



ECC 203 : Electromagnetics and Radiating Systems

Wire Antenna : Dipole Antenna

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Contents



- Wire Antennas
 - Why study wire antennas?
 - Infinitesimal Dipole
 - Finite Length Dipole
 - Half-Wavelength Dipole
 - Monopole



Why Study Wire Antennas?



Why Study Wire Antennas?



https://en.wikipedia.org/wiki/Whip_antenna

Why Study Wire Antennas?

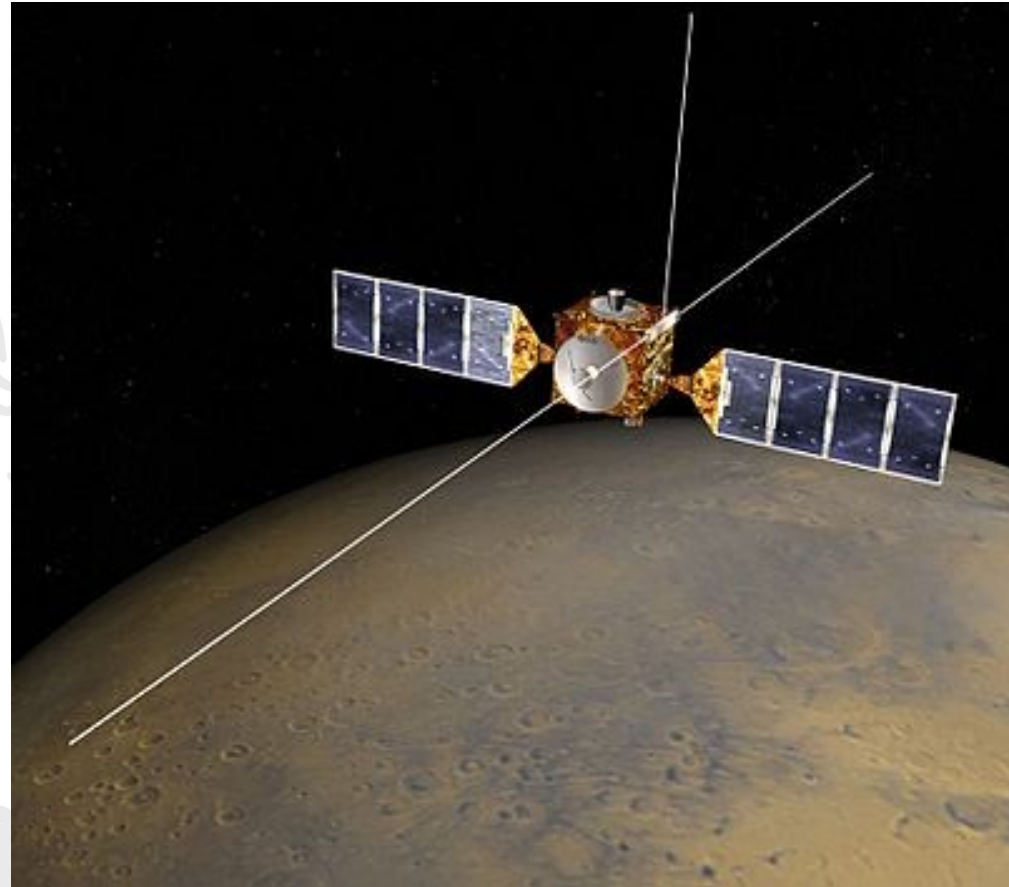


<https://www.army-technology.com/contractors/antennas/cojot2/>

Why Study Wire Antennas?



<https://directory.eoportal.org/web/eoportal/satellite-missions/h/hit-sat>



https://en.wikipedia.org/wiki/Mars_Express

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Introduction

Sources of Antenna → J - vector electric current density (A/m^2)
Radiation Fields M - vector magnetic current density (V/m^2)



Some problems involving electric currents can be cast in equivalent forms involving magnetic currents (the use of magnetic currents is simply a mathematical tool, they have never been proven to exist).



Introduction

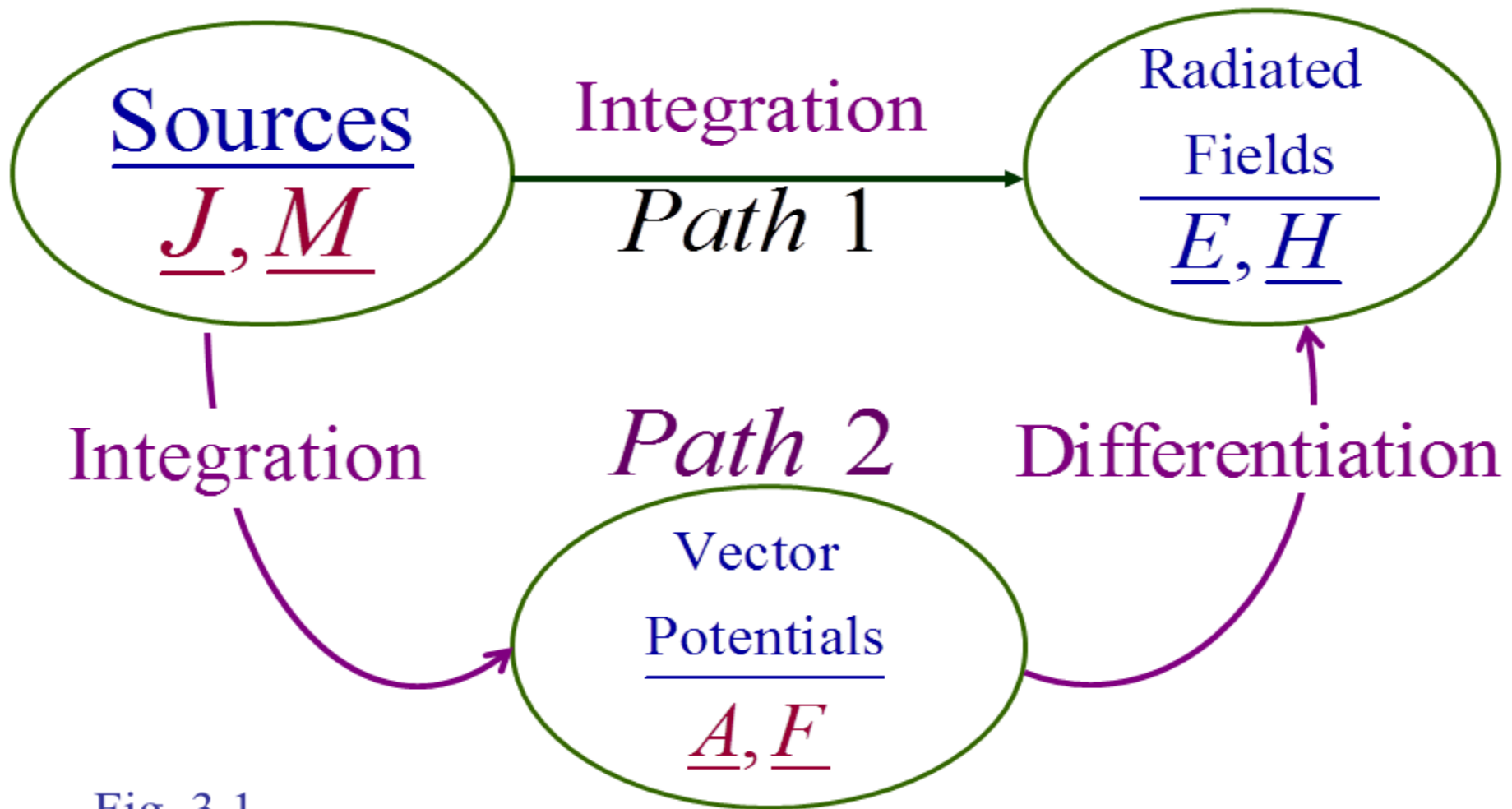
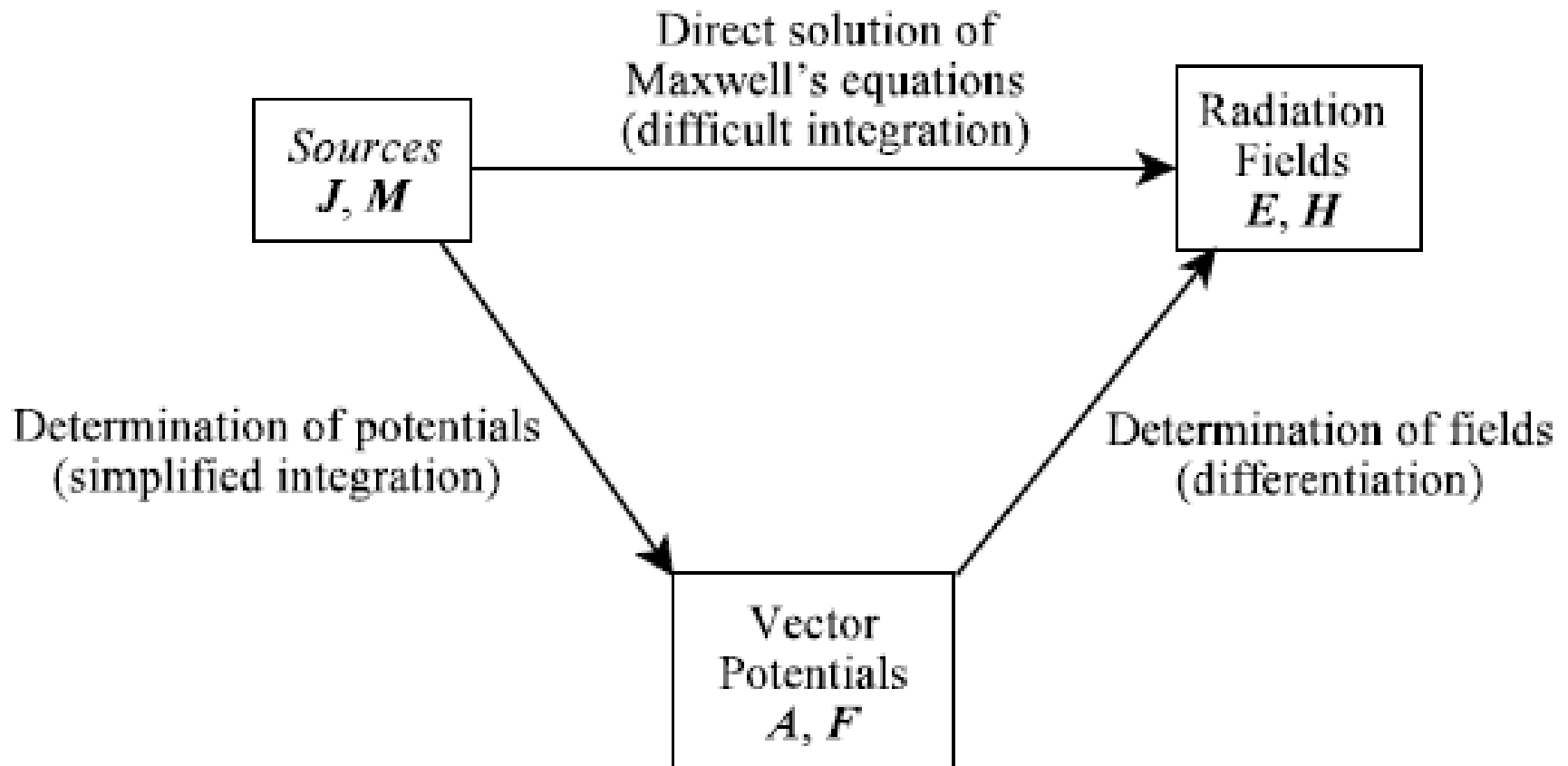
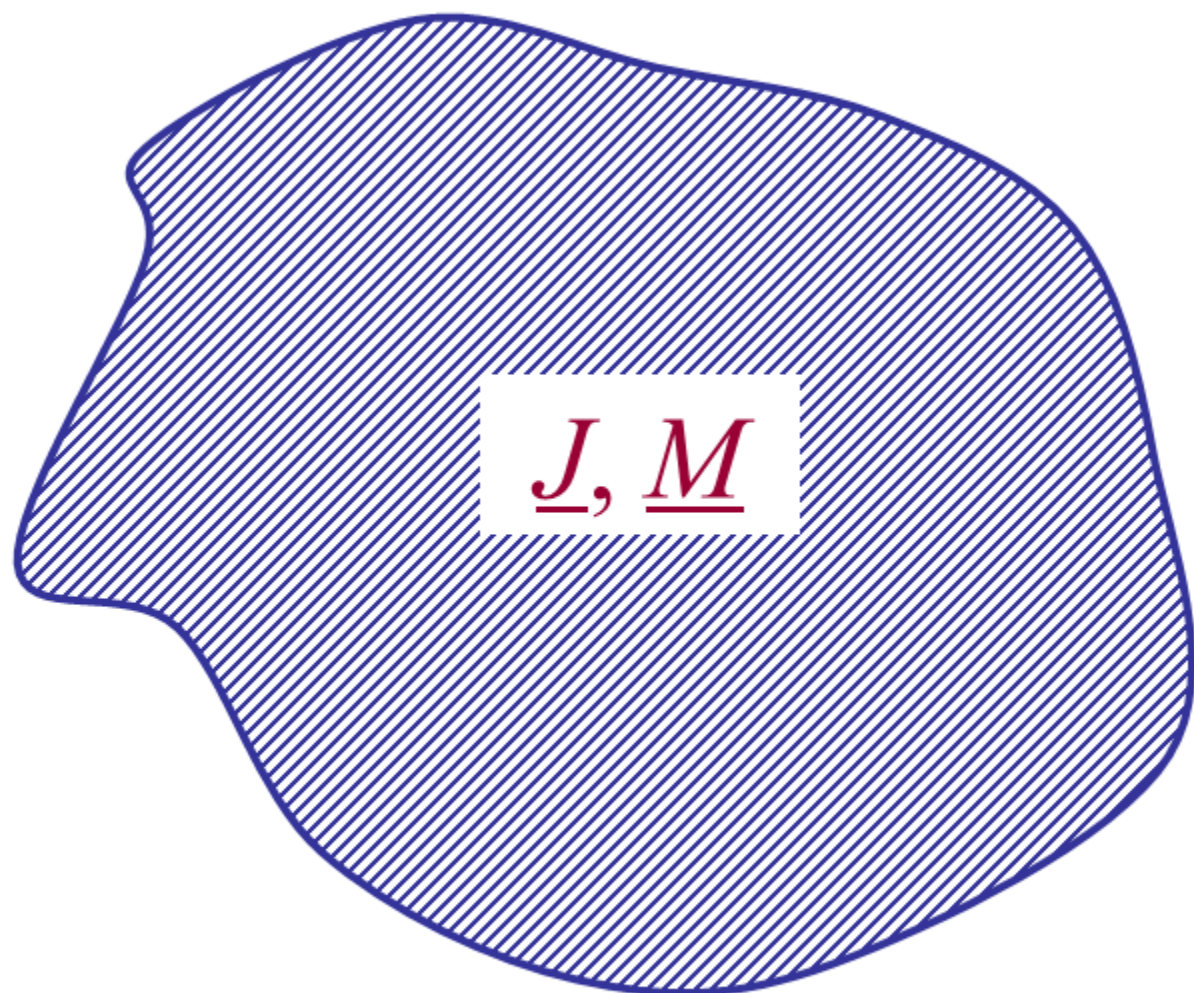


