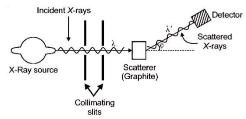
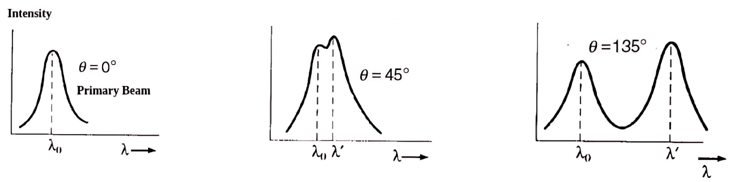
Compton's experiment :



λ_0 : Wavelength of incident radiation



$$\Delta\lambda = \lambda_1 - \lambda_0 = 2\lambda_c Sin^2\theta/2$$

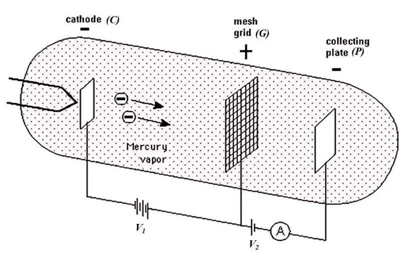
Where $\lambda_c = \frac{h}{m_0 c}$ is called Compton wavelength of Electron

This explains the presence of the shifted $\lambda = \lambda_1$ in the scattered radiation

If $\Delta\lambda$ is very small,
then $\lambda_1 \approx \lambda_0$ (**No shift in wavelength**)

Can the Compton Effect be observed with visible light?

Franck and Hertz experiment : (Energy quantization in atoms)

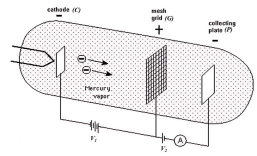


Electrons emitted from the heated cathode C and accelerated towards a wire grid G, maintained at a +ve potential V_1 with respect to C.

The electrons accelerated by potential V_1, attain a K.E. $\frac{1}{2}mv^2 = eV_1$

Some electrons pass through the grid and are collected by plate P causing a current I to flow in the collector circuit.

Franck and Hertz experiment :



The collector P is kept at a slightly lower potential $V_2 = V_1 - \Delta V$ $(\Delta V << V_1)$ than the grid.

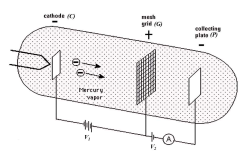
The small retarding potential ΔV between the grid and the collector reduces the K.E. of electrons (but cannot stop them)

Now the tube is filled with Hg vapour

Electrons collide with Hg atoms (elastically), but may not have enough energy to excite them
(Elastic Collision: electrons deflected but retain the same K.E.)

In contrast, if an electron collides inelastically, it loses energy (E, say and thereby excites the Hg atom.

Then K.E. of electron : $\frac{1}{2}\mathrm{mv}^2 = \mathrm{eV}_1 - E$



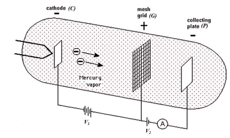
In contrast, if an electron collides inelastically, it loses energy (E, say), and thereby exciting the Hg atom.

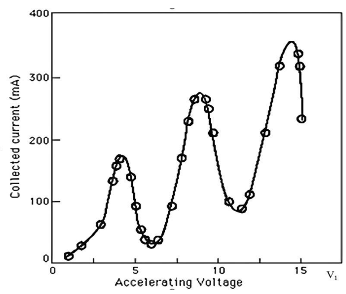
Then K.E. of electron : $\frac{1}{2}\mathrm{mv}^2 = \mathrm{eV}_1 - E$

Now if $eV_1 = E$ or even a little larger then the retarding potential ΔV will be enough to prevent the electron from reaching the plate (drop in current)

Experiment: Increase V_1 gradually from 0 to higher values.

Franck-Hertz experiment :





- ✓ Electrons are accelerated in the Franck-Hertz apparatus, and the collected current rises with accelerated voltage.
- ✓ The first peak is the first excited level of Hg (4.9 eV above ground state)
- ✓ 4.9 volt excited state corresponds to a strong line in the ultraviolet emission spectrum of mercury at 254 nm (a 4.9eV photon).
- ✓ Drops in the collected current occur at multiples of 4.9 volts since an accelerated electron, which has 4.9 eV of energy removed in a collision, can be re-accelerated to produce other such collisions at multiples of 4.9 volts.

