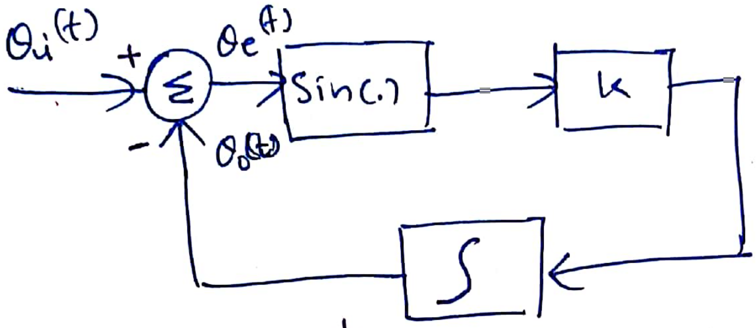
Non-linear Model for the first-order PLL



$$\theta_o(t) = K \int_0^t \sin[\theta_e(\tau)]\, d\tau$$

$$\frac{d\theta_o(t)}{dt} = K \sin[\theta_e(t)] \quad \text{— (1)}$$

$$\theta_e(t) = \theta_i(t) - \theta_o(t)$$

$$\therefore \frac{d\theta_o(t)}{dt} = \frac{d\theta_i(t)}{dt} - \frac{d\theta_e(t)}{dt} \quad \text{— (2)}$$

From (1) & (2),

$$\frac{d\theta_e(t)}{dt} + K \sin[\theta_e(t)] = \frac{d\theta_i(t)}{dt} \quad \text{— (3)}$$

The operation of the PLL is governed by this non-linear differential equation.

Consider a ~~step~~ linear change in the phase of the i/p signal,

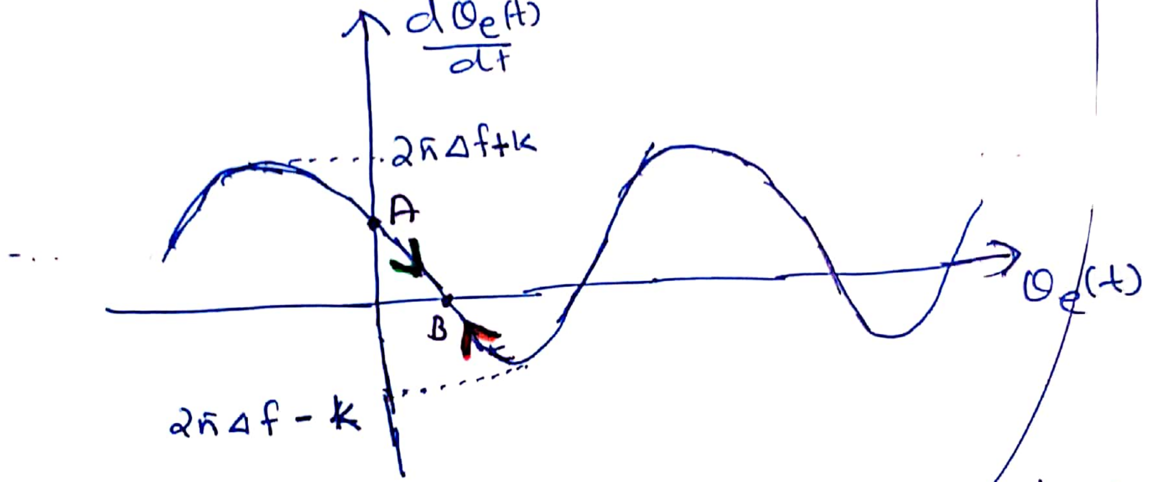
$$\theta_i(t) = 2\pi \Delta f\, t\, u(t)$$

$$\frac{d\theta_i(t)}{dt} = 2\pi \Delta f\, u(t)$$

The governing equation of the PLL for $t > 0$ is

$$\frac{d\theta_e(t)}{dt} + K \sin[\theta_e(t)] = 2\pi \Delta f.$$

We can analyze the operation of the PLL using the plot of $\frac{d\theta_e(t)}{dt}$ vs. $\theta_e(t)$. This is called the 'phase-plane plot'.



