ECC 203 : Electromagnetics and Radiating Systems

Antenna Parameters 2

Gowrish Basavarajappa

Asst. Professor, ECE Dept., IIT Roorkee

gowrish.b@ece.iitr.ac.in

www.gowrish.in

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- Antenna Parameters
  - Directivity
    - Directional Patterns
    - Omnidirectional Patterns
  - Antenna Efficiency
  - Gain, Realized Gain
  - Beam Efficiency
  - Bandwidth
  - Polarization
    - Linear, Circular, and Elliptical Polarizations
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Directivity

- *directivity of an antenna is defined as the ratio of the radiation intensity in a given direction from the antenna to the radiation intensity averaged over all directions.*
- *The average radiation intensity is equal to the total power radiated by the antenna divided by 4π.*
- *If the direction is not specified, the direction of maximum radiation intensity is implied.*

$$D = \frac{U(\theta,\phi)}{U_o} = \frac{4\pi U(\theta,\phi)}{P_{rad}} \qquad \text{(2-16)}$$

$$D_{\max} = D_o = \frac{U_{\max}}{U_o} = \frac{4\pi U_{\max}}{P_{rad}} \qquad \text{(2-16a)}$$

$$\boxed{D(\text{dB}) = 10\log_{10}[D(\text{dimensionless})]}$$

Directivity

$$D = \text{directivity (dimensionless)}$$
$$D_0 = \text{maximum directivity (dimensionless)}$$
$$U = \text{radiation intensity (W/unit solid angle)}$$
$$U_{\max} = \text{maximum radiation intensity}$$
$$U_o = \text{radiation intensity of isotropic}$$
$$P_{rad} = \text{radiated power (W)}$$

- *For an isotropic source, it is very obvious from (2-16) or (2-16a) that the directivity is unity since U, U_{max}, and U_0 are all equal to each other.*

Directivity

- *the directivity is a "figure of merit" describing how well the radiator directs energy in a certain direction.*

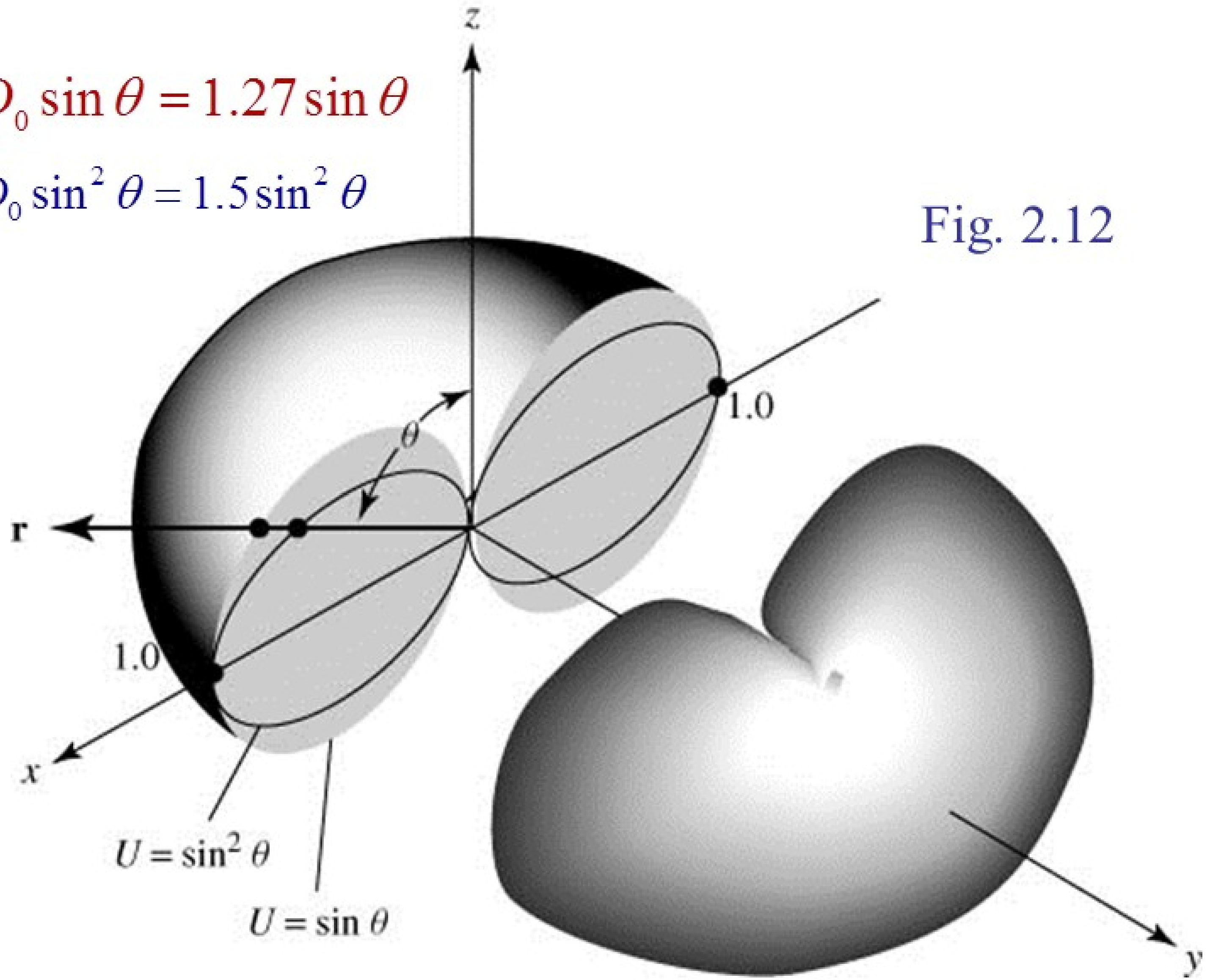
- *The directivity of an isotropic source is unity since its power is radiated equally well in all directions. For all other sources, the maximum directivity will always be greater than unity, and it is a relative "figure of merit" which gives an indication of the directional properties of the antenna as compared with those of an isotropic source.*

Three-Dimensional Radiation Patterns

$$D = D_0 \sin\theta = 1.27 \sin\theta$$

$$D = D_0 \sin^2\theta = 1.5 \sin^2\theta$$

Fig. 2.12



Two-Dimensional Directivity Pattern

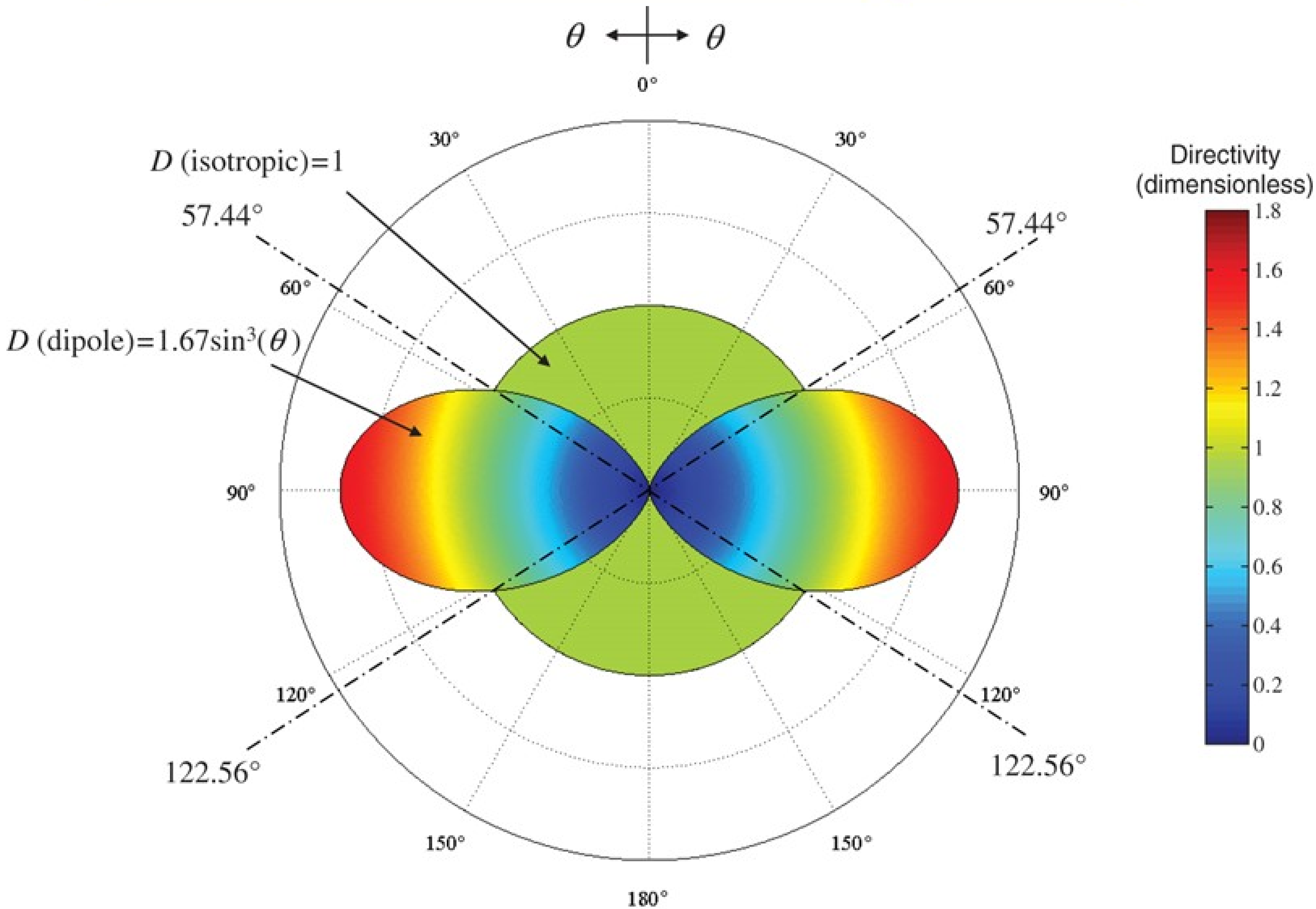


Fig. 2.13(a)

