Phase-Locked Loop (PLL)

The PLL is a feedback control system that can synchronize the instantaneous phase of an oscillator output with that of a reference signal.

The main components of a PLL are:

1) Voltage-Controlled Oscillator (VCO)
2) Phase detector or Phase comparator
3) Loop filter

VCO

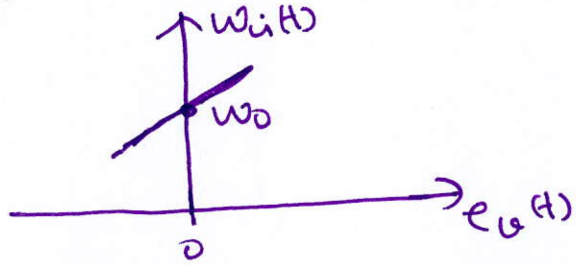
The instantaneous frequency of VCO output is proportional to the control input voltage.

$$\boxed{w_i(t) = w_0 + K_v \cdot e_v(t)} \quad (1)$$

$w_i(t)$ - instantaneous frequency of the VCO output

K_v - VCO constant

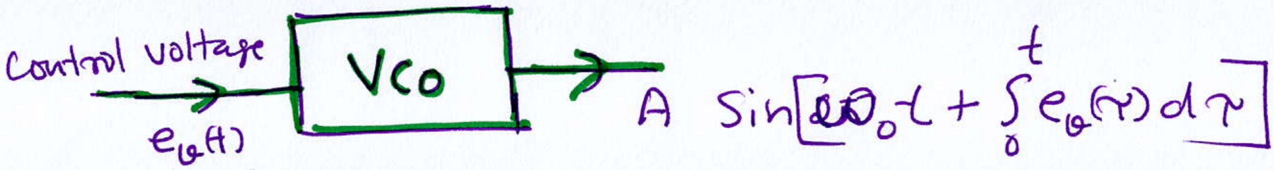
$e_v(t)$ - Control Voltage



w_0, the frequency of the VCO output when the control voltage is zero, is called the free-running frequency of the VCO.

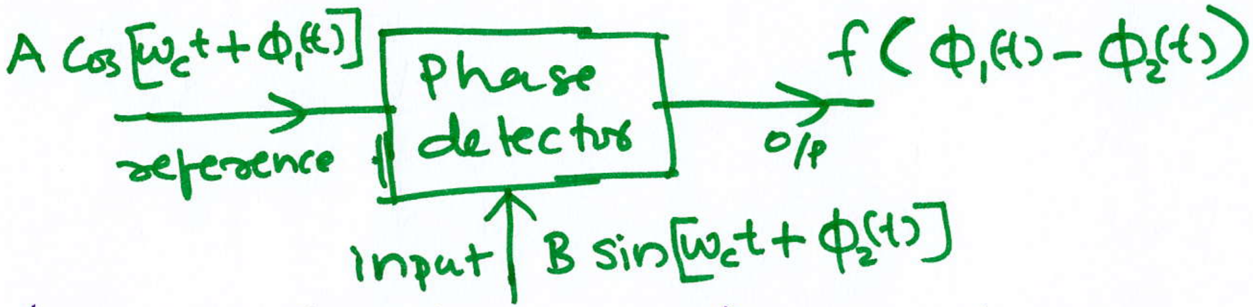
Instantaneous phase

$$\boxed{\int_0^t w_i(\tau)\, d\tau = w_0 t + K_v \int_0^t e_v(\tau)\, d\tau} \quad (2)$$

