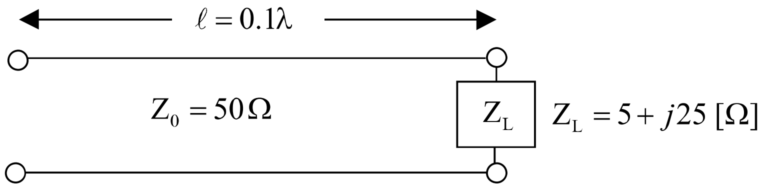
Smith Chart Problems



1. The 0.1λ length line shown has a characteristic impedance of 50Ω and is terminated with a load impedance of $Z_L = 5 + j25\Omega$.

   (a) Locate $z_L = \dfrac{Z_L}{Z_0} = 0.1 + j0.5$ on the Smith chart.

   See the point plotted on the Smith chart.

   (b) What is the impedance at $\ell = 0.1\lambda$?

   Since we want to move away from the load (i.e., toward the generator), read read 0.074λ on the WAVELENGTHS TOWARD GENERATOR scale and add $\ell = 0.1\lambda$ to obtain 0.174λ on the WAVELENGTHS TOWARD GENERATOR scale. A radial line from the center of the chart intersects the constant reflection coefficient magnitude circle at $z = 0.38 + j1.88$. Hence $Z = zZ_0 = 50(0.38 + j1.88) = 19 + 94\Omega$.

   (c) What is the VSWR on the line?

   Find VSWR $= z_{\text{max}} = 13$ on the horizontal line to the right of the chart's center. Or use the SWR scale on the chart.

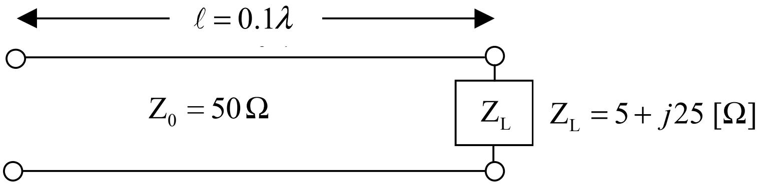
   (d) What is Γ_L?

   From the REFLECTION COEFFICIENT scale below the chart, find $|\Gamma_L| = 0.855$. From the ANGLE OF REFLECTION COEFFICIENT scale on the perimeter of the chart, find the angle of $\Gamma_L = 126.5°$. Hence $\Gamma_L = 0.855e^{j126.5°}$.