Three-dimensional Directivity Pattern

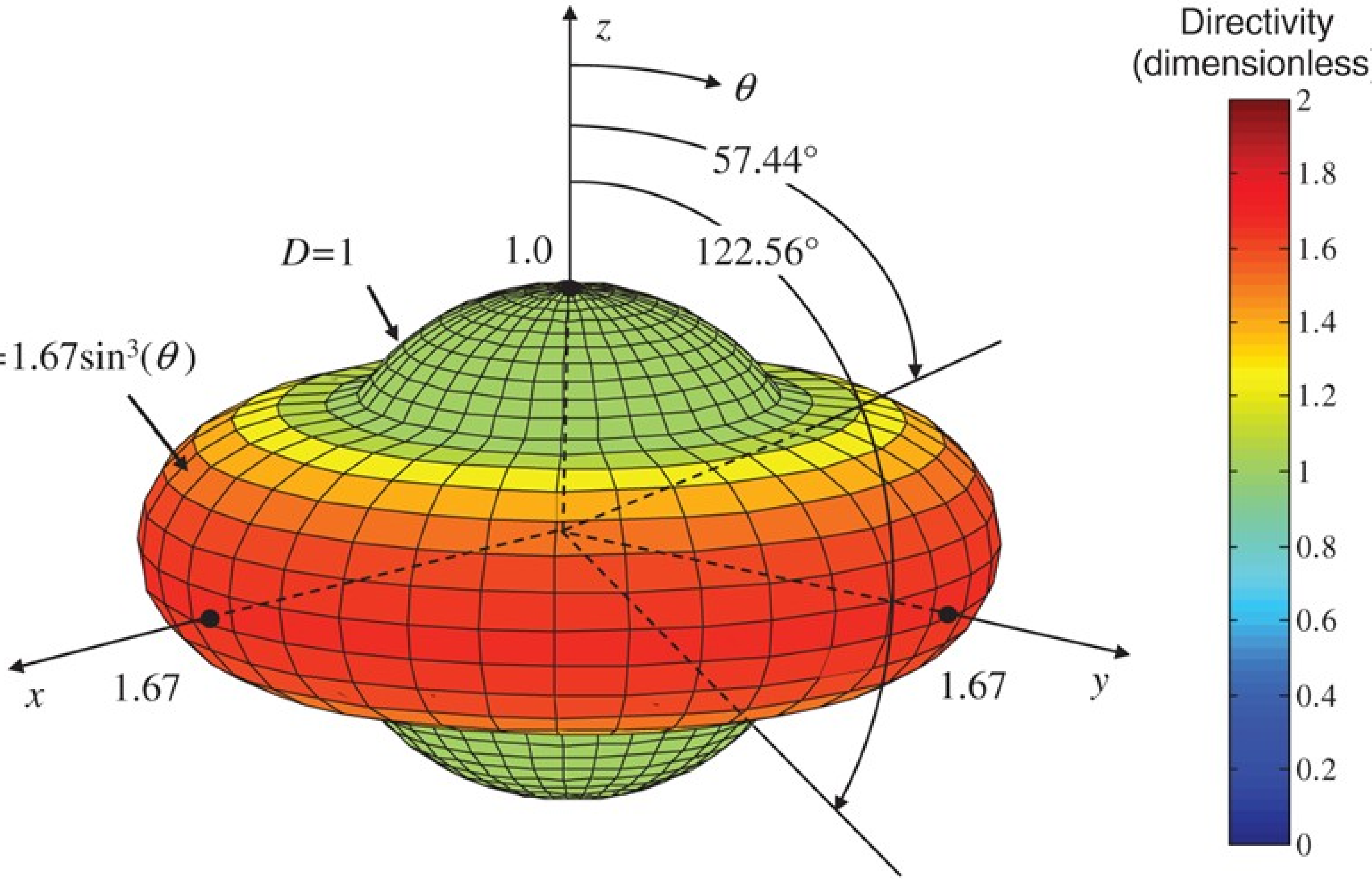


Fig. 2.13(b)

Directivity

The radial component of the radiated power density of an infinitesimal linear dipole of length $l \ll \lambda$ is given by

$$\mathbf{W}_{av} = \hat{\mathbf{a}}_r W_r = \hat{\mathbf{a}}_r A_0 \frac{\sin^2\theta}{r^2} \quad (\mathrm{W/m^2})$$

where A_0 is the peak value of the power density, θ is the usual spherical coordinate, and $\hat{\mathbf{a}}_r$ is the radial unit vector. Determine the maximum directivity of the antenna and express the directivity as a function of the directional angles θ and ϕ.

Solution: The radiation intensity is given by

$$U = r^2 W_r = A_0 \sin^2\theta$$

The maximum radiation is directed along $\theta = \pi/2$. Thus

$$U_{max} = A_0$$

Directivity

The total radiated power is given by

$$P_{\text{rad}} = \oiint_{\Omega} U\, d\Omega = A_0 \int_0^{2\pi} \int_0^{\pi} \sin^2\theta \sin\theta \, d\theta \, d\phi = A_0 \left(\frac{8\pi}{3}\right)$$

Using (2-16a), we find that the maximum directivity is equal to

$$D_0 = \frac{4\pi U_{\max}}{P_{\text{rad}}} = \frac{4\pi A_0}{\frac{8\pi}{3}(A_0)} = \frac{3}{2}$$

$$D = D_0 \sin^2\theta = 1.5 \sin^2\theta$$

Directivity

- *Instead of using the exact expression of to compute the directivity, it is often convenient to derive simpler expressions, even if they are approximate, to compute the directivity. These can also be used for design purposes.*

- *The beam solid angle Ω_A is defined as the solid angle through which all the power of the antenna would flow if its radiation intensity is constant (and equal to the maximum value of U) for all angles within Ω_A.*

Directional Patterns

$$U(\theta,\phi)=\begin{cases} B_o \cos^n(\theta) & \begin{cases} 0\le\theta\le\pi/2 \\ 0\le\phi\le 2\pi \end{cases} \\ 0 & \text{Elsewhere} \end{cases} \qquad (2\text{-}31)$$

$$n=1,2,3\ldots10,15,20$$

Non-Symmetrical Pattern

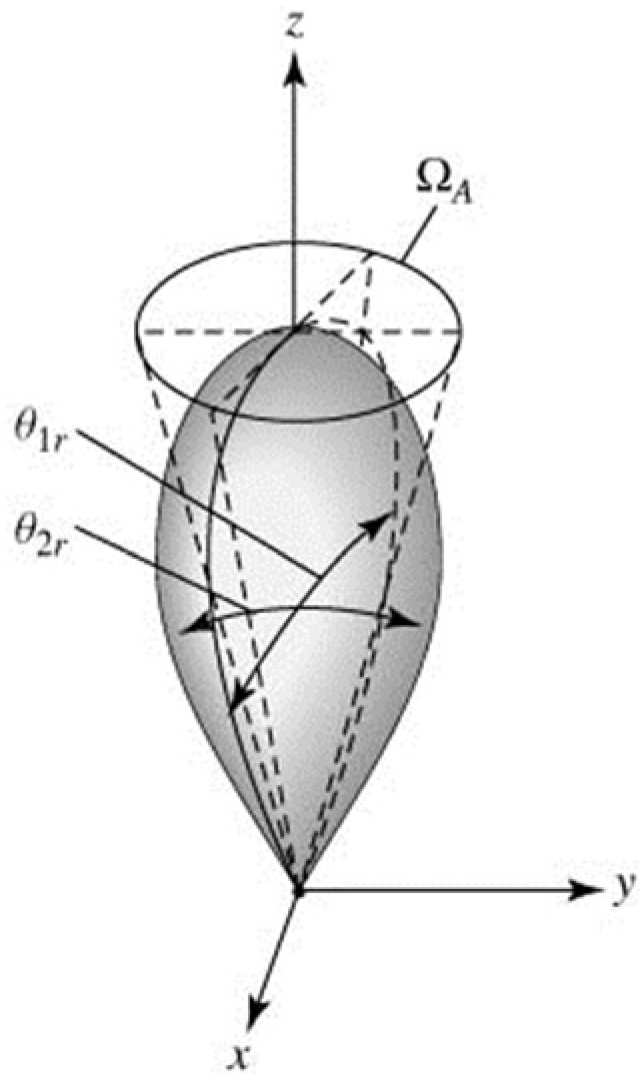


Fig. 2.14(a)

Kraus

$$\boxed{\Omega_A = \int_0^{2\pi}\int_0^{\pi} F_n(\theta,\phi)\sin\theta\, d\theta\, d\phi \simeq \Theta_{1r}\Theta_{2r}} \quad \text{(2-26a)}$$

$$\boxed{D_0 = \frac{4\pi}{\Omega_A} \simeq \frac{4\pi}{\Theta_{1r}\Theta_{2r}} = \frac{41{,}253}{\Theta_{1d}\Theta_{2d}}} \quad \text{(2-26)}$$

$$D_0 \simeq \frac{4\pi}{\Theta_{1r}\Theta_{2r}} = \frac{4\pi\left(180/\pi\right)^2}{\Theta_{1d}\Theta_{2d}} = \frac{41{,}253}{\Theta_{1d}\Theta_{2d}} \quad \text{(2-27)}$$

Θ_{1r} = half-power beamwidth in one plane (rad)

Θ_{2r} = half-power beamwidth in a plane at a right angle to the other (rad)

Tai & Pereira

$$\frac{1}{D_0} = \frac{1}{2}\left(\frac{1}{D_1} + \frac{1}{D_2}\right)$$ Arithmetic mean (2-29)

$$\boxed{\begin{aligned} D_0 &= \frac{4\pi}{\Omega_A} \simeq \frac{32\ln(2)}{\Theta_{1r}^2 + \Theta_{2r}^2} = \frac{22.181}{\Theta_{1r}^2 + \Theta_{2r}^2} \quad &(2\text{-}30a) \\ D_0 &= \frac{72,815}{\Theta_{1d}^2 + \Theta_{2d}^2} \quad &(2\text{-}30b) \end{aligned}}$$

$$D_0 = \frac{4\pi}{\Omega_A} = \frac{U_{\max}}{U_0} = 4\pi \frac{U_{\max}}{P_{rad}}$$

