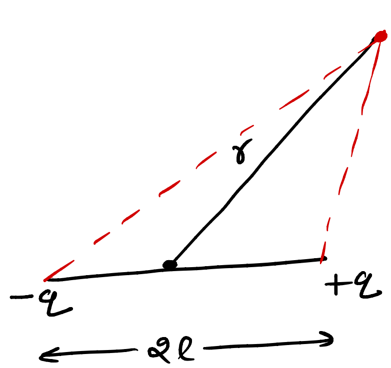
Lecture - 09

22/08/2023.

Electric dipole :- Two point charges of equal magnitude but opposite sign, seperated by a small distance.



define $\boxed{\bar{p} = q \times 2l}$

p is called the dipole moment.

Homework :-

$$\boxed{V(r,\theta) = \frac{1}{4\pi\epsilon_0}\frac{p\cos\theta}{r^2}}$$

In the case of Monopole $V \propto \frac{1}{r}$

dipole $V \propto \frac{1}{r^2}$

quadrupole $V \propto \frac{1}{r^3}$

Octapole $V \propto \frac{1}{r^4}$.

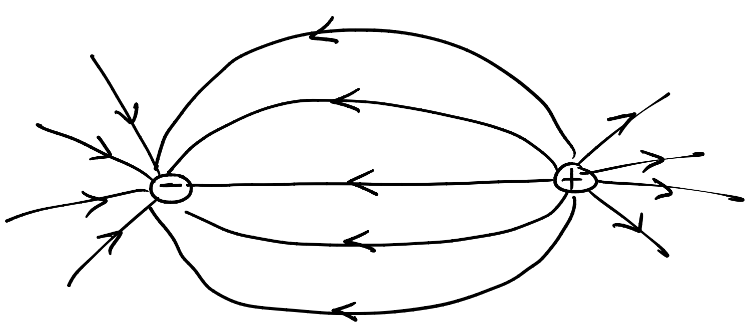
$$E = -\nabla V$$

$$E_r = -\frac{\partial V}{\partial r} = \frac{1}{4\pi\epsilon_0}\frac{2p\cos\theta}{r^3}$$

$$E_\theta = -\frac{1}{r}\frac{\partial V}{\partial \theta} = \frac{1}{4\pi\epsilon_0}\frac{p\sin\theta}{r^3}$$

$$\bar{E} = E_r\hat{r} + E_\theta\hat{\theta}$$

$$\bar{E} = \frac{1}{4\pi\epsilon_0}\frac{2p\cos\theta}{r^3}\hat{r} + \frac{1}{4\pi\epsilon_0}\frac{p\sin\theta}{r^3}\hat{\theta} \quad \checkmark$$



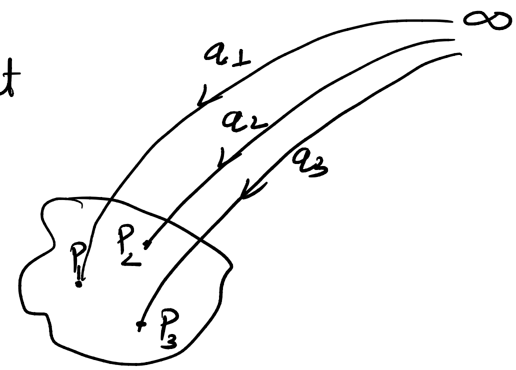
$$\nabla\times\bar{E} = \frac{1}{1\cdot r\cdot r\sin\theta}\cdot\frac{1}{4\pi\epsilon_0}p\cdot\begin{vmatrix}\hat{r} & r\hat{\theta} & r\sin\theta\,\hat{\varphi}\\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\varphi}\\ \frac{2\cos\theta}{r^3} & \frac{r\sin\theta}{r^3} & 0\end{vmatrix}$$

$$\Rightarrow \frac{p}{4\pi\epsilon_0 r^2\sin\theta}\left[\hat{r}(0) - r\hat{\theta}(0) + r\sin\theta\,\hat{\varphi}\left(\cancel{-\frac{2\sin\theta}{r^3}} + \cancel{\frac{2\sin\theta}{r^3}}\right)\right]$$

$$\Rightarrow 0.$$

Energy of point charge distribution :→

* How much work is required to assemble point charges?



