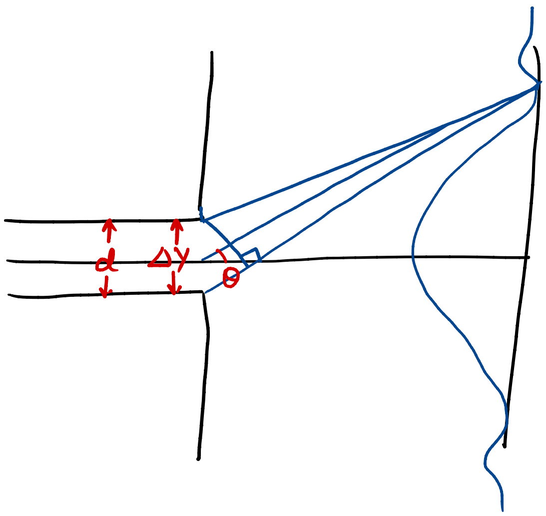
Lecture-26

17/10/23.

Heisenberg Uncertainty Principle ⇒

§ Diffraction of a beam of e^- by a slit ⇒



* The I^{st} minima of the pattern is obtained for $n=1$

$$d \sin\theta = \lambda$$

* e^- have passed through the slit, so the uncertainty in determing the position of $e^- = \Delta y$.

$$\therefore \quad \Delta y \sin\theta = \lambda$$

$$\Delta y = \frac{\lambda}{\sin\theta} \quad \text{——— (1)}$$

* Initially the e^- has no component of momentum along y-axis. After the slit the e^- aquire an additional component of momentum along y axis. If p is the momentum of e^- then the y component will be $p\sin\theta$

The e^- may be anywhere within the pattern from $(-\theta$ to $\theta)$. So.

$$\Delta p_y = p\sin\theta - (-p\sin\theta)$$

$$= 2p\sin\theta$$

$$\Delta p_y = 2\frac{h}{\lambda}\sin\theta \quad \text{——— (2)}$$

$$\therefore \quad \Delta y \cdot \Delta p_y = \frac{h}{\sin\theta} \cdot 2\frac{h}{\lambda}\sin\theta$$

$$= 2h$$

$$\geqslant \frac{\hbar}{2}$$

Application of uncertainty principle ⇒

① Argument that e^- cannot exist in the nucleus ⇒

* Experimental observation show that the e^- in an atom posses energy < 4 MeV.

$$\Delta q \cdot \Delta p \approx \hbar$$

$$\Delta p \sim \frac{1.055 \times 10^{-34}}{10^{-14}}$$

$$\Delta p \sim 10^{-20} \text{ kg m/sec.}$$

$$\Delta T = \frac{(\Delta p)^2}{2m} \sim \frac{(10^{-20})^2}{2 \times 9 \times 10^{-31}} \text{ Jule}$$

$$\sim \frac{10^{-40}}{2 \times 9 \times 10^{-31} \times 1.6 \times 10^{-19}} \text{ eV}$$

$$\sim \frac{10^{10}}{2 \times 9 \times 1.6} \sim 400 \text{ MeV.}$$

$R = R_0 A^{1/3}$
$R_0 = 1.2$ fermi
A is nucleon no.
Let say $A = 64$.
$R = 1.2 \times 10^{-15} \times 6$
$\therefore 2R \sim 10^{-14}$

⇒ This indicate that e^- cannot exist in the nucleus.

② Minimum energy of a harmonic Oscillator ⇒

* Let say a particle of mass m execute simple harmonic motion along X axis. The uncertainty in the position of particle is Δx.

$$\therefore \Delta p = \frac{\hbar}{\Delta x}$$

Then uncertainty in total energy

$$\Delta E = K.E + P.E.$$

$$\Delta E = \frac{(\Delta p)^2}{2m} + \frac{1}{2} k (\Delta x)^2$$

$$\Delta E = \frac{\hbar^2}{8m(\Delta x)^2} + \frac{1}{2} k (\Delta x)^2 \quad \text{——— ①}$$

For Δx, let us minimize this energy,

$$\frac{d\,\Delta E}{d\,\Delta x} = 0$$

$$\frac{-\hbar^2}{4m(\Delta x)^3} + k\,\Delta x = 0$$

$$\therefore \Delta x = \left(\frac{\hbar^2}{4mk}\right)^{1/4} \quad \text{put in } ①$$

$$\Delta E = \frac{\hbar^2}{8m\left(\frac{\hbar^2}{4mk}\right)^{1/2}} + \frac{1}{2}k\left(\frac{\hbar^2}{4mk}\right)^{1/2}$$

$$= \frac{\hbar}{2}\left(\frac{k}{m}\right)^{1/2}$$

$$= \frac{1}{2}\hbar\omega$$

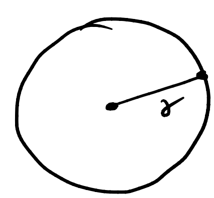
$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$\omega = \sqrt{k/m}$$

$$\boxed{E_{min} = \frac{1}{2}\hbar\omega}$$

③ Ground state energy of H atom :→

$$\Delta E = \Delta T + \Delta V$$



$$\Delta E = \frac{(\Delta p)^2}{2m} - \frac{e^2}{4\pi\epsilon_0(\Delta q)}$$

$$= \frac{\hbar^2}{(\Delta q)^2\cdot 2m} - \frac{e^2}{4\pi\epsilon_0\,\Delta q} \quad \text{——— } ①$$

for minimum uncertainty -

$$\frac{d\,\Delta E}{d\,\Delta q} = 0$$

$$0 = -\frac{\hbar^2}{m\,(\Delta q)^3} + \frac{e^2}{4\pi\epsilon_0(\Delta q)^2}$$

$$\Delta q = \frac{4\pi\epsilon_0 \hbar^2}{m e^2} = 0.053 \text{ nm.} \quad [\text{Bohr Radius}]$$

put in ①-

$$\Delta E = \frac{\hbar^2}{2m\left(\frac{4\pi\epsilon_0 \hbar^2}{me^2}\right)^2} - \frac{e^2}{4\pi\epsilon_0\left(\frac{4\pi\epsilon_0 \hbar^2}{me^2}\right)}$$

$$= -\frac{m}{2\hbar^2}\left(\frac{e^2}{4\pi\epsilon_0}\right)^2$$

$$= -13.6 \text{ ev} \quad [\text{Bohr energy}].$$