Control Theory - Home Work 2

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1 Introduction

Consider spring-mass-damper system and complete the following tasks:

2 Task 1

Task: design PD-controller that tracks time varying reference states i.e. $\begin{bmatrix} x & \dot{x} \end{bmatrix}^T$ as closely as possible. Test your controller on different trajectories;

First of all, what is PD control for our system?

PD control is a control in form $u = e \cdot k_p + \dot{e} \cdot k_d$, where e is an error function: $e = x^* - x$, where x^* is our desired value.

Thus out system takes form:

$$\dot{X} = AX + B \begin{bmatrix} k_d & k_p \\ 0 & 0 \end{bmatrix} X,$$

where $X = \begin{bmatrix} \dot{x} \\ x \end{bmatrix}$, $B = \begin{bmatrix} \frac{1}{m} \\ 0 \end{bmatrix}$

Let's consider different trajectories of the system.

The trajectory is defined my this set of parameters: m, b, k, x_0, $\dot{x_0}$, desired values (x^* and $\dot{x^*}$) and the controller parameters (k_p and k_d).

Let's try a few sets of parameters:

$x^* = 1$, $\dot{x^*} = 0$ - for all tests

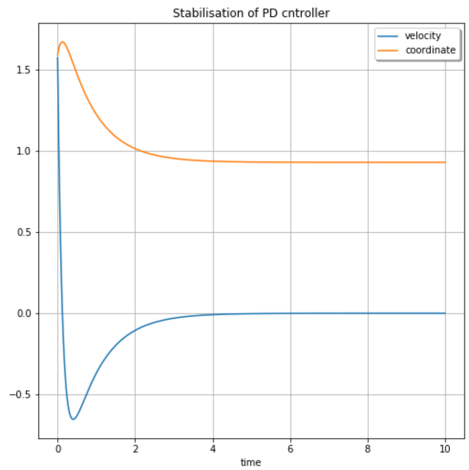

Figure 1: $m = 0.1$, $k = 0.5$, $b = 0.07$, $k_p = 0.9$, $k_d = 0.4$

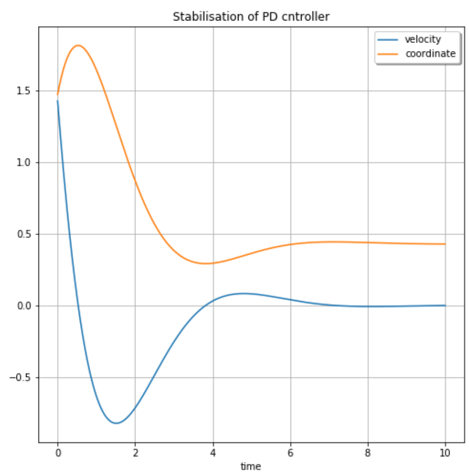

Figure 2: $m = 1$, $k = 0.8$, $b = 0.6$, $k_p = 0.6$, $k_d = 0.8$

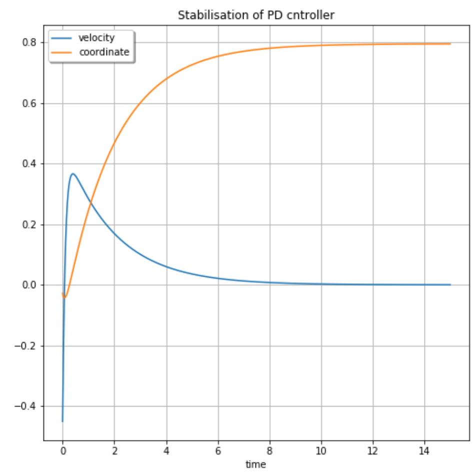

Figure 3: $m = 0.1$, $k = 0.1$, $b = 0.2$, $k_p = 0.4$, $k_d = 0.8$, $x_0 < 0$

At the figure 1 the smooth stopping around the desired values can be seen, at the figure 2 there are a smooth oscillations and the coordinate is quite far from the desired value, while at the graph 3 there is a growing without oscillations according to the coordinate and some oscillations of the velocity.

3 Task 2 and 3

Task 2: find k_p and k_d such that there are no oscillations and no overshootings in the system. Prove it on step input signal;
Task 3: prove that spring-mass-damper system is stable with PD-controller for k_p and k_d of your choice.

First of all, lets use some parameters from the previous examples to reach graphs without oscillations and overshooting.

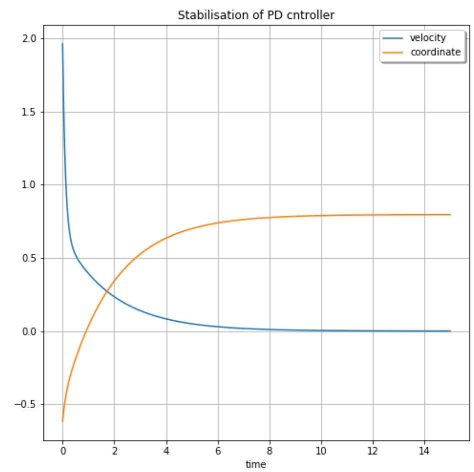

Figure 4: $m = 0.1$, $k = 0.1$, $b = 0.2$, $k_p = 0.4$, $k_d = 0.8$, $x_0 < 0$, $\dot{x^*} > 0$

The parameters on the figure 4 are the same as on graph 3, but the velocity, which is now greater than 0 initially.

Let's check if the system is stable.

The first way to deal with it is to check eigenvalues of the resulted matrix.

$$\dot{X} = AX + B\begin{bmatrix} -k_d & -k_p \\ 0 & 0 \end{bmatrix} X + \begin{bmatrix} k_p x^* + k_d \dot{x^*} \\ 0 \end{bmatrix}$$

$$\dot{X} = (A + B\begin{bmatrix} -k_d & -k_p \\ 0 & 0 \end{bmatrix})X + \begin{bmatrix} k_p x^* + k_d \dot{x^*} \\ 0 \end{bmatrix}$$

$$A = \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 10 \\ 0 \end{bmatrix}$$

Now let's try to find eigenvalues of a resulted matrix:

$$Det(\begin{bmatrix} -2 - 10k_d - \lambda & 1 - 10k_p \\ 1 & -\lambda \end{bmatrix}) = \lambda^2 + \lambda(2 + 10k_d) + 10k_p + 1 = 0$$

$$k_p = 0.4, k_d = 0.8$$

$$D = 100 - 20 = 80 < 9^2$$

$$\lambda \approx \frac{-10 \pm 9}{2} < 0$$

Lambda is less then zero, thus, the given system is stable.

Another way to check the stability of the system is Bode plot:

