ECC 203 : Electromagnetics and Radiating Systems

Electrostatics

Gowrish Basavarajappa

Asst. Professor, ECE Dept., IIT Roorkee

gowrish.b@ece.iitr.ac.in

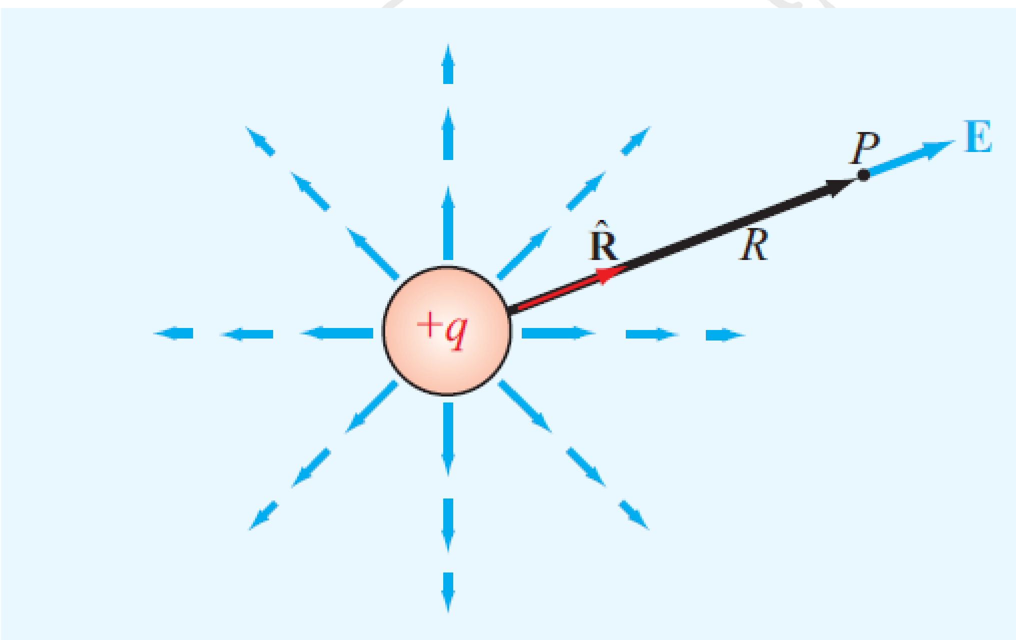
www.gowrish.in

Coulomb’s Law

- An isolated charge *q* induces an electric field **E** at every point in space, and at any specific point *P*, **E** is given by

$$\mathbf{E} = \hat{\mathbf{R}} \, \frac{q}{4\pi\epsilon R^2} \qquad \text{(V/m)},$$

- where ˆ**R** is a unit vector pointing from *q* to *P, R* is the distance between them, and is the electrical permittivity of the medium containing the observation point *P*.



$$\mathbf{E}_1 = \frac{q_1(\mathbf{R} - \mathbf{R}_1)}{4\pi\epsilon|\mathbf{R} - \mathbf{R}_1|^3} \qquad \text{(V/m)}$$

