Deep Learning Home Work 1

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Q1. a)
$$f(x) = \begin{cases} h & \text{if } 0 < x < \delta \\ 0 & \text{otherwise} \end{cases}$$

A Simple Neural Network with 2 hidden neurons can realize the above function using a step activation on the hidden neurons. Below is the network architecture: where X is the input

w1, w2, w3 and w4 are the weights connecting input to hidden neurons and output neuron.

b1, b2, b3 are the 3 biases.

$$z1 = \sigma(w1.X + b1)$$

$$z2 = \sigma(w2.X + b2)$$

$$z3 = w3.z1 + w4.z2 + b3$$

in above equations σ represents unit step function.

The values for all variables which when input to the Neural Networks can realize the orignal box function are:

$$b3 = -h$$

$$w3 = w4 = h$$

$$w1 = 1, w2 = -1$$

$$b1 = -1, b2 = (\delta - 1)$$

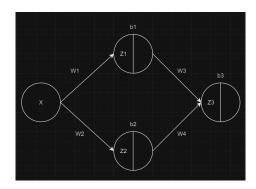


Figure 1: 1.a NN Architecture

For any given value of X we can solve the box function using these above values as parameters.

- b) For any function f(x) defined over interval [-B, B] can be approximated by a neural network with one hidden layer because of flexibility derived by the neural network. Hidden layer with enough neurons can capture complex patterns within the inputs. By adjusting the parameters like number of neurons, weights and biases we can approximate f(x) with high precision. As shown in sub-answer (a), we can divide the interval [-B, B] into multiple segments and let them be represented by multiple hidden neurons and using activation function like step activation, weights and biases we can capture complex functions. For example: we divided the condition $0 < x < \delta$ into 2 segments: x > 0 and $\delta x > 0$ to capture the step function f(x) in question 1.a
- c) Yes, the argument in b can be extended to a case of d-dimensional input, where input is a vector X:
- Neural Network with hidden layers can learn complex patterns in input data like text or audio as each neuron in the hidden layer can catch different simple pattern / feature, allowing neural networks to perform on complex dataset with high-dimensional data.
- Neural network can handle data with varying dimensions , which means a neural network can handle d-dimensional dataset input as input vector and model its function f(x).

Practical Issues:

- Dimensionality: Increase in input vector dimension can increase the computation required by the network exponentially which increases the resources like machinery cost and time for training and testing.
- Overfitting: More complex dataset requires more neurons to capture complex features, but this can lead to overfitting of the model on the training dataset and not generalize well.
- Interpret-ability: Small networks can be understood by humans but large networks have complex internal working and no human can understand this working of model or how it is generating the output. Hence in case of fin tuning the model it is impossible for humans to directly do it.

Q2.
$$y = softmax(z)$$

$$softmax(z_i) = \frac{e^{z_i}}{\sum_{j=0}^{N} e^{z_j}}$$

$$y_i = \frac{e^{z_i}}{\sum_{j=0}^{N} e^{z_j}}$$

$$\log y_i = \log \left(\frac{e^{z_i}}{\sum_{j=0}^N e^{z_j}}\right)$$

$$\log y_i = \log e^{z_i} - \log \sum_{j=0}^N e^{z_j}$$

$$\frac{\partial \log y_i}{\partial z_j} = \frac{\partial z_i}{\partial z_j} - \frac{\partial \log \sum_{j=0}^N e^{z_j}}{\partial z_j}$$

$$\frac{\partial \log y_i}{\partial z_j} = \frac{\partial z_i}{\partial z_j} - \frac{e^{z_j}}{\sum_{j=0}^N e^{z_j}}$$

$$\frac{\partial \log y_i}{\partial z_j} = \frac{\partial z_i}{\partial z_j} - softmax(z_j)$$

$$\frac{\partial \log y_i}{\partial z_j} = \frac{\partial z_i}{\partial z_j} - y_j$$

 $\frac{\partial z_i}{\partial z_j}=\delta_{ij}$ because as per given condition $\frac{\partial z_i}{\partial z_j}=1$ when i != j

$$\frac{1}{y_i} \frac{\partial y_i}{\partial z_j} = \delta_{ij} - y_j$$

$$\frac{\partial y_i}{\partial z_j} = y_i (\delta_{ij} - y_j)$$

Hence Proved.

```
In [34]:
```

```
import numpy as np
import matplotlib.pyplot as plt
from tensorflow import keras
from tensorflow.keras import layers, models, datasets
from sklearn.model_selection import train_test_split
```

