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Q. A hypothetical isotropic antenna is radiating in free-space. At a distance of 100 m from the antenna, the total electric field (E_θ) is measured to be 5 V/m. Find the (a) power density (W_{rad}) (b) power radiated (P_{rad}).

$$(a) \quad \underline{W}_{\text{rad}} = \frac{1}{2} [\underline{E} \times \underline{H}^*] = \frac{E^2}{2\eta} \hat{a}_r = \frac{5^2 \hat{a}_r}{2(120\pi)} = 0.03315 \hat{a}_r \text{ Watts/m}^2$$

$$\begin{aligned} (b) \quad P_{\text{rad}} &= \oint_S W_{\text{rad}} dS = \int_0^{2\pi} \int_0^\pi (0.03315) (r^2 \sin\theta d\theta d\phi) \\ &= \int_0^{2\pi} \int_0^\pi (0.03315) (100)^2 \sin\theta d\theta d\phi \\ &= 2\pi (0.03315) (100)^2 \int_0^\pi \sin\theta d\theta = 2\pi (0.03315) (100)^2 \cdot 2 \\ &= 4165.75 \text{ watts} \end{aligned}$$

Q. A beam antenna has half-power beamwidths of 30 deg and 35 deg in perpendicular planes intersecting at the maximum of the mainbeam. Find its approximate directivity and maximum effective aperture (in λ^2) using:

(a) Kraus' and (b) Tai and Pereira's formulas.

The minor lobes are very small and can be neglected.

$$(a) D_0 \simeq \frac{41,253}{\theta_{1d} \theta_{2d}} = \frac{41,253}{30(35)} = 39.29 = 15.94 \text{ dB}$$

$$A_{em} = \frac{\lambda^2}{4\pi} D_0$$

$$(b) D_0 \simeq \frac{72,815}{\theta_{1d}^2 + \theta_{2d}^2} = \frac{72,815}{(30)^2 + (35)^2} = 34.27 = 15.35 \text{ dB}$$

$$A_{em} = \frac{\lambda^2}{4\pi} D_0$$

Q. The normalized radiation intensity of an antenna is rotationally symmetric in ϕ , and it is represented by

$$U = \begin{cases} 1 & 0^\circ \leq \theta < 30^\circ \\ 0.5 & 30^\circ \leq \theta < 60^\circ \\ 0.1 & 60^\circ \leq \theta < 90^\circ \\ 0 & 90^\circ \leq \theta \leq 180^\circ \end{cases}$$

What is the directivity (above isotropic) of the antenna (in dB)?

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$$(a) D_0 = \frac{4\pi U_{max}}{P_{rad}} = \frac{U_{max}}{U_0}$$

$$P_{rad} = \int_0^{2\pi} \int_0^\pi U \sin\theta d\theta d\phi = 2\pi \int_0^\pi U \sin\theta d\theta = 2\pi \left\{ \int_0^{30^\circ} \sin\theta d\theta + \int_{30^\circ}^{60^\circ} (0.5) \sin\theta d\theta + \int_{60^\circ}^{90^\circ} (0.1) \sin\theta d\theta \right\} = 2\pi \left\{ (-\cos\theta) \Big|_0^{30^\circ} + \left(-\frac{\cos\theta}{2}\right) \Big|_{30^\circ}^{60^\circ} + (-0.1 \cos\theta) \Big|_{60^\circ}^{90^\circ} \right\}$$

(Continued)

$$(Cont'd) = 2\pi \left\{ (-0.866 + 1) + \left(\frac{-0.5 + 0.866}{2} \right) + \left(\frac{-0 + 0.5}{10} \right) \right\}$$

$$P_{rad} = 2\pi \{ -0.866 + 1 - 0.25 + 0.433 + 0.05 \} = 2\pi (0.367) \\ = 0.734 \cdot \pi = 2.3059$$

$$D_0 = \frac{1(4\pi)}{2.3059} = 5.4496 = 7.3636 \text{ dB}$$

Q. The normalized radiation intensity of an antenna is symmetric, and it can be approximated by :

$$U(\theta) = \begin{cases} 1 & 0^\circ \leq \theta < 30^\circ \\ \frac{\cos(\theta)}{0.866} & 30^\circ \leq \theta < 90^\circ \\ 0 & 90^\circ \leq \theta \leq 180^\circ \end{cases}$$

and it is independent of ϕ . Find the approximate directivity using Kraus' formula.