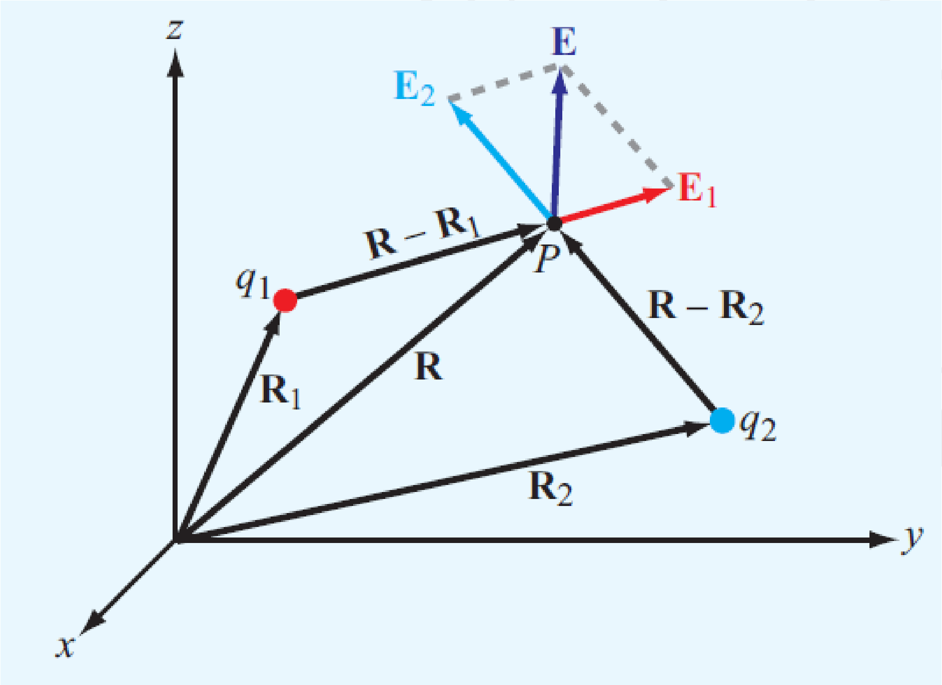
Coulomb’s Law

- In the presence of an electric field **E** at a given point in space, which may be due to a single charge or a distribution of charges, the force acting on a test charge *q’* when placed at *P*, is

$$\mathbf{F} = q'\mathbf{E} \qquad \text{(N)}$$

Electric Field Due to Multiple Point Charges

- The electric field obeys the ***principle of linear superposition***.

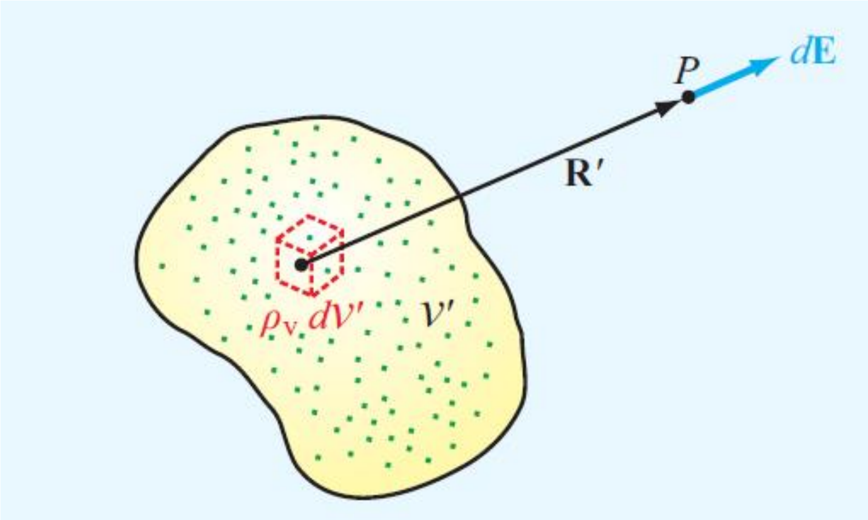


$$\mathbf{E} = \frac{1}{4\pi\epsilon}\sum_{i=1}^{N}\frac{q_i(\mathbf{R}-\mathbf{R}_i)}{|\mathbf{R}-\mathbf{R}_i|^3} \qquad \text{(V/m)}$$

Coulomb's Law

Electric Field Due to a Charge Distribution

- The electric field obeys the ***principle of linear superposition***.



$$\mathbf{E} = \int_{\mathcal{V}'} d\mathbf{E} = \frac{1}{4\pi\epsilon} \int_{\mathcal{V}'} \hat{\mathbf{R}}' \, \frac{\rho_v \, d\mathcal{V}'}{R'^2}$$

(volume distribution)

$$\mathbf{E} = \frac{1}{4\pi\epsilon} \int_{S'} \hat{\mathbf{R}}' \, \frac{\rho_s \, ds'}{R'^2}$$

(surface distribution)

$$\mathbf{E} = \frac{1}{4\pi\epsilon} \int_{l'} \hat{\mathbf{R}}' \, \frac{\rho_\ell \, dl'}{R'^2}$$

(line distribution)

Coulomb's Law

Material other than vacuum / air

- For a material with electrical permittivity , the electric field quantities **D** and **E** are related by

$$\mathbf{D} = \epsilon \mathbf{E}$$

$$\epsilon = \epsilon_r \epsilon_0$$

$$\epsilon_0 = 8.85 \times 10^{-12} \approx (1/36\pi) \times 10^{-9} \qquad \text{(F/m)}$$

- is the electrical permittivity of free space, and ε_r is called the ***relative permittivity*** (or ***dielectric constant***) of the material.

