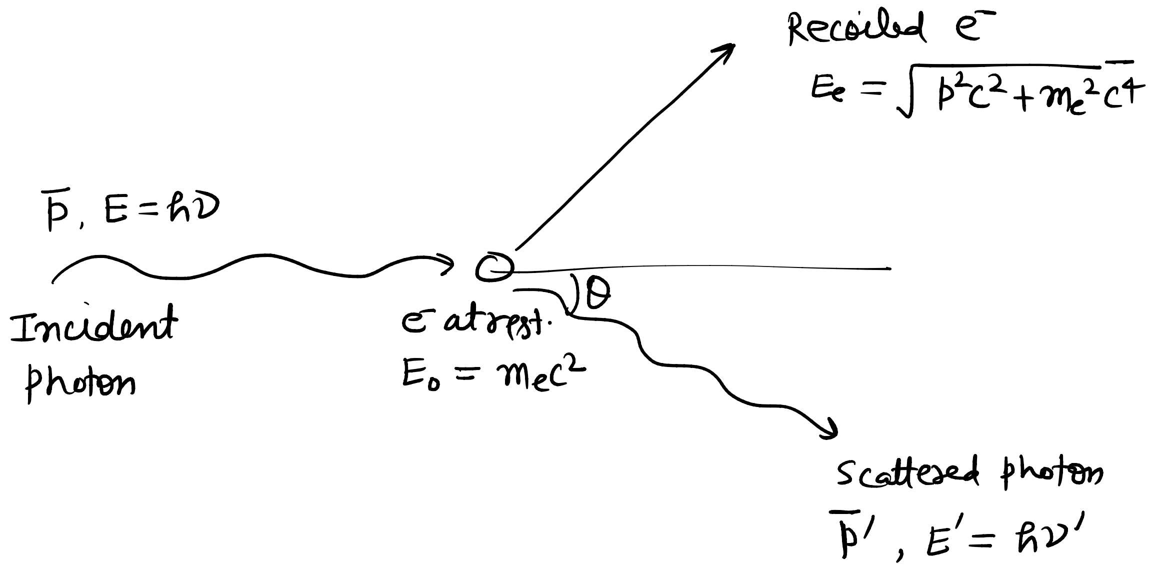
Lecture-22

6/10/2023.

Compton effect ⇒

It provide the conformation of the particle aspect of radiation.



Observation :→ Wavelength of the scattered radiation is larger than the wavelength of the incident radiation.

This can be explained only by assuming that X-ray photons behave like particles.

Classical physics ⇒

X ray wavelength/frequency ⟶ e^- oscillate with same frequency ⟶ this will radiate light with same frequency/wavelength ⟶ XX

* Conservation of momentum ⇒

$$\bar{p} = \bar{p}_e + \bar{p}'$$

$$\bar{p}_e = \bar{p} - \bar{p}'$$

$$p_e^2 = (p - p')^2$$

$$= p^2 + p'^2 - 2pp'\cos\theta$$

$$= \frac{h^2}{c^2}\left[\nu^2 + \nu'^2 - 2\nu\nu'\cos\theta\right] \quad \text{——— (1)}$$

* Conservation of energy ⇒

$$E + E_0 = E' + E_e$$

$$h\nu + m_e c^2 = h\nu' + \sqrt{p_e^2 c^2 + m_e^2 c^4}$$

$$h\nu - h\nu' + m_e c^2 = \sqrt{h^2(\nu^2 + \nu'^2 - 2\nu\nu'\cos\theta) + m_e^2 c^4}$$

$$\left[\nu - \nu' + \frac{m_e c^2}{h}\right]^2 = \nu^2 + \nu'^2 - 2\nu\nu'\cos\theta + \frac{m_e^2 c^4}{h^2}.$$

$$\cancel{\nu^2} + \cancel{\nu'^2} + \cancel{\frac{m_e^2 c^4}{h^2}} - 2\nu\nu' - \frac{2\nu' m_0 c^2}{h} + \frac{2\nu m_0 c^2}{h}$$

$$= \cancel{\nu^2} + \cancel{\nu'^2} - 2\nu\nu'\cos\theta + \cancel{\frac{m_e^2 c^4}{h^2}}$$

$$\frac{2 m_0 c^2}{h}\left[\nu - \nu'\right] = 2\nu\nu'\left[1 - \cos\theta\right]$$

$$\frac{\nu - \nu'}{\nu\nu'} = \frac{h}{m_0 c^2}\left[1 - \cos\theta\right]$$

$$\frac{1}{\nu'} - \frac{1}{\nu} = \frac{h}{m_0 c^2}\left[1 - \cos\theta\right]$$

$$\boxed{\lambda' - \lambda = \underbrace{\frac{h}{m_0 c}}_{\Downarrow} \left[1 - \cos\theta\right] = 2\lambda_c \sin^2\theta/2}$$

λ_c = Compton wavelength

* The compton effect confirms that photon behave like particle & the collide with e^- like material particles.

Q.1 ⇒ X-rays of wavelength 10.0 pm are scattered from a target.

(a) ⇒ Find the wavelength of the x-rays scattered through 45°

(b) ⇒ Find the maximum wavelength present in the scattered X rays.

(c) ⇒ Find the maximum K.E. of the recoil e^-.

A ⇒ (a) ⇒ $\lambda = 10$ pm, $\lambda' = ?$ $\quad \theta = 45°$

$$\lambda' - \lambda = \frac{h}{m_0 c}(1-\cos\theta)$$

$$\lambda' = 10\times10^{-12} + \frac{6.62\times10^{-34}}{9.1\times10^{-31}\times 3\times10^{8}}[1-\cos 45^\circ]$$

$$= 10\times10^{-12} + .7\times10^{-12}$$

$$\lambda = 10.7 \text{ pm}$$

(b) $\Rightarrow$ $\lambda' = \lambda + \frac{h}{m_0 c}[1-\cos\theta]$ for max $\lambda' \Longrightarrow \theta = 180^\circ$

$$\lambda' = \lambda + \frac{2h}{m_0 c}.$$

$$= 10\times10^{-12} + \frac{2\times 6.62\times10^{-34}}{9.1\times10^{-31}\times3\times10^{8}}$$

$$= 10\times10^{-12} + 4.9\times10^{-12} = 14.9 \text{ pm}$$

(C) $\Rightarrow$ $K.E. = h\nu - h\nu' = hc\left[\frac{1}{\lambda} - \frac{1}{\lambda'}\right]$

$$K.E = \frac{6.62\times10^{-34}\times3\times10^{8}}{10^{-12}}\left[\frac{1}{10} - \frac{1}{14.9}\right]$$

$$= 6.54\times10^{-15} \text{ J}.$$

$$= 40.8 \text{ kev}.$$

Homework:

Franck & Hertz experiment $\Rightarrow$

Energy quantization in atom.