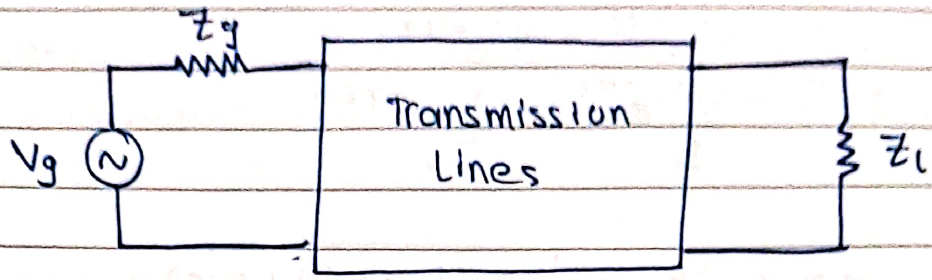
$\nabla \cdot D = \rho$

$D = \epsilon E + P$

ECC-203
Transmission lines



Transmission lines are two-port network connecting generator to load.

Effects of transmission lines like phase shift & reflections become significant when $\boxed{l \geq 10^{-3}\lambda}$

Transmission Lines → Transverse Electromagnetic (TEM)
E & H are perpendicular to the wave propagation dir^n
(have 2 parallel conductors)
Ex. Coaxial line, Two-wire line, Parallel-Plate line, Microstrip line.

↓

Higher Order Lines
A significant component in dir^n of wave propogation
ex. optical Fibre

Lumped Element Circuit Model

We break the transmission line in components with:

R : series Resistance (Ω/m)

L : Series Inductance (H/m)

G : shunt conductance (S/m)

C : Shunt capacitance (F/m)

By applying KCL & KVL to the lumped element model, we get Telegrapher's equations.

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}$$

$$I(z) = \frac{V_0^+}{Z_0} e^{-\gamma z} - \frac{V_0^-}{Z_0} e^{+\gamma z}$$

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$$v(z,t) = \mathrm{Re}\{V(z)\, e^{j\omega t}\}$$

Transmission line Effects

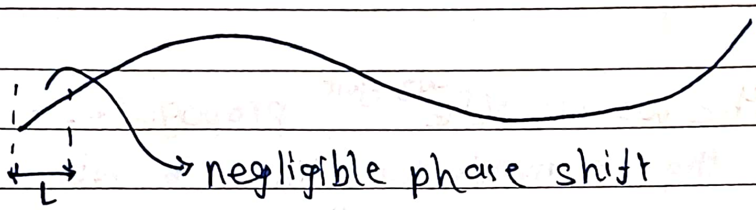
In normal wires, we assume that the voltage given on one end immediately reflects on the other end with exact same amplitude, but when length of wires is longer than "1% of wavelength" transmission line effects play a role:

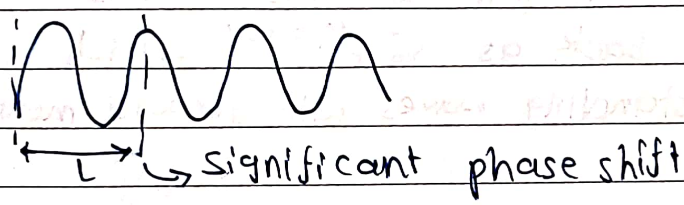
→ Signal delay / Phase shift
→ Reflections
→ Attenuation
→ Dispersion

Signal Delay / Phase Shift

when a wave travels, if its wavelength is large compared to the length of line, there is negligible phase shifting between the 2 ends of the line.

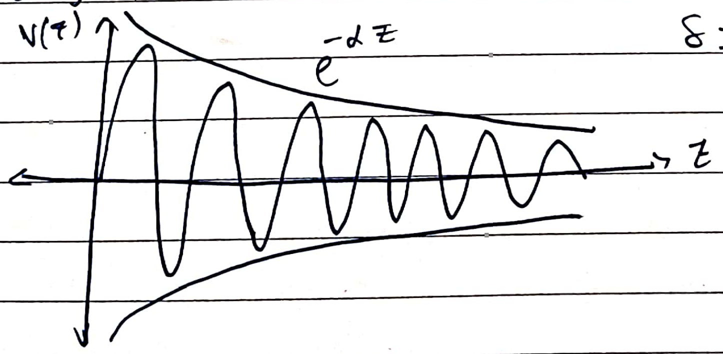
But when the wavelength is comparable to the length of wire, there is a significant phase difference b/w the 2 ends.





