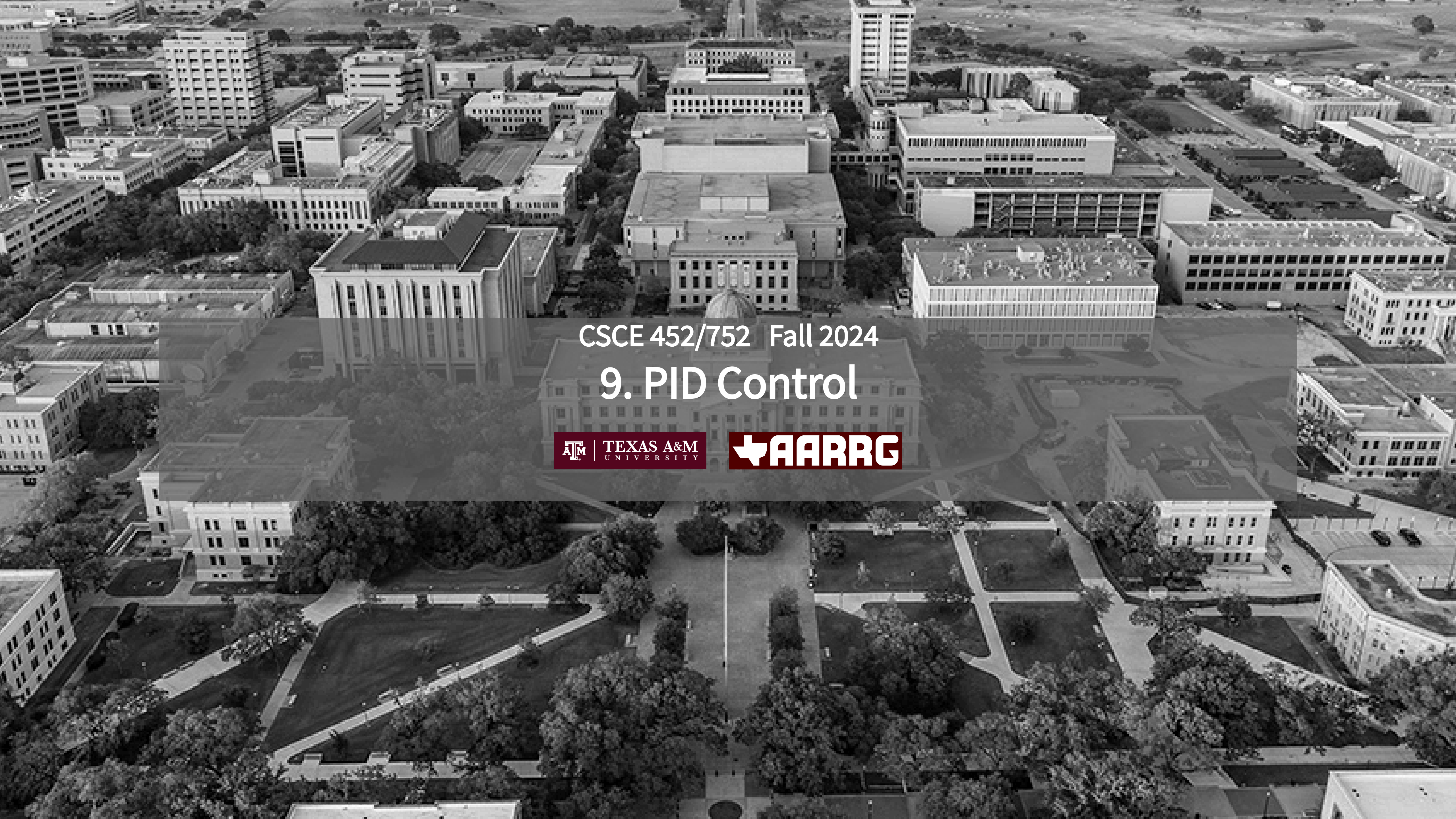


CSCE 452/752 Fall 2024

9. PID Control

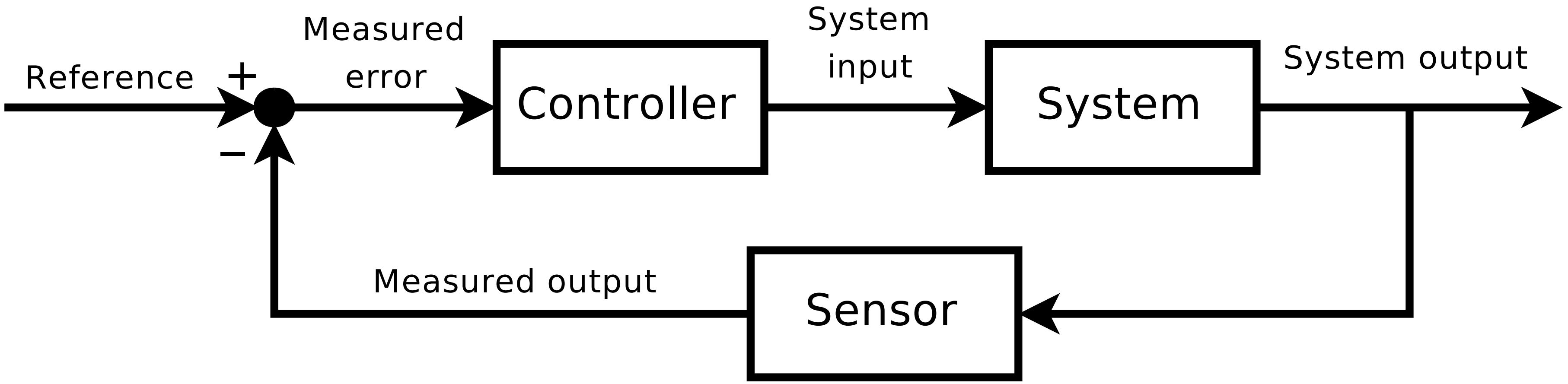
TEXAS A&M UNIVERSITY

AARRG

Introduction

Definition

Control theory refers to methods for regulating the behavior of dynamical systems.



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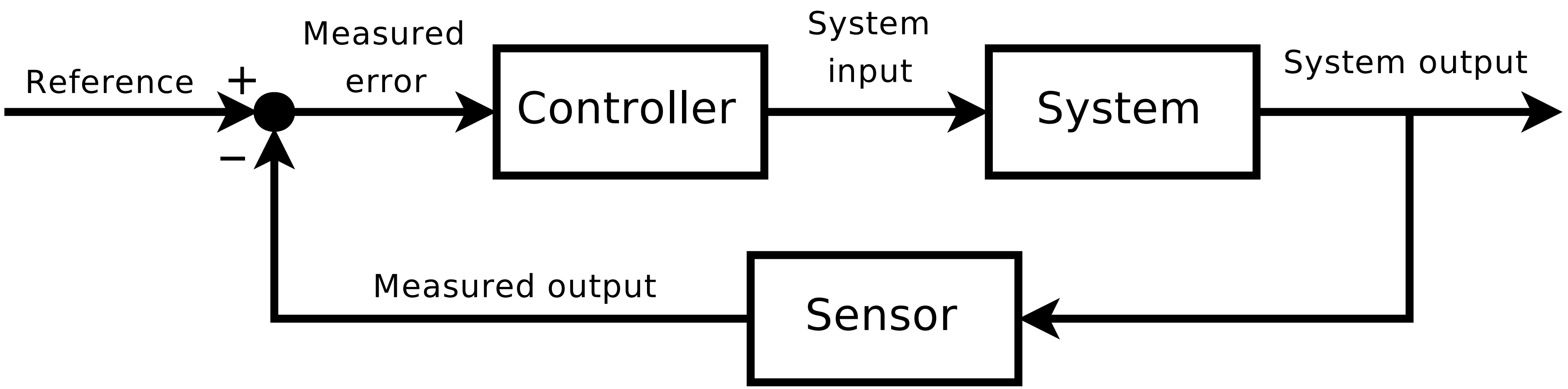

We could do a whole ~~course~~ minor on control.

Notation

Notation:

- X: state space
- $x(t)$: state at time t
- U: input space / action space / control space
- $u(t)$: input / action / control at time t

The **system model** describes how the state changes:

$$\dot{x} = f(x, u)$$

Example

A differential drive robot rotating in place:

- $X = [0, 2\pi)$ (all orientations)
- $x(t)$ (orientation at time t)
- $U = \mathbb{R}$ (all possible angular velocities)
- $u(t)$ (angular velocity at time t)
- $\dot{x} = u$

Goal

The goal is to **stabilize** the state to a **set point**.

The **error** is the difference between the state and the set point:

$$e(t) = x(t) - s$$

We want to choose actions that drive the state to the set point and keep it there.

Is this easy?