In [56]:

```
(train_images, train_labels), (test_images, test_labels) = datasets.fashion_mnist.load_d
ata()
```

In the below code I have combined both the train and test set and shuffled this combined dataset. After the suffling, the datset is again split into train and test set with a ratio of 0.3, I did this to get random train and test dataset to get a more accuarate results

In [57]:

In the below code I have normalized the pixel intensities so that the value so the model is not sensitve to perticular intensities and neglect some.

```
In [58]:
```

```
train_images = train_images.astype('float32') / 255.0
test_images = test_images.astype('float32') / 255.0
```

Below code is the main Neurla network model with 3 hidden layers of size: 256, 128, and 64 neuron. each hidden layer has a activation of REIu and the final output layer has 10 output neurons and softmax activation. The input image of 28 * 28 is flattend into array of 784 values. THe model is trained with Adam optimization and crossentropy loss function.

In [67]:

```
model = models.Sequential([
    layers.Flatten(input_shape=(28, 28)),
    layers.Dense(256, activation='relu'),
    layers.Dense(128, activation='relu'),
    layers.Dense(64, activation='relu'),
    layers.Dense(10, activation='softmax')
])
```

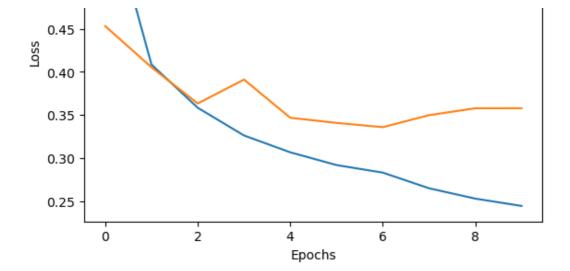
In [68]:

```
metrics=['accuracy'])
In [69]:
history = model.fit(train images, train labels, epochs= 10, batch size= 128, validation
split=0.2)
Epoch 1/10
307/307 [============== ] - 12s 38ms/step - loss: 0.6035 - accuracy: 0.787
0 - val loss: 0.4529 - val accuracy: 0.8348
6 - val loss: 0.4052 - val accuracy: 0.8507
Epoch 3/10
307/307 [============== ] - 11s 35ms/step - loss: 0.3582 - accuracy: 0.868
9 - val loss: 0.3632 - val accuracy: 0.8643
Epoch 4/10
0 - val loss: 0.3910 - val accuracy: 0.8556
Epoch 5/10
8 - val loss: 0.3466 - val accuracy: 0.8748
Epoch 6/10
4 - val loss: 0.3407 - val accuracy: 0.8756
Epoch 7/10
2 - val loss: 0.3357 - val accuracy: 0.8774
Epoch 8/10
307/307 [=============== ] - 8s 26ms/step - loss: 0.2648 - accuracy: 0.9014
- val loss: 0.3496 - val accuracy: 0.8712
Epoch 9/10
5 - val loss: 0.3576 - val accuracy: 0.8708
Epoch 10/10
6 - val loss: 0.3577 - val accuracy: 0.8728
In [70]:
test loss, test accuracy = model.evaluate(test images, test labels)
print(f"Test Accuracy: {test accuracy}")
print(f"Test Loss: {test loss}")
Test Accuracy: 0.8809047341346741
Test Loss: 0.35619986057281494
-Above we can see that the test dataset is used to test the above trained model and we get a accuaray of
88.09% and a loss of 0.35 on the test dataset. -Below is a graph plot of the train set loss and Test set loss. As we
can see the train test keeps decreasing weheras the test loss first decreases fast and after some epochs it
continues to decrease at a very slow rate or even increase sometimes.
In [71]:
plt.plot(history.history['loss'], label='Train Loss')
plt.plot(history.history['val loss'], label='Validation Loss')
plt.xlabel('Epochs')
plt.ylabel('Loss')
plt.legend()
plt.show()

    Train Loss

  0.60
                                      Validation Loss
  0.55
```

0.50



In [75]:

```
sample_indices = np.random.choice(test_images.shape[0], 3)
sample_images = test_images[sample_indices]
sample_labels = test_labels[sample_indices]
```

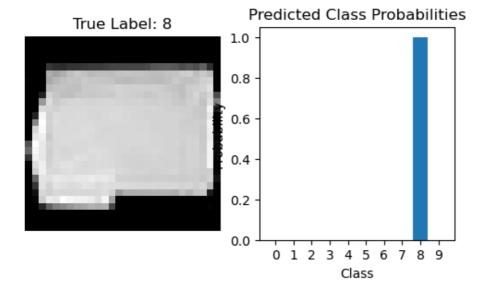
Below we can see 3 images, their actual class and predicted class. as we can see each image has proababilities for each class and the highest class is labeled as the image class for the prediction. This is done using softmax activation.

