* No. of allowed values of r in the range r to $r+dr$: $N(r)dr$
_______ ν _______ ν to $\nu+d\nu$: $N(\nu)d\nu$

$\nu = \frac{cr}{2a}$

2-D case

$$N(r)dr = N(\nu)d\nu = \frac{1}{4}(2\pi r dr) = \frac{\pi r dr}{2} = \frac{\pi}{2}\left(\frac{\cancel{c}r}{2a}\right)\frac{\cancel{c}}{2a} = \frac{\pi}{2}\frac{2a\nu}{c}\frac{2a d\nu}{c}$$

$$\boxed{\frac{2\pi a^2 \nu d\nu}{c^2}} = \boxed{\frac{2a^2\nu\pi d\nu}{c^2}}$$

* With two independent perpendicular direct^{n}s of polarisat^n of wave: $N(r)dr = \frac{4\pi a^2 \nu d\nu}{c^2}$

$\nu = \frac{cr}{2a}$, $r = \frac{2a\nu}{c}$

3-D case

$$N(r)dr = N(\nu)d\nu = \frac{1}{8}(4\pi r^2 dr) = \frac{\pi}{2}\frac{4a^2\nu^2}{c^2}\frac{2a d\nu}{c} = \frac{4\pi a^3 \nu^2 d\nu}{c^3}$$

$$= \boxed{\frac{8\pi a^3 \nu^2 d\nu}{c^3}}$$ ← No. of electromagnetic modes in a cubical cavity of side a whose frequency lie b/w ν & $\nu+d\nu$

* Avg. Energy per degree of freedom $= \frac{1}{2}kT$
→ So avg energy of 2 deg of free $= \boxed{\bar{\varepsilon} = kT}$ ← classical average energy

* Energy density in frequency interval ν to $\nu+d\nu$
$$u(\nu,T)d\nu = \frac{\bar{\varepsilon}N(\nu)d\nu}{\text{Vol}^m \text{ of } c}$$

$\varepsilon = h\nu$

$$\boxed{u(\nu,T)d\nu =} \frac{8\pi(kT)\nu^2 d\nu}{c^3(e^{\frac{h\nu}{kT}}-1)} = \frac{8\pi h\nu^3 d\nu}{c^3(e^{\frac{h\nu}{kT}}-1)}$$

Energy density in interval ν to $\nu+d\nu$

Energy density $u \to J/m^3$
$$u(T) = \int_0^\infty u(\nu,T)d\nu.$$

* Average energy per standing wave (planck)
$$\boxed{\bar{\varepsilon} = \frac{h\nu}{e^{\frac{h\nu}{kT}}-1}}$$

* Wien's displacement law:- $\lambda_m T = \text{Cons}$

K^{th} order stationary

$t_1, t_2 \; - - \quad - \; - t_k$

$F_{x(t_1) \cdots x(t_k)}(x_1 \cdots x_k) = F_{x(t_1+\tau) \cdots x(t_k+\tau)}(x_1 \cdots x_k) \quad \forall t_1 \cdots t_k, \tau.$

↳ Strict sense stationary RP.

Franck and Hertz experiment

$$\boxed{\frac{1}{2} m v^2 = e V_s}$$

wave properties of matter : de Broglie's hypothesis.

* Dual ~~nature of~~

Energy of photon $\rightarrow E = h\nu$

magnitude of magnitude $\rightarrow p = \frac{h\nu}{c} = \frac{h}{\lambda}$

* ~~For a free material particle, associate~~

$\nu = \frac{E}{h}$, $E = \frac{hc}{\lambda}$

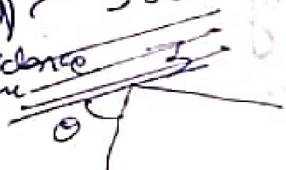
* Davisson - Germer experiment (For X - Rays).

→ interplanar spacing

→ See Bragg's law

Bragg's law → $2d\sin\theta = n\lambda$

↳ Incidence angle

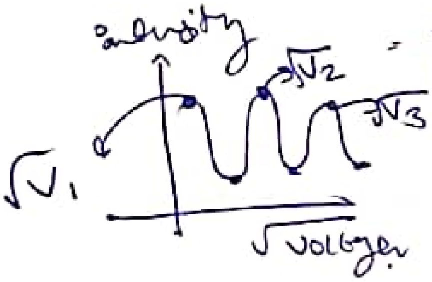


$\lambda = \frac{2d\sin\theta}{n}$

$2d\sin\theta$ = extra path travelled

For constructive interference this should be equal to $n\lambda$

$\frac{h}{p} = \frac{h}{\sqrt{2meV}} = \frac{2d\sin\theta}{n}$



* Evolution of wave f^n.

$$y = A\cos(kx-\omega t)$$

$$y = Re\left(Ae^{j(kx-\omega t)}\right)$$

solution of wave eq^n

$$\psi = Ae^{j(kx-\omega t)}$$

$$\omega \to 2\pi\nu \;:\; k = \frac{2\pi}{\lambda}$$

$$\psi = Ae^{j\left(\frac{2\pi x}{\lambda} - 2\pi\nu t\right)}$$

(r,t) → x here

$$= \boxed{Ae^{\frac{j}{\hbar}(px-Et)}}$$ ← wave f^n of a particle moving in x direction.

$$\frac{\partial\psi}{\partial x} = \frac{jp}{\hbar}\psi$$

$$\hat{p} = -j\hbar\frac{\partial}{\partial x}$$

$$\frac{\partial^2\psi}{\partial x^2} = \frac{jp}{\hbar}\frac{jp}{\hbar}\psi$$

$$\frac{\partial^2\psi}{\partial x^2} = \frac{-p^2\psi}{\hbar^2}$$

$$\boxed{\hat{p}^2 = \hbar^2\frac{\partial^2}{\partial x^2}}$$

$$\frac{\partial\psi}{\partial t} = -\frac{Ej}{\hbar}\psi$$ similarly.

$$\boxed{\hat{E} = j\hbar\frac{\partial}{\partial t}}$$

Postulates of Quantum mechanics

① Descript^n of state of a system.

* At a time t → state of a physical system is defined as by $\Psi(\vec{r},t)$

* $\Psi^*(\vec{r},t)\,\Psi(\vec{r},t) = |\Psi|^2$ → probability distribut^n.

② * The evolut^n of wavef^n is governed by schrodinger eq^n.

It can have variety of soln; including complex ones.

③ Descript^n of physical quantities.

Every physical quantity in quantum mechanics is described by an operator.

measurable

$$\hat{H} = \frac{\hat{p}^2}{2m} + \hat{V} = \text{Energy of system.}$$

$\hat{p}$ → momentum operat^n.

$\hat{V}$ → potential energy operator.

** $\boxed{\hat{H}\Psi = E\Psi.}$

** Eigenvalue of Hamiltonian operator : Energy

Eigenvector

Energy operator (Hamiltonian)

Compton effect formulae

λ_0 → initial λ

λ_1 → final λ

① $\lambda_1 - \lambda_0 = \frac{h}{m_e c}(1-\cos\theta)$

② $E = h\nu = \frac{hc}{\lambda}$

$E' = \frac{E}{1+\frac{E}{m_e c^2}(1-\cos\theta)}$ ← energy of incident photon.

③ KE of recoil electron

$K = E - E' = E\left(1 - \frac{1}{1+\frac{E}{m_e c^2}(1-\cos\theta)}\right)$

④ $\tan\phi = \frac{E'\sin\theta}{E - E'\cos\theta}$

ϕ → electron angle

θ ← photon angle

⑤ $p_e = \frac{1}{c}\sqrt{E^2 + E'^2 - 2EE'\cos\theta}$

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$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2m}{\hbar^2}(\hat{E} - \hat{V})\psi = 0$$ time independent / steady state schrodinger eqn in 1D.

In 3D → $\boxed{\underset{\sim}{\nabla}^2 \psi + \frac{2m}{\hbar^2}(\hat{E} - \hat{V})\psi = 0}$ **

we find in the same way that of a scalar in spatial coordinate system.

