ES112 Lab and Practice Problems

September 23, 2015

1. Write down a recursive function to calculate $\binom{n}{k}$, using the following definition – $\binom{n}{0} = 1$ and $\binom{n}{n} = 1$ and $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$.

2. Use the following definition to write a function that detects whether a string is a palindrome:

   - Any string of length 0 or length 1 is a palindrome.
   - A string `s` is a palindrome if `s[0] == s[-1]` and `s[1:-1]` is a palindrome.

3. Extend the above function so that non-alphabetic characters inside the string are ignored. For instance, "Madam, I'm Adam" and "Do geese see god?" should be palindromes as well.

4. Write down a function that takes as parameter two strings `s` and `t` and can say whether `s` is a subsequence of `t`. For instance `abcd` is a subsequence of `aXYZbymzcU817djak`. Notice that the characters of `s` are present in order in `t` but not necessarily consecutively.

5. Write down a function that takes as input n and uses recursion (not loops) to calculate the sum $\sum_{i=1}^{n} \sqrt{i}$.

6. Consider two vectors defined in $\Re^n$, denoted by $x = (x_1, x_2, \ldots, x_n)$ and $y = (y_1, y_2, \ldots, y_n)$. The Euclidean distance between two such vectors x and y is defined as the following:

$$\|x - y\|_2 = \left(\sum_{i=1}^{n} (x_i - y_i)^2 \right)^{1/2}.$$

   Write down a function `calculateEuclidean(x, y)` that takes two lists `x` and `y` of same size and returns the Euclidean distance between them.

7. Compose a function that reverses the order of a one-dimensional array of floats. Do not create another array to hold the result.

8. The greatest common divisor (gcd) of two positive integers is the largest integer that divides evenly into both of them. For example, the greatest common divisor of 102 and 68 is 34 since both 102 and 68 are multiples of 34, but no integer larger than 34 divides evenly into 102 and 68. We can efficiently compute the gcd using the following property, which holds for positive integers p and q — If $q = 0$, the gcd is p. If $p > q$, the gcd of p and q is the same as the gcd of q and $p\%q$.

9. Consider the following famous unsolved problem in number theory, called the Collatz problem or $3n + 1$ problem: Given a number n, we do the following operations. If n is even, we set $n = n/2$, else we set $n = (3n + 1)/2$. The claim is that we always end up in $n = 1$. Write a function that given an input n prints out the Collatz sequence starting from n and ends when it reaches 1.

10. Compose a function that accepts a date as input and writes the day of the week on which that date falls. Your program should accept three command-line arguments: `m` (month), `d` (day), and `y` (year). For `m` use 1 for January, 2 for February, and so forth. Use the following formulas, for the Gregorian calendar. All the divisions are integer divisions.

$$\begin{aligned} y_0 &= y - (14 - m)/12 \\ x &= y_0 + y_0/4 - y_0/100 + y_0/400 \\ m_0 &= m + 12 * ((14 - m)/12) - 2 \\ d_0 &= (d + x + (31 * m_0)/12) \bmod 7 \end{aligned}$$

The output will be d_0 which is a value between 0 to 7. Convert 0 to Sunday, 1 to Monday, 2 for Tuesday, and so forth.