---

1. $\Omega_A = \int_0^{2\pi}\int_0^{\pi} F_n(\theta,\phi)\sin(\theta)\,d\theta d\phi$ (<u>Exact</u>)

2. $\Omega_A \simeq \Theta_{1r}\Theta_{2r} \Rightarrow D_o \simeq D_o \simeq \dfrac{4\pi}{\Theta_{1r}\Theta_{2r}} = \dfrac{41,253}{\Theta_{1d}\Theta_{2d}}$ (Kraus)

3. $\Omega_A \simeq \dfrac{\Theta_{1r}^2 + \Theta_{2r}^2}{22.181}(4\pi) \Rightarrow D_o \simeq \dfrac{22.181}{\Theta_{1r}^2 + \Theta_{2r}^2}$ (T&P)

$$D_o \simeq \frac{72,815}{\Theta_{1d}^2 + \Theta_{2d}^2} \quad \text{(T\&P)}$$

Exact & Approximate Directivities

Table 2.1

n	Exact Equation (2-22)	Kraus Equation (2-26)	Kraus % Error	Tai and Pereira Equation (2-30a)	Tai and Pereira % Error
1	4	2.86	−28.50	2.53	−36.75
2	6	5.09	−15.27	4.49	−25.17
3	8	7.35	−8.12	6.48	−19.00
4	10	9.61	−3.90	8.48	−15.20
5	12	11.87	$n \approx 5.5$ −1.08	10.47	−12.75
6	14	14.13	+0.93	12.46	−11.00
7	16	16.39	+2.48	14.47	−9.56
8	18	18.66	+3.68	16.47	−8.50
9	20	20.93	+4.64	18.47	−7.65
10	22	23.19	+5.41	20.47	−6.96
11.28	24.56	26.08	+6.24	23.02	−6.24
15	32	34.52	+7.88	30.46	−4.81
20	42	45.89	+9.26	40.46	−3.67

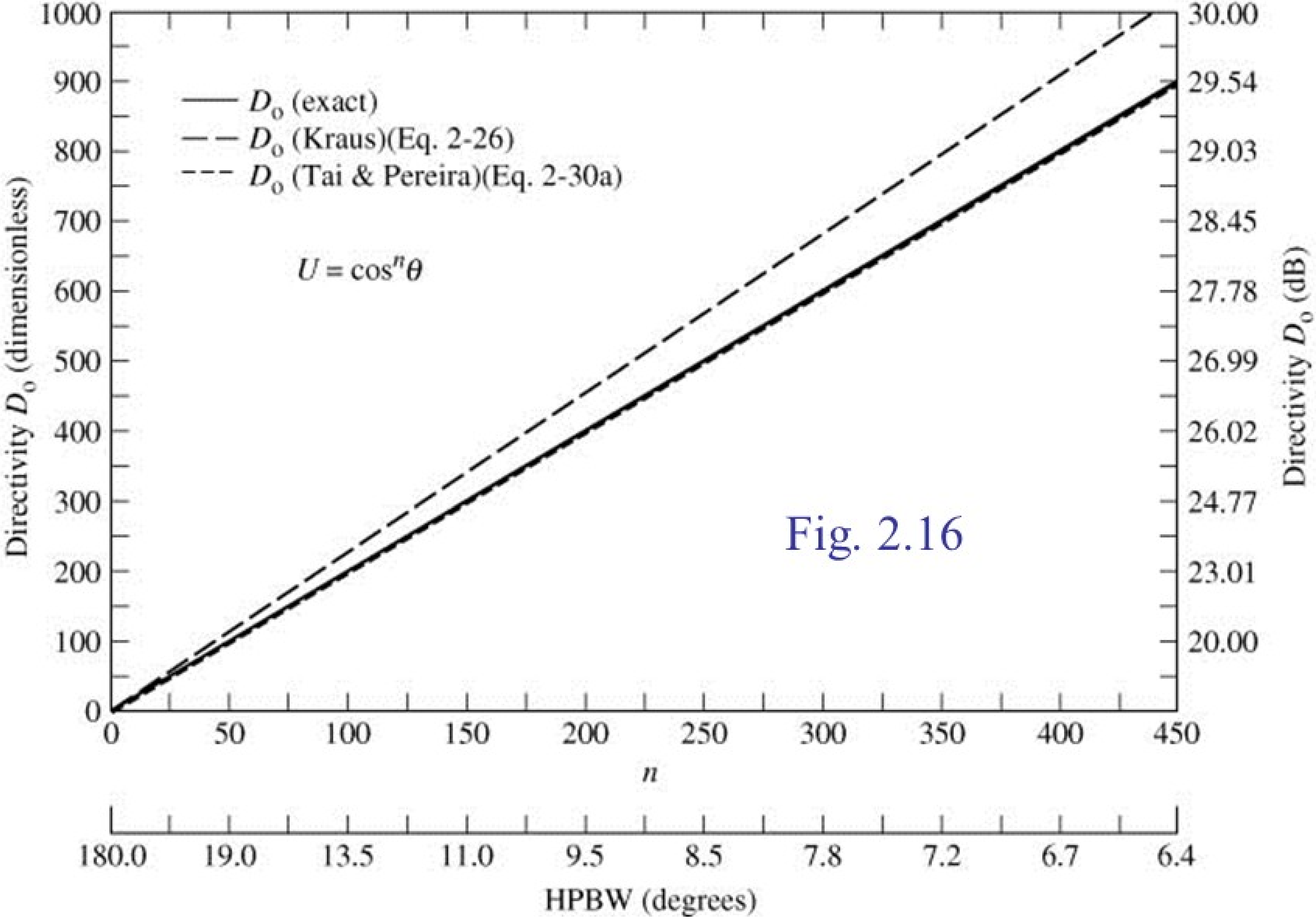

1000
900
800
700
600
500
400
300
200
100
0
D_0 (exact)
D_0 (Kraus)(Eq. 2-26)
D_0 (Tai & Pereira)(Eq. 2-30a)
$U = \cos^n\theta$
Directivity D_0 (dimensionless)
30.00
29.54
29.03
28.45
27.78
26.99
26.02
24.77
23.01
20.00
Directivity D_0 (dB)
0
50
100
150
200
250
300
350
400
450
n
180.0
19.0
13.5
11.0
9.5
8.5
7.8
7.2
6.7
6.4
HPBW (degrees)

Fig. 2.16

Example 2.7:

$$U = \begin{cases} B_o \cos^4\theta & 0 \le \theta \le \pi/2 \\ & 0 \le \phi \le 2\pi \\ 0 & \pi/2 \le \theta \le \pi \\ & 0 \le \phi \le 2\pi \end{cases}$$

Find Directivity : Exact and Approximate

Solution:

$$P_{rad} = \int_0^{2\pi}\int_0^{\pi/2} U \sin\theta d\theta d\phi = B_o \int_0^{2\pi}\int_0^{\pi/2} \cos^4\theta \sin\theta d\theta d\phi$$

$$P_{rad} = 2\pi B_o \int_0^{\pi/2} \cos^4\theta \sin\theta d\theta = \frac{2\pi}{5} B_o$$

$$D_0(\text{exact}) = \frac{4\pi U_{\max}}{P_{rad}} = \frac{4\pi(5)B_0}{2\pi B_0}$$

$$D_0(\text{exact}) = 10 \text{ (dimensionless)} = 10 \text{ dB}$$

Approximate:

To find the HPBW, you set

$$\cos^4\theta_h = 0.5 \Rightarrow \theta_h = \cos^{-1}(0.5)^{1/4}$$

$$\theta_h = 0.57185 \text{ radians} = 32.765^{\circ}$$

Because of the symmetry of the pattern

$$\Theta_{1r} = \Theta_{2r} = 2(0.57185) = 1.1437 \text{ radians} = 65.53^{\circ}$$

Using the previous results, the following approximate directivities are obtained:

$$D_0(\text{Kraus}) \simeq \frac{4\pi}{(1.1437)^2} = 9.61$$

$$D_0(\text{Kraus}) \simeq 9.61 \text{ (dimensionless)} = 9.83 \text{ dB}$$

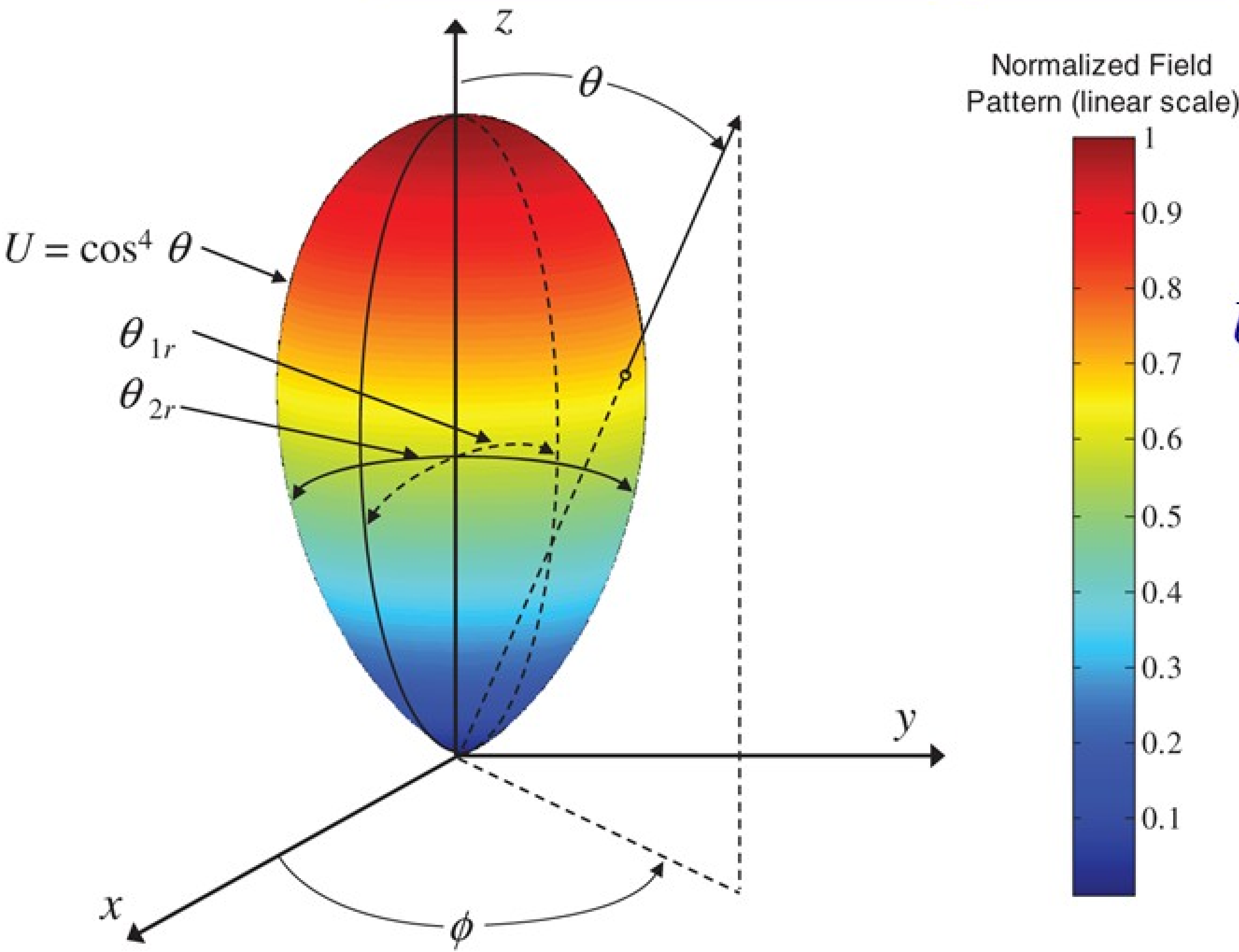
$$(-3.9\% \text{ Error})$$

$$D_0(\text{T\&P}) \simeq \frac{22.181}{2(1.1437)^2} = 8.4787$$

$$D_0(\text{T\&P}) \simeq 8.4787 \text{ (dimensionless)} = 9.283 \text{ dB}$$

$$(-15.21\% \text{ Error})$$

Radiation Intensity Pattern



$$U(\theta,\phi) = \cos^4 \theta$$

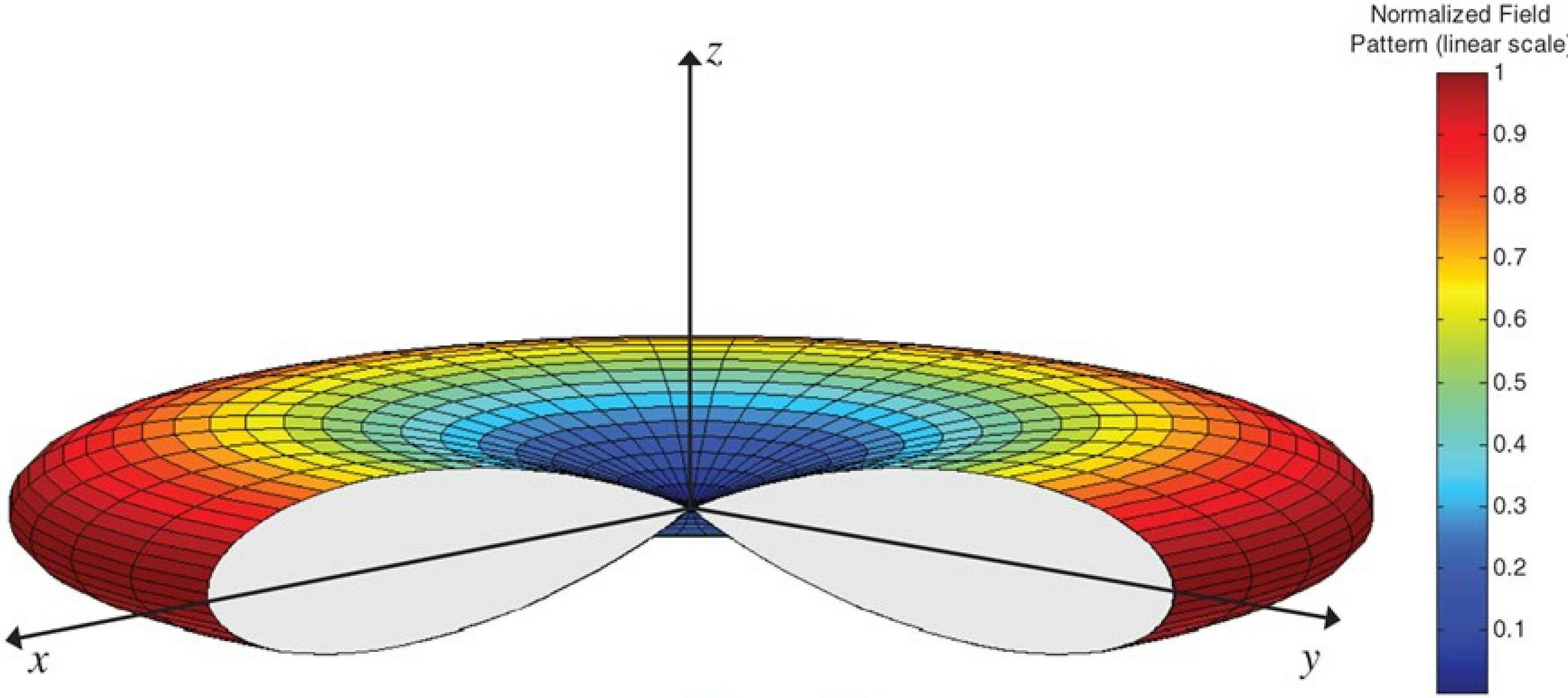
$$0 \le \theta \le 90^o$$

$$0 \le \phi \le 360^o$$

Fig. 2.15

Omindirectional
Patterns

Omnidirectional Pattern Without Minor Lobes



$$U \approx \left|\sin^n(\theta)\right| \left\} \begin{array}{l} 0 \le \theta \le \pi \\ 0 \le \phi \le 2\pi \end{array} \right.$$

Fig. 2.17(b)

Omnidirectional Patterns

$$U = \left|\sin^n(\theta)\right| \left\} \begin{matrix} 0 \le \theta \le \pi \\ 0 \le \phi \le 2\pi \end{matrix} \right. \qquad (2\text{-}32)$$

Directivity:

McDonald

$$D_0 = \frac{101}{\text{HPBW(degrees)} - 0.0027[\text{HPBW(degrees)}]^2} \qquad (2\text{-}33a)$$

Pozar

$$D_o = -172.4 + 191\sqrt{0.818 + \frac{1}{\text{HPBW(degrees)}}} \qquad (2\text{-}33b)$$

Exact And Approximate Directivities

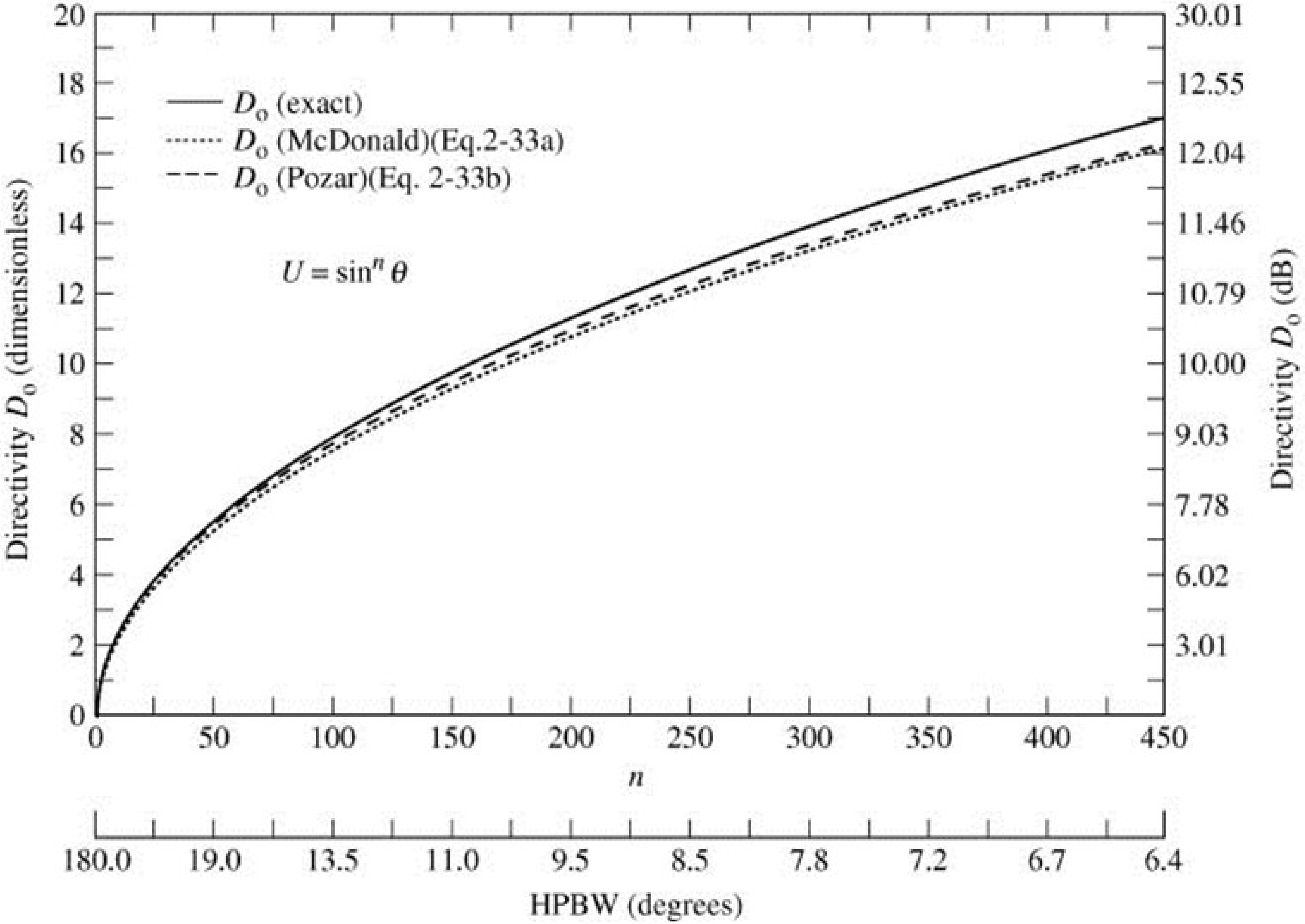


Fig. 2.18

Directivity

Design an antenna with omnidirectional amplitude pattern with a half-power beamwidth of 90°. Express its radiation intensity by $U = \sin^n \theta$. Determine the value of n and attempt to identify elements that exhibit such a pattern. Determine the directivity of the antenna using (2-16a), (2-33a), and (2-33b).