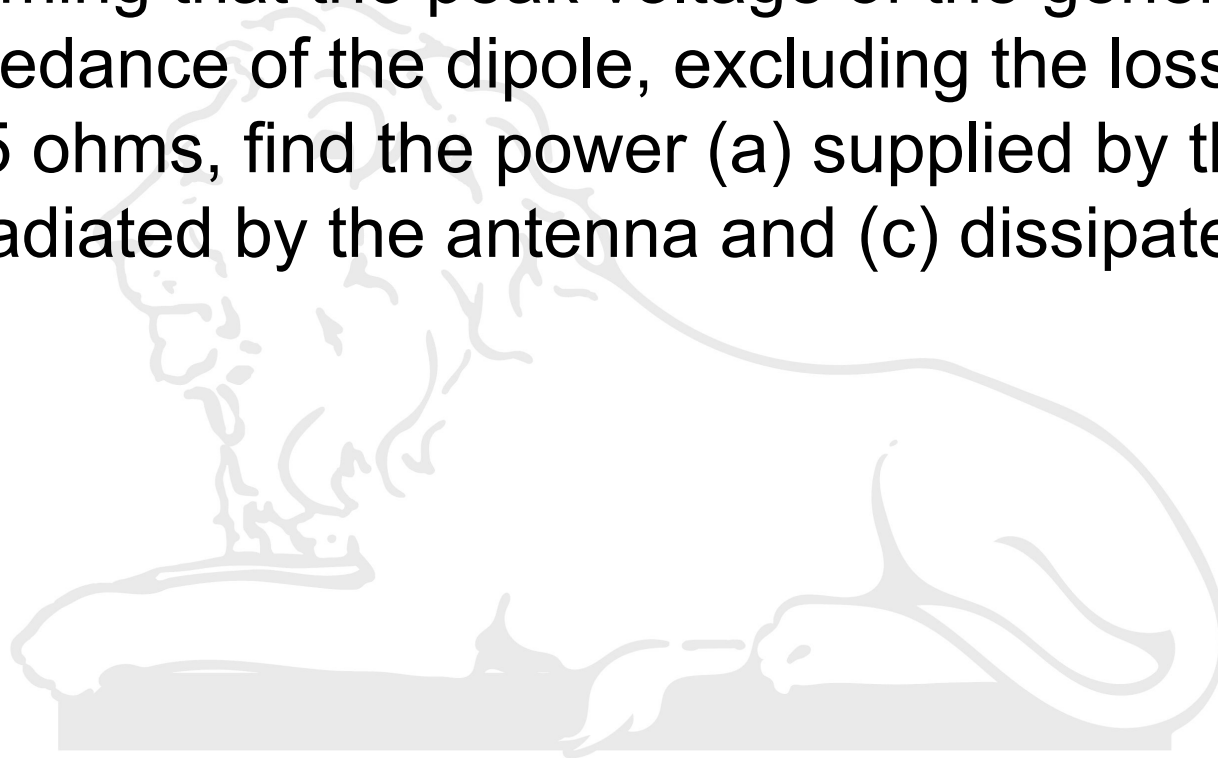
$$(b) \mu = \frac{\cos(\theta)}{0.866} = 0.5 \Rightarrow \cos \theta = 0.5(0.866) = 0.433, \theta = \cos^{-1}(0.433) = 64.34^\circ$$

$$\Theta_{1r} = 2(64.34) = 128.68^\circ = 2.246 \text{ rad} = \Theta_{2r}$$

$$D_0 \simeq \frac{4\pi}{\Theta_{1r} \Theta_{2r}} = \frac{4\pi}{(2.246)^2} = 2.4912 = 3.9641 \text{ dB}$$



Q. A $\lambda/2$ dipole, with a total loss resistance of 1 ohm, is connected to a generator whose internal impedance is $50 + j25$ ohms. Assuming that the peak voltage of the generator is 2 V and the impedance of the dipole, excluding the loss resistance, is $73 + j42.5$ ohms, find the power (a) supplied by the source (real) , (b) radiated by the antenna and (c) dissipated by the antenna.

