Covariance and Correlation coefficient

PROBABILITY AND STATISTICS FOR ECE

Covariance

- Covariance is a measure of the joint variability of two random variables.
- It is used to indicate if the variables tend to show similar/opposite behavior
- Covariance is also used when calculating variance of sum of R.V.s

Covariance

Recall:

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&\mathbf{Var}[X+Y] \\
=&\mathbf{E}\{[(X+Y)-(\mu_X+\mu_Y)]^2\} \\
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Define:

$$\mathbf{Cov}[X,Y]=\mathbf{E}[XY]-\mu_X\mu_Y$$

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Another definition:

$$\mathbf{Cov}[X, Y] = \mathbf{E}[(X - \mu_X)(Y - \mu_Y)]$$

This is also called $(1, 1)$th central moment

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Covariance Properties

- $\mathbf{Cov}[X, a] = 0$
- $\mathbf{Cov}[X, X] = \mathbf{Var}[X]$
- $\mathbf{Cov}[X, Y] = \mathbf{Cov}[Y, X]$
- if $\mathbf{Cov}[X, Y] = 0$, X and Y are orthogonal
- When $\mu_X = 0$ or $\mu_X = 0$, $\mathbf{Cov}[X, Y] = \mathbf{E}[XY]$
- When X, Y are s.i., then $\mathbf{E}[XY] = \mathbf{E}[X]\mathbf{E}[X] \Rightarrow \mathbf{Cov}[X, Y] = 0$

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Covariance Matirx

For random vector

$$\begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{bmatrix}$$

We can calculate the covariance matrix by intuitively generalize covariance into multiple dimensions

$$\Sigma = \begin{bmatrix} \mathbf{Cov}[X_1, X_1] & \mathbf{Cov}[X_1, X_2] & \dots & \mathbf{Cov}[X_1, X_n] \\ \mathbf{Cov}[X_2, X_1] & \mathbf{Cov}[X_2, X_2] & \dots & \mathbf{Cov}[X_2, X_n] \\ \vdots & \ddots & \dots & \vdots \\ \mathbf{Cov}[X_n, X_1] & \mathbf{Cov}[X_n, X_2] & \dots & \mathbf{Cov}[X_n, X_n] \end{bmatrix}$$

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Correlation Coefficient

- Covariance indicates how correlated are two R.V.
- But not comparable
- $\mathbf{Cov}[X, Y] = 1$, $\mathbf{Cov}[Z, W] = 10$, Z, W more correlated?

Correlation Coefficient

Define Pearson correlation coefficient

$$\rho_{XY} = \frac{\mathbf{Cov}[X, Y]}{\sigma_X \sigma_Y}$$

This can be calculated by **numpy.corrcoef**

- ρ_{XY} is a measure of the dependence between X and Y
- $-1 \leq \rho_{XY} \leq 1$ (Without proof)
- $\rho_{XY} = \pm 1$ if X and Y are linearly related
- If X and Y are s.i., they are uncorrelated, $\mathbf{Cov}[X, Y] = \rho_{XY} = 0$
- The converse is NOT TRUE

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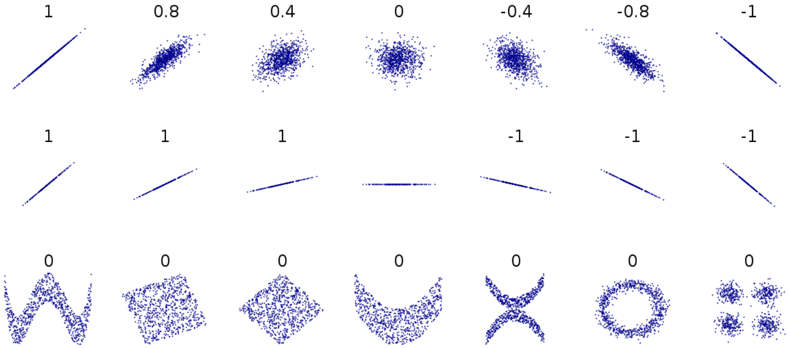