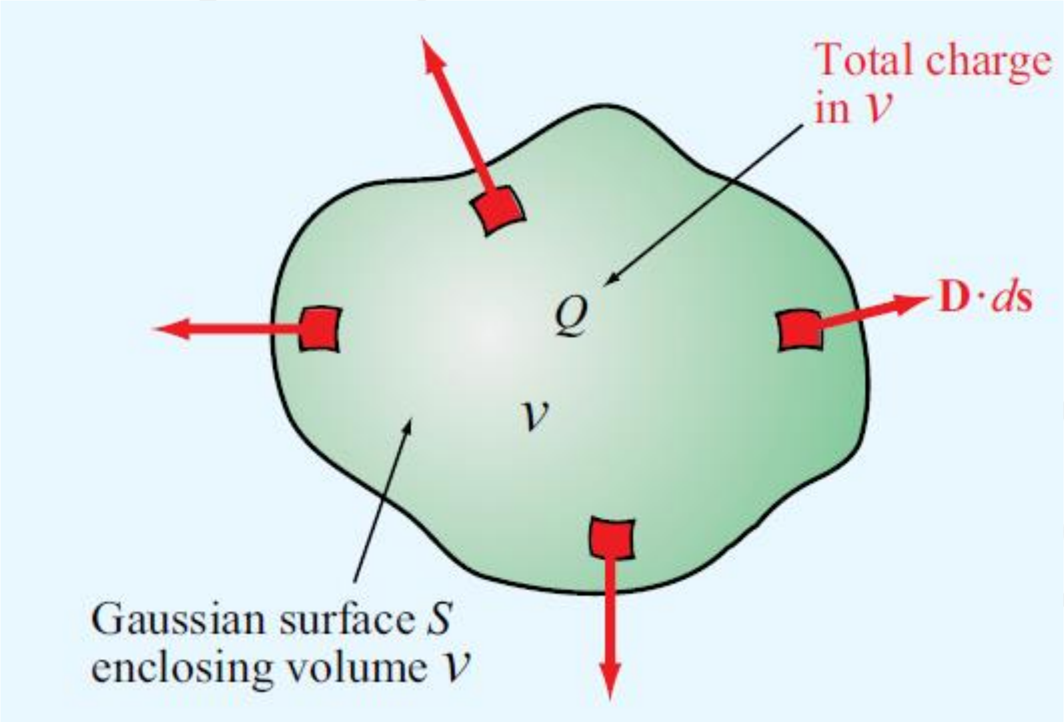
Gauss's Law

$$\nabla \cdot \mathbf{D} = \rho_v,$$

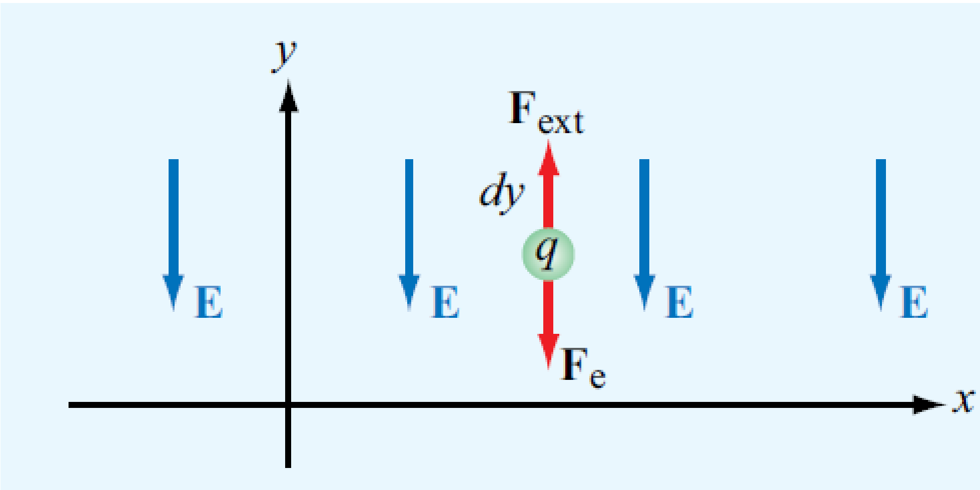
(differential form of Gauss's law)

$$\oint_S \mathbf{D} \cdot d\mathbf{s} = Q.$$

(integral form of Gauss's law)



