Quantum Theory

19-22 Rayleigh-Jeans Law, number density of standing waves in 1-, 2-, and 3-D, Planck's law of blackbody radiation, Stefan's law, Wein's displacement law.

23-27 Photoelectric effect, Compton effect, Frank-Hertz experiment, de Broglie waves, wave packet, phase and group velocities, Davisson-Germer experiment, uncertainty principle (single slit thought experiment).

28-32 Basic postulates of quantum mechanics and physical meaning of wave functions, Schrödinger wave equation, stationary states, expectation values, probability current density.

33-42 Particle in a 1-D box, 1-D step potential, reflection and transmission by a barrier and tunneling, 1-D linear harmonic oscillator (direct solution but comparison with classical physics is required).

Concepts of Modern Physics, Arthur Beiser, Tata McGraw Hill, *8th ed* (2024).

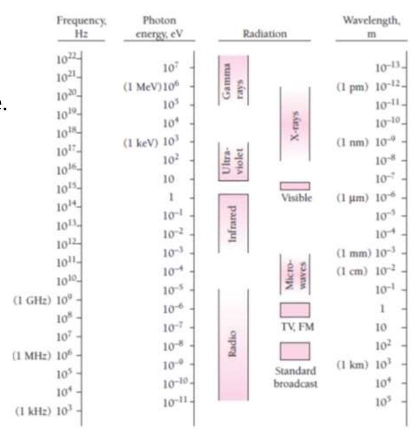
Maxwell predicted that EM waves should exist that travel with the speed of light

Heinrich Hertz, through his experiment, determined the **wavelength** and **speed of the waves** he generated

➔ Showed that it has both electric and magnetic components
➔ Found that they could be reflected, refracted, and diffracted

- ✓ Light is not the only example of an EM wave.
- ✓ All waves have the same fundamental nature.
- ✓ Their interactions with matter depend upon their frequencies

Interference and diffraction are found only in waves !!



From the classical theory, it was established that
Light consisted of EM waves that obeyed Maxwell's theory

Quantum Theory

Quantum theory arose from the study of radiation emitted from heated bodies

The first sign that something was seriously amiss came from attempts to understand the origin of the radiation emitted by bodies of matter.

- **The ability of a body to radiate is closely related to its ability to absorb radiation.**
- **It is expected!**

➔ Since a body at a constant temperature is in thermal equilibrium with its surroundings, it must absorb energy from them at the same rate as it emits energy.

- It can be considered that an ideal body is one that absorbs *all* radiation incident upon it, regardless of frequency.

➔ Such a body is called a **blackbody**

Point of introducing idealized blackbody in the discussion of thermal radiation is that we can now disregard the precise nature of whatever is radiating, since <u>all blackbodies behave identically</u>!

Generalisation of classical physics

➔ Classical laws should come out as limiting cases

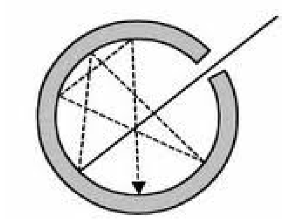
Thermal radiation:
The radiation emitted by a body as a result of its temperature is called thermal radiation

Perfect Blackbody:
Absorbs all thermal radiation incident on it.

Example: Lamp black, nearly a perfect blackbody

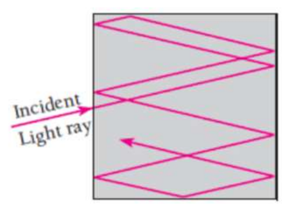
But, how does one construct a **blackbody** for experimental purposes in the laboratory?

Cavity : with walls blackened by lamp black



- Radiation incident at the small hole
- Reflected back and forth by the walls
- Eventually absorbed by the walls of the cavity

A negligible amount of radiation is reflected back through the hole



Essentially, all the radiation incident on the hole is absorbed
➔ property of a perfect blackbody

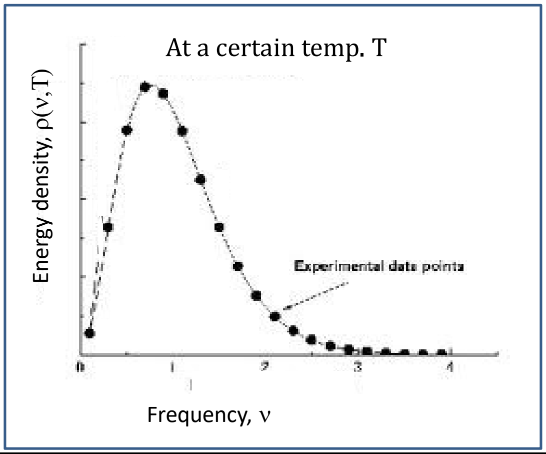
The cavity walls are constantly emitting and absorbing radiation
➔This radiation (blackbody radiation) is our interest

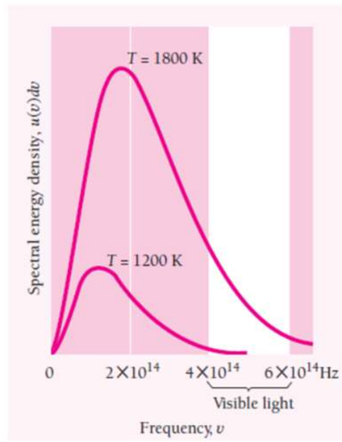
Theory of cavity radiation

The energy density of cavity radiation (or blackbody radiation) :

$$\rho(\nu, T)$$

frequency Temperature

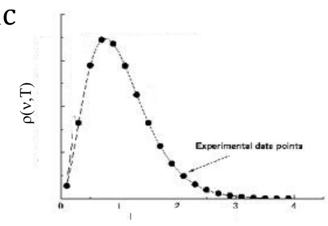




Theory of cavity radiation

Explanations from classical electromagnetic theory (classical physics)

1) Wilhelm Wien
2) Rayleigh and Jeans

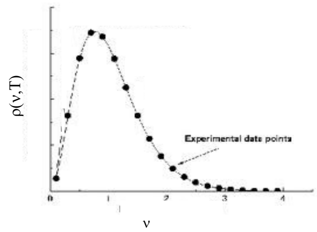


Wien's frequency distribution Law:
Conjectured that the energy density can be related to Maxwell's velocity distribution function for molecules of an ideal gas.
(*not to be confused with Wien's displacement law*)

$$\rho(\nu, T) = a\, \nu^3 e^{-b\nu/T}, \quad a, b \text{ are constants}$$

This formula agrees well with data at high frequencies only.

