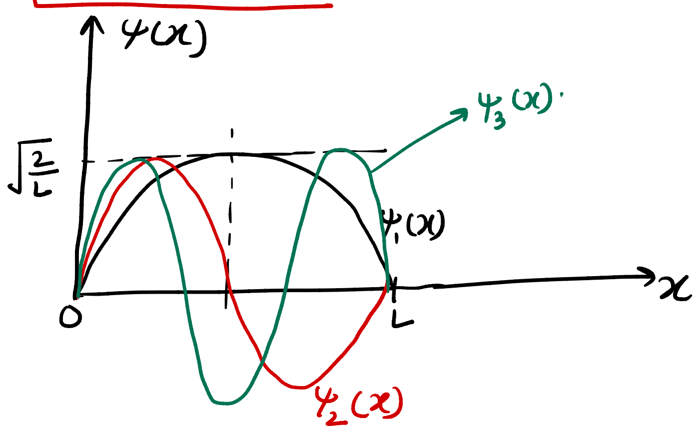
Lecture-33

2/11/2023

Particle in infinite Pot. Well :→

$$\boxed{\psi_n(x) = \sqrt{\frac{2}{L}}\,\sin\frac{n\pi x}{L}}$$

$$\boxed{E_n = \frac{n^2 h^2}{8 m L^2}}$$



$$\boxed{\int \psi_m^*(x)\,\psi_n(x)\,dx = \delta_{mn}}$$

Orthonormality

where, $\delta_{mn} = \begin{cases} 1 & m = n \\ 0 & m \neq n \end{cases}$

Most general

$$\psi(x,t) = \sum_{n=1}^{\infty} \psi_n(x)\cdot e^{-\frac{iE_n t}{\hbar}}$$

$$= \sqrt{\frac{2}{L}} \sum_{n=1}^{\infty} \sin\frac{n\pi x}{L}\cdot e^{-\frac{iE_n t}{\hbar}}$$

Finite Pot. Well :→

⋮

$$\psi_I(x) = A_1 e^{k_1 x} \quad \text{——— (1)}$$

$$\psi_{II}(x) = B_1 \sin k_2 x + B_2 \cos k_2 x \quad \text{——— (2)}$$

$$\psi_{III}(x) = C_2 e^{-k_1 x} \quad \text{——— (3)}$$

$$k_1 = \frac{\sqrt{2m(V_0 - E)}}{\hbar}, \qquad k_2 = \frac{\sqrt{2mE}}{\hbar}$$

$$\frac{\partial \psi_I}{\partial x} = A_1 k_1 e^{k_1 x}$$

$$\frac{\partial \psi_{II}}{\partial x} = B_1 k_2 \cos k_2 x - B_2 k_2 \sin k_2 x$$

$$\frac{\partial \psi_{III}}{\partial x} = -C_2 k_1 e^{-k_1 x}$$

B.C :→ $\psi_I(0) = \psi_{II}(0)$.

$$A_1 = B_2 \qquad \text{④}$$

$$\frac{\partial \psi_I}{\partial x}(0) = \frac{\partial \psi_{II}}{\partial x}(0).$$

$$A_1 k_1 = B_1 k_2 \qquad \text{⑤}$$

$$\psi_{II}(L) = \psi_{III}(L).$$

$$B_1 \sin k_2 L + B_2 \cos k_2 L = C_2 e^{-k_1 L}$$

$$B_2 \cdot \frac{k_1}{k_2} \sin k_2 L + B_2 \cos k_2 L = C_2 e^{-k_1 L} \qquad \text{⑥}$$

$$\frac{\partial \psi_{II}}{\partial x}(L) = \frac{\partial \psi_{III}}{\partial x}(L).$$

$$B_1 k_2 \cos k_2 L - B_2 k_2 \sin k_2 L = -C_2 k_1 e^{-k_1 L}$$

$$B_2 \frac{k_1}{\cancel{k_2}} \cdot \cancel{k_2} \cos k_2 L - B_2 k_2 \sin k_2 L = -C_2 k_1 e^{-k_1 L} \qquad \text{⑦}$$

⑥/⑦

$$\frac{\frac{k_1}{k_2} \sin k_2 L + \cos k_2 L}{k_1 \cos k_2 L - k_2 \sin k_2 L} = -\frac{1}{k_1}$$

$$\Rightarrow k_1^2 \sin k_2 L + k_1 k_2 \cos k_2 L = -k_1 k_2 \cos k_2 L + k_2^2 \sin k_2 L.$$

$\div \cos k_2 L$

$$\Rightarrow k_1^2 \tan k_2 L + k_1 k_2 = -k_1 k_2 + k_2^2 \tan k_2 L$$

$$(k_2^2 - k_1)^2 \tan k_2 L = 2 k_1 k_2$$

$$\boxed{\tan k_2 L = \frac{2 k_1 k_2}{k_2^2 - k_1^2}}$$

L.H.S $\Rightarrow$ $\tan\left(\frac{L\sqrt{2mE}}{\hbar}\right)$

R.H.S $\Rightarrow$

$$\frac{2\frac{\sqrt{2m(V_0-E)}}{\cancel{\hbar}}\cdot\frac{\sqrt{2mE}}{\cancel{\hbar}}}{\frac{2mE}{\cancel{\hbar^2}}-\frac{2m(V_0-E)}{\cancel{\hbar^2}}} = \frac{2\sqrt{\cancel{2m}(V_0-E)}\sqrt{\cancel{2m}E}}{\cancel{2m}(E-V_0+E)}$$

$$= \frac{2\sqrt{E(V_0-E)}}{(2E-V_0)}$$

$$\therefore \boxed{\tan\left(\frac{L\sqrt{2mE}}{\hbar}\right) = \frac{2\sqrt{E(V_0-E)}}{(2E-V_0)}}$$

Transcendental eq^n, it can be solved graphically or numerically.

Plot L.H.S & R.H.S [Homework]

Look in the Book

$\Rightarrow$ Energy quantization.