* for time independent $E \rightarrow$ energy

—— dependent $E \rightarrow$ operator $i\hbar \frac{\partial}{\partial t}$.

$$\langle p\rangle = \int_{-\infty}^{\infty} \psi^* \hat{p}\, \psi\, dx = \int_{-\infty}^{\infty} \psi^* \left(\frac{\hbar}{i}\frac{\partial}{\partial x}\right)\psi\, dx = \frac{\hbar}{i}\int_{-\infty}^{\infty} \psi^* \frac{\partial \psi}{\partial x}\, dx$$

$$\langle E\rangle = \int_{-\infty}^{\infty} \psi^* \hat{E}\, \psi\, dx = \int \psi^* \left(i\hbar\frac{\partial}{\partial t}\right)\psi\, dx = i\hbar\int_{-\infty}^{\infty} \psi^* \frac{\partial \psi}{\partial t}\, dx$$

~~$\langle p\rangle = \int_{-\infty}^{\infty}$~~

* Hermitian operator can also be complex so

$$\int_{-\infty}^{\infty} \psi_i^* \hat{\Omega}\, \psi_j\, d\tau = \int_{-\infty}^{\infty} \psi_i \hat{\Omega}^* \psi_j^*\, dV^*$$

* The eigen funct's of Hermitian operators are orthonormal, i.e. The product of funct's integrate over all space to either 0 or 1

$\hat{\Omega}\psi_n$ all eig. f^n form orthonormal

$$\int_{-\infty}^{\infty} \psi_i^* \psi_j\, d\tau = \delta_{ij} \text{ (Kronecker delta)}$$

may have more than one eigenvector.

$$\delta_{ij} = \begin{cases} 0 & \text{when } i \neq j \text{ orthogonal} \\ 1 & \text{when } i = j \text{ (normal)} \end{cases}$$

Exercise - $\langle \hat{x}\hat{p}_x - \hat{p}_x\hat{x}\rangle$

Conservation of probability

If a particle is described by a normalised wavef^n.

$$\int |\psi(r,t)|^2\, dV = 1$$

Dirac Brackets

Expectation value

$$\int \psi^* \hat{\Omega} \psi \, d\tau = \langle \psi^* | \hat{\Omega} | \psi \rangle$$

$$\int \psi^* \psi \, d\tau = \langle \psi^* | \psi \rangle$$

$|\psi\rangle \rightarrow$ ket, denotes the state described by ψ.

$\langle \psi^* |$ is called bra, and denotes the complex conjugate of f^n ψ^*.

Theorems of QM.

* Eigenvalues of Hermitian operators are real.

$$\hat{\Omega} \psi_n = \omega_n \psi_n$$

* Orthogonality theorem.

Eigen f^ns corresponding to diff eigenvalues for same Hermitian operator are orthogonal. $\langle \psi_j^* \psi_i \rangle = 0$

$$\hat{\Omega} \psi_n = \omega_n \psi_n$$

$$\int_{-\infty}^{\infty} \psi_j^* \psi_i \, d\tau = \delta_{ij}$$

$$\delta_{ij} = 0, \; i \neq j$$
$$1, \; i = j.$$

* Commuting operators have simultaneous eigen f^ns.

$$[\hat{A}, \hat{B}] f = \hat{A}\hat{B} f = \hat{B}\hat{A} f.$$

$$(\hat{A}\hat{B} - \hat{B}\hat{A}) f = 0$$

$$\hat{A}\hat{B} - \hat{B}\hat{A} = \hat{0}$$

$\hat{0} \rightarrow$ null operator

if $\hat{0} f = 0$

$[\hat{A}, \hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A} = 0 \Rightarrow$ operators ~~don't~~ commute. (leads to Heisenberg uncertainty principle.

$\neq 0 \Rightarrow$ operators commute. don't.

* $\boxed{\text{Let } \psi_i \text{ be eigen} f^n \text{ for } \hat{B} \text{ so that : } \hat{B}\psi_i = b_i \psi_i.}$ ★★★

where all b_i are diff, meaning that the eigenf^n are nondegenerate.

when $[\hat{A}, \hat{B}] = \hat{0}$ $\hat{B}(\hat{A}\psi) = \hat{A}\hat{B}\psi_i = \hat{A} b_i \psi_i = b_i(\hat{A}\psi_i)$

eigenvalue

$\hookrightarrow$ eigenf^n of $\hat{B}$

Ques 3

$$\lambda T = c = \cancel{2/0}\ 2.898\times10^{-3}\ m\,K$$

$$(\lambda)(1000) = c$$

$$\lambda = 2.898\times10^{-6}\ m.$$

$$\cancel{\nu = \frac{3\times10^{8}}{2.898\times10^{-6}}}$$

$$E(\nu) = \frac{8\pi h\nu^{3}}{c^{3}\left(e^{\frac{h\nu}{kT}}-1\right)}$$

$$\frac{dE}{d\nu} = \frac{3\cdot 8\pi h\nu^{2}}{c^{3}\left(e^{\frac{h\nu}{kT}}-1\right)} - \frac{8\pi h\nu^{3}}{c^{3}\left(e^{\frac{h\nu}{kT}}-1\right)^{2}}\left(e^{\frac{h\nu}{kT}}\right)\left(\frac{h}{kT}\right) = 0.$$

$$\nu_{max} = 6.25\times10^{13}\ sec^{-1}$$

$$\lambda = \frac{3\times10^{8}}{6.25\times10^{13}} = 10^{-5}\times\frac{3}{6.25} = 0.48\times10^{-5}$$

$$= \boxed{4800\ nm} \rightarrow \text{Infrared.}$$

Ques 4

Intensity $= 1.4\times10^{3}$ | $r_0 = 1.5\times10^{11}$, $r_s = 7\times10^{8}$

Power will be same.

$$1.4\times10^{3}\times 4\pi(r_0^{2}) = 4\pi(r_s)^{2}\,\sigma T^{4}$$

$T = 5.8\times10^{3}$ K from Here.

Ques 5

$$= \frac{(273+35)^{4}-(273+34)^{4}}{(273+35)^{4}}\times100 \approx 1.3\%$$

Ques 6

$$E(\nu) = \frac{8\pi h\nu^{3}}{c^{3}\left(e^{\frac{h\nu}{kT}}-1\right)},\quad \nu \leftrightarrow \frac{c}{\lambda}$$

Definition

$$E(\lambda) = \frac{8\pi}{\lambda^{3}\left(e^{\frac{hc}{\lambda kT}}-1\right)} \quad dC$$

$$-\frac{\hbar^2}{2m}\hat{E}\psi = \frac{\partial^2\psi(x)}{\partial x^2} \rightarrow \frac{\partial^2\psi(x)}{\partial x^2} + \frac{\hbar^2 E}{2m}\psi = 0$$

$$\nabla^2\psi + \frac{2m}{\hbar^2}(\hat{E} - \hat{V}_0)\psi = 0$$

k^2

$$\psi(x) = A\sin kx + B\cos kx, \quad k = \frac{\sqrt{2mE}}{\hbar}$$

$$\frac{\partial^2\psi}{\partial x^2} + k^2\psi = 0$$

at the boundaries $x = \pm\frac{a}{2}$

$\psi = A\sin kx + B\cos kx \rightarrow$ General

* See the example

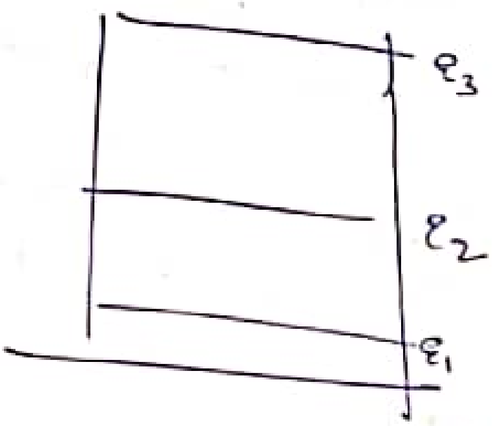
$\psi(x) = 0 \rightarrow$ this gives

$$E_n = \frac{\hbar^2 k_n^2}{2m} = \frac{\pi^2\hbar^2 n^2}{2ma^2}$$

$$E_1 = \frac{\pi^2\hbar^2}{2ma^2} \text{ and so on.}$$

$$E_2 = \frac{2\pi^2\hbar^2}{ma^2}$$

$$E_3 = \frac{9\pi^2\hbar^2}{2ma^2}$$



* The infinite square well potential

$$V(x) = \begin{cases} \infty & \frac{a}{2} < x,\ x < -\frac{a}{2} \\ 0 & |x| < \frac{a}{2} \end{cases}$$