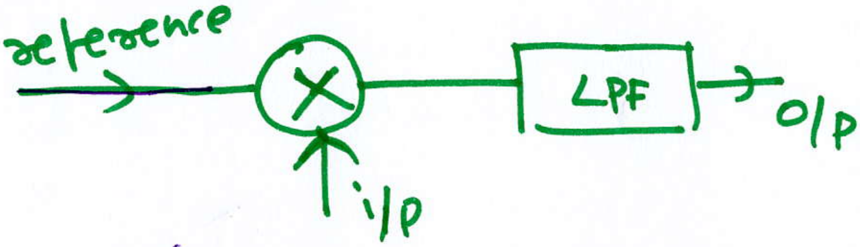


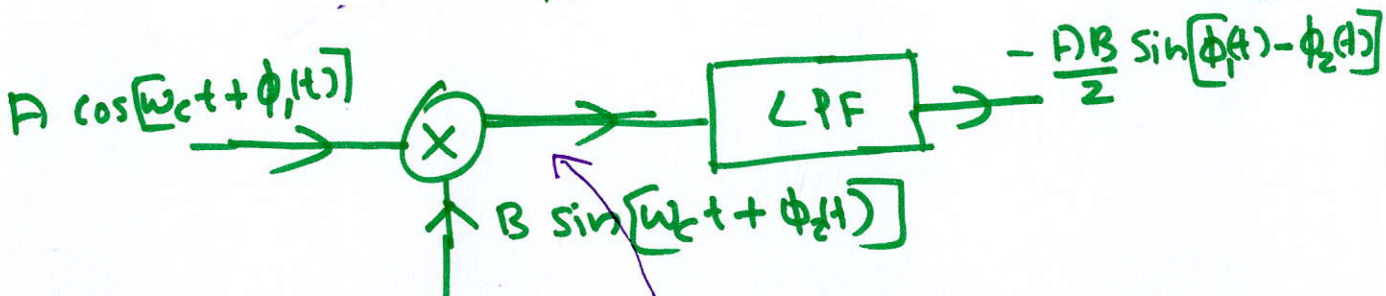
Phase Detector

The phase detector compares the instantaneous phase of the input signal with that of a reference signal and produces an output that is a function of the phase difference.



We consider the following realization of phase detector.



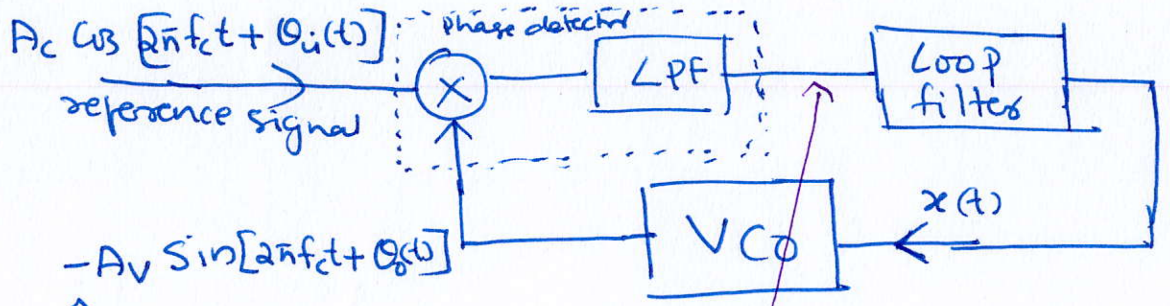


The mixer (multiplier) output is

$$\underbrace{\frac{AB}{2} \sin\left[2\omega_c t + \phi_1(t) + \phi_2(t)\right]}_{\text{removed by the LPF}} - \frac{AB}{2} \sin\left[\phi_1(t) - \phi_2(t)\right]$$

The loop filter is a low-pass filter whose role is explained later

Block diagram of PLL



↑
-ve sign is to make the product modulator o/p +ve.

The phase detector o/p is given by

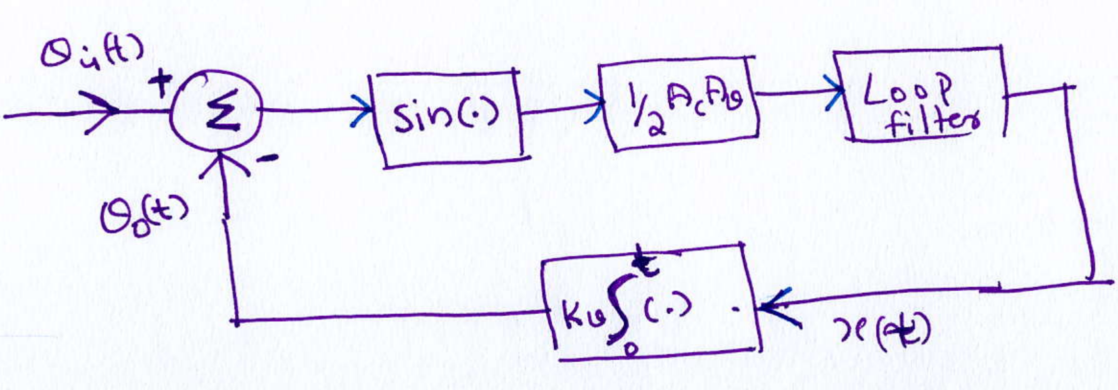
$$\frac{A_c A_v}{2} \sin[\theta_i(t) - \theta_o(t)]$$

