ES 311 Heat and Mass Transfer
Mini-Project 01 Final Report

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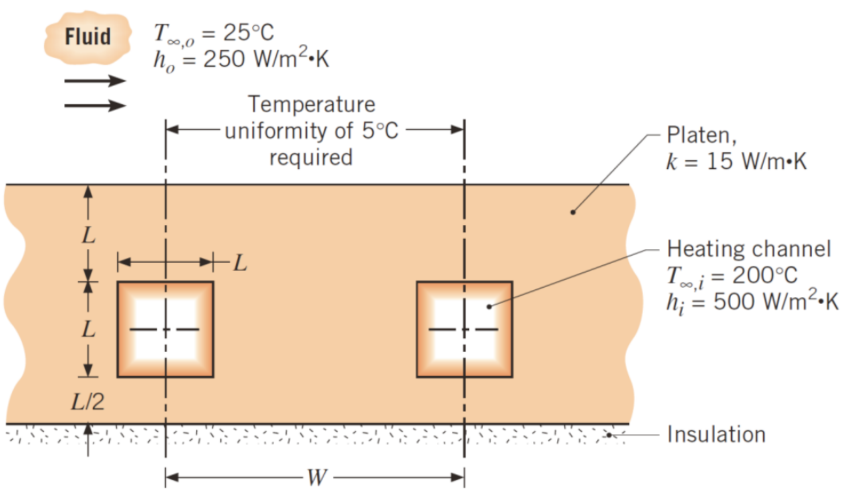
Problem 1. A platen of thermal conductivity k = 15 W/m-K is heated by flow of a hot fluid through channels of width L = 20 mm, with $T_{\infty,i} = 200$ C and $h_i = 500$ W/m2-K. The upper surface of the platen is used to heat a process fluid at $T_{\infty,o} = 25$ C with a convection coefficient of $h_o = 250$ W/m^2-K. The lower surface of the platen is insulated. Initially, the platen is at the process fluid temperature $T_i = T_{\infty,o} = 25$ C. The hot fluid is switched on at t = 0. To heat the process fluid uniformly, the temperature of the platens upper surface must be uniform to within 5 C. Use implicit time marching to solve this as a transient conduction problem. Start with an initial guess of W = 2L. Assume that the platen is very large in the direction perpendicular to the page.



1 Solution

The problem under consideration is solved by using implicit time marching which is one of the Finite Differences Method to solve transient conduction problem. First of all we observe that the domain of platen under consideration is symmetric about the central axis of the platen taken perpendicular to the insulated surface, hence by considering just one half of the platen we can efficiently solve the entire problem. Also by symmetry of platen

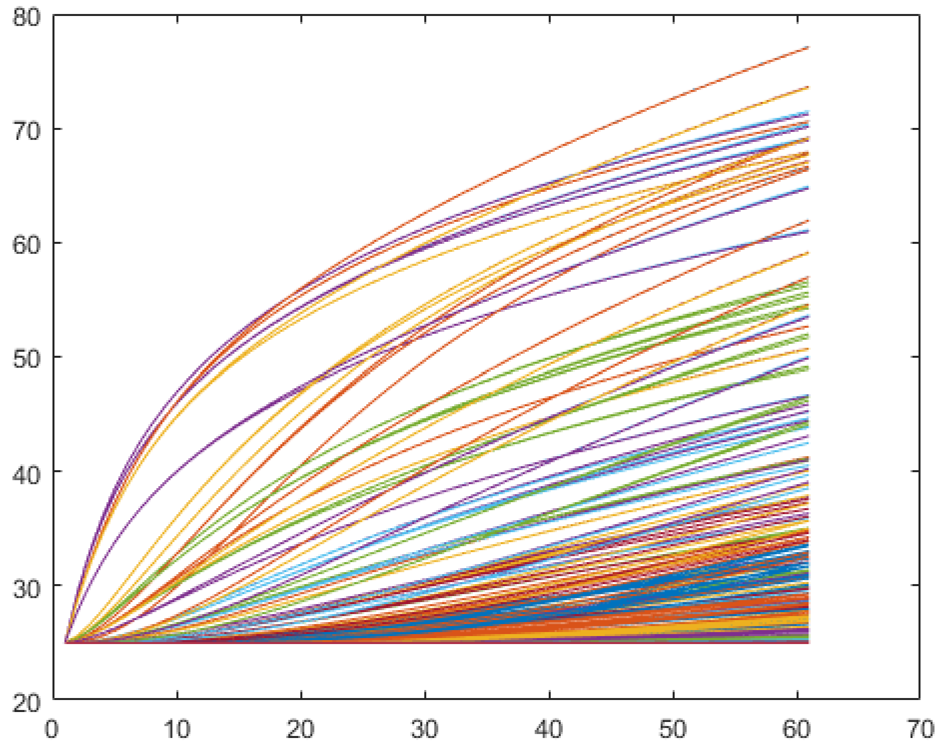
itself, the surface corresponding to the plane of symmetry acts as an insulated surface. Also as the length of the platen is not definite we can assume that the effects of the hot fluid are felt only upto the surface at a distance of LD = 6*(W+L); as a result of which the surface at a distance of LD can be considered as a convective surface.
According to the finite difference methods, for different nodal points different cases were needed to be made. A list of these points are:

