INDIAN INSTITUTE OF TECHNOLOGY ROORKEE



ECC 203: Electromagnetics and Radiating Systems

Magnetostatics

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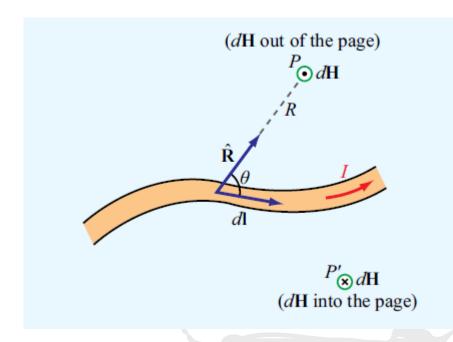
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Biot-Savart Law



$$d\mathbf{H} = \frac{I}{4\pi} \frac{d\mathbf{l} \times \hat{\mathbf{R}}}{R^2} \qquad (A/m)$$



$$\mathbf{H} = \frac{I}{4\pi} \int_{l} \frac{d\mathbf{l} \times \hat{\mathbf{R}}}{R^2}$$
 (A/m)

$$\mathbf{H} = \frac{1}{4\pi} \int_{S} \frac{\mathbf{J}_{s} \times \hat{\mathbf{R}}}{R^{2}} ds,$$

(surface current)

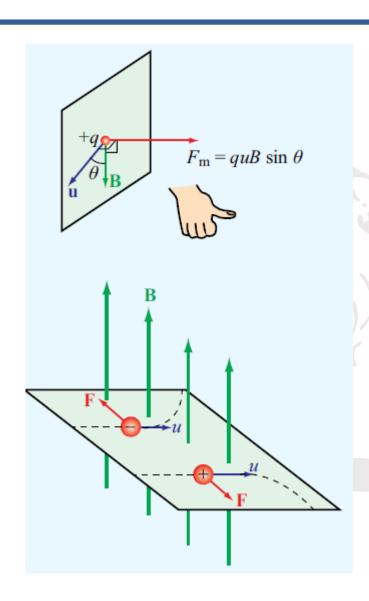
$$\mathbf{H} = \frac{1}{4\pi} \int_{\mathcal{V}} \frac{\mathbf{J} \times \hat{\mathbf{R}}}{R^2} \, d\mathcal{V}.$$

(volume current)

$$B = \mu H$$

Biot-Savart Law





$$\mathbf{F}_{\mathrm{m}} = q\mathbf{u} \times \mathbf{B}$$

$$\mathbf{F} = \mathbf{F}_{e} + \mathbf{F}_{m}$$
$$= q\mathbf{E} + q\mathbf{u} \times \mathbf{B} = q(\mathbf{E} + \mathbf{u} \times \mathbf{B})$$

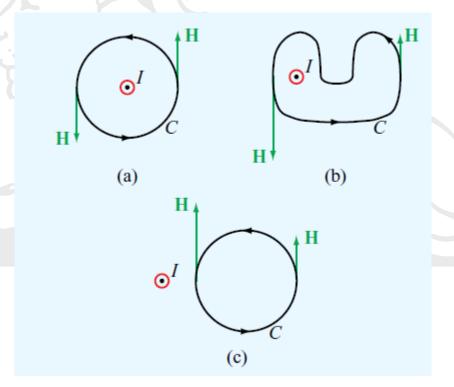
Lorentz force

Ampere' Law



$$\nabla \times \mathbf{H} = \mathbf{J} \quad \Longleftrightarrow \quad \oint_C \mathbf{H} \cdot d\boldsymbol{\ell} = I,$$

Ampere's law states that the line integral of **H** around a closed path is equal to the current traversing the surface bounded by that path



Gauss's Law for Magnetism



$$\nabla \cdot \mathbf{B} = 0 \quad \Longleftrightarrow \quad \oint_{S} \mathbf{B} \cdot d\mathbf{s} = 0$$

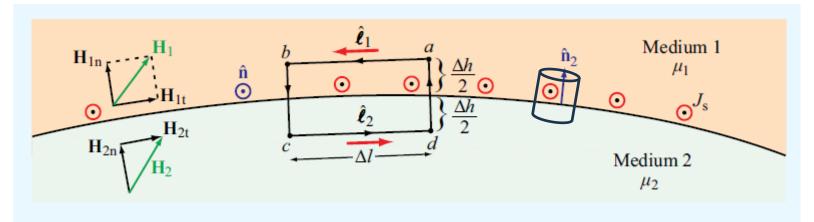
Note that the right-hand side of Gauss's law for magnetism is zero, reflecting the fact that the magnetic equivalence of an electric point charge does not exist in nature → NO MAGNETIC MONOPOLE in NATURE (YET !!!)

Magnetic Field Intensity / Magnetic Flux Density always form closed loop.

https://em8e.eecs.umich.edu/jsmodules/ulaby_modules.html

Magnetic Boundary Condition: Normal





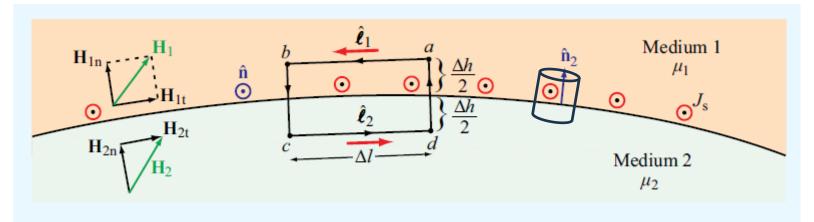
$$\oint_{S} \mathbf{B} \cdot d\mathbf{s} = 0 \quad \Longrightarrow \quad B_{1n} = B_{2n}.$$

▶ Thus the normal component of **B** is continuous across the boundary between two adjacent media. ◀

$$\mu_1 H_{1n} = \mu_2 H_{2n}$$

Magnetic Boundary Condition: Tangential





$$\oint_C \mathbf{H} \cdot d\mathbf{l} = \int_a^b \mathbf{H}_1 \cdot \hat{\boldsymbol{\ell}}_1 \, d\ell + \int_c^d \mathbf{H}_2 \cdot \hat{\boldsymbol{\ell}}_2 \, d\ell = I$$

$$(\mathbf{H}_1 - \mathbf{H}_2) \cdot \hat{\boldsymbol{\ell}}_1 \ \Delta l = \mathbf{J}_s \cdot \hat{\mathbf{n}} \ \Delta l.$$

$$\hat{\mathbf{n}}_2 \times (\mathbf{H}_1 - \mathbf{H}_2) = \mathbf{J}_s$$

Thank You

Questions?