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UNIVERSIDAD AUTÓNOMA DE NUEVO LEÓN

Universidad Autonoma de Nuevo León

Facultad de Ingenieria Mecánica y Eléctrica

Modelado y simulación de sistemas

Generación de variables aleatorias no-uniformes

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Generación de variables aleatorias no-uniformes

Metodo de transformada inversa

Distribución exponencial.

$$x = -\frac{1}{\lambda} \ln R$$

donde =

x = variable aleatoria

λ = media estadistica de la variable aleatoria

Ln = logaritmo natural

R = numero rectangular previamente aprobado.

Distribución uniforme

$$x = a + (b-a) R$$

donde:

a = valor minimo

b = valor maximo

Distribución empírica

$$x = \sqrt{2R} \quad \text{si } R \leq 1/2$$

$$x = 2R \quad \text{si } R > 1/2$$

Distribución poisson

$$f(x) = \frac{e^{-\lambda} \lambda^x}{x!}$$

Distribución exponencial

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Ejemplo:

rectangulares : 0.76971 , 0.82534 , 0.63249 , 0.11073

3 minutos y llegan 4 clientes.

T_A - tiempo de atención a cada cliente

$T_{A1} = -\frac{1}{3} Ln(0.76971) = 0.087247152$ min

$T_{A2} = -\frac{1}{3} Ln(0.82534) = 0.063986618$ min

$T_{A3} = -\frac{1}{3} Ln(0.63249) = 0.152696956$ min

$T_{A4} = -\frac{1}{3} Ln(0.11073) = 0.733553491$ min

T_{TC} (tiempo total del cajero) = $T_{A1} + T_{A2} + T_{A3} + T_{A4}$

$T_{TC} = 1.037448427$ min

T_{PA} (tiempo promedio de atención) = $\frac{T_{TC}}{\text{No. de clientes}}$

$T_{PA} = 0.259361054$ min

Distribución uniforme 15 10 25

Ejemplo

$x = a + (b-a)R$

Entre 2 y 4 minutos para 4 refrescos

0.76971, 0.82534, 0.63249, 0.11073

$a = 2$, $b = 4$, T_E - tiempo de envasamiento

$T_{E1} = 2 + (4-2)(0.76971) = 3.53942$ min

$T_{E2} = 2 + (4-2)(0.82534) = 3.65068$ min

$T_{E3} = 2 + (4-2)(0.63249) = 3.26498$ min

$T_{E4} = 2 + (4-2)(0.11073) = 2.22146$ min

Distribución empírica 17 / 10 / 25

Transmitir 4 mensajes de texto

Rectangulares: 0.76971, 0.82534, 0.63249, 0.11073

$T_1 = 2(0.76971) = 1.53942$ min

$T_2 = 2(0.82534) = 1.65068$ min

$T_3 = 2(0.63249) = 1.26498$ min

$T_4 = \sqrt{2(0.11073)} = 0.470895367$ min

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Metodo de transformada inversa

$\lambda = 5$ min para Distribución exponencial

Tiempo de refresco entre 5 y 8 minutos

Resolver por Distribución empírica

a)	b)	c)
0.1331	0.9959	0.3621
0.2389	0.3091	0.3830
0.2054	0.3014	0.4015
0.1700	0.2944	0.4177
0.2703	0.2649	0.4319
0.3508	0.6699	0.0359
0.3264	0.9299	0.0753
0.2994	0.8294	0.1141
0.3729	0.7348	0.1517
0.4099	0.3185	0.1879

Distribución Exponencial

a) $\lambda = 5$ min

$T_1 = -\frac{1}{5} \ln(0.1331) = 0.403309$ min

$T_2 = -\frac{1}{5} \ln(0.2389) = 0.286342$ min

$T_3 = -\frac{1}{5} \ln(0.2054) = 0.316559$ min

$T_4 = -\frac{1}{5} \ln(0.1700) = 0.354391$ min

$T_5 = -\frac{1}{5} \ln(0.2703) = 0.261644$ min

$T_6 = -\frac{1}{5} \ln(0.3508) = 0.209507$ min

$T_7 = -\frac{1}{5} \ln(0.3264) = 0.223926$ min

$T_8 = -\frac{1}{5} \ln(0.2094) = 0.241194$ min

$T_9 = -\frac{1}{5} \ln(0.3729) = 0.197288$ min

$T_{10} = -\frac{1}{5} \ln(0.4099) = 0.178368$ min

$T_{Total} = 0.403369 + 0.286342 + 0.316559 + 0.354391 + 0.261644$
$0.209507 + 0.223926 + 0.241194 + 0.197288 + 0.178368$