This experiment was a strong confirmation of the idea of quantized atomic energy levels.

Wave properties of matter: de Broglie's *hypothesis*

Dual nature of electromagnetic radiation:

1. Wave (Interference, Diffraction, etc.)
2. Particle properties (Photoelectric effect, Compton effect, etc.)

Louis de Broglie made a hypothesis that "**Material particles might also possess wave-like properties**". So like radiation, they have a dual nature. **Wave-particle duality would be a universal characteristic of nature.**

Energy of photon $E = h\nu$

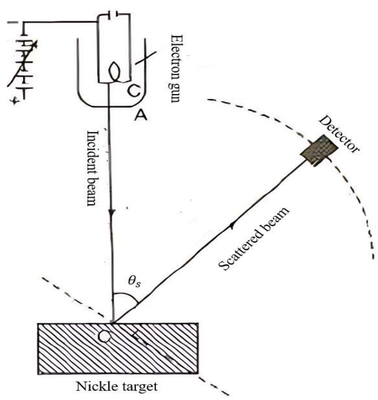
Magnitude of momentum $p = \frac{h\nu}{c} = \frac{h}{\lambda}$

For a free material particle, associated matter waves also have frequency $\boldsymbol{\nu}$ and wavelength $\boldsymbol{\lambda}$ related, respectively, to the energy $\boldsymbol{E}$ and the magnitude $\boldsymbol{p}$ of the momentum of the particle

$$\boldsymbol{\nu = \frac{E}{h}} \text{ and } \boldsymbol{\lambda = \frac{h}{p}} = \textbf{de-Broglie wavelength of particle}$$

Experimental confirmation of de Broglie's hypothesis

Davisson-Germer Experiment



Davisson and Germer were studying the scattering of electrons from a solid

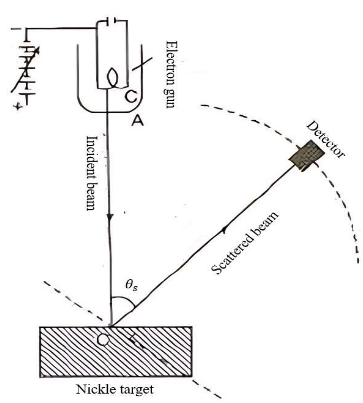
Initially, they observed that the intensity of the scattered beam does not vary significantly with angle.

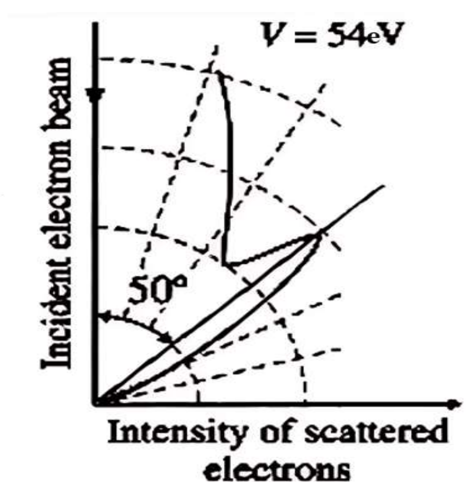
!! PROBLEM !!
Air went inside the apparatus

Nickel→oxidised→heated to remove the oxide

!! RESUMED THE MEASUREMENT AGAIN !!

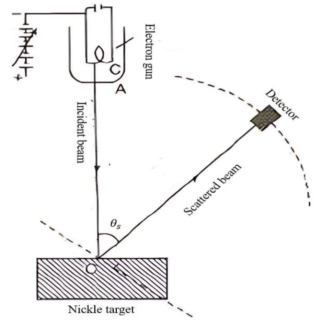
!! SURPRISE !!