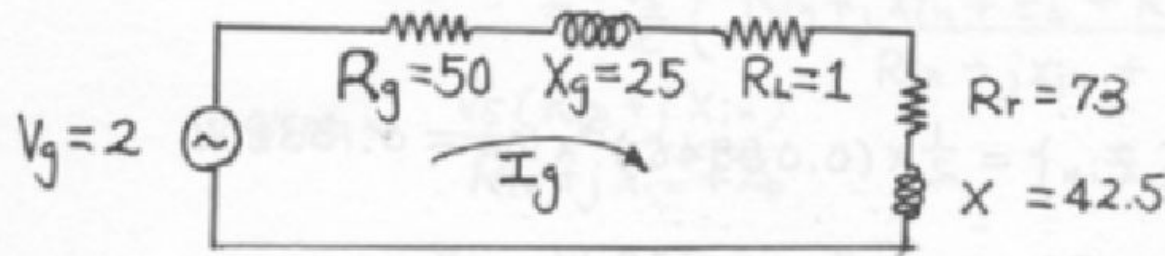


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$$I_g = \frac{2}{(50+1+73) + j(25+42.5)} = \frac{2}{124+j67.5}$$

$$= (12.442 - j6.7724) \times 10^{-3} = 14.166 \times 10^{-3} \angle -28.56^\circ$$



$$(a). P_s = \frac{1}{2} \operatorname{Re}(V_g \cdot I_g^*) = \operatorname{Re}(12.442 + j6.7724) \times 10^{-3} = 12.442 \times 10^{-3} \text{ W}$$

$$(b). P_r = \frac{1}{2} |I_g|^2 R_L = 7.325 \times 10^{-3} \text{ W}$$

$$(c). P_L = \frac{1}{2} |I_g|^2 R_L = 0.1003 \times 10^{-3} \text{ W}$$

The remaining supplied power is dissipated as heat in the internal resistor of the generator, or

$$P_g = \frac{1}{2} |I_g|^2 R_g = 5.0169 \times 10^{-3} \text{ W}$$

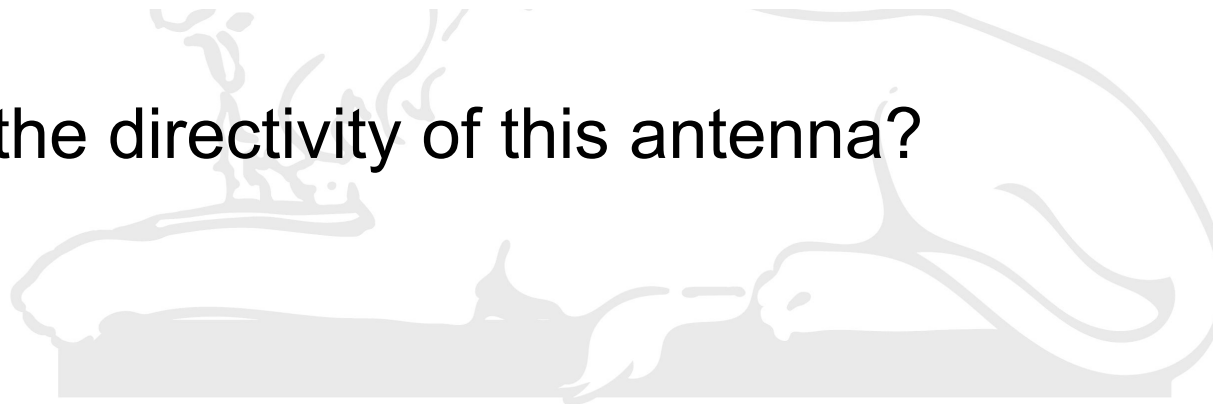
Thus

$$P_r + P_L + P_g = (7.325 + 0.1003 + 5.0169) \times 10^{-3} = 12.4422 \times 10^{-3} = P_s$$

Q. The E-field pattern of an antenna, independent of ϕ , varies as follows:

$$E = \begin{cases} 1 & 0^\circ \leq \theta \leq 45^\circ \\ 0 & 45^\circ < \theta \leq 90^\circ \\ \frac{1}{2} & 90^\circ < \theta \leq 180^\circ \end{cases}$$

(a) What is the directivity of this antenna?