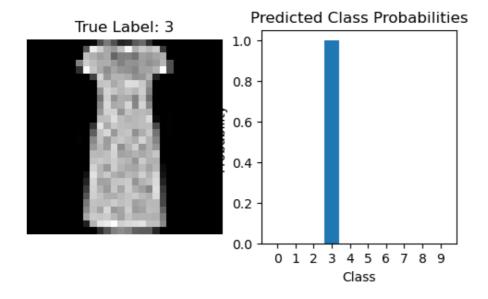
In [76]:

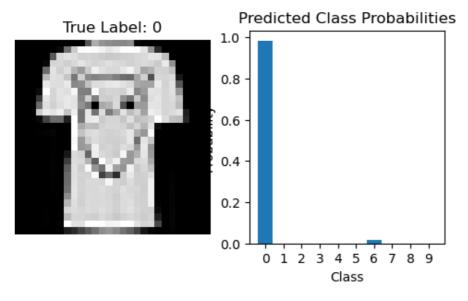
In [77]:

```
for i in range(3):
    plt.figure(figsize=(6, 3))
    plt.subplot(1, 2, 1)
    plt.imshow(sample_images[i], cmap='gray')
    plt.title(f"True Label: {sample_labels[i]}")
    plt.axis('off')

    plt.subplot(1, 2, 2)
    plt.bar(range(10), predictions[i])
    plt.title("Predicted Class Probabilities")
    plt.xlabel("Class")
    plt.ylabel("Probability")
    plt.xticks(range(10), [str(i) for i in range(10)])
    plt.show()
```







In []:

In this problem we will train a neural network from scratch using numpy. In practice, you will never need to do this (you'd just use TensorFlow or PyTorch). But hopefully this will give us a sense of what's happening under the hood.

For training/testing, we will use the standard MNIST benchmark consisting of images of handwritten images.

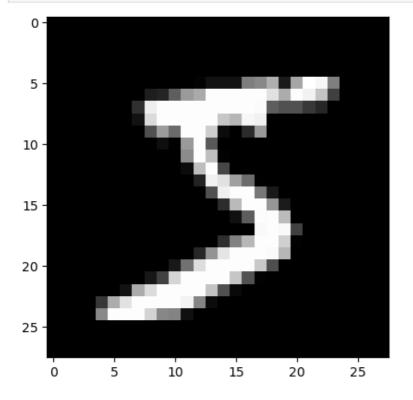
In the second demo, we worked with autodiff. Autodiff enables us to implicitly store how to calculate the gradient when we call backward. We implemented some basic operations (addition, multiplication, power, and ReLU). In this homework problem, you will implement backprop for more complicated operations directly. Instead of using autodiff, you will manually compute the gradient of the loss function for each parameter.

In [217]:

```
import tensorflow as tf
import matplotlib.pyplot as plt

(x_train, y_train), (x_test, y_test) = tf.keras.datasets.mnist.load_data(path="mnist.npz")

plt.imshow(x_train[0],cmap='gray');
```



Loading MNIST is the only place where we will use TensorFlow; the rest of the code will be pure numpy.

Let us now set up a few helper functions. We will use sigmoid activations for neurons, the softmax activation for the last layer, and the cross entropy loss.

In [218]:

```
import numpy as np

def sigmoid(x):
    # Numerically stable sigmoid function based on
    # http://timvieira.github.io/blog/post/2014/02/11/exp-normalize-trick/

x = np.clip(x, -500, 500) # We get an overflow warning without this

return np.where(
    x >= 0,
    1 / (1 + np.exp(-x)),
    np.exp(x) / (1 + np.exp(x))
```

```
def dsigmoid(x): # Derivative of sigmoid
  return sigmoid(x) * (1 - sigmoid(x))
def softmax(x):
  # Numerically stable softmax based on (same source as sigmoid)
  # http://timvieira.github.io/blog/post/2014/02/11/exp-normalize-trick/
 b = x.max()
  y = np.exp(x - b)
 return y / y.sum()
def cross entropy loss(y, yHat):
  return -np.sum(y * np.log(yHat))
def integer to one hot(x, max):
  # x: integer to convert to one hot encoding
  # max: the size of the one hot encoded array
  result = np.zeros(10)
  result[x] = 1
  return result
```