Define Spearman's rank correlation coefficient

$$r_S = \rho_{rg_X rg_Y} = \frac{\mathbf{Cov}[rg_X, rg_Y]}{\sigma_{rg_X}\sigma_{rg_Y}}$$

as the Pearson correlation coefficient between the ranked variables.
This can be calculated by **scipy.stats.spearmanr**

- Pearson's correlation works well if the relationship between variables is linear and if the variables are roughly normal
- Spearman's rank correlation mitigates the effect of outliers and skewed distributions

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- Correlation coefficients measure the strength and sign of a relationship
- Not the slope
- A common way to show the slop is linear least squares fit
- We assume each y_i is roughly equal to $\alpha + \beta x_i$

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$$\epsilon_i = \alpha + \beta x_i - y_i \neq 0$$

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- We want to minimize the squared residual:

$$\min_{\alpha,\beta} \sum \epsilon_i^2$$

- $\hat{\alpha}, \hat{\beta}$ that minimize squared residual can be calculated easily
- Compute sample means $\bar{x}, \bar{y}, \mathbf{Var}[X], \mathbf{Cov}[X,Y]$
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$$\begin{cases} \hat{\beta} = & \frac{\mathbf{Cov}[X,Y]}{\mathbf{Var}[X]} \\ \hat{\alpha} = & \bar{y} - \hat{\beta}\bar{x} \end{cases}$$

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Goodness of Fit

- Having a linear model fit to the data, we may want to know how good is it
- One common measure is predictive power of a model
- Also commonly known as "R-squared"

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- $\mathbf{Var}[\epsilon] = \frac{1}{n}\sum(\hat{\alpha} + \hat{\beta}x_i - y_i)^2$
  is the MSE when estimating Y using the linear model
- ${}^{\mathbf{Var}[\epsilon]}/_{\mathbf{Var}[Y]}$ shows the performance difference caused by introducing linear model
- $R^2 = 0.60$, we can say the model explains 60% of the variability
- Or more precisely, reduces the MSE of prediction by 60%
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- $\mathbf{Var}[Y] = \frac{1}{n}\sum(\bar{y} - y_i)^2$
  is the MSE when estimating Y using $\bar{y}$
- $\mathbf{Var}[\epsilon] = \frac{1}{n}\sum(\hat{\alpha} + \hat{\beta}x_i - y_i)^2$
  is the MSE when estimating Y using the linear model
- ${}^{\mathbf{Var}[\epsilon]}/_{\mathbf{Var}[Y]}$ shows the performance difference caused by introducing linear model
- $R^2 = 0.60$, we can say the model explains 60% of the variability
- Or more precisely, reduces the MSE of prediction by 60%
- $R^2 = \rho^2$ (Can you show it in Jupyter notebook?)

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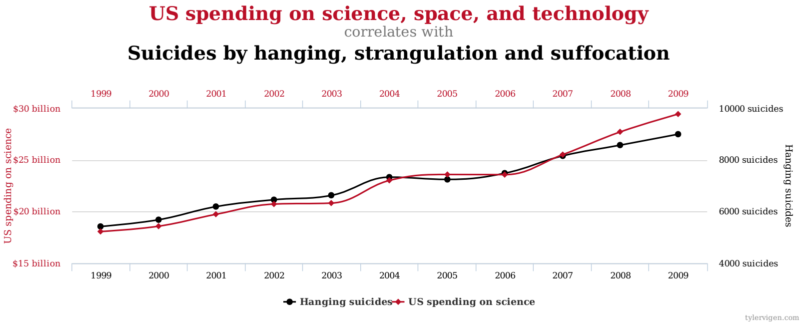
Correlation and Causation



