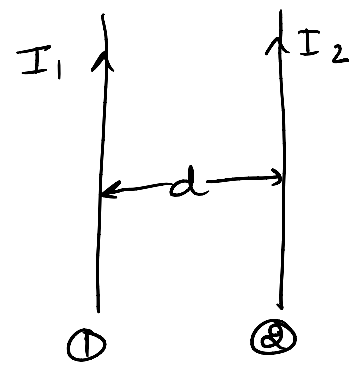
Lecture-13

02/09/2023

* Force b/w two long, ||el wire, a distance 'd' apart, carrying current I_1 & I_2 ⇒



Field at ② due to ① is-

$$B = \frac{\mu_0}{2\pi}\frac{I_1}{d}$$ (points into the page)

Force b/w them-

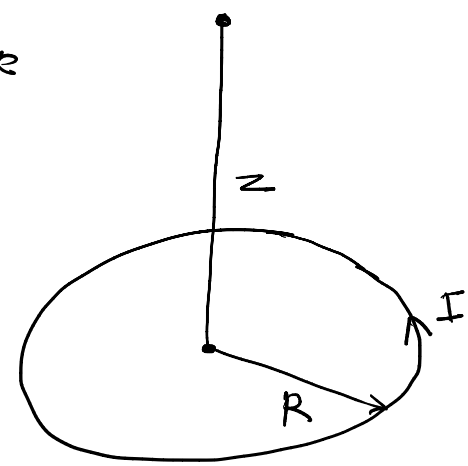
$$F = I_2 \int dl \times \frac{\mu_0}{2\pi}\frac{I_1}{d}$$

$$= \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d}\int dl$$

Force per unit length

$$\boxed{f = \frac{F}{L} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d}}$$

Homework Q ⇒ Find the magnetic field a distance z above the center of a circular loop of radius R, which carries a steady current I.

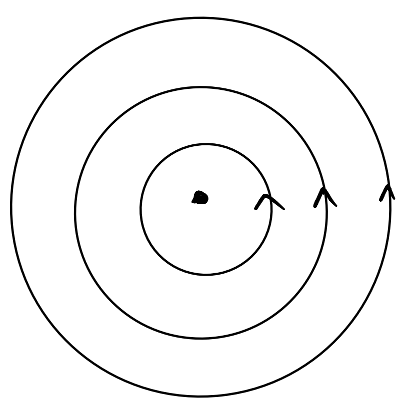


Ans ⇒ $$B(z) = \frac{\mu_0 I}{2}\frac{R^2}{(R^2+z^2)^{3/2}}.$$

Magnetic field of an infinite straight wire ⇒

$$B = \frac{\mu_0}{2\pi}\frac{i}{R}$$

* Obviously, this field has non-zero curl.



* Circulation ⇒

$$\oint B\cdot dl = \oint \frac{\mu_0}{2\pi}\frac{i}{R}\, dl$$
$$= \frac{\mu_0}{2\pi}\frac{i}{R}\oint dl$$
$$= \frac{\mu_0}{\cancel{2\pi}}\frac{i}{\cancel{R}}\cdot \cancel{2\pi R}$$
$$= \mu_0 I$$

$$\boxed{\oint B\cdot dl = \mu_0 I_{encl.}}$$

I_{encl} is total current enclosed by the integration path.

* This is known as Ampere's law.

$$\oint B\cdot dl = \mu_0\int J\cdot ds$$
$$\int (\nabla\times B)\cdot ds = \mu_0\int J\cdot ds$$

$$\boxed{\nabla\times B = \mu_0 J}$$

$$\boxed{I = \int \underbrace{J}_{\text{current density}}\cdot ds}$$

Ampere's law in differential form.