Fig. 3.1

Introduction



A - magnetic vector potential (due to J)
 F - electric vector potential (due to M)



$$\underline{J} \text{ (A/m}^2\text{)}$$

$$\underline{M} \text{ (V/m}^2\text{)}$$

When

J has units of A/m^2

M has units of V/m^2

Then

$$\underline{A} = \frac{\mu}{4\pi} \iiint_V \underline{J} \frac{e^{-jkR}}{R} dv' \quad (3-27)$$

$$\underline{F} = \frac{\varepsilon}{4\pi} \iiint_V \underline{M} \frac{e^{-jkR}}{R} dv' \quad (3-28)$$

Dipole and Geometry

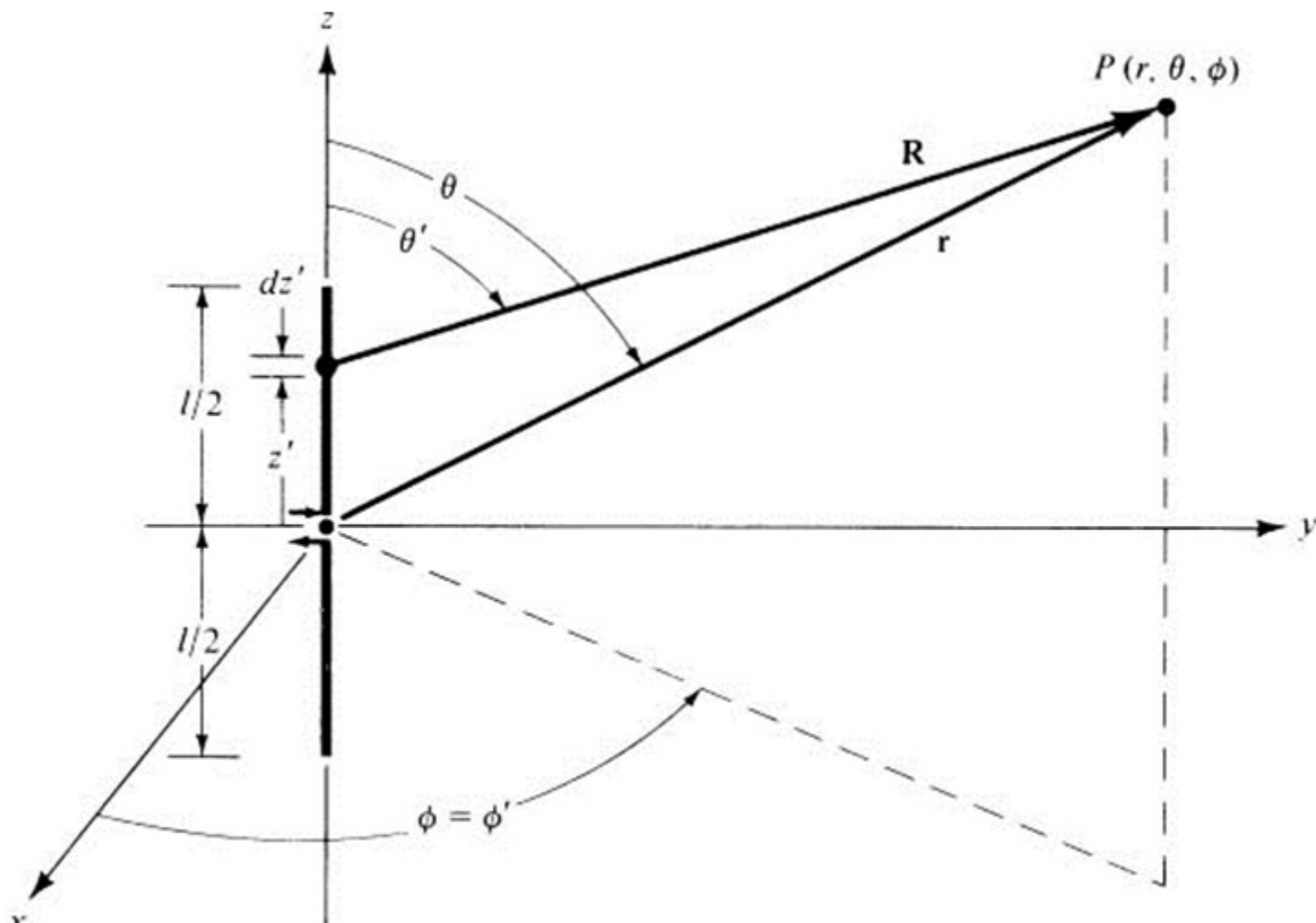


Fig. 4.4a

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Infinitesimal Dipole (top hat) ($\ell \leq \lambda / 50$)

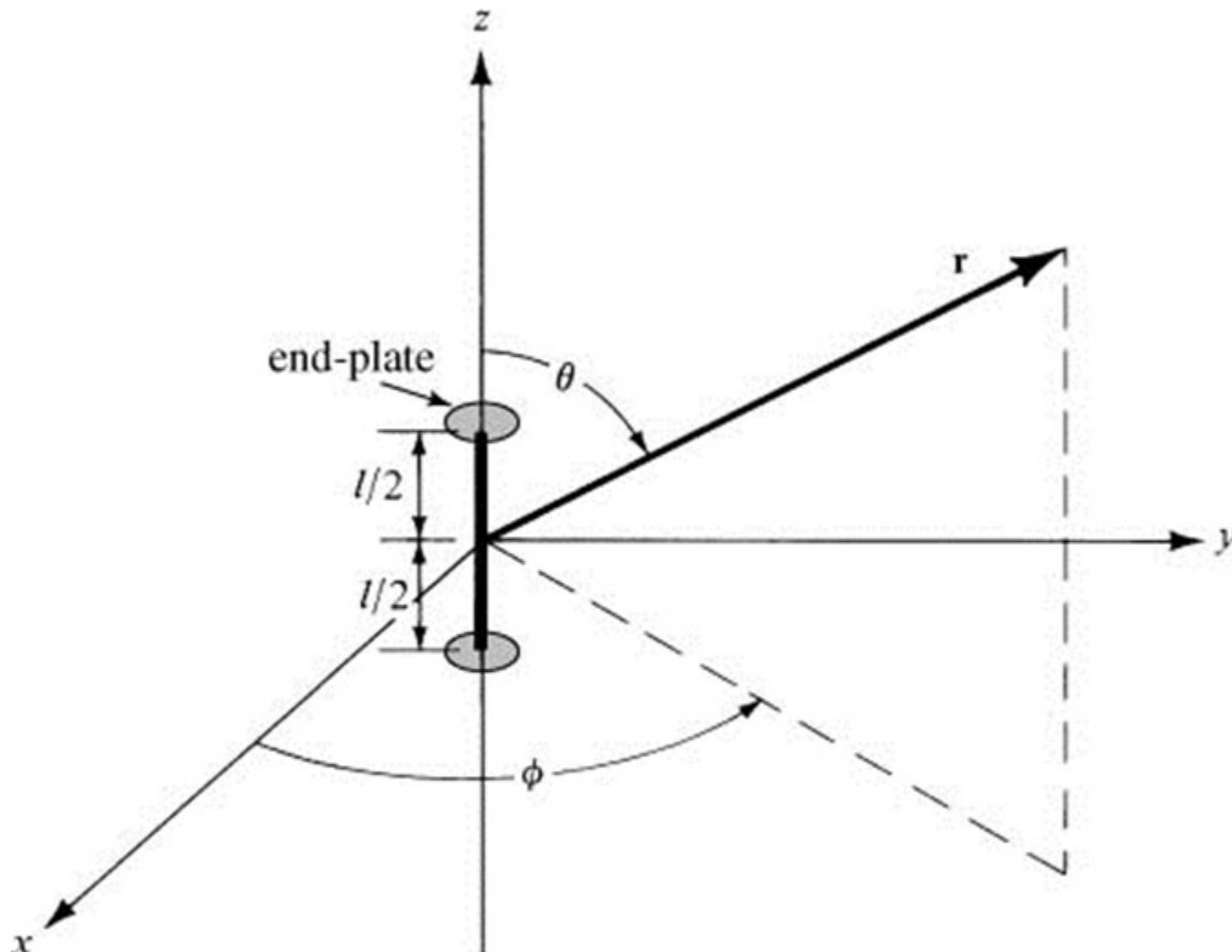


Fig. 4.1a

Electric Field Orientation

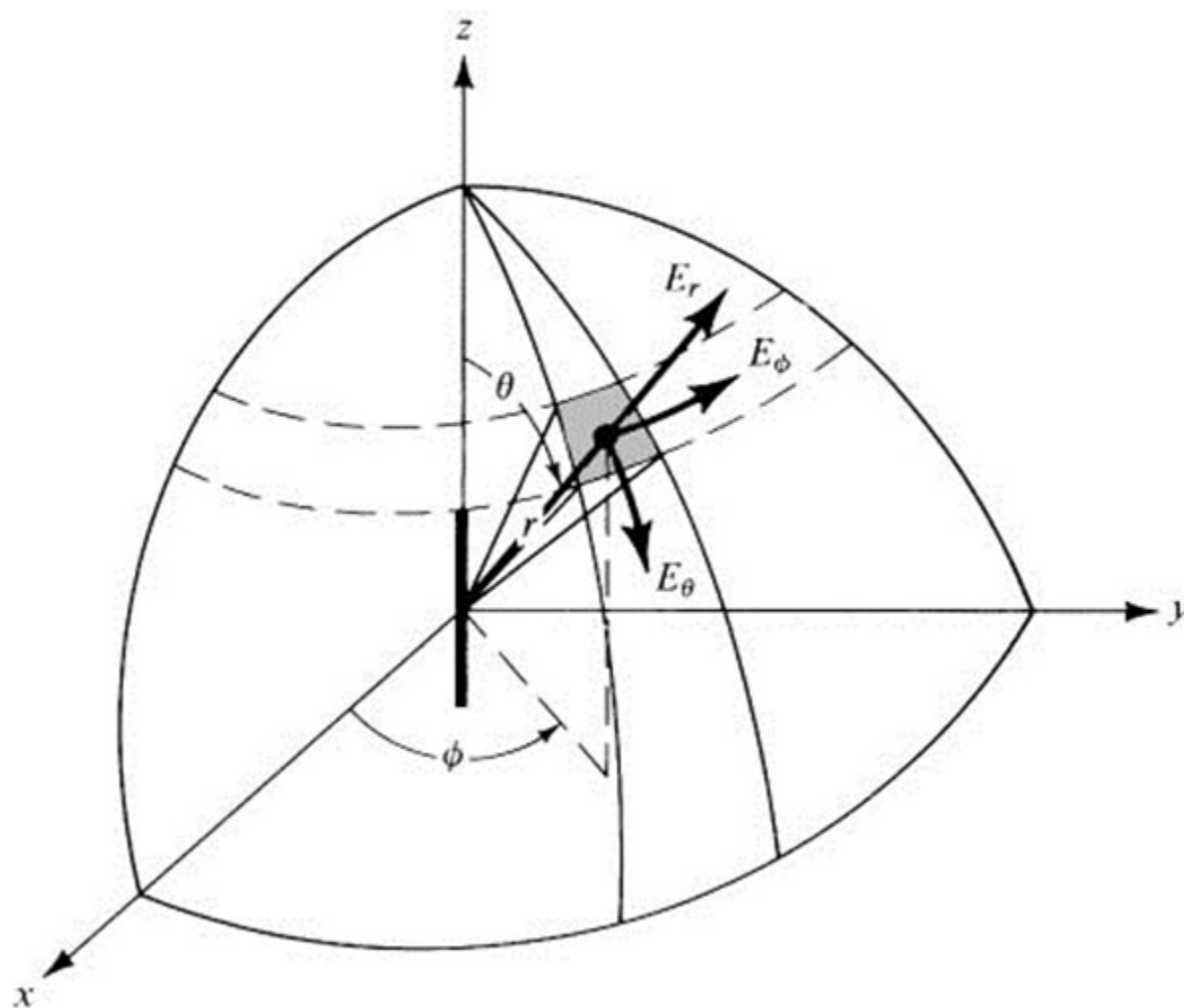


Fig. 4.1b

When

J has units of A/m^2

M has units of V/m^2

Then

$$\underline{A} = \frac{\mu}{4\pi} \iiint_V \underline{J} \frac{e^{-jkR}}{R} dv' \quad (3-27)$$

$$\underline{F} = \frac{\varepsilon}{4\pi} \iiint_V \underline{M} \frac{e^{-jkR}}{R} dv' \quad (3-28)$$