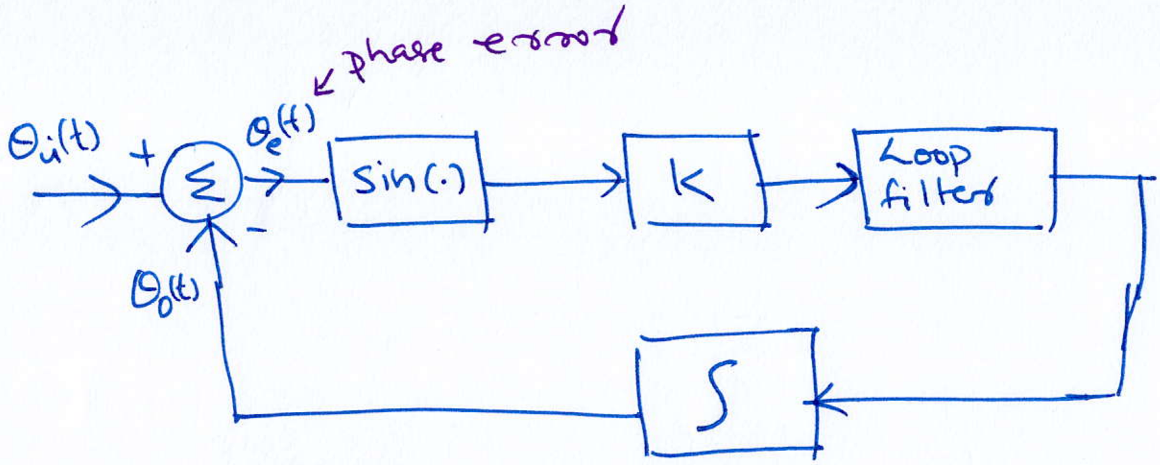
~~The relation between the $\theta_o(t)$ and $\theta_o(t)$ and $x(t)$ are~~

$\theta_o(t)$ and the VCO control i/p $x(t)$ are related by

$$\boxed{\theta_o(t) = K_v \int_0^t x(\tau)\, d\tau}$$

Since we are interested in the relation between $\theta_i(t)$ and $\theta_o(t)$, the PLL can be modeled as





$$\boxed{K = \frac{1}{2} A_c A_v K_v \text{ is the loop gain}}$$

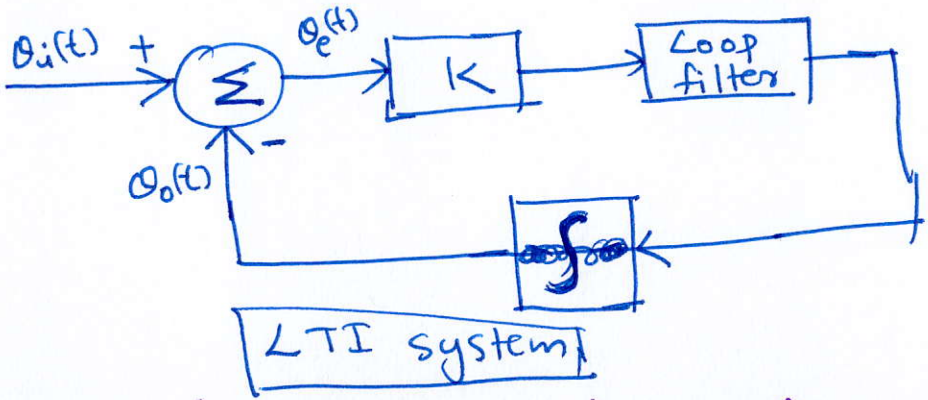
This is a negative feedback control system. The analysis of this system is difficult due to the sin(.) non-linearity.