Taking divergence.

$$\underbrace{\nabla\cdot(\nabla\times B)}_{0} = \mu_0\,\nabla\cdot J$$

$$\Rightarrow \nabla\cdot J = 0$$

$\nabla\cdot J + \frac{\partial\rho}{\partial t} = 0$

only if $\underbrace{\frac{\partial\rho}{\partial t}}_{0} = 0$

in Magnetostatic

Application of Ampere's law ⇒

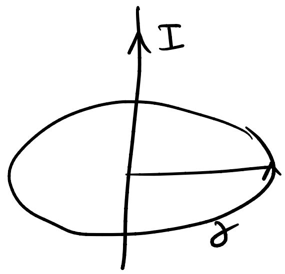
① Infinite line current

② Infinite sheet current

③ A Solenoid

④ A Toroid.

(i) ⇒ Infinite line current ⇒

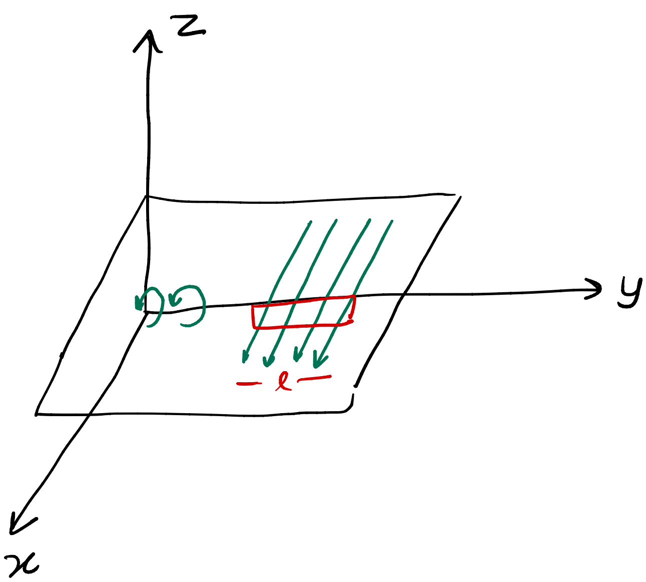


$$\oint B \cdot dl = \mu_0 I_{enc}.$$

$$B \cdot 2\pi r = \mu_0 I$$

$$B = \frac{\mu_0}{2\pi} \frac{I}{r}$$

(ii) ⇒ Infinite sheet current ⇒



* Find the magnetic field of an infinite current $K = K\hat{x}$, flowing over the xy plane.

* x component → No
z 〃 → No.

Any vertical contribution from a filament at $+y$ is canceled by the corresponding filament at $-y$.

* B can have only y- component.

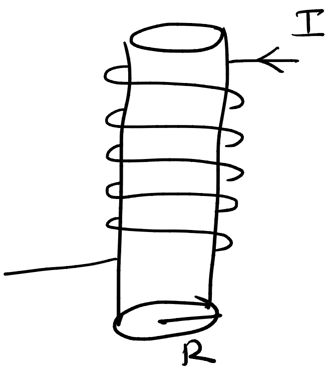
$$\oint B \cdot dl = \mu_0 I_{enc}$$

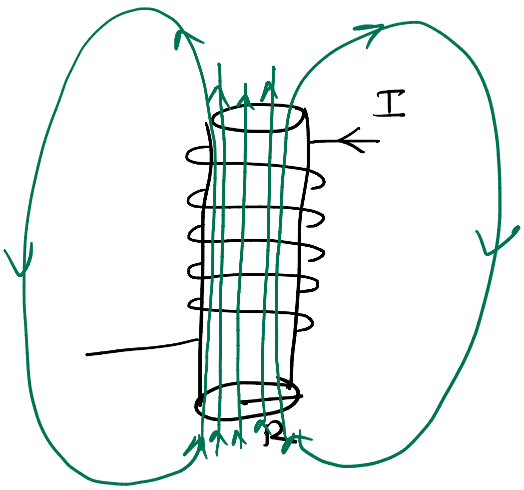
$$B \cdot 2l = \mu_0 K l$$

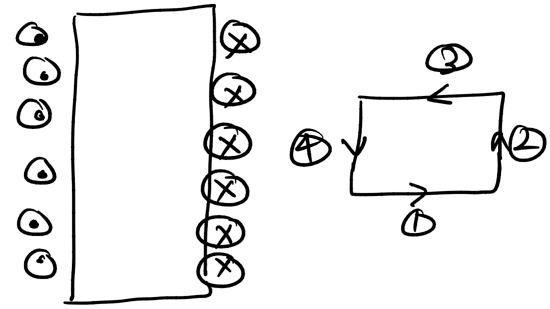
[uniform surface current $= Kl$]