In the rotation example, suppose:

- Starting state: $x(0) = \pi/2$
- Set point: $s = 0$
- System model: $\dot{x} = u$

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Result:

After $t = 2$, the system stays at the set point $x(t) = 0$.

Does this work?

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In our example:

- acceleration
- wheel slip
- miscalibration of the motors
- mismeasurement of the wheels
- uneven floor
- small delays executing the command
- ...

Feedback

Feedback controllers can respond to states that we didn't expect to reach.

Basic idea: Actions depend on states:

$$u : X \to U$$

Back to our example

In the rotation example, suppose:

- Starting state: $x(0) = \pi/2$
- Set point: $s = 0$
- System model: $\dot{x} = u$

Choose an **closed loop control law**:

$$u(x) = -x$$

Result:

Result: From **any** state, the system moves toward the set point.

PID control

One simple and commonly used type of feedback controller is the **proportional-integral-derivative (PID) controller**:

$$u(t) = K_p e(t) + K_i \int_0^t e(\tau)\mathrm{d}\tau + K_d \frac{\mathrm{d}e}{\mathrm{d}t}$$

This consists of:

- A **proportional term** to account for present errors.
- An **integral term** to account for past errors.
- An **derivative term** to account for future errors.
- A **gain** parameter for each term to calibrate the behavior.

Various special cases, such as P or PD controllers, can be formed by setting one or more of the gains to 0.

Discrete time PID

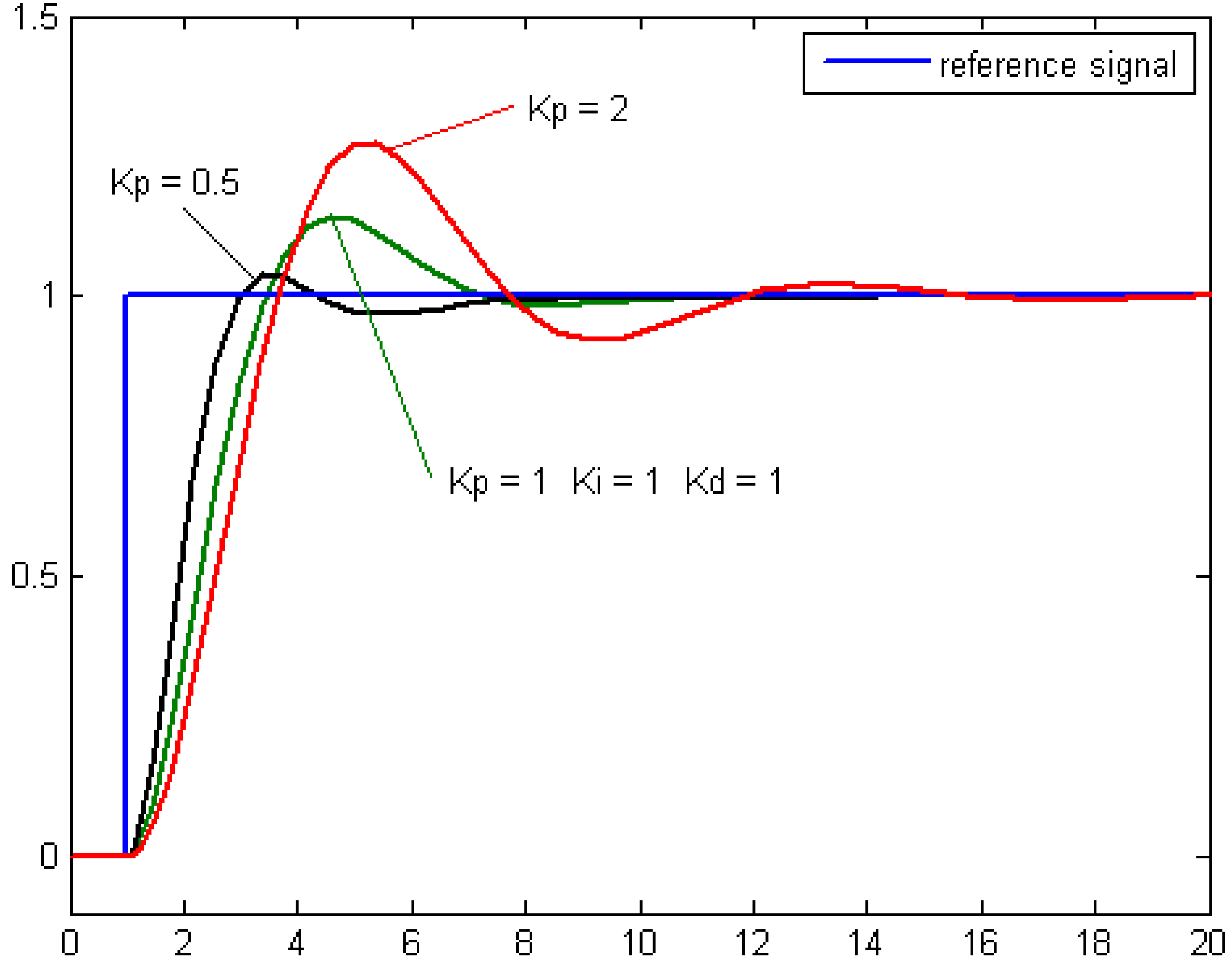
A **discrete-time system** is one in which we can only measure the state at fixed intervals.