Assume that the PLL is in the locked state at $t = 0$ with $\theta_e(0) = 0$.

At $t = 0^+$

$$\frac{d\theta_e(t)}{dt} = 2\pi \Delta f$$

This corresponds to the ~~po~~ operating point 'A' in the phase-plane plot.

For $t > 0$, the PLL tries to track down the instantaneous phase of the input signal $[\theta_i(t)]$ and the operating point moves along the phase-plane plot.

The trajectory of the operating point depends on the sign of $\frac{d\theta_e(t)}{dt}$.

Note that dt, a time increment, is always a positive quantity.

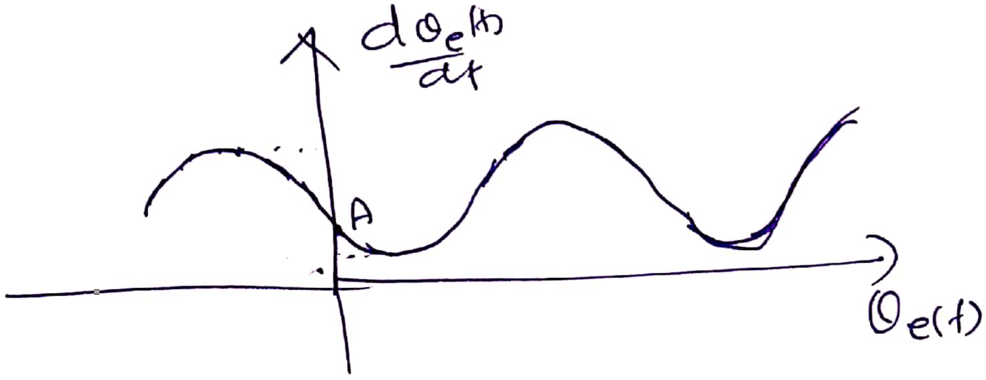
In the upper-half of the phase-plane plot (i.e., $\frac{d\theta_e(t)}{dt} > 0$), the operating point moves in the increasing direction of $\theta_e(t)$, $\left(\frac{d\theta_e(t)}{dt} > 0\right)$. On the other hand, in the lower-half of the phase-plane plot (i.e., $\frac{d\theta_e(t)}{dt} < 0$), the operating point moves in the decreasing direction of $\theta_e(t)$ i.e., ~~left to~~ from right to left.

In the steady-state i.e., $\frac{d\theta_e(t)}{dt} = 0$, the operating point is at 'B' in the phase-plane plot.

In the above analysis, it is assumed that $\boxed{2\pi\Delta f - K < 0}$.

If this condition is not satisfied, i.e., when $\boxed{\Delta f > \frac{K}{2\pi}}$

the phase-plane plot is



In this case, the loop will never lock because the point $\frac{d\theta_e(t)}{dt} = 0$ is not in the operating curve.

The maximum value of Δf for the loop to lock, or

$$\Delta f = \frac{K}{2\pi}$$

is called the lock-range of the first-order PLL.

Applications of PLL

PLL is one of the most versatile circuit blocks used in both communication and instrumentation systems. It is widely used in cell phones, radios, computers, and storage devices. We discuss few applications of PLL.

