Question 3: In a nonconducting medium with $\varepsilon = 16\,\varepsilon_0$ and $\mu = \mu_0$, the electric field intensity of an electromagnetic wave is

$$\mathbf{E}(z,t) = \hat{\mathbf{x}}\, 10 \sin(10^{10} t - kz) \qquad \text{(V/m)}.$$

Determine the associated magnetic field intensity **H** and find the value of *k*.

Solution: We begin by finding the phasor $\widetilde{\mathbf{E}}(z)$ of $\mathbf{E}(z,t)$. Since $\mathbf{E}(z,t)$ is given as a sine function and phasors are defined in this book with reference to the cosine function, we rewrite As,

$$\mathbf{E}(z,t) = \hat{\mathbf{x}}\, 10 \cos(10^{10} t - kz - \pi/2) \qquad \text{(V/m)}$$
$$= \Re\mathfrak{e}\left[\widetilde{\mathbf{E}}(z)\, e^{j\omega t}\right],$$

with $\omega = 10^{10}$ (rad/s) and $\widetilde{\mathbf{E}}(z) = \hat{\mathbf{x}}\, 10 e^{-jkz} e^{-j\pi/2} = -\hat{\mathbf{x}} j 10 e^{-jkz}$.

To find both $\widetilde{\mathbf{H}}(z)$ and *k*, we will perform a "circle": we will use the given expression for $\widetilde{\mathbf{E}}(z)$ in Faraday's law to find $\widetilde{\mathbf{H}}(z)$; then we will use $\widetilde{\mathbf{H}}(z)$ in Ampere's law to find $\widetilde{\mathbf{E}}(z)$, which we will then compare with the original expression for $\widetilde{\mathbf{E}}(z)$; and the comparison will yield the value of *k*. Application of gives

$$\widetilde{\mathbf{H}}(z) = -\frac{1}{j\omega\mu} \nabla \times \widetilde{\mathbf{E}}$$
$$= -\frac{1}{j\omega\mu} \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ -j10e^{-jkz} & 0 & 0 \end{vmatrix}$$
$$= -\frac{1}{j\omega\mu}\left[\hat{\mathbf{y}} \frac{\partial}{\partial z}(-j10e^{-jkz})\right]$$
$$= -\hat{\mathbf{y}} j \frac{10k}{\omega\mu} e^{-jkz}.$$

So far, we have used first equation for $\widetilde{\mathbf{E}}(z)$ to find $\widetilde{\mathbf{H}}(z)$, but *k* remains unknown. To find *k*, we use $\widetilde{\mathbf{H}}(z)$ to find $\widetilde{\mathbf{E}}(z)$

$$\widetilde{\mathbf{E}}(z) = \frac{1}{j\omega\epsilon} \nabla \times \widetilde{\mathbf{H}}$$
$$= \frac{1}{j\omega\epsilon}\left[-\hat{\mathbf{x}} \frac{\partial}{\partial z}\left(-j \frac{10k}{\omega\mu} e^{-jkz}\right)\right]$$
$$= -\hat{\mathbf{x}} j \frac{10k^2}{\omega^2\mu\epsilon} e^{-jkz}.$$

$$k = \omega\sqrt{\mu\epsilon}$$
$$= 4\omega\sqrt{\mu_0\epsilon_0}$$
$$= \frac{4\omega}{c} = \frac{4 \times 10^{10}}{3 \times 10^8} = 133 \text{ (rad/m)}.$$

On comparison both the terms of $\widetilde{\mathbf{E}}(z)$

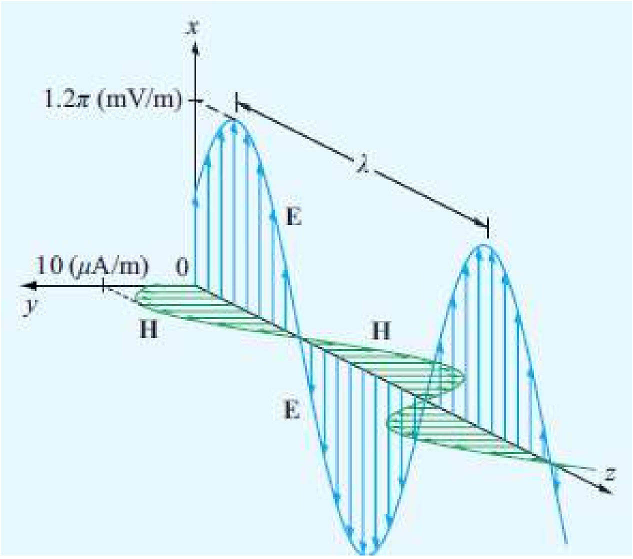
$$k^2 = \omega^2 \mu\epsilon,$$

With *k* known, the instantaneous magnetic field intensity is then given by

$$\mathbf{H}(z,t) = \Re\mathfrak{e}\left[\widetilde{\mathbf{H}}(z)\, e^{j\omega t}\right]$$
$$= \Re\mathfrak{e}\left[-\hat{\mathbf{y}} j \frac{10k}{\omega\mu} e^{-jkz} e^{j\omega t}\right]$$
$$= \hat{\mathbf{y}}\, 0.11 \sin(10^{10} t - 133z) \qquad \text{(A/m)}.$$