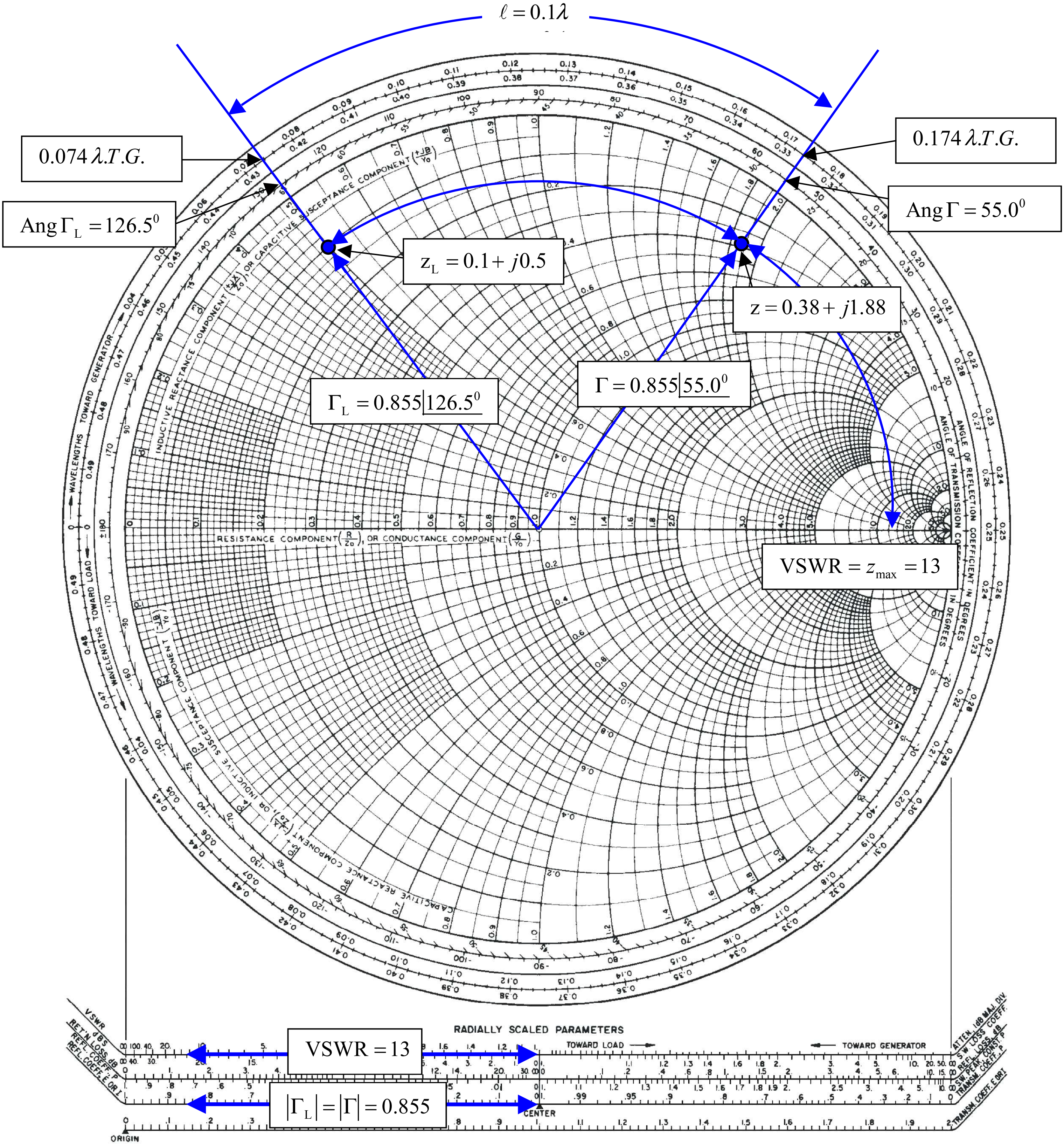
   (e) What is Γ at $\ell = 0.1\lambda$ from the load?

   Note that $|\Gamma| = |\Gamma_L| = 0.855$. Read the angle of the reflection coefficient from the ANGLE OF REFLECTION COEFFICIENT scale as $55.0°$. Hence $\Gamma = 0.855e^{j55.0°}$.

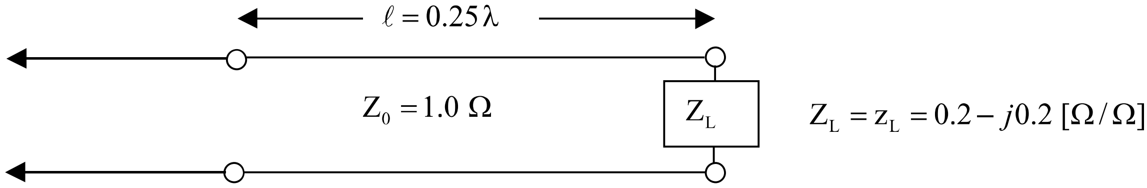
Problem 1



$\ell = 0.1\lambda$
$Z_0 = 50\,\Omega$
Z_L
$Z_L = 5 + j25$ [Ω]