Theory of cavity radiation



Explanations from classical electromagnetic theory (classical physics)

1) Wilhelm Wien
2) Rayleigh and Jeans

Rayleigh-Jeans Law:

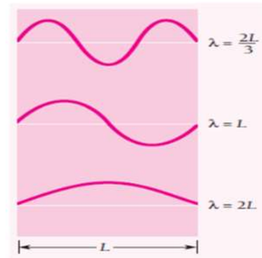
First due to Rayleigh and later modified by James Jeans
Developed rigorously from classical physics
Involves no conjectures or arbitrary constants

Calculated the **Density of standing waves in a cavity**

Energy density = (Density of standing waves in a cavity) × (Classical average energy per standing wave)

$$\rho(\nu,T) = \frac{8\pi kT}{c^3}\,\nu^2,\quad k \text{ is the Boltzmann constant}$$

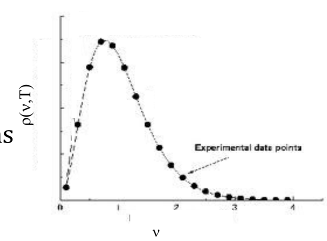
This formula agrees well with the data only at <u>low frequencies</u>



EM radiation in a cavity whose walls are perfect reflectors consists of standing waves that have nodes at the walls, which restricts their possible wavelengths.

Theory of cavity radiation

Rayleigh-Jeans Law:



First due to Rayleigh and later modified by James Jeans
Developed rigorously from classical physics
Involves no conjectures or arbitrary constants

$$\rho(\nu,T) = \frac{8\pi kT}{c^3}\,\nu^2,\quad k \text{ is the Boltzmann constant}$$

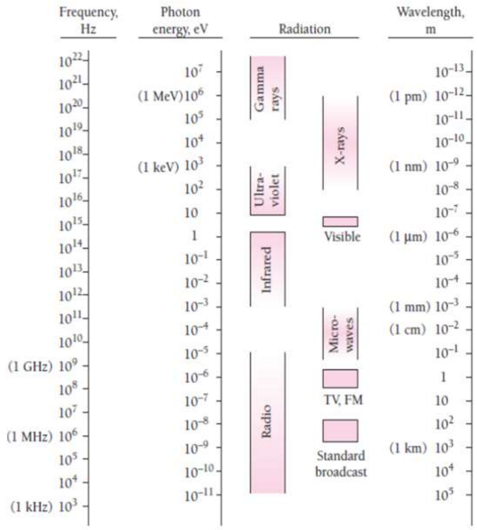
With R–J law: $\rho(\nu,T) \to \infty$ as $\nu \to \infty$

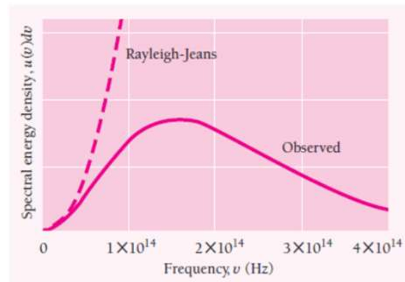
In reality, of course, the energy density (and radiation rate) falls to 0 as $\nu \to \infty$

Discrepancy

This discrepancy is known as the **Ultraviolet catastrophe**

R-J formula agrees well with the data only at <u>low frequencies</u>.

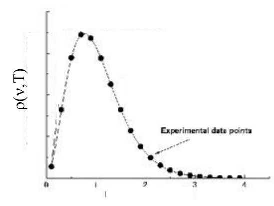




Comparison of the Rayleigh-Jeans formula of the radiation from a blackbody at 1500 K with the observed spectrum.

The failure of classical physics led Planck to the discovery that radiation is emitted in quanta whose energy is *hν*.

Wien's frequency distribution formula



$\rho(\nu, T) = a\, \nu^3 e^{-b\nu/T}$, a, b are constants

It agrees well with the data at high frequencies only.

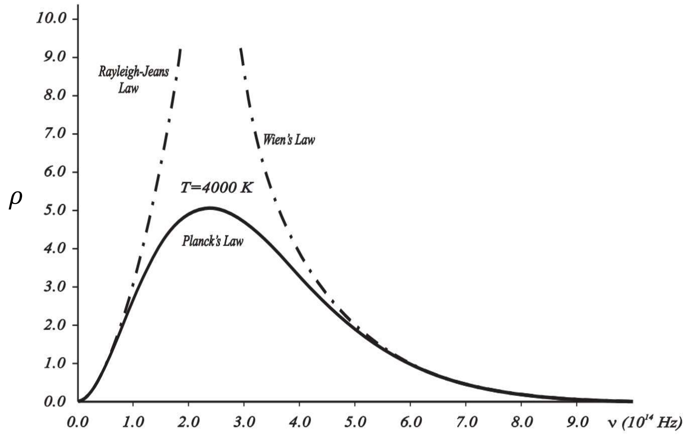
Rayleigh-Jeans' Law:

$$\rho(\nu, T) = \frac{8\pi k T}{c^3}\, \nu^2, \quad k \text{ is the Boltzmann constant}$$

This formula agrees well with the data only at low frequencies.

Max Planck (attempt around 1900):
***Interpolation* between these two classical limits and put forward a third formula**

$$\rho(\nu, T) = a\, \nu^3 \frac{1}{e^{b\nu/T} - 1}, \quad a, b \text{ are constants}$$



Max Planck (Interpolation):
Interpolation between these two classical limits, and put forward a third formula

$$\rho(\nu, T) = a\, \nu^3 \frac{1}{e^{b\nu/T} - 1}, \quad a, b \text{ are constants}$$

➔ Agrees quite well with the data.

Max Planck (Interpolation): Agrees quite well with data.

$$\rho(\nu, T) = a\, \nu^3 \frac{1}{e^{b\nu/T} - 1}, \quad a, b \text{ are constants}$$

Soon after this, Planck showed that

If energy were emitted, *not continuously* but in *packets* of

E = nhν   where, n =1, 2, 3, ...   h = constant

Then, the spectral energy density can be rigorously derived without any arbitrary constants

$$\boxed{\rho(\nu, T) = \frac{8\pi h \nu^3}{c^3} \frac{1}{e^{h\nu/kT} - 1}}$$

Bose-Einstein Distribution Functions for BOSONS. Photons are BOSONS with spin 1

Although Planck got the right formula, it had flaws!!