Attenuation

The amplitude of the wave decreases as the wave propogates through transmission lines due to losses, amplitude decreases by a factor $e^{-\alpha}$.



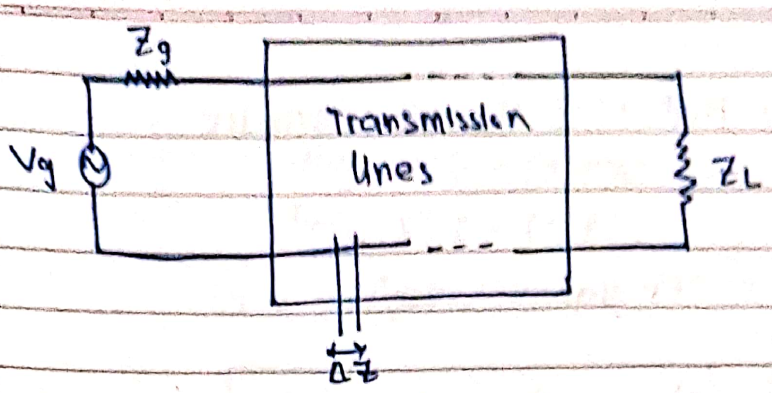
$\delta = \frac{1}{\alpha}$

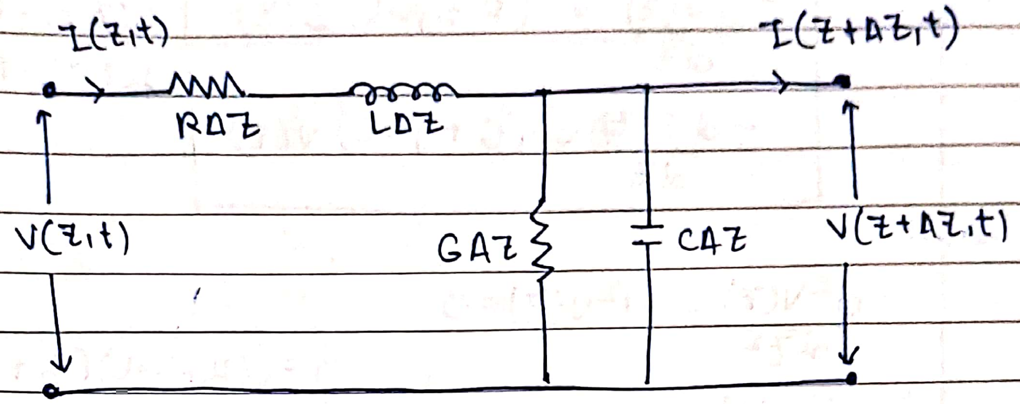
Dispersion

The propogation constant $\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$ depends on the frequency ω of the propogating wave, a digital square pulse consists of various frequencies combined so different propogation constants are offered for different frequencies, so the signal gets distorted as it travels.

Reflections

When the wave $V_0^+ e^{-\gamma z + j\omega t}$ propogates through the transmission lines & hits the load, a part of the wave is reflected back as $V_0^- e^{+\gamma z + j\omega t}$ which creates standing waves with forward moving wave





$$V(z+\Delta z,t) = V(z,t) - I(z,t)R\Delta z - L\Delta z\frac{\partial I(z,t)}{\partial t}$$

$$\Rightarrow \frac{V(z+\Delta z,t) - V(z,t)}{\Delta z} = -I(z,t)R - L\frac{\partial I(z,t)}{\partial t}$$

$$\Rightarrow \boxed{\frac{\partial V(z,t)}{\partial z} = -I(z,t)R - L\frac{\partial I(z,t)}{\partial t}} \quad -(1)$$

$$I(z+\Delta z,t) = I(z,t) - G\Delta z V(z+\Delta z,t) - C\Delta z\frac{\partial V(z+\Delta z,t)}{\partial t}$$

$$\Rightarrow \frac{I(z+\Delta z,t) - I(z,t)}{\Delta z} = -GV(z+\Delta z,t) - C\frac{\partial V(z+\Delta z,t)}{\partial t}$$

$$\Rightarrow \boxed{\frac{\partial I(z,t)}{\partial z} = -GV(z,t) - C\frac{\partial V(z,t)}{\partial t}} \quad -(2)$$