Solution: Since the half-power beamwidth is 90°, the angle at which the half-power point occurs is $\theta = 45°$. Thus

$$U(\theta = 45°) = 0.5 = \sin^n(45°) = (0.707)^n$$

or

$$n = 2$$

Therefore, the radiation intensity of the omnidirectional antenna is represented by $U = \sin^2 \theta$.

Directivity

Using the definition of (2-16a), the exact directivity is

$$U_{\text{max}} = 1$$

$$P_{\text{rad}} = \int_0^{2\pi} \int_0^{\pi} \sin^2\theta \, \sin\theta \, d\theta \, d\phi = \frac{8\pi}{3}$$

$$D_0 = \frac{4\pi}{8\pi/3} = \frac{3}{2} = 1.761 \text{ dB}$$

Since the half-power beamwidth is equal to 90°, then the directivity based on (2-33a) is equal to

$$D_0 = \frac{101}{90 - 0.0027(90)^2} = 1.4825 = 1.71 \text{ dB}$$

while that based on (2-33b) is equal to

$$D_0 = -172.4 + 191\sqrt{0.818 + 1/90} = 1.516 = 1.807 \text{ dB}$$

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Antenna Efficiency

- *The total antenna efficiency e_0 is used to take into account losses at the input terminals and within the structure of the antenna.*

- *Such losses may be due,*

1. *reflections because of the mismatch between the transmission line and the antenna*
2. *I^2R losses (conduction and dielectric)*

Antenna Reference Terminals

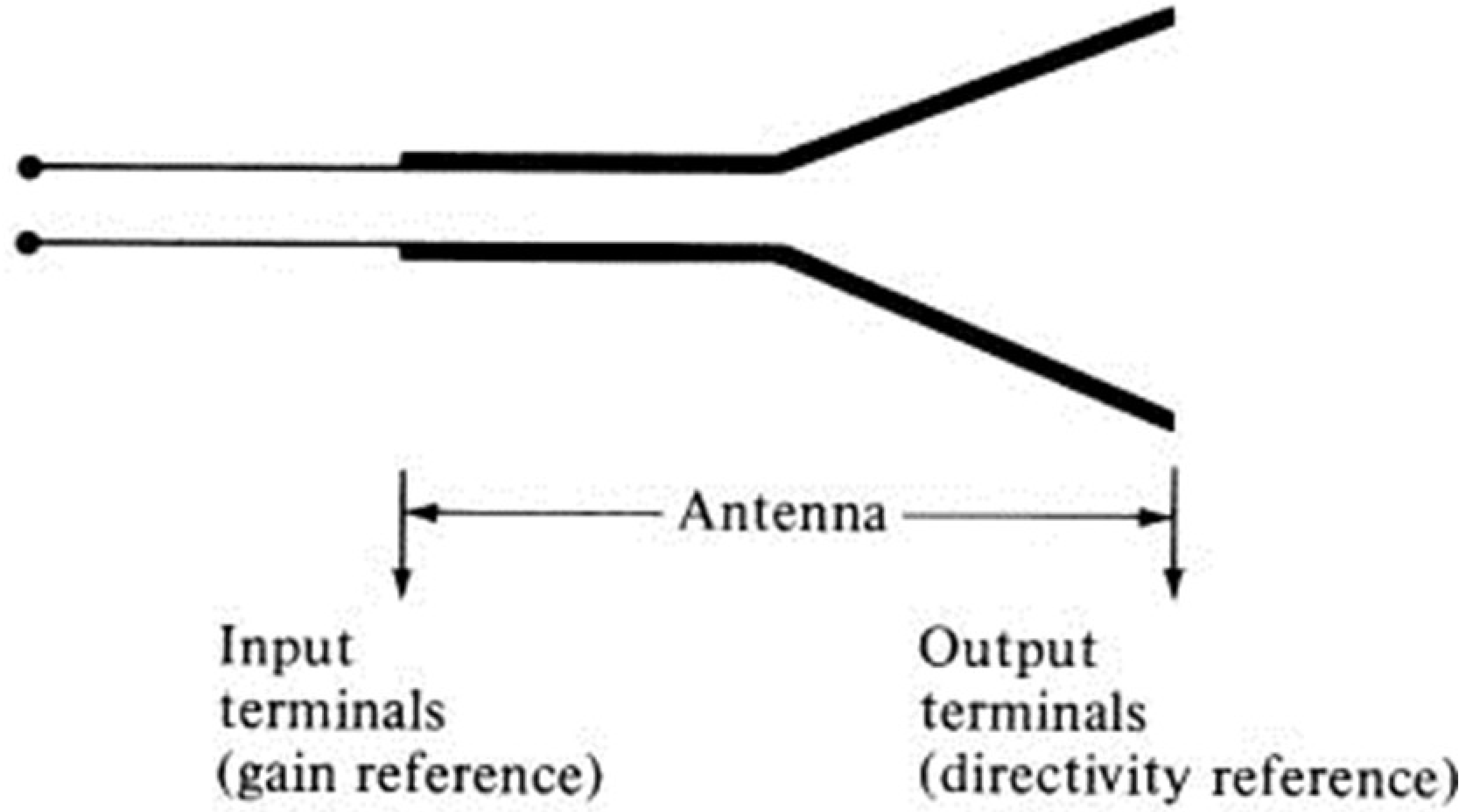
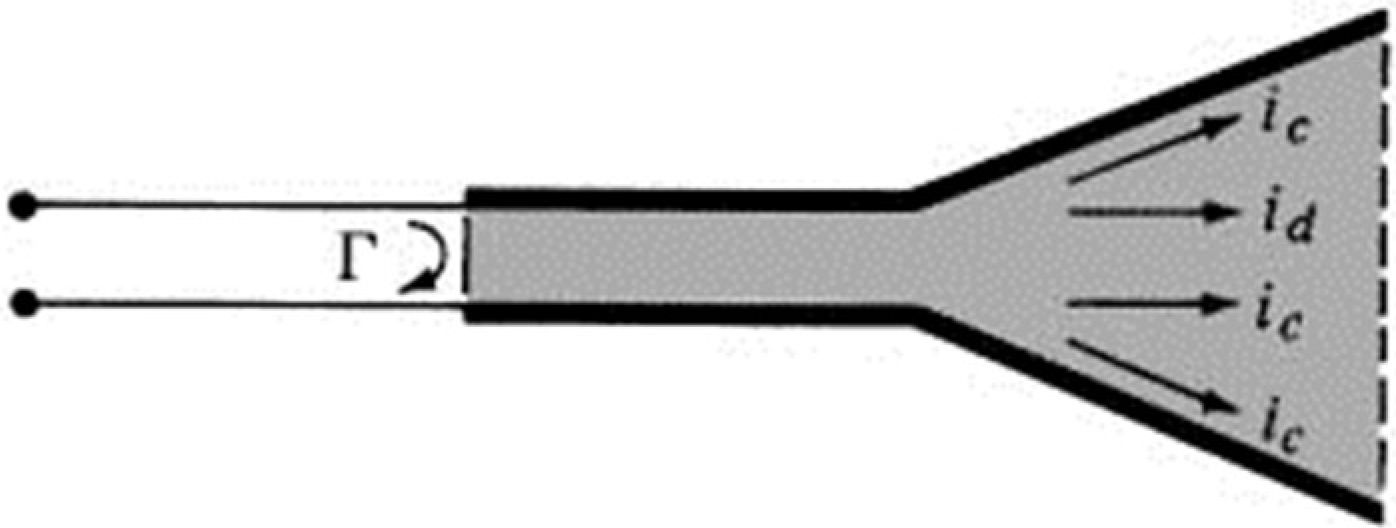


Fig. 2.22(a)

Reflection, Conduction, And Dielectric Losses



(b) Reflection, conduction, and dielectric losses

Fig. 2.22(b)