Limiting cases of Planck's Law

$$\rho(\nu,T) = \frac{8\pi h}{c^3}\nu^3 \frac{1}{e^{h\nu/kT}-1}$$

At large frequencies, $h\nu >> kT$

$$e^{h\nu/kT} - 1 = e^{h\nu/kT}$$

$$\rho(\nu,T) = \frac{8\pi h}{c^3}\nu^3 e^{-h\nu/kT}$$

Compare with Wien's frequency distribution formula

$\rho(\nu,T) = a\ \nu^3 e^{-b\nu/T}$, a, b are constants

$$\boxed{a = \frac{8\pi h}{c^3},\quad b = h/k}$$

Limiting cases of Planck's Law

$$\rho(\nu,T) = \frac{8\pi h\nu^3}{c^3}\frac{1}{e^{h\nu/kT}-1}$$

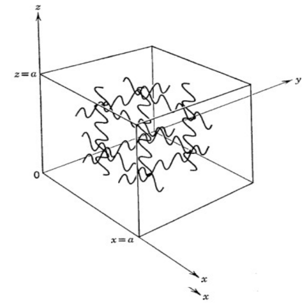
At small frequencies, $h\nu << kT$

$$e^x = 1 + x + \frac{x^2}{2} + \ldots \qquad = 1 + x + O(x^2)$$

$$\rho(\nu,T) \simeq \frac{8\pi h}{c^3}\nu^3 \frac{1}{1+\frac{h\nu}{kT}-1}$$

$$= \frac{8\pi kT}{c^3}\nu^2 \quad \text{(Rayleigh–Jeans)}$$

Rayleigh-Jeans Law



- ✓ A cavity with metallic walls is heated uniformly to a temperature T
- ✓ The walls emit EM radiation

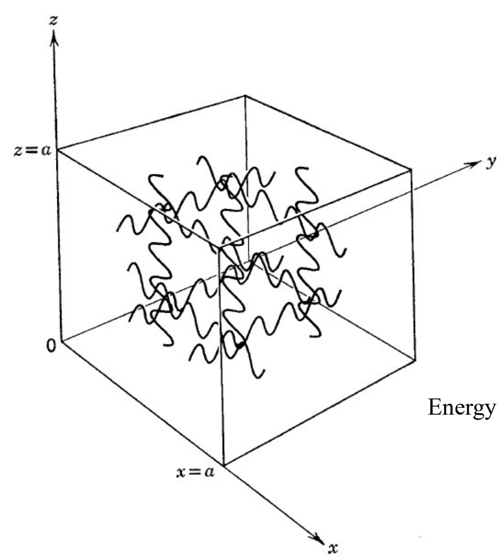
R-J : classical electromagnetic theory :
Radiation inside the cavity exists in the form of standing waves with nodes at the metallic surface

By geometrical arguments:

1. Count the number of standing waves within the frequency interval **ν & ν + dν**
   (frequency dependence of waves)
2. Use the kinetic theory (classical) to calculate the average total energy of these waves

Energy density in frequency interval **ν & ν + dν**

$$\mathrm{u}(\nu, T)\mathrm{d}\nu = \frac{\bar{\epsilon}\, N(\nu)d\nu}{Volume\ of\ cavity}$$

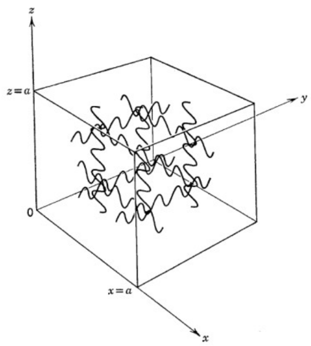


Energy density in frequency interval **ν & ν + dν**

$$\mathrm{u}(\nu, T)\mathrm{d}\nu = \frac{\bar{\epsilon}\, N(\nu)d\nu}{Volume\ of\ cavity}$$

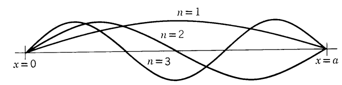
A metallic walled cubical cavity filled with electromagnetic radiation, showing three noninterfering components of that radiation bouncing back and forth between the walls and forming standing waves with nodes at each wall.

Count of standing waves within the frequency interval ν & ν + dν



For a node to occur at each wall
Path length from wall to wall = integer × half-wavelength

In 1-D: $2a/\lambda = n$



In 3-D : $\left(\frac{2a}{\lambda}\right)^2 = n_x^2 + n_y^2 + n_z^2$

Allowed frequencies : $\nu = \frac{c}{2a}\sqrt{n_x^2 + n_y^2 + n_z^2} = \frac{c}{2a} r$

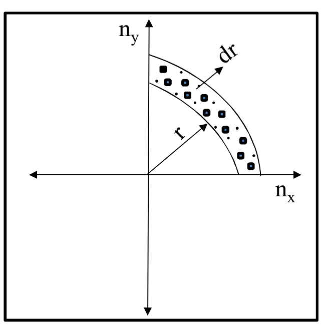
where, $r^2 = n_x^2 + n_y^2 + n_z^2$

Therefore, no. of *allowed frequencies* == no. of *allowed values* of ***r***

How to count *r*?

