Assignment #5

(Sep 01, 2025)

Maxwell Equations, E.M. waves: Sadiku Chap 9, 10, Griffiths Chap 7-9

1. Using Maxwell's equations derive the wave equations for electric and magnetic fields and show that in free space electromagnetic waves travel with velocity of light.

2. In free space,

$$\mathbf{E} = \frac{50}{\rho}\cos(10^8 t - kz)\,\mathbf{a}_\rho \text{ V/m}$$

Find k, $\mathbf{J}_d$, and $\mathbf{H}$.

3. In a certain region,

$$\mathbf{J} = (2y\,\mathbf{a}_x + xz\,\mathbf{a}_y + z^3\,\mathbf{a}_z)\sin 10^4 t \text{ A/m}^2$$

find ρ_v if $\rho_v(x,\ y,\ 0,\ t) = 0$.

4. Check the following fields are genuine EM fields (i.e., they satisfy Maxwell's equations). Assume that the fields exist in charge-free regions.

(a) $\mathbf{A} = 40\sin(\omega t + 10x)\,\mathbf{a}_z$

(b) $\mathbf{B} = \dfrac{10}{\rho}\cos(\omega t - 2\rho)\,\mathbf{a}_\phi$

(c) $\mathbf{C} = \left(3\rho^2\cot\phi\,\mathbf{a}_\rho + \dfrac{\cos\phi}{\rho}\,\mathbf{a}_\phi\right)\sin\omega t$

(d) $\mathbf{D} = \dfrac{1}{r}\sin\theta\sin(\omega t - 5r)\,\mathbf{a}_\theta$

5. Find the state of polarization when the following two plane polarized waves superimpose.

(a) $E_x = E_0\cos(\omega t + kz) \quad E_y = \dfrac{E_0}{\sqrt{2}}\cos(\omega t + kz + \pi)$

(b) $E_x = E_0\cos(kz - \omega t + \dfrac{\pi}{3}) \quad E_y = E_0\cos(kz - \omega t - \dfrac{\pi}{6})$

(c) $E_x = E_0\cos(kz - \omega t + \dfrac{\pi}{4}) \quad E_y = \dfrac{E_0}{\sqrt{2}}\cos(kz - \omega t)$

(d) $E_x = E_0\cos(\omega t + kz) \quad E_y = E_0\cos(\omega t + kz)$

6. A plane harmonic electromagnetic wave propagates in vacuum. The electric field of the wave has amplitude 50 mV/m and frequency 100 MHz. Calculate the root mean square (*rms*) value of the displacement current and the intensity of the wave.

7. Consider an electromagnetic wave polarized in the y-direction propagates in the positive x-direction in vacuum. Using the Maxwell's equation show that

$$\frac{\partial B}{\partial t} = -\frac{\partial E}{\partial x} \quad \text{and} \quad \frac{\partial E}{\partial t} = -c^2\frac{\partial B}{\partial x}$$

8. An electromagnetic wave with electric field $\mathbf{E} = 30\cos(\omega t - z)\,\mathbf{a}_x$ V/m propagating in air hits normally a lossless dielectric medium ($\mu = \mu_0$ and $\varepsilon = 4\varepsilon_0$) at $z = 0$. Calculate:

(a) the amplitude of electric fields of the reflected ad transmitted waves

(b) reflection and transmission coefficients

(c) the increment in wavelength when the wave enters into the dielectric medium.

9. A uniform plane wave in a certain medium ($\mu = \mu_0$, $\varepsilon = 4\varepsilon_0$) is given by

$$\mathbf{E} = 12\cos(\omega t - 40\pi x)\mathbf{a}_z \quad \text{V/m}$$

(a) Find ω.

(b) If the wave is normally incident on a dielectric ($\mu = \mu_0$, $\varepsilon = 3.2\varepsilon_0$), determine E_r and E_t.

10. An electromagnetic plane wave of wavelength $\lambda = 500\,\text{nm}$ with a circular cross-section of diameter $d = 2\,\text{mm}$ propagates in free space carrying power $P = 5\,\text{mW}$. The wave is linearly polarized along the x-axis and travels in the $+z$ direction. Determine the complete expressions for the electric and magnetic fields of the wave.