EEE-101 - Control Systems Engineering

Tutorial - 2

Laplace transform of some standard signals and a few related identities are provided in tabular form below for your ready reference. The tutorial questions follow from the next page.

Item no.	$f(t)$	$F(s)$
1.	$\delta(t)$: unit impulse	1
2.	$u(t)$: unit step	$\frac{1}{s}$
3.	$tu(t)$: unit ramp	$\frac{1}{s^2}$
4.	$t^n u(t)$	$\frac{n!}{s^{n+1}}$
5.	$e^{-at}u(t)$	$\frac{1}{s+a}$
6.	$\sin \omega t u(t)$	$\frac{\omega}{s^2+\omega^2}$
7.	$\cos \omega t u(t)$	$\frac{s}{s^2+\omega^2}$

Table 1: Laplace transform table

Item no.	Theorem		Name
1.	$\mathscr{L}[f(t)] = F(s)$	$= \int_{0-}^{\infty} f(t)e^{-st}dt$	Definition
2.	$\mathscr{L}[kf(t)]$	$= kF(s)$	Linearity theorem
3.	$\mathscr{L}[f_1(t)+f_2(t)]$	$= F_1(s)+F_2(s)$	Linearity theorem
4.	$\mathscr{L}[e^{-at}f(t)]$	$= F(s+a)$	Frequency shift theorem
5.	$\mathscr{L}[f(t-T)]$	$= e^{-sT}F(s)$	Time shift theorem
6.	$\mathscr{L}[f(at)]$	$= \frac{1}{a}F\left(\frac{s}{a}\right)$	Scaling theorem
7.	$\mathscr{L}\left[\frac{df}{dt}\right]$	$= sF(s) - f(0-)$	Differentiation theorem
8.	$\mathscr{L}\left[\frac{d^2f}{dt^2}\right]$	$= s^2F(s) - sf(0-) - f'(0-)$	Differentiation theorem
9.	$\mathscr{L}\left[\frac{d^nf}{dt^n}\right]$	$= s^nF(s) - \sum_{k=1}^{n} s^{n-k}f^{k-1}(0-)$	Differentiation theorem
10.	$\mathscr{L}\left[\int_{0-}^{t} f(\tau)d\tau\right]$	$= \frac{F(s)}{s}$	Integration theorem
11.	$f(\infty)$	$= \lim_{s\to 0} sF(s)$	Final value theorem[1]
12.	$f(0+)$	$= \lim_{s\to\infty} sF(s)$	Initial value theorem[2]

[1]For this theorem to yield correct finite results, all roots of the denominator of $F(s)$ must have negative real parts, and no more than one can be at the origin.
[2]For this theorem to be valid, $f(t)$ must be continuous or have a step discontinuity at $t = 0$ (that is, no impulses or their derivatives at $t = 0$).

Table 2: Laplace transform theorems

1. Let $G(s) = \dfrac{1}{(s+1)(s+4)(s^2+2s+2)^2}$, and $Y(s) = G(s)R(s)$. Find $y(t)$, considering zero initial conditions and $R(s) = \mathcal{L}[e^{-t}u(t)]$ where $u(t)$ is the unit step signal.

2. Write the differential equation which describes the following system

$$G(s) = \frac{(s^2+3s+7)(s+2)}{(s+3)(s+4)(s^2+2s+100)}.$$

3. A system with impulse response

$$g(t) = e^{-t}(1 - sin\ t)$$

is subjected to a unit-step input. Find the transfer function and therefrom obtain the steady-state value of the output.

[Hint: Apply final value theorem (Item no. 11 in Table 2) to obtain the *steady-state value*]

4. Consider the system, as shown in Fig. 1, is described by the differential equation

$$\ddot{y}(t) + 7\dot{y}(t) + 10y(t) = \dot{r}(t) + 3r(t).$$

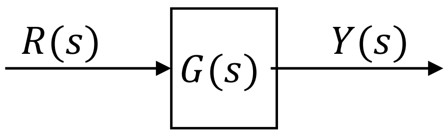

Figure 1

(a) Find the transfer function $G(s)$.

(b) Find the system response $y(t)$ to an input $r(t) = e^{-t}u(t)$ assuming zero initial conditions.

(c) Repeat part (b) when the initial conditions are $y(0) = 1$, $\dot{y}(0) = 1/2$.

[$u(t)$ represents unit-step input.]

5. Find out the transfer function from $v(t)$ to $i(t)$ for the following circuit

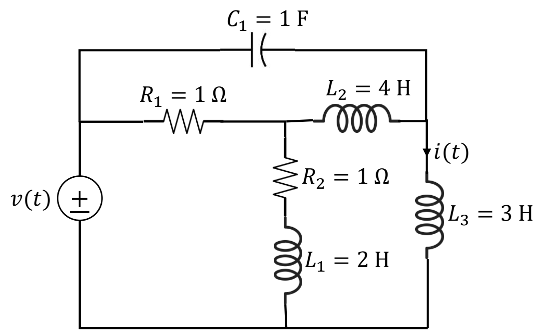

Figure 2

6. Find the transfer function $\dfrac{V_0(s)}{V_i(s)}$ for the following circuit

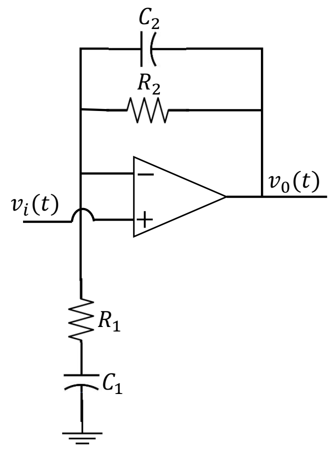

Figure 3