Assignment #2

(Aug 05, 2025)

Vectors Calculus, Sadiku Chap 3, Griffiths Chap 1

1. Given that $\rho_s = x^2 + xy$, calculate $\int_S \rho_s \, dS$ over the region $y \le x^2$, $0 < x < 1$.

2. Given that $\mathbf{H} = x^2\mathbf{a}_x + y^2\mathbf{a}_y$, evaluate $\displaystyle\int_L \mathbf{H} \cdot d\mathbf{l}$, where L is along the curve $y = x^2$ from $(0, 0)$ to $(1, 1)$.

3. If the integral $\displaystyle\int_A^B \mathbf{F} \cdot d\mathbf{l}$ is regarded as the work done in moving a particle from A to B, find the work done by the force field

$$\mathbf{F} = 2xy\,\mathbf{a}_x + (x^2 - z^2)\,\mathbf{a}_y - 3xz^2\,\mathbf{a}_z$$

on a particle that travels from $A(0, 0, 0)$ to $B(2, 1, 3)$ along

(a) The segment $(0, 0, 0) \to (0, 1, 0) \to (2, 1, 0) \to (2, 1, 3)$

(b) The straight line $(0, 0, 0)$ to $(2, 1, 3)$

4. Let $\mathbf{A} = 2xy\,\mathbf{a}_x + xz\,\mathbf{a}_y - y\,\mathbf{a}_z$. Evaluate $\displaystyle\int_{\mathrm{v}} \nabla \cdot \mathbf{A}\, dv$ over:

(a) A rectangular region $0 \le x \le 2$, $0 \le y \le 2$, $0 \le z \le 2$

(b) A cylindrical region $\rho \le 3$, $0 \le z \le 5$

(c) A spherical region $r \le 4$

5. Let $\mathbf{D} = 2\rho z^2\,\mathbf{a}_\rho + \rho\cos^2\phi\,\mathbf{a}_z$. Evaluate

(a) $\oint_S \mathbf{D} \cdot d\mathbf{S}$

(b) $\int_{\mathrm{v}} \nabla \cdot \mathbf{D}\, dv$

over the region defined by $2 \le \rho \le 5$, $-1 \le z \le 1$, $0 < \phi < 2\pi$.

6. Determine the flux of $\mathbf{D} = \rho^2\cos^2\phi\,\mathbf{a}_\rho + z\sin\phi\,\mathbf{a}_\phi$ over the closed surface of the cylinder $0 \le z \le 1$, $\rho = 4$. Verify the divergence theorem for this case.

7. Apply the divergence theorem to evaluate $\displaystyle\oint_S \mathbf{A} \cdot d\mathbf{S}$ where $\mathbf{A} = x^2\mathbf{a}_x + y^2\mathbf{a}_y + z^2\mathbf{a}_z$ and S is the surface of the solid bounded by the cylinder $\rho = 1$ and planes $z = 2$ and $z = 4$.

8. Calculate the circulation of $\mathbf{A} = \rho\cos\phi\,\mathbf{a}_\rho + z\sin\phi\,\mathbf{a}_z$ around the edge L of the wedge defined by $0 \le \rho \le 2$, $0 \le \phi \le 60°$, $z = 0$ as shown in Fig. 1.

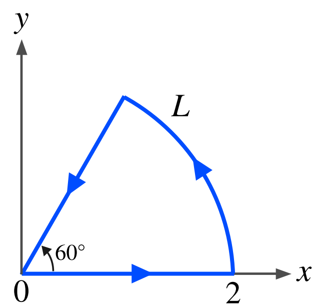

Fig. 1

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9. If $\mathbf{H} = 10\cos\theta\,\mathbf{a}_r$, evaluate $\int_S \mathbf{H}\cdot d\mathbf{S}$ over a hemisphere defined by $r = 1$, $0 < \phi < 2\pi$, $0 < \theta < \pi/2$.

10. Given that $\mathbf{F} = x^2 y\,\mathbf{a}_x - y\,\mathbf{a}_y$, find

(a) $\oint_L \mathbf{F}\cdot d\mathbf{l}$ where L is shown in Fig. 2.

(b) $\int_S (\nabla\times\mathbf{F})\cdot d\mathbf{S}$ where S is the area bounded by L.

(c) Is Stoke's theorem satisfied?

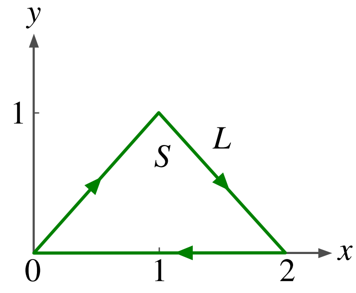

Fig. 2

11. Let $\mathbf{A} = \rho\sin\phi\,\mathbf{a}_\rho + \rho^2\,\mathbf{a}_\phi$, evaluate $\oint_L \mathbf{A}\cdot d\mathbf{l}$ if L is the contour of Fig. 3.

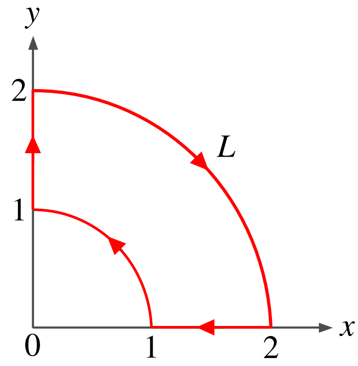

Fig. 3

12. A vector field is given by $\mathbf{Q} = \dfrac{\sqrt{x^2+y^2+z^2}}{\sqrt{x^2+y^2}}\left[(x-y)\mathbf{a}_x + (x+y)\mathbf{a}_y\right]$.

Evaluate the following integrals:

(a) $\int_L \mathbf{Q}\cdot d\mathbf{l}$, where L is the circular edge of the volume in the form of an ice cream cone shown in Figure 4.

(b) $\int_{S_1} (\nabla\times\mathbf{Q})\cdot d\mathbf{S}$, where S_1 is the top surface of the volume.

(c) $\int_{S_2} (\nabla\times\mathbf{Q})\cdot d\mathbf{S}$, where S_2 is the slanting surface of the volume.

(d) $\displaystyle\int_{S_1} \mathbf{Q}\cdot d\mathbf{S}$

(e) $\displaystyle\int_{S_2} \mathbf{Q}\cdot d\mathbf{S}$

(f) $\displaystyle\int_v \nabla\cdot\mathbf{Q}\,dv$

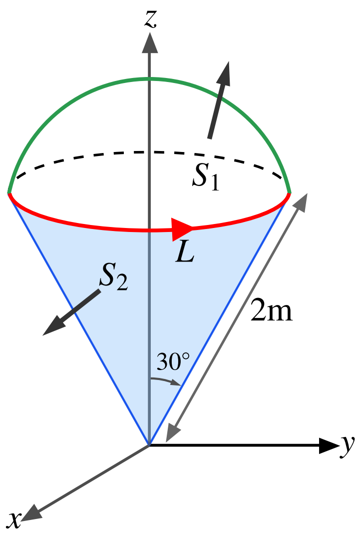

Fig. 4

How do your results in parts (a) to (f) compare?