Dipole and Geometry

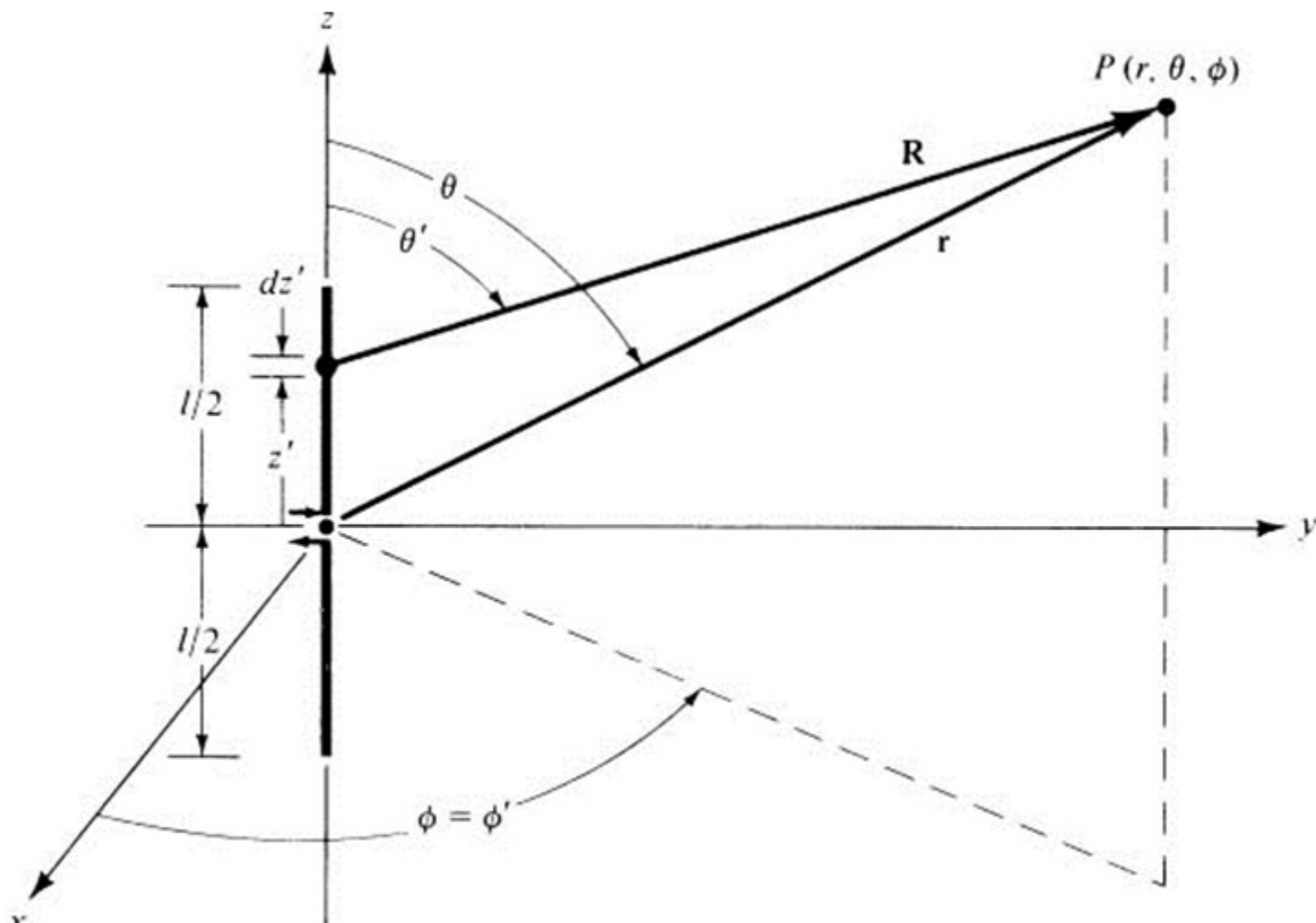


Fig. 4.4a

Vector Potential

$$\underline{A} = \frac{\mu}{4\pi} \int_{-\ell/2}^{\ell/2} \underline{I}_e(z') \frac{e^{-jkR}}{R} d\ell'$$

Approximation: $R \approx r$

$$\underline{A} = \frac{\mu}{4\pi} \int_{-\ell/2}^{\ell/2} \hat{a}_z I_o \frac{e^{-jkr}}{r} dz'$$

$$\underline{A} = \hat{a}_z \frac{\mu I_o}{4\pi} \frac{e^{-jkr}}{r} \int_{-\ell/2}^{\ell/2} dz'$$

$$\underline{A} = \hat{a}_z \frac{\mu I_o e^{-jkr}}{4\pi r} (\ell) \quad (4-4)$$

$$\underline{A} = \hat{a}_z \frac{\mu I_o \ell}{4\pi} \frac{e^{-jkr}}{r}$$

$$\underline{A} = \hat{a}_z \frac{\mu I_o \ell}{4\pi r} e^{-jkr} = \hat{a}_z A_z$$

$$A_z = \frac{\mu I_o \ell}{4\pi r} e^{-jkr}; \quad A_x = A_y = 0$$

$$\begin{pmatrix} A_r \\ A_\theta \\ A_\phi \end{pmatrix} = \underbrace{\begin{pmatrix} \sin \theta \cos \phi & \sin \theta \sin \phi & \cos \theta \\ \cos \theta \cos \phi & \cos \theta \sin \phi & -\sin \theta \\ -\sin \phi & \cos \phi & 0 \end{pmatrix}}_{[T]_{rs}} \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix}$$

$$[A]_s = [T]_{rs} [A]_r$$

$$\underline{A} = \hat{a}_z A_z = \hat{a}_z \frac{\mu I_o \ell}{4\pi r} e^{-jkr}$$

$$A_r = A_z \cos \theta = \frac{\mu I_o \ell}{4\pi r} \cos \theta e^{-jkr} \quad (4-6a)$$

$$A_\theta = -A_z \sin \theta = -\frac{\mu I_o \ell}{4\pi r} \sin \theta e^{-jkr} \quad (4-6b)$$

$$A_\phi = 0 \quad (4-6c)$$

$$\underline{H} = \frac{1}{\mu} \nabla \times \underline{A} = \hat{a}_\phi \frac{1}{\mu r} \left[\frac{\partial}{\partial r} (r A_\theta) - \frac{\partial A_r}{\partial \theta} \right] \quad (4-7)$$

$$H_r = H_\theta = 0 \quad (4-8a)$$

$$H_\phi = j \frac{k I_e \ell}{4\pi r} \sin \theta \left[1 + \frac{1}{jkr} \right] e^{-jkr} \quad (4-8b)$$

$$\underline{E} = -j\omega \underline{A} - j \frac{1}{\omega\mu\epsilon} \nabla(\nabla \cdot \underline{A}) \quad (4-9)$$

$$E_r = \eta \frac{I_e \ell \cos \theta}{2\pi r^2} \left[1 + \frac{1}{jkr} \right] e^{-jkr} \quad (4-10a)$$

$$E_\theta = j\eta \frac{kI_e \ell \sin \theta}{4\pi r} \left[1 + \frac{1}{jkr} - \frac{1}{(kr)^2} \right] e^{-jkr} \quad (4-10b)$$

$$E_\phi = 0 \quad (4-10c)$$

$$E_r = \eta \frac{I_o \ell}{2\pi r^2} \cos \theta \left[1 + \frac{1}{jkr} \right] e^{-jkr} \quad (4-10a)$$

$$E_\theta = j\eta \frac{kI_o \ell}{4\pi r} \sin \theta \left[1 + \frac{1}{jkr} - \frac{1}{(kr)^2} \right] e^{-jkr} \quad (4-10b)$$

$$E_\phi = H_r = H_\theta = 0 \quad (4-10c), (4-8a)$$

$$H_\phi = j \frac{kI_o \ell}{4\pi r} \left[1 + \frac{1}{jkr} \right] \sin \theta e^{-jkr} \quad (4-8b)$$

$$\begin{aligned}
\underline{W} &= \frac{1}{2} \underline{E} \times \underline{H}^* = \frac{1}{2} [\hat{a}_r E_r + \hat{a}_\theta E_\theta] \times (\hat{a}_\phi H_\phi^*) \\
\underline{W} &= \frac{1}{2} \left(\underbrace{\hat{a}_r \times \hat{a}_\phi}_{-\hat{a}_\theta} \right) E_r H_\phi^* + \frac{1}{2} \left(\underbrace{\hat{a}_\theta \times \hat{a}_\phi}_{\hat{a}_r} \right) E_\theta H_\phi^* \\
\underline{W} &= \frac{1}{2} \left[\underbrace{\hat{a}_r E_\theta H_\phi^*}_{\underline{W}_r} - \underbrace{\hat{a}_\theta E_r H_\phi^*}_{\underline{W}_\theta} \right] \quad (4-11) \\
\underline{W} &= \hat{a}_r W_r + \hat{a}_\theta W_\theta
\end{aligned}$$

$$\underline{W} = \hat{a}_r W_r + \hat{a}_\theta W_\theta$$

$$W_r = \frac{1}{2} E_\theta H_\phi^* = \frac{\eta}{8} \left| \frac{I_0 l}{\lambda} \right|^2 \frac{\sin^2 \theta}{r^2} \left[1 - j \frac{1}{(kr)^3} \right] \quad (4-12a)$$

$$W_\theta = \frac{1}{2} E_r H_\phi^* = j\eta \frac{k |I_0 l|^2 \cos \theta \sin \theta}{16\pi^2 r^3} \left[1 + \frac{1}{(kr)^2} \right] \quad (4-12b)$$

The W_θ component, which is purely imaginary, varies primarily (for small kr) as $1/r^5$, and the imaginary part of the W_r component varies primarily (for small kr) also as $1/r^5$, then these components are primarily dominant inside the radian sphere ($r \leq \lambda/2\pi$). Thus the power density is primarily imaginary, pulsates back and forth, and it is basically trapped inside the radian sphere. A small amount of real power density “escapes/leaks” outside the radian sphere, and it is represented by the real part of W_r which varies as $1/r^2$. This eventually leads to the real power radiated and found in the far-field.

$$P = \oiint_S \underline{W} \cdot d\underline{s}$$

$$P = \int_0^{2\pi} \int_0^{\pi} (\hat{a}_r W_r + \hat{a}_\theta W_\theta) \cdot \underbrace{\hat{a}_r r^2 \sin \theta d\theta d\phi}_{d\underline{s}}$$

$$P = \int_0^{2\pi} \int_0^{\pi} W_r r^2 \sin \theta d\theta d\phi$$

$$P = \eta \left(\frac{\pi}{3} \right) \left| \frac{I_0 l}{\lambda} \right|^2 \left[1 - j \frac{1}{(kr)^3} \right] \quad (4-14)$$

Outwardly Flowing Complex Power from Infinitesimal Dipole

time-average
radiated power

time-average
reactive power
in radial direction

$$P = \frac{1}{2} \iint_S \underline{E} \times \underline{H}^* \cdot d\underline{S} = \underbrace{P_{rad}}_{\text{time-average radiated power}} + \underbrace{j2\omega (\tilde{W}_m - \tilde{W}_e)}_{\text{time-average reactive power in radial direction}}$$
$$= \eta \left(\frac{\pi}{3} \right) \left| \frac{I_0 l}{\lambda} \right|^2 \left[1 - j \frac{1}{(kr)^3} \right] \quad (4-15)$$