- One for each of the four points at the corner of the domain.
- All the points on the left edge of the domain.
- All the points on the right edge of the domain.
- All the points on the top edge of the domain.
- All the points on the bottom edge of the domain.
- All the points on the corner of the heating channel.
- All the points on the left edge of the heating channel.
- All the points on the right edge of the heating channel.
- All the points on the top edge of the heating channel.
- All the points on the bottom edge of the heating channel.
- Rest other nodal points.

Energy Balance equation was applied for each nodal point in each of the above cases which resulted in matrices A, B and C which follow the same terminology as discussed in the lectures. The equation which we derived in the lecture is as follows:

$$A_{NXN}T_{NX1}^{p+1} = B_{NXN}T_{NX1}^{p} + C_{NX1}$$

From the above equation we can get T_{NX1}^{p+1} by pre-multiplying the inverse of A on both sides of the equation. The MATLAB code for the above solution is attached with title Platen.



Above figure is a plot of Temperature VS Time for each of the nodal points considered for Finite Difference Analysis using implicit time marching method to solve the above problem. It can inferred from the figure that by 30 seconds almost every nodal point in the domain reaches steady state which is evident from the fact that near 60 mark in the above graph which corresponds to the number of time steps with each time step equal to 0.5 seconds, the line graph becomes almost horizontal which signifies that temperature is not changing evidently with time. Hence the domain or system has reached steady state.

To check for the temperature uniformity at the surface, the program at every instant of time checks the difference between the temperature of the first nodal point and temperature of the nodal point on the upper surface and above the left corner of the heating channel. If at some time instant the temperature difference is more than 5^oC it stores that time instant in an array named **uu**. Hence on running the program for n=1, 2 and 3 no time instants were to be found in array **uu**. But some time instants were found for n=4 which are **uu** = [55,56,57,58,59,60,61]; hence it can be inferred that for W=4L the temperature uniformity condition is not maintained. **Please check the above in the Platen.m file by changing n=1, 2, 3 and 4**

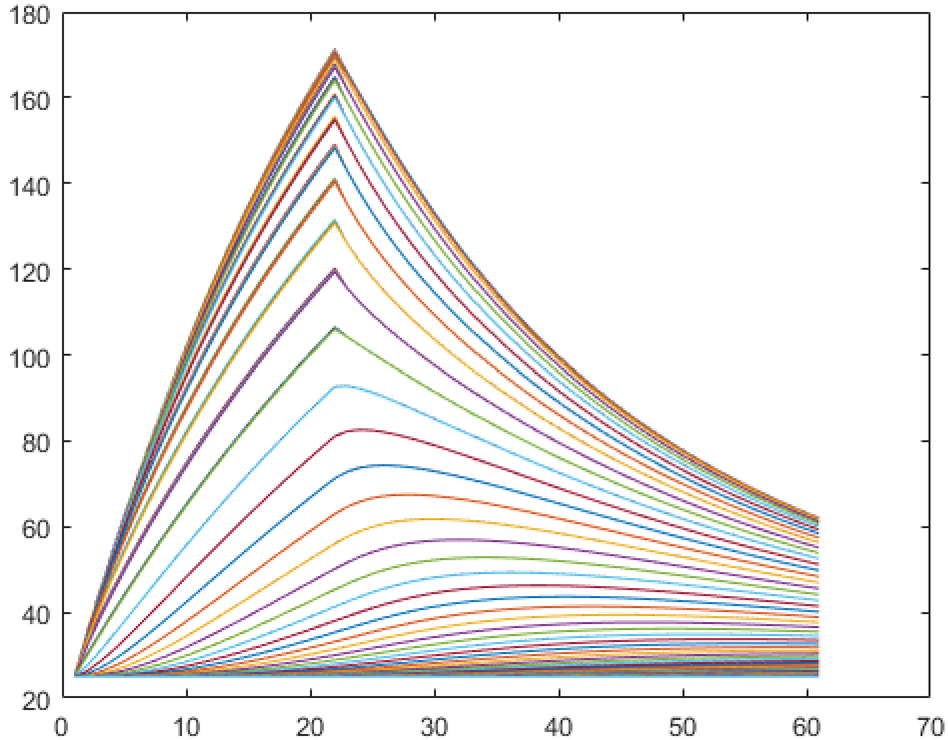
From the first figure it was inferred that steady state of the system is reached. This temperature was obtained by applying energy balance equation and not heat diffusion equation, hence inherently the energy balance equation is satisfied which is proved by