$\ell = 0.1\lambda$
$0.074\,\lambda T.G.$
$0.174\,\lambda T.G.$
$\text{Ang}\,\Gamma_L = 126.5^0$
$\text{Ang}\,\Gamma = 55.0^0$
$z_L = 0.1 + j0.5$
$z = 0.38 + j1.88$
$\Gamma_L = 0.855\angle 126.5^0$
$\Gamma = 0.855\angle 55.0^0$
$\text{VSWR} = z_{max} = 13$
RESISTANCE COMPONENT (R/Zo), OR CONDUCTANCE COMPONENT (G/Yo)
INDUCTIVE REACTANCE COMPONENT (+jX/Zo), OR CAPACITIVE SUSCEPTANCE COMPONENT (+jB/Yo)
CAPACITIVE REACTANCE COMPONENT (-jX/Zo), OR INDUCTIVE SUSCEPTANCE COMPONENT (-jB/Yo)
WAVELENGTHS TOWARD GENERATOR
WAVELENGTHS TOWARD LOAD
ANGLE OF REFLECTION COEFFICIENT IN DEGREES
ANGLE OF TRANSMISSION COEFFICIENT IN DEGREES
RADIALLY SCALED PARAMETERS
TOWARD LOAD
TOWARD GENERATOR
$\text{VSWR} = 13$
$|\Gamma_L| = |\Gamma| = 0.855$
CENTER
ORIGIN



2. A transmission line has $Z_0 = 1.0$, $Z_L = z_L = 0.2 - j0.2\Omega$.

   (a) What is z at $\ell = \dfrac{\lambda}{4} = 0.25\lambda$?

   From the chart, read 0.467λ from the WAVELENGTHS TOWARD GENERATOR SCALE. Add 0.25λ to obtain 0.717λ on the WAVELENGTHS TOWARD GENERATOR SCALE. This is not on the chart, but since it repeats every half wavelength, it is the same as $0.717\lambda - 0.500\lambda = 0.217\lambda$. Drawing a radial line from the center of the chart, we find an intersection with the constant reflection coefficient magnitude circle at $z = Z = 2.5 + j2.5$.

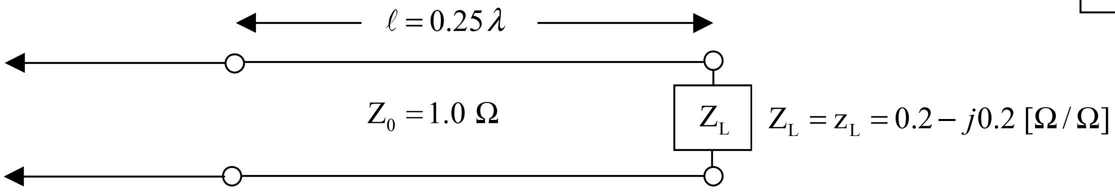
   (b) What is the VSWR on the line?

   From the intersection of the constant reflection coefficient circle with the right hand side of the horizontal axis, read VSWR $= z_{\text{max}} = 5.3$.