In that case, we can use discrete approximations:

$$u_k = K_p e_k + K_i \left(\Delta t \sum_{i=1}^{k} e_i \right) + K_d \left(\frac{e_k - e_{k-1}}{\Delta t} \right)$$

PID Tuning

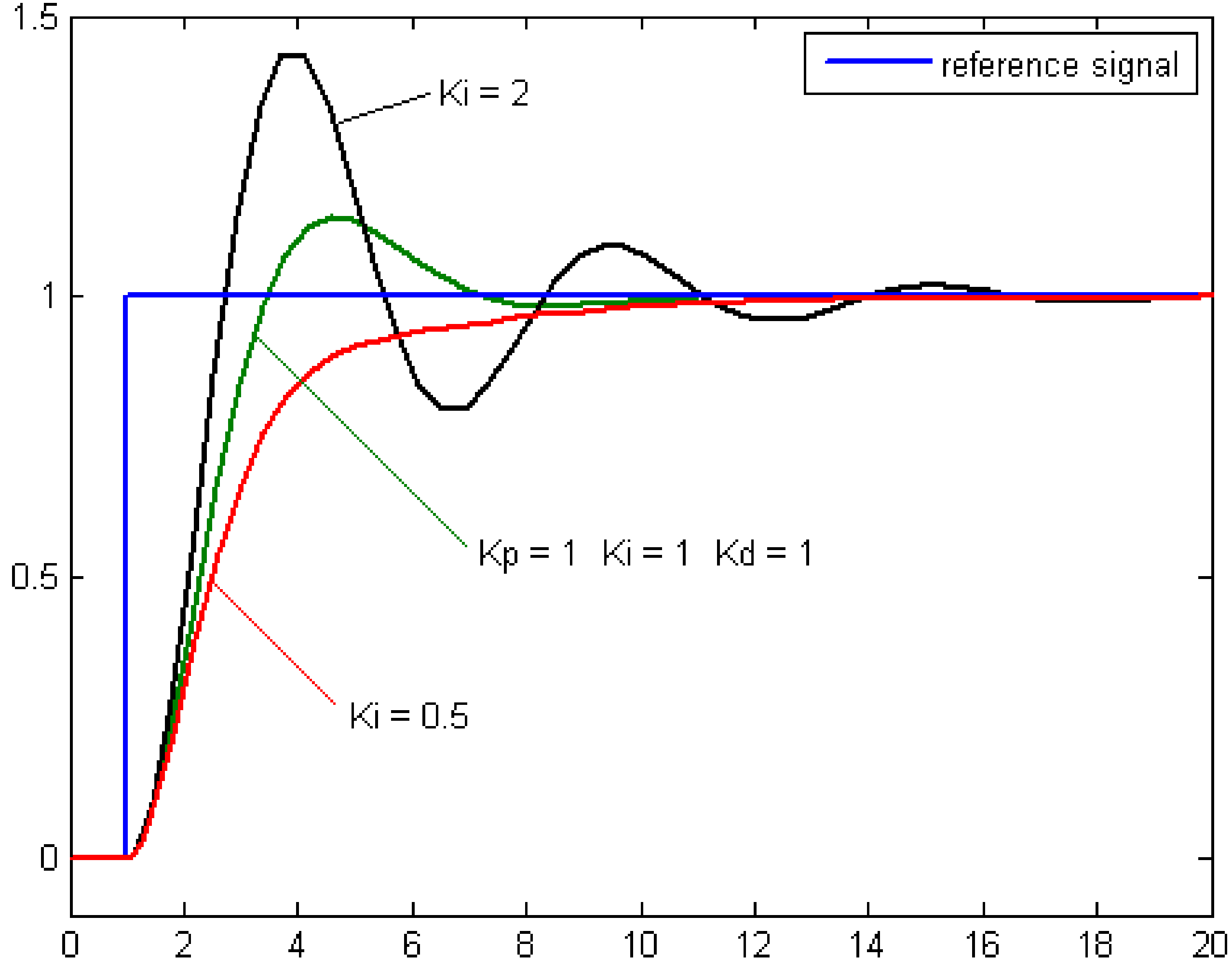
Changes to K_p:



[en.wikipedia.org]

PID Tuning

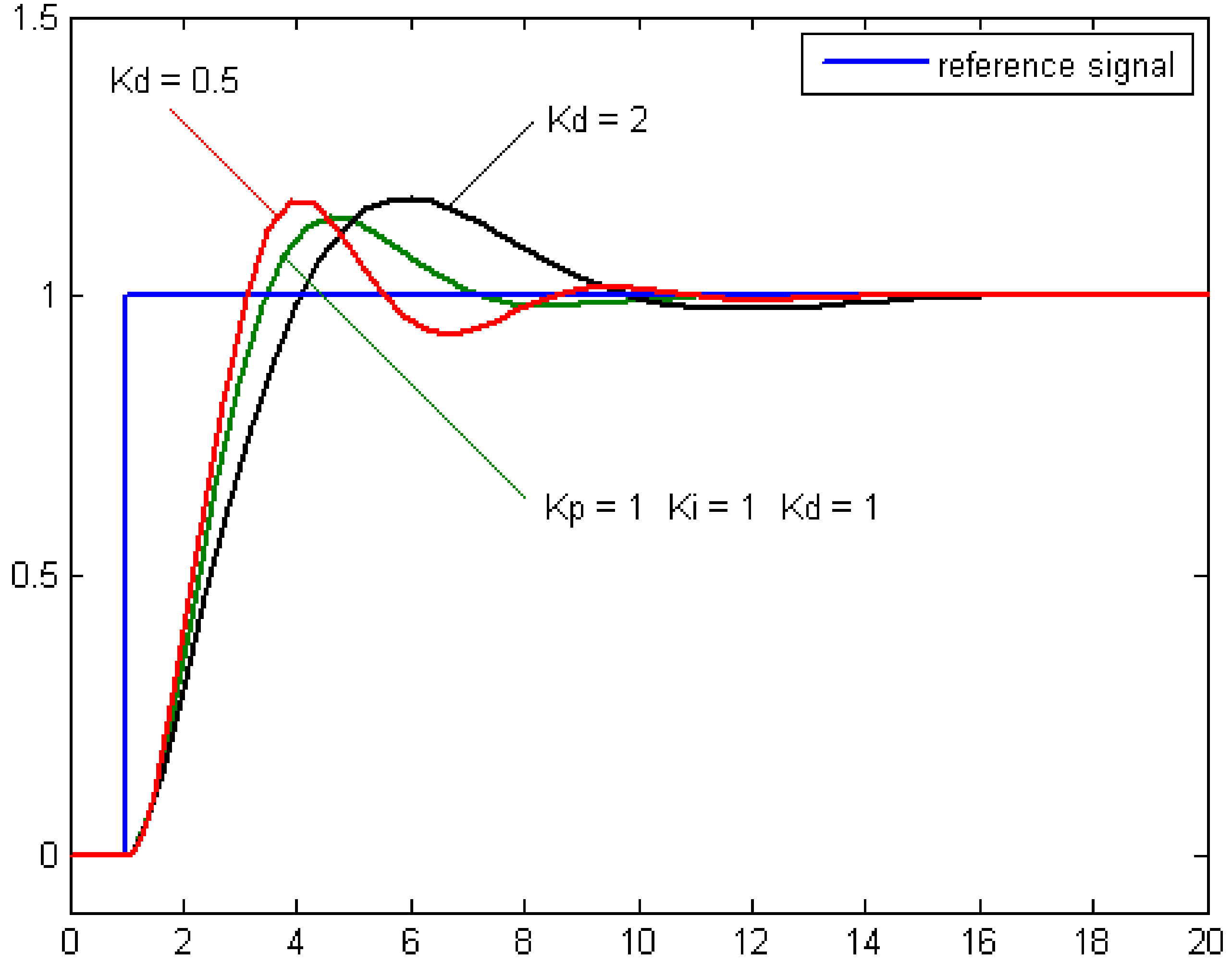
Changes to K_i:



[en.wikipedia.org]

PID Tuning

Changes to K_d:



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