Outwardly Flowing Complex Power from Infinitesimal Dipole

$$P = P_{rad} + j2\omega(\tilde{W}_m - \tilde{W}_e)$$

$$P_{rad} = \eta \left(\frac{\pi}{3} \right) \left| \frac{I_0 l}{\lambda} \right|^2$$

remains constant
as r increases

(4-16)

$$2\omega(\tilde{W}_m - \tilde{W}_e) = -\eta \left(\frac{\pi}{3} \right) \left| \frac{I_0 l}{\lambda} \right|^2 \frac{1}{(kr)^3}$$

(4-17)

decreases rapidly
as r increases

$$P_{rad} = \eta \left(\frac{\pi}{3} \right) \left| \frac{I_o \ell}{\lambda} \right|^2 = \frac{1}{2} |I_o|^2 R_r \quad (4-18)$$

$$R_r = \frac{2P_{rad}}{|I_o|^2} = \eta \left(\frac{2\pi}{3} \right) \left| \frac{\ell}{\lambda} \right|^2 = 80\pi \left(\frac{\ell}{\lambda} \right)^2$$

$$R_r = 80\pi^2 \left(\frac{\ell}{\lambda} \right)^2 \quad (4-19)$$

$$\ell \leq \lambda / 50$$

Evaluation of Directivity

$$\underline{W}_{rad} = \underline{W}_{av} = \hat{a}_r \frac{\eta}{8} \left| \frac{I_o \ell}{\lambda} \right|^2 \frac{\sin^2 \theta}{r^2} = \hat{a}_r \frac{\eta}{2} \left| \frac{k I_o \ell}{4\pi} \right|^2 \frac{\sin^2 \theta}{r^2}$$

$$W_{rad} = \frac{\eta}{8} \left| \frac{k I_o}{\lambda} \right|^2 \frac{\sin^2 \theta}{r^2}$$

$$U = r^2 W_{rad} = \frac{\eta}{8} \left| \frac{k I_o}{\lambda} \right|^2 \sin^2 \theta$$

$$D_o = \frac{U_{\max}}{U_o} = \frac{4\pi U_{\max}}{P_{rad}} = \frac{4\pi \frac{\eta}{8} \left| \frac{k I_o}{\lambda} \right|^2}{\eta \left(\frac{\pi}{3} \right) \left| \frac{I_o \ell}{\lambda} \right|^2} = \frac{3}{2} \quad (4-31)$$

Directivity, Normalized Power Pattern, and Radiation Resistance for Infinitesimal Dipole

$$D_0 = \frac{3}{2} = 1.76 \text{ dB}$$

$$U_n = \sin^2(\theta)$$

$$R_r = 80\pi^2 \left(\frac{\ell}{\lambda} \right)^2$$

assumes medium
is free space

$$A_{em} = \frac{\lambda^2}{4\pi} D_o = \frac{\lambda^2}{4\pi} \left(\frac{3}{2} \right) = \frac{3}{8\pi} \lambda^2$$

$$A_{em} = \frac{3}{8\pi} \lambda^2 = 0.119 \lambda^2 \quad (4-32)$$

3-D Radiation Pattern of Infinitesimal Dipole

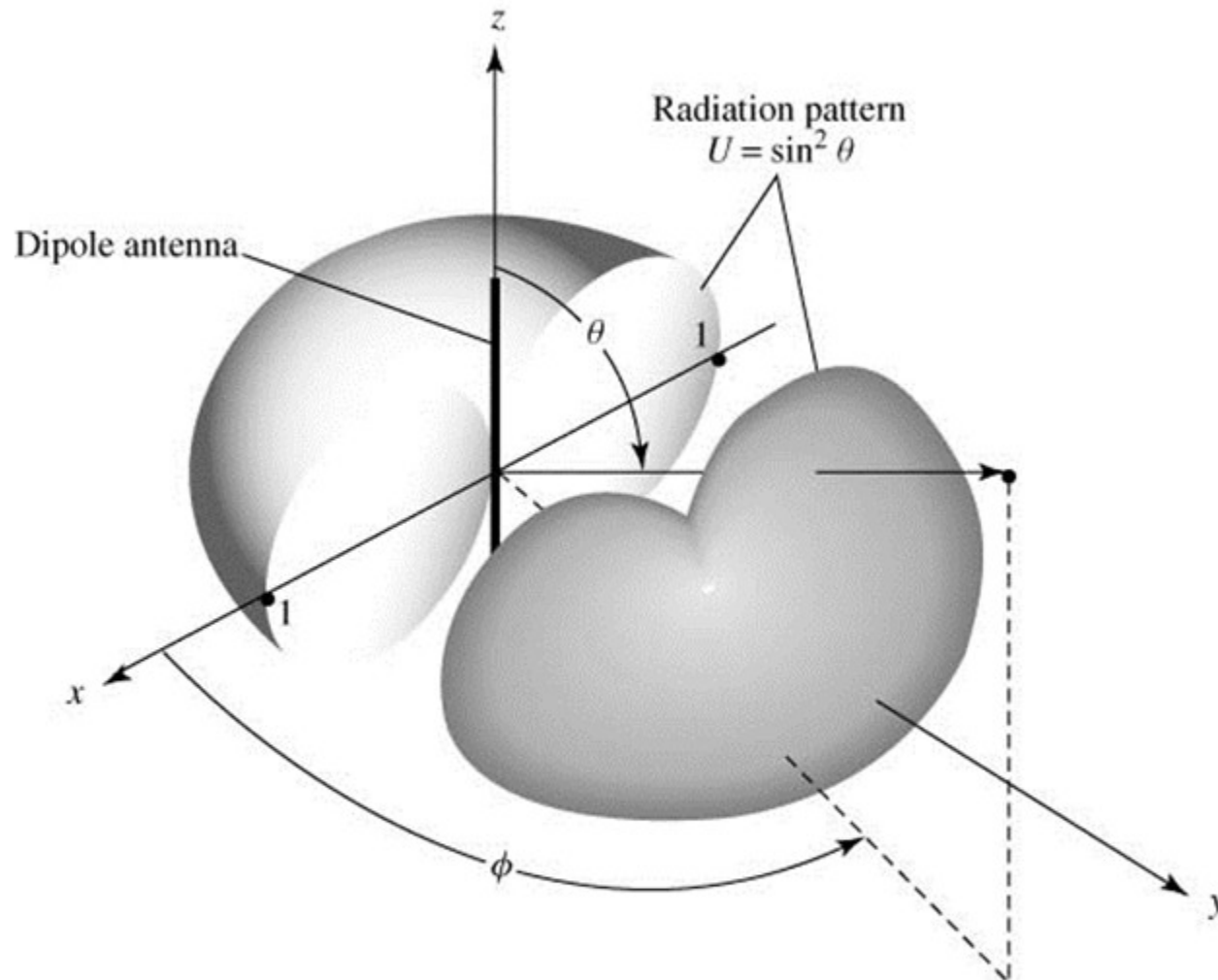


Fig. 4.3

Example 4.1

Find the radiation resistance of an infinitesimal dipole whose overall length is $l = \lambda/50$.

Solution: Using (4-19)

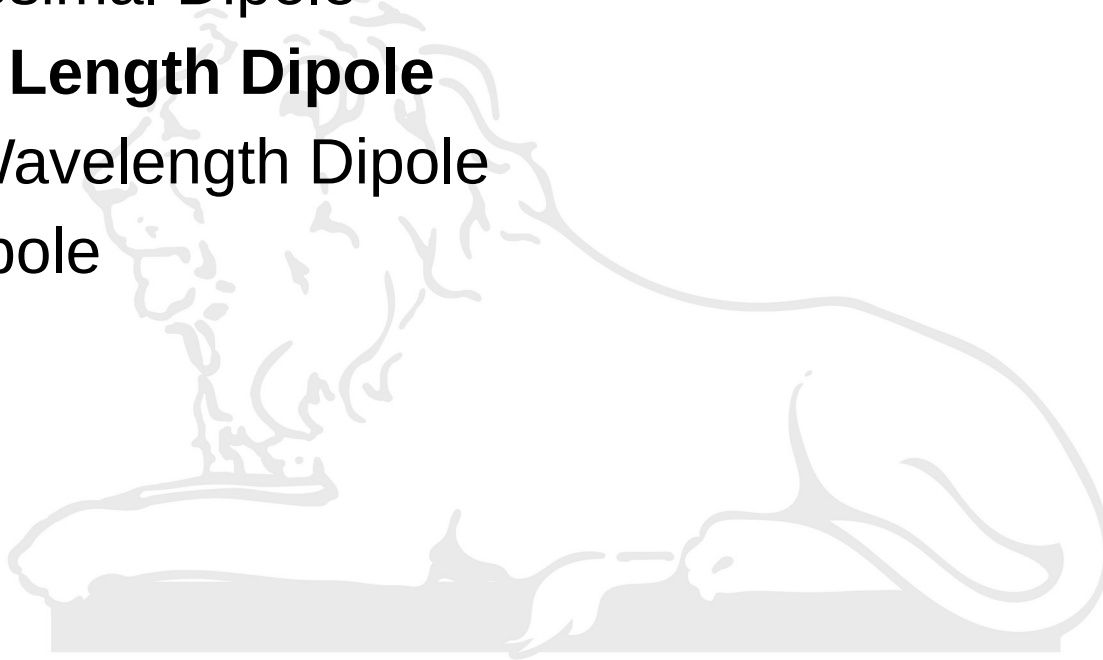
$$R_r = 80\pi^2 \left(\frac{l}{\lambda}\right)^2 = 80\pi^2 \left(\frac{1}{50}\right)^2 = 0.316 \text{ ohms}$$

Since the radiation resistance of an infinitesimal dipole is about 0.3 ohms, it will present a very large mismatch when connected to practical transmission lines, many of which have characteristic impedances of 50 or 75 ohms. The reflection efficiency (e_r) and hence the overall efficiency (e_0) will be very small.