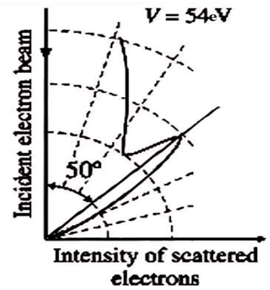




An accelerating voltage of 54 volts gave a peak at a scattering angle of 50°

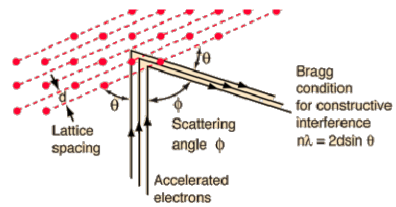
At high temperatures, many small individual crystals of Nickel formed a single large crystal of Nickel



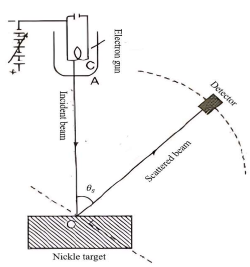


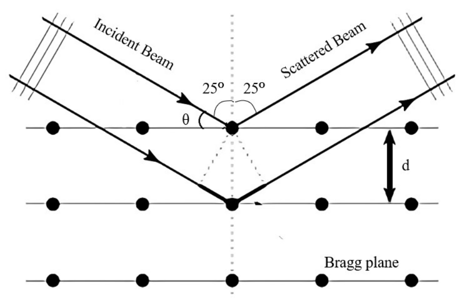
This could be explained by assuming that electron waves were being diffracted just like X-rays by planes of atoms in a crystal.

Bragg's Law $2dSin\theta = n\lambda$



The angle θ in the Bragg law corresponding to that scattering angle is 65°

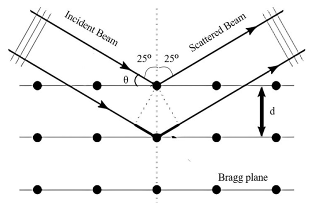


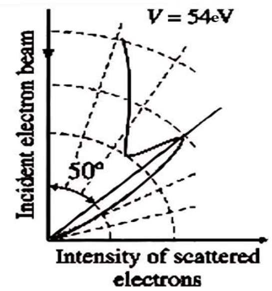


$$\boldsymbol{\theta = 90^o - 25^o = 65^o}$$

In the particular case of 54 eV,
a sharp maxima in the electron distribution occurred at 50º

The angle of incidence relative to the Bragg planes was ≈ 65⁰





The spacing of the planes measured by the X-ray technique is $d = 0.091$nm

The Bragg equation for maxima in the diffraction pattern is $n\lambda = 2dSin\theta$

For $n = 1$

$$\boldsymbol{\lambda = 2\,(0.091)\,Sin65^0 = 0.165\ \text{nm}}$$

Now, if we use the formula to calculate the wavelength of an electron:

$$\boldsymbol{\lambda = \frac{h}{mv} = \frac{h}{p}}$$

Energy of electron = 54 eV

Kinetic Energy:
$k = \frac{1}{2}mv^2 = \frac{p^2}{2m}$

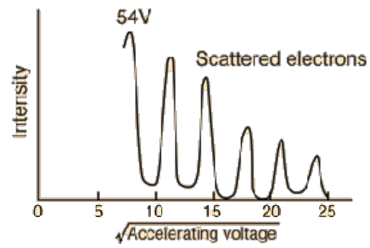
$$p = \sqrt{2mk}$$

$$= (2 \times 9.1 \times 10^{-31} \times 54 \times 1.6 \times 10^{-19})^{1/2}$$
$$= 4.0 \times 10^{-24}\ \text{kg m/s}$$

$$\lambda = \frac{h}{p} = \frac{6.626\times10^{-34}}{4.0\times10^{-24}} = 1.66\ \times 10^{-10}\text{m} = 0.166\ \text{nm}$$

➔ Which agrees with the observed wavelength

for n = 1 $\lambda = 2\ (0.091)\ Sin65^0 = 0.165$ nm



For the electron, $m = 9.1 \times 10^{-31}$ kg, accelerated by the potential difference V, we can write

$$\lambda = \frac{h}{p} = \frac{h}{(2mE)^{1/2}} = \frac{h}{(2meV)^{1/2}} = \frac{12.3}{(V(volts))^{1/2}}\ \text{Å} = \frac{2d\sin\theta}{n}$$

For n=1, 2, 3 ➔ $\sqrt{V}$ = 7.36, 14.7, and 22,
which appear to agree with the first, third, and fifth peaks.