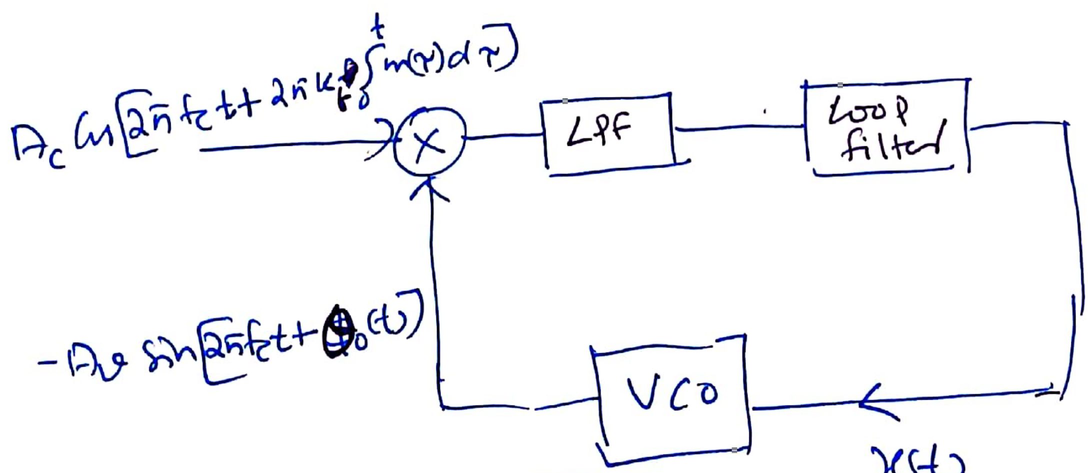
1. FM Demodulation.

