Control Theory Assignment 5

Alina Bogdanova BS18-04

April 2020

1 Introduction

This homework is related to the parameter estimation task: both static and dynamic. The system, that will be considered is a two link manipulator.

2 Tasks solution

2.1 Formulas

Task: choose physically adequate values for kinematic (l_1 , l_2) and dynamic parameters (l_{c1} , l_{c2} , m_1 , m_2)

Solution

As known, from the course of physics:

$I_i = \frac{1}{3} m_i l_{ci}^2$ - inertia moment

Let's find matrices M, C and g from the given source:

Given:

$$d_{11}\ddot{q_1} + d_{12}\ddot{q_2} + c_{121}\dot{q_1}\dot{q_2} + c_{211}\dot{q_2}\dot{q_1} + c_{221}\dot{q_2}^2 + \phi_1 = \tau_1$$
$$d_{21}\ddot{q_1} + d_{22}\ddot{q_2} + c_{112}\dot{q_1}^2 + \phi_2 = \tau_2$$

Can be written as:

$$\begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix} \ddot{q} + \begin{bmatrix} c_{121}\dot{q_2} & c_{211}\dot{q_1} \\ c_{112}\dot{q_1} & 0 \end{bmatrix} \dot{q} + \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} = \tau$$

where

$$d_{11} = m_1 l_{c1}^2 + m_2(l_1^2 + l_{c2}^2 + 2l_1 l_{c2}^2 + 2l_1 l_{c2} cos(q_2)) + \frac{1}{3} m_1 l_{c1}^2 + \frac{1}{3} m_2 l_{c2}^2$$

$$d_{12} = d_{21} = m_2(l_{c2}^2 + l_1 l_{c2} cos(q_2)) + \frac{1}{3} m_2 l_{c2}^2$$

$$d_{22} = \frac{4}{3} m_2 l_{c2}^2$$

thus,

$$\boxed{M(q) = \begin{bmatrix} m_2l_1(l_1+2l_{c2}^2+2l_{c2}cos(q_2)) + \frac{4}{3}m_1l_{c1}^2 + \frac{4}{3}m_2l_{c2}^2 & m_2l_1l_{c2}cos(q_2) + \frac{4}{3}m_2l_{c2}^2 \\ m_2l_1l_{c2}cos(q_2) + \frac{4}{3}m_2l_{c2}^2 & \frac{4}{3}m_2l_{c2}^2 \end{bmatrix}}$$

It is known, that

$$c_{121} = c_{211} = -m_2l_1l_{c2}sin(q_2) = -c_{112} = h$$

So,

$$\boxed{C(q,\dot{q}) = \begin{bmatrix} h\dot{q}_2 & h(\dot{q}_2+\dot{q}_1) \\ -h\dot{q}_1 & 0 \end{bmatrix}}$$

Using, the fact that

$$\phi_1 = (m_1l_{c1} + m_2l_1)gcos(q_1) + m_2l_{c2}gcos(q_1+q_2)$$

$$\phi_2 = m_2l_{c2}cos(q_1+q_2)$$

We can conclude that

$$\boxed{g(q) = \begin{bmatrix} (m_1l_{c1} + m_2l_1)gcos(q_1) + m_2l_{c2}gcos(q_1+q_2) \\ m_2l_{c2}cos(q_1+q_2) \end{bmatrix}}$$

Also it s possible to find that:

$$Y(q,\dot{q},\ddot{q}) = \begin{bmatrix} \ddot{q}_2 & cos(q_2)(2\ddot{q}_1+\ddot{q}_2) + sin(q_2)(\dot{q}_1{}^2 - 2\dot{q}_1\dot{q}_2) & \ddot{q}_2 & gcos(q_1) & gcos(q_1+q_2) \\ 0 & cos(q_2)\ddot{q}_1 + sin(q_2)\dot{q}_1{}^2 & \ddot{q}_2 & 0 & gcos(q_1+q_2) \end{bmatrix}$$

and

$$\boxed{\Theta = \pi = \begin{bmatrix} m_2l_1^2 + \frac{4}{3}m_1l_{c1}^2 + \frac{4}{3}m_2l_{c2}^2 \\ m_2l_1l_{c2} \\ m_2l_1l_{c2} \\ m_1l_{c1} + m_2l_1 \\ m_2l_2 \end{bmatrix}}$$

Criteria of choosing parameters:

$$0 < l_{ci} < l_i$$

2.2 Simulation

Task: Using expressions for $M(q)$, $C(q,\dot{q})$ and $g(q)$ from referenced material simulate the system;

Solution

Simulation was done with initial parameters:

$l_1 = 0.5$, $l_2 = 0.5$, $l_{c1} = 0.25$, $l_{c2} = 0.25$, $m_1 = 0.1$ and $m_2 = 0.2$
The simulation itself is simply a solution of the differential equation:

$$\begin{bmatrix} \ddot{q} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} M^{-1}(q)(\tau - g(q) - C(q, \dot{q}))\dot{q} \\ \dot{q} \end{bmatrix}$$