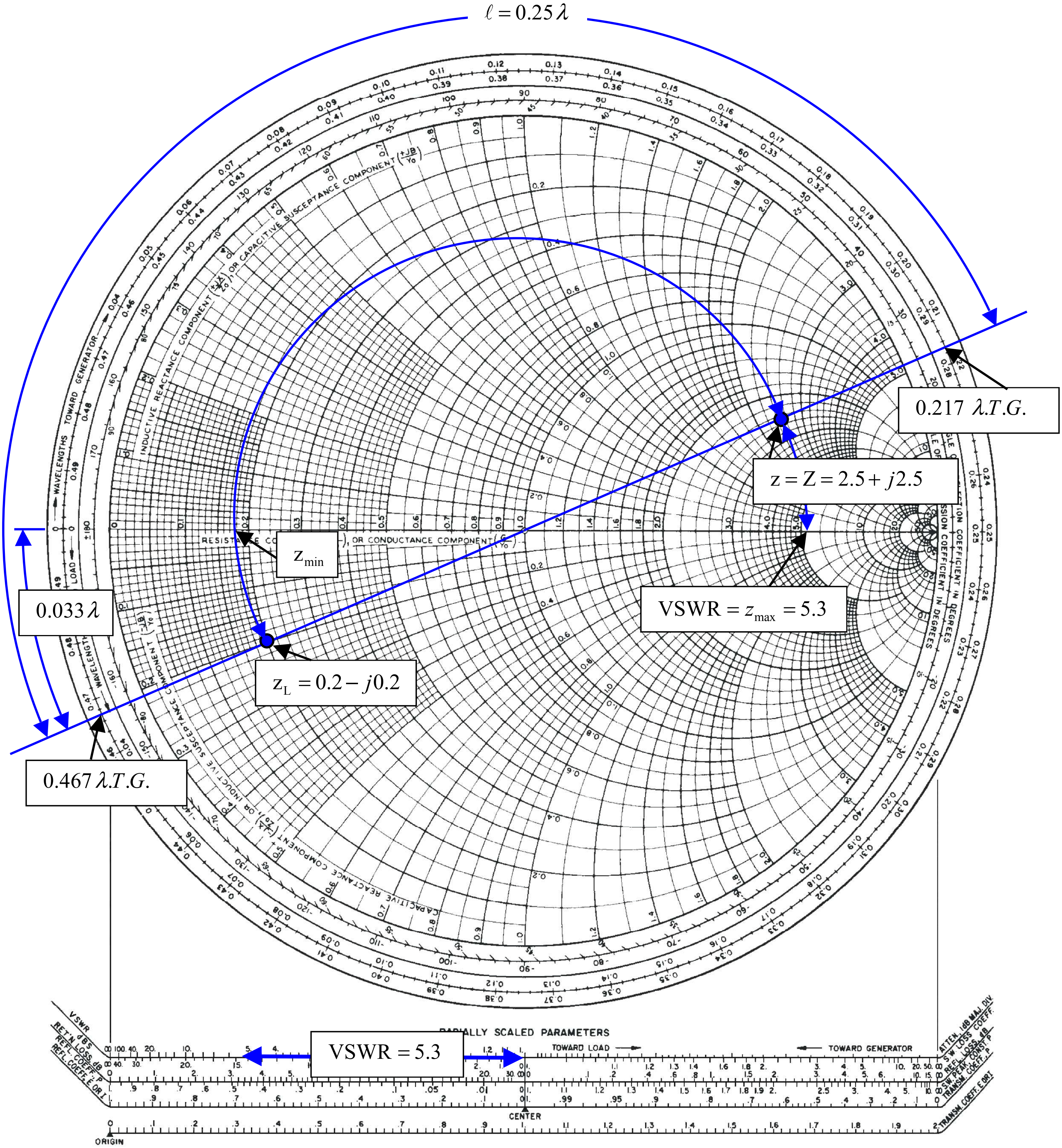
   (c) How far from the load is the first voltage minimum? The first current minimum?

   The voltage minimum occurs at z_{min} which is at a distance of $0.500\lambda - 0.467\lambda = 0.033\lambda$ from the load. Or read this distance directly on the WAVELENGTHS TOWARD LOAD scale.
   The current minimum occurs at z_{max} which is a quarter of a wavelength farther down the line or at $0.033\lambda + 0.25\lambda = 0.283\lambda$ from the load.

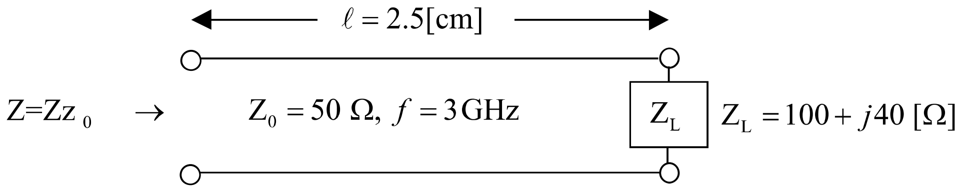
Problem 2



$\ell = 0.25\lambda$
$Z_0 = 1.0\ \Omega$
Z_L
$Z_L = z_L = 0.2 - j0.2\ [\Omega/\Omega]$