Here

$$V(x) = \begin{cases} 0 & 0 < x < L \\ \infty & \text{elsewhere} \end{cases}$$

$$\nabla^2\psi + \frac{2m}{\hbar^2}(\hat{E})\psi = 0$$

$$\psi = A\sin\frac{\sqrt{2mE}}{\hbar}x$$

$\psi = 0$ at $x = L$

$$S \Rightarrow \frac{\sqrt{2mE}}{\hbar}L = n\pi, \quad n = 1, 2, 3 -$$

Energy eigenvalues: $E_n = n^2\pi^2\hbar^2/2mL^2, \ n = 1, 2, 3$

$\psi_2 =$

$\psi_2(x) = A\sin k_2 x + B\cos k_2 x = Ae^{ik_2x} + Be^{-ik_2x}$

$\psi_1(x) = A\sin k_1 x + B\cos k_1 x. = Ce^{ik_1x} + \cancel{De^{-ik_1x}} \to 0$

$* \cancel{Ae^{ikx} + Ce}$

~~Reflect~~

Reflectⁿ & Transmission coefficient:-

$$R = \left| \frac{\text{reflected current density}}{\text{incident current density}} \right| = \frac{|J_{reflected}|}{|J_{incident}|}$$

$$T = \left| \frac{\text{Transmitted current density}}{\text{incident current density}} \right| = \left| \frac{J_{transmitted}}{J_{incident}} \right|$$

$$J = \frac{i\hbar}{2m}\left(\psi\nabla\psi^* - \psi^*\nabla\psi\right).$$

$\psi_i = Ae^{ik_2x}$

$\frac{\partial \psi_i}{\partial x} = Aik_2 e^{ik_2x}$

$$J_{incident} = \frac{i\hbar}{2m}\left(-Ae^{ik_2x}\, A^* i k_2 e^{-ik_2x} - A^* e^{-ik_2x}\, A i k_2 e^{ik_2x}\right)$$

$$= \frac{i\hbar}{2m}\left(|A|^2 ik_2 + |A|^2 ik_2\right)$$

$$= \left(i\, i k_2 |A|^2 \frac{i\hbar}{2m}\right)(-1)$$

$$= \frac{k_2\hbar}{m}|A|^2$$

Similarly $J_{reflected} = -\frac{\hbar k_2}{m}|B|^2$

$J_{transmitted} = \frac{\hbar k_2}{m}|C|^2$

at $x = L$ $\psi(L) = 0$

so $A\sin(kL) = 0$

$kL = n\pi$

$k = \frac{n\pi}{L}$

as $k^2 = \frac{2mE}{\hbar^2}$

~~$E = 2m$~~

$E = \frac{k^2\hbar^2}{2m}$

~~$\hbar$~~ $= k = \frac{n\pi}{L}$

so $E_n = \frac{\cancel{k^2}\hbar^2}{2m}$

$$\boxed{E_n = \frac{n^2\pi^2\hbar^2}{2mL^2}} \quad **$$

so $\int_{-\infty}^{\infty} |\psi(x)|^2 dx = 1$ must satisfy. } prob. of finding an electron in the chosen limits.

we will get normalised ψ from this.

so $\psi(x) = A\sin\left(\frac{n\pi x}{L}\right)$

$$\boxed{\text{~~not~~ number of nodes} = n-1} \quad *****$$

$$\int_0^L A^2\sin^2\left(\frac{n\pi x}{L}\right)dx = 1$$

$$\left(\frac{A^2}{2}\right)\int_0^L \left(1 - \cos\left(\frac{2n\pi x}{L}\right)\right) dx = 1$$

$$\left(\frac{A^2}{2}\right)\left(L - \frac{LA}{2n\pi}(0)\right) = 1$$

$$A = \sqrt{\frac{2}{L}}$$

$$-\frac{\hbar}{2m}\psi\nabla^2\psi^* + \psi V\psi^* = -i\hbar\,\psi\frac{\partial\psi^*}{\partial t}$$

↳ after subtracting.

$$-\frac{\hbar^2}{2m}\left[\psi^*\nabla^2\psi - \psi\nabla^2\psi^*\right] = -i\hbar\left(\psi^*\frac{\partial\psi}{\partial t} + \psi\frac{\partial\psi^*}{\partial t}\right)$$

The rate of change of probability $= \frac{\partial}{\partial t}\int_V \rho(\vec{r},t)\,dv = \frac{\partial}{\partial t}$

$= \int_V \psi$

★ J is called probability current density.

$$\boxed{\vec{J} = \frac{i\hbar}{2m}\left(\psi\nabla\psi^* - \psi^*\nabla\psi\right)}$$

★ $\psi(\vec{r},t) = C\,\Upsilon(\vec{r})\,e^{-\frac{iEt}{\hbar}}$

↳ $= 1$ when normalising.

$E_0 + m_e c^2 = E_2 + \sqrt{p_1^2 c^2 + m_e c^4}$

$\frac{hc}{\lambda_0} + m_e c^2 = \frac{hc}{\lambda_1} + \sqrt{\left(\frac{h}{\lambda_1}\frac{\sin\theta}{\sin\phi}\right)^2 c^2 + m_e c^4}$

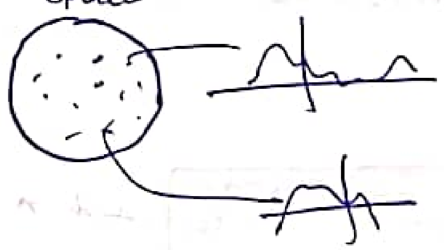
$\lambda_1 - \lambda_0 = \frac{h}{m_e c}(1-\cos\theta)$

$KE_e = E_0 - E_1$, $\Delta\lambda = \lambda_1 - \lambda_0 = \lambda_e(1-\cos\theta)$

$\lambda_e = \frac{h}{m_e c}$ → compton wavelength of an electron.

Random processes

Sample space



Random process by choosing the point.

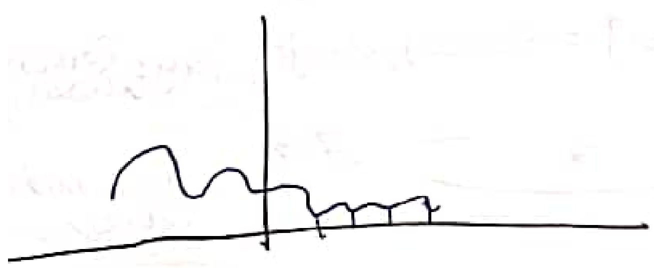
$A\cos(\omega_0 t + \Theta)$

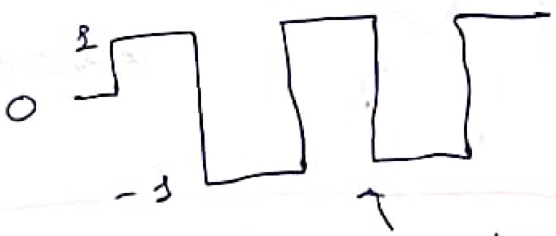
$\Theta \sim U(-\pi, \pi)$
↑
random.

Noise

1 1 0 1 1

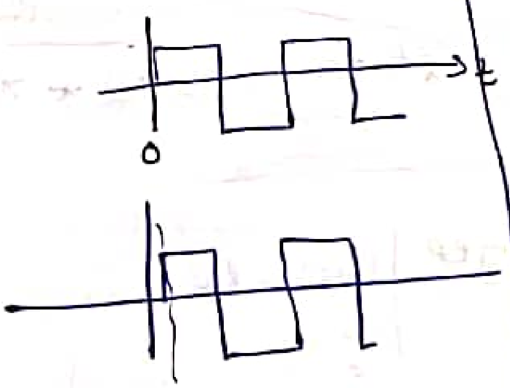
$x(t)$





Descrete random process.

