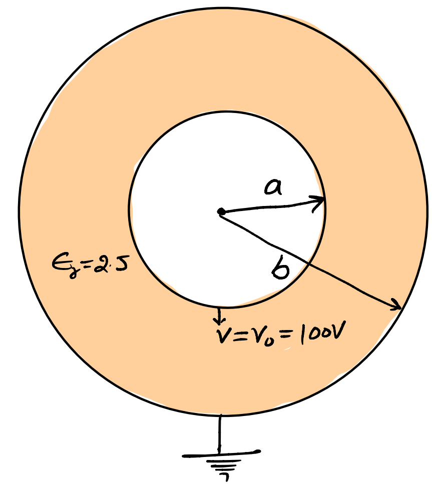
Lecture - 12 01/09/23.

Q.2 ⇒ Conducting spherical shells with radii $a = 10cm$ & $b = 30cm$ are maintained at a pot difference of 100V such that $V(r=b) = 0$ & $V(r=a) = 100V$.

Determine V & $\bar{E}$ in the region b/w the shells. If $\varepsilon_r = 2.5$ in the region, determine the total charge induced on the shells & the capacitance of the conductor.



Ans ⇒ Since V depends only on r -

$$\nabla^2 V = \frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{dV}{dr}\right) = 0$$

$$\frac{d}{dr}\left(r^2 \frac{dV}{dr}\right) = 0$$

$$r^2 \frac{dV}{dr} = A \quad \Rightarrow \quad \frac{dV}{dr} = \frac{A}{r^2}$$

$$\therefore \quad V = -\frac{A}{r} + B$$

B.C ⇒ at $r=b$, $V=0$ ⇒ $B = A/b$

at $r=a$, $V = V_0$ ⇒ $V_0 = A\left[\frac{1}{b} - \frac{1}{a}\right]$ ⇒ $A = \frac{V_0}{\frac{1}{b} - \frac{1}{a}}$

$$\therefore V = \frac{V_0\left(\frac{1}{r} - \frac{1}{b}\right)}{\left(\frac{1}{a} - \frac{1}{b}\right)}$$

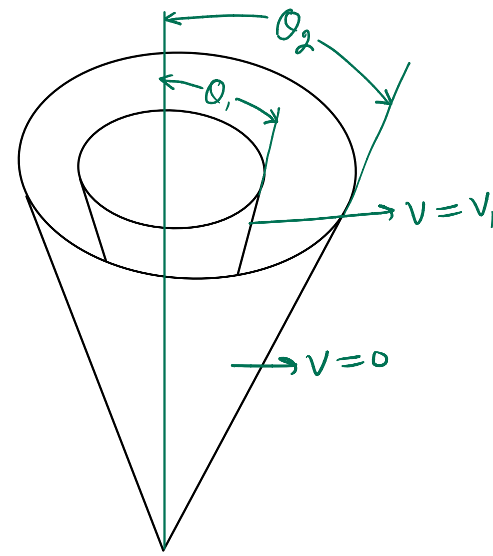
$$E = -\nabla V = -\frac{dV}{dr}\hat{r} \Rightarrow \frac{-A}{r^2} = \frac{V_0}{r^2\left(\frac{1}{a} - \frac{1}{b}\right)}\hat{r}$$

charge $\int E \cdot ds = \frac{q}{\varepsilon}$

$$q = \varepsilon_0 \varepsilon_r \int \frac{V_0}{\cancel{r^2}\left(\frac{1}{a} - \frac{1}{b}\right)} \cancel{r \cdot r} \sin\theta \, d\theta \, d\phi = \frac{\varepsilon_0 \varepsilon_r V_0}{\left(\frac{1}{a} - \frac{1}{b}\right)} \cdot 4\pi$$

$$C = \frac{Q}{V_0} = \frac{4\pi\varepsilon}{\left(\frac{1}{a} - \frac{1}{b}\right)}$$

Q.3 ⇒ Solve Laplace's eq^n for the region b/w coaxial cones as shown. A pot. V_1 is assumed at θ_1 & $V=0$ at θ_2. The cone vertices are insulated at $r=0$.



Ans ⇒ $$\frac{1}{r^2 \sin\theta} \frac{d}{d\theta}\left(\sin\theta \frac{dV}{d\theta}\right) = 0$$