$\ell = 0.25\lambda$
$0.217\ \lambda.T.G.$
$z = Z = 2.5 + j2.5$
z_{min}
0.033λ
$\mathrm{VSWR} = z_{max} = 5.3$
$z_L = 0.2 - j0.2$
$0.467\ \lambda.T.G.$
$\mathrm{VSWR} = 5.3$
RADIALLY SCALED PARAMETERS
TOWARD LOAD
TOWARD GENERATOR
CENTER
ORIGIN



3. The air-filled two-wire line has a characteristic impedance of 50Ω and is operated at $f = 3$ GHz. The load is $Z_L = 100 + j40\Omega$.

   (a) For the line above, find z_L on the chart.

   The normalized load is $z_L = \dfrac{Z_L}{Z_0} = \dfrac{100 + j40}{50} = 2.0 + j0.8$. See the Smith chart for location of point.

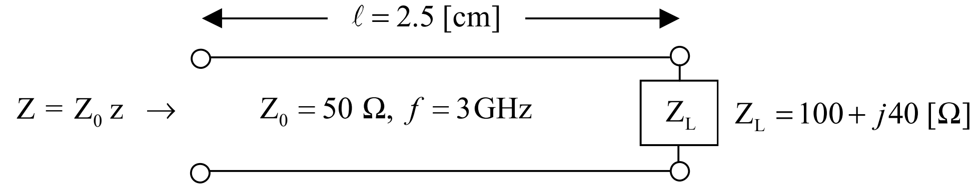
   (b) What is the line impedance 2.5 cm from the load?

   Note that $\lambda = \dfrac{c}{f} = \dfrac{3 \times 10^{10}\ \mathrm{cm/sec}}{3 \times 10^{9}\ \mathrm{Hz}} = 10$ cm. Since we are going to move toward the generator (away from the load), at the normalized load position, first read 0.217λ on the WAVELENGTHS TOWARD GENERATOR scale. Then add 2.5 cm/10 cm $= 0.25\lambda$ to this value to obtain 0.467λ on the WAVELENGTHS TOWARD GENERATOR scale. A radial line from the center at this point intersects the constant reflection coefficient magnitude circle at $z = 0.43 - j0.17$, so $Z = zZ_0 = 50(0.43 + j0.17) = 21.5 - j8.5\Omega$.