What gives the second, fourth, and sixth peaks?

They originate from a different set of planes in the crystal.

Those peaks satisfy a sequence 2,3,4, suggesting that the first peak of that series would have been at 5.85

DE BROGLIE'S WAVES

A moving body behaves in certain ways as if it has a wave nature !!

A photon of light of frequency ν has the momentum ⇨ $p = \frac{h\nu}{c} = \frac{h}{\lambda}$

Photon wavelength $\lambda = \frac{h}{p}$

De Broglie suggested it as a general principle that **applies to both material particles and photons.**

If the momentum of a particle of mass m and velocity v is $p = \gamma mv$,
De Broglie wavelength

$$\boxed{\lambda = \frac{h}{\gamma mv}}$$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

γ is the relativistic factor

Matter Waves

In light waves, the Electric field (**E**) and Magnetic field (**B**) vary

What does vary in matter waves/de Broglie waves?

The wavefunction Ψ $\rightarrow$
Related to the probability of finding a body at a point (x,y,z) in space at a time t.

Physical significance:
$|\Psi|^2$ $\rightarrow$ absolute value of wavefunction $\rightarrow$ probability density $\rightarrow$ proportional to the probability of experimentally finding the body described by the wavefunction Ψ at point (x, y, z) at time t

How fast do the de Broglie waves travel ?

Since we associate a de Broglie wave with a moving body, we expect that this wave has the same velocity as that of the body.

Let us see if this is true!

If we call the **de Broglie wave velocity =** v_p

Then, $\boxed{v_p = \nu\lambda}$

de Broglie wavelength (we defined earlier) $\boxed{\lambda = \frac{h}{\gamma m v}}$

Find the frequency Expression:
Equate the quantum expression $E = h\nu$ with
The relativistic formula for total energy $E = \gamma mc^2$

$$h\nu = \gamma mc^2$$

$$\nu = \frac{\gamma mc^2}{h}$$

De Broglie phase velocity

$$v_p = \nu\lambda = \left(\frac{\gamma mc^2}{h}\right)\left(\frac{h}{\gamma mv}\right) = \frac{c^2}{v}$$

➔ Particle velocity (v) must be less than the velocity of light c

➔ de Broglie waves always travel faster than light! STRANGE!

To understand this unexpected result, we must examine the distinction between **phase velocity** and **group velocity.**
(Phase velocity is what we have been calling wave velocity.)

If we choose $t = 0$ when the displacement y of the string at $x = 0$ is a maximum, its displacement at any future time t at the same place is given by

$$y = A \cos 2\pi\nu t$$

A is the amplitude of the vibrations and ν their frequency.

Assume that the wave has some speed v_p that depends on the properties of the string.

Wave formula

$$y = A \cos 2\pi\nu\left(t - \frac{x}{v_p}\right)$$

$$y = A \cos 2\pi\left(\nu t - \frac{\nu x}{v_p}\right)$$

Since the wave speed v_p is given by $v_p = \nu\lambda$ we have

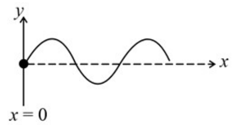
$$y = A \cos 2\pi\left(\nu t - \frac{x}{\lambda}\right)$$

Angular frequency $\omega = 2\pi\nu$ **Wave number** $k = \frac{2\pi}{\lambda} = \frac{\omega}{v_p}$

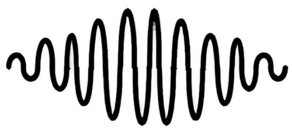
$$y = A \cos (\omega t - kx)$$

Representing de-Broglie Waves

Not as



A group of waves need not have the same velocity as the waves themselves



Wave Packet

Speed of wave group =?

[wave packets]
[wave groups]

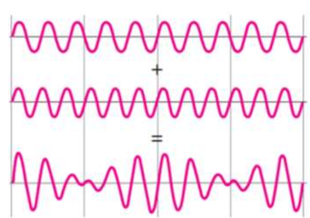
Reminds us of "beats"
: Superposition of waves of different frequencies

To describe a wave group mathematically
➔ take a superposition of individual waves of different wavelengths
➔ whose interference with one another results in the variation in amplitude that defines the group shape.