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$$E = \begin{cases} 1 & 0 < \theta \leq 45^\circ \\ 0 & 45^\circ < \theta \leq 90^\circ \\ \frac{1}{2} & 90^\circ < \theta \leq 180^\circ \end{cases}$$

$$(a) \quad U = \frac{r^2 E^2}{2\eta} = \frac{r^2 |E|^2}{\eta}, \quad U_{\max} = \frac{r^2}{\eta} = \frac{1}{120\pi}$$



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(Cont'd)

$$P_{\text{rad}} = \frac{r^2}{\eta} \int_0^{2\pi} d\phi \left[\int_0^{45^\circ} \sin\theta d\theta + \int_{90^\circ}^{180^\circ} \frac{1}{4} \sin\theta d\theta \right]$$

$$= \frac{r^2}{\eta} [2\pi] \left[-\cos\theta \Big|_0^{45^\circ} + \frac{1}{4} (-\cos\theta) \Big|_{90^\circ}^{180^\circ} \right]$$

$$= \frac{2 r^2 \pi}{\eta} \left[-\cos 45^\circ + \cos 0^\circ - \frac{1}{4} \cos 180^\circ + \frac{1}{4} \cos 90^\circ \right]$$

$$P_{\text{rad}} = 0.54289 \frac{2\pi r^2}{\eta}$$

$$D = \frac{4\pi U_{\text{max}}}{P_{\text{rad}}} = \frac{4\pi \frac{r^2}{\eta}}{0.54289 (2\pi) r^2 / \eta} = 3.684$$

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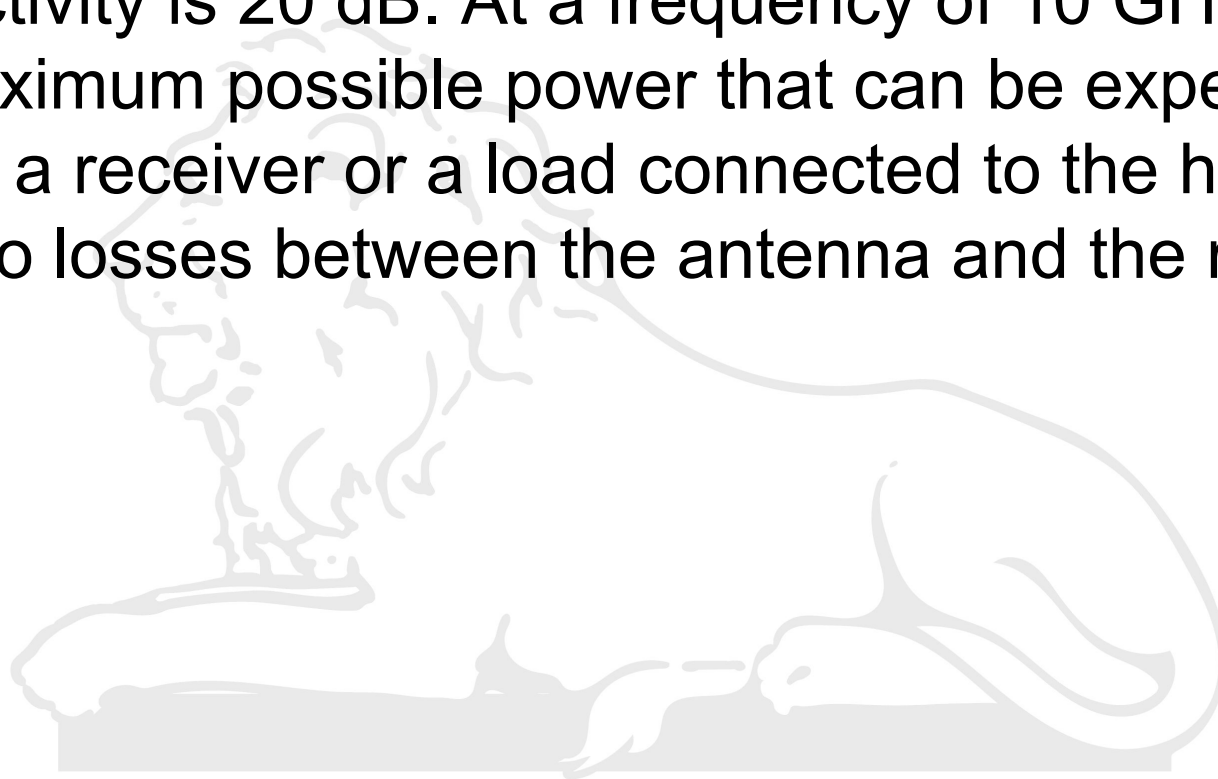
Q. An antenna has a maximum effective aperture of 2.147 m^2 at its operating frequency of 100 MHz . It has no conduction or dielectric losses. The input impedance of the antenna itself is 75 ohms , and it is connected to a 50-ohm transmission line. Find the directivity of the antenna system (“system” meaning includes any effects of connection to the transmission line). Assume no polarization losses.