inferring that steady state is achieved by the system.
Total heat transferred to the process fluid can be obtained in steady state by calculating the heat transferred by the heating channel to the platen in steady state which would be equal as the state is steady: **s=1.1048e+06**. Refer to the Platen.m MATLAB code for the same.

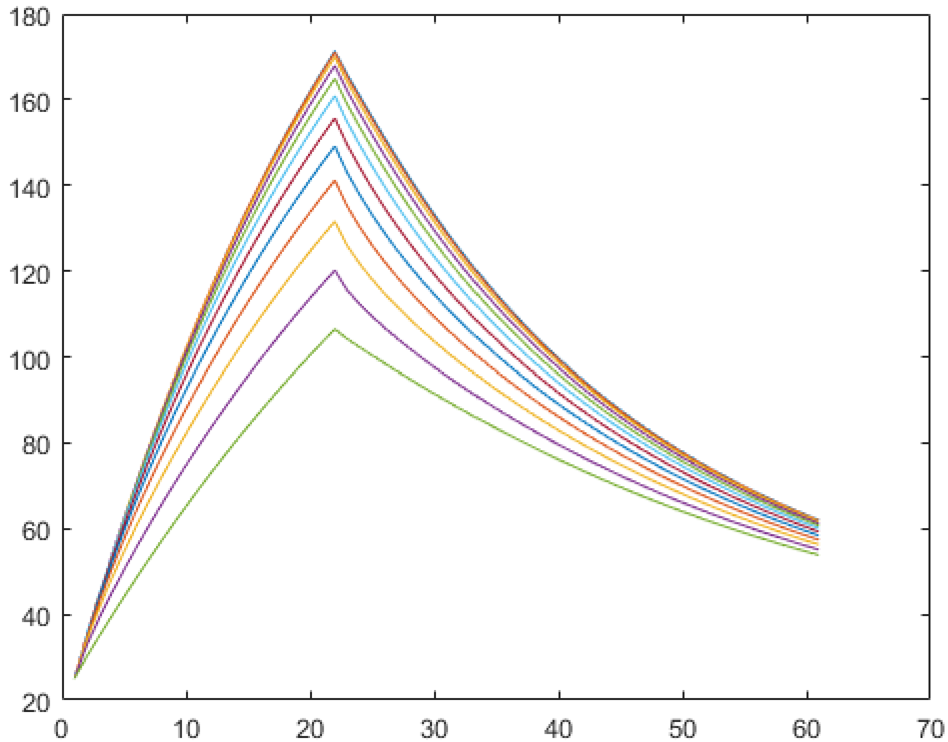
Problem 2. Complete the code that was written by the instructor to solve the example transient conduction problem discussed in class. Add (1) explicit time marching scheme option and (2) a simplified implicit time marching scheme by utilizing the Bi_y = (h * d / k) $\leq$ 0.1 (thereby removing the discretization in the y-direction).

