Assignment 1

Due Date: November 21.

Some Important Points

1. Make a group of at most 2 people. The group members remain same till the end of this course.
2. Collaboration is encouraged. However, each group should write its final answer separately.
3. Please mention names of all the people with whom you have discussed the question.

1. (5 points) You are given two 2-3-4 tree tree A and B each having n elements, such that all the elements in A are less than all the elements in B. Describe a procedure which joins these two trees into a single 2-3-4 tree (a single 2-3-4 tree C that contain all elements of A and B, destroying old versions of A and B) and takes time $O(\log n)$.

2. (5 points) Let T be a red black tree storing n items and let k be an item in T. Show how to construct from T, in $O(\log n)$ time, two red black tree T' and T'' such that T' contains all the keys of T less than k and T'' contains all the keys of T greater than k (and T is destroyed).

3. (5 points) In a directed graph each graph has a direction, for example edge (u, v) implies that this edge *starts* from u and ends at v. The adjacency matrix representation of a directed graph is as follows:

$$A_{u,v} = \begin{cases} 1 & \text{if there exists an edge } (u,v)\text{, an edge that starts from } u \text{ and ends at } v \\ 0 & \text{otherwise} \end{cases}$$

   The in-degree of a vertex is the number of edges that end at that vertex. Similarly the out degree of a vertex is the total number of edges that start from a vertex. A directed graph G contains a *universal sink* if there exists a vertex with in-degree $n - 1$ and out-degree 0. Given a adjacency matrix representation of a directed graph G, in $O(n)$ time, find if it contains a universal sink.

4. (5 points) In a tree, let $\delta(u, v)$ denote the shortest distance between u and v. The diameter of a tree $T = (V, E)$ is defined as $\max_{u,v \in V}\{\delta(u, v)\}$, that is the largest of all shortest-path distances in the tree. Give an efficient algorithm to find the diameter of the tree.