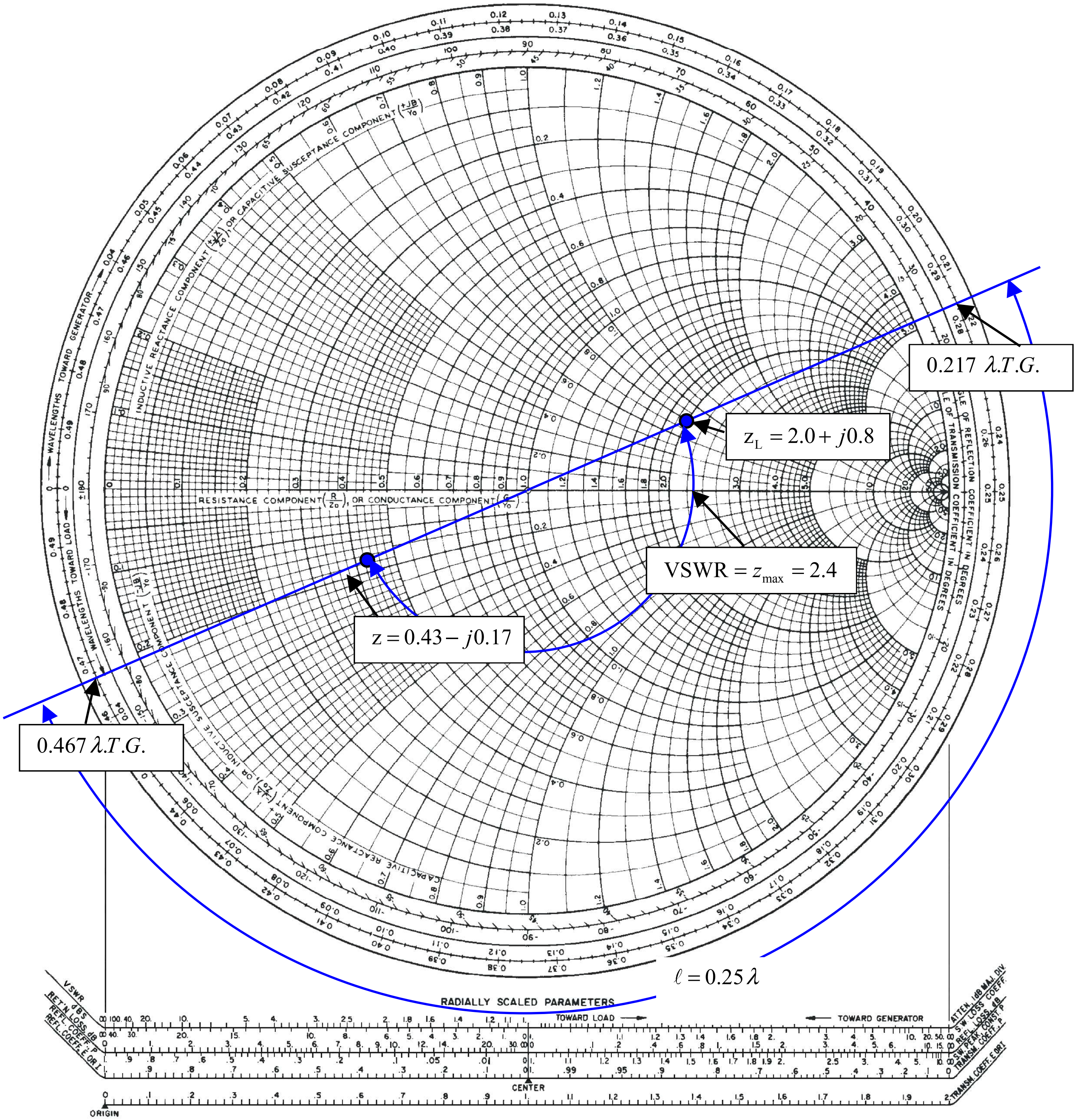
   (c) What is the VSWR on the line?

   From the intersection of the reflection coefficient circle and the horizontal axis on the right hand side of the chart, read VSWR $= 2.4$. Or use the SWR scale below the chart.

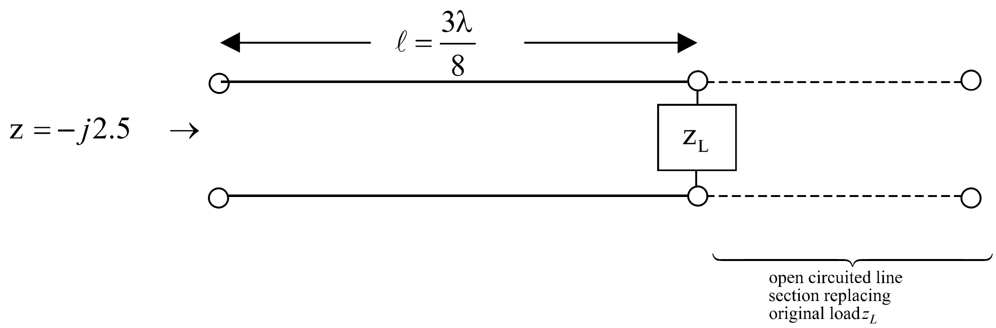
Problem 3



ℓ = 2.5 [cm]
Z = Z0 z →
Z0 = 50 Ω, f = 3GHz
ZL
ZL = 100 + j40 [Ω]