We note that *k* has the same expression as the phase constant of a lossless transmission line

Question 4: The electric field of a 1 MHz plane wave traveling in the $+z$ direction in air points along the x direction. If this field reaches a peak value of 1.2π (mV/m) at $t = 0$ and $z = 50$ m, obtain expressions for $\mathbf{E}(z, t)$ and $\mathbf{H}(z, t)$ and then plot them as a function of z at $t = 0$.



Solution: At $f = 1$ MHz, the wavelength in air is $\lambda = \frac{c}{f} = \frac{3 \times 10^8}{1 \times 10^6} = 300$ m,

and the corresponding wavenumber is $k = (2\pi/300)$ (rad/m).
The general expression for an x-directed electric field traveling in the $+z$ direction is given by $\mathbf{E}(z, t) = \hat{\mathbf{x}}|E_{x0}^{+}|\cos(\omega t - kz + \phi^{+})$

$$= \hat{\mathbf{x}}\, 1.2\pi \cos\left(2\pi \times 10^6 t - \frac{2\pi z}{300} + \phi^{+}\right) \quad \text{(mV/m)}.$$

The field $\mathbf{E}(z, t)$ is maximum when the argument of the cosine function equals zero or a multiple of 2π. At $t = 0$ and $z = 50$ m, this condition yields

$$-\frac{2\pi \times 50}{300} + \phi^{+} = 0 \qquad \text{or} \qquad \phi^{+} = \frac{\pi}{3}.$$

Hence,

$$\mathbf{E}(z, t) = \hat{\mathbf{x}}\, 1.2\pi \cos\left(2\pi \times 10^6 t - \frac{2\pi z}{300} + \frac{\pi}{3}\right) \quad \text{(mV/m)}$$

and

$$\mathbf{H}(z, t) = \hat{\mathbf{y}}\, \frac{E(z, t)}{\eta_0}$$

$$= \hat{\mathbf{y}}\, 10 \cos\left(2\pi \times 10^6 t - \frac{2\pi z}{300} + \frac{\pi}{3}\right) \quad (\mu\text{A/m})$$

where we have used the approximation $\eta_0 \approx 120\pi$ ().

At $t = 0$,

$$\mathbf{E}(z, 0) = \hat{\mathbf{x}}\, 1.2\pi \cos\left(\frac{2\pi z}{300} - \frac{\pi}{3}\right) \quad \text{(mV/m)},$$

$$\mathbf{H}(z, 0) = \hat{\mathbf{y}}\, 10 \cos\left(\frac{2\pi z}{300} - \frac{\pi}{3}\right) \quad (\mu\text{A/m}).$$

Plots of $\mathbf{E}(z, 0)$ and $\mathbf{H}(z, 0)$ as a function of z are shown in Figure

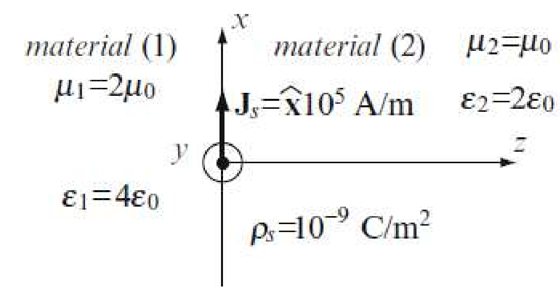
Question 5: An interface between two general materials contains both a current density given as $\mathbf{J}_s = \hat{\mathbf{x}}10^5$ A/m and a uniform surface charge density given as $\mathbf{J}_s = \hat{\mathbf{x}}10^5$ A/m . The static magnetic field intensity and static electric field intensity in material (1) are $\mathbf{H}_1 = \hat{\mathbf{x}}10^5 + \hat{\mathbf{y}}10^5 - \hat{\mathbf{z}}10^5 \quad [\mathrm{A/m}], \qquad \mathbf{E}_1 = \hat{\mathbf{x}}100 + \hat{\mathbf{y}}20 - \hat{\mathbf{z}}100 \quad [\mathrm{V/m}]$

For the material properties given in Figure, ($\mu_1 = 2\mu_0$, $\mu_2 = \mu_0$ [H/m], $\varepsilon_1 = 4\varepsilon_0$, and $\varepsilon_2 = 2\varepsilon_0$ [F/m]), find:

(a) The electric field intensity in material (2).

(b) The magnetic flux density in material (2).

Note: Static electric and magnetic fields are independent of each other.



