The development of the world reflected in the history of the world

Algebra

1:Abel transformation

$A_k = \sum_{i=1}^{k} a_i \quad\quad B_k = \sum_{i=1}^{k} b_i$

$a_k = A_k - A_{k-1} \quad\quad b_k = B_k - B_{k-1}$

$$\begin{aligned} \Rightarrow \sum_{i=1}^{k} a_i b_i &= a_1 b_1 + a_2 b_2 + \ldots + a_k b_k \\ &= a_1(B_1) + a_2(B_2 - B_1) + \ldots a_k(B_k - B_{k-1}) \\ &= B_1(a_1 - a_2) + \ldots B_{k-1}(a_{k-1} - a_k) + B_k a_k \\ &= -\sum_{i=1}^{k-1} B_i(a_{i+1} - a_i) + a_k B_k \end{aligned}$$

Similarly:$\sum_{i=1}^{k} a_i b_i = a_1 b_1 + a_2 b_2 + \ldots + a_k b_k = -\sum_{i=1}^{k-1} A_i(b_{i+1} - b_i) + b_k A_k$

$$\Rightarrow \sum_{i=1}^{k-1} [A_i(b_{i+1} - b_i) - B_i(a_{i+1} - a_i)] = A_k b_k - B_k a_k$$

$$\sum_{i=1}^{k-1} [A_i b_{i+1} - B_i a_{i+1} - (A_i b_i - B_i a_i)] = A_k b_k - B_k a_k$$

$$\sum_{i=1}^{k-1} \begin{vmatrix} A_i & B_i \\ a_{i+1} & b_{i+1} \end{vmatrix} - \sum_{i=1}^{k-1} \begin{vmatrix} A_i & B_i \\ a_i & b_i \end{vmatrix} = \begin{vmatrix} A_k & B_k \\ a_k & b_k \end{vmatrix}$$

$def: \Delta(a_{i+1}) = a_{i+1} - a_i \quad\quad \Delta(b_{i+1}) = b_{i+1} - b_i$

$\Rightarrow \Delta(A_i) = A_i - A_{i-1} = a_i \quad\quad \Delta(B_i) = B_i - B_{i-1} = b_i$

$\sum_{i=1}^{k} a_i \Delta(B_i) = b_k A_k - \sum_{i=1}^{k-1} A_i \Delta(b_{i+1})$

$Let\ \ u = \Delta(B_i) = b_i$, $v = A_i$,then $\Delta(v) = a_i$,$\Delta(b_{i+1}) = \Delta(u)$

$$\Rightarrow \sum_{i=1}^{k} u\Delta(v) = uv - \sum_{i=1}^{k-1} v\Delta(u)$$

$$\simeq \int u\mathrm{d}v = uv - \int v\mathrm{d}u$$

Analysis

Geometry