* Binary wave

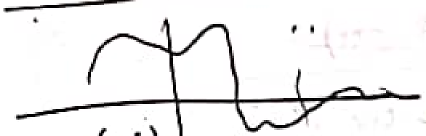


δt → time delay.

$X(t_0)$

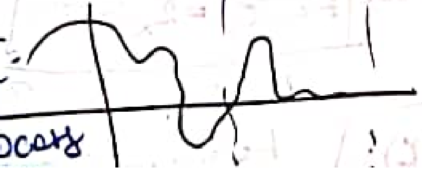
$F_{X(t_0)}(x) = P[X(t_0) \le x]$

$f_{X(t_0)}(x) = \frac{d}{dx} F_{X(t_0)}(x)$



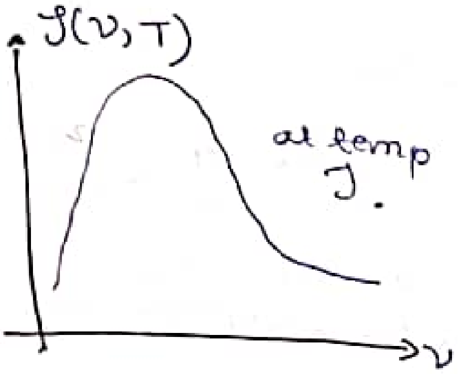
$F_{X(t_0)}(x) \neq F_{X(t_0+\tau)}(x)$ $\forall \tau$
↓
~~first order~~ non RP.

$F_{X(t_0)}(x) = F_{X(t_0+\tau)}(x)$
⇓
first order stationary random process



Quantum theory

* Absorbtⁿ ability is related to emissivity.
* Thermal equilibrium :- absorbing and radiating at the same rate.
* Black body :- absorbs all radiatⁿ incident upon it.
* Thermal radiatⁿ → Radiatⁿ emitted by body as a result of its temperature.



Theory of cavity radiatⁿ

Explanatⁿ from the classic electromagnetic theory
1) Wilhelm wien
2) Rayleigh and Jeans.

* Wein's frequency distributⁿ law :- $S(\nu,T) = a\nu^3 e^{-\frac{b\nu}{T}}$ } only for high frequency.

* Rayleigh Jeans law :- $S(\nu,T) = \frac{8\pi k T \nu^2}{c^3}$ } at low frequency

* Max planck :- $\underset{=}{\overset{u(\nu,T)}{}}\; S(\nu,T) = \frac{8\pi h \nu^3}{c^3} \frac{1}{(e^{\frac{h\nu}{kT}} - 1)}$ → Energy per unit volume per unit frequency.

$h\nu \gg kT$ → wein's freq.

$h\nu \ll kT$ → Rayl- - .

Rayleigh - Jeans law

→ Energy density in frequency interval ν & $\nu + d\nu$:- $u(\nu,T)d\nu = \frac{\bar{E}\, N(\nu)d\nu}{\text{Volume of Cavity}}$

allowed frequency = $\frac{cr}{2a}$

Stefan boltzmann Law

* Total spectral energy density of blackbody radiatⁿ.

$\longrightarrow$ Energy/(volume·frequency).

$$u_{tot} = \int_0^\infty u(\nu)\,d\nu$$

$$= \frac{8\pi h}{c^3}\int_0^\infty \frac{\nu^3\,d\nu}{e^{h\nu/kT}-1}$$

$$= \frac{8\pi h}{c^3}\ \boxed{\frac{8\pi^5 k^4 T^4}{15c^3h^3} = u_{Tot} = aT^4}$$

* Spectral Radiancy $\rightarrow$ Power per unit area in frequency of interval ν to $\nu + d\nu$ at temperature T.

$\hookrightarrow R_T(\nu)d\nu \rightarrow$ Tells how much power per unit area comes from a frequency band at ν of width $d\nu$.

$$R_T(\nu)d\nu \propto u(\nu)d\nu$$

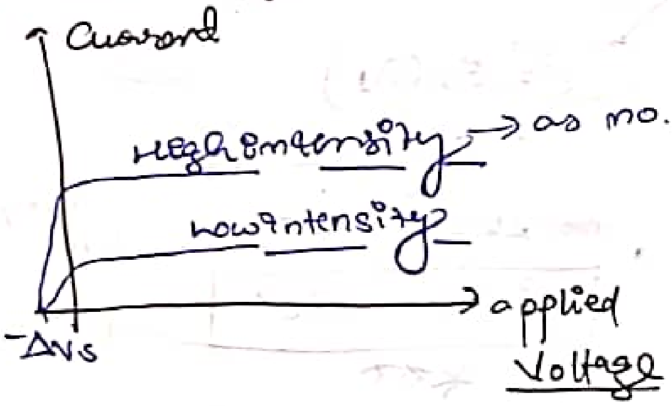
$$R_T(\nu)d\nu = \frac{ec}{4}\,u(\nu)d\nu$$

$e \rightarrow$ emissivity.

Radiancy $\rightarrow R_T$ = Total power per unit area across all frequencies and all directⁿs.

$$\boxed{R_T = \sigma e T^4}$$

* Photoelectric effect + photoelectrons.



$\rightarrow$ as no. of photoelectrons ↑↑.

De Broglie's wave

** For de broglie wave $\lambda \propto \frac{1}{v}$ **

*Photon of freq ν has momentum $\boxed{p = \frac{h\nu}{c}}$

If the momentum of a particle of mass m and velocity v is $\boxed{p = \gamma m v}$.

relativistic factor $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

Matter waves

In light waves, the Electric field(E) and magnetic field (B) vary
what does vary in matter waves (de broglie waves)?

The wave funct^n Ψ → Related to the probability of finding a body at a point (x,y,z) in space at time t.

Physical significance:

$|\Psi|^2$ → absolute value of wavef^n → probability density.
→ proport^{n}al to the probability of experimentally finding the body described by the wavef^n Ψ at point (x,y,z) at time t.

How fast do de broglie waves travel.
→ Since we associate a de broglie's wave with a moving body, we expect that this wave has same velocity as that of the body.

de Broglie wave velocity $= v_p$.

$V_p = \nu\lambda$

$\lambda = \frac{h}{\gamma m v}$ ← velocity

Find the frequency expression
Equate the quantum expression $E = h\nu$ with
Total energy formula $E = \gamma mc^2$

$\nu = \frac{\gamma mc^2}{h}$

classical wave → energy is emitted or absorbed continuously.

Planck's wave → Energy is emitted or absorbed in packets.

Classical wave	Planck's wave
* $\cancel{\bar{E}} = \cancel{KT}$	* $\cancel{\bar{E} = \frac{h\nu}{e^{h\nu/KT}-1}}$
* $\cancel{v_p}$ $v = \lambda \nu$	* $v_p = \lambda \nu$
* Spectral Energy density $u(\nu,T) = \cancel{8\pi} \frac{8\pi \nu^2 KT}{c^3}$	* spectral energy density $u(\nu,T) = \frac{8\pi h\nu^3}{c^3} \frac{1}{e^{h\nu/KT}-1}$
* $\bar{E} = KT$ ↓ average energy per oscillator | per mode	* $\bar{E} = \frac{h\nu}{e^{\frac{h\nu}{KT}}-1}$ ↓ "

① Phase velocity (v_p)

velocity at which a particular phase (like crest or through) of wave travels

$\cancel{v_p = \frac{\omega}{K}}$ $v_p = \frac{\omega \to 2\pi\nu}{k \to 2\pi/\lambda} = \frac{\nu\lambda}{\cancel{\pi}}$

for light in vaccum $v_p = \frac{c}{n} = c$ as $n = 1$.

for a de Broglie wave: $v_p = \frac{E \to h\nu}{P \to \frac{h}{\lambda}} = \lambda\nu$ too.

Doesn't represent energy or information speed.

② Group velocity (v_g)

→ speed at which the wave packet (envelope) or energy | information travels.

$v_g = \frac{d\omega}{dK}$ }

For a non relativistic → $\boxed{v_g = \frac{d\omega}{dk} = \frac{\hbar k}{m} = \frac{p}{m} = v_p}$ for relativistic also this is equal to v_p.

