Homework 3

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1 Question 1

You are controlling a very small differential drive robot whose wheels are 4cm apart.

The robot starts at $(x_s, y_s, \theta_s) = (5cm, 5cm, 0)$ and wants to travel to $(x_g, y_g, \theta_g) = (10cm, 10cm, \pi/2)$

The velocity of each wheel must remain between $-2cm/s$ and $2cm/s$.

Give a sequence of one or more actions, each of the form $(v_l, v_r, \Delta t)$, to move the robot from its starting point to its goal.

1.1 Solution

For the differential drive robot whose wheels are 4cm apart to travel from $(x_s, y_s, \theta_s) = (5cm, 5cm, 0)$ to $(x_g, y_g, \theta_g) = (10cm, 10cm, \pi/2)$, only 3 actions are required:

1. Move forward along the x-axis from $(5cm, 5cm)$ to $(10cm, 5cm)$:
   - Since the robot should move in a straight line along the positive direction, $v_l = v_r = 2cm/s$.
   - The distance to travel is
     $$d = 10cm - 5cm = 5cm$$
     , with a velocity of $2cm/s$, it requires
     $$\Delta t = \frac{d}{v} = 2.5s$$
     to reach $(10cm, 5cm)$.
   - Action $u_1 : (v_l, v_r, \Delta t) = (2cm/s, 2cm/s, 2.5s)$
2. Rotate counter-clockwise from $\theta = 0$ to $\theta = \pi/2$
   - Since we want the robot to rotate in place, the distance (R) from the center of the robot to ICC is 0. According to the definition of angular velocity:
     $$\omega = \frac{v_r}{R + \frac{l}{2}} = \frac{v_l}{R - \frac{l}{2}}$$
     , where l is the distance between the wheels. If $R = 0$, then we can obtain $v_l = -v_r$. Furthermore, since we want the robot to rotate counter-clockwise, the wheels' velocity must be set to $(v_l, v_r) = (-2, 2)$ respectively.
   - To calculate the angular velocity, we have:
     $$\omega = \frac{v_r - v_l}{l} = \frac{2 - (-2)}{4} = 1 \text{ rad/s}$$

- Time to rotate by $\Delta\theta = \pi/2$:

$$\Delta t = \frac{\Delta\theta}{\omega} = \frac{\pi/2}{1 \text{ rad/s}} = \pi/2 \text{ s}$$

- Action $u_2 : (v_l, v_r, \Delta t) = (-2cm/s, 2cm/s, \pi/2s)$

3. Move forward along the y-axis from $(10cm, 5cm)$ to $(10cm, 10cm)$:
   - Similar to action u_1, for the robot to travel in a straight line along the positive y direction, $v_l = v_r = 2cm/s$.
   - Since the distance d is also $5cm$, the time required $\Delta t = 2.5s$.
   - Action $u_3 : (v_l, v_r, \Delta t) = (2cm/s, 2cm/s, 2.5s)$