$$\sin\theta \frac{dV}{d\theta} = A$$

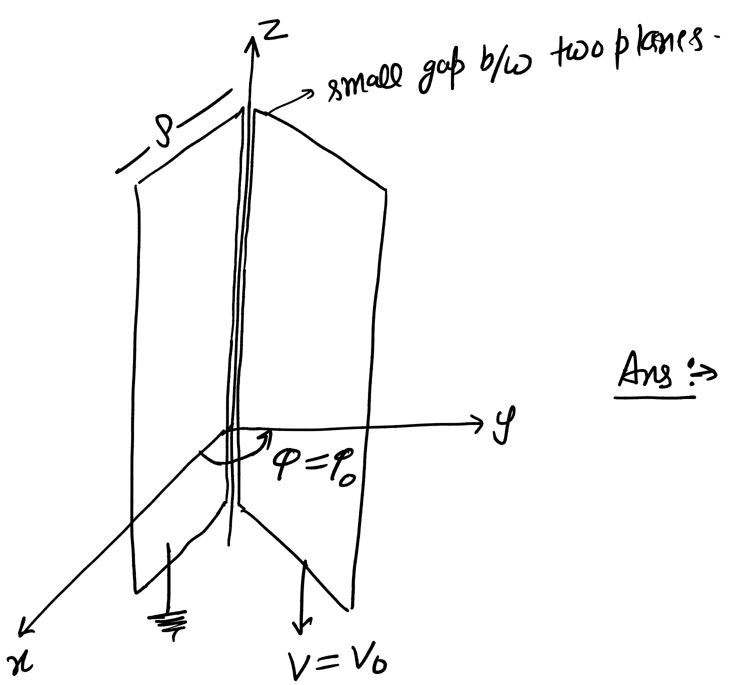
$$V = A \ln(\tan \theta/2) + B$$

∴ B.C ⇒ $$V_1 = A \ln(\tan \theta_1/2) + B$$
$$0 = A \ln(\tan \theta_2/2) + B$$

$$\Rightarrow V = V_1 \frac{\ln(\tan \theta/2) - \ln(\tan \theta_2/2)}{\ln(\tan \theta_1/2) - \ln(\tan \theta_2/2)}$$

Homework:

Q.4 ⇒ Two semi-infinite conducting planes at $\varphi=0$ & $\varphi=\pi/6$ are seperated by an infinitesimal insulating gap. If $V(\varphi=0)=0$ & $V(\varphi=\pi/6)=100\,V$, calculate V & E in the region b/w the planes. [Please see the figure].



Ans ⇒ $$V = \frac{600}{\pi}\varphi$$

$$E = -\frac{600}{\pi\rho}\hat{\varphi}$$

Magnetostatics :

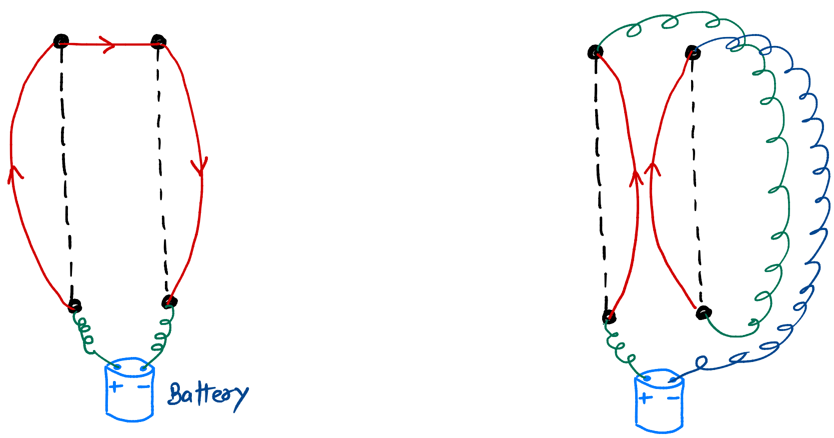
* In magnetostatic, the charges moves with const. velocity.
* Magnetostatic field is produced by a const current flow. (direct current).

* Force on an electric charge depends not only on where it is, but also on how fast it is moving.

* Every point in space is characterized by two vector fields which determine the force on any charge.

Electric force → independent of motion of charge.

Magnetic force → depend on the velocity.



Observations :

* Current in opposite direction repel.
* Current in same " attract.

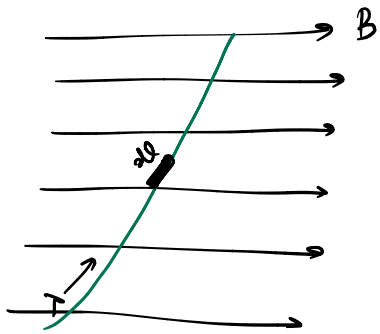
* Magnetic force : Magnetic force on a charge Q moving with velocity $\bar{v}$ in a magnetic field B is

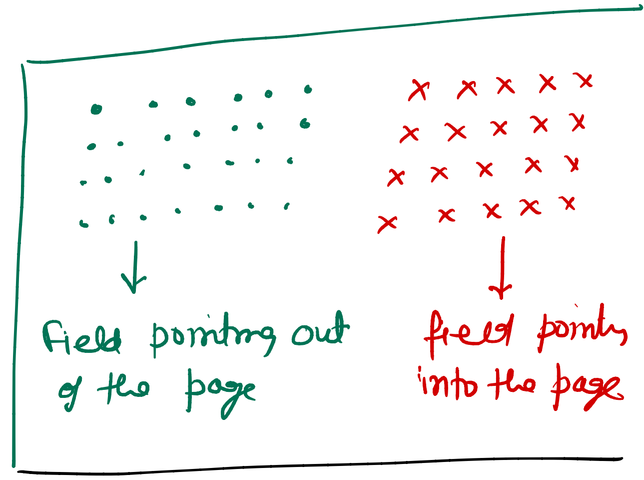
$$\boxed{F_{mag} = Q(\bar{v} \times \bar{B})}$$ known as Lorentz law.