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Finite Dipole Geometry

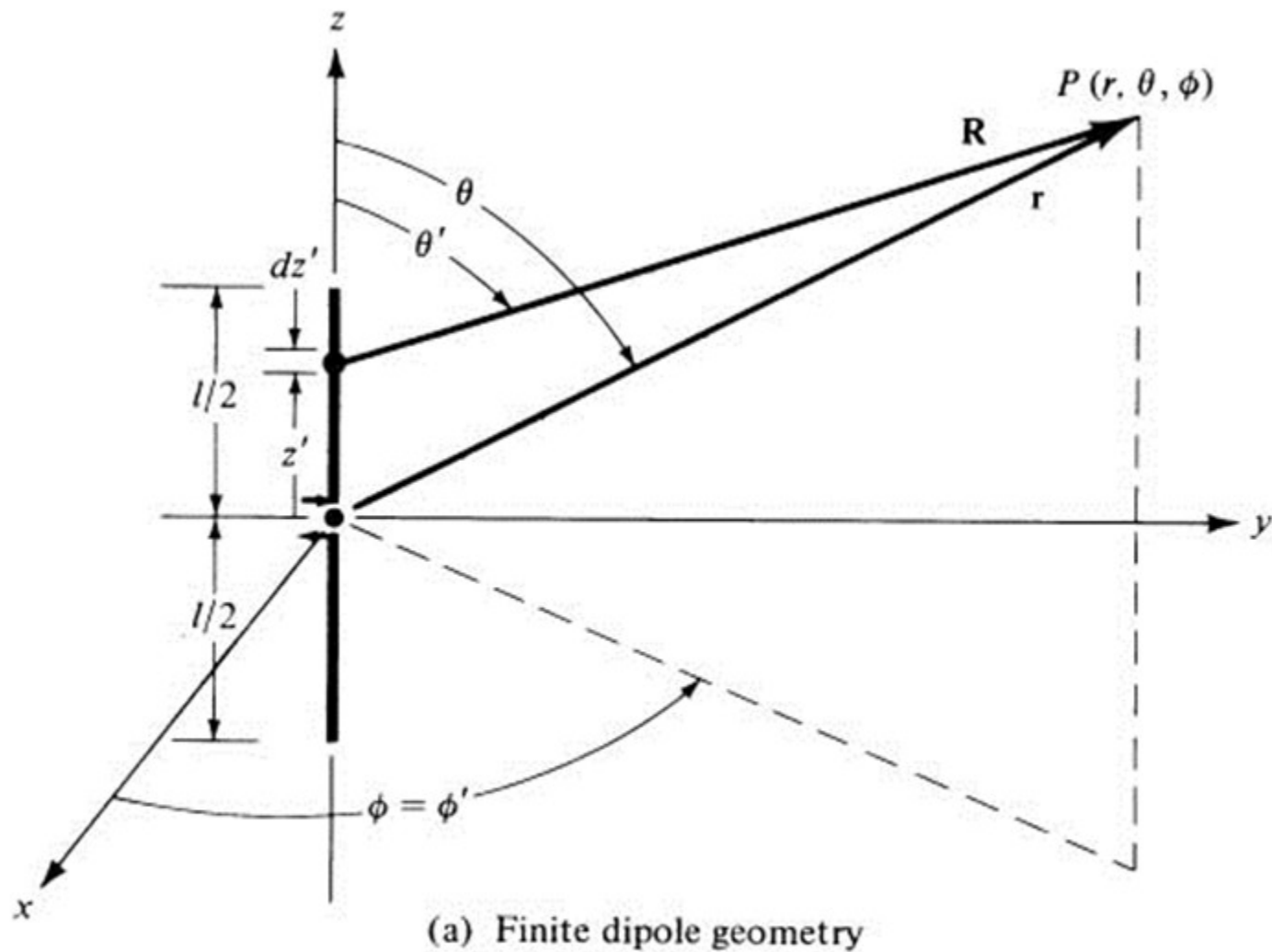


Fig. 4.5a

Finite Dipole Geometry & Far-Field Approximations

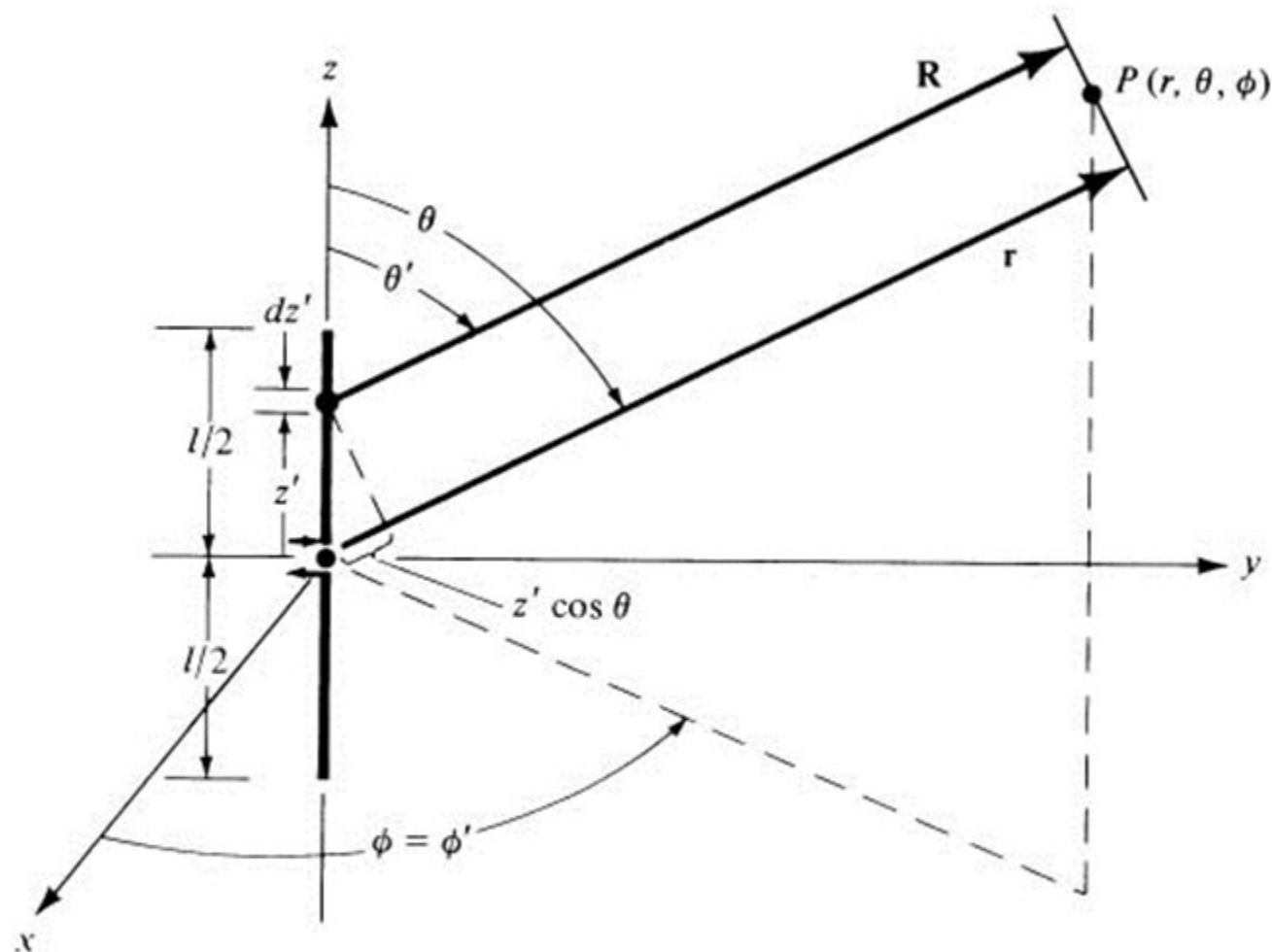


Fig. 4.5b

Phase Variations

$$R \cong r - z' \cos \theta$$

Amplitude Variations

$$R \cong r$$

Far-Field Region

$$A_z = \frac{\mu}{4\pi} \int_{-l/2}^{+l/2} I_e(z') \frac{e^{-jkR}}{R} dz'$$

$$A_z \simeq \frac{\mu}{4\pi} \int_{-l/2}^{+l/2} I_e(z') \frac{e^{-jk(r-z'\cos\theta)}}{r} dz'$$

$$A_z = \frac{\mu e^{-jkr}}{4\pi r} \int_{-l/2}^{+l/2} I_e(z') e^{jkz'\cos\theta} dz'$$

Current Distributions Along the Length of a Linear Wire Antenna

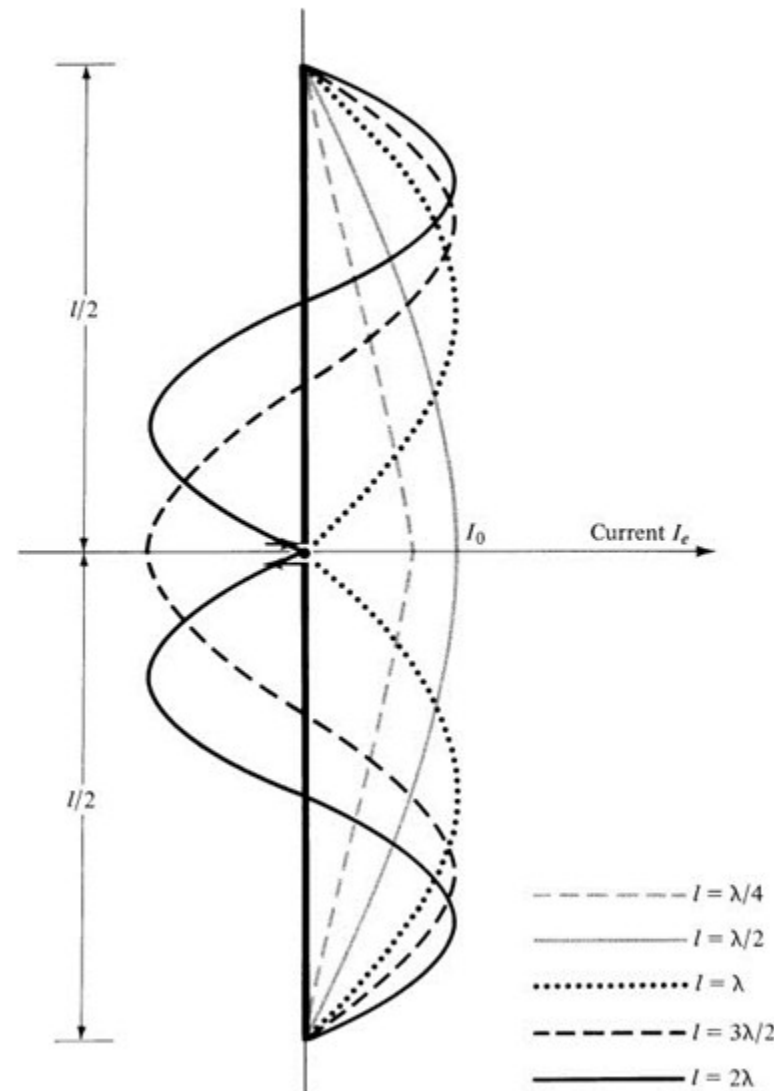


Fig. 4.8

Idealized Current Distribution Of Linear Dipole

$$I_e(z') = \begin{cases} I_o \sin \left[k \left(\frac{\ell}{2} - z' \right) \right] & 0 \leq z' \leq +\ell / 2 \\ I_o \sin \left[k \left(\frac{\ell}{2} + z' \right) \right] & -\ell / 2 \leq z' \leq 0 \end{cases} \quad (4-56)$$

$$\begin{aligned}
 E_{\theta} = & j\eta \frac{ke^{-jk r}}{4\pi r} \sin \theta \\
 & \times \left\{ \int_{-l/2}^0 I_o \sin \left[k \left(\frac{l}{2} + z' \right) \right] e^{+jkz' \cos \theta} dz' \right. \\
 & \left. + \int_0^{+l/2} I_o \sin \left[k \left(\frac{l}{2} - z' \right) \right] e^{+jkz' \cos \theta} dz' \right\}
 \end{aligned}
 \tag{4-60}$$

$$\begin{aligned}
 & \int e^{\alpha x} \sin[\beta x + \gamma] dx \\
 &= \frac{e^{\alpha x}}{\alpha^2 + \beta^2} [\alpha \sin(\beta x + \gamma) - \beta \cos(\beta x + \gamma)]
 \end{aligned}
 \tag{4-61}$$

$$\alpha = \pm jk \cos \theta, \quad \beta = \pm k, \quad \gamma = \frac{kl}{2}$$

(4-61a,b,c)

$$E_{\theta} = j\eta \frac{I_o e^{-jkr}}{2\pi r} \left[\frac{\cos\left(\frac{k\ell}{2} \cos \theta\right) - \cos\left(\frac{k\ell}{2}\right)}{\sin \theta} \right] \quad (4-62a)$$

$$E_{\theta} \cong C \left[\underbrace{\frac{\cos\left(\frac{k\ell}{2} \cos \theta\right) - \cos\left(\frac{k\ell}{2}\right)}{\sin \theta}}_{\text{Field Pattern}} \right]$$

$$H_{\phi} \cong \frac{E_{\theta}}{\eta}, \quad C = j\eta \frac{I_o e^{-jkr}}{2\pi r}$$

$$\begin{aligned}
\underline{W}_{av} &= \frac{1}{2} \text{Re}[\underline{E} \times \underline{H}^*] = \frac{1}{2} \text{Re}[\hat{a}_\theta E_\theta \times \hat{a}_\phi H_\phi^*] \\
&= \hat{a}_r \frac{1}{2} \text{Re}(E_\theta H_\phi^*) = \hat{a}_r \frac{1}{2} \text{Re}\left(E_\theta \frac{E_\theta^*}{\eta}\right) \\
\underline{W}_{av} &= \hat{a}_r \frac{1}{2\eta} \text{Re}(E_\theta E_\theta^*) = \hat{a}_r \frac{1}{2\eta} \text{Re}(|E_\theta|^2) \\
\underline{W}_{av} &= \hat{a}_r \eta \frac{|I_0|^2}{8\pi^2 r^2} \left[\frac{\cos\left(\frac{kl}{2} \cos \theta\right) - \cos\left(\frac{kl}{2}\right)}{\sin \theta} \right]^2 \quad (4-63)
\end{aligned}$$

$$W_{av} = W_{rad} = \eta \frac{|I_0|^2}{8\pi^2 r^2} \left[\frac{\cos\left(\frac{kl}{2} \cos \theta\right) - \cos\left(\frac{kl}{2}\right)}{\sin \theta} \right]^2 \quad (4-63)$$

$$U_{rad} = r^2 W_{av} = \underbrace{\eta \frac{|I_0|^2}{8\pi^2}}_{B_0} \left[\frac{\cos\left(\frac{kl}{2} \cos \theta\right) - \cos\left(\frac{kl}{2}\right)}{\sin \theta} \right]^2 \quad (4-64)$$

$$U_{rad} = B_0 \underbrace{\left[\frac{\cos\left(\frac{kl}{2} \cos \theta\right) - \cos\left(\frac{kl}{2}\right)}{\sin \theta} \right]^2}_{\text{Power Pattern}}$$

Elevation Plane Amplitude Patterns for a Thin Dipole with Sinusoidal Current Distribution ($l = \ll \lambda, \lambda/4, \lambda/2, 3\lambda/4, \lambda$)