* Total work in positioning three charges.

* First we position q_1 then q_2 & then q_3 :→

$$W_E = W_1 + W_2 + W_3$$

$$= 0 + q_2 \underbrace{V_{21}}_{\text{pot. on 2 due to 1.}} + q_3 (V_{31} + V_{32}) \quad \text{——— (1)}$$

* Now let us reverse the order.

$$W_E = W_3 + W_2 + W_1$$

$$= 0 + q_2 V_{23} + q_1 (V_{12} + V_{13}) \quad \text{——— (2)}$$

(1) + (2)

$$2W_E = q_1(V_{12} + V_{13}) + q_2(V_{21} + V_{23}) + q_3(V_{31} + V_{32})$$

$$2W_E = q_1 V_1 + q_2 V_2 + q_3 \underbrace{V_3}_{\text{total pot. on } q_3}$$

$$\boxed{W_E = \frac{1}{2} \sum_i^n q_i V_i}$$

* Energy of a continuous charge distribution :→

$$\boxed{W_E = \frac{1}{2} \int \underbrace{\rho}_{\text{Volume charge density}} V \, d\tau}$$

$$W = \frac{1}{2} \int \lambda V \, dl$$
↳ line charge distribution

$$W = \frac{1}{2} \int \sigma V \, ds$$
↳ Surface " "

Home work.

$$\boxed{W = \frac{1}{2} \varepsilon_0 \int_{\text{all space}} E^2 \, d\tau}$$

Q.1 ⇒ Find the energy stored in a uniformly charged solid sphere of radius R & charge q.

A ⇒ $W = \frac{1}{2}\varepsilon_0 \int E^2 d\tau$ [Integral over all volume]

$$= \frac{1}{2}\varepsilon_0 \frac{1}{(4\pi\varepsilon_0)^2}\left[\int_0^R \left(\frac{qr}{R^3}\right)^2 4\pi r^2 dr + \int_R^\infty \frac{q^2}{r^4} 4\pi r^2 dr\right]$$

$$= \frac{1}{2}\varepsilon_0 \frac{1}{(4\pi\varepsilon_0)^2}\left[\frac{4\pi}{R^6}\left(\frac{r^5}{5}\right)_0^R + 4\pi q^2\left(-\frac{1}{r}\right)_R^\infty\right]$$

$$\Rightarrow \frac{1}{2}\frac{1}{4\pi\varepsilon_0} q^2\left[\frac{1}{5R} + \frac{1}{R}\right]$$

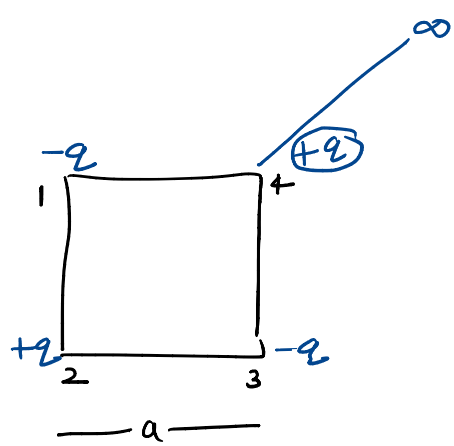
$$\Rightarrow \frac{1}{4\pi\varepsilon_0}\frac{3}{5}\frac{q^2}{R}$$ A.

Homework ⇒

Q.2 ⇒ Find the energy of a uniformly charge spherical shell of Total charge q & radius R.

A ⇒ $\frac{1}{4\pi\varepsilon_0}\frac{q^2}{2R}$ A.

Q.3 ⇒ Three charges are situated at the corner of a square of side a.



(a) ⇒ How much work does it take to bring in another charge +q, from far away & place it in the forth corner?

(b) ⇒ How much work does it take to assemble the whole configuration of four charges?

