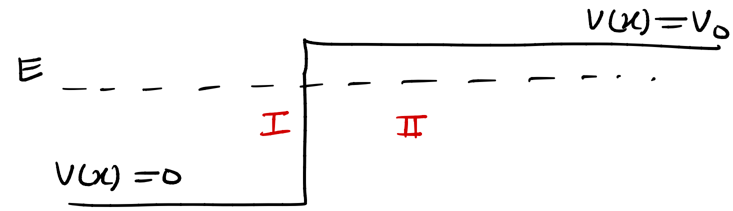
Lecture-35

7/11/2023.

Potential Step :→



$$\frac{d^2\psi_I}{dx^2} + \frac{2m}{\hbar^2} E\,\psi_I = 0 \qquad (1)$$

$$\frac{d^2\psi_{II}}{dx^2} + \frac{2m}{\hbar^2}(E-V)\,\psi_{II} = 0$$

$$\frac{d^2\psi}{dx^2} - \frac{2m}{\hbar^2}(V-E)\,\psi = 0 \qquad (2)$$

$$k_1 = \frac{\sqrt{2mE}}{\hbar}$$

$$k_2 = \frac{\sqrt{2m(V-E)}}{\hbar}$$

$$\left(\frac{d^2}{dx^2} + k_1^2\right)\psi_I = 0$$

$$\left(\frac{d^2}{dx^2} - k_2^2\right)\psi_{II} = 0$$

$$\psi_I(x) = A e^{ik_1 x} + B e^{-ik_1 x} \qquad (3)$$

$$\psi_{II}(x) = C e^{-k_2 x} \qquad (4)$$

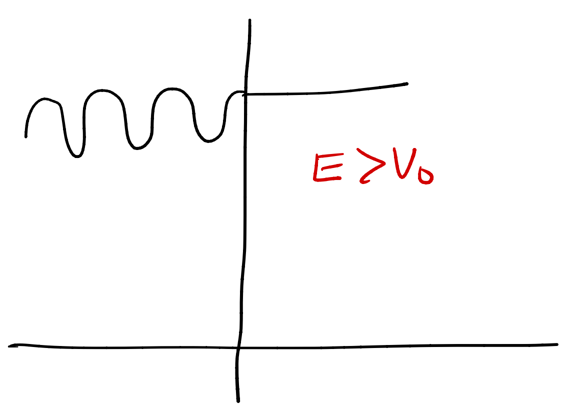
$$J_{incident} = \frac{\hbar}{m} k_1 |A|^2$$

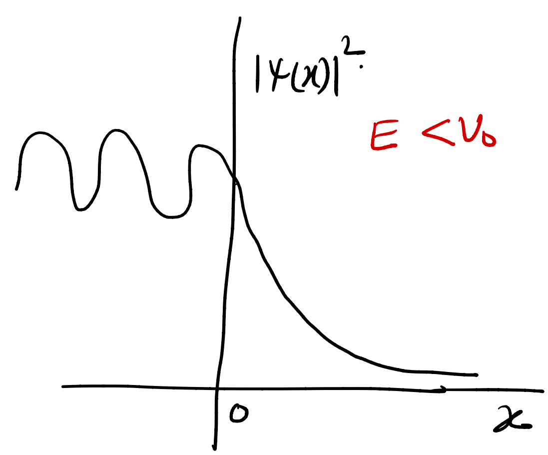
$$J_{transmitted} = \frac{\hbar}{2mi}\left(\psi^* \nabla\psi - \psi \nabla\psi^*\right). \qquad \psi_{II} = Ce^{-k_2 x}.$$

$$= \frac{\hbar}{2mi}\left[C^* e^{-k_2 x} C(-k_2) e^{-k_2 x} - C e^{-k_2 x} C^* (-k_2) e^{-k_2 x}\right]$$

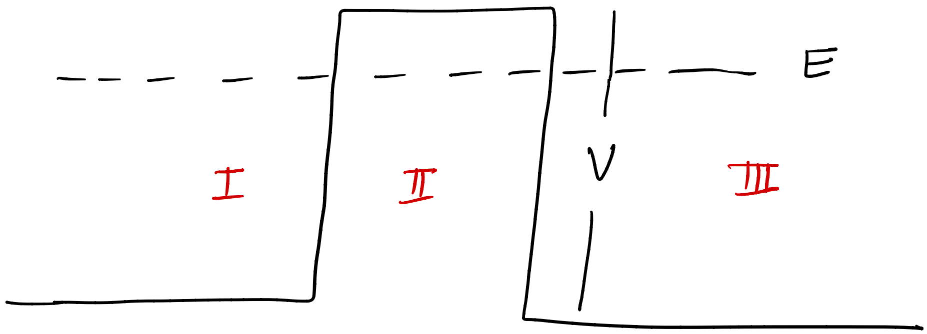
$$= 0$$

$$\therefore \left.\begin{aligned} T &= 0 \\ R &= 1 \end{aligned}\right\}$$





The Potential Barrier & Tunneling effect ⇒



$$\frac{\partial^2 \psi_I}{\partial x^2} + \frac{2m}{\hbar^2} E \psi_I = 0 \qquad \text{——— (1)}$$