Antenna Efficiency e_o

$$e_o = e_r \boxed{e_c e_d} = e_r \boxed{e_{cd}} \quad (2\text{-}44)$$

$$e_o = (1 - |\Gamma_{in}|^2) e_{cd} \quad (2\text{-}45)$$

$e_o =$ Total efficiency

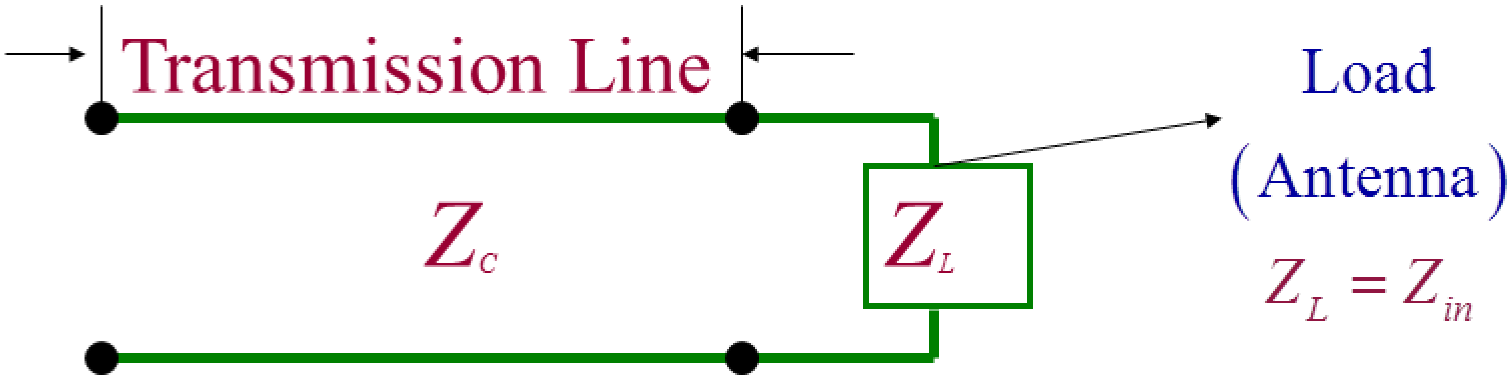
$e_r =$ Reflection efficiency

$e_{cd} =$ Radiation efficiency

Transmission Line and Load

Z_c = Characteristic Impedance of Line

Z_L = Load Impedance



$$\Gamma_{in} = \frac{Z_L - Z_c}{Z_L + Z_c} = \frac{Z_{in} - Z_c}{Z_{in} + Z_c}$$

$$e_r = (1 - |\Gamma_{in}|^2)$$

$$\Gamma_{in} = \frac{Z_{in} - Z_c}{Z_{in} + Z_c}$$

$$\mathrm{VSWR} = \frac{1 + |\Gamma_{in}|}{1 - |\Gamma_{in}|}$$

$$|\Gamma_{in}| = \frac{\mathrm{VSWR} - 1}{\mathrm{VSWR} + 1}$$

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Gain, Realized Gain

- *Gain of an antenna (in a given direction) is defined as "the ratio of the intensity, in a given direction, to the radiation intensity that would be obtained if the power accepted by the antenna were radiated isotropically. The radiation intensity corresponding to the isotropically radiated power is equal to the power accepted (input) by the antenna divided by 4π".*

$$\text{Gain} = G = 4\pi \frac{\text{Radiation intensity}}{\text{Total input (accepted) power}}$$

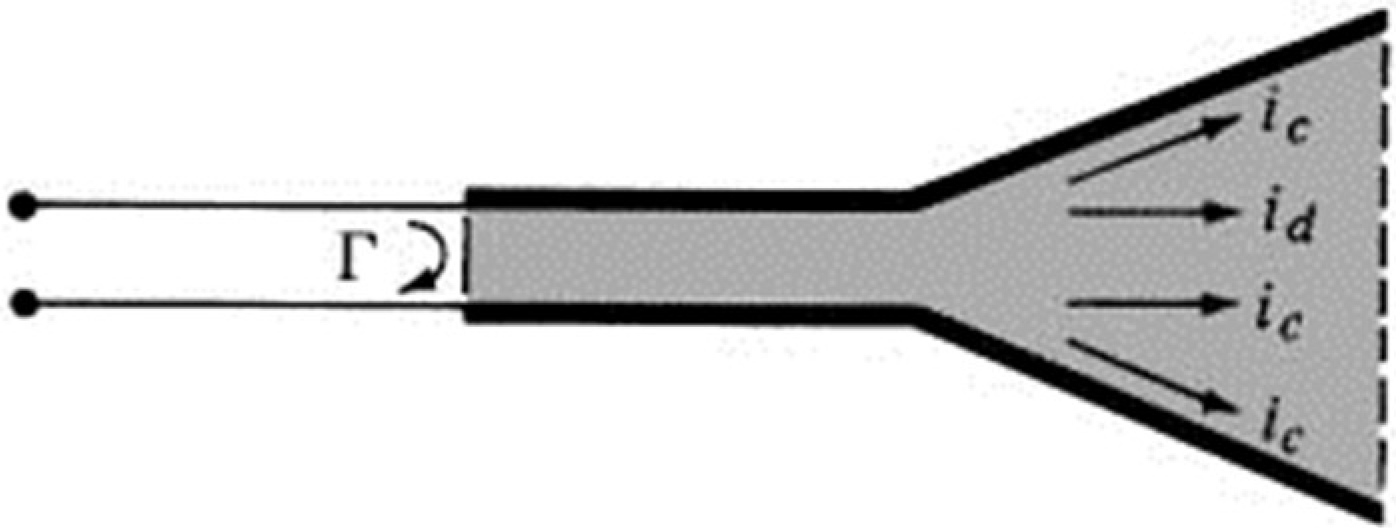
$$G = 4\pi \frac{U(\theta,\phi)}{P_{in}} \tag{2-46a}$$

$$P_{rad} = e_{cd} P_{in} \Rightarrow P_{in} = \frac{P_{rad}}{e_{cd}} \tag{2-47}$$

$$G = 4\pi \frac{U(\theta,\phi)}{P_{rad}/e_{cd}} = e_{cd} \underbrace{\left[4\pi \frac{U(\theta,\phi)}{P_{rad}}\right]}_{D} \tag{2-48}$$

$$G = e_{cd} D \tag{2-49}$$

Reflection, Conduction, And Dielectric Losses



(b) Reflection, conduction, and dielectric losses

Fig. 2.22(b)

$$\boxed{G_o = e_{cd} D_o} \tag{2-49a}$$

$$e_{cd} = e_c e_d = \text{Radiation efficiency}$$

$$e_c = \text{Conduction efficiency}$$

$$e_d = \text{Dielectric efficiency}$$

$$\boxed{\begin{aligned} G &= e_{cd} D \\ G_o &= e_{cd} D_o \end{aligned}}$$

$$G_o(dB) = 10\log_{10}\left[e_{cd} D_o\right]$$

$$G_o(dB) = 10\log_{10}(e_{cd}) + 10\log_{10}(D_o)$$

Gain, Realized Gain

- *According to the IEEE Standards, "gain does not include losses arising from impedance mismatches (reflection losses) and polarization mismatches (losses)."*

- *realized gain (G_{re}), that also takes into account the reflection/mismatch losses*

Realized Gain G_{re}

$$G_{re}(\theta,\phi) = e_o D(\theta,\phi) = e_r e_{cd} D(\theta,\phi)$$

$$G_{re}(\theta,\phi) = (1-|\Gamma_{in}|^2) e_{cd} D(\theta,\phi) \qquad \text{(2-49b)}$$

$$e_o = \text{antenna total efficiency}$$

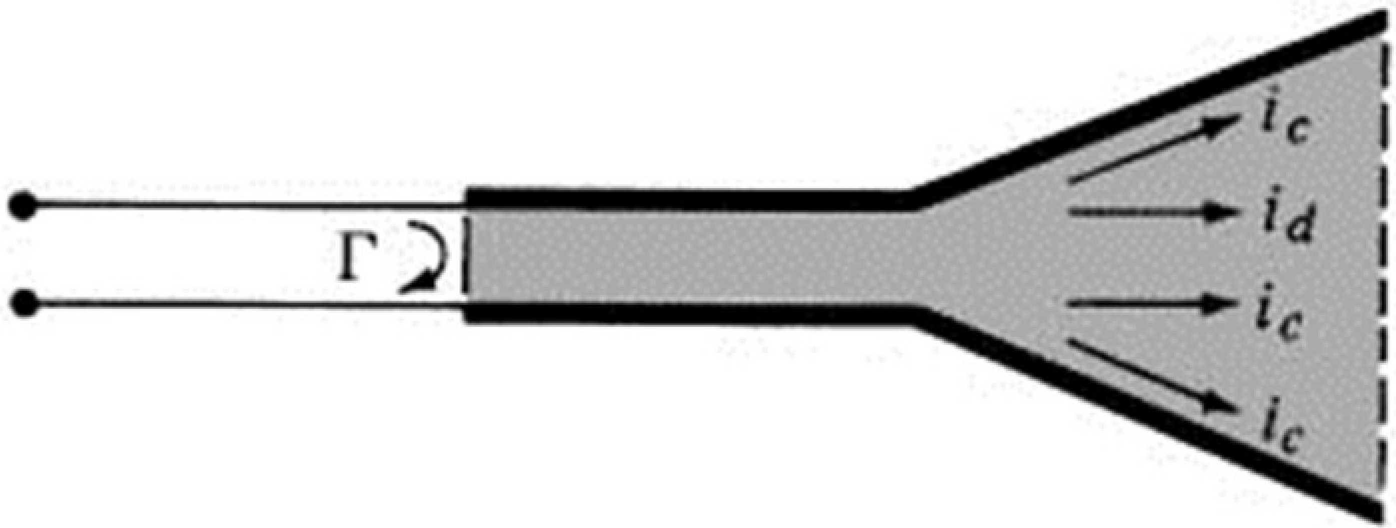
$$e_r = (1-|\Gamma_{in}|^2) = \text{Reflection efficiency}$$

$$e_{cd} = e_c e_d = \text{Radiation efficiency}$$

$$e_r = \text{Conduction efficiency}$$

$$e_d = \text{Dielectric efficiency}$$

Reflection, Conduction, And Dielectric Losses



(b) Reflection, conduction, and dielectric losses

Fig. 2.22(b)

Gain, Realized Gain

A lossless resonant half-wavelength dipole antenna, with input impedance of 73 ohms, is connected to a transmission line whose characteristic impedance is 50 ohms. Assuming that the pattern of the antenna is given approximately by

$$U = B_0 \sin^3 \theta$$

find the maximum realized gain of this antenna.

Solution: Let us first compute the maximum directivity of the antenna. For this

$$U|_{\max} = U_{\max} = B_0$$

$$P_{\text{rad}} = \int_0^{2\pi} \int_0^{\pi} U(\theta, \phi) \sin\theta \, d\theta \, d\phi = 2\pi B_0 \int_0^{\pi} \sin^4 \theta \, d\theta = B_0 \left(\frac{3\pi^2}{4} \right)$$

$$D_0 = 4\pi \frac{U_{\max}}{P_{\text{rad}}} = \frac{16}{3\pi} = 1.697$$

Since the antenna was stated to be lossless, then the radiation efficiency $e_{cd} = 1$.

