Lecture-18

29/09/23.

Q.1 ⇒ An electric field in free space is given by $E = 50 \cos(10^8 t + \beta x)\, a_y$ V/m

(a) ⇒ Find the direction of wave propagation.

(b) ⇒ Calculate β & the time it takes to travel a distance of $\lambda/2$.

(c) ⇒ Sketch the wave at $t = 0$, $T/4$ & $T/2$.

Ans ⇒ (a) ⇒ Due to +ve sign in $(\omega t + \beta x)$, the wave is travelling in $-a_x$ direction

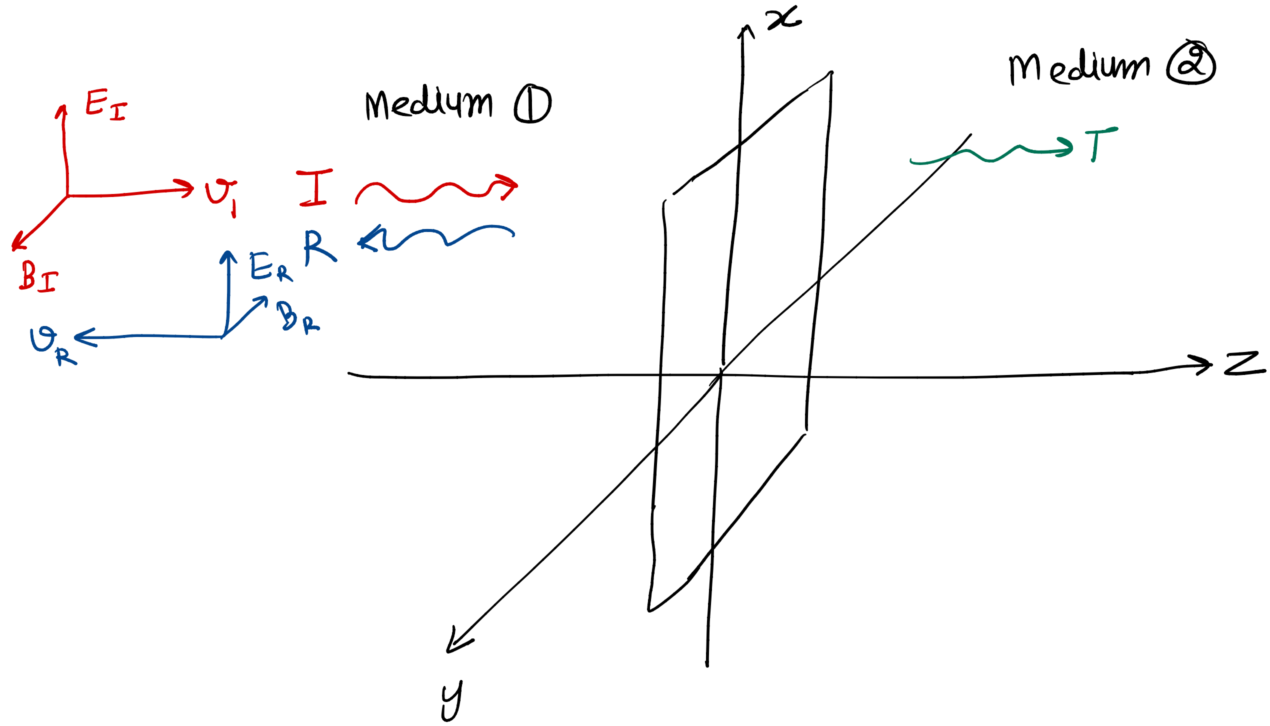
(B) ⇒ $\beta = \omega/c \Rightarrow \dfrac{10^8}{3 \times 10^8} = 1/3$ rad/m.

$t_1 = T/2 = \frac{1}{2}\, 2\pi/\omega = 31.42$ nsec.

(C) ⇒ ______ Homework

P.T.O.

Reflection & Transmission of plane wave at Normal incidence :→



* xy plane is the boundary of two linear medium.
* A plane wave of frequency ω, travelling in the z direction & polarized in the x direction, approaches the interface from the left.

Incident wave →

$$\left.\begin{aligned} \vec{E}_I(z,t) &= E_{0I}\, e^{i(k_1 z-\omega t)}\,\hat{x} \\ \vec{B}_I(z,t) &= \frac{E_{0I}}{v_1}\, e^{i(k_1 z-\omega t)}\,\hat{y} \end{aligned}\right\} \text{——} ①$$

Reflected wave :→

$$\left.\begin{aligned} \vec{E}_R(z,t) &= E_{0R}\, e^{i(-k_1 z-\omega t)}\,\hat{x} \\ \vec{B}_R(z,t) &= -\frac{E_{0R}}{v_1}\, e^{i(-k_1 z-\omega t)}\,\hat{y} \end{aligned}\right\} \text{——} ②$$