Linear approximation of PLL

When $\theta_i(t) \approx \theta_0(t)$

$$\sin[\theta_i(t) - \theta_0(t)] \approx \theta_i(t) - \theta_0(t)$$

This approximation results in the following linear model.

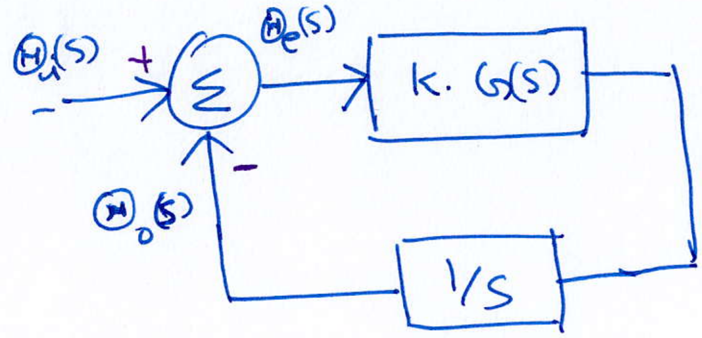


We can analyze the system in the Laplace-domain.

$$\theta_i(t) \xrightarrow{LT} \Theta_i(s) \;;\; \theta_0(t) \xrightarrow{LT} \Theta_0(s)$$

integrator $\longrightarrow$ $\boxed{1/s}$

Assuming that the system function of the loop filter is $G(s)$, the s-domain model of linearized ~~a~~ PLL ~~is~~ can be represented as



The transfer function $H(s) = \dfrac{\Theta_o(s)}{\Theta_i(s)}$

$$\Theta_o(s) = \left[\Theta_i(s) - \Theta_o(s)\right] \cdot \frac{K \cdot G(s)}{s}$$

$$\therefore H(s) = \frac{K\,G(s)}{s + K\,G(s)} \quad (3)$$

The error-transfer function

$$H_e(s) = \frac{\Theta_e(s)}{\Theta_i(s)} = \frac{\Theta_i(s) - \Theta_o(s)}{\Theta_i(s)}$$

$$\therefore H_e(s) = 1 - H(s)$$

$$= \frac{s}{s + K\,G(s)} \quad (4)$$

First-order PLL

For first-order PLL, $G(s) = 1$

$$\therefore H(s) = \frac{K}{s+K} \quad (5) \qquad H_e(s) = \frac{s}{s+K} \quad (6)$$

Let us now analyze how a first-order PLL synchronize $\theta_i(t)$ and $\theta_o(t)$.

Remember,

instantaneous phase of the input reference signal

$$2\pi f_c t + \theta_i(t)$$

instantaneous phase of the ~~out~~ VCO output

$$2\pi f_c t + \theta_o(t)$$

instantaneous phase error

$$\theta_e(t) = [2\pi f_c t + \theta_i(t)] - [2\pi f_c t + \theta_o(t)]$$

$$= \theta_i(t) - \theta_o(t)$$

First, we consider how the PLL tracks a sudden change (step change) in the phase of the reference signal.

