GHz$. Propagation is in free space and z is the vertical direction. Assume plane waves and lossless propagation. Calculate:

(a) The wave number for the wave.

(b) The electric field intensity of the wave in phasor form.

Solution:

The free-space wave number is calculated from the intrinsic impedance of free space which is known. With the intrinsic impedance, we can calculate the magnetic field intensity, using Faraday's law in Cartesian coordinates:

(a)

$$\eta = \frac{E_x^+(z)}{H_y^+(z)} = \frac{\omega\mu}{k} = \sqrt{\frac{\mu}{\varepsilon}} \quad [\Omega]$$

$$\eta_0 = \frac{\omega\mu_0}{k_0} \quad \rightarrow \quad k_0 = \frac{\omega\mu_0}{\eta_0} = \frac{2 \times \pi \times 3 \times 10^{10} \times 4 \times \pi \times 10^{-7}}{377} = 628.3 \quad \left[\frac{\text{rad}}{\text{m}}\right]$$

(b) The magnitude of the electric field intensity can be written as:

$$H_y^+(z) = \frac{1}{\eta} E_x^+(z) \quad \left[\frac{\text{A}}{\text{m}}\right]$$

$$|\mathbf{E}| = \eta_0 |\mathbf{H}| = 377 \times 25 = 9425 \quad \left[\frac{\text{V}}{\text{m}}\right]$$

However, to find the direction of the electric field intensity, we must use Ampere's law in lossless media and in the frequency domain, written here in component form:

$$\hat{\mathbf{x}}\left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right) + \hat{\mathbf{y}}\left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right) + \hat{\mathbf{z}}\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right) = j\omega\varepsilon_0(\hat{\mathbf{x}} E_x + \hat{\mathbf{y}} E_y + \hat{\mathbf{z}} E_z)$$

Because **H** has only a component in the x direction and only varies with z, only the derivative $\partial H_x/\partial z$ is nonzero on the left-hand side. This term is in the y direction; therefore, the right-hand side can only have a y-directed component:

$$\hat{\mathbf{y}} \frac{\partial H_x}{\partial z} = \hat{\mathbf{y}}\, j\omega\varepsilon_0 E_y$$

To calculate the derivative of **H** with respect to z, we write **H** in phasor form:

$$\mathbf{H} = \hat{\mathbf{x}} H_0 e^{-jk_0 z} \quad [\text{A/m}]$$

The electric field intensity is therefore

$$\mathbf{E} = \hat{\mathbf{y}} \frac{1}{j\omega\varepsilon_0} \frac{\partial H_x}{\partial z} = \hat{\mathbf{y}} \frac{-jk_0}{j\omega\varepsilon_0} H_0 e^{-jk_0 z} = -\hat{\mathbf{y}} \frac{k_0}{\omega\varepsilon_0} H_0 e^{-jk_0 z} = -\hat{\mathbf{y}} \frac{628.3}{2 \times \pi \times 3 \times 10^{10} \times 8.854 \times 10^{-12}} \times 25 e^{-j628.3z}$$

$$= -\hat{\mathbf{y}}\, 9425 e^{-j628.3z} \quad \left[\frac{\text{V}}{\text{m}}\right]$$

Q.4: Consider a plane wave that generates an electric field intensity $E = -\hat{y}E_0\cos(wt - kz)$ [V/m], where $E_0 = 1{,}000$ V/m, and $f = 300$ MHz. Propagation in free space:

(a) Calculate the instantaneous and time-averaged power densities in the wave.

(b) Suppose a receiving dish antenna is 1 m in diameter. How much power is received by the receiving antenna if the surface of the dish is perpendicular to the direction of propagation of the wave?

Solution 4: (a) The electric field intensity has only a y component and varies only with z: $-\hat{\mathbf{x}}\dfrac{\partial E_y}{\partial z} = \hat{\mathbf{x}}\, j\omega\mu H_x$

Using the phasor form of E : $\mathbf{E} = -\hat{\mathbf{y}}\, E_0 e^{-jkz} \quad \left[\dfrac{\mathrm{V}}{\mathrm{m}}\right]$

we get, $\mathbf{H} = \hat{\mathbf{x}}\,\dfrac{k}{\omega\mu_0} E_0 e^{-jkz} = \hat{\mathbf{x}}\,\dfrac{1}{\eta_0} E_0 e^{-jkz} \quad \left[\dfrac{\mathrm{A}}{\mathrm{m}}\right]$