* $v_p = \cancel{\frac{1}{\beta}}\frac{\omega}{k} = \frac{\nu}{\lambda}$

* $v_g = \frac{d\omega}{dK}$

* $v_p = v_g$ ← classical waves even when $n \neq 1$.

* $v_p \neq v_g$ In dispersive or planck waves. ($n = n(f,\lambda)$)

* For matter waves, $v_g = v_{particle}$, $v_p = \frac{c^2}{v_g}$. ← For relativistic

For non-relativistic matter $\boxed{v_g = 2v_p}$

* The max born interpretation of a wavef^n placed 4 restrictions on an acceptable wavef^n :-

A wavef^n must be → (i) continuous
(ii) have a continuous slope.
(iii) be single valued.
(iv) be square integrable.

wavef^n = ψ

probability density f^n = $\psi^*\psi = |\psi|^2$

$\int_{-\infty}^{\infty} |\psi|^2 dV = N^2$ → normalisation constant

$\int_{-\infty}^{\infty} |\frac{\psi}{N}|^2 dV = 1$

$\int_{-\infty}^{\infty} \psi_n(x)^* \psi_n^*(x) dx = 1$

$\psi_n(x)$ = normalised wave f^n.

well behaved wavefunct^n's

~~* 4 must $\psi \to 0$ as x, y, z~~ $\to \infty$ ✓

* All observables in classical mechanics have a corresponding linear hermitian operator in quantum mechanics $\hat{\Omega} f(x) = g(x)$

operator, f^n, new f^n

Linear operator $\hat{\Omega}(\alpha f + \beta g) = \alpha \hat{\Omega} f + \beta \hat{\Omega} g$

* All hermitian operators have the eigenfunct^n-eigenvalue pair

$\hat{\Omega} f(x) = \omega f(x)$ → eigenvalue.

eigenvector / f^n

→ say $\hat{\Omega} = \frac{\partial}{\partial x}$

$f(x) = x^3$

so $\omega f(x) = g(x)$
$= 3x^2$.

* $\hat{\Omega}\psi_n = \omega_n \psi_n$ → eigenf^n. (n) → quantum number

→ n shows entire family of solut^n's

Hermitian operator; eigenvalue

Physical quantities	Operator
posit^n coordinate: x	$\hat{x}$
posit^n vector: $\vec{r}$	$\vec{r}$
x component of momentum.	$-i\hbar\frac{\partial}{\partial x}$
momentum p.	$-i\hbar\vec{\nabla}$
kinetic energy $\frac{p^2}{2m}$	$\frac{-\hbar^2}{2m}\nabla^2$
Potential energy: $V(\vec{r},t)$	$\hat{V}(r,t)$
Total energy: $\frac{p^2}{2m} + V(r,t)$	$-\frac{\hbar^2}{2m}\nabla^2 + \hat{V}(r,t)$

* Average or expectation value.

$P(\vec{r},t)dv = \psi^*(\vec{r},t)\psi(\vec{r},t)dv$

= The probability of finding the particle in volume dv about point $\vec{r}$, at time t.

The expectation value of position vector

$= \langle \vec{r} \rangle = \int \vec{r} P(\vec{r},t) dv$

$= \int \psi^*(\vec{r},t)\hat{r}\psi(\vec{r},t)dv$ — operator of that

| average position of particle.

Physical meaning → value of $\vec{r}$ on a very large number of equivalent identically prepared independent systems represented by ψ.

* Average energy of Planck's formula.

$$\bar{E} = \frac{\sum_{n=0}^{\infty} E P(E)}{\sum_{n=0}^{\infty} P(E) = 1}$$

$$P(E) = \frac{e^{-E/kT}}{kT}$$

$\boxed{P(E)dE \rightarrow \text{prob of finding a given entity of a system with energy in interval } E \text{ to } E+dE.}$

$E = nh\nu$

* the expectation value of an arbitrary f^n $f(\vec{r},t) = f(x,y,z,t)$ is

$\langle f(\vec{r},t)\rangle = \int \psi^*(\vec{r},t) f(\vec{r},t) \psi(\vec{r},t) dv$

* Similarly expectation value of A is:

$\langle A \rangle = \int \psi^*(\vec{r},T)\hat{A}\psi(\vec{r},T)dv$

$\langle p_{x,y,z} \rangle = \int \psi^*(\vec{r},t)\left(-i\hbar\frac{\partial}{\partial x,y,z}\right)\psi(\vec{r},T)dv$

$\langle \vec{p} \rangle = \int \psi^*(\vec{r},t)(-i\hbar\vec{\nabla})(\psi(\vec{r},t))dv$

$$\bar{x} = \frac{\int_{-\infty}^{\infty} x P(x,t)dx}{\int P(x,t)dx} = \frac{\int_{-\infty}^{\infty}\psi^* x \psi}{\int \psi^*\psi}$$

Commutators and uncertainity

~~Theorem :~~

* Consider particle, in +x direct^n moving.

let the wave fn be Ne^{ikx} with $N \rightarrow$ normalisat^n factor

$$|\psi|^2 = N^2$$

* So probability density is independent of x, there is an equal probability of finding the particle elsewhere on x-axis.

* In other words the posit^n of particle can't be predicted.

$[\hat{p}, \hat{x}] = i\hbar$ or see what we get! $\Rightarrow$ operators donot commute.

$\Rightarrow$ the observables (Energy & time) cannot be known with precision at same time.

Example

$E[\hat{E}, \hat{t}] = \int_{-\infty}^{\infty} \psi^* ($

$\langle (-\frac{\hbar^2}{2m}\nabla^2 + \hat{V})t$

$\hat{E} = i\hbar \frac{\partial}{\partial t}$ **

$\int_{-\infty}^{\infty} \psi (x - i\hbar \frac{\partial}{\partial x} x$

$\int \psi^* (-i\hbar \nabla (x\psi) + \frac{x \partial \nabla \psi}{i\hbar}) d\tau$

~~$\int_{-\infty}^{\infty} \psi^* (-i\hbar(\psi + x\nabla\psi - x$~~

$\int_{-\infty}^{\infty} \psi^* (-i\hbar) ($

$i\hbar \int_{-\infty}^{\infty} \psi^* (-\psi + x\nabla\psi$

$\int_{-\infty}^{\infty} \psi^* (-i\hbar (\frac{\partial}{}$

$-i\hbar \int_{-\infty}^{\infty} \psi^* (\frac{\partial x(\psi)}{\partial x} - x \frac{\partial \psi}{\partial x}) d$

$\psi + x$

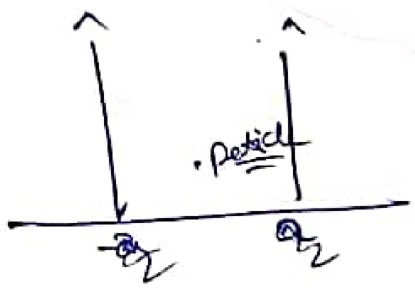
observable

$$\boxed{\begin{aligned} \Delta A &= \sqrt{\langle \hat{A}^2 \rangle - \langle \hat{A} \rangle^2} \\ \langle A^2 \rangle &= \int \psi^* \hat{A}^2 \psi \, d\tau \\ \langle A \rangle^2 &= \left(\int \psi A^* \psi \, d\tau\right)^2 \end{aligned}}$$

operator of A.