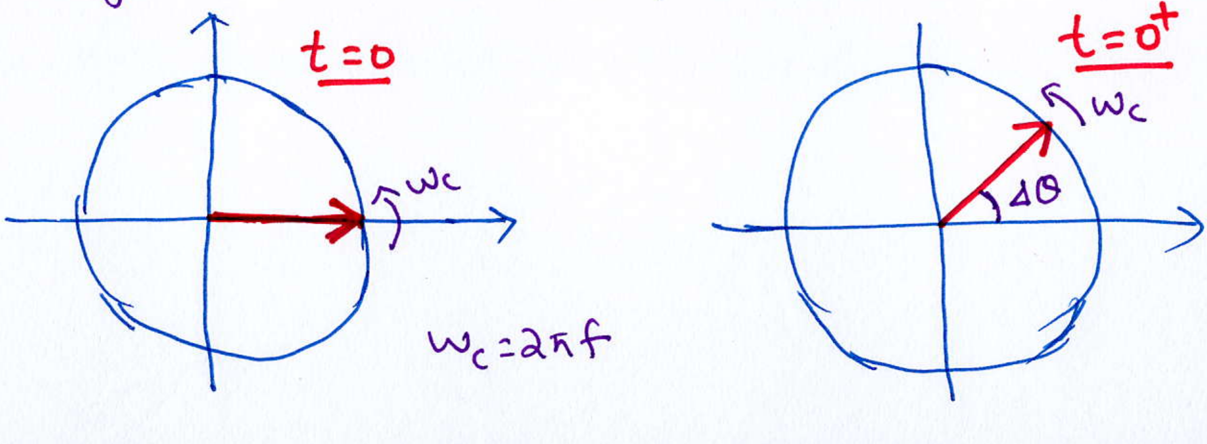
Let $\theta_i(t) = \Delta\theta\, u(t)$

i.e. the reference signal is

$$A \cos[2\pi f_c t + \Delta\theta\, u(t)]$$

where $u(t)$ is the unit step function

The phasor corresponding to this reference signal can be represented as



$w_c = 2\pi f$

$$\theta_i(t) = \Delta\theta \, u(t)$$

$$\boxed{u(t) \xrightarrow{LT} \frac{1}{s}}$$

$$\Theta_i(s) = \frac{\Delta\theta}{s}$$

$$H(s) = \frac{\Theta_o(s)}{\Theta_i(s)} = \frac{K}{s+K}$$ from 5

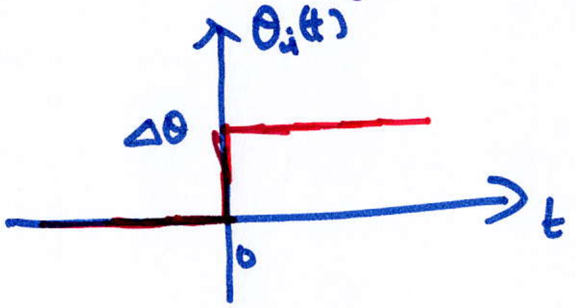
$$\therefore \Theta_o(s) = H(s) \, \Theta_i(s) = \frac{K}{s+K} \cdot \frac{\Delta\theta}{s}$$

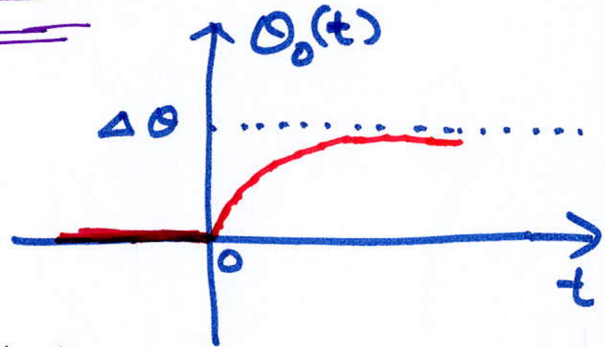
$$\Theta_o(s) = \Delta\theta \left[\frac{1}{s} - \frac{1}{s+K} \right]$$

$$\boxed{e^{-Kt} u(t) \xrightarrow{LT} \frac{1}{s+K}}$$

Taking the inverse LT,

$$\underline{\underline{\theta_o(t) = \Delta\theta \left[1 - e^{-Kt}\right] u(t)}}$$





In the steady statate

$$\underline{\underline{\lim_{t\to\infty} \theta_o(t) = \Delta\theta}}$$

$$\therefore \underline{\underline{\lim_{t\to\infty} \theta_e(t) = 0}}$$ $\boxed{\text{phase error}}$

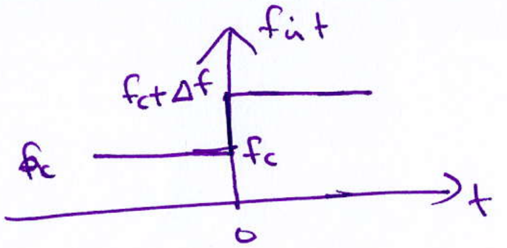
Thus, the first-order PLL can track a sudden change in phase, with the output phase converging to the input phase exponentially fast. The steady state phase error is <u>zero</u>.

