ECC-201: Fundamentals of Communication Systems

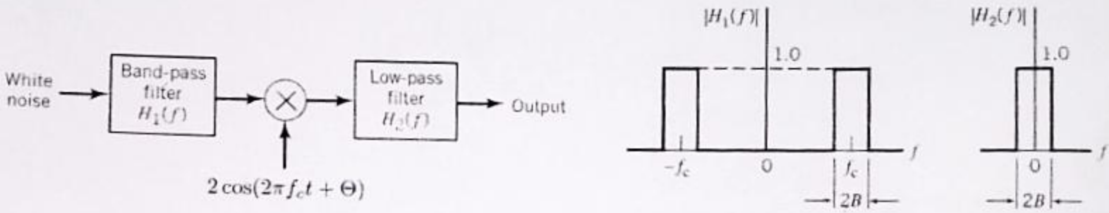
Autumn Semester 2025-26 **Tutorial Sheet:6**

1. A random process is defined as $Y(t) = X(t) + X(t-T)$, where $X(t)$ is a WSS random process with autocorrelation $R_X(\tau)$ and power spectral density $S_x(f)$.
   (a) Show that $Y(t)$ is a WSS random process.
   (b) Find the autocorrelation function and PSD of $Y(t)$.

2. Consider the random process:
$$X(t) = A, \quad \forall t$$
where A is a random variable uniformly distributed between 0 and 10. Determine the power spectral density of $X(t)$ from the basic definition of power spectral density (Do not use Wiener-Khinchin theorem).

3. Consider that a WSS random process $X(t)$ is applied to a mixer with a sinusoidal input with random phase. The output process of the mixer is given by
$$Y(t) = X(t)\cos(2\pi f_0 t + \Theta),$$
where Θ is a random variable uniformly distributed in the interval $(0, 2\pi)$. Further, the sources of $X(t)$ and $\cos(2\pi f_0 t + \Theta)$ are independent. Is $Y(t)$ a WSS process?

4. White Gaussian noise of zero mean and power spectral density $N_0/2$ is applied to the filtering scheme shown in the figure. The random variable Θ is uniformly distributed in the range



$[0, 2\pi]$.
   i. Find the power spectral density of the output process $N(t)$.
   ii. Find the average power of $N(t)$.
   iii. What is the maximum sampling rate at which $N(t)$ can be sampled so that the resulting samples are independent? Justify your answer.

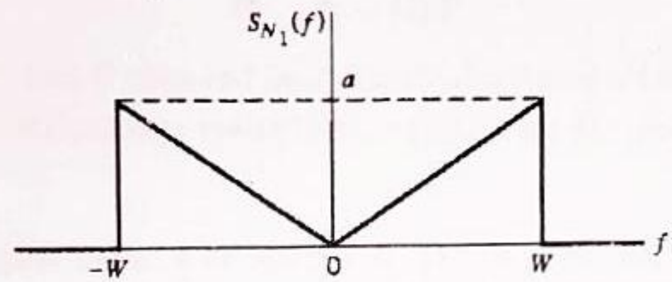
5. A pair of noise processes $N_1(t)$ and $N_2(t)$ are related by

$$N_2(t) = N_1(t)\cos(2\pi f_c t + \Theta) - N_1(t)\sin(2\pi f_c t + \Theta)$$

where f_c is a constant, and θ is the value of a random variable Θ whose pdf is given by

$$f_\Theta(\theta) = \begin{cases} \frac{1}{2\pi}, & 0 \le \theta \le 2\pi \\ 0, & \text{otherwise} \end{cases} .$$

The noise process $N_1(t)$ is stationary and its power spectral density is as shown in the figure. Find and plot the corresponding power spectral density of $N_2(t)$.



6. A random telegraph signal $X(t)$ is characterized by the autocorrelation function

$$R_X(\tau) = \exp(-2\nu|\tau|),$$

where ν is a constant. This signal is applied to a low-pass RC filter (with impulse response $h(t)$). Determine the autocorrelation function and the power spectral density of the output $Y(t)$ of the RC filter.