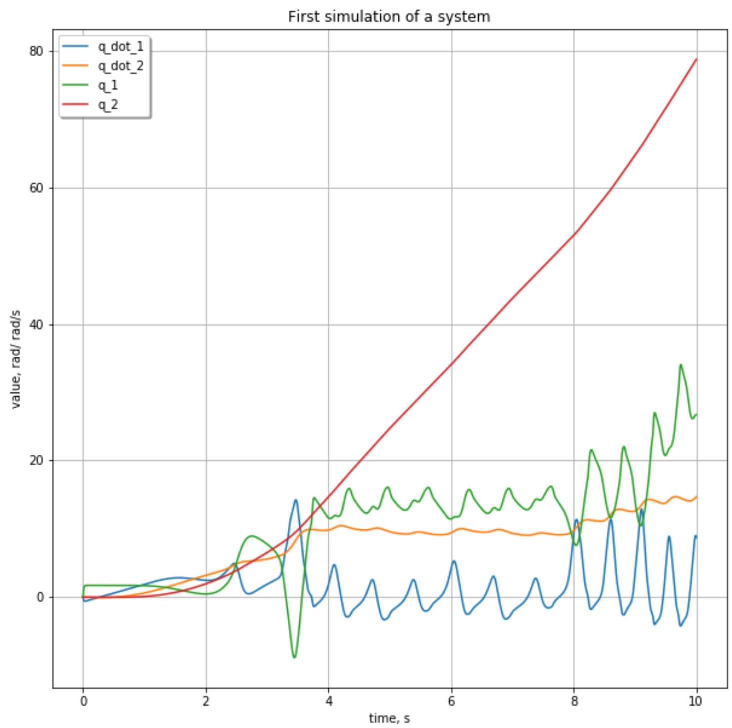

Figure 1: Result of the simulation

At the figure 1, it is possible to see, that the system diverges(q_2) and oscillates(q_1).

2.3 PD controller and parameter estimation

Task: perform parameter estimation of the robot:

1. design PD controller for each joint of the robot $\tau = K_p(q^* - q) + K_d(\dot{q}^* - \dot{q})$;

2. make the robot to track $q^*(t) = \begin{bmatrix} cos(t) & cos(t) \end{bmatrix}^T$ for 5 seconds, record q(t) and $\dot{q}(t)$;

3. using expression for $Y(q, \dot{q}, \ddot{q})$ from [1] estimate parameters π by means least squares;

4. for different trajectory perform validation of estimated parameters $\hat{\pi}$

Solution:

The coefficients for the controller were selected as a constants: $K_p = 10$, $K_d = 1$.

During the calculations $\tau = K_p(cos(t) - q) + K_d(-sin(t) - \dot{q}))$

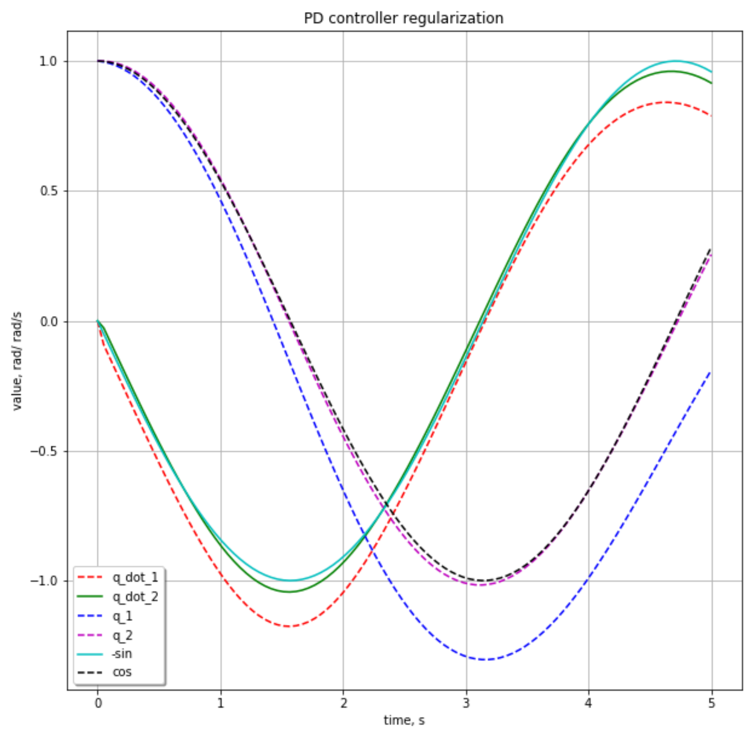

Figure 2: PD controller with variable limits

The next thing to be done is to estimate the parameters.

Using least squares parameter estimation, this list of parameters was estimated:

$\left[[0.14583311 \quad 0.02499993 \quad 0.06666665 \quad 0.15000293 \quad 0.09999943 \right]$

While the real one is

$\left[[0.14583333 \quad 0.025 \quad 0.06666667 \quad 0.0.15 \quad 0.15 \right]$

So the resulted error is extremely low, MSE metric is equal to 0.00249994

3 Conclusion

Even static parameter estimation can lead to good precision.