If the velocities of the waves are the same
➔ The velocity with which the wave group travels is the common phase velocity.

However, if the phase velocity varies with wavelength
➔ Different Individual waves do not proceed together
➔ This situation is called **dispersion**

As a result, the wave group has a velocity different from the phase velocities of the waves that make it up
➔ **case of de Broglie waves**



Find the velocity v_g with which a wave group travels

Suppose the wave group arises from the combination of two waves that have the same amplitude A but differ by an amount $\Delta\omega$ in angular frequency and an amount Δk in wave number.

We may represent the original waves by the formulas

$$y_1 = A \cos(\omega t - kx)$$
$$y_2 = A \cos[(\omega + \Delta\omega)t - (k + \Delta k)x]$$

The resultant displacement y at any time t and any position x is the sum of y_1 and y_2.

Use the following

$$\begin{cases} \cos\alpha + \cos\beta = 2\cos\frac{1}{2}(\alpha+\beta)\cos\frac{1}{2}(\alpha-\beta) \\ \cos(-\theta) = \cos\theta \end{cases}$$

$$\begin{aligned} y &= y_1 + y_2 \\ &= 2A\cos\tfrac{1}{2}[(2\omega + \Delta\omega)t - (2k + \Delta k)x]\cos\tfrac{1}{2}(\Delta\omega\, t - \Delta k\, x) \end{aligned}$$

Since $\Delta\omega$ and Δk are small compared with ω and k respectively,

$$2\omega + \Delta\omega \approx 2\omega$$
$$2k + \Delta k \approx 2k$$

$$y = 2A\cos(\omega t - kx)\cos\left(\frac{\Delta\omega}{2}t - \frac{\Delta k}{2}x\right)$$

It represents a wave of angular frequency ω and wave number k that has superimposed upon it a modulation of angular frequency $\frac{1}{2}\Delta\omega$ and of wave number $\frac{1}{2}\Delta k$

Phase velocity $v_p = \dfrac{\omega}{k}$ **Group velocity** $v_g = \dfrac{\Delta\omega}{\Delta k}$

When ω and k have continuous spreads instead of the two values in the preceding discussion, the group velocity is instead given by

$$\boxed{v_g = \frac{d\omega}{dk}}$$

The **angular frequency** and **wave number** of the de Broglie waves associated with a body of mass ***m*** moving with the velocity ***υ*** are

$$\omega = 2\pi\nu = \frac{2\pi\gamma mc^2}{h}$$

$$= \frac{2\pi mc^2}{h\sqrt{1 - \upsilon^2/c^2}}$$

Angular frequency of de Broglie waves

$$k = \frac{2\pi}{\lambda} = \frac{2\pi\gamma m\upsilon}{h}$$

$$= \frac{2\pi m\upsilon}{h\sqrt{1 - \upsilon^2/c^2}}$$

Wave number of de Broglie waves

Both ω and k are functions of the body's velocity υ.

The group velocity υ_g of the de Broglie waves associated with the body is

$$\upsilon_g = \frac{d\omega}{dk} = \frac{d\omega/d\upsilon}{dk/d\upsilon}$$

Now $$\frac{d\omega}{d\upsilon} = \frac{2\pi m\upsilon}{h(1 - \upsilon^2/c^2)^{3/2}}$$

$$\frac{dk}{d\upsilon} = \frac{2\pi m}{h(1 - \upsilon^2/c^2)^{3/2}}$$

De Broglie group velocity $$\boxed{\upsilon_g = \upsilon}$$ ⇨ **The de Broglie wave group associated with a moving body travels with the same velocity as the body.**

The phase velocity v_p of de Broglie waves is, as we found earlier,

$$v_p = \frac{\omega}{k} = \frac{c^2}{v}$$

This exceeds both the velocity of the body v and the velocity of light c, since $v < c$.

However, v_p has no physical significance because the motion of the wave group, not the motion of the individual waves that make up the group, corresponds to the motion of the body, and $v_g < c$ as it should be.