TABLE A6.1 *Fourier-Transform Theorems*

Property	*Mathematical Description*
1. Linearity	$ag_1(t) + bg_2(t) \rightleftharpoons aG_1(f) + bG_2(f)$ where a and b are constants
2. Dilation (time scaling)	$g(at) \rightleftharpoons \frac{1}{\lvert a\rvert}G\left(\frac{f}{a}\right)$ where a is a constant
3. Duality	If $g(t) \rightleftharpoons G(f)$, then $G(t) \rightleftharpoons g(-f)$
4. Time shifting	$g(t - t_0) \rightleftharpoons G(f)\exp(-j2\pi f t_0)$
5. Frequency shifting	$\exp(j2\pi f_c t)g(t) \rightleftharpoons G(f - f_c)$
6. Area under $g(t)$	$\int_{-\infty}^{\infty} g(t)\,dt = G(0)$
7. Area under $G(f)$	$g(0) = \int_{-\infty}^{\infty} G(f)\,df$
8. Differentiation in the time domain	$\frac{d}{dt}g(t) \rightleftharpoons j2\pi f G(f)$
9. Integration in the time domain	$\int_{-\infty}^{t} g(\tau)\,d\tau \rightleftharpoons \frac{1}{j2\pi f}G(f) + \frac{G(0)}{2}\delta(f)$
10. Conjugate functions	If $g(t) \rightleftharpoons G(f)$, then $g^*(t) \rightleftharpoons G^*(-f)$
11. Multiplication in the time domain	$g_1(t)g_2(t) \rightleftharpoons \int_{-\infty}^{\infty} G_1(\lambda)G_2(f-\lambda)\,d\lambda$
12. Convolution in the time domain	$\int_{-\infty}^{\infty} g_1(\tau)g_2(t-\tau)\,d\tau \rightleftharpoons G_1(f)G_2(f)$
13. Correlation theorem	$\int_{-\infty}^{\infty} g_1(t)g_2^*(t-\tau)\,dt \rightleftharpoons G_1(f)G_2^*(f)$
14. Rayleigh's energy theorem	$\int_{-\infty}^{\infty} \lvert g(t)\rvert^2\,dt = \int_{-\infty}^{\infty} \lvert G(f)\rvert^2\,df$

TABLE A6.2 *Fourier-Transform Pairs*

Time Function	*Fourier Transform*
$\mathrm{rect}\left(\frac{t}{T}\right)$	$T\,\mathrm{sinc}\,(fT)$
$\mathrm{sinc}\,(2Wt)$	$\frac{1}{2W}\mathrm{rect}\left(\frac{f}{2W}\right)$
$\exp(-at)u(t), \quad a > 0$	$\frac{1}{a + j2\pi f}$
$\exp(-a\lvert t\rvert), \quad a > 0$	$\frac{2a}{a^2 + (2\pi f)^2}$
$\exp(-\pi t^2)$	$\exp(-\pi f^2)$
$\begin{cases} 1 - \frac{\lvert t\rvert}{T}, & \lvert t\rvert < T \\ 0, & \lvert t\rvert \geq T \end{cases}$	$T\,\mathrm{sinc}^2(fT)$
$\delta(t)$	1
1	$\delta(f)$
$\delta(t - t_0)$	$\exp(-j2\pi f t_0)$
$\exp(j2\pi f_c t)$	$\delta(f - f_c)$
$\cos(2\pi f_c t)$	$\frac{1}{2}[\delta(f - f_c) + \delta(f + f_c)]$
$\sin(2\pi f_c t)$	$\frac{1}{2j}[\delta(f - f_c) - \delta(f + f_c)]$
$\mathrm{sgn}(t)$	$\frac{1}{j\pi f}$
$\frac{1}{\pi t}$	$-j\,\mathrm{sgn}(f)$
$u(t)$	$\frac{1}{2}\delta(f) + \frac{1}{j2\pi f}$
$\sum_{i=-\infty}^{\infty} \delta(t - iT_0)$	$\frac{1}{T_0}\sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{T_0}\right)$

TABLE A6.3 *Hilbert-Transform Pairs*[a]

Time Function	*Hilbert Transform*
$m(t)\cos(2\pi f_c t)$	$m(t)\sin(2\pi f_c t)$
$m(t)\sin(2\pi f_c t)$	$-m(t)\cos(2\pi f_c t)$
$\cos(2\pi f_c t)$	$\sin(2\pi f_c t)$
$\sin(2\pi f_c t)$	$-\cos(2\pi f_c t)$
$\delta(t)$	$\frac{1}{\pi t}$
$\frac{1}{t}$	$-\pi\delta(t)$

TABLE A6.4 *Trigonometric Identities*

$$\exp(\pm j\theta) = \cos\theta \pm j\sin\theta$$
$$\cos\theta = \tfrac{1}{2}[\exp(j\theta) + \exp(-j\theta)]$$
$$\sin\theta = \frac{1}{2j}[\exp(j\theta) - \exp(-j\theta)]$$
$$\sin^2\theta + \cos^2\theta = 1$$
$$\cos^2\theta - \sin^2\theta = \cos(2\theta)$$
$$\cos^2\theta = \tfrac{1}{2}[1 + \cos(2\theta)]$$
$$\sin^2\theta = \tfrac{1}{2}[1 - \cos(2\theta)]$$
$$2\sin\theta\cos\theta = \sin(2\theta)$$
$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$
$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$
$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$
$$\sin\alpha\sin\beta = \tfrac{1}{2}[\cos(\alpha - \beta) - \cos(\alpha + \beta)]$$
$$\cos\alpha\cos\beta = \tfrac{1}{2}[\cos(\alpha - \beta) + \cos(\alpha + \beta)]$$
$$\sin\alpha\cos\beta = \tfrac{1}{2}[\sin(\alpha - \beta) + \sin(\alpha + \beta)]$$