HPBW

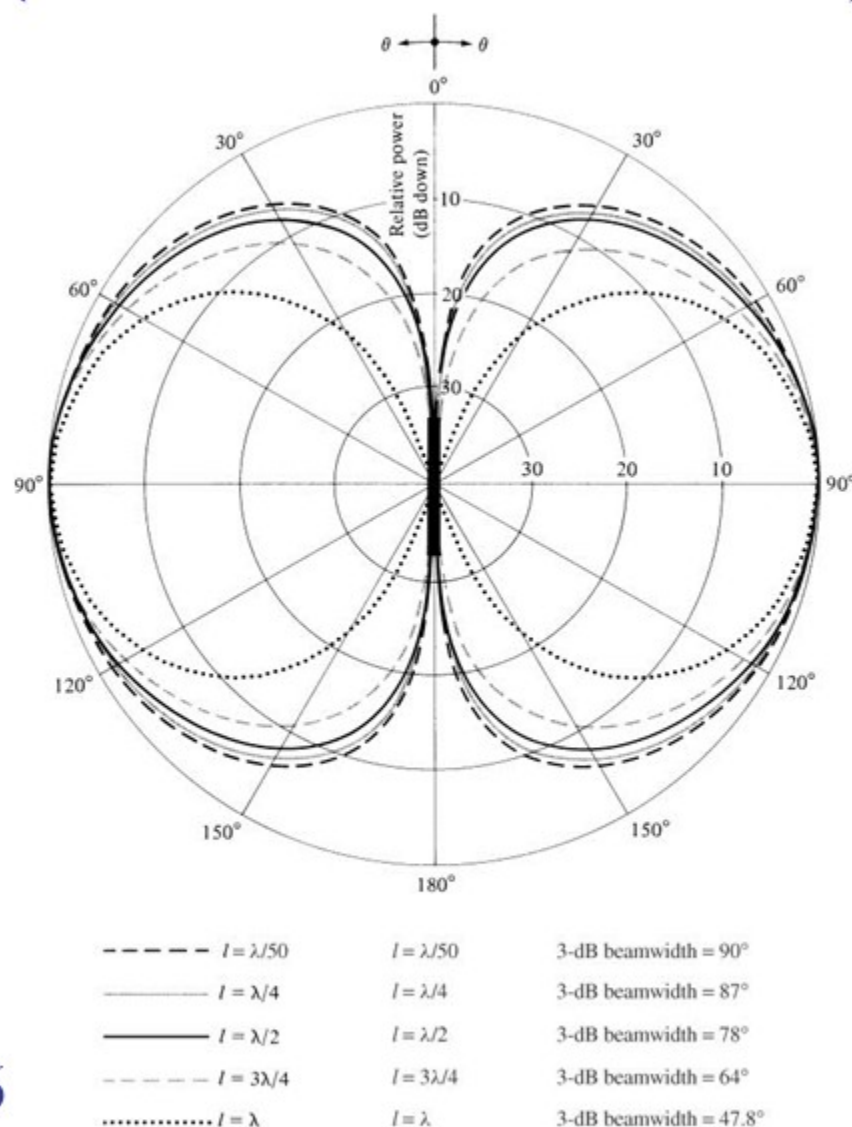
1. $l \leq \frac{\lambda}{50}$: HPBW = 90°
2. $l \leq \frac{\lambda}{2}$: HPBW = 74.93°
3. $l \leq \lambda$: HPBW = 47.8°

$$\frac{\lambda}{50} \leq l \leq \lambda$$

$$90^\circ \geq \text{HPBW} \geq 47.8^\circ$$

$$\Delta(\text{HPBW}) = 42.2^\circ$$

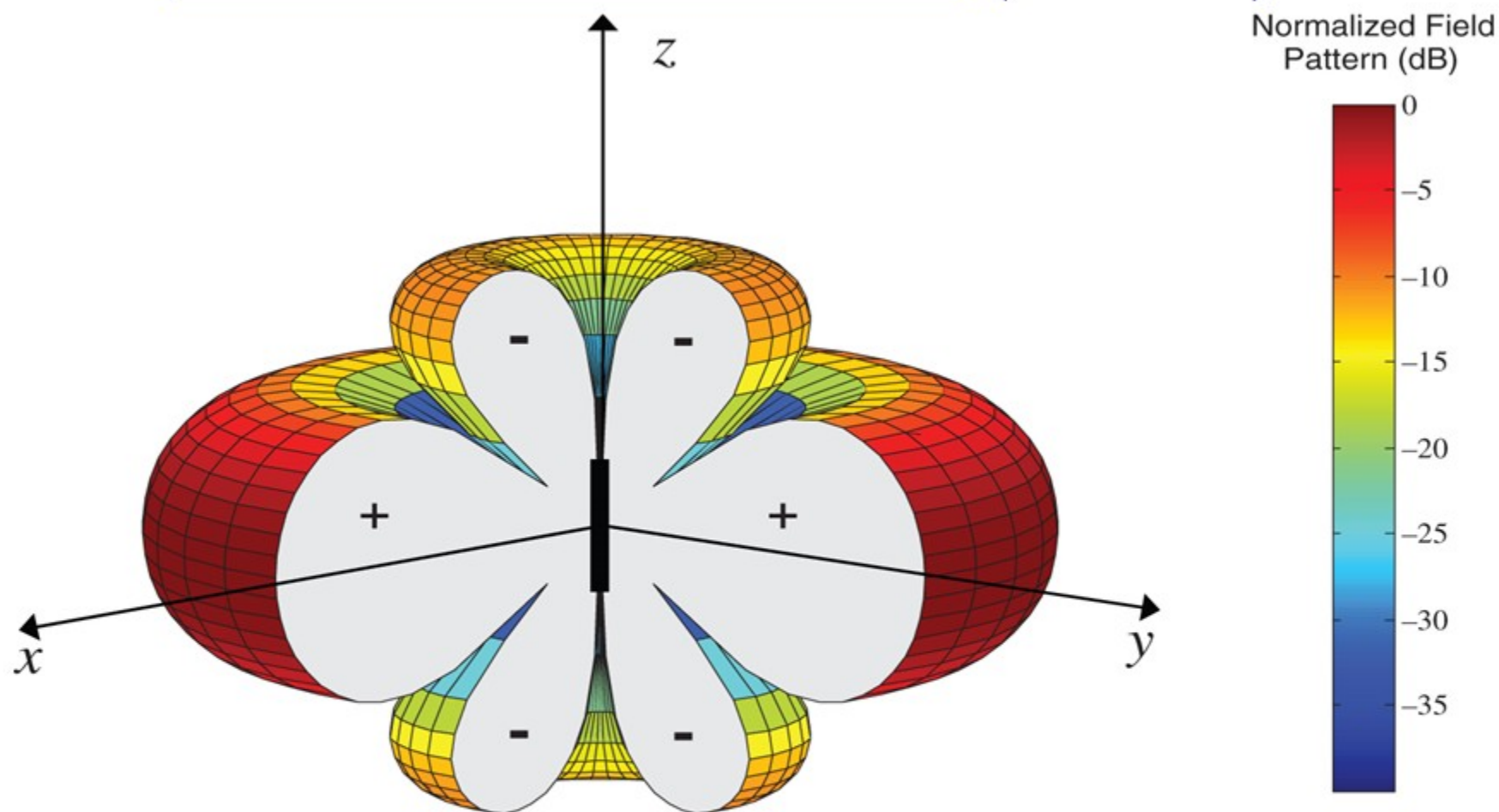
Fig. 4.6



Chapter 4

Linear Wire Antennas

Three-Dimensional Pattern ($l=1.25\lambda$)



(a) Three-dimensional

Fig. 4.7a

Two-Dimensional Pattern ($l=1.25\lambda$)

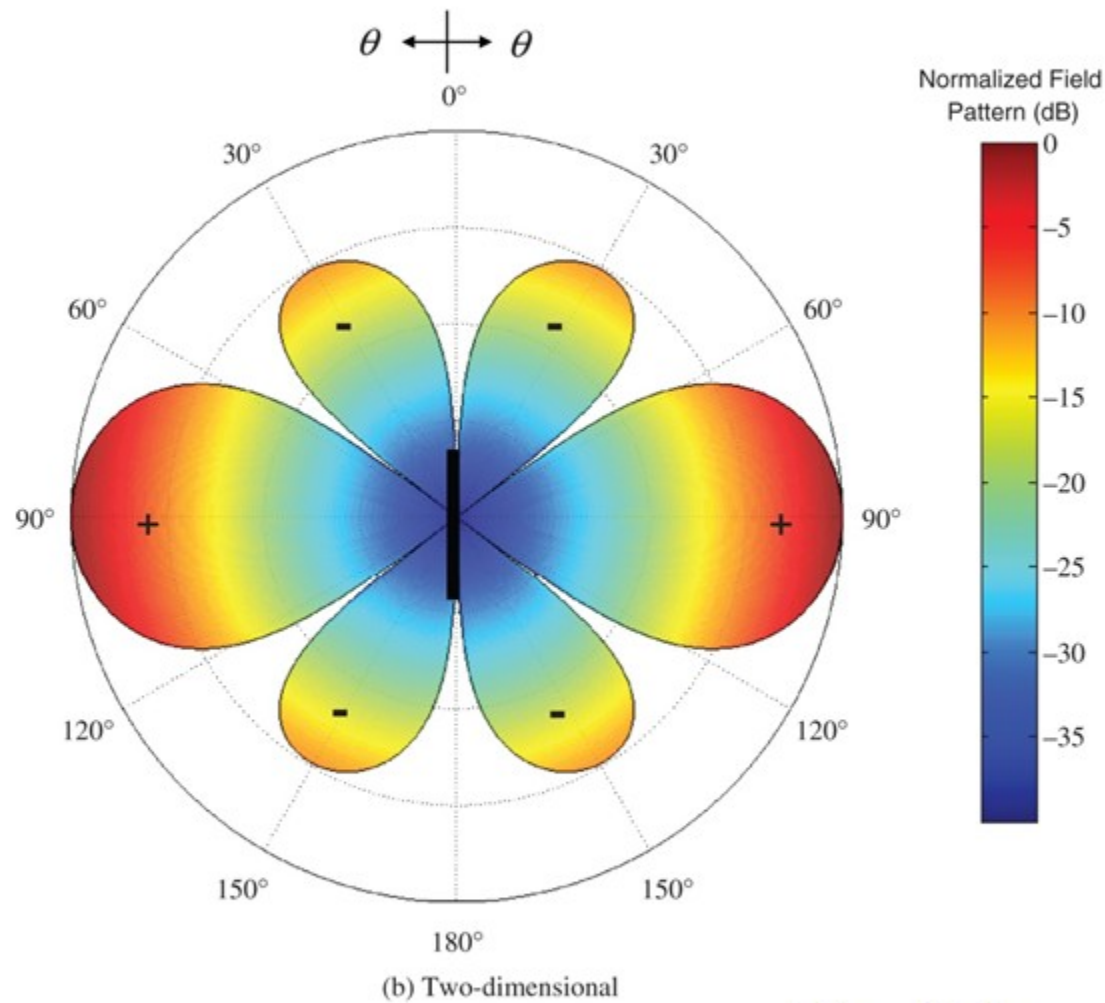
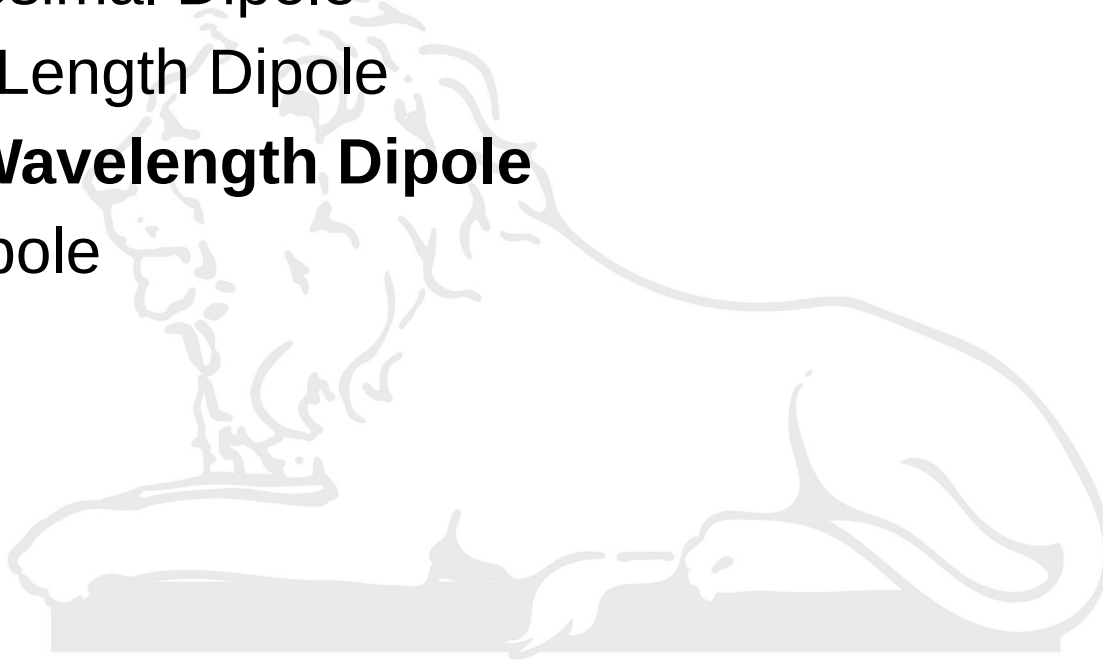


