Lecture-11

25/08/2023.

Laplacian ⇒ $\nabla^2 T$ or $\nabla^2 V$ is called Laplacian of V.

$$\boxed{\nabla^2 V = 0}$$

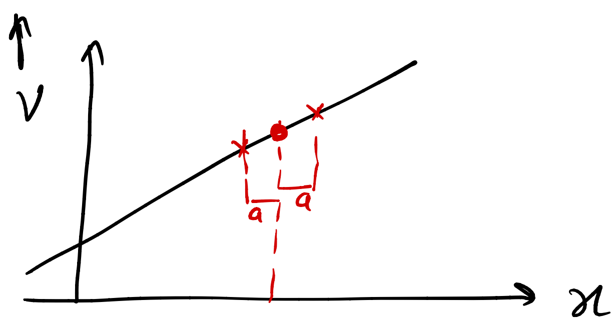
is called Laplacian eq^n. & it's sol^n is called harmonic fu^n.

For 1D ⇒

$$\frac{d^2V}{dx^2} = 0$$

$$\frac{dV}{dx} = a$$

$$V = ax + b$$



Conclusions ⇒ ① $V(x) = \frac{V(x+a) + V(x-a)}{2}$

$V(x)$ is the average of $V(x+a)$ & $V(x-a)$.

② V has no local maxima or minima

⇒ Extreme values of V must occur at the end point.

* Acc. to Laplace eq^n V_3 cannot have local max or min. & all extrema occure on the boundaries.

$$\nabla^2 T = \frac{1}{h_1 h_2 h_3}\left[\frac{\partial}{\partial u_1}\left(\frac{h_2 h_3}{h_1}\frac{\partial T}{\partial u_1}\right) + \frac{\partial}{\partial u_2}\left(\frac{h_3 h_1}{h_2}\frac{\partial T}{\partial u_2}\right) + \frac{\partial}{\partial u_3}\left(\frac{h_1 h_2}{h_3}\frac{\partial T}{\partial u_3}\right)\right]$$

Electrostatic Boundary value Problem ⇒

* Given $Q \longrightarrow$ determine $\begin{matrix} V \\ E \end{matrix}$

We use Colomb law or Gauss's law.

* Then, $E = -\nabla V$

§ Practical problems ⇒ Charge & pot are known at some boundary & we want to find E & V throughout the region.

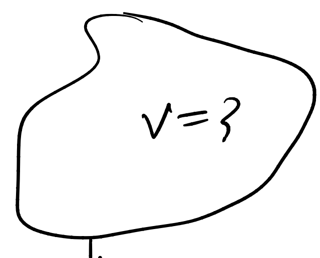
§ They can be tackled using -

1. Laplace's eqn.
2. Poisson's eqn
3. Method of Images.

* <u>Uniqueness theorem</u> ⇒ The soln to the Laplace eqn in some volume is <u>uniquely determined</u> if the pot. is specified on the boundary surface.

<u>Proof</u> ⇒ Let us say there are two soln V_1 & V_2 to the laplace eqn.

<u>Then V_1 must be equal to V_2</u>



V is specified

$V_1 = V_2$

* $\nabla^2 V_1 = 0$ & $\nabla^2 V_2 = 0$

Let say $V_3 = V_1 - V_2$

$$\therefore \nabla^2 V_3 = \nabla^2 V_1 - \nabla^2 V_2 = 0$$

∴ V_3 satisfies laplace eqn, V_3 can not have local max or minima & all extrema occurs on the boundaries Therefore. V_3 must be zero everywhere. Hence <u>$V_1 = V_2$</u>

Q.1 ⇒ Find the potential function and the electric field intensity for the region b/w two concentric right circular cylinders, where, $V=0$ at $r=1\,mm$ & $V=150\,V$ at $r=20\,mm$.

A ⇒

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dV}{dr}\right)=0$$

$$r\frac{dV}{dr}=A$$

$$V=A\ln r+B$$

$$V=50.1\ln r+345.9\ V$$ Ans.

$$E=\frac{50.1}{r}(\hat{a}_r)\ V/m$$ Ans.