$$A_{em} = 2.147 = \left(\frac{\lambda^2}{4\pi}\right) \cdot e_{cd} \cdot (1 - |\Gamma|^2) \cdot |\hat{\rho}_w \cdot \hat{\rho}_a|^2 \cdot D_o$$

$$\Gamma = \frac{75 - 50}{75 + 50} = 0.2 \quad ; \quad \lambda = \frac{3 \times 10^8}{100 \times 10^6} = 3 \text{ m}$$

$$\therefore D_o = \frac{2.147}{\frac{3^2}{4\pi} [(1 - (0.2)^2)]} = 3.125$$

Q. An incoming wave, with a uniform power density equal to 10^{-3} W/m^2 is incident normally upon a lossless horn antenna whose directivity is 20 dB. At a frequency of 10 GHz, determine the very maximum possible power that can be expected to be delivered to a receiver or a load connected to the horn antenna. There are no losses between the antenna and the receiver or load.



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$$W_i = 10^{-3} \text{ W/m}^2$$

$$A_{em} = \frac{\lambda^2}{4\pi} D_o, \quad D_o = 20 \text{ dB} = 10 \log_{10} X \Rightarrow X = 100$$

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{10 \times 10^9} = 0.03 \text{ m} = 3 \times 10^{-2} \text{ m}$$

$$A_{em} = \frac{(3 \times 10^{-2})^2}{4\pi} \cdot 100 = \frac{9 \times 10^{-4}}{4\pi} \cdot (100) = 0.716 \times 10^{-2} = 7.16 \times 10^{-3}$$

$$P_{rec} = 10^{-3} \cdot \left(\frac{9 \times 10^{-2}}{4\pi} \right) = \frac{9 \times 10^{-5}}{4\pi} = 0.716 \times 10^{-5} = 7.16 \times 10^{-6} \text{ Watts}$$

$$P_{rec} = 7.16 \times 10^{-6} \text{ watts.}$$

Q. A communication satellite is in stationary (synchronous) orbit about the earth (assume altitude of 22,300 statute miles). Its transmitter generates 8.0 W. Assume the transmitting antenna is isotropic. Its signal is received by the 210-ft diameter tracking paraboloidal antenna on the earth at the NASA tracking station at Goldstone, California. Also assume no resistive losses in either antenna, perfect polarization match, and perfect impedance match at both antennas. At a frequency of 2 GHz, determine the:

- (a) power density (in W/m^2) incident on the receiving antenna.
- (b) power received by the ground-based antenna whose gain is 60 dB.

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1 status mile = 1609.3 meter, 22,300 (status miles) = 3.588739×10^7 m

$$a. \quad P_i = \frac{P_{rad}}{4\pi R^2} = \frac{8 \times 10^{-14}}{4\pi \times (3.58874)^2} = 4.943 \times 10^{-16} \text{ Watts/m}^2.$$

$$b. \quad A_{em} = \frac{\lambda^2}{4\pi} D_o, \quad (\leftarrow D_o = 60 \text{ dB}, = 10^6) \\ (\leftarrow \lambda = 0.15 \text{ m})$$

$$A_{em} = \frac{(0.15)^2}{4\pi} \cdot 10^6 = 1790.493 \text{ m}^2$$

$$P_{received} = A_{em} \cdot P_i = (1790.493) \cdot (4.943 \times 10^{-16}) \\ = 8.85 \times 10^{-13} \text{ watts.}$$