We know that V $ I are periodic in time, so $V(z,t) = V(z)\,e^{j\omega t}$

$$I(z,t) = I(z)\,e^{j\omega t}$$

putting in telegrapher's eq^n:

$$\boxed{\begin{aligned} -\frac{dV(z)}{dz} &= (R + j\omega L)\,I(z) \\ -\frac{dI(z)}{dz} &= (G + j\omega C)\,V(z) \end{aligned}}$$

$V(z,t) = Re\{V(z)e^{j\omega t}\}$
$I(z,t) = Re\{I(z)e^{j\omega t}\}$

$$\frac{d^2V(z)}{dz^2} - \gamma^2 V(z) = 0$$

$$\frac{d^2I(z)}{dz^2} - \gamma^2 I(z) = 0$$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = \alpha + j\beta$$

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$$

$$I(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}$$

$$= \frac{V_0^+}{Z_0} e^{-\gamma z} - \frac{V_0^-}{Z_0} e^{\gamma z}$$

For a lossless transmissio line: $R = 0,\ G = 0$

$\alpha = 0,\ \beta = \omega\sqrt{LC}$

$Z_0 = \sqrt{\frac{L}{C}}$

$\boxed{v_p = \frac{\omega}{\beta}}$ Phase velocity

In EM wave propogating in air: $R = 0$

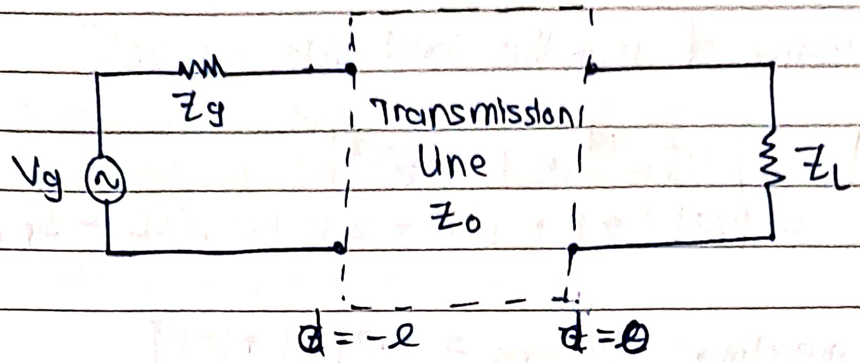
$G = \sigma$

$L = \mu$

$C = \varepsilon$

If v_p is independent of ω, then the line is non-dispersive.

Reflection Coefficient



$$V(z) = V_0^+ e^{\gamma d} + V_0^- e^{-\gamma d}$$

$$I(z) = \frac{V_0^+}{Z_0} e^{\gamma d} - \frac{V_0^-}{Z_0} e^{-\gamma d}$$

at $d = 0$: $V_L = V_0^+ + V_0^-$

$$I_L = I_0^+ - I_0^- = \frac{V_0^+}{Z_0} - \frac{V_0^-}{Z_0}$$

$$Z_L = \frac{V_L}{I_L} = \left(\frac{V_0^+ + V_0^-}{V_0^+ - V_0^-}\right) Z_0$$

$$\boxed{\Gamma = \frac{V_0^-}{V_0^+} = \frac{Z_L/Z_0 - 1}{Z_L/Z_0 + 1} = \frac{z_L - 1}{z_L + 1}}$$

Normalized load impedce

$$z_L = \frac{Z_L}{Z_0}$$

As the reflection coefficient increases, the reflected wave forms a standing wave along with the incident, this results in oscillation of energy in the transmission lines rather than transfer of energy.