The instantaneous power density emitted by the antenna is given by the Poynting vector as:

$$\mathcal{P} = \mathbf{E}\times\mathbf{H} = [-\hat{\mathbf{y}}\, E_0\cos(\omega t - kz)] \times \left[\hat{\mathbf{x}}\frac{1}{\eta_0}E_0\cos(\omega t - kz)\right] = \hat{\mathbf{z}}\,\frac{E_0^2}{\eta_0}\cos^2(\omega t - kz) \quad \left[\frac{\mathrm{W}}{\mathrm{m}^2}\right]$$

The time-averaged power density is found by integrating the instantaneous power density over one cycle of the wave ($T = {}^1/_f = 2\pi\omega$) :

$$\mathcal{P}_{av}(z) = \hat{\mathbf{z}}\frac{1}{T}\int_0^T \frac{E_0^2}{\eta_0}\cos^2(\omega t - kz)dt = \hat{\mathbf{z}}\frac{1}{T}\frac{E_0^2}{\eta_0}\int_0^T \left[\frac{1}{2} + \frac{1}{2}\cos 2(\omega t - kz)\right]dt$$

$$= \hat{\mathbf{z}}\frac{1}{T}\frac{E_0^2}{\eta_0}\int_0^T \frac{dt}{2} + \hat{\mathbf{z}}\frac{1}{T}\frac{E_0^2}{\eta_0}\int_0^T \left[\frac{1}{2}\cos 2(\omega t - kz)\right]dt \quad \left[\frac{\mathrm{W}}{\mathrm{m}^2}\right]$$

The second integral is zero and the first equals T/2. The time-averaged power density is therefore

$$\mathcal{P}_{av}(z) = \hat{\mathbf{z}}\,\frac{E_0^2}{2\eta_0} = \hat{\mathbf{z}}\,\frac{1000^2}{2\times 377} = \hat{\mathbf{z}}\,1326.26 \quad \left[\frac{\mathrm{W}}{\mathrm{m}^2}\right]$$

(b) The power received by the antenna equals the power density multiplied by the surface of the antenna. Thus, for a dish of diameter d=1 m, the instantaneous power received is

$$P(t) = |\mathcal{P}(z,t)|S = \frac{E_0^2\pi d^2}{4\eta_0}\cos^2(\omega t - kz) = \frac{1000^2\times\pi\times 1}{4\times 377}\cos^2(6\pi\times 10^8 t - kz)$$
$$= 2083.28\cos^2(6\pi\times 10^8 t - kz) \quad [\mathrm{W}]$$

The time-averaged power received is

$$P_{av} = |\mathcal{P}_{av}|S = \frac{E_0^2\pi d^2}{8\eta_0} = 1326.26\times\pi\times(0.5)^2 = 1041.61 \quad [\mathrm{W}]$$

Q.5: Find the polarization of the wave in each case:

(a) $\mathbf{E}(z,t) = -\hat{\mathbf{y}}\,25e^{-0.001z}\cos\left(10^3 t - 1000z\right) \quad [\text{V/m}]$.
(b) $\mathbf{H}(z) = -\hat{\mathbf{x}}\,H_0 e^{-j\beta z} + \hat{\mathbf{y}}\,2H_0 e^{-j\beta z} \quad [\text{A/m}]$.
(c) $\mathbf{H}(z) = -\hat{\mathbf{y}}\,H_0 e^{-j\beta z} + j\hat{\mathbf{x}}\,H_1 e^{-j\beta z} \quad [\text{A/m}]$.

Solution:

(a) **Step a:** rewrite field in the time domain form.
Step b: We set z =0:

$$\mathbf{E}(z=0,t) = -\hat{\mathbf{y}}\,25\cos\left(10^3 t\right) \quad [\text{V/m}]$$

Step c: As time changes, the vector E may be either in the positive y or negative y direction. The field is linearly polarized in the y direction.
Step d: For a linearly polarized wave, there can be no rotation. This can be seen from the fact that for any two values of t the vector remains on the y axis.