Q. A series of microwave repeater links operating at 10 GHz are used to relay television signals into a valley that is surrounded by steep mountain ranges. Each repeater consists of a receiver, transmitter, antennas, and associated equipment. The transmitting and receiving antennas are identical horns, each having gain over isotropic of 15 dB. The repeaters are separated in distance by 10 km. For acceptable signal-to-noise ratio, the power received at each repeater must be greater than 10 nW. Loss due to polarization mismatch is not expected to exceed 3 dB. Assume matched loads and free-space propagation conditions. Determine the minimum transmitter power that should be used.

$$f = 10 \text{ GHz}, \rightarrow \lambda = \frac{3 \times 10^8}{10^{10}} = 0.03 \text{ m}$$

$$G_{\text{ot}} = G_{\text{or}} = 15 \text{ dB} = 10^{1.5} = 31.62$$

$$R = 10 \text{ km} = 10^4 \text{ m}$$

$$P_r \geq 10 \text{ nW} = 10^{-8} \text{ W}$$

$$|\hat{p}_t \cdot \hat{p}_r|^2 = -3 \text{ dB} = \frac{1}{2}$$

Frills Transmission Equation:

$$\begin{aligned} \frac{P_r}{P_t} &= G_{\text{ot}} \cdot G_{\text{or}} \cdot \left(\frac{\lambda}{4\pi R} \right)^2 \cdot |\hat{p}_t \cdot \hat{p}_r|^2 \\ &= (10^{1.5})^2 \cdot \left(\frac{0.03}{4\pi \times 10^4} \right)^2 \cdot \left(\frac{1}{2} \right) = 2.85 \times 10^{-11} \end{aligned}$$

$$P_t = \frac{P_r}{2.85 \times 10^{-11}}$$

$$P_r \geq 10^{-8} \text{ W} \rightarrow (P_t)_{\text{min}} = 351 \text{ W}$$

Q. In a long-range microwave communication system operating at 9 GHz, the transmitting and receiving antennas are identical, and they are separated by 10,000 m. To meet the signal-to-noise ratio of the receiver, the received power must be at least $10 \mu\text{W}$. Assuming the two antennas are aligned for maximum reception to each other, including being polarization matched, what should the gains (in dB) of the transmitting and receiving antennas be when the input power to the transmitting antenna is 10 W?



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$$\frac{P_r}{P_t} = \left(\frac{\lambda}{4\pi R} \right)^2 \cdot G_{ot} \cdot G_{or}, \quad \lambda = \frac{3 \times 10^8}{9 \times 10^9} = \frac{3 \times 10^8}{90 \times 10^8} = \frac{1}{30}$$

$$R = 10,000 \text{ meter} = \frac{10,000}{1/30} \lambda = 3 \times 10^5 \lambda$$

$$\frac{P_r}{P_t} = \left[\frac{\lambda}{4\pi (3 \times 10^5 \lambda)} \right]^2 \cdot G_o^2 = \frac{10 \times 10^{-6}}{10} = 10^{-6}$$

$$G_o^2 = 10^{-6} (4\pi \times 3 \times 10^5)^2$$

$$G_o = 10^{-3} (4\pi \times 3 \times 10^5) = 12\pi \times 10^2 = 1200\pi$$

$$G_o = 1200\pi = 3,769.91 = 10 \log_{10}(3,769.91) \text{ dB}$$

$$G_o = 3,769.91 = 35.76 \text{ dB}$$

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Q. A radar antenna, used for both transmitting and receiving, has a gain of 150 (dimensionless) at its operating frequency of 5 GHz. It transmits 100 kW, and is aligned for maximum directional radiation and reception to a target 1 km away having a radar cross section of 3 m². The received signal matches the polarization of the transmitted signal. Find the received power.

$$P_r = P_t \cdot \sigma \cdot \frac{G_{ot} \cdot G_{or}}{4\pi} \cdot \left[\frac{\lambda}{4\pi R_1 \cdot R_2} \right]^2, \quad \lambda = \frac{3 \times 10^8}{5 \times 10^9} = 0.06 \text{ m}$$

$$P_r = 10^5 \cdot (3) \cdot \frac{150^2}{4\pi} \cdot \left[\frac{0.06}{4\pi (10^6)} \right]^2$$

$$P_r = 1.22 \times 10^{-8} \text{ Watts}$$

Thank You



Questions?