2 Implicit Solution

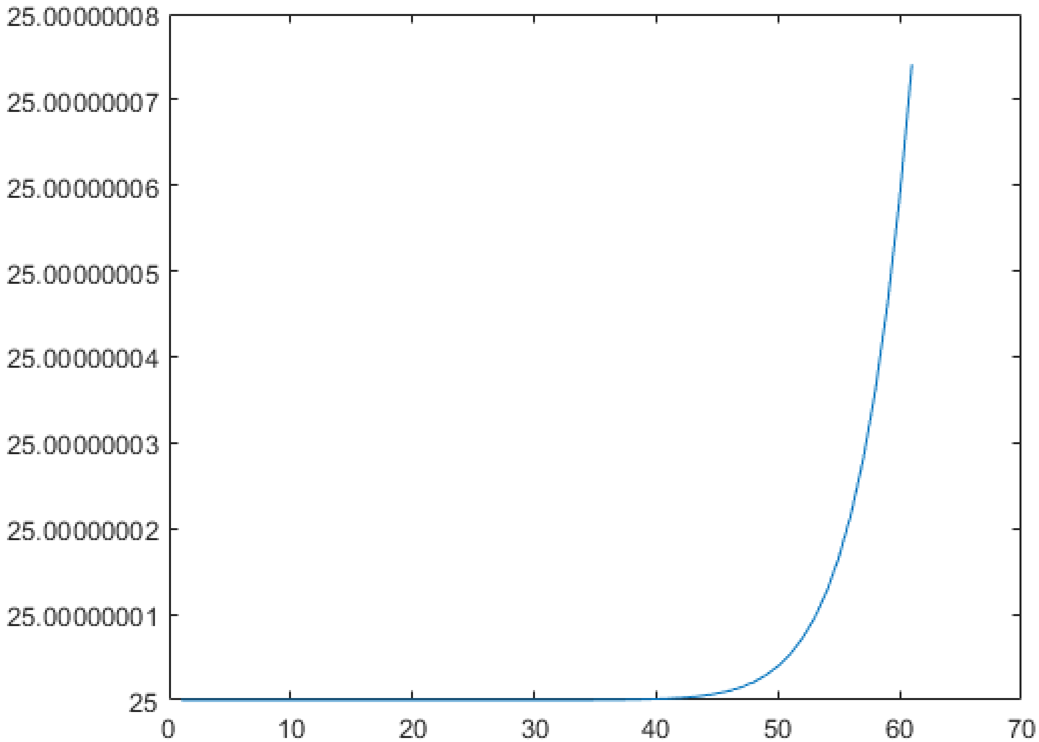
The solution to the above problem is obtained using implicit as well as explicit time marching method of Finite Difference Method. Observing the given system, it becomes evident that it is symmetric about an axis which divides the system into two equal halves. The surface corresponding to the above axis acts as an insulated surface. Similarly from the previous problem as the length of the system is not definite we can contain a finite domain of length 220 mm which signifies that the effects of the processes is negligible at 220 mm. As a result of which the surface at 220 mm acts as a convective surface. The implicit equations are solved in the same manner as in the previous problem i.e. by pre-multiplying the inverse of A on both sides of the equation. **Please check the code for the same in the LaserHeating_posted.m file.**



Above figure is a plot of Temperature VS time for each of the nodal points considered in the domain. As it is evident from the above figure that during the entire process the maximum temperature of any nodal point in the domain at any time instant is always less than 200^oC; hence the material safety criteria is satisfied.



Above figure is a plot of Temperature VS time for the nodal points 1 to 12 which corresponds to the surface in contact with the plastic film in the domain. As is evident from the above figure apart from the nodal point 12 every other nodal point which is in contact with the plastic film has temperature greater that 90^oC for at-least 10 seconds. Hence the adhesion criteria is also more or less satisfied.



Above figure is a plot of Temperature VS Time for the nodal point 333 which is present on the rightmost edge of the domain which we considered as not being much affected by the processes happening. It can be inferred from the graph that the temperature of the nodal point 333 remains constant throughout then changes by an order of 0.00000001 as a minute effect of the process is experienced. Hence the system or domain size as well as boundary condition assumptions are valid.