OK, we are now ready to build and train our model. The input is an image of size 28x28, and the output is one of 10 classes. So, first:

Q1. Initialize a 2-hidden layer neural network with 32 neurons in each hidden layer, i.e., your layer sizes should be:

```
784 -> 32 -> 32 -> 10
```

If the layer is n_{in} your layer weights should be initialized by sampling from a normal distribution with mean $\times n_{out}$

zero and variance $1/\max(n_{in},...)$

 n_{out})

In the Below Code we initalize weights and biases for the Neural Netowork: -We first define the layer size for each layer -The weights are intialized such that the initalization is from a normal distribution with zero mean and variance of 1

```
/max(n_{in}, n_{out})
```

- The size of each weight matrix is (size of input layer, size of output layer) for example: weight matrix which connects input to 1st hidden layer: (32, 784)
- The size of each bias matrix is (size of the output layer, 1) for example: bias matrix which connects input to 1st hidden layer: (32, 1)

```
In [219]:
```

```
import math

# Initialize weights of each layer with a normal distribution of mean 0 and
# standard deviation 1/sqrt(n), where n is the number of inputs.
# This means the weighted input will be a random variable itself with mean
# 0 and standard deviation close to 1 (if biases are initialized as 0, standard
# deviation will be exactly 1)

from numpy.random import default_rng

rng = default_rng(80085)
# Define the sizes of the layers in the neural network
layer_sizes = [784, 32, 32, 10]

# Initialize weights and biases for each layer
weights = [] # List to store weight matrices for each layer
biases = [] # List to store bias vectors for each layer
```

```
# Loop through each layer (except the output layer)
for i in range(len(layer sizes) - 1):
   # Initialize weights
   weight shape = (layer sizes[i+1], layer sizes[i]) # Define the shape of the weight
matrix
   weight variance = 1.0 / np.sqrt(max(weight shape)) # He initialization for weights
    weight values = np.random.normal(loc=0.0, scale=weight variance, size=weight shape)
    weights.append(weight values) # Add the weight matrix to the list of weights
    # Initialize biases
    bias shape = (layer sizes[i+1], 1) # Define the shape of the bias vector
    bias values = np.zeros(bias shape) # Initialize biases to zero
    biases.append(bias values) # Add the bias vector to the list of biases
# Display the initialized weights and biases
for i, (w, b) in enumerate(zip(weights, biases), 1):
    print(f"Layer {i}:")
    print(f"Weights shape: {w.shape}") # Display the shape of the weight matrix
    print(f"Biases shape: {b.shape}") # Display the shape of the bias vector
    print()
for i in range(len(weights)):
    print(f"Layer: {i + 1}")
    print(weights[i].shape)
Layer 1:
Weights shape: (32, 784)
Biases shape: (32, 1)
Layer 2:
Weights shape: (32, 32)
Biases shape: (32, 1)
Layer 3:
Weights shape: (10, 32)
Biases shape: (10, 1)
Layer: 1
(32, 784)
Layer: 2
(32, 32)
```

Next, we will set up the forward pass. We will implement this by looping over the layers and successively computing the activations of each layer.

Q2. Implement the forward pass for a single sample, and for the entire dataset.

Right now, your network weights should be random, so doing a forward pass with the data should not give you any meaningful information. Therefore, in the last line, when you calculate test accuracy, it should be somewhere around 1/10 (i.e., a random guess).

Below function of one_hot_softmax is a function which takes the ouptput given by the neural network as the input and outputs an array of 0s and 1 which has the max probability in the original input to the function. example: input: [0.2, 0.6, 0.2, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0] output: [0, 1, 0, 0, 0, 0, 0, 0, 0, 0]

```
In [220]:
```

Layer: 3 (10, 32)

```
def one_hot_softmax(softmax_output):
    # Create one-hot encoding
    max_indices = np.argmax(softmax_output)
    one_hot_output = np.zeros_like(softmax_output)
    one_hot_output[max_indices] = 1
    return one_hot_output
```