Charts from the book **Spurious Correlations**
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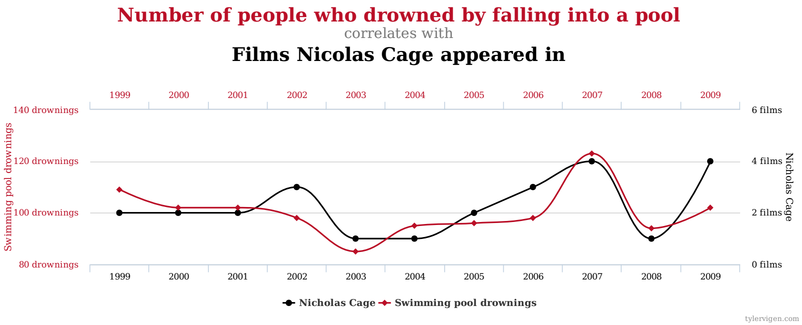
Correlation and Causation



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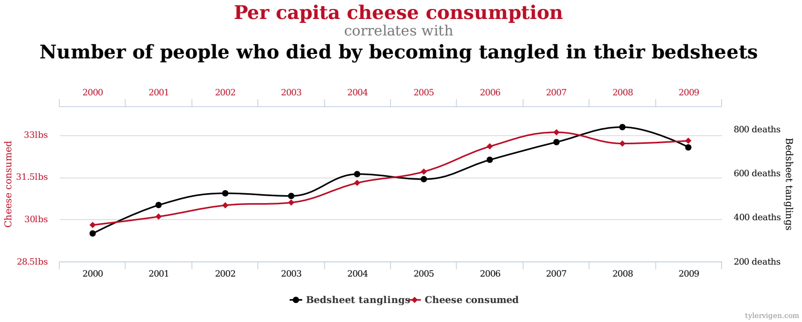

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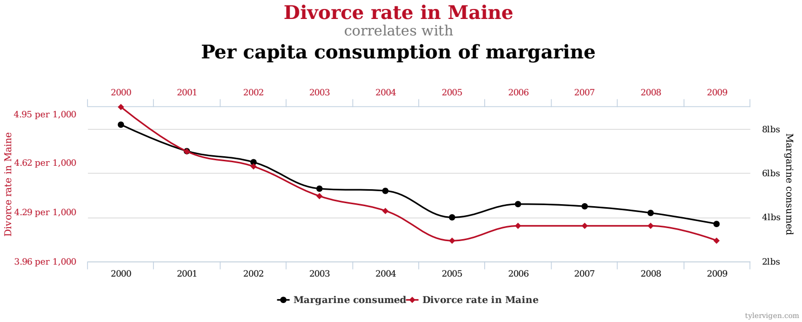

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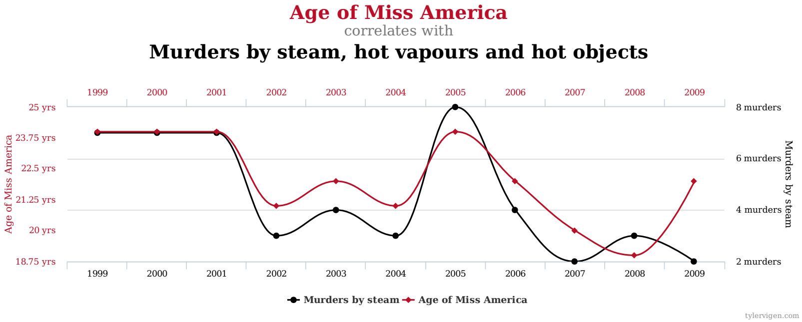

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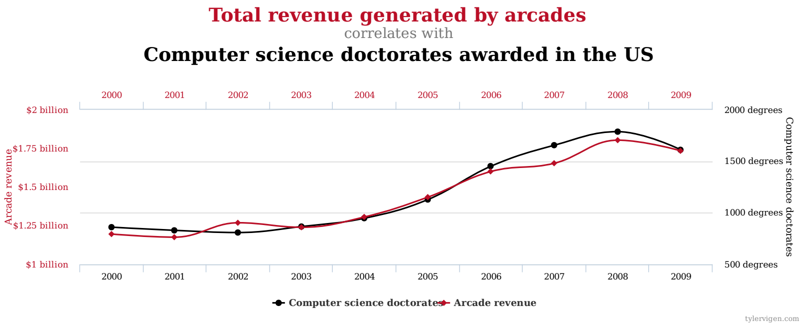