3 Explicit Solution

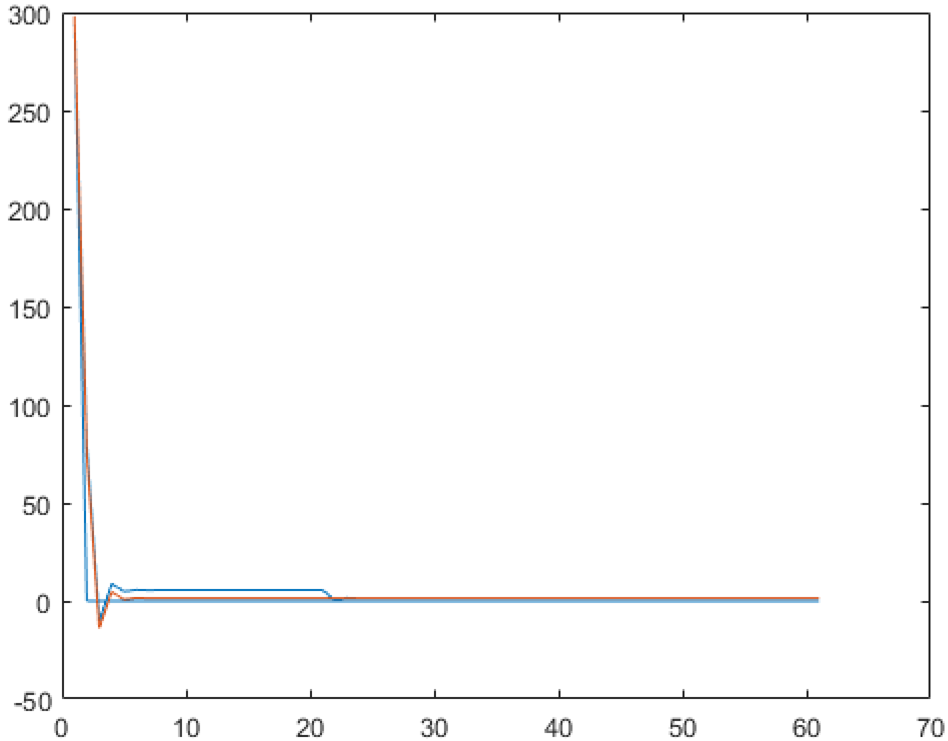
For solving the given problem using explicit time marching method, the entire domain was discretized only in the x direction and not in the y direction as Bi_y = (h * d / k) ≤ 0.1. Thereafter as the entire domain was discretized in X direction three cases were observed:

- The nodal point to the leftmost edge of the domain.
- The nodal point to the rightmost edge of the domain.
- Other remaining nodal points.

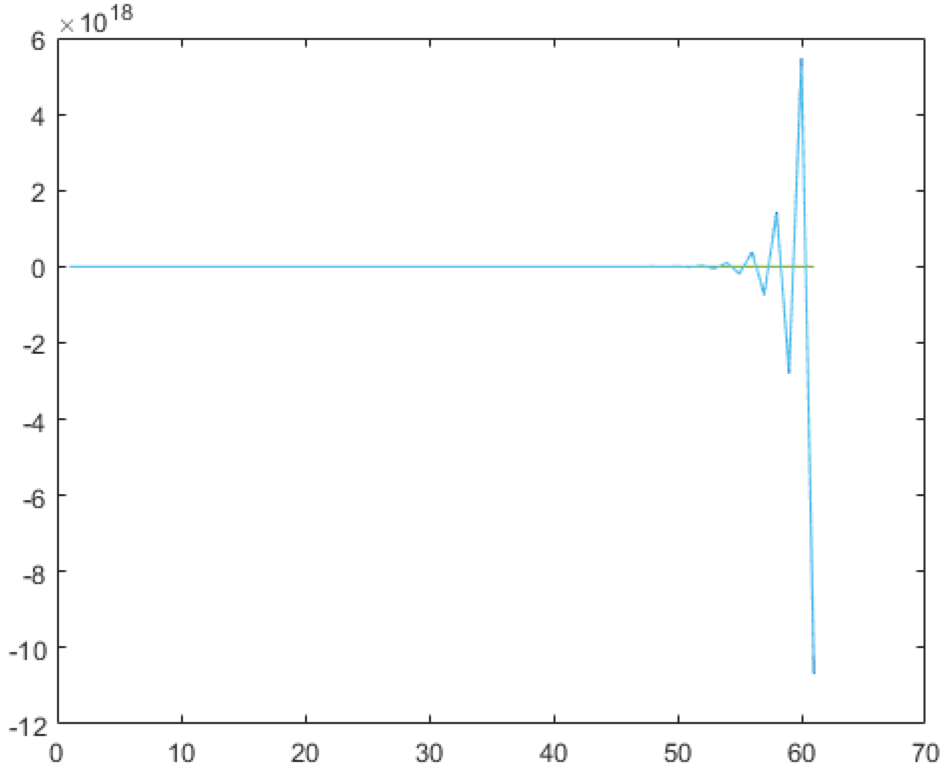
Thereafter Energy Balance Equation was applied at each of the nodal points in an explicit manner. Thereafter the equations were solved one by one in a sequence which resulted into a temperature matrix TE.