Electric Scalar Potential



To move the charge along the positive *y* direction (against the force $\mathbf{F}_e$), we need to provide an external force $\mathbf{F}_{ext}$ to counteract $\mathbf{F}_e$, which requires the expenditure of energy.

To move *q* without acceleration (at constant speed), the net force acting on the charge must be zero, which means that $\mathbf{F}_{ext} + \mathbf{F}_e = 0$, or

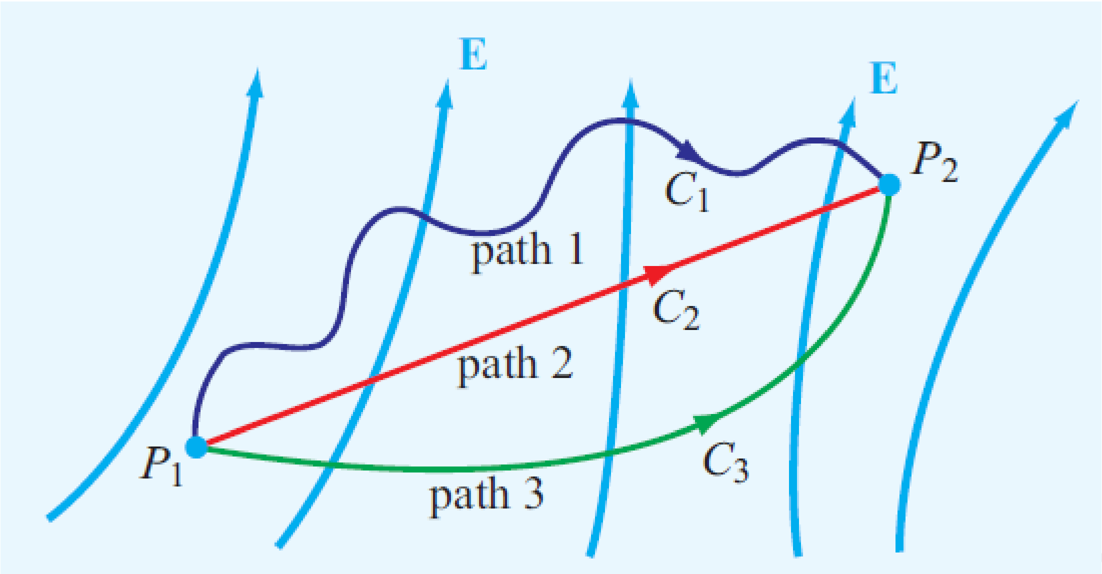
$$\mathbf{F}_{ext} = -\mathbf{F}_e = -q\mathbf{E}.$$

$$dW = \mathbf{F}_{ext} \cdot d\mathbf{l} = -q\mathbf{E} \cdot d\mathbf{l}$$

The differential electric potential energy *dW* per unit charge
is called the ***differential electric potential*** (or differential
voltage) *dV* . That is,

$$dV = \frac{dW}{q} = -\mathbf{E} \cdot d\mathbf{l} \qquad \text{(J/C or V)}$$

Electric Scalar Potential



$$\int_{P_1}^{P_2} dV = -\int_{P_1}^{P_2} \mathbf{E} \cdot d\mathbf{l},$$

$$V_{21} = V_2 - V_1 = -\int_{P_1}^{P_2} \mathbf{E} \cdot d\mathbf{l},$$

the line integral of the electrostatic field E around any closed contour C is zero

$$\oint_C \mathbf{E} \cdot d\mathbf{l} = 0 \qquad \text{(electrostatics).}$$

A vector field whose line integral along any closed path is zero is called a ***conservative*** or an ***irrotational*** field. Hence, the electrostatic field **E** is conservative