Applications of Quantum mechanics

particle in a BOX-1D



$V(x) = \infty \quad x < -\frac{a}{2}$ or $x > \frac{a}{2}$

$\quad 0 \quad |x| < \frac{a}{2}$

$\psi = 0 \quad , |x| \geq \frac{a}{2}$

Task to find ψ , $|x| < \frac{a}{2}$

Simplifying eqⁿs gives us

$$\boxed{\tan(K_2 L) = \frac{2K_1K_2}{K_2^2 - K_1^2}}$$ ** ** **

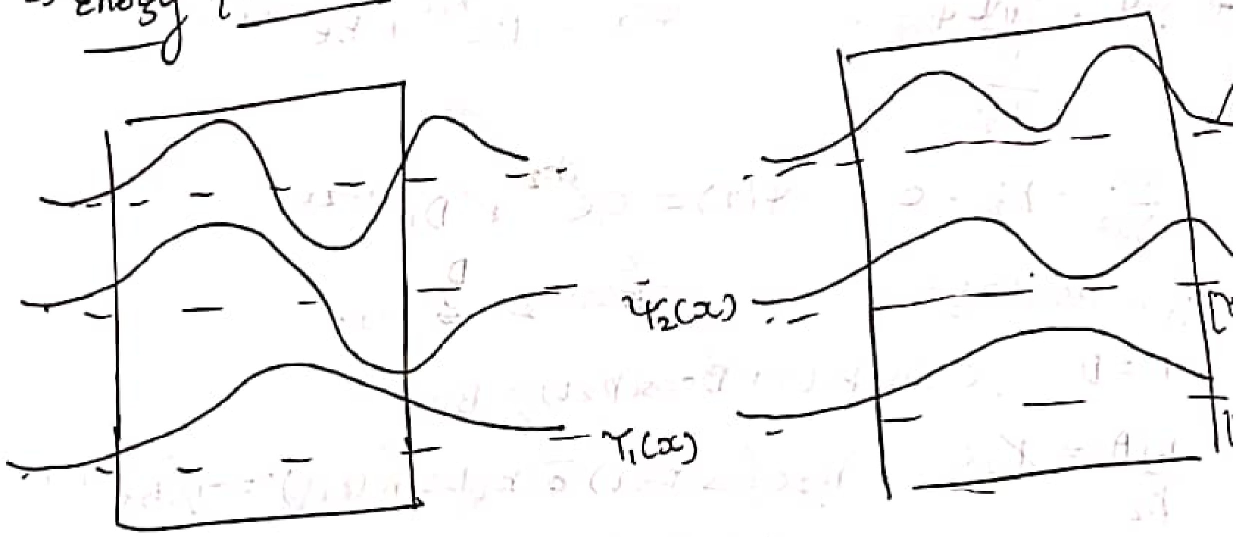
→ ~~This will give one~~ Each point where the particle is allowed to have the value of E where he can survive in to the well.

$$K_2 = \sqrt{\frac{2mE}{\hbar^2}}, \quad K_1 = \sqrt{\frac{2m(V-E)}{\hbar^2}}$$

This is transcendental eqⁿ, meaning it can't be solved analytically for E - only graphically or numerically.

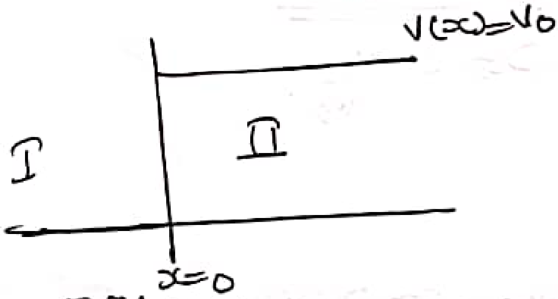
* Bound state wave-functⁿs.

⇒ Energy quantizatⁿ



Potential step:-

*

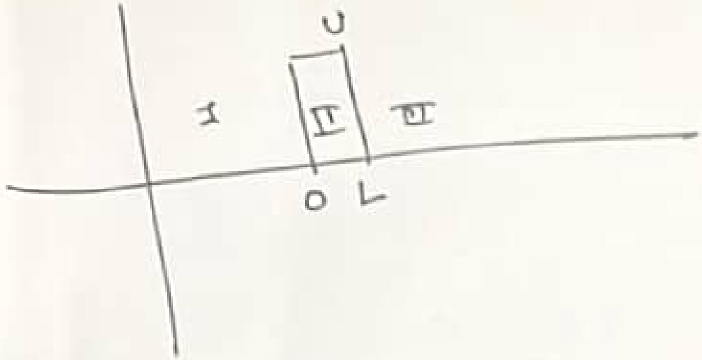


Eqⁿ for $x<0$

$$\frac{\partial^2\psi}{\partial x^2} + \underbrace{\frac{2mE}{\hbar^2}}_{K_2^2}\psi = 0$$

Eqⁿ for $x>0$

$$\frac{\partial^2\psi}{\partial x^2} + \underbrace{\frac{2m(E-V)}{\hbar^2}}_{K_1^2}\psi = 0$$



* Schrodinger eq^n for reg^n II

$$\frac{\partial^2\psi}{\partial x^2} + \frac{2m}{\hbar^2}(E-U)\psi = 0$$

$$= \frac{\partial^2\psi}{\partial x^2} - \underbrace{\frac{2m}{\hbar^2}(U-E)}_{k_1^2}\psi = 0$$

Since $U > E$.

$$\psi_{II} = Ce^{-k_1 x} + De^{k_1 x}$$

$$\psi_I = Ae^{ik_2 x} + Be^{-ik_2 x}$$

$$\psi_{III} = Fe^{ik_2 x}$$

__applying Boundary condit^ns__

$$A + B = C + D.$$

$$ik_2 A - ik_2 B = -k_1 C + k_1 D$$

$$Ce^{-k_1 L} + De^{k_1 L} = Fe^{ik_2 L}$$

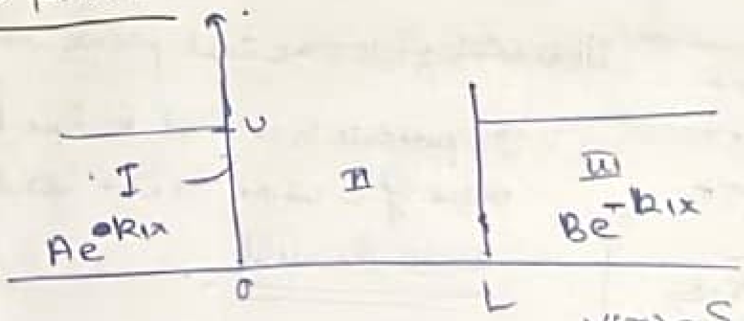
$$-k_1 Ce^{-k_1 L} + k_1 De^{k_1 L} = ik_2 Fe^{ik_2 L}$$

$$\frac{A}{F} = \left[\frac{1}{2} + \frac{i}{4}\left(\frac{k_1}{k_2} - \frac{k_2}{k_1}\right)\right] e^{(ik_2 + k_1)L} \quad \Bigg| \quad k_1 = \frac{\sqrt{2m(U-E)}}{\hbar}$$

$$+ \left[\frac{1}{2} - \frac{i}{4}\left(\frac{k_1}{k_2} - \frac{k_2}{k_1}\right)\right] e^{(ik_2 - k_1)L}$$

$$k_2 = \sqrt{\frac{2mE}{\hbar^2}}$$

Finite potential well



$$V(x) = \begin{cases} N_0 & (-\infty, 0) \\ 0 & (0, L) \\ N_0 & (L, \infty) \end{cases}$$

$$\frac{\partial^2\psi}{\partial x^2} + \frac{2m}{\hbar^2}(E-U)\psi = 0$$

$$\frac{\partial^2\psi}{\partial x^2} - a^2\psi = 0 \quad \begin{matrix} x<0 \\ x>L \end{matrix}$$

$$a = \sqrt{\frac{2m(U-E)}{\hbar}}$$

$$\frac{\partial^2\psi}{dx^2} + k^2 \cancel{(E-V_0)}\psi = 0 \quad \text{I}, \text{III}$$

$$\frac{\partial^2\psi}{dx^2} - \underbrace{\frac{2m(N_0-E)}{\hbar^2}}_{k_1^2}\psi = 0$$

$$\frac{\partial^2\psi}{dx^2} - k_1^2\psi = 0$$

$$\psi(x) = Ae^{k_1x} + Be^{-k_1x}$$

II $$\frac{\partial^2\psi}{\partial x^2} + \underbrace{\frac{2mE}{\hbar^2}}_{k_2^2}\psi = 0$$

$$\frac{\partial^2\psi}{\partial x^2} + k_2^2 = 0 \qquad \psi(x) = Ce^{ik_2x} + De^{-ik_2x}$$

$$= C\sin k_2x + D\cos k_2x.$$

From BC at $x=0$.