Following are the implications of using explicit time marching:

- If the Grid Fourier Number Inequality is not satisfied then the solution blows up i.e. the solution of temperatures become numerically unstable.



Above figure is the plot of Temperature VS Time for nodal points which were selected randomly by the program. Here number of nodal points in x direction i.e. **nx=51**.

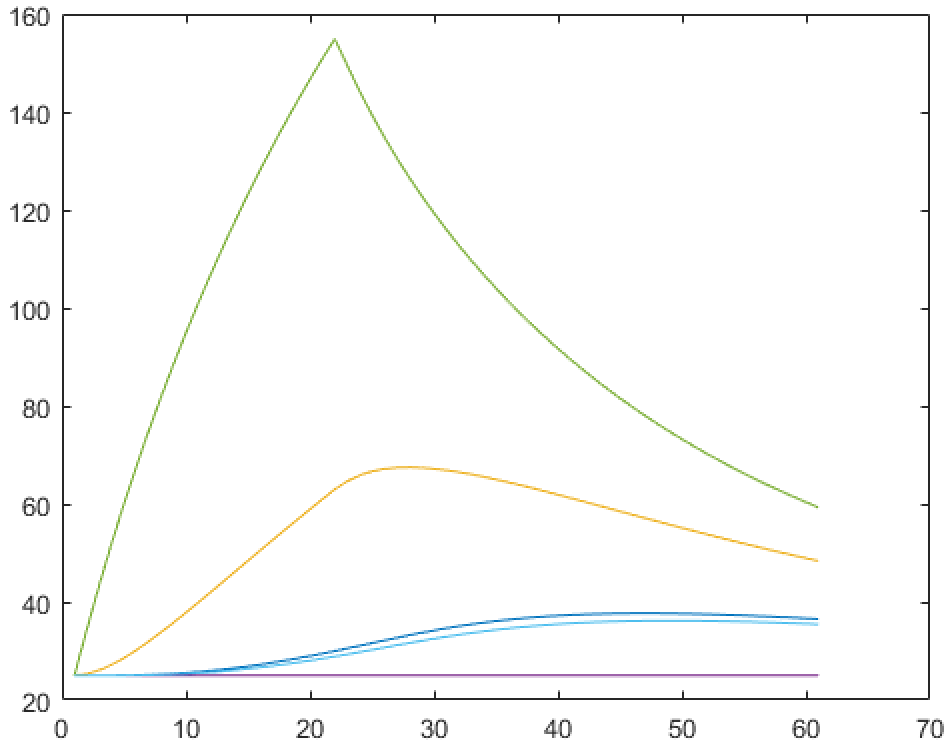


Above figure is the plot of Temperature VS Time for nodal points which were selected randomly by the program. Here number of nodal points in x direction i.e. **nx=111**.

It is evident that as time step dt or Δt becomes larger the solution to the explicit time marching method blows up i.e. it becomes numerically unstable. Here for creating an equivalent effect of increasing Δt , we have increase number of nodal points in the x direction which in turn decreases the length of discretization in x direction i.e. Δx for the same Δt which in turn is resulting in increasing of Grid Fourier time number and hence the numerical system becomes unstable.

In explicit as well as implicit time marching method by utilizing the knowledge about Biot number, much computational resources can be saved as by having the knowledge of Biot number, the number of discretization decreases which leads to less number of nodal points which in turn leads to less number of Energy Balance or Heat diffusion equations to be solved. Hence having the knowledge of Biot number decreases the computational resources significantly.

To justify the above thing to boss, below figure is more than satisfactory.



Above figure depicts the Temperature VS Time plots of nodal points present in a single x-plane (one whose normal is along the x direction) for the implicit time marching case which we performed ignoring the Biot number in y direction. However the plot with cyan color corresponds to Temperature VS Time plot at the same location but for the explicit time marching case in which we acknowledged the value of Boit number in y direction. As the plots are very similar it can be concluded that by having the knowledge of Biot number we can hence reduce the number of discretization which in turn would not affect the final solution drastically.