

Homework 7

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1 Question 1

April wants to use a discrete-time PID controller to control her robot. She selects 200 for the set point, sets all three gain parameters to 2, and samples the state once per second (so that $\Delta t = 1$). When April executes this controller, the first three states are 205, 204, and 198. Compute the output of the controller after this third sensor reading. Show your work and mark your final answer clearly.

1.1 Solution

From the problem statement, we know the following:

1. $K_p = K_i = K_d = 2$
2. $\Delta t = 1$
3. $s = 200$
4. $x_1 = 205, x_2 = 204, x_3 = 198$

The discrete PID controller formula is given by:

$$u_k = K_p e_k + K_i \left(\Delta t \sum_{i=1}^k e_i \right) + K_d \left(\frac{e_k - e_{k-1}}{\Delta t} \right) \quad (1)$$

The error term can be calculated using:

$$e_k = x_k - s \quad (2)$$

By substituting the known values into these equations, we can compute each term at $k = 3$.

$$e_1 = 205 - 200 = 5, \quad e_2 = 204 - 200 = 4, \quad e_3 = 198 - 200 = -2 \quad (3)$$

- Proportional term: $K_p \times e_3 = 2 \times (-2) = -4$
- Integral term: $K_i \times (\Delta t \sum_{i=1}^k e_i) = 2 \times (1 \times (5 + 4 - 2)) = 14$
- Derivative term: $K_d \times \left(\frac{e_k - e_{k-1}}{\Delta t} \right) = 2 \times \left(\frac{-2 - 4}{1} \right) = -12$

Therefore, we can obtain the output from the controller:

$$u_3 = -4 + 14 - 12 = -2 \quad (4)$$