$A = D$ $\qquad C\sin(k_2L) + D\cos(k_2L) = Be^{-k_1L}$

$\frac{k_1A}{k_2} = C$ $\qquad k_2C\cos(k_2L) - k_2D\sin(k_2L) = -k_1Be^{-k_1L}$

taking derivative.

$K_1A = K_2C$. ← after derivative

Substituting $A = D \Rightarrow C = \frac{K_1D}{K_2}$

(b) BC at $x=L$.

$$Be^{-K_1L} = C\sin K_2L + D\cos K_2L$$

$$-K_1Be^{-K_1L} = K_2C\cos(K_2L) - K_2D\sin K_2L.$$

De broglie phase velocity $v_p = \nu\lambda = \left(\frac{\gamma mc^2}{h}\right)\left(\frac{h}{\gamma mv}\right) = \frac{c^2}{v}$
phase.

→ particle velocity (v) must be less than c.

→ De Broglie waves travel faster than light.

* Representing De-Broglie waves.

* A group of waves need not to have same velocity as waves themselves.

To describe a wave group mathmatically.
→ Take superposit^n of individual waves of diff wavelength.
→ whose interference with one another results in variat^n in amp. the defines group shape.

If the velocities of waves are same.
→ The velocity with which the wave group travels is common phase velocity

However if v_p varies with λ.
→ Different individual waves do not proceed together.
→ This is called dispersion.
As a result, the wave group has a velocity diff from the phase velocities of the waves that make it up.

* ~~The~~ Find the velocity v_g with which a wave group travels.

*suppose we have two waves

$$y_1 = A\cos(\omega t - kx)$$

$$y_2 = A\cos((\omega+\Delta\omega)t - (k+\Delta k)x)$$

$$y = y_1 + y_2$$
$$= 2A\cos\left[\frac{1}{2}(2\omega+\Delta\omega)t - (2k+\Delta k)x\right)\cos\left(\frac{1}{2}(\Delta\omega t - \Delta k x)\right)$$

Since $\Delta\omega$ & Δk are very small.

$$2\omega + \Delta\omega \approx 2\omega$$
$$2k + \Delta k \approx 2k$$
$$y = 2A\cos(\omega t - kx)\cos\left(\frac{\Delta\omega t}{2} - \frac{\Delta k}{2}x\right)$$

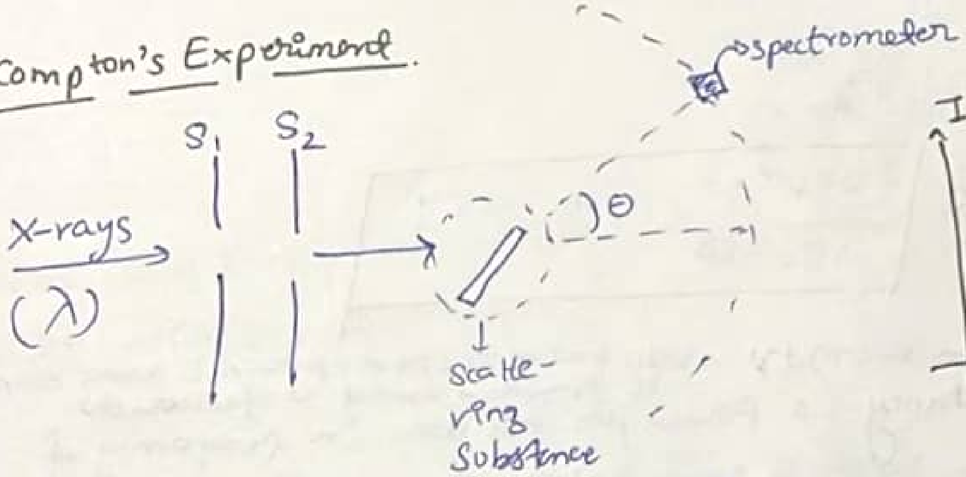
It represents a wave of ang. freq ω & wave num. k that has superimposed upon it a modulat^n of angular freq. $\frac{\Delta\omega}{2}$ & wave no. $\frac{\Delta k}{2}$

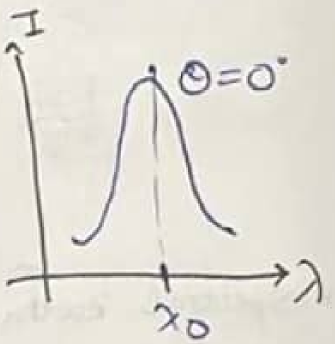
$v_p = \frac{\omega}{k}$, $v_g = \frac{\Delta\omega}{\Delta k}$ or $d\omega/dk$

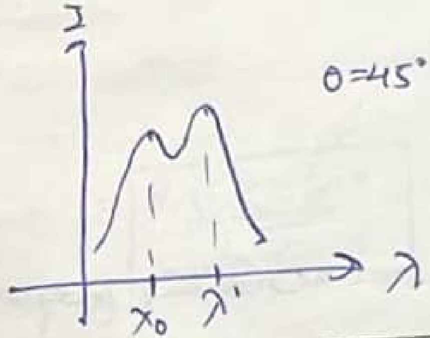
* To pull an electron from a metal surface it takes about half the energy to pull an electron from a free atom of metal.

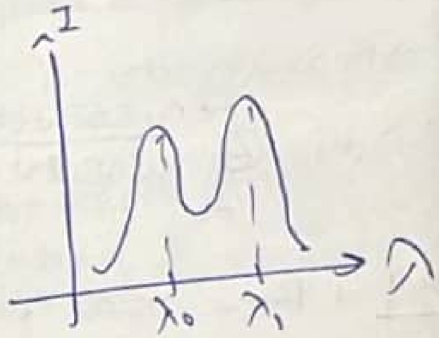
$$\underset{WF}{\phi} = \frac{IE}{2}$$

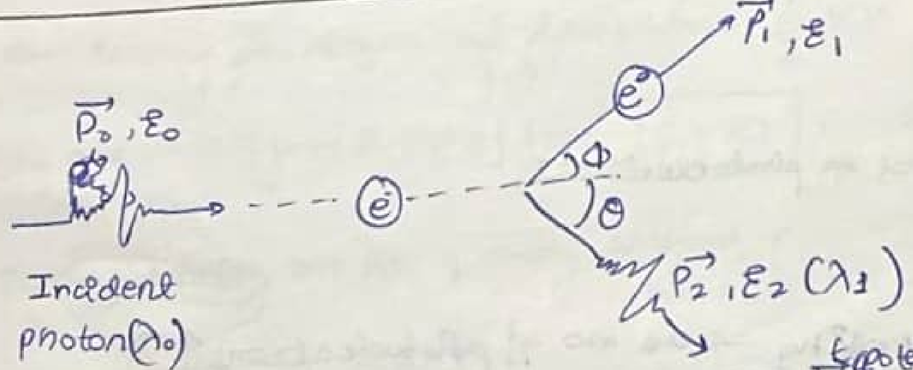
* Compton's Experiment.











$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}, \quad \mathcal{E} = mc^2, \quad \vec{P} = m\vec{v}$$

$$\approx m_0\left(1 + \frac{v^2}{2c^2}\right)$$

Total energy

$$KE = \mathcal{E} - m_0c^2$$

$$\boxed{\mathcal{E}^2 = p^2c^2 + m_0^2c^4}$$

***

For photon

$m_0 = 0$

$\mathcal{E} = h\nu$

$p = \frac{h}{\lambda}$

$$\vec{P_0} = \vec{P_1} + \vec{P_2}$$

$$P_0 = P_1\cos\phi + P_2\cos\theta, \quad P_1\sin\phi = P_2\sin\theta$$

$$P_1 = \frac{h}{\lambda_1}\frac{\sin\theta}{\sin\phi}$$

$$\frac{h}{\lambda_0} = \frac{h\sin\theta\cos\phi}{\lambda_1 \sin\phi} + \frac{h}{\lambda_1}\cos\theta \Rightarrow \frac{1}{\lambda_0} = \frac{1}{\lambda_1}\left(\frac{\sin\theta}{\tan\phi} + \cos\theta\right) \Rightarrow \lambda_1 = \lambda_0(\sin\theta\cot\phi + \cos\theta)$$

