DEPARTMENT OF MATHEMATICS, IIT ROORKEE
MAB-103: Numerical Methods

Assignment-4 Eigenvalues and Eigenvectors by Power Method Session: 2025-26

1. Find the largest eigenvalue and corresponding eigenvector of the following matrices using power method:

   (a) $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$, with initial vector $X_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, correct to 2D.

   (b) $\begin{bmatrix} 4 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & -1 \end{bmatrix}$, with initial vector $X_0 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, correct to 2D.

   (c) $\begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, with initial vector $X_0 = (1, 0, 0)^T$, correct to 1D.

   (d) $\begin{bmatrix} 4 & 2 & 2 \\ 2 & 5 & 1 \\ 2 & 1 & 6 \end{bmatrix}$, with initial vector $X_0 = (1, 1, 1)^T$, correct to 2D.

2. Using power method, obtain the largest eigenvalue and corresponding eigenvector for the system of equations,

$$(2 - \lambda)x_1 - x_2 = 0,$$
$$-x_1 + (2 - \lambda)x_2 - x_3 = 0,$$
$$-x_2 + (2 - \lambda)x_3 = 0,$$

   starting with $X_0 = (1, 0, 0)^T$.

3. Find the smallest eigenvalue and corresponding eigenvector of the matrix

$$\begin{bmatrix} 4 & 2 & 2 \\ 2 & 5 & 1 \\ 2 & 1 & 6 \end{bmatrix},$$

   with initial vector $X_0 = (1, 1, 1)^T$, correct to 3D.

4. Find the smallest eigenvalue and corresponding eigenvector of the matrix A, whose inverse is

$$\begin{bmatrix} 0.75 & 0.5 & 0.25 \\ 0.5 & 1.0 & 0.5 \\ 0.25 & 0.5 & 0.75 \end{bmatrix},$$

   with initial vector $X_0 = (1, 0, 0)^T$, correct to 3D.

5. Starting with the initial vector $(1, 1, 1)^T$, find the numerically dominant eigenvalue and the associated eigenvector of the matrix,

$$\begin{bmatrix} 3 & -1 & 0 \\ -2 & 4 & -3 \\ 0 & -1 & 1 \end{bmatrix},$$

by retaining the numerical values to 3D or correct to 1D.

6. (a) Complete six iterations of the power method to approximate a dominant eigenvector of the matrix

$$\begin{bmatrix} 2 & -12 \\ 1 & -5 \end{bmatrix},$$

with initial vector $X_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, correct to 2D.

(b) Use the result to approximate the dominant eigenvalue of the given matrix using Rayleigh quotient.

Answers:

(1) (a) 6.00, $(1.00, 0.25)^T$.

(b) 4.00, $(1.00, 0.00, 0.00)^T$.

(c) 4.0, $(1.0, 0.5, 0.0)^T$.

(d) 8.39, $(0.81, 0.77, 1.00)^T$.

(2) $\lambda = 3.41$, $X = (0.73, -1.00, 0.69)^T$.

(3) $\lambda = 2.13$, $X = (1.00, -0.57, -0.37)^T$.

(4) $\lambda = 0.585$, $X = (0.707, 1.000, 0.707)^T$.

(5) $\lambda = 5.478$, $(-0.404, 1.000, -0.223)^T$, or $\lambda = 5.5$, $(-0.4, 1.0, -0.2)^T$.

(6) (a) $(2.99, 1..00)^T$, (b) -2.01.