$Z_L = Z_0$	$\Gamma = 0$
$Z_L = 0$	$\Gamma = -1$
$Z_L = \infty$	$\Gamma = 1$

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Voltage Standing Wave Ratio (SWR):

Considering d from the load side, we get

$$|V(d)| = \left| V_0^+ e^{\gamma d} + \Gamma V_0^+ e^{-\gamma d} \right|$$
$$= |V_0^+| \left(1 + |\Gamma|^2 + 2|\Gamma|\cos(2\beta d - \theta_r)\right)^{1/2}$$

For maxima: $|V|_{max} = |V_0^+|(1+|\Gamma|)$

$$d_{max} = \frac{\theta_r \lambda}{4\pi} + \frac{n\lambda}{2}$$

$$\Gamma = |\Gamma| e^{j\theta_r}$$

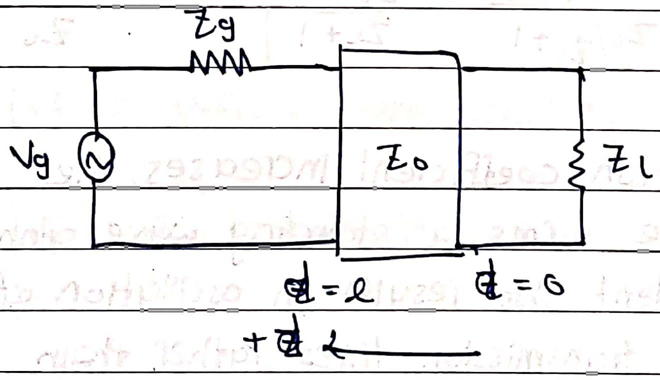
For minima: $|V|_{min} = |V_0^+|(1-|\Gamma|)$

$$d_{min} = d_{max} \pm \lambda/4$$

$$\boxed{S = \frac{|V|_{max}}{|V|_{min}} = \frac{1+|\Gamma|}{1-|\Gamma|}}$$

$\Gamma : 0 \rightarrow 1$
$S : 1 \rightarrow \infty$

Note: wave is moving in $+d$ direction.



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$$Z(d) = Z_0 \left[\frac{1 + \Gamma e^{-j2\beta d}}{1 - \Gamma e^{-j2\beta d}} \right]$$

Here d is the distance from the load terminal on the transmission line.

$$Z(0) = Z_0 \left(\frac{1 + \Gamma}{1 - \Gamma} \right) = Z_L$$

$$Z_{in} = Z(L) = Z_0 \left(\frac{e^{j\beta L} + \Gamma e^{-j\beta L}}{e^{j\beta L} - \Gamma e^{-j\beta L}} \right)$$

putting $\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$,

$$\boxed{\begin{gathered} Z_{out} = Z_L \\ Z_{in} = Z_0 \left(\frac{Z_L + jZ_0 \tan\beta L}{Z_0 + jZ_L \tan\beta L} \right) \end{gathered}}$$

$$Z_{in} = Z_0 \left(\frac{z_L + j\tan\beta L}{1 + jz_L \tan\beta L} \right)$$

Complete Transmission Line Sol^n

$$V(d) = V_0^+ e^{\gamma d} + V_0^- e^{-\gamma d} \qquad V_0^- = \Gamma V_0^+$$

$$= V_0^+ \left(e^{\gamma d} + \Gamma e^{-\gamma d} \right)$$

$$V(L) = V_{in} = V_0^+ \left(e^{\gamma L} + \Gamma e^{-\gamma L} \right)$$

$$V_{in} = \frac{V_g Z_{in}}{Z_g + Z_{in}} \Rightarrow V_0^+ = \left(\frac{V_g Z_{in}}{Z_g + Z_{in}} \right) \frac{1}{e^{\gamma L} + \Gamma e^{-\gamma L}}$$

$$V(d) = \left(\frac{V_g Z_{in}}{Z_g + Z_{in}} \right) \left(\frac{e^{\gamma d} + \Gamma e^{-\gamma d}}{e^{\gamma L} + \Gamma e^{-\gamma L}} \right)$$

$$Z_{in} = Z_0 \left(\frac{z_L + \tanh(\gamma L)}{1 + z_L \tanh(\gamma L)} \right)$$