The angular frequency and wave number of the de Broglie waves associated with a body of mass m moving with velocity v are

$$\omega = 2\pi\nu = 2\pi \frac{\gamma m c^2}{h}$$

$$= \frac{2\pi m c^2}{h\sqrt{1-\left(\frac{v}{c}\right)^2}}$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi \gamma m v}{h} = \frac{2\pi m v}{h\sqrt{1-\left(\frac{v}{c}\right)^2}}$$

$$k = \frac{2\pi}{h}\gamma m v = \frac{2\pi m v}{h\sqrt{1-\frac{v^2}{c^2}}}$$

$$v_g = \frac{d\omega}{dk} = \frac{\frac{d\omega}{dv}}{\frac{dk}{dv}}$$

So $\frac{d\omega}{dv} = \frac{2\pi m c^3}{(h)}\left(-\frac{1}{2}\right)\left(c^2 - v^2\right)^{3/2}\left(-2v\right)$

$$= \frac{2\pi m c^3 v}{h\left(c^2 - v^2\right)^{3/2}}$$

$$\frac{dk}{dv} = \frac{2\pi m}{h}\left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} + \frac{v}{(2)\left(1-\frac{v^2}{c^2}\right)^{3/2}}\frac{(-2v)}{c^2}\right)$$

$$= \frac{2\pi m}{h}\left(1 + \frac{v^2}{c^2}\right)\left(\frac{1}{\left(1-\frac{v^2}{c^2}\right)^{3/2}}\right)$$

$$= \frac{2\pi m}{h\left(1-\frac{v^2}{c^2}\right)^{3/2}}$$

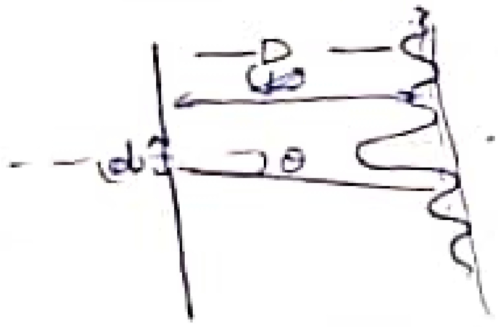
$\boxed{v_g = v \text{ we get}}$ → The de Broglie group wave velocity of a moving body is same as that of

*** Physically a de Broglie wave group is associated to a moving body.

Read Single Slit

* single slit diffraction

* Condition for minima: $d\sin\theta = m\lambda$, $m = \pm1, \pm2, \ldots$
d - slit width



* Central maximum width = $\beta = \frac{2\lambda}{d}$, $\frac{2D\lambda}{d}$, $w = \frac{2D\lambda}{d}$

θ_{max} $\beta = \frac{2\lambda}{d} \rightarrow$ angular width

* Condition for secondary maxima: $d\sin\theta = \frac{(2n+1)\lambda}{2}$, $m = 1, 2, 3 \ldots$

* Intensity distribution formula:-

$$I(\theta) = I_0\left(\frac{\sin(\beta)}{\beta}\right)^2, \quad \beta = \frac{\pi d\sin\theta}{\lambda} \quad ****$$

f^n of θ

Heisenberg uncertainty principles

(i) Position momentum uncertainty

$$\boxed{\Delta x \Delta p \geq \frac{h}{4\pi}}$$

(ii) Energy - time uncertainty

$$\boxed{\Delta E \Delta t \geq \frac{h}{4\pi}}$$

$\psi_n = A \sin \sqrt{\frac{2mE_n}{\hbar}}\, x$

$= A \sin \frac{n\pi x}{L}$

$\int |\psi_n|^2 dx = \int_0^L |\psi_n|^2 dx = A^2 \int \sin^2\left(\frac{n\pi x}{L}\right) dx = 1$

find A. $A = \sqrt{\frac{2}{L}}$

Que: find the probab of a particle being in range 0.45L and 0.55L for the ground state.

$\frac{2}{L} \int_{0.45L}^{0.55L} \sin^2\left(\frac{\pi x}{L}\right) dx$

$\frac{2}{L}\left(\frac{1-\cos\left(\frac{2\pi x}{L}\right)}{2}\right)$

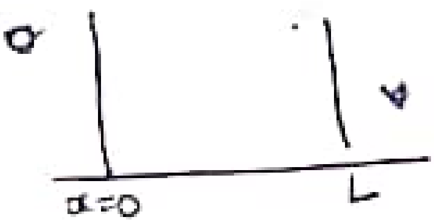
$\left(\frac{1}{L}\right)(0.1L)\left(-\frac{1}{2}\right.$

$\frac{1}{L}\left(0.1L - \left(\frac{L}{2\pi}\right)\left(\sin(2\pi(0.55)) - \sin(2\pi(0.45))\right)\right)$ 1.1π, 0.9π

$0.1 - \frac{1}{2\pi}\left(\sin(1.1\pi) - \sin(0.9\pi)\right)$

$= 19.8\%$

Particle in 1-D box



$V(x) = \begin{cases} 0, & 0<x<L \\ \infty, & \text{elsewhere} \end{cases}$

$\psi(x) \begin{cases} 0, & \text{elsewhere} \\ ?, & 0<x<L \end{cases}$

→ Now Time independent eqn.

$\nabla^2\psi + \frac{2m}{\hbar^2}(E - V^0)\psi = 0$

so $\frac{\partial^2\psi}{\partial x^2} + \underbrace{\frac{2mE}{\hbar^2}}_{\to k^2}\psi = 0$ so general soln:- $\psi(x) = A\sin kx + B\cos kx$

$\psi(0)=0$ so $B=0$

- Find the momentum of the particle in 1-D box

$$k^2 = \frac{2mE}{\hbar^2}$$

$$E = \frac{\hbar^2 k^2}{2m}$$

$$E = \frac{\hbar^2 k^2}{2m} \quad \frac{n^2\pi^2}{L^2}$$

$$E_n = \frac{n^2\pi^2\hbar^2}{2mL^2}.$$

$$\langle p \rangle = \int_{-\infty}^{\infty} \psi^* \hat{p}\, \psi\, dx = \int_{-\infty}^{\infty} \psi^* \left(-i\hbar \frac{\partial}{\partial x}\right) \psi\, dx$$

$$= (A^2)(-i\hbar) \int \sin\left(\frac{n\pi x}{L}\right)\left(\frac{n\pi}{L}\right)\cos\left(\frac{n\pi x}{L}\right) dx$$

$$= -i\hbar \frac{2}{L}\left(\frac{n\pi}{L}\right) \int_0^L \sin\left(\frac{n\pi x}{L}\right)\cos\left(\frac{n\pi x}{L}\right) dx$$

$$= \frac{-2n\pi i\hbar}{L^2}\left(\frac{\sin^2 \frac{n\pi}{L}x}{2n\pi}\right)\Bigg|_0^L$$

$$= \frac{-i\hbar}{L} \qquad = 0.$$

$\langle p \rangle = 0 \rightarrow$ strange, seems strange.

$$E = \frac{p^2}{2m} \quad \boxed{\text{as } p = \hbar k \;\big|\; \text{so } p = \pm\sqrt{2mE_n}}$$

$$\boxed{P_n = \pm\sqrt{2mE_n} = \pm\frac{n\pi\hbar}{L}}$$

$$\text{So } P_{avg} = \frac{\frac{n\pi\hbar}{L} - \frac{n\pi\hbar}{L}}{2} = 0$$

- Momentum eigenfn

as a Hermitian eigenfn satisfies $\hat{\Omega}\psi_n(x) = \omega_n\psi_n(x)$

$$\text{So } \boxed{\hat{p}\,\Phi_p(x) = p\,\Phi_p(x)}$$

$$-i\hbar \frac{\partial \phi_p(x)}{\partial x} = p\phi_p(x)$$

$$\frac{\partial \phi_p(x)}{\partial x} = \frac{i p \phi_p(x)}{\hbar}$$

$$\boxed{\phi_p(x) = A e^{\frac{ipx}{\hbar}}}$$

superposition of + mom

$$\psi(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right) = \sqrt{\frac{2}{L}}\frac{1}{2i}\left(e^{\frac{in\pi x}{L}} - e^{-\frac{in\pi x}{L}}\right)$$