Fig. 4.7b

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Half-Wavelength Dipole $(l=\lambda/2)$

Half-Wavelength Dipole ($l = \lambda/2$)

$$E_{\theta} \simeq j\eta \frac{I_0 e^{-jkr}}{2\pi r} \underbrace{\left[\frac{\cos\left(\frac{\pi}{2} \cos \theta\right)}{\sin \theta} \right]}_{\text{Field Pattern}} \quad (4-84)$$

$$H_{\phi} \simeq j \frac{I_0 e^{-jkr}}{2\pi r} \underbrace{\left[\frac{\cos\left(\frac{\pi}{2} \cos \theta\right)}{\sin \theta} \right]}_{\text{Field Pattern}} = \frac{E_{\theta}}{\eta} \quad (4-85)$$

$$W_{av} = \eta \frac{|I_0|^2}{8\pi^2 r^2} \underbrace{\left[\frac{\cos\left(\frac{\pi}{2} \cos \theta\right)}{\sin \theta} \right]^2}_{\text{Power Pattern}} \simeq \eta \frac{|I_0|^2}{8\pi^2 r^2} \sin^3 \theta \quad (4-86)$$

$$U = r^2 W_{av} = \eta \frac{|I_0|^2}{8\pi^2} \underbrace{\left[\frac{\cos\left(\frac{\pi}{2} \cos \theta\right)}{\sin \theta} \right]^2}_{\text{Power Pattern}} \simeq \eta \frac{|I_0|^2}{8\pi^2} \sin^3 \theta \quad (4-87)$$

$$P_{rad} = \eta \frac{|I_0|^2}{4\pi} \int_0^\pi \frac{\cos^2\left(\frac{\pi}{2} \cos \theta\right)}{\sin \theta} d\theta \quad (4-88)$$

$$P_{rad} = \eta \frac{|I_0|^2}{8\pi} \int_0^{2\pi} \left(\frac{1 - \cos y}{y} \right) dy = \eta \frac{|I_0|^2}{8\pi} C_{in}(2\pi) \quad (4-89)$$

$$C_{in}(2\pi) = 0.577 + \ln(2\pi) - C_i(2\pi)$$

$$C_{in}(2\pi) = 0.577 + 1.838 - (-0.02) \simeq 2.435 \quad (4-90)$$

$$D_0 = \frac{4\pi U_{\max}}{P_{rad}} = 4\pi \frac{U|_{\theta=\pi/2}}{P_{rad}} \quad (4-91)$$

$$D_0 = \frac{4}{C_{in}(2\pi)} = \frac{4}{2.435} = 1.643$$

$$A_{em} = \frac{\lambda^2}{4\pi} D_0 = \frac{\lambda^2}{4\pi} (1.643) = 0.13\lambda^2 \quad (4-92)$$

$$R_r = \frac{2P_{rad}}{|I_0|^2} \frac{\eta}{4\pi} C_{in}(2\pi) \simeq \frac{120\pi}{4\pi} (2.435) \simeq 73.05 \quad (4-93)$$

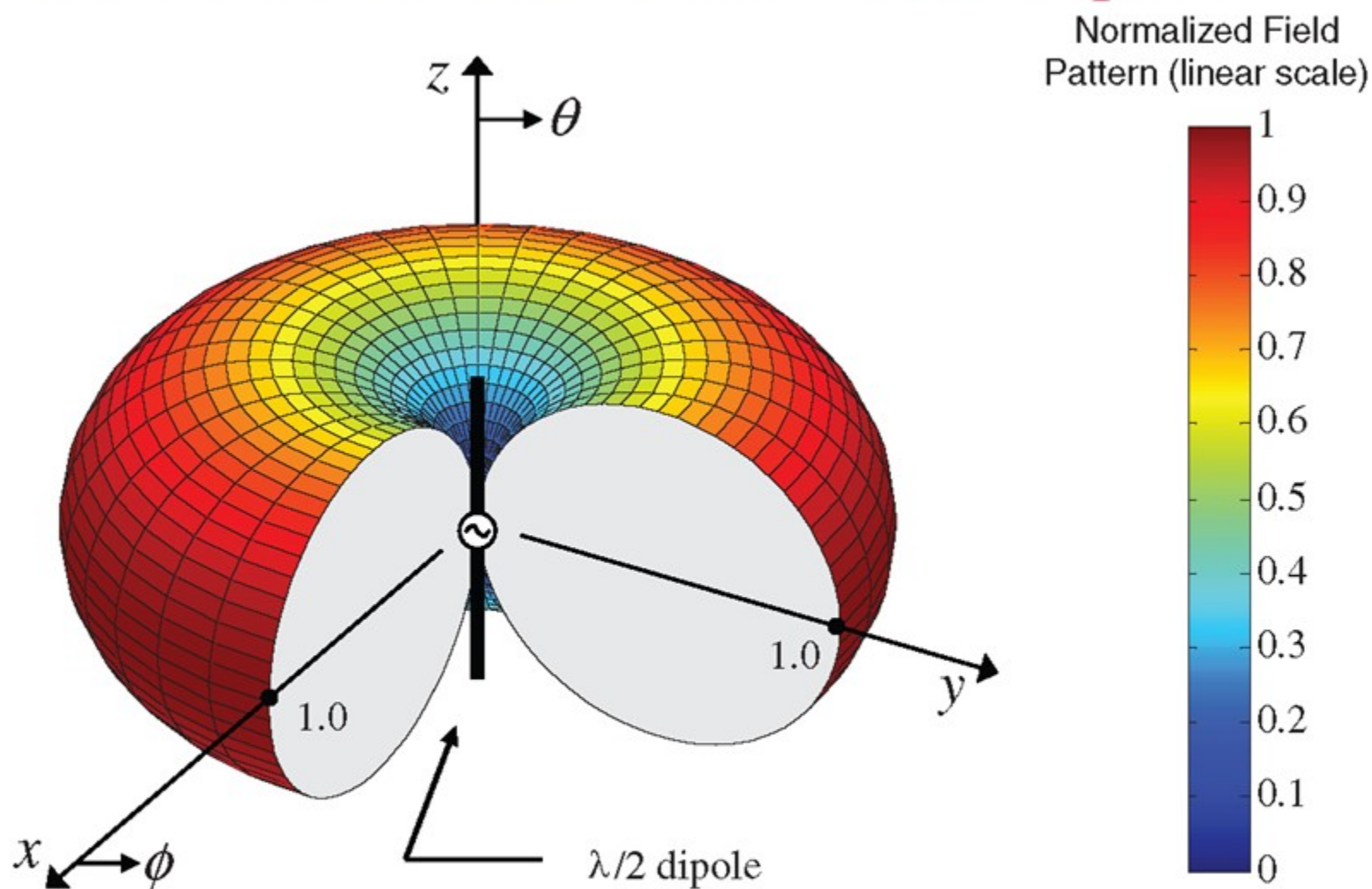
Half-Wavelength Dipole

- The input impedance of the half-wavelength dipole may be obtained as

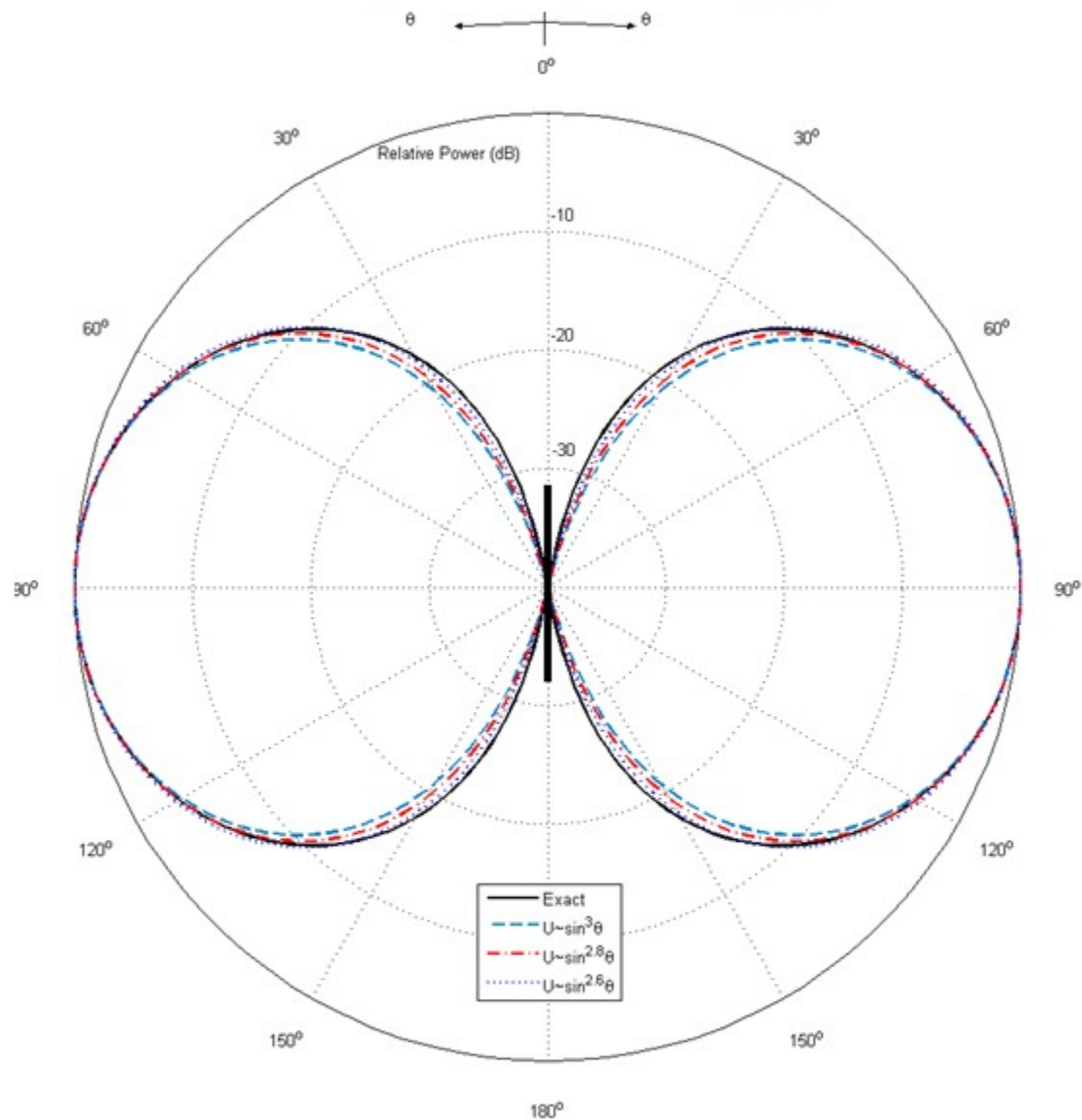
$$Z_{in} = 73.1 + j42.5 \Omega$$

- To eliminate the imaginary part of the input impedance, we can reduce the antenna length slightly. Depending on the radius of the wire, “resonance” occurs when the length is about 0.47λ to 0.48λ .