If an FM modulated signal

$$s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\,d\tau\right]$$

is the i/p to ~~the~~ a PLL,

$$\theta_i(t) = 2\pi k_f \int_0^t m(\tau)\,d\tau$$



$$\boxed{\theta_o(t) = K_v \int_0^t x(\tau)\,d\tau}$$

$$\therefore \frac{d\theta_o(t)}{dt} = \cancel{x(t)}\, K_v \cdot x(t)$$

When the loop is locked,

$$\theta_i(t) \approx \theta_o(t)$$

$$\Rightarrow \frac{d\theta_i(t)}{dt} \approx \frac{d\theta_o(t)}{dt}$$

$$\therefore \boxed{2\pi k_f\, m(t) = k_v \cdot x(t)}$$

The VCO input $x(t)$ is a scaled version of the message signal $m(t)$

2. PLL as frequency synthesizer

$\boxed{\text{Section 3.5.1 of Madhow's book}}$

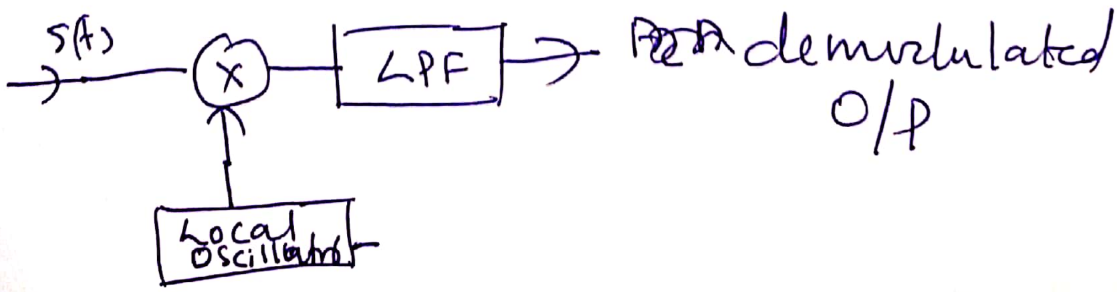
3. ~~Costa's Loop.~~

3. Carrier Recovery & Demodulation of DSB-SC Signals.

DSB-SC modulated signal,

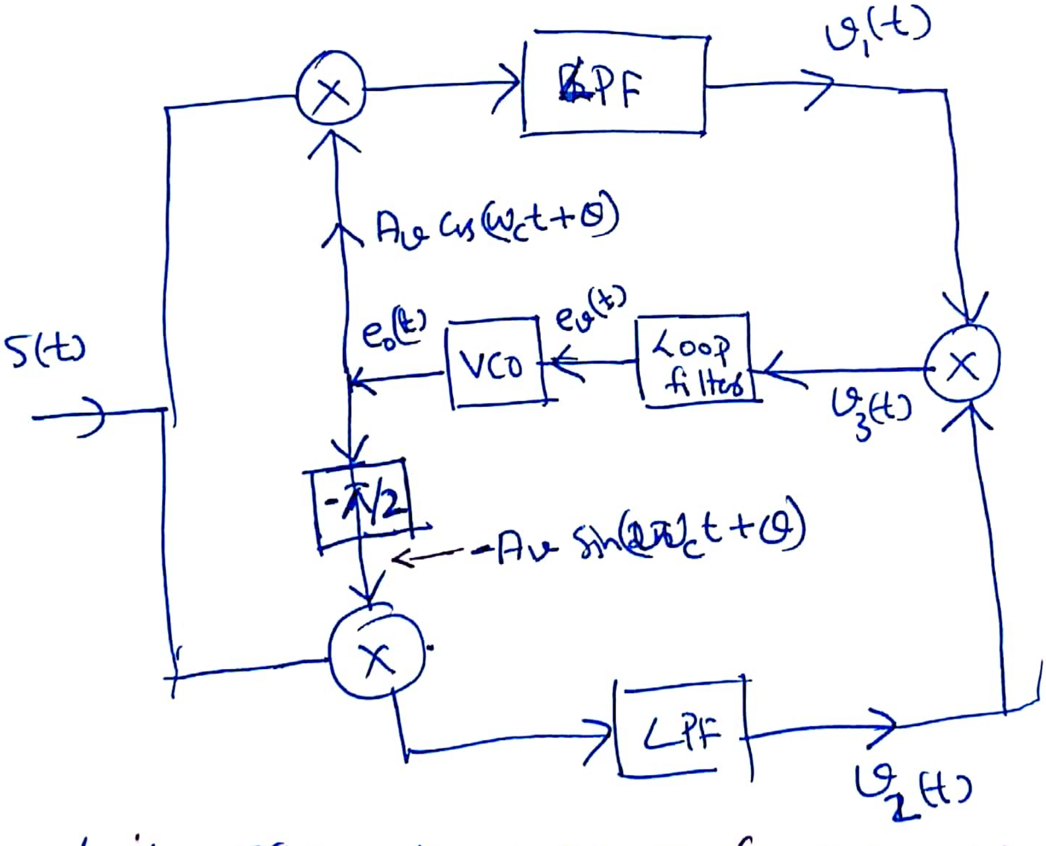
$$s(t) = A_c\, m(t) \cos(2\pi f_c t)$$

We have earlier discussed about synchronous ~~detect~~ demodulation of the DSB-SC signals.



The drawback of synchronous demodulation is that the local oscillator frequency ~~need to~~ and phase need to be synchronized with that of the carrier signal.

In practical applications, a PLL known as the Costa's PLL ~~or Cost~~ or the Costa's Loop is used for this purpose.



Let $s(t) = A_c\, m(t) \cos(2\pi f_c t + \phi)$

$e_0(t) = A_v \cos(2\pi f_c t + \theta)$

$$v_1(t) = \frac{1}{2} A_c A_v \cos(\phi - \theta)\; m(t)$$

$$v_2(t) = \frac{1}{2} A_c A_v \sin(\phi - \theta)\; m(t)$$

$$v_3(t) = \frac{1}{4} (A_c A_c)^2 \sin(2\psi)\; m^2(t)$$

$$\boxed{\text{where } \psi = (\phi - \theta)}$$

The loop filter is a narrow-band LPF, where o/p is given by

$$e_o(t) = K \sin(2\psi),$$

where $K = \frac{1}{8} (A_c A_v)^2 \langle m^2(t) \rangle$

$\langle m^2(t) \rangle$ is the average value of $m^2(t)$

Hint: Find the spectrum of $v_3(t)$

When the loop is locked,

i.e., $\psi = (\phi - \theta) \approx 0$

$$v_1(t) \approx \frac{1}{2} A_c A_v\; m(t)$$

is the scaled version of the ~~[illegible]~~ modulated signal ~~[illegible]~~ $m(t)$

Note that $e_o(t) = A_v \cos(\omega_c t + \theta)$

$= A_v \cos(\omega_c t + \phi)$ ⇐ when the loop is locked

is synchronized with the carrier signal. Hence it can be used for carrier recovery