Transmitted wave :→

$$\left.\begin{aligned} \vec{E}_T(z,t) &= E_{0T}\, e^{i(k_2 z-\omega t)}\,\hat{x} \\ \vec{B}_T(z,t) &= \frac{E_{0T}}{v_2}\, e^{i(k_2 z-\omega t)}\,\hat{y} \end{aligned}\right\} \text{——} ③$$

Boundary Condⁿ [at z=0]

$$E_1^{\parallel} = E_2^{\parallel} \text{ —— } (a_1) \qquad \& \qquad D_1^{\perp} = D_2^{\perp} \text{ —— } (a_2)$$

$$B_1^{\perp} = B_2^{\perp} \text{ —— } (a_3) \qquad\qquad H_1^{\parallel} = H_2^{\parallel} \text{ —— } (a_4)$$

(a_2) & (a_3) will give you $[0=0]$

Eq^n (a_1) $\Rightarrow$

$$E_{0I}\, e^{i(k_1 z-\omega t)}\,\hat{x} + E_{0R}\, e^{i(-k_1 z-\omega t)}\,\hat{x}$$

$$= E_{0T}\, e^{i(k_2 z-\omega t)}\,\hat{x}$$

[Put $z=0$]

$$E_{0I} + E_{0R} = E_{0T} \quad \text{——— (1)}$$

Eq^n (a_4) $\Rightarrow$ $H_1^{\parallel} = H_2^{\parallel}$

$$\frac{1}{\mu_1} B_1^{\parallel} = \frac{1}{\mu_2} B_2^{\parallel}$$

$$\frac{1}{\mu_1}\left[\frac{1}{v_1} E_{0I}\, e^{i(k_1 z-\omega t)}\,\hat{y} - \frac{1}{v_1} E_{0R}\, e^{i(-k_1 z-\omega t)}\,\hat{y}\right]$$

$$= \frac{1}{\mu_2}\left[\frac{1}{v_2} E_{0T}\, e^{i(k_2 z-\omega t)}\,\hat{y}\right]$$

[Put $z=0$]

$$\frac{1}{\mu_1}\left[\frac{1}{v_1} E_{0I} - \frac{1}{v_1} E_{0R}\right] = \frac{1}{\mu_2}\left[\frac{1}{v_2} E_{0T}\right]$$

$$E_{0I} - E_{0R} = \frac{\mu_1 v_1}{\mu_2 v_2} E_{0T}$$

$$\beta = \frac{\mu_1 v_1}{\mu_2 v_2} = \frac{\mu_1 n_2}{\mu_2 n_1}$$

$$E_{0I} - E_{0R} = \beta E_{0T} \quad \text{——— (2)}$$

$$E_{0I} + E_{0R} = E_{0T} \quad \text{——— (1)} \quad \times \beta$$

$$E_{0I} - E_{0R} = \beta E_{0T} \quad \text{——— (2)}$$

substract

$$\beta E_{0I} - E_{0I} + \beta E_{0R} + E_{0R} = 0$$

$$E_{0R}[\beta+1] = E_{0I}[1-\beta]$$

$$E_{0R} = \left(\frac{1-\beta}{1+\beta}\right) E_{0I}$$

Similarly

$$E_{0T} = \left(\frac{2}{1+\beta}\right) E_{0I}$$

——— (3)

For Nonmagnetic medium $\mu_1 = \mu_0 = \mu_2$

$$\beta = \frac{v_1}{v_2} = \frac{n_2}{n_1}$$

Hence,

$$E_{0R} = \left(\frac{v_2 - v_1}{v_2 + v_1}\right) E_{0I} \quad \& \quad E_{0T} = \left(\frac{2v_2}{v_2 + v_1}\right) E_{0I}$$

in term of refractive index.

$$\boxed{E_{0R} = \left(\frac{n_1 - n_2}{n_1 + n_2}\right) E_{0I}} \quad \& \quad \boxed{E_{0T} = \left(\frac{2n_1}{n_1 + n_2}\right) E_{0I}}$$

Reflection coefficient $R = \frac{I_R}{I_I}$ &

Transmission " $T = \frac{I_T}{I_I}$.

$$\boxed{I = \frac{1}{2} \varepsilon v E_0^2}$$

$$\boxed{R = \frac{I_R}{I_I} = \left(\frac{E_{0R}}{E_{0I}}\right)^2 = \frac{(n_1 - n_2)^2}{(n_1 + n_2)^2}}$$

$$T = \frac{I_T}{I_I} = \frac{\varepsilon_2 v_2}{\varepsilon_1 v_1} \left(\frac{E_{0T}}{E_{0I}}\right)^2 = \boxed{\frac{4 n_1 n_2}{(n_1 + n_2)^2} = T}$$

$$\Rightarrow R + T = 1$$

Ex:→ When light passes from air ($n=1$) into glass ($n_2 = 1.5$)

Show $R = 4\%$, & $T = 96\%$.