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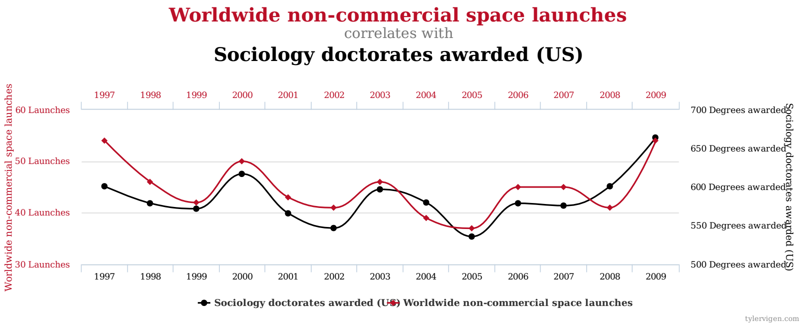

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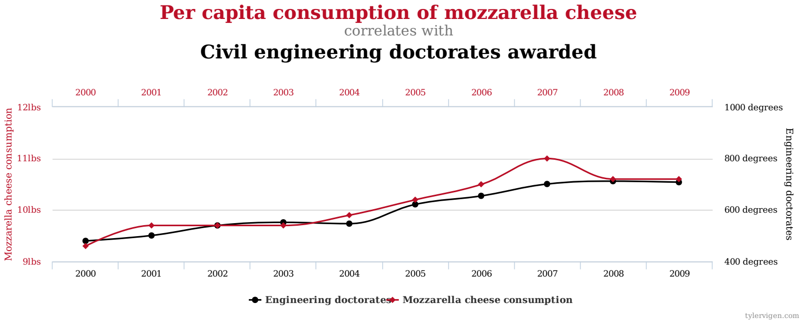

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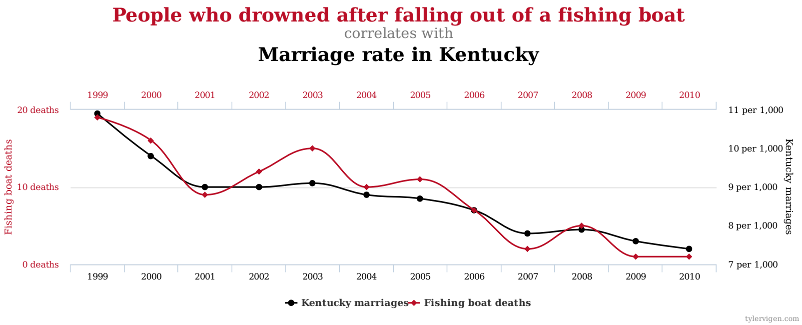

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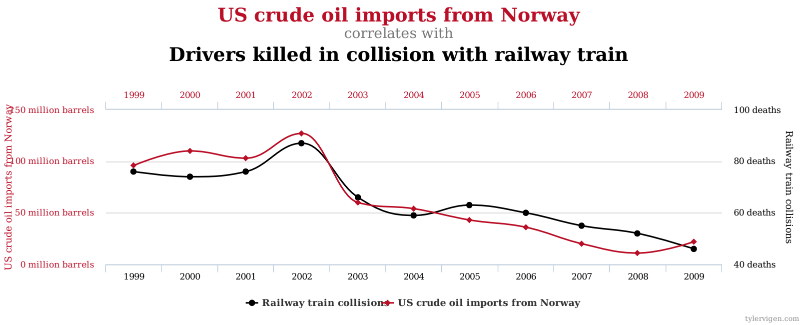