The feed_forward_sample: This function takes a sample image and its actual label. It flattens the image into a

array and inputs it into the neural netowork. The for loop iterates over all the layers of the network and compute values on each layer using previously defined weights and bias and use sigmoid activation function for all layers accept last layer. The last layer has the activation function of softmax. Finally we calcualte the loss as cross entropy loss and we return the prediction for the given array

Feed Forward Dataset function: a for loop is run over all example and each example is send to the feed forward sample function with its actual label

```
In [221]:
def feed forward sample(sample, y):
  """ Forward pass through the neural network.
     sample: 1D numpy array. The input sample (an MNIST digit).
      label: An integer from 0 to 9.
    Returns: the cross entropy loss, most likely class
     # Reshape the input sample into a column vector
  sample = np.array(sample.flatten()).reshape(784, 1)
  a = {1: sample} # Initialize the activation dictionary with the input sample
  z = {} # Initialize the dictionary to store the weighted inputs
     # Forward pass through the network
  for l in range(1, len(weights)):
     node in = a[1]
      # Compute the weighted inputs for each layer and apply the activation function
      z[1 + 1] = weights[1 - 1].dot(node in) + biases[1 - 1]
      a[1 + 1] = sigmoid(z[1 + 1])
 # Output layer calculations
  z[4] = weights[2].dot(a[3]) + biases[2]
  a[4] = softmax(z[4])
# Convert the true label 'y' into a one-hot encoded vector
  y = integer_to_one_hot(y, 10)
    # Compute the cross-entropy loss between the predicted and true labels
  loss = cross entropy loss(y, a[4].flatten())
    # Extract the output of the softmax layer (most likely class probabilities)
  one_hot_guess = a[4].flatten()
    # Convert the softmax output into a one-hot representation
  one_hot_output = one_hot_softmax(one hot guess)
  return loss, one hot output
def feed forward dataset(x, y):
 losses = np.empty(x.shape[0])
  one_hot_guesses = np.empty((x.shape[0], 10))
  for i in range(len(x)):
      losses[i] , one hot quesses[i] = feed forward sample(x[i] , y[i])
  y one hot = np.zeros((y.size, 10))
  y one hot[np.arange(y.size), y] = 1
 correct guesses = np.sum(y one hot * one hot guesses)
  correct guess percent = format((correct guesses / y.shape[0]) * 100, ".2f")
  print("\nAverage loss:", np.round(np.average(losses), decimals=2))
  print("Accuracy (# of correct guesses):", correct_guesses, "/", y.shape[0], "(", corre
ct_guess_percent, "%)")
def feed_forward_training_data():
 print("Feeding forward all training data...")
  feed forward dataset(x train, y train)
  print("")
def feed forward test data():
 print("Feeding forward all test data...")
  feed forward dataset(x_test, y_test)
  print("")
feed forward test data()
```

```
Feeding forward all test data...

Average loss: 2.55

Accuracy (# of correct guesses): 966.0 / 10000 ( 9.66 %)
```

OK, now we will implement the backward pass using backpropagation. We will keep it simple and just do training sample-by-sample (no minibatching, no randomness).

Q3: Compute the gradient of all the weights and biases by backpropagating derivatives all the way from the output to the first layer.

init_tri_values function: This function is used to initlaize the change is weights and biases. THe initialization is array of zeros and same size as previously initalized weights and biases.

calculate_out_layer_delta function: This function caluclate the delta for the output layer. Delta can aslo be said as the diffenriation of y with respect to z.

calculate_hidden_delta: This function calculate the delta for hidden layer. This delta depends on the delta from the next layer, weight and z value of the current layer.

```
In [222]:
```

```
def init_tri_values(nn_structure):
    tri_W = {}
    tri_b = {}
    for 1 in range(0, len(nn_structure) - 1):
        tri_W[1] = np.zeros((nn_structure[l+1], nn_structure[l]))
        tri_b[1] = np.zeros((nn_structure[l+1], 1))
    return tri_W, tri_b

def calculate_out_layer_delta(y, a_out, z_out):
    # delta^(n1) = -(y_i - a_i^(n1)) * f'(z_i^(n1))
    return - (y-a_out) * dsigmoid(z_out)

def calculate_hidden_delta(delta_plus_1, w_l, z_l):
    # delta^(l) = (transpose(W^(l)) * delta^(l+1)) * f'(z^(l))
    return w_l.T.dot(delta_plus_1) * dsigmoid(z_l)
```