In the presence of both electric & magnetic field.

$$\boxed{\bar{F} = Q(\bar{E} + (\bar{v} \times \bar{B}))}$$

* Force on a current carrying wire :→





Magnetic force on a segment of current carrying wire :→

$$F = \int (v \times B)\, dq$$

$$= \int (\underline{v} \times B)\, \underline{\lambda}\, dl$$

$$= \int (\bar{I} \times \bar{B})\, dl$$

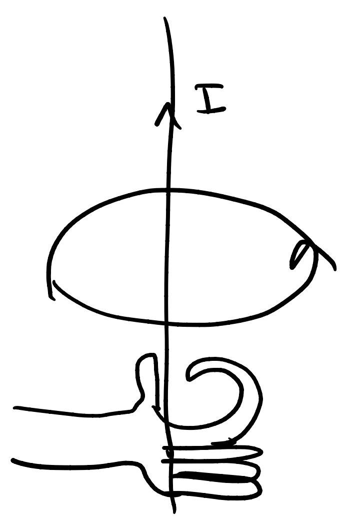
$$= \int I\,(d\bar{l} \times \bar{B})$$

$$\boxed{F = I \int d\bar{l} \times \bar{B}}$$

$$F = BIl \sin\theta$$

* Direction of magnetic field :→

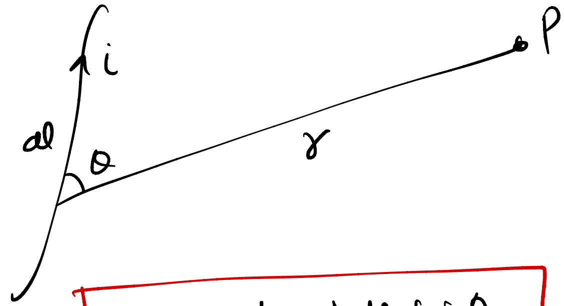
Right hand rule :→



* If you grab the wire with your right hand
  → Thumb in the direction of current
  ⟹ Your finger will curl around the direction of magnetic field.

* Laws governing magnetostatic fields :-

① Biot - Savart's law :⇒



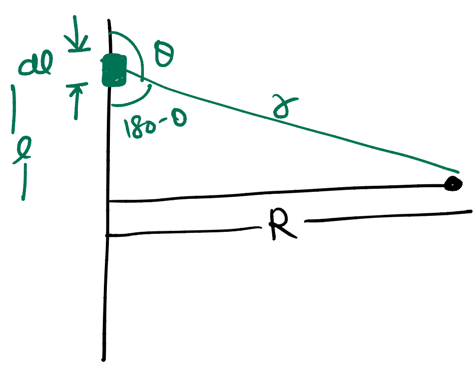
$$\boxed{dB = \frac{\mu_0}{4\pi}\frac{i\,dl\sin\theta}{r^2}}$$

$$\bar{B} = \frac{\mu_0}{4\pi} I \int \frac{dl \times \hat{r}}{r^2}$$

μ_0 is called permeability of free space.

$$\frac{\mu_0}{4\pi} = 10^{-7}\ N/A^2.$$

Ex :- Magnetic field at a distance 'R' away from a long straight wire carrying a steady current I.



* $\sin(180-\theta) = \frac{R}{r}$

$\sin\theta = \frac{R}{r}$

* $\tan(180-\theta) = \frac{R}{l}$

$-\tan\theta = R/l$

$l = -R\cot\theta$

$dl = R\,\mathrm{cosec}^2\theta\, d\theta.$

$$dB = \frac{\mu_0}{4\pi}\frac{i\,dl\sin\theta}{r^2}$$

$$dB = \frac{\mu_0}{4\pi}\frac{i\,\cancel{R}\,\cancel{\mathrm{cosec}^2\theta}\,d\theta\,\sin\theta}{R^{\cancel{2}}\,\cancel{\mathrm{cosec}^2\theta}}$$

$$B = \frac{\mu_0}{4\pi}\frac{i}{R}\int_{\theta_1}^{\theta_2}\sin\theta\, d\theta$$

$$= \frac{\mu_0}{4\pi}\frac{i}{R}\left[\cos\theta\right]_0^{\pi}$$

$$\boxed{B = \frac{\mu_0}{2\pi}\frac{i}{R}}$$