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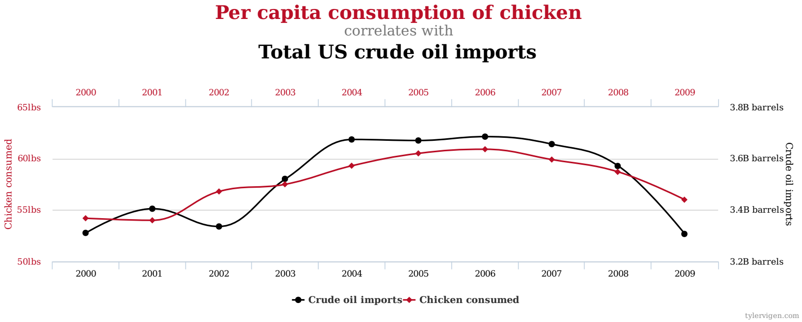

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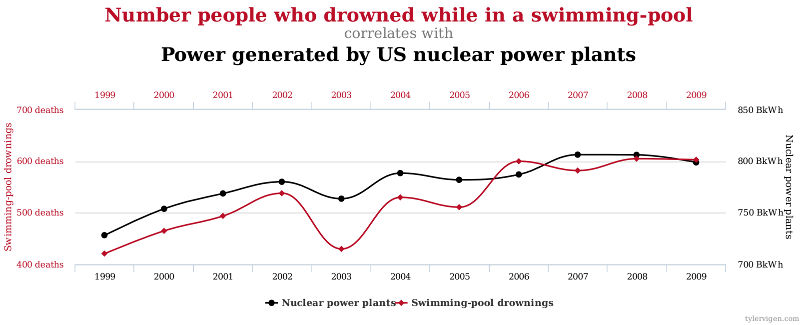

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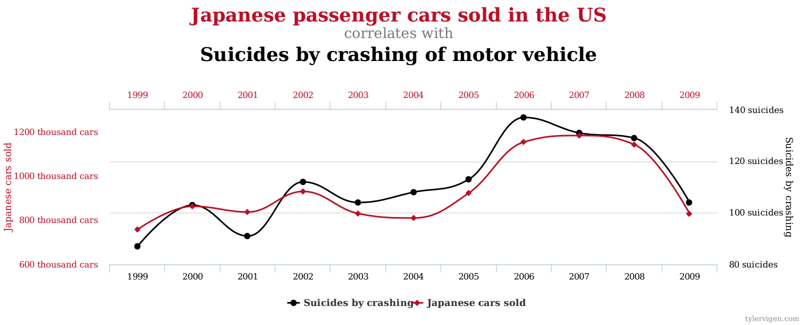

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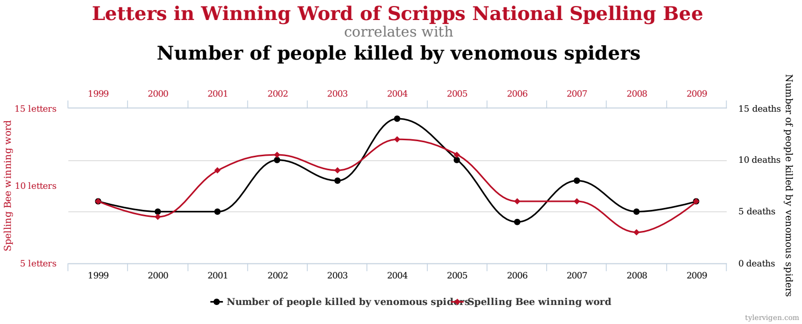

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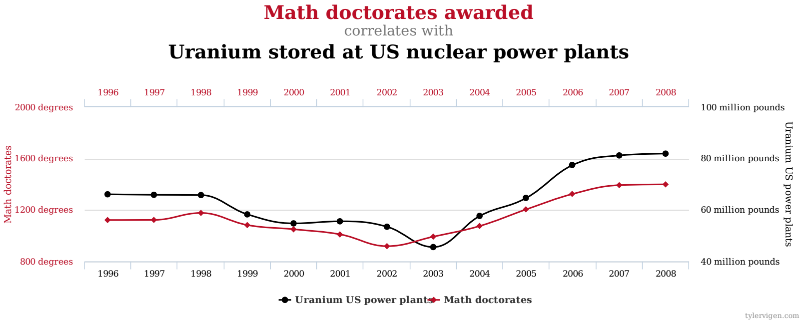

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