0.217 λT.G.
zL = 2.0 + j0.8
VSWR = zmax = 2.4
z = 0.43 − j0.17
0.467 λT.G.
ℓ = 0.25λ
RESISTANCE COMPONENT (R/Z0), OR CONDUCTANCE COMPONENT (G/Y0)
INDUCTIVE REACTANCE COMPONENT (+jX/Z0), OR CAPACITIVE SUSCEPTANCE COMPONENT (+jB/Y0)
CAPACITIVE REACTANCE COMPONENT (−jX/Z0), OR INDUCTIVE SUSCEPTANCE COMPONENT (−jB/Y0)
WAVELENGTHS TOWARD GENERATOR
WAVELENGTHS TOWARD LOAD
ANGLE OF REFLECTION COEFFICIENT IN DEGREES
ANGLE OF TRANSMISSION COEFFICIENT IN DEGREES
RADIALLY SCALED PARAMETERS
TOWARD LOAD
TOWARD GENERATOR
CENTER
ORIGIN



4. The line shown is $3/8\lambda$ long and its normalized input impedance is $z = -j2.5$.

   (a) What is the normalized receiving or load end impedance, z_L?

   Since we must move toward the load to find the load impedance, locate $z = -j2.5$ on the chart and find the intersection on the WAVELENGTHS TOWARD LOAD scale at 0.189λ. Add the distance $3/8\lambda = 0.375\lambda$ to get 0.064λ on the WAVELENGTHS TOWARD LOAD scale. At the intersection of the line from the center of the chart and the constant reflection coefficient circle, read $z_L = -j0.425$.

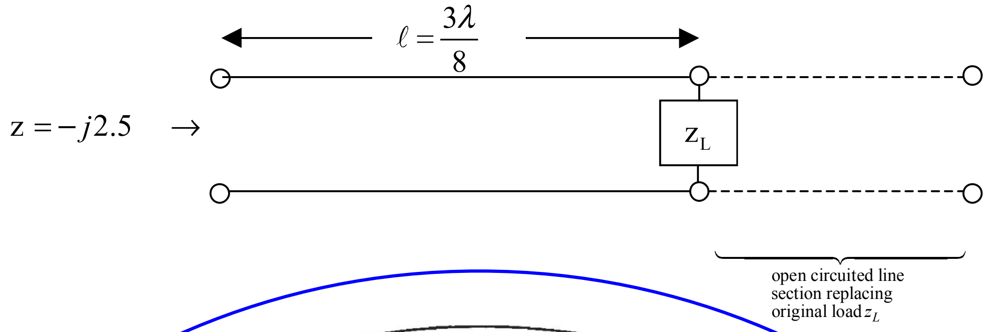
   (b) What is the distance from the load to the first voltage minimum?

   The voltage minimum occurs at the impedance minimum on the horizontal line to the left of the chart's center. This distance can be read directly off the WAVELENGTHS TOWARD LOAD scale (why?) as 0.064λ. Note that the line impedance at the voltage minimum would be that of a short circuit.