Electric Scalar Potential

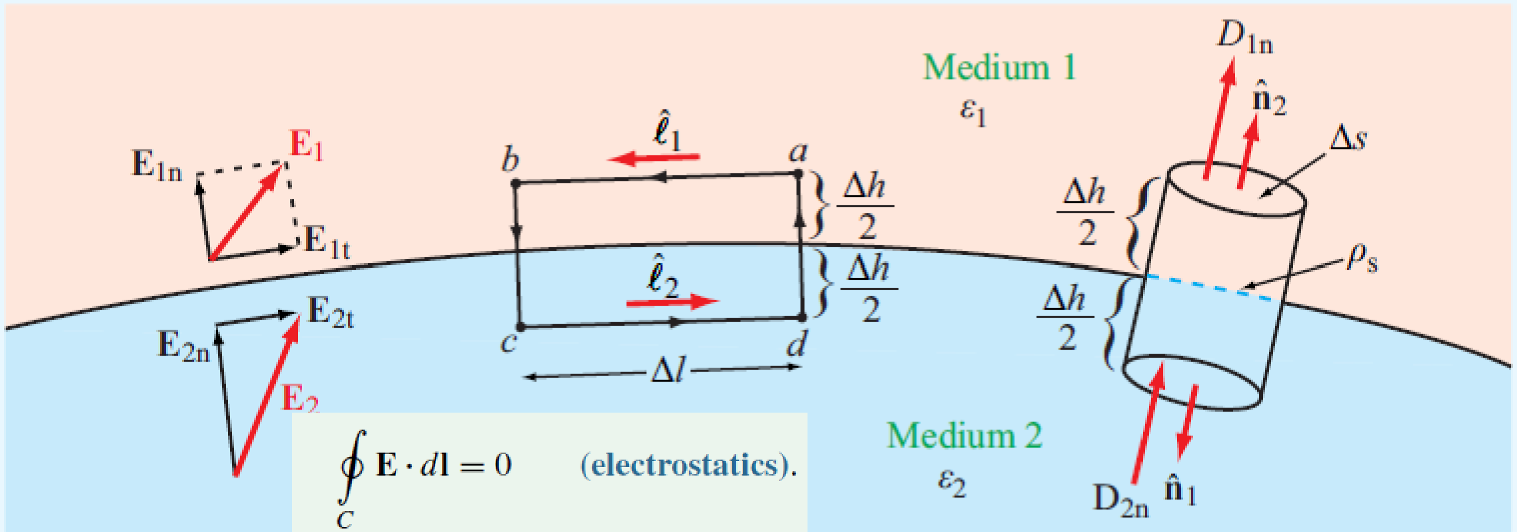
Electric Field as a Function of Electric Potential

$$\mathbf{E} = -\nabla V$$

- This differential relationship between V and **E** allows us to determine **E** for any charge distribution by first calculating V and then taking the negative gradient of V to find **E**

- The expressions for V , involve scalar sums and scalar integrals, and as such are usually much easier to evaluate than the vector sums and integrals in the expressions for **E** derived on the basis of Coulomb’s law. Thus, even though the electric potential approach for finding **E** is a two-step process, it is conceptually and computationally simpler to apply than the direct method based on Coulomb’s law

Electric Boundary Condition : Tangential



$$\oint_C \mathbf{E} \cdot d\mathbf{l} = 0 \qquad \text{(electrostatics)}.$$

$$\oint_C \mathbf{E} \cdot d\mathbf{l} = \int_a^b \mathbf{E}_1 \cdot \hat{\boldsymbol{\ell}}_1 \, dl + \int_c^d \mathbf{E}_2 \cdot \hat{\boldsymbol{\ell}}_2 \, dl = 0,$$

$$\mathbf{E}_1 = \mathbf{E}_{1t} + \mathbf{E}_{1n}$$

$$\mathbf{E}_2 = \mathbf{E}_{2t} + \mathbf{E}_{2n}$$

Noting that $\hat{\boldsymbol{\ell}}_1 = -\hat{\boldsymbol{\ell}}_2$, it follows that