Gain, Realized Gain

Thus, the total maximum gain is equal to

$$G_0 = e_{cd}D_0 = 1(1.697) = 1.697$$

$$G_0(\text{dB}) = 10\log_{10}(1.697) = 2.297$$

which is identical to the directivity because the antenna is lossless.

There is another loss factor which is not taken into account in the gain. That is the loss due to reflection or mismatch losses between the antenna (load) and the transmission line. This loss is accounted for by the reflection efficiency of (2-44) or (2-45), and it is equal to

$$e_r = (1 - |\Gamma|^2) = \left(1 - \left|\frac{73 - 50}{73 + 50}\right|^2\right) = 0.965$$

$$e_r(\text{dB}) = 10\log_{10}(0.965) = -0.155$$

Therefore the overall efficiency is

$$e_0 = e_r e_{cd} = 0.965$$

$$e_0(\text{dB}) = -0.155$$

Thus, the overall losses are equal to 0.155 dB. The maximum realized gain is equal to

$$G_{re0} = e_0 D_0 = 0.965(1.697) = 1.6376$$

$$G_{re0}(\text{dB}) = 10\log_{10}(1.6376) = 2.142$$

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Bandwidth

- *The bandwidth of an antenna is defined as "the range of frequencies within which the performance of the antenna, with respect to some characteristic, conforms to a specified standard."*
- *The bandwidth can be considered to be the range of frequencies, on either side of a center frequency, where the antenna characteristics (such as input impedance, pattern, beamwidth, polarization, side lobe level, gain, beam direction, radiation efficiency) are within an acceptable value of those at the center frequency.*
- *For broadband antennas, the bandwidth is usually expressed as the ratio of the upper-to-lower frequencies of acceptable operation. For example, a 10:1 bandwidth indicates that the upper frequency is 10 times greater than the lower.*
- *For narrowband antennas, the bandwidth is expressed as a percentage of the frequency difference (upper minus lower) over the center frequency of the bandwidth. For example, a 5% bandwidth indicates that the frequency range of acceptable operation is 5% of the bandwidth center frequency.*

I. Pattern Bandwidth

A. Directivity/Gain

B. Side lobe level

C. Beamwidth

D. Polarization

E. Beam direction

II. Impedance Bandwidth

A. Input impedance

B. Radiation efficiency

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Polarization

- *The electric (or magnetic) field intensity of a uniform plane wave has a direction in space. This direction may either be constant or may change as the wave propagates.*
- *The polarization of a plane wave is "the figure traced by the tip of the electric field vector as a function of time, at a fixed point in space."*

Linearly Polarized Wave

$$\mathbf{E} = \mathbf{a}_x E_x$$

linearly polarized in the x direction.

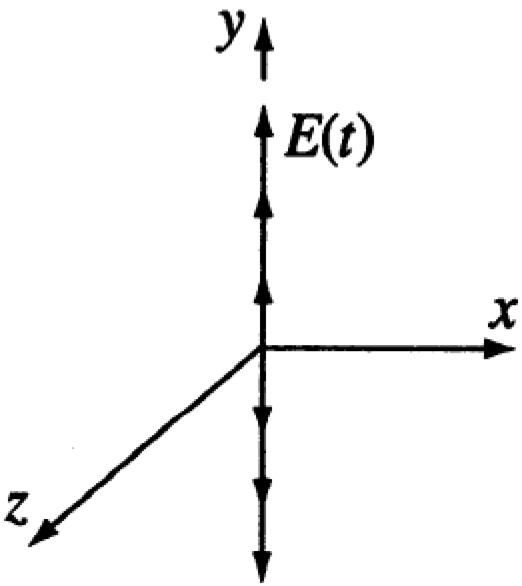
$$\mathbf{E} = \hat{\mathbf{y}} E_y(z) = \hat{\mathbf{y}} E_y e^{-\gamma z}$$

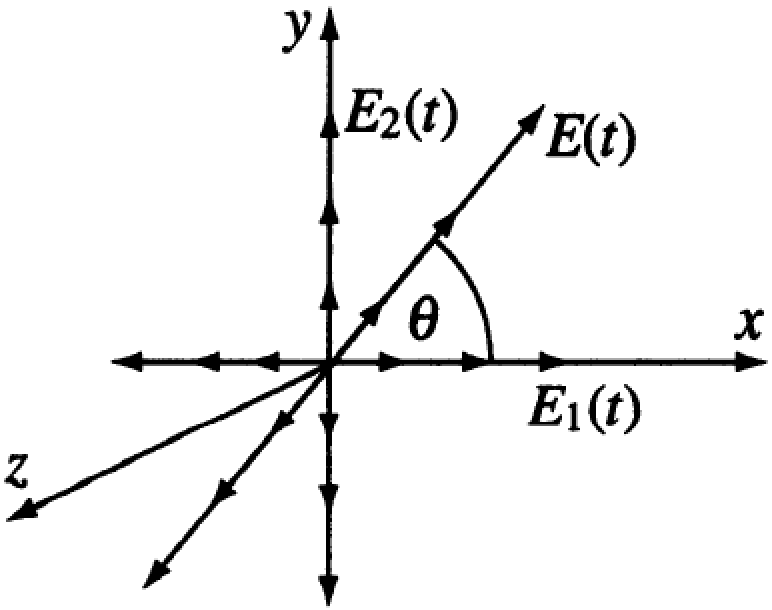
linearly polarized in the y direction.

$$\mathbf{E}(z,t) = \hat{\mathbf{x}} E_x e^{-\alpha z} \cos(\omega t - \beta z) + \hat{\mathbf{y}} E_y e^{-\alpha z} \cos(\omega t - \beta z)$$

linearly polarized in the θ = arctan(|Ey|/|Ex|) direction.

Linearly Polarized Wave:

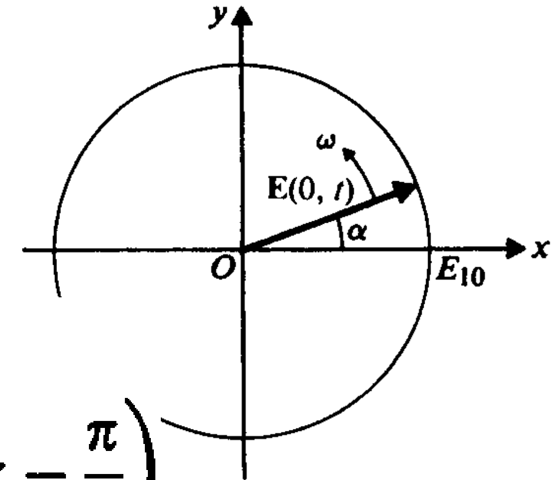




Polarization

Elliptically and Circularly Polarized

$$\mathbf{E}(z) = \mathbf{a}_x E_1(z) + \mathbf{a}_y E_2(z)$$
$$= \mathbf{a}_x E_{10} e^{-jkz} - \mathbf{a}_y j E_{20} e^{-jkz}$$



$$\mathbf{E}(z, t) = \mathscr{Re}\{[\mathbf{a}_x E_1(z) + \mathbf{a}_y E_2(z)]e^{j\omega t}\}$$
$$= \mathbf{a}_x E_{10} \cos(\omega t - kz) + \mathbf{a}_y E_{20} \cos\left(\omega t - kz - \frac{\pi}{2}\right)$$

$$\mathbf{E}(0, t) = \mathbf{a}_x E_1(0, t) + \mathbf{a}_y E_2(0, t)$$
$$= \mathbf{a}_x E_{10} \cos \omega t + \mathbf{a}_y E_{20} \sin \omega t$$

Hence **E**, which is the sum of two linearly polarized waves in both space and time quadrature, is ***elliptically polarized*** if $E_{20} \neq E_{10}$, and is ***circularly polarized*** if $E_{20} = E_{10}$. This is a ***right-hand*** or ***positive circularly polarized wave.***

Elliptically and Circularly Polarized Wave:

$$\mathbf{E}(z) = \mathbf{a}_x E_{10} e^{-jkz} + \mathbf{a}_y j E_{20} e^{-jkz}$$

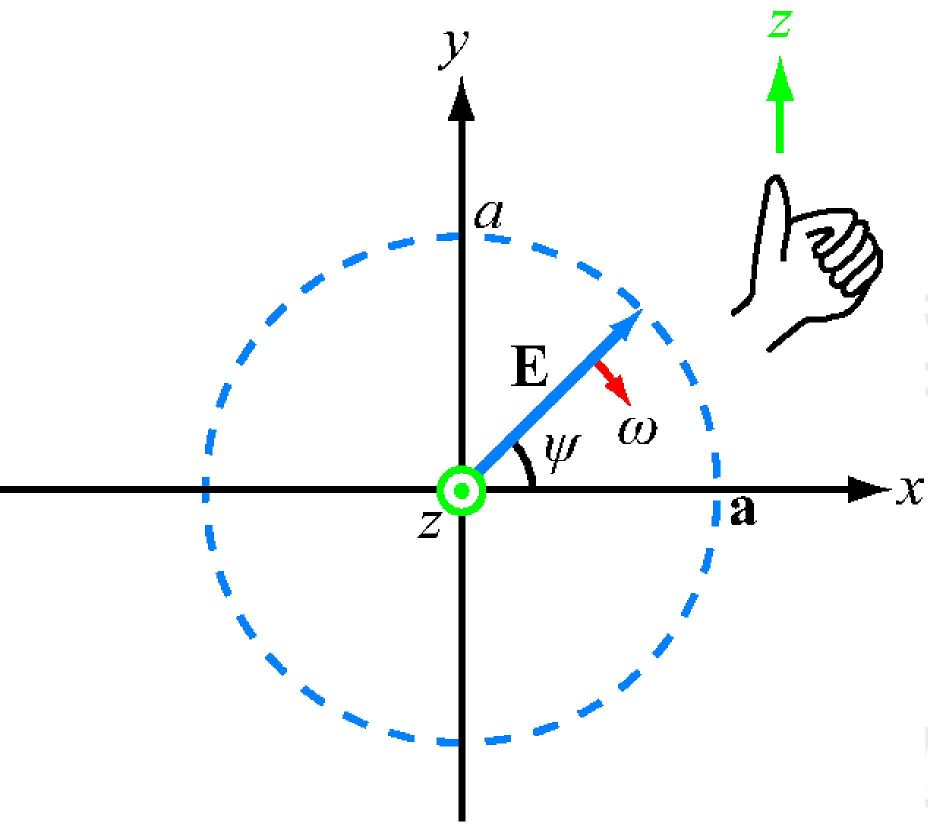
$$\mathbf{E}(z, t) = \mathscr{R}e\{[\mathbf{a}_x E_1(z) + \mathbf{a}_y E_2(z)]e^{j\omega t}\}$$

$$\mathbf{E}(0, t) = \mathbf{a}_x E_{10} \cos \omega t - \mathbf{a}_y E_{20} \sin \omega t$$

