Lecture-37 10/11/2023.

Q ⇒ Consider a photon that scatters from an electron at rest. If the compton wavelength shift is observed to be triple the wavelength of the incident photon & the photon scatters at 60°, calculate

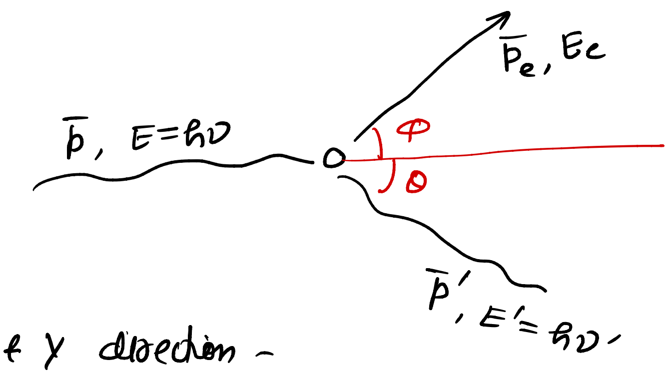
(a) The wavelength of incident photon

(b) The energy of the recoiling e^-.

(c) The angle at which the e^- scatters.

A ⇒

$$\boxed{\lambda' - \lambda = \frac{h}{m_e c}(1-\cos\theta)}$$



Conservation of Momentum along X & Y direction -

$$\left.\begin{aligned} p &= p_e\cos\varphi + p'\cos\theta \\ 0 &= p_e\sin\varphi - p'\sin\theta \end{aligned}\right\}$$

$$\left.\begin{aligned} p_e\cos\varphi &= p - p'\cos\theta \\ p_e\sin\varphi &= p'\sin\theta \end{aligned}\right\} \begin{aligned} &\longrightarrow (1) \\ &\longrightarrow (2) \end{aligned}$$

$$\tan\varphi = \frac{\sin\theta}{\frac{p}{p'} - \cos\theta} \qquad (2)/(1)$$

$$\because p = h/\lambda$$

$$\boxed{\tan\varphi = \frac{\sin\theta}{\frac{\lambda'}{\lambda} - \cos\theta}}$$

(a) ⇒ $3\lambda = \frac{h}{mc}(1-\cos 60°) \Rightarrow \lambda = 4.04\times10^{-13}\ m$ A

(b) ⇒ Energy Conservation

$$\frac{hc}{\lambda} = \frac{hc}{\lambda'} + K.E. \Rightarrow K.E \Rightarrow hc\left[\frac{1}{\lambda} - \frac{1}{\underset{4\lambda}{\lambda'}}\right] = \frac{3hc}{4\lambda} = 2.3\ MeV \text{ A}$$

(C) ⇒ $\tan\varphi = \dfrac{\sin 60^\circ}{4 - \cos 60^\circ} \Longrightarrow \varphi = 13.9^\circ$

Ground State of H atom ⇒

$$\boxed{\psi_{100} = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}} \quad \text{———} \; ①$$

$a_0 = \dfrac{\hbar^2}{me^2}$

§ Most probable distance ⇒

$$\psi^*\psi = \frac{1}{\pi a_0^3} e^{-2r/a_0}$$

$$d\tau = 1 \cdot r \cdot r\sin\theta \; dr\, d\theta\, d\varphi$$

$$\psi^*\psi\, d\tau = \frac{1}{\pi a_0^3} e^{-2r/a_0} r^2 \underline{\sin\theta}\; dr\, \underline{d\theta}\, \underline{d\varphi}$$

$$P(r)dr = \frac{1}{\pi a_0^3} \cdot 4\pi\, e^{-2r/a_0} r^2\, dr$$

$$P(r)\,dr = \frac{4}{a_0^3} e^{-2r/a_0} r^2.$$

To maximize $P(r)$:-

$$\frac{d}{dr}P(r) = \frac{d}{dr}\left(\frac{4}{a_0^3} e^{-2r/a_0} r^2\right) = 0$$

$$\Rightarrow \frac{4}{a_0^3}\left[e^{-2r/a_0} \cdot 2r + r^2 \cdot e^{-2r/a_0}\left(-\frac{2}{a_0}\right)\right] = 0$$

$$\Rightarrow \cancel{\frac{8}{a_0^3}}\left[r e^{-2r/a_0} - \frac{r^2}{a_0} e^{-2r/a_0}\right] = 0$$

$$\Rightarrow \cancel{r e^{-2r/a_0}}\left[1 - \frac{r}{a_0}\right] = 0$$

$$\Rightarrow 1 - \frac{r}{a_0} = 0$$

$$\boxed{r = a_0}$$ Bohr radius.

§ Average distance $\langle r \rangle$ in ground state is

$$\langle r \rangle = \int \psi^* r \psi \, d\tau$$

$$= \int \frac{4}{a_0^3} e^{-2r/a_0} \cdot r^2 \cdot r \, dr$$

$$= \frac{4}{a_0^3} \int_0^\infty e^{-2r/a_0} r^3 \, dr$$

$$= \frac{4}{a_0^3} \frac{\lfloor 3}{\left(\frac{2}{a_0}\right)^4}$$

$$\boxed{\langle r \rangle = 3 \frac{a_0}{2}}$$

$$\boxed{\int_0^\infty r^n e^{-\alpha r} dr = \frac{\lfloor n}{\alpha^{n+1}}}$$