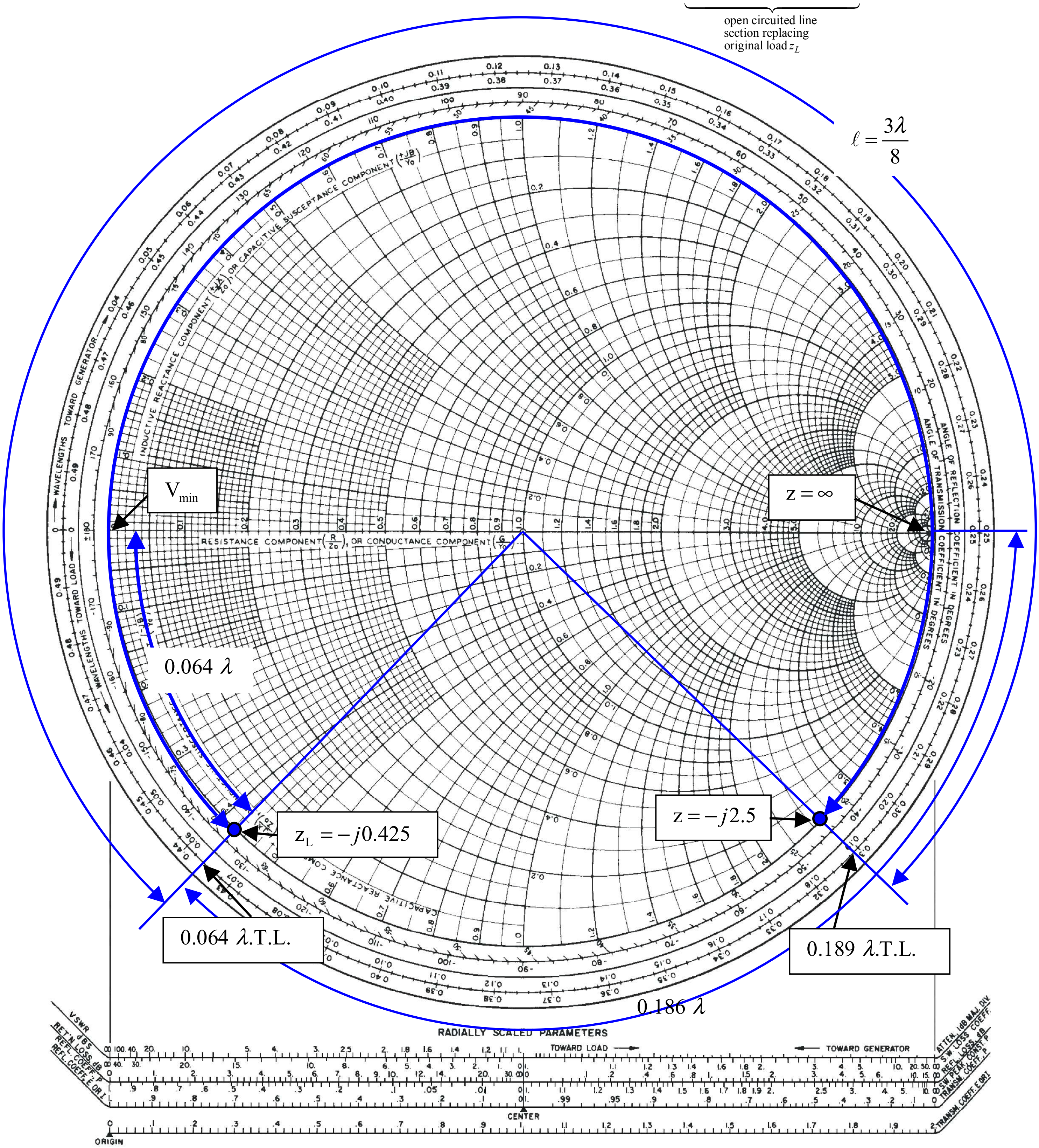
   (c) What length of open-circuited line could be used to replace z_L?

   From the position of z_L, we want to move toward the new load (i.e., toward the end of the open-circuited line) until we reach the open circuit condition $z = \infty$. This occurs at 0.25λ on the WAVELENGTHS TOWARD LOAD scale and the total distance moved is hence $0.25\lambda - 0.064\lambda = 0.186\lambda$, which is the length of open-circuited line needed to replace z_L.

Problem 4



$\ell = \frac{3\lambda}{8}$
$z = -j2.5 \quad \rightarrow$
z_L
open circuited line section replacing original load z_L



$\ell = \frac{3\lambda}{8}$
V_{min}
$z = \infty$
$0.064\ \lambda$
$z_L = -j0.425$
$z = -j2.5$
$0.064\ \lambda$.T.L.
$0.189\ \lambda$.T.L.
$0.186\ \lambda$
RESISTANCE COMPONENT $\left(\frac{R}{Z_o}\right)$, OR CONDUCTANCE COMPONENT $\left(\frac{G}{Y_o}\right)$
RADIALLY SCALED PARAMETERS
TOWARD LOAD
TOWARD GENERATOR
CENTER
ORIGIN