Three-Dimensional Pattern of $\lambda/2$ Dipole



Half-Wavelength Dipole Approximations



Contents



- Wire Antennas
 - Why Study Wire Antennas?
 - Infinitesimal Dipole
 - Finite Length Dipole
 - Half-Wavelength Dipole
 - **Monopole**



Image Theory

Electric Conductor

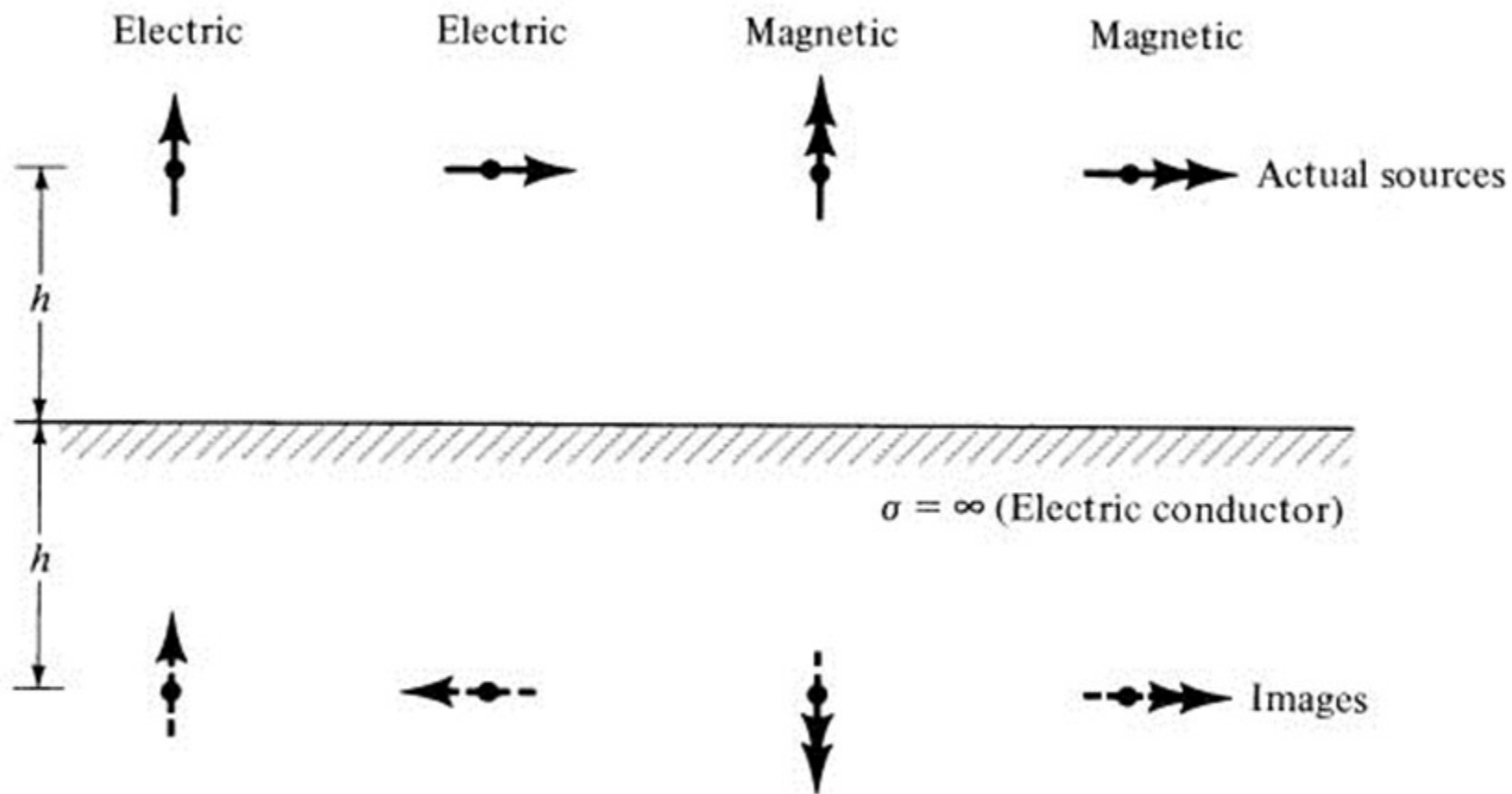
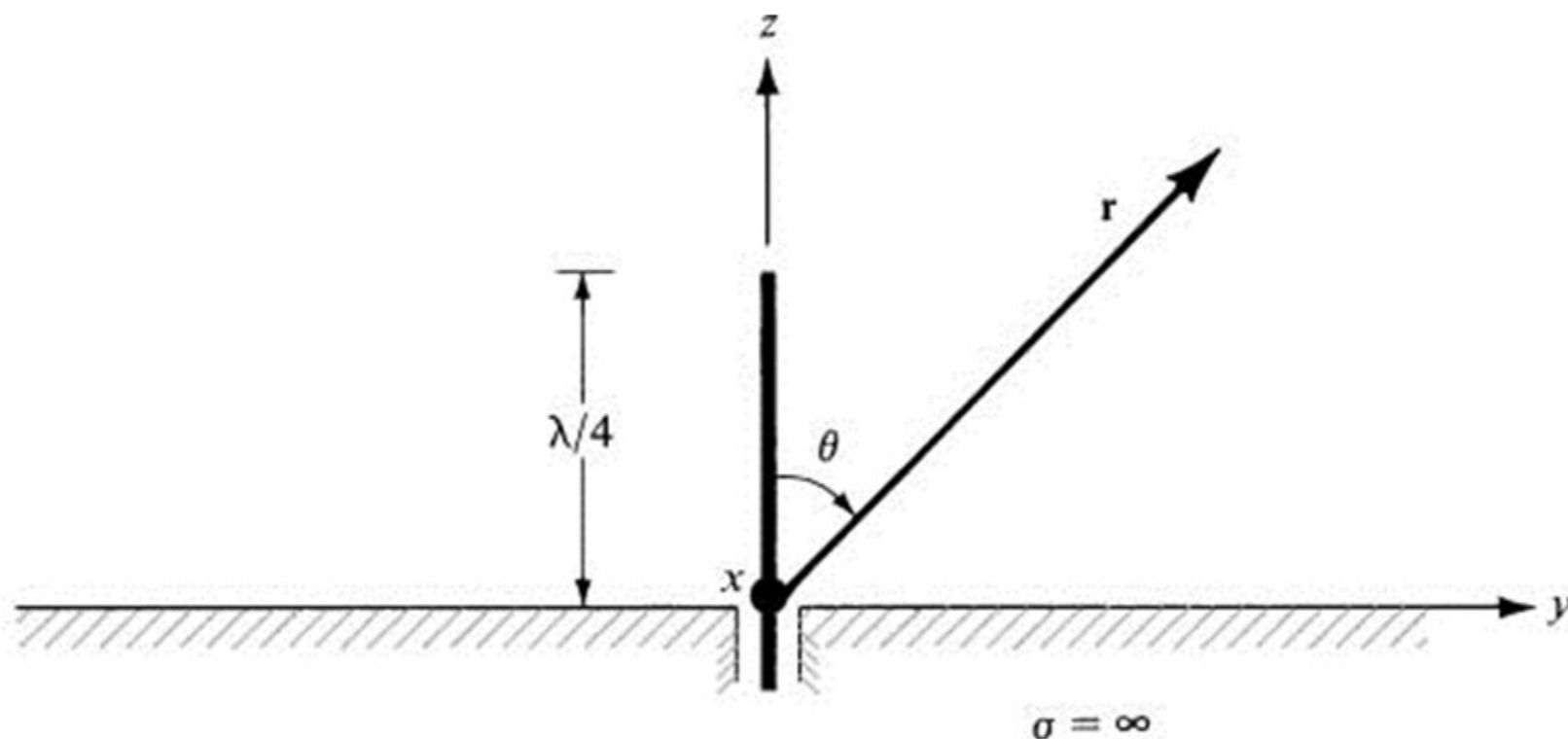


Fig. 4.16(a)

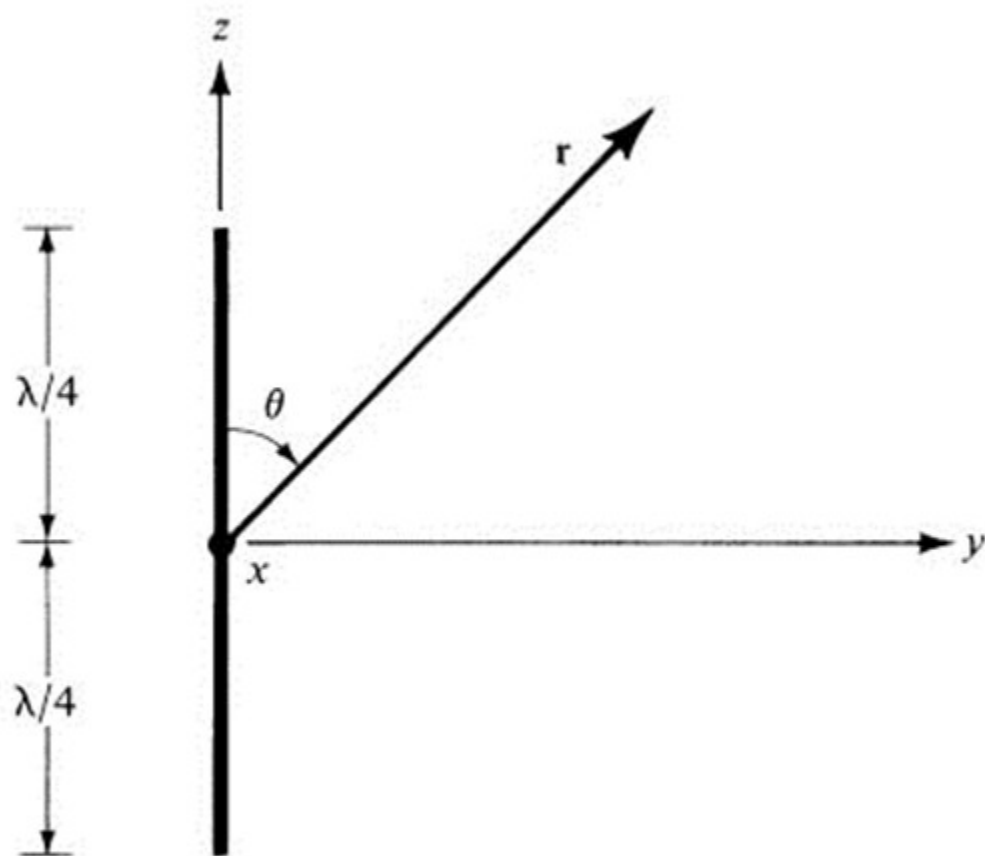
Monopoles and Dipoles



(a) $\lambda/4$ monopole on infinite electric conductor

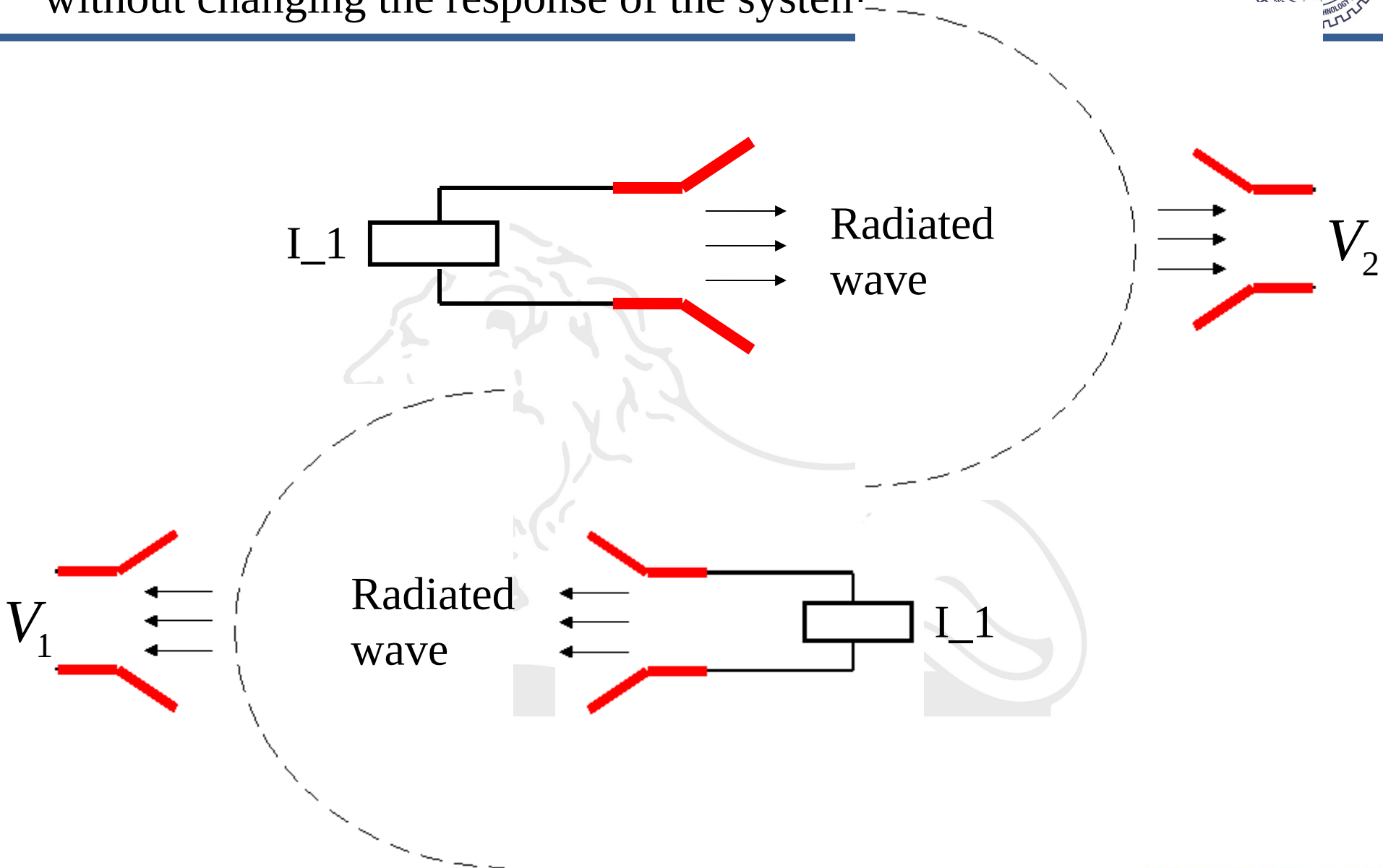
Fig. 4.22(a)

Monopoles and Dipoles



(b) Equivalent of $\lambda/4$ monopole on infinite electric conductor

The source and the detecting antenna can be interchanged without changing the response of the system



**Thank
You**

**Question
s?**