Note: Increasing the loop gain K increases the rate of convergence of the output phase.

We now consider how the first-order PLL tracks a step change in the frequency of the reference signal.

Let the instantaneous frequency of the reference signal is

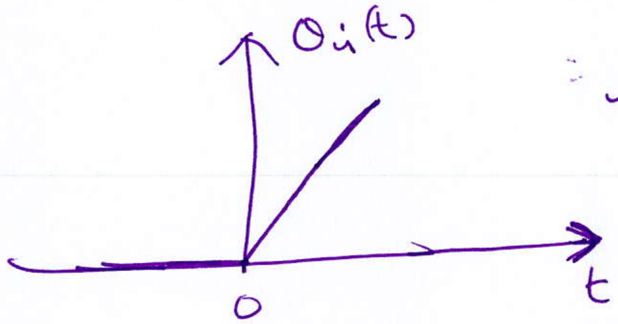
$$f_i(t) = f_c + \Delta f\, u(t)$$



The reference signal is

$$A_c \cos\left[2\pi f_c t + 2\pi \Delta f\, t\, u(t)\right]$$

$$\therefore\ \theta_i(t) = 2\pi \Delta f\, t\, u(t)$$



i.e. linear change in the phase.

$$\theta_i(t) = 2\pi \Delta f\, t\, u(t)$$

$$\boxed{t\, u(t) \xrightarrow{LT} \frac{1}{s^2}}$$

$$\Theta_i(s) = \frac{2\pi \Delta f}{s^2} \qquad (7)$$

$$\therefore\ \Theta_o(s) = H(s) \cdot \Theta_i(s)$$

$$\Theta_o(s) = \frac{K}{s+K} \cdot \frac{2\pi \Delta f}{s^2} \qquad (8)$$

from (5)

Let us first consider how the PLL tracks the ~~chan~~ instantaneous frequency.

Instantaneos frequency of the VCO output,

$$f_0(t) = \frac{1}{2\pi}\frac{d}{dt}\left[2\pi f_c t + \theta_0(t)\right]$$

$$= f_c + \frac{1}{2\pi}\frac{d\,\theta_0(t)}{dt}$$

$$\theta_0(t) \xrightarrow{LT} \Theta_0(s)$$

$$\frac{d}{dt}\theta_0(t) \xrightarrow{LT} s\,\Theta_0(s)$$

From 8,

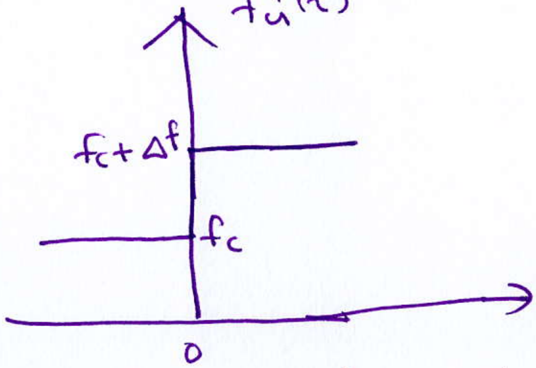
$$\frac{1}{2\pi}\frac{d}{dt}\theta_0(t) \xrightarrow{LT} \frac{K}{s+K}\cdot\frac{\Delta f}{s} = \Delta f\left(\frac{1}{s} - \frac{1}{s+K}\right)$$