2-D case: **Allowed frequencies :** $\nu = \frac{c}{2a}\sqrt{n_x^2 + n_y^2} = \frac{c}{2a} r$

where, $r^2 = n_x^2 + n_y^2$



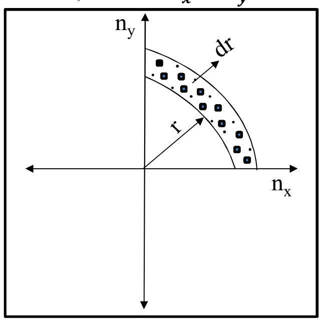
No. of points in *shaded* area
= Area × no. of points per unit area (=1)
= Area

No. of points in *shaded* area
= no. of *allowed values* of r in the range r to $r + dr$, : $N(r)\,dr$
= no. of *allowed frequencies* in the range **ν to ν + dν** : $N(\nu)\,d\nu$

2-D case: **Allowed frequencies**

$$\boldsymbol{\nu = \frac{c}{2a}\sqrt{n_x^2 + n_y^2} = \frac{c}{2a} r}$$

where, $\boldsymbol{r^2 = n_x^2 + n_y^2}$



No. of points in *shaded* area
= No. of *allowed values* of *r* in the range *r* to *r* +*dr*, : *N(r)dr*
= no. of *allowed frequencies* in the range **ν to ν+dν** : *N(ν)dν*

$$\mathrm{N}(r)dr = \frac{1}{4}(2\pi r)dr$$

$$= \frac{\pi}{2} r dr$$

$$= \frac{\pi}{2}\left(\frac{2a}{c}\nu\right)\left(\frac{2a}{c}d\nu\right)$$

$$\mathrm{N}(\nu)d\nu = \frac{2\pi}{c^2}\, a^2 \nu d\nu$$

With two independent perpendicular directions of polarisation of the wave:
Multiply the above by 2

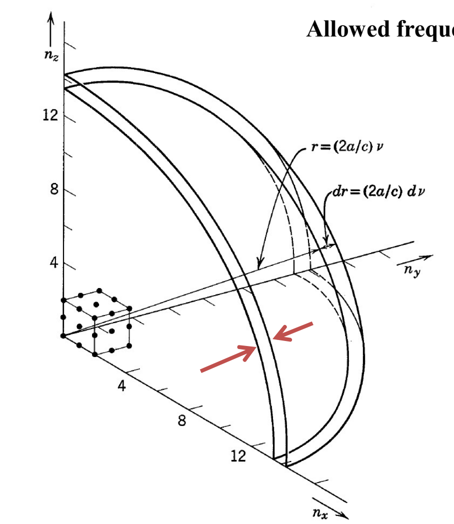
$$\mathrm{N}(r)dr = \frac{4\pi}{c^2}\, a^2 \nu d\nu$$

No. density of standing waves : $\dfrac{N(\nu)d\nu}{a^2}$

3-D case:

Allowed frequencies : $\nu = \dfrac{c}{2a}\sqrt{n_x^2 + n_y^2 + n_z^2} = \dfrac{c}{2a} r$

where, $r^2 = n_x^2 + n_y^2 + n_z^2$



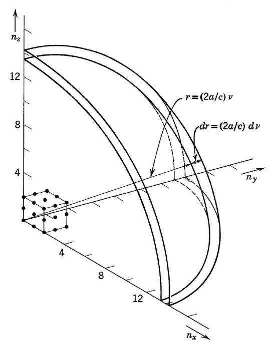
No. of *allowed frequencies*: *N(ν)dν*
= no. of *allowed values* of ***r*** between radii
r to *r* +*dr* : *N(r)dr*

The allowed frequencies in a three-dimensional cavity in the form of a cube of edge length "*a*" are determined by three indices n_x , n_y, n_z, which can each assume only Integral values.

<u>3-D case:</u>

Allowed frequencies : $\nu = \frac{c}{2a}\sqrt{n_x^2 + n_y^2 + n_z^2} = \frac{c}{2a}r$

where, $r^2 = n_x^2 + n_y^2 + n_z^2$



No. of *<u>allowed frequencies</u>* : $N(\nu)d\nu$
= no. of *<u>allowed values</u>* of ***r*** between radii
r to *r* +*dr* : *N(r)dr*

$$N(r)dr = \frac{1}{8}(4\pi r^2)dr$$

$$= \frac{\pi}{2}\left(\frac{2a\nu}{c}\right)^2 \cdot \frac{2a}{c}d\nu$$

$$= \frac{4\pi}{c^3}a^3\,\nu^2\,d\nu$$

Also taking into account two perpendicular directions of polarisation: multiply above by 2

$$N(\nu)d\nu = \frac{8\pi}{c^3}a^3\,\nu^2\,d\nu$$

Number density of standing waves : $\frac{N(\nu)d\nu}{a^3}$

Final step: average energy per standing wave

Classical theory:
Each wave originates due to the thermal motion of an *oscillator* in the cavity wall.

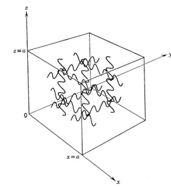
Such an oscillator has two degrees of freedom
Kinetic energy
Potential energy

<u>Classical Equipartition theorem of energy</u>:
Average energy per degree of freedom of an entity in thermal equilibrium at temperature T = ½ kT (k → Boltzmann constant)

Therefore, average energy (corresponding to two degrees of freedom) $\overline{\epsilon} = kT$

Remember:
The Classical Equipartition theorem of energy is valid for a continuous distribution of energies.

Rayleigh-Jeans Law



By geometrical arguments:

Count the number of standing waves in the frequency interval **ν & ν + dν**

Use the kinetic theory (classical) to calculate the average total energy of these waves

Energy density in frequency interval **ν to ν + dν** $\mathrm{u}(\nu, T)\mathrm{d}\nu = \dfrac{\bar{\epsilon}\, N(\nu)d}{Volume\ of\ cavity}$

$$u(\nu, T)d\nu = \frac{kT}{a^3}\frac{8\pi}{c^3}a^3\,\nu^2\,d\nu$$

$$= \frac{8\pi kT}{c^3}\nu^2\,d\nu$$

Average energy per standing wave (Planck)

Each wave originates due to the thermal motion of an ‘oscillator’ in the cavity wall.