(b) The magnetic field intensity has two components: one in the y direction and one in the x direction with amplitude half that of the y component. The two components are in phase; therefore, the polarization is linear, but for proper characterization, we must first find the electric field intensity. This is found from Ampere's law:

$$\nabla \times \mathbf{H} = \hat{\mathbf{x}}\left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right) + \hat{\mathbf{y}}\left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right) + \hat{\mathbf{z}}\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right) = j\omega\varepsilon\mathbf{E}$$

With $H_z = 0$, $\partial H_y/\partial x = 0$, and $\partial H_x/\partial y = 0$ and calculating the derivatives $\partial H_x/\partial z$ and $\partial H_y/\partial z$, we get

$$j\omega\varepsilon\mathbf{E} = \hat{\mathbf{x}}\,j\beta 2H_0 e^{-j\beta z} + \hat{\mathbf{y}}\,j\beta H_0 e^{-j\beta z}$$

Dividing both sides by $j\omega\varepsilon$ and setting $\eta = \beta/\omega\varepsilon$ gives the expression for the electric field intensity:

$$\mathbf{E} = \hat{\mathbf{x}}\,\eta 2H_0 e^{-j\beta z} + \hat{\mathbf{y}}\,\eta H_0 e^{-j\beta z} \quad [\text{V/m}]$$

Now, we apply steps **a** through **d** to find the polarization and sense of rotation (if any).
Step a: The vector **E** is written in the time domain:

$$\mathbf{E}(z,t) = \text{Re}\left\{\hat{\mathbf{x}}\,\eta 2H_0 e^{-j\beta z} e^{-j\beta t} + \hat{\mathbf{y}}\,\eta H_0 e^{-j\beta t}\right\} = \hat{\mathbf{x}}\,\eta 2H_0 \cos(\omega t - \beta z) + \hat{\mathbf{y}}\,\eta H_0 \cos(\omega t - \beta z) \quad [\text{V/m}]$$

Step b: We set $z = 0$:

$$\mathbf{E}(z=0,t) = \hat{\mathbf{x}}\,\eta 2H_0 \cos(\omega t) + \hat{\mathbf{y}}\,\eta H_0 \cos(\omega t) \quad [\text{V/m}]$$

Step c: At $t = 0$, the vector **E** has components $2\eta H_0$ in the positive x direction and ηH_0 in the positive y direction. This ratio remains constant as t changes. Thus, **E** is linearly polarized at an angle equal to $\tan^{-1}(Hy/Hx) = \tan^{-1}(1/2) = 26°34'$ with respect to the positive x axis

(c) First, we find the electric field intensity. Applying Ampere's law as in **(b)**, setting $H_z = 0$, $\partial H_y/\partial x = \partial H_x/\partial y = 0$, and calculating the derivatives $\partial H_x/\partial z$ and $\partial H_y/\partial z$ gives

$$j\omega\varepsilon\mathbf{E} = -\hat{\mathbf{x}}\, j\beta H_0 e^{-j\beta z} + \hat{\mathbf{y}}\, j\beta H_1 \mathrm{e}^{-j\beta z}$$

Now, we divide by $j\omega\varepsilon$ and set $\eta = \beta/\omega\varepsilon$:

$$\mathbf{E} = -\hat{\mathbf{x}}\,\eta H_0 e^{-j\beta z} - \hat{\mathbf{y}}\, j\eta H_1 e^{-j\beta z} \quad [\mathrm{V/m}]$$

Step a: The electric field intensity in the time domain is

$$\mathbf{E}(z,t) = \mathrm{Re}\left\{-\hat{\mathbf{x}}\,\eta H_0 e^{-j\beta z} e^{+j\omega t} - \hat{\mathbf{y}}\, j\eta H_1 e^{-j\beta z} e^{+j\omega t}\right\} = -\hat{\mathbf{x}}\,\eta H_0 \cos(\omega t - \beta z) - \hat{\mathbf{y}}\,\eta H_1 \cos(\omega t - \beta z + \pi/2) \quad [\mathrm{V/m}]$$

where $j = e^{j\pi/2}$ was used.

Step b: Setting $z = 0$ gives

$$\mathbf{E}(z=0,t) = -\hat{\mathbf{x}}\,\eta H_0 \cos(\omega t) - \hat{\mathbf{y}}\,\eta H_1 \cos(\omega t + \pi/2) = -\hat{\mathbf{x}}\,\eta H_0 \cos(\omega t) + \hat{\mathbf{y}}\,\eta H_1 \sin(\omega t) \quad [\mathrm{V/m}]$$

This is clearly an elliptically polarized wave, since $H_0 \neq H_1$

Step c: As t changes, the vector E describes an ellipse.

Step d: The rotation of **E** is found by setting $\omega t = 0$ and $\omega t = \pi/2$. These give

$$\mathbf{E}(z=0, \omega t = 0) = -\hat{\mathbf{x}}\,\eta H_0, \quad \mathbf{E}(z=0, \omega t = \pi/2) = +\hat{\mathbf{y}}\,\eta H_1 \quad [\mathrm{V/m}]$$

This indicates rotation in the clockwise direction. The wave is left elliptically polarized.