taking the inverse LT, we get

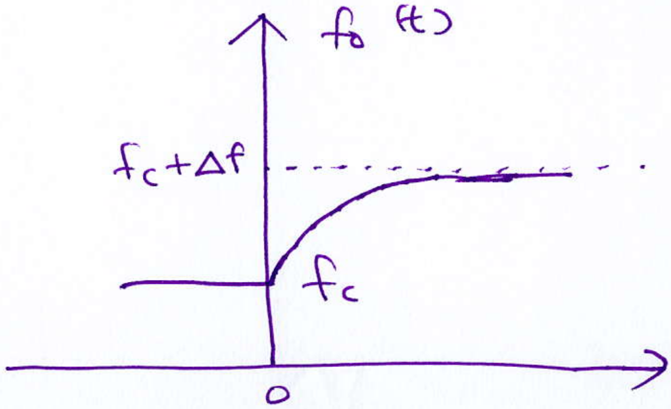
$\therefore$

$$\frac{1}{2\pi}\frac{d}{dt}\theta_0(t) = \Delta f\left(1 - e^{-Kt}\right)u(t)$$

$$\therefore\ f_0(t) = f_c + \Delta f\left(1 - e^{-Kt}\right)u(t)$$



instantaneous freq: of the reference signal



instantaneous freq: of the VCO o/p

$\therefore$ The first order PLL tracks step change in frequency, converging exponentially fast.

~~It is state~~

We now analyze ~~the~~ how the PLL tracks instantaneous phase.

$$H_e(s) = \frac{\Theta_e(s)}{\Theta_i(s)} = \frac{\cancel{k}s}{s+k} \quad \swarrow \text{from (6)}$$

$$\Theta_e(s) = \frac{\cancel{k}s}{s+k}\,\Theta_i(s) = \frac{\cancel{k}\,s}{s+k}\cdot\frac{2\pi\Delta f}{s^2} \quad \swarrow \text{from (8)}$$

By using the final value theorem

$$\lim_{t\to\infty}\theta_e(t) = \lim_{s\to 0} s\,\Theta_e(s)$$

$$= \frac{2\pi\Delta f}{K}$$

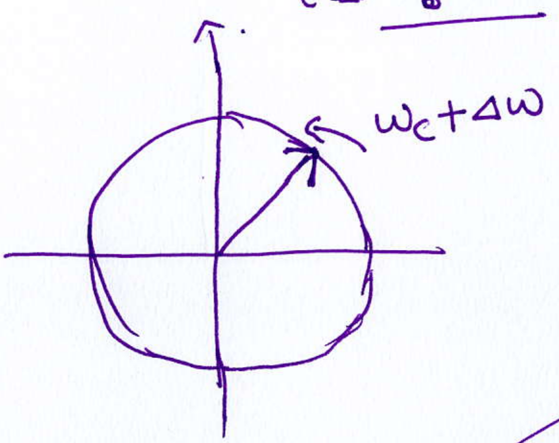
i.e. there is ~~a~~ steady state phase error.

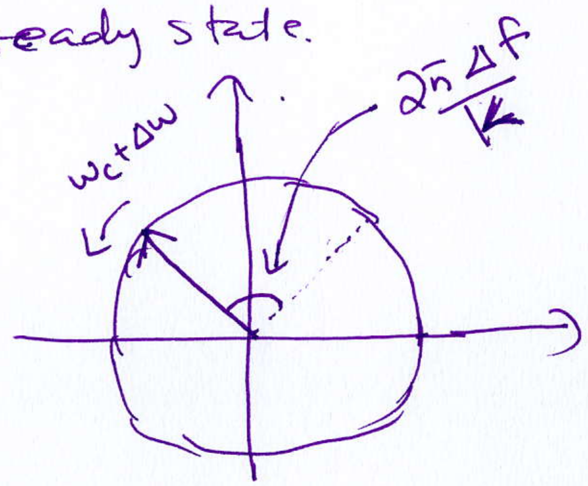
We can summarize,

If there is a step change in frequency ~~of~~ the reference signal, the first-order PLL tracks the change in frequency, but there is a steady state phase error.

In terms of phasor

$t = t_0 >> 0$ ← steady state.





$$\boxed{\begin{aligned} w_c &= 2\pi f_c \\ \Delta w &= 2\pi\Delta f \end{aligned}}$$

This problem, i.e. steadystate phase error, can be ~~bas~~ solved if we use a second order PLL.

The transfer function ~~G(s)~~ of ~~the~~ a second order PLL is given by

$$G(s) = \frac{s+a}{s}$$

Exercise: Show that the steady-state phase error ($\lim_{t \to \infty} \theta_e(t)$) is zero if ~~we use as~~ for second order PLL