Then R-J used the Classical Equipartition theorem of energy: valid for a continuous distribution of energies

Quantum theory:

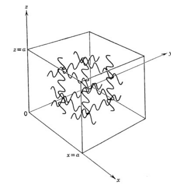
Planck: The ‘oscillators’ no longer have a continuous distribution of energies
.... but energies chosen from a discrete set

$E = nh\nu, \quad n = 1, 2, 3, \ldots \quad h = \text{constant}$

Under this assumption, Planck showed that the average energy of an ‘oscillator’ is

$$\boxed{\bar{\epsilon} = \frac{h\nu}{e^{h\nu/kT} - 1}}$$

Planck's Law



By geometrical arguments:

Count the number of standing waves in the frequency interval **ν** & **ν + dν**

Thee average energy of these waves

Energy density in frequency interval **ν** to **ν + dν** $\quad \mathrm{u}(\nu, T)\mathrm{d}\nu = \dfrac{\bar{\epsilon}\, N(\nu)d}{Volume\ of\ cavity}$

$$\bar{\epsilon} = \frac{h\nu}{e^{h\nu/kT} - 1}$$

$$N(\nu)d\nu = \frac{8\pi a^3}{c^3}\nu^2\, d\nu$$

$$\boxed{u(\nu, T) = \frac{8\pi h}{c^3}\frac{\nu^3}{e^{h\nu/kT} - 1}}$$

Derivation of Wien's Law and Stefan-Boltzmann Law from Planck's Law

Derivation of Wien's displacement Law

Start from Planck's Law $\quad u(\nu, T)d\nu = \dfrac{8\pi h}{c^3}\dfrac{\nu^3}{e^{h\nu/kT} - 1}d\nu$

Express in terms of λ : $\quad u(\lambda)d\lambda = -u(\nu)d\nu$

$$= -\frac{8\pi h}{c^3}\frac{c^3}{\lambda^3}\frac{1}{e^{hc/\lambda kT} - 1}\left(-\frac{c}{\lambda^2}d\lambda\right)$$

$$= \frac{8\pi hc}{\lambda^5}\frac{d\lambda}{e^{hc/\lambda kT} - 1}$$

At a given temperature (T=Constant), find λ_{max} for which the energy density is maximum

Derivation of Wien's displacement Law

$$u(\lambda)d\lambda = \frac{8\pi hc}{\lambda^5}\frac{d\lambda}{e^{hc/\lambda kT}-1}$$

$$\frac{du(\lambda)}{d\lambda}_{at\,\lambda_{max}} = 0$$

$$\frac{du(\lambda)}{d\lambda} = 8\pi hc\left[-\frac{5}{\lambda^6}\frac{1}{e^{hc/\lambda kT}-1} + \frac{1}{\lambda^5}\frac{-e^{hc/\lambda kT}}{(e^{hc/\lambda kT}-1)^2}\left(\frac{-hc}{\lambda^2 kT}\right)\right] = 0$$

$\frac{1}{\lambda^6}\frac{8\pi hc}{e^{hc/\lambda kT}-1}$ is the common factor (eliminated)

$$-5 + \frac{hc}{\lambda kT}\frac{e^{hc/\lambda kT}}{(e^{hc/\lambda kT}-1)} = 0$$

Put $\frac{hc}{\lambda kT} = x$

$$-5 + x\frac{e^x}{(e^x-1)} = 0$$

$$-5(e^x-1) + xe^x = 0$$

$$(1-e^{-x}) = \frac{x}{5}$$

Above Eqn. is Transcendental Eqn. (No analytic Solution) which have only numerical solutions.

x=4.965 (another solution is x = 0)

$$x = \frac{hc}{\lambda kT} = 4.965$$

$$\lambda_m T = 0.2014\frac{hc}{k} = 2.878\times10^{-3}\text{ m.K}$$

$\lambda_m T$ = constant **Wien's displacement Law**

Stefan-Boltzmann Law

Total spectral energy density of blackbody radiation

$$u_{tot} = \int_0^\infty u(\nu)d\nu$$

$$= \frac{8\pi h}{c^3}\int_0^\infty \frac{\nu^3}{(e^{h\nu/kT}-1)}d\nu$$

Put $x = \frac{h\nu}{kT}$ $dx = \frac{hd\nu}{kT}$ $\nu = \frac{kTx}{h}$

$$= \frac{8\pi h}{c^3}\left(\frac{kT}{h}\right)^4\int_0^\infty \frac{x^3}{(e^x-1)}dx$$

$$\int_0^\infty \frac{x^3}{(e^x-1)}dx = \frac{\pi^4}{15}$$

$$= \frac{8\pi^5 k^4}{15c^3h^3}T^4 = a\,T^4$$

$$\boxed{u_{tot} \propto T^4}$$

In the spectral distribution of black body radiation,
***Spectral Radiancy*_ $R_T(\nu)\,d\nu$** is defined as
Energy emitted per unit time, per unit area in frequency interval ν & $\nu + d\nu$ at temperature T

$$R_T(\nu)d\nu \propto u(\nu)d\nu$$

$$R_T(\nu)d\nu = e\frac{c}{4}u(\nu)d\nu$$

Emissivity

Emissivity (e) depends on the nature of the radiating surface,
➔ e ranges between 0 & 1
0 for a perfect reflector (which does not radiate at all)
1, for a blackbody.
➔ Some typical values of e are:
0.07 for polished steel,
0.6 for oxidized copper and brass,
0.97 for matte black paint.

Radiancy $R_T = \int_0^\infty R_T(\nu)d\nu = \frac{ec}{4}\int_0^\infty u(\nu)d\nu$

Energy emitted per unit time, per unit area at temp. T

Energy emitted per unit time, per unit area at temp. T

$$R_T = \int_0^\infty R_T(v)dv = \frac{e\,c}{4}\int_0^\infty u(v)dv$$

$$= \frac{eca}{4}T^4$$

$$\boxed{R_T = e\sigma T^4}$$ **Stefan-Boltzmann Law**

σ =**Stefan-Boltzmann constant =** $\frac{ca}{4}$

$$= \frac{2\pi^5 k^4}{15c^2h^3}$$

= 5.670 x 10^{-8} W/m^2 .K^4

Derivation of the average energy in Planck's formula

Average energy:
$$\bar{\mathscr{E}} = \frac{\sum_{n=0}^{\infty} \mathscr{E} P(\mathscr{E})}{\sum_{n=0}^{\infty} P(\mathscr{E})}$$

$$P(\mathscr{E}) = \frac{e^{-\mathscr{E}/kT}}{kT}$$

$P(\mathscr{E})\,d\mathscr{E}$ is the probability of finding a given entity of a system with energy in the interval between $\mathscr{E}$ and $\mathscr{E} + d\mathscr{E}$,

$\mathscr{E} = nhv$ where $n = 0, 1, 2, 3, \ldots.$

$$\bar{\mathscr{E}} = \frac{\sum_{n=0}^{\infty} \frac{nh\nu}{kT} e^{-nh\nu/kT}}{\sum_{n=0}^{\infty} \frac{1}{kT} e^{-nh\nu/kT}} = kT \frac{\sum_{n=0}^{\infty} n\alpha e^{-n\alpha}}{\sum_{n=0}^{\infty} e^{-n\alpha}} \qquad \text{where } \alpha = \frac{h\nu}{kT}$$