$T_{Total} = 2.672528$ min

$T_{promedio} = \frac{2.672528}{10} = 0.2672528$ min

Distribución Exponencial

b) $\lambda = 5 \text{ min}$

$T_1 = -\frac{1}{5} \ln(0.9959) = 0.00082168 \text{ min}$

$T_2 = -\frac{1}{5} \ln(0.3091) = 0.234818 \text{ min}$

$T_3 = -\frac{1}{5} \ln(0.3014) = 0.239863 \text{ min}$

$T_4 = -\frac{1}{5} \ln(0.2944) = 0.244563 \text{ min}$

$T_5 = -\frac{1}{5} \ln(0.2649) = 0.265680 \text{ min}$

$T_6 = -\frac{1}{5} \ln(0.6699) = 0.080125 \text{ min}$

$T_7 = -\frac{1}{5} \ln(0.9299) = 0.014535 \text{ min}$

$T_8 = -\frac{1}{5} \ln(0.8294) = 0.037410 \text{ min}$

$T_9 = -\frac{1}{5} \ln(0.7348) = 0.061631 \text{ min}$

$T_{10} = -\frac{1}{5} \ln(0.3185) = 0.228826 \text{ min}$

$T_{Total} = 0.00082168 + 0.234818 + 0.239863 + 0.244563 + 0.265680 + 0.080125 + 0.014535 + 0.037410 + 0.061631 + 0.228826$

$T_{Total} = 1.408272 \text{ min}$

$T_{Promedio} = \frac{1.408272}{10} = 0.1408272 \text{ min}$

Distribución Exponencial

c) $\lambda = 5 min$

$T_1 = -\frac{1}{5} \ln(0.3621) = 0.203166 \text{ min}$

$T_2 = -\frac{1}{5} \ln(0.3830) = 0.191944 \text{ min}$

$T_3 = -\frac{1}{5} \ln(0.4015) = 0.185095 \text{ min}$

$T_4 = -\frac{1}{5} \ln(0.4177) = 0.174598 \text{ min}$

$T_5 = -\frac{1}{5} \ln(0.4319) = 0.167912 \text{ min}$

$T_6 = -\frac{1}{5} \ln(0.0359) = 0.665403 \text{ min}$

$T_7 = -\frac{1}{5} \ln(0.0753) = 0.517255 \text{ min}$

$T_8 = -\frac{1}{5} \ln(0.1141) = 0.434136 \text{ min}$

$T_9 = -\frac{1}{5} \ln(0.1517) = 0.377190 \text{ min}$

$T_{10} = -\frac{1}{5} \ln(0.1879) = 0.334369 \text{ min}$

$T_{Total} = 0.203166 + 0.191944 + 0.185095 + 0.174598 + 0.167912 + 0.665403 + 0.517255 + 0.434136 + 0.377190 + 0.334369.$

$T_{Total} = 3.251048 \text{ min}$

$T_{Promedio} = \frac{3.251048}{10} = 0.3251048 \text{ min}$

Distribución Uniforme

a) Tiempo de refresco entre 5 y 8 minutos

$x = a + (b-a)R$, $a=5$ y $b=8$

$T_1 = 5+(8-5)(0.1331) = 5.3993$ min

$T_2 = 5+(8-5)(0.2389) = 5.7167$ min

$T_3 = 5+(8-5)(0.2054) = 5.6162$ min

$T_4 = 5+(8-5)(0.1700) = 5.51$ min

$T_5 = 5+(8-5)(0.2703) = 5.8109$ min

$T_6 = 5+(8-5)(0.3508) = 6.0524$ min

$T_7 = 5+(8-5)(0.3264) = 5.9792$ min

$T_8 = 5+(8-5)(0.2994) = 5.8982$ min

$T_9 = 5+(8-5)(0.3729) = 6.1187$ min

$T_{10} = 5+(8-5)(0.4099) = 6.2297$ min

b) Tiempo de refresco entre 5 y 8 minutos

$x = a + (b-a)R$, $a=5$ y $b=8$

$T_1 = 5+(8-5)(0.9959) = 7.9877$ min

$T_2 = 5+(8-5)(0.3091) = 5.9273$ min

$T_3 = 5+(8-5)(0.3014) = 5.9042$ min

$T_4 = 5+(8-5)(0.2944) = 5.8832$ min

$T_5 = 5+(8-5)(0.2649) = 5.7947$ min

$T_6 = 5+(8-5)(0.6699) = 7.0097$ min

$T_7 = 5+(8-5)(0.9299) = 7.7897$ min

$T_8 = 5+(8-5)(0.8294) = 7.4882$ min

$T_9 = 5+(8-5)(0.7348) = 7.2044$ min

$T_{10} = 5+(8-5)(0.3185) = 5.9555$ min

Distribución Uniforme

c) Tiempo de refresco entre 5 y 8 minutos.