$$B = \begin{cases} \frac{\mu_0 K}{2} \hat{y} & \text{for } z < 0 \\ -\frac{\mu_0 K}{2} \hat{y} & \text{for } z > 0 \end{cases}$$

(iii) ⇒ <u>A very long solenoid ⇒</u> Consists of n close winding per unit length on a cylinder of radius R





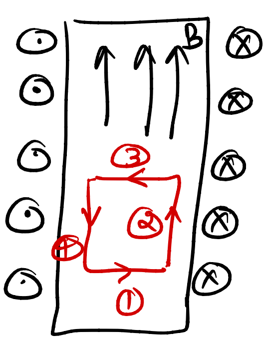


$$\int_1 B\cdot dl \quad \& \quad \int_3 B\cdot dl = 0$$

$$\therefore \int_2 B\cdot dl + \int_4 B\cdot dl = 0 \qquad \left[\ \oint B\cdot dl = 0\right.$$

$$\therefore B_2 = B_4$$

Magnetic field outside does not depend on the distance from axis.
However
As distance $\longrightarrow \infty$ $B \rightarrow 0$ therefore
B must be zero everywhere



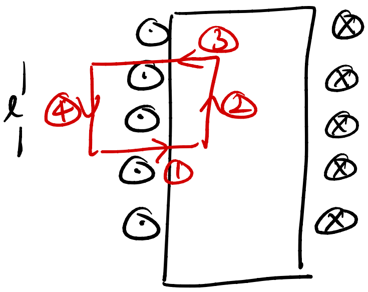
* For the loop inside ⇒

$$\int_1 B\cdot dl = 0 \quad \& \quad \int_3 B\cdot dl = 0$$

$$\therefore \int_2 B\cdot dl + \int_4 B\cdot dl = 0$$

$$B_{at\,2} = B_{at\,4}$$

B is const inside. $B = ?$



$$\int_1 B\cdot dl + \int_2 B\cdot dl + \int_3 B\cdot dl + \cancel{\int_4 B\cdot dl} = \mu_0 I n l$$

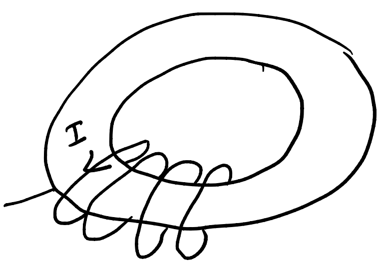
$= 0$ $\quad = 0 \quad = 0$ outside $B = 0$.

$$\int_1 B\cdot dl = \mu_0 I n l$$

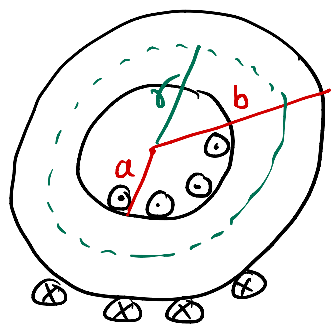
$$B\cancel{l} = \mu_0 I n \cancel{l}$$

$$B = \begin{cases} \mu_0 n I \ \hat{z} & \text{inside solinoid} \\ 0 & \text{outside} \end{cases}$$

④ A toroidal coil :- consits of a circular ring or donet around which a long wire is wrapped.



$\Rightarrow$



$$\int B\cdot dl = \mu_0 I_{encl}.$$

$$B\cdot 2\pi r = \mu_0 NI$$

$$B = \begin{cases} \dfrac{\mu_0 NI}{2\pi r} & \text{inside} \quad (a<r<b) \\ 0 & \text{outside \& inside } (r<a \end{cases}$$