Below code is a algorithm of backpropogation using gradient descent. we first calcualte the forward pass prediction and based on the prediction and actual label, we calcualte output layer delta and loop it backwards to get all the change in weights and biases and deltas for each layer. After the loop we update the weights and biases.

```
In [223]:
```

```
def train_one_sample(sample, y, learning_rate=0.003):
   # Flatten the input sample and reshape it into a column vector
 sample = np.array(sample.flatten()).reshape(784, 1)
 a = {1: sample} # Initialize the activation dictionary with the input sample
 z = \{\} # Initialize the dictionary to store the weighted inputs
 delta = \{\}
 for 1 in range(1, len(weights)):
     node in = a[1]
      # Calculate the weighted inputs for each layer and apply the activation function
     z[l+1] = weights[l-1].dot(node in) + biases[l-1]
     a[1 + 1] = sigmoid(z[1 + 1])
    # Output layer calculations
 z[4] = weights[2].dot(a[3]) + biases[2]
 a[4] = softmax(z[4])
  # Convert the true label 'y' into a one-hot encoded vector
 y = (integer_to_one_hot(y, 10)).reshape(10,1)
 loss = cross entropy loss(y, a[4]) # Compute the cross-entropy loss between the pre
dicted and true labels
 yHat = a[4]
```

```
# Backward pass
 tri W, tri b = init tri values(layer sizes) # Initialize the gradients for weights and
biases
 for 1 in range(len(layer sizes), 0, -1): # Iterate through the layers in reverse order
     if l == len(layer sizes):
           # Calculate the error at the output layer
          delta[l] = calculate out layer delta(y, a[l], z[l])
      else:
          if 1 > 1:
               # Calculate the error for hidden layers
              delta[1] = calculate \ hidden \ delta(delta[1 + 1], \ weights[1 - 1], \ z[1])
               # Accumulate gradients for weights and biases
          tri W[l - 1] += delta[l+1].dot(a[l].T)
          tri b[l - 1] += delta[l + 1]
 # Update weights and biases using gradient descent
 for 1 in range(len(weights), 1 , -1):
      weights[l - 1] += -learning\_rate * (tri <math>W[l - 1])
      biases[l - 1] += -learning_rate * (tri_b[l - 1])
 return weights, biases
```

Finally, train for 3 epochs by looping over the entire training dataset 3 times.

Q4. Train your model for 3 epochs.

Below code calls the above train example code to train over all training example and get a near optimal weights and biases values which are then test on the test dataset and this porcess continues for 3 epochs.

```
In [224]:
def train one epoch(learning rate=0.003):
 print ("Training for one epoch over the training dataset...")
  # Loop over the entire training dataset and train each sample
  # Q4. Write the training loop over the epoch here.
 for j in range(len(x train)):
    train_one_sample(x_train[j], y_train[j])
 print("Finished training.\n")
# Function to perform forward pass on the test dataset
feed_forward_test_data()
# Function to perform testing and training
def test_and_train():
   # Train the model for one epoch
  train one epoch()
   # Perform forward pass on the test data
  feed forward test data()
# Perform testing and training for a certain number of epochs
for i in range(3):
  test_and train()
Feeding forward all test data...
Average loss: 2.55
Accuracy (# of correct guesses): 966.0 / 10000 ( 9.66 %)
***********
Training for one epoch over the training dataset...
Finished training.
Feeding forward all test data...
Average loss: 1.79
Accuracy (# of correct guesses): 5378.0 / 10000 ( 53.78 %)
```

As Seen above the loss decreases after each epoch and the correct predciton increases after each epoch whihch signifies that our neural network is working properly

That's it!

Your code is probably very time- and memory-inefficient; that's ok. There is a ton of optimization under the hood in professional deep learning frameworks which we won't get into.

If everything is working well, you should be able to raise the accuracy from $\sim 10\%$ to $\sim 70\%$ accuracy after 3 epochs.

Attributes & Values for Satyrium spini (Spine Hairstreak):

- Name: Satyrium spini (Spine Hairstreak)
- Wingspan: 1.5 to 2 inches (3.8 to 5.1 cm)
- Color: Dark brown or black with blue and orange markings
- Underside of Wings: Mottled brown with orange and white markings
- Distinctive Features: Spine-like projection at the end of the hind wing
- Habitat: Forests, fields, and gardens
- Nectar Sources: Variety of flowers
- Attracted to: Bright, open areas