A ⇒ (a) ⇒ $W_4 \Rightarrow \frac{1}{4\pi\varepsilon_0} q\left[\frac{q_1}{r_{14}} + \frac{q_2}{r_{24}} + \frac{q_3}{r_{34}}\right]$

$$= \frac{1}{4\pi\varepsilon_0} q\left[\frac{-q}{a} + \frac{q}{a\sqrt{2}} - \frac{q}{a}\right] = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a}\left(-2+\frac{1}{\sqrt{2}}\right)$$ A

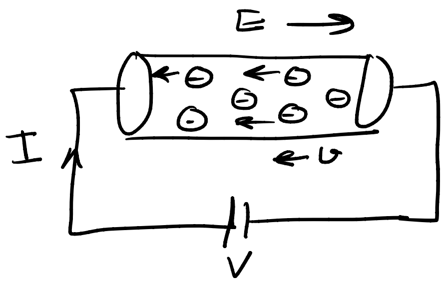
(b) $\Rightarrow$ $W_{total} = W_1 + W_2 + W_3 + W_4$

$$= 0 + \frac{1}{4\pi\epsilon_0}\left[-\frac{q^2}{a}\right] + \frac{1}{4\pi\epsilon_0}\, -q\left[-\frac{q}{a\sqrt{2}} + \frac{q}{a}\right] + W_4$$

$$\Rightarrow \frac{1}{4\pi\epsilon_0}\frac{2q^2}{a}\left[-2+\frac{1}{\sqrt{2}}\right]$$

Current density J $\Rightarrow$

Current per unit area.



$$\bar{J} = \frac{I}{S}\hat{s} = \frac{dI}{dS}\hat{s}$$

$$dI = J \cdot ds$$

$$\boxed{I = \int J \cdot d\bar{s}}$$

Let say v is the velocity of the flow of charge & ρ is the density.

$$dQ = \rho \underbrace{S\,dx}_{\text{volume}}$$

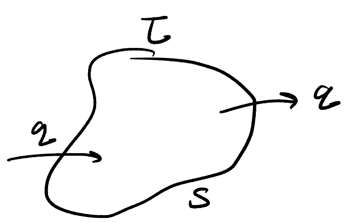
$$\frac{dQ}{dt} = \rho S \frac{dx}{dt}$$

$$I = \rho S v$$

$$\frac{I}{S} = \rho v$$

$$\boxed{J = \rho v}$$

#



The current flowing out through the boundary S

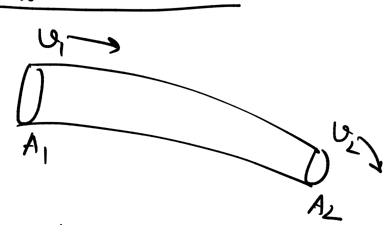
$$I = -\frac{dq}{dt} = -\frac{d}{dt}\int \rho\, d\tau \quad \text{—— (a)}$$

Acc. to the definition of current density

$$I = \oint_S J \cdot ds$$

$$= \int \nabla \cdot J\, d\tau \quad \text{—— (b)}$$

Mass conservation $\Rightarrow$



Mass coming in $\Rightarrow \rho A_1 v_1 \Delta t$

Mass going out $\Rightarrow \rho A_2 v_2 \Delta t$

For steady flow [No accumulation of Mass]

$$\rho A_1 v_1 \Delta t = \rho A_2 v_2 \Delta t$$

$$A_1 v_1 = A_2 v_2$$

$$\int \nabla \cdot J = -\int \frac{\partial \rho}{\partial t} \, d\tau$$

$$\boxed{\nabla \cdot J + \frac{\partial \rho}{\partial t} = 0}$$

This is the statement of local conservation of charge. This is the continuity eqn.

Homework.

Q.1 ⇒ For static (time-independent) fields, which of the following current densities are possible?

(a) ⇒ $J = 2x^3 y \, a_x + 4x^2 z^2 a_y - 6x^2 yz \, a_z$

(b) ⇒ $J = xy \, a_x + y(z+1) \, a_y - 2y \, a_z$

(c) ⇒ $J = \frac{z^2}{\rho} a_\rho + z \cos\varphi \, a_z$

(d) ⇒ $J = \frac{\sin\theta}{r^2} a_r$

A ⇒ for static field $\frac{\partial \rho}{\partial t} = 0$

$\therefore \nabla \cdot J = 0$