$x = a + (b-a)R$, $a=5$ y $b=8$.

$T_1 = 5 + (8-5)(0.3621) = 6.0863$ min

$T_2 = 5 + (8-5)(0.3830) = 6.149$ min

$T_3 = 5 + (8-5)(0.4015) = 6.2045$ min

$T_4 = 5 + (8-5)(0.4177) = 6.2531$ min

$T_5 = 5 + (8-5)(0.4319) = 6.2957$ min

$T_6 = 5 + (8-5)(0.0359) = 5.1077$ min

$T_7 = 5 + (8-5)(0.0753) = 5.2259$ min

$T_8 = 5 + (8-5)(0.1141) = 5.3423$ min

$T_9 = 5 + (8-5)(0.1517) = 5.4551$ min

$T_{10} = 5 + (8-5)(0.1879) = 5.5637$ min

Distribución Empírica

a)

$T_1 = \sqrt{2(0.1331)} = 0.515945$ min

$T_2 = \sqrt{2(0.2389)} = 0.691230$ min

$T_3 = \sqrt{2(0.2054)} = 0.640936$ min

$T_4 = \sqrt{2(0.1700)} = 0.583095$ min

$T_5 = \sqrt{2(0.2703)} = 0.735255$ min

$T_6 = \sqrt{2(0.3508)} = 0.837615$ min

$T_7 = \sqrt{2(0.3264)} = 0.807960$ min

$T_8 = \sqrt{2(0.2994)} = 0.773821$ min

$T_9 = \sqrt{2(0.3729)} = 0.863597$ min

$T_{10} = \sqrt{2(0.4099)} = 0.905428$ min

$T_{Total} = 0.515945 + 0.691230 + 0.640936 + 0.583095 + 0.735255 + 0.837615 + 0.807960 + 0.773821 + 0.863597 + 0.905428$

$T_{Total} = 7.354882$ min $\quad T_{Promedio} = \frac{7.354882}{10} = 0.7354882$ min

b)

$T_1 = 2(0.9959) = 1.9918$ min

$T_2 = \sqrt{2(0.3091)} = 0.786256$ min

$T_3 = \sqrt{2(0.3014)} = 0.776401$ min

$T_4 = \sqrt{2(0.2944)} = 0.767333$ min

$T_5 = \sqrt{2(0.2649)} = 0.727873$ min

$T_6 = 2(0.6699) = 1.3398$ min

$T_7 = 2(0.9299) = 1.8598$ min

$T_8 = 2(0.8294) = 1.6588$ min

$T_9 = 2(0.7348) = 1.4696$ min

$T_{10} = \sqrt{2(0.3185)} = 0.798122$ min

$T_{Total} = 1.9918 + 0.786256 + 0.776401 + 0.767383 + 0.727873 + 1.3398 + 1.8598 + 1.6588 + 1.4696 + 0.798122$

$T_{Total} = 12.175785$ min $\quad T_{Promedio} = \frac{12.175785}{10} = 1.2175785$ min

Distribución Empírica

c)

$T_1 = \sqrt{2(0.3621)} = 0.850999$ min

$T_2 = \sqrt{2(0.3830)} = 0.875214$ min

$T_3 = \sqrt{2(0.4015)} = 0.896102$ min

$T_4 = \sqrt{2(0.4179)} = 0.914002$ min

$T_5 = \sqrt{2(0.4319)} = 0.929408$ min

$T_6 = \sqrt{2(0.0359)} = 0.267955$ min

$T_7 = \sqrt{2(0.0753)} = 0.388072$ min

$T_8 = \sqrt{2(0.1141)} = 0.477702$ min

$T_9 = \sqrt{2(0.1517)} = 0.550817$ min

$T_{10} = \sqrt{2(0.1879)} = 0.613025$ min

$T_{Total} = 0.850999 + 0.875214 + 0.896102 + 0.914002 + 0.929408 + 0.267955 + 0.388072 + 0.477702 + 0.550817 + 0.613025$