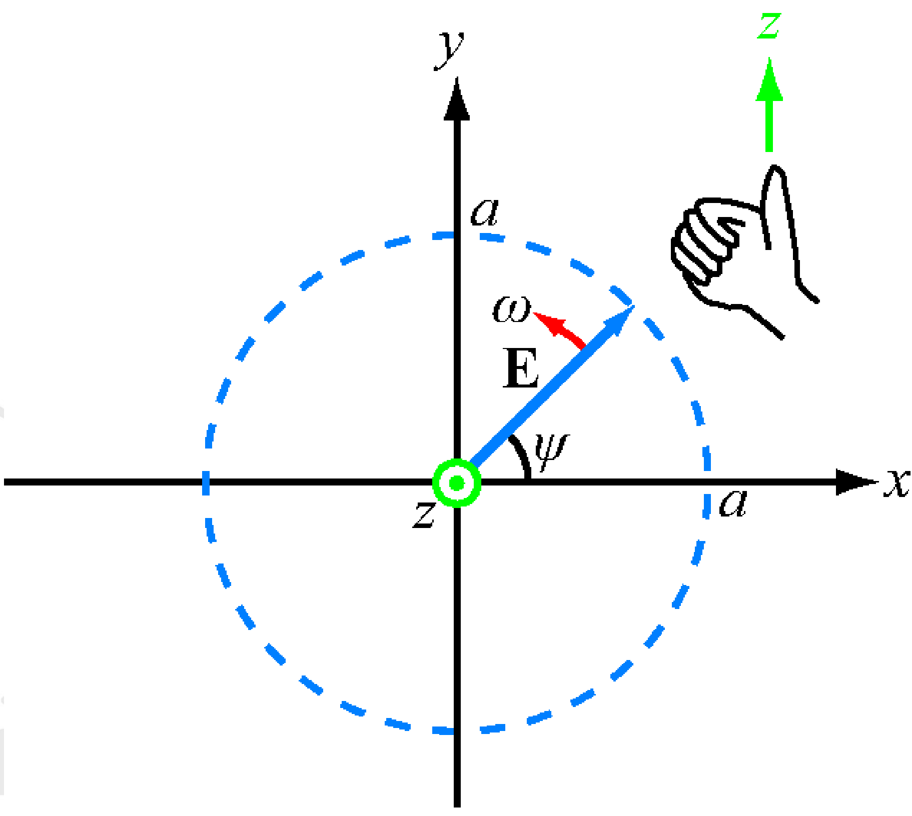
If $E_{20} = E_{10}$, **E** will be circularly polarized, and its angle measured from the x-axis at $z = 0$ will now be $-\omega t$, indicating that **E** will rotate with an angular velocity ω in a *clockwise* direction; this is a ***left-hand*** or ***negative circularly polarized wave***.

Polarization

Elliptically and Circularly Polarized



(a) LHC polarization



(b) RHC polarization

Polarization

- *In general, the polarization of the receiving antenna will not be the same as the polarization of the incoming (incident) wave. This is commonly stated as “polarization mismatch.”*
- *The amount of power extracted by the antenna from the incoming signal will not be maximum because of the polarization loss.*
- *the polarization loss can be taken into account by introducing a polarization loss factor (PLF).*

Polarization Loss Factor (PLF):

$$\mathrm{PLF} = |\hat{\rho}_w \cdot \hat{\rho}_a|^2 = |\cos\psi_p|^2 \qquad (2\text{-}71)$$

Antenna: Polarization determined in it transmitting mode

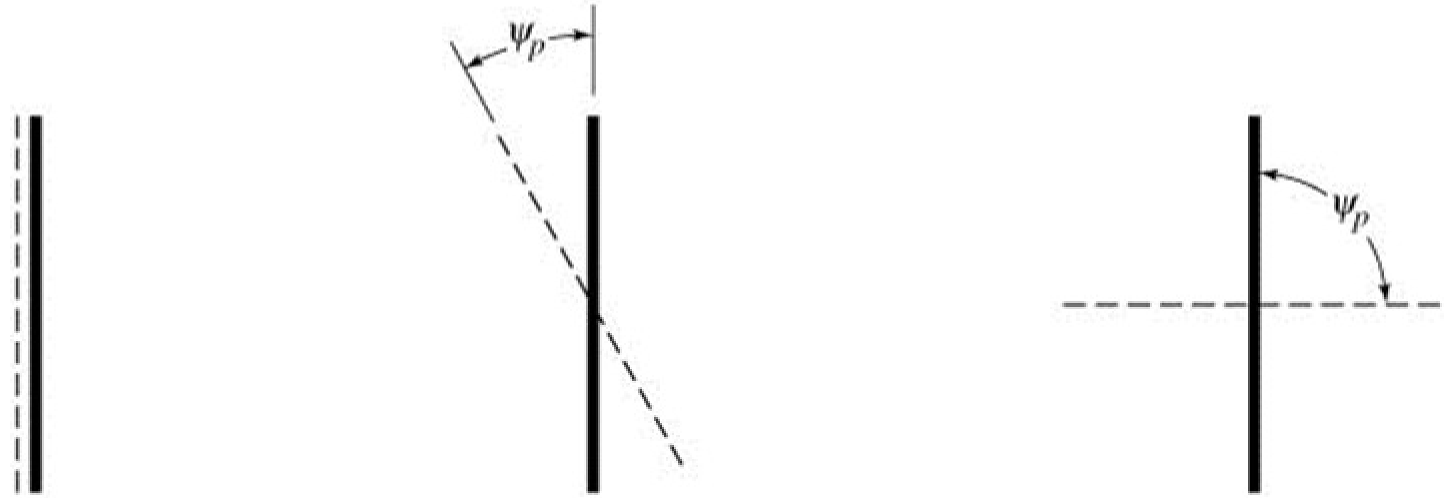
Incident Wave: Polarization determined in direction of wave travel

$$0 \le \mathrm{PLF} \le 1$$

$$\mathrm{PLF(dB)} = 10\log_{10}|\hat{\rho}_w \cdot \hat{\rho}_a|^2 = 20\log_{10}|\hat{\rho}_w \cdot \hat{\rho}_a|$$

$$-\infty \le \mathrm{PLF(dB)} \le 0$$

PLF for Transmitting and Receiving Linear Antennas



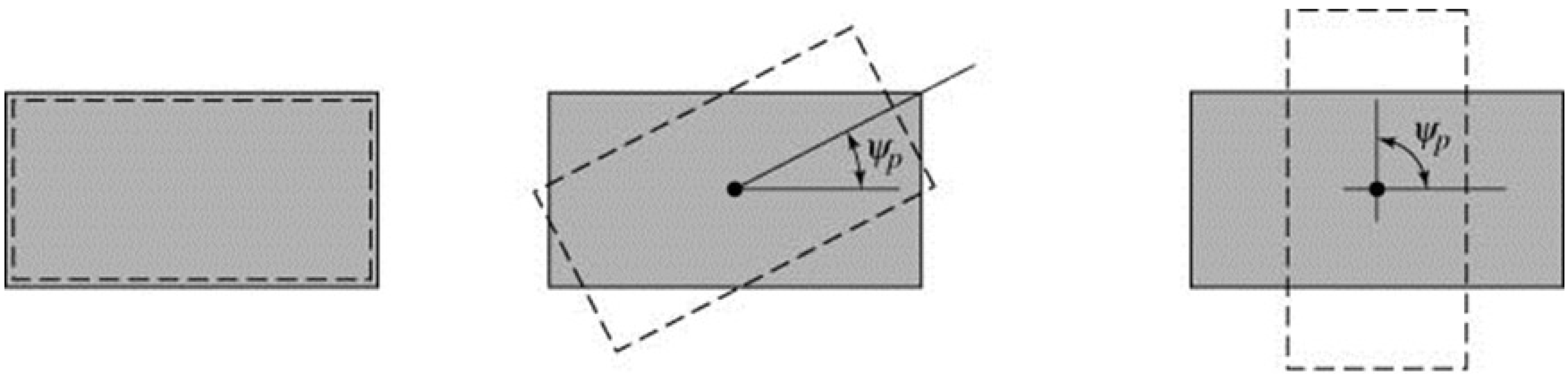
$\text{PLF} = |\hat{\boldsymbol{\rho}}_w \cdot \hat{\boldsymbol{\rho}}_a|^2 = 1$ (aligned)

$\text{PLF} = |\hat{\boldsymbol{\rho}}_w \cdot \hat{\boldsymbol{\rho}}_a|^2 = \cos^2 \psi_p$ (rotated)

$\text{PLF} = |\hat{\boldsymbol{\rho}}_w \cdot \hat{\boldsymbol{\rho}}_a|^2 = 0$ (orthogonal)

Fig. 2.25b

PLF for Transmitting and Receiving Aperture Antennas



$\text{PLF} = |\hat{\rho}_w \cdot \hat{\rho}_a|^2 = 1$
(aligned)

$\text{PLF} = |\hat{\rho}_w \cdot \hat{\rho}_a|^2 = \cos^2 \psi_p$
(rotated)

$\text{PLF} = |\hat{\rho}_w \cdot \hat{\rho}_a|^2 = 0$
(orthogonal)

Fig. 2.25a

Thank You

Questions?