$$\frac{\partial^2 \psi_{II}}{\partial x^2} + \frac{2m}{\hbar^2} (E-V)\psi_{II} = 0$$

$$\frac{\partial^2 \psi_{II}}{\partial x^2} - \frac{2m}{\hbar^2} (V-E)\psi_{II} = 0 \qquad \text{——— (2)}$$

$$\frac{\partial^2 \psi_{III}}{\partial x^2} + \frac{2m}{\hbar^2} E \psi_{III} = 0 \qquad \text{——— (3)}$$

$$k_1 = \frac{\sqrt{2mE}}{\hbar} \quad \& \quad k_2 = \frac{\sqrt{2m(V-E)}}{\hbar}$$

$$\frac{\partial^2 \psi_I}{\partial x^2} + k_1^2 \psi_I = 0 \qquad \text{——— (4)}$$

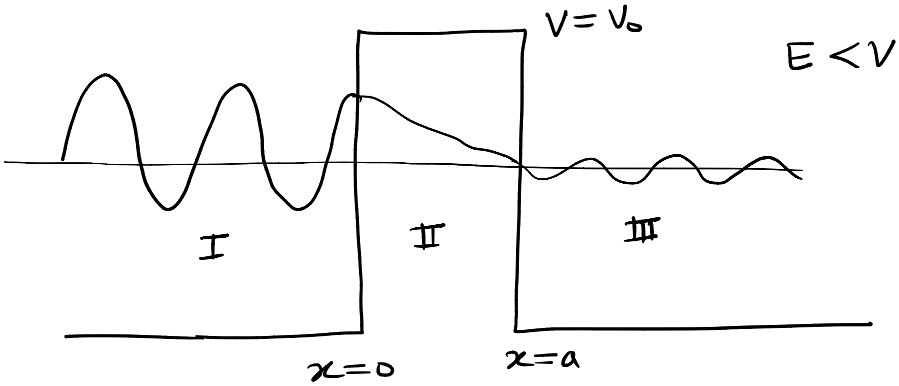
$$\frac{\partial^2 \psi_{II}}{\partial x^2} - k_2^2 \psi_{II} = 0 \qquad \text{——— (5)}$$

$$\frac{\partial^2 \psi_{III}}{\partial x^2} + k_1^2 \psi_{III} = 0 \qquad \text{——— (6)}$$

$$\psi_I = A e^{ik_1x} + B e^{-ik_1x}$$

$$\psi_{II} = C e^{k_2x} + D e^{-k_2x}$$

$$\psi_{III} = E e^{ik_1x} + \cancel{F e^{-ik_1x}}$$



$$R = \frac{|B|^2}{|A|^2} \quad \& \quad T = \frac{|E|^2}{|A|^2}.$$

$$\psi_{I} = \psi_{II} \quad \& \quad \frac{\partial \psi_{I}}{\partial x} = \frac{\partial \psi_{II}}{\partial x} \quad \Big/ \; x=0$$

$$\psi_{II} = \psi_{III} \quad \& \quad \frac{\partial \psi_{II}}{\partial x} = \frac{\partial \psi_{III}}{\partial x} \quad \Big/ \; x=a.$$

$$A + B = C + D. \quad \text{———} \quad (7)$$

$$ik_1 (A - B) = k_2 (C - D) \quad \text{———} \quad (8)$$

$$C e^{k_2 a} + D e^{-k_2 a} = E e^{ik_1 a} \quad \text{———} \quad (9)$$

$$k_2 (C e^{k_2 a} - D e^{-k_2 a}) = ik_1 E e^{ik_1 a} \quad \text{———} \quad (10)$$

On solving $\Rightarrow$

$$\boxed{T = \frac{16E}{V_0}\left(1 - \frac{E}{V_0}\right) e^{-2k_2 a}}$$

$$\boxed{T \sim e^{-2k_2 a}}$$

$$T = e^{-\frac{2a}{\hbar}\sqrt{2m(V-E)}}$$

Q :⇒ Calculate the probability of transmission of α particle through the rectangular barrier indicated by

$V_0 = 2\,eV$

$E = 1\,eV$

barrier width $= 1\,\mathring{A}$

mass of α particle $= 6.4 \times 10^{-27}$ kg.

A ⇒ $$\boxed{T = \frac{16E(V_0 - E)}{V_0^2}\, e^{-\frac{2a}{\hbar}\sqrt{2m(V_0 - E)}}}$$

$$= \frac{16 \times 1.6 \cancel{\times 10^{-19}} \times (1 \times 1.6 \cancel{\times 10^{-19}})}{(2 \times 1.6 \cancel{\times 10^{-19}})^2} \times \exp\left[-\frac{2 \times 1 \times 10^{-10}}{1.05 \times 10^{-34}} \times \sqrt{2 \times 6.4 \times 10^{-27} \times 1.6 \times 10^{-19}}\right]$$

$$= 4 \times e^{-86}$$ A