$T_{Total} = 5.833888$ min

$T_{Promedio} = \frac{5.833888}{10} = 0.5833888$ min

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Distribución Poisson

$$f(x) = \frac{e^{-\lambda} \lambda^x}{x!}$$

Ejemplo: Una semana de trabajo de 5 a 7 días, donde $\lambda = 5$

X	f(x)	F(x)
0	$e^{-5} 5^0/0! = 0.006737946999$	$0 + 0.006737946999 = 0.006737946999$
1	$e^{-5} 5^1/1! = 0.033689735$	$0.006737946999 + 0.033689735 = 0.040427682$
2	$e^{-5} 5^2/2! = 0.084224337$	$0.040427682 + 0.084224337 = 0.124652019$
3	$e^{-5} 5^3/3! = 0.140373895$	$0.124652019 + 0.140373895 = 0.265025914$
4	$e^{-5} 5^4/4! = 0.175467369$	$0.265025914 + 0.175467369 = 0.440493283$
5	$e^{-5} 5^5/5! = 0.175467369$	$0.440493283 + 0.175467369 = 0.615960652$
6	$e^{-5} 5^6/6! = 0.146222808$	$0.615960652 + 0.146222808 = 0.76218346$
7	$e^{-5} 5^7/7! = 0.104444863$	$0.76218346 + 0.104444863 = 0.866628323$
8	$e^{-5} 5^8/8! = 0.065278039$	$0.866628323 + 0.065278039 = 0.931906362$
9	$e^{-5} 5^9/9! = 0.036265577$	$0.931906362 + 0.036265577 = 0.968171939$
10	$e^{-5} 5^{10}/10! = 0.018132788$	$0.968171939 + 0.018132788 = 0.986304727$
11	$e^{-5} 5^{11}/11! = 0.008242176 6$	$0.986304727 + 0.008242176 6 = 0.994546903$
12	$e^{-5} 5^{12}/12! = 0.003434240 2$	$0.994546903 + 0.003434240 2 = 0.997981143$
13	$e^{-5} 5^{13}/13! = 0.001320861 6$	$0.997981143 + 0.001320861 6 = 0.999302004$
14	$e^{-5} 5^{14}/14! = 0.000471736$	$0.999302004 + 0.000471736 = 0.999773374$
15	$e^{-5} 5^{15}/15! = 0.000157245 4$	$0.999773374 + 0.000157245 4 = 0.999930985$

Distribución de Poisson

Ejemplo: Trabajan de 4 a 7 días, donde $\lambda = 4$

x	$f(x)$	$F(x)$
0	$e^{-4}\ 4^0/0! = 0.018315638$	0.018315638
1	$e^{-4}\ 4^1/1! = 0.073262555$	$0.018315638 + 0.073262555 = 0.091578193$
2	$e^{-4}\ 4^2/2! = 0.146525111$	$0.091578193 + 0.146525111 = 0.238103304$
3	$e^{-4}\ 4^3/3! = 0.195366814$	$0.238103304 + 0.195366814 = 0.433470118$
4	$e^{-4}\ 4^4/4! = 0.195366814$	$0.433470118 + 0.195366814 = 0.628836932$
5	$e^{-4}\ 4^5/5! = 0.156293451$	$0.628836932 + 0.156293451 = 0.785130383$
6	$e^{-4}\ 4^6/6! = 0.104195634$	$0.785130383 + 0.104195634 = 0.889326017$
7	$e^{-4}\ 4^7/7! = 0.059540362$	$0.889326017 + 0.059540362 = 0.948866379$
8	$e^{-4}\ 4^8/8! = 0.029770181$	$0.948866379 + 0.029770181 = 0.978636560$
9	$e^{-4}\ 4^9/9! = 0.013231191$	$0.978636560 + 0.013231191 = 0.991867751$
10	$e^{-4}\ 4^{10}/10! = 0.005292476$	$0.991867751 + 0.005292476 = 0.997160227$
11	$e^{-4}\ 4^{11}/11! = 0.001924536$	$0.997160227 + 0.001924536 = 0.999084763$
12	$e^{-4}\ 4^{12}/12! = 0.000641512$	$0.999084763 + 0.000641512 = 0.999726275$
13	$e^{-4}\ 4^{13}/13! = 0.000197388$	$0.999726275 + 0.000197388 = 0.999923663$