Solution: Since both current densities and charge densities exist on the interface, we must use the general interface conditions

(a) The tangential and normal vector components of **E** in material (1) are

$$\mathbf{E}_{1t} = \hat{\mathbf{x}}100 + \hat{\mathbf{y}}20, \qquad \mathbf{E}_{1n} = -\hat{\mathbf{z}}100 \qquad [\mathrm{V/m}]$$

The tangential component of the electric field intensity is continuous across the interface:

$$\mathbf{E}_{2t} = \mathbf{E}_{1t} = \hat{\mathbf{x}}100 + \hat{\mathbf{y}}20 \qquad [\mathrm{V/m}]$$

The normal component of the electric field intensity is discontinuous across the interface:

$$\varepsilon_1 E_{1n} - \varepsilon_2 E_{2n} = \rho_s \quad \rightarrow \quad E_{2n} = \frac{\varepsilon_1 E_{1n} - \rho_s}{\varepsilon_2} \qquad [\mathrm{V/m}]$$

where we assume E_{1n} points away from the interface and E_{2n} points toward the interface. This gives

$$E_{2n} = \frac{4\varepsilon_0(-100) - 10^{-9}}{2\varepsilon_0} = -200 - \frac{10^{-9}}{2 \times 8.854 \times 10^{-12}} = -256.47 \qquad \left[\frac{\mathrm{V}}{\mathrm{m}}\right]$$

Thus, the electric field intensity in material (2) is

$$\mathbf{E}_2 = \mathbf{E}_{2t} + \mathbf{E}_{2n} = \hat{\mathbf{x}}100 + \hat{\mathbf{y}}20 - \hat{\mathbf{z}}256.47 \qquad \left[\frac{\mathrm{V}}{\mathrm{m}}\right]$$

(b) First, we write the magnetic flux density ($\mathbf{B} = \mu\mathbf{H}$)

$$\mathbf{B}_1 = \hat{\mathbf{x}}2\mu_0 \times 10^5 + \hat{\mathbf{y}}2\mu_0 \times 10^5 - \hat{\mathbf{z}}2\mu_0 \times 10^5 \quad [\mathrm{T}]$$

For convenience we separate the magnetic flux density into its tangential and normal components as follows:

$$\mathbf{B}_{1t} = \mu_1\mathbf{H}_{1t} = \hat{\mathbf{x}}2\mu_0 \times 10^5 + \hat{\mathbf{y}}2\mu_0 \times 10^5, \quad \mathbf{B}_{1n} = \mu_1\mathbf{H}_{1n} = -\hat{\mathbf{z}}2\mu_0 \times 10^5 \quad [\mathrm{T}]$$

The tangential component of the magnetic flux density is discontinuous across the interface

$$\hat{\mathbf{n}} \times \left(\frac{\mathbf{B}_1}{\mu_1} - \frac{\mathbf{B}_2}{\mu_2}\right) = \mathbf{J}_s \quad \rightarrow \quad \hat{\mathbf{n}} \times \mathbf{B}_2 = -\mathbf{B}_{2t} = \mu_2\left(\frac{\hat{\mathbf{n}} \times \mathbf{B}_1}{\mu_1} - \mathbf{J}_s\right) \quad [\mathrm{T}]$$

Since the normal must point into medium (1), we write $\hat{\mathbf{n}} = -\hat{\mathbf{z}}$ and get

$$-\hat{\mathbf{z}} \times \mathbf{B}_2 = \mathbf{B}_{2t} = \mu_2 \left[\frac{-\hat{\mathbf{z}} \times \mathbf{B}_1}{\mu_1} - \mathbf{J}_s \right] \quad [\mathrm{T}]$$

or

$$\mathbf{B}_{2t} = \mu_2 \left(\frac{\hat{\mathbf{z}} \times \mathbf{B}_1}{\mu_1} + \mathbf{J}_s \right) \quad [\mathrm{T}]$$

Substituting for $\mathbf{B}_1$ and $\mathbf{J}_s$,

$$\mathbf{B}_{2t} = \mu_0 \left(\frac{\hat{\mathbf{z}} \times (\hat{\mathbf{x}}2\mu_0 \times 10^5 + \hat{\mathbf{y}}2\mu_0 \times 10^5 - \hat{\mathbf{z}}2\mu_0 \times 10^5)}{2\mu_0} + \hat{\mathbf{x}} \times 10^5 \right) = \hat{\mathbf{y}}10^5\mu_0 \quad [\mathrm{T}]$$

The normal component of $\mathbf{B}$ is continuous across the interface:

$$\mathbf{B}_{2n} = \mathbf{B}_{1n} = -\hat{\mathbf{z}}2 \times 10^5\mu_0 \quad [\mathrm{T}]$$

Thus, the magnetic flux density in material (2) is

$$\mathbf{B}_2 = \hat{\mathbf{y}}10^5\mu_0 - \hat{\mathbf{z}}2 \times 10^5\mu_0 \quad [\mathrm{T}]$$