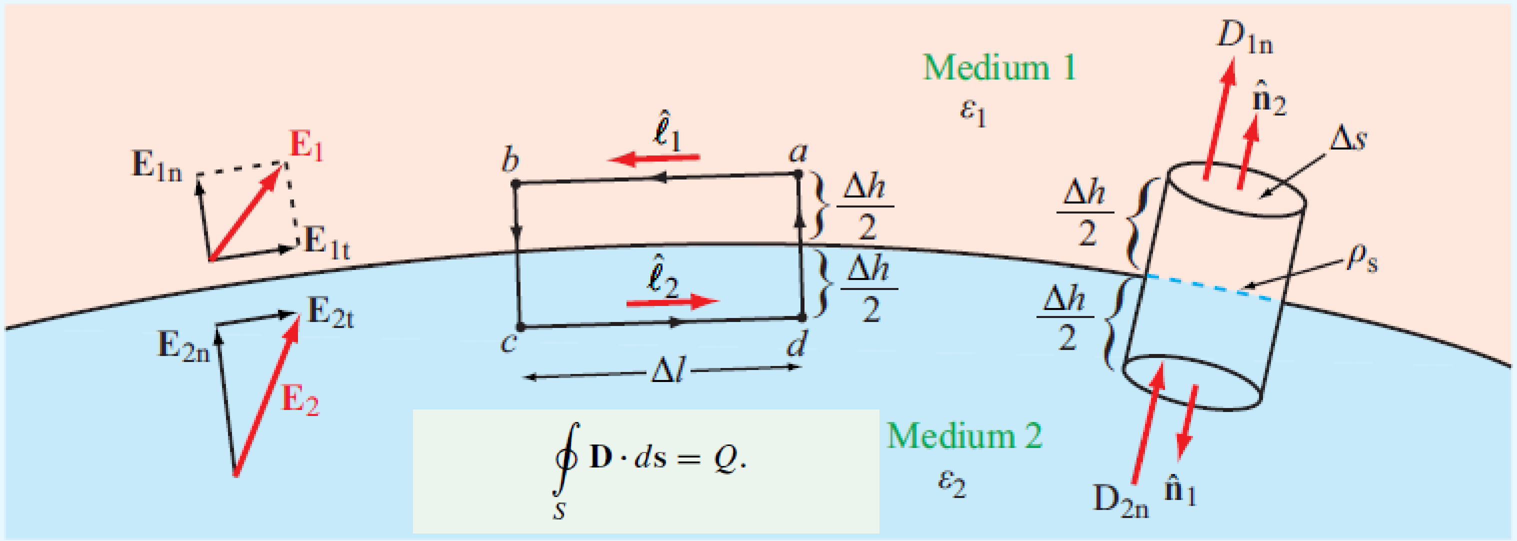
$$(\mathbf{E}_1 - \mathbf{E}_2) \cdot \hat{\boldsymbol{\ell}}_1 = 0$$

$$\mathbf{E}_{1t} = \mathbf{E}_{2t} \qquad \text{(V/m)}.$$

▶ Thus, the tangential component of the electric field is **continuous** across the boundary between any two media. ◀

$$\frac{\mathbf{D}_{1t}}{\epsilon_1} = \frac{\mathbf{D}_{2t}}{\epsilon_2}$$

Electric Boundary Condition : Normal



$$\oint_S \mathbf{D} \cdot d\mathbf{s} = \int_{\text{top}} \mathbf{D}_1 \cdot \hat{\mathbf{n}}_2 \, ds + \int_{\text{bottom}} \mathbf{D}_2 \cdot \hat{\mathbf{n}}_1 \, ds = \rho_s \, \Delta s$$

$$D_{1n} - D_{2n} = \rho_s \qquad (\text{C/m}^2)$$

$$\hat{\mathbf{n}}_1 = -\hat{\mathbf{n}}_2$$

▶ The normal component of **D** changes abruptly at a charged boundary between two different media in an amount equal to the surface charge density. ◀

$$\hat{\mathbf{n}}_2 \cdot (\mathbf{D}_1 - \mathbf{D}_2) = \rho_s \qquad (\text{C/m}^2).$$

$$\epsilon_1 E_{1n} - \epsilon_2 E_{2n} = \rho_s$$

Thank You

Questions?

https://em8e.eecs.umich.edu/jsmodules/ulaby_modules.html