This, in turn, can be evaluated most easily by noting that

$$-\alpha \frac{d}{d\alpha} \ln \sum_{n=0}^{\infty} e^{-n\alpha} = \frac{-\alpha \frac{d}{d\alpha} \sum_{n=0}^{\infty} e^{-n\alpha}}{\sum_{n=0}^{\infty} e^{-n\alpha}} = \frac{-\sum_{n=0}^{\infty} \alpha \frac{d}{d\alpha} e^{-n\alpha}}{\sum_{n=0}^{\infty} e^{-n\alpha}} = \frac{\sum_{n=0}^{\infty} n\alpha e^{-n\alpha}}{\sum_{n=0}^{\infty} e^{-n\alpha}}$$

so that

$$\bar{\mathscr{E}} = kT\left(-\alpha \frac{d}{d\alpha} \ln \sum_{n=0}^{\infty} e^{-n\alpha}\right) = -h\nu \frac{d}{d\alpha} \ln \sum_{n=0}^{\infty} e^{-n\alpha}$$

Now

$$\sum_{n=0}^{\infty} e^{-n\alpha} = 1 + e^{-\alpha} + e^{-2\alpha} + e^{-3\alpha} + \cdots$$
$$= 1 + X + X^2 + X^3 + \cdots \qquad \text{where } X = e^{-\alpha}$$

$$\bar{\mathscr{E}} = kT\left(-\alpha \frac{d}{d\alpha} \ln \sum_{n=0}^{\infty} e^{-n\alpha}\right) = -h\nu \frac{d}{d\alpha} \ln \sum_{n=0}^{\infty} e^{-n\alpha}$$

Now

$$\sum_{n=0}^{\infty} e^{-n\alpha} = 1 + e^{-\alpha} + e^{-2\alpha} + e^{-3\alpha} + \cdots$$
$$= 1 + X + X^2 + X^3 + \cdots \qquad \text{where } X = e^{-\alpha}$$

but

$$(1 - X)^{-1} = 1 + X + X^2 + X^3 + \cdots$$

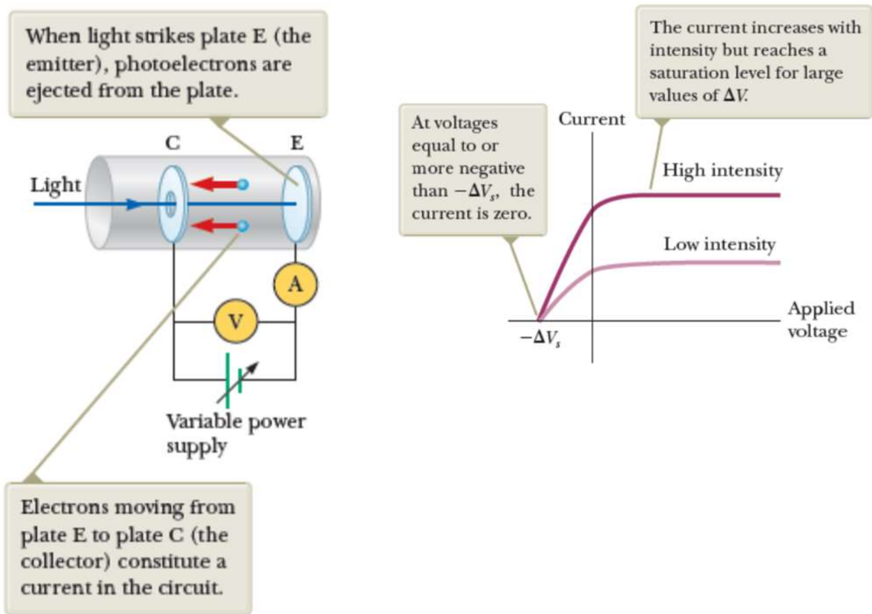
so

$$\bar{\mathscr{E}} = -h\nu \frac{d}{d\alpha} \ln (1 - e^{-\alpha})^{-1}$$

$$= \frac{-h\nu}{(1 - e^{-\alpha})^{-1}} (-1)(1 - e^{-\alpha})^{-2} e^{-\alpha}$$

$$= \frac{h\nu e^{-\alpha}}{1 - e^{-\alpha}} = \frac{h\nu}{e^{\alpha} - 1} = \frac{h\nu}{e^{h\nu/kT} - 1}$$

Photoelectric Effect & Photoelectrons



Apparently, the existence of the photoelectric effect is not surprising!
➔ Light waves carry energy, and some of the energy absorbed by the metal may somehow concentrate on individual electrons and reappear as their kinetic energy.

But three experimental findings show that no such simple explanation is possible

❖ **Within the limits of experimental accuracy (about 10^{-9} s), there is no time interval between the arrival of light at a metal surface and the emission of photoelectrons.**

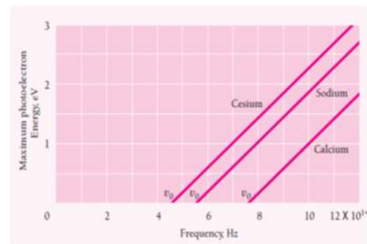
- Since the energy in an EM wave is supposed to be spread across the wave fronts, a finite time should elapse before an individual electron accumulates enough energy (several eV) to leave the metal.
- A detectable photoelectron current results when 10^{-6} W/m^2 of EM energy is absorbed by a sodium surface.
- A layer of sodium 1 atom thick and 1 m^2 in area contains about 10^{19} atoms. If the incident light is absorbed in the uppermost atomic layer, each atom receives energy at an average rate of 10^{-25} W.
- At this rate, over a month would be needed for an atom to accumulate the energy of the magnitude that photoelectrons from a sodium surface are observed to have.

❖ **A bright light yields more photoelectrons than a dim one of the same frequency, but the electron energies remain the same.**

The EM theory of light, on the contrary, predicts that the more intense the light, the greater the energies of the electrons.

❖ **The higher the frequency of the light, the more energy the photoelectrons have.**

- Blue light results in faster electrons than red light.
- At frequencies below a certain critical frequency ν_0, which is characteristic of each particular metal, no electrons are emitted.
- Above ν_0, the photoelectrons range in energy from 0 to a maximum value that increases linearly with increasing frequency
- This observation, also, cannot be explained by the EM theory of light.



Quantum theory of light… needed?

Einstein realized that the photoelectric effect could be understood if the energy in light is not spread out over wave fronts but is concentrated in small packets, or **photons.**

$$\textbf{Photon energy} = h\nu$$

The three experimental observations listed above follow directly from Einstein's hypothesis.

(1) Because EM wave energy is concentrated in photons and not spread out, there should be no delay in the emission of photoelectrons.

(2) All photons of a frequency ν have the same energy, so changing the intensity of a monochromatic light beam will change the number of photoelectrons but not their energies.

(3) The higher the frequency, the greater the photon energy $h\nu$, and so the more energy the photoelectrons have.

What is the meaning of the critical frequency ν_0 below which no photoelectrons are emitted?

$$\phi = h\nu_0$$

Work function

Metal	Symbol	Work Function, eV
Cesium	Cs	1.9
Potassium	K	2.2
Sodium	Na	2.3
Lithium	Li	2.5
Calcium	Ca	3.2
Copper	Cu	4.7
Silver	Ag	4.7
Platinum	Pt	6.4

To pull an electron from a metal surface generally takes about half of what is needed to pull an electron from a free atom of that metal

Example Cs: the ionization energy of Cs is 3.9 eV compared with its work function of 1.9 eV

The photoelectric effect is a phenomenon of the visible and ultraviolet regions.

$$\boxed{h\nu = \mathrm{KE}_{\max} + \phi}$$

Compton Effect (Particle nature of EM radiation)

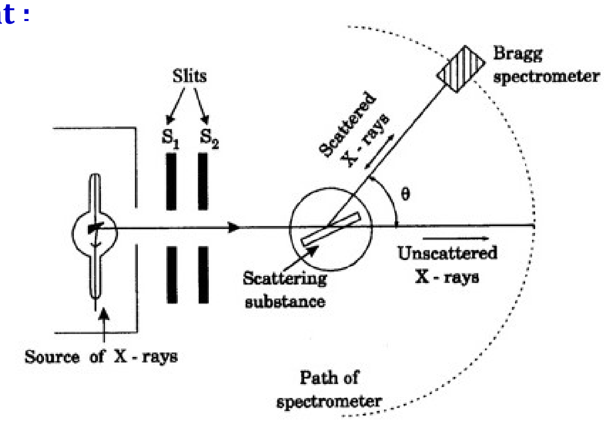
What was known before Compton (~1923) :

- ✓ Scattering of soft X-rays by various substances
- ✓ Results interpreted by J.J. Thompson's Classical theory

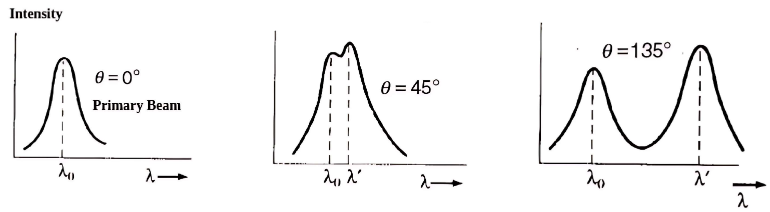
EM radiation (X-rays) hits electron
Electron vibrates with the same frequency as the incident radiation
An oscillating electron, in turn, emits radiation of the *same frequency* (as incident radiation)
Net effect : incident radiation is scattered with no change in frequency (Thompson scattering)

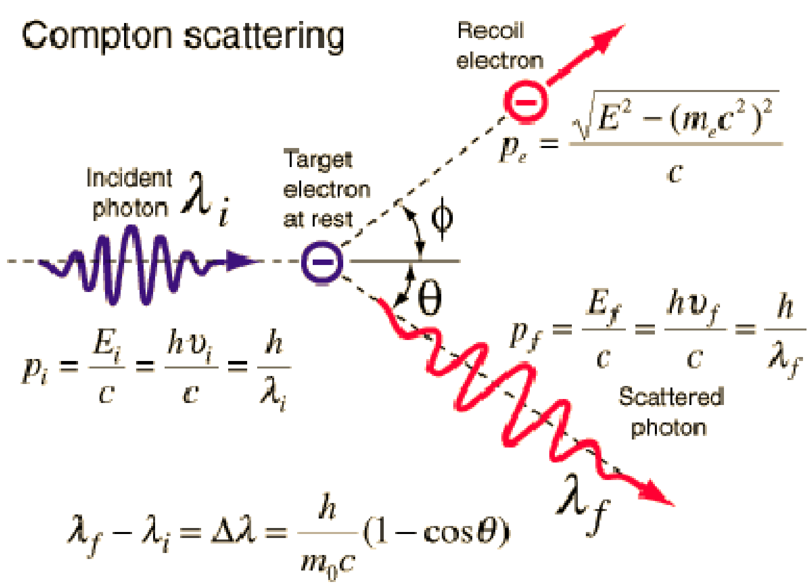
However, there were problems with hard X-rays (larger frequencies)

Compton's experiment :



λ_0: Wavelength of incident radiation



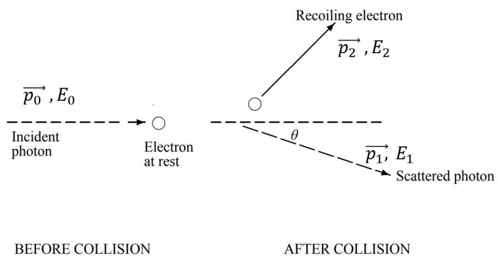


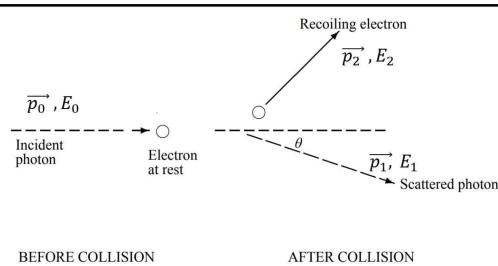
Compton's explanation

Modified wavelength (λ_1) arises due to scattering of *X-ray photons* by loosely bound electrons.

EM radiation: photons: considered as particles, $E = h\nu$

If the energy of X-ray photon (hard X ray) is ***large*** compared to binding energy of electrons, then these electrons can be approximately treated as *free*





We will use relativistic kinematics in this explanation as the velocity of electrons after recoil may ***not*** be small

Rest mass

Total energy of a particle : $$E = mc^2 = \frac{m_0 c^2}{\sqrt{1-\frac{v^2}{c^2}}}$$

Corresponding momentum : $$\vec{p} = m\vec{v} = \frac{m_0 \vec{v}}{\sqrt{1-\frac{v^2}{c^2}}}$$

Rest mass

Total energy of a particle : $E = mc^2 = \dfrac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}}$

Corresponding momentum : $\vec{p} = m\vec{v} = \dfrac{m_0\vec{v}}{\sqrt{1-\frac{v^2}{c^2}}}$

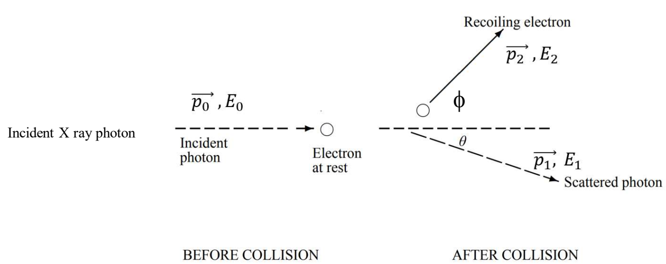
$$\mathrm{p}^2\mathrm{c}^2 = \frac{m_0^2v^2c^2}{1-\frac{v^2}{c^2}}$$

Relativistic energy-momentum relation $\mathrm{E}^2 = \mathrm{p}^2\mathrm{c}^2 + \mathrm{m}_0^2\mathrm{c}^4$

Kinetic energy : $T = E - m_0c^2$

Photon has rest mass (m_0)= 0 and E= $h\nu = hc/\lambda$ and momentum $p = E/c = h/\lambda$

Compton's explanation :



"Before"

$$\vec{p}_0 = \frac{h}{\lambda_0}$$

$$E_0 = \frac{hc}{\lambda_0}$$

"After"

Scattered photon : $\vec{p}_1 = \dfrac{h}{\lambda_1}$, $E_1 = \dfrac{hc}{\lambda_1}$

Recoil electron : $\vec{p}_2$

"Before"

$\vec{p}_0 = \frac{h}{\lambda_0}$

$E_0 = \frac{hc}{\lambda_0}$

"After"

Scattered photon : $\vec{p}_1 = \frac{h}{\lambda_1}$ $E_1 = \frac{hc}{\lambda_1}$

Recoil electron : $\vec{p}_2$

Conservation of momentum (Before & After) :

$$\vec{p}_0 = \vec{p}_1 + \vec{p}_2$$

$$p_0 = p_1\, Cos\theta + p_2\, Cos\phi \quad (1) \rightarrow \quad p_2\, Cos\phi = p_0 - p_1\, Cos\theta$$

$$0 = p_1\, Sin\theta - p_2\, Sin\phi \quad (2) \rightarrow \quad p_2\, Sin\phi = p_1\, Sin\theta$$

Squaring and adding the above TWO equations

$$p_2^2 = p_0^2 + p_1^2 Cos^2\theta + p_1^2\, Sin^2\theta - 2p_1p_0Cos\theta$$

$$p_2^2 = p_0^2 + p_1^2 - 2p_1p_0Cos\theta \quad (3)$$

"Before"

$\vec{p}_0 = \frac{h}{\lambda_0}$

$E_0 = \frac{hc}{\lambda_0}$

"After"

Scattered photon : $\vec{p}_1 = \frac{h}{\lambda_1}$ $E_1 = \frac{hc}{\lambda_1}$

Recoil electron : $\vec{p}_2$

Conservation of energy (Before and After):

$$E_0 + m_0c^2 = E_1 + \sqrt{p_2^2c^2 + m_0^2c^4} \quad (4)$$

Photon energy

Rest mass energy of electron

Kinetic energy of electron after collision : $T_2 = (m_0^2c^4 + p_2^2c^2)^{1/2} - \mathrm{m_0c^2}$

$$= E_0 - E_1 = c(p_0 - p_1)$$

$$m_0^2c^4 + p_2^2c^2 = (\,T_2 + \mathrm{m_0c^2})^2$$

$$= m_0^2c^4 + c^2(p_0 - p_1)^2 + 2m_0c^2.c(p_0 - p_1)$$

$$p_2^2 = (p_0 - p_1)^2 + 2m_0c(p_0 - p_1) \quad (5)$$

"Before"

$\vec{p}_0 = \frac{h}{\lambda_0}$

$E_0 = \frac{hc}{\lambda_0}$

"After"

Scattered photon : $\vec{p}_1 = \frac{h}{\lambda_1}$ $E_1 = \frac{hc}{\lambda_1}$

Recoil electron : $\vec{p}_2$

Conservation of momentum : $p_2^2 = p_0^2 + p_1^2 - 2p_1p_0Cos\theta$

Conservation of energy : $p_2^2 = (p_0 - p_1)^2 + 2m_0c(p_0 - p_1)$

Equating both equation we get:

$$p_0^2 + p_1^2 - 2p_1p_0Cos\theta = (p_0 - p_1)^2 + 2m_0c(p_0 - p_1)$$

we get:

$$m_0c(p_0 - p_1) = 2p_1p_0Sin^2\theta/2$$

"Before"

$\vec{p}_0 = \frac{h}{\lambda_0}$

$E_0 = \frac{hc}{\lambda_0}$

"After"

Scattered photon : $\vec{p}_1 = \frac{h}{\lambda_1}$ $E_1 = \frac{hc}{\lambda_2}$

Recoil electron : $\vec{p}_2$

$$m_0c(p_0 - p_1) = 2p_1p_0Sin^2\theta/2$$

Multiply both side by $h/m_0 c\, p_0 p_1$

we get:

$$\frac{h}{p_1} - \frac{h}{p_0} = 2\frac{h}{m_0c}Sin^2\theta/2$$

$$\boxed{\Delta\lambda = \lambda_1 - \lambda_0 = 2\lambda_c Sin^2\theta/2 \;=\; \lambda_c\,(1- cos\theta)}$$

Compton equation

Where $\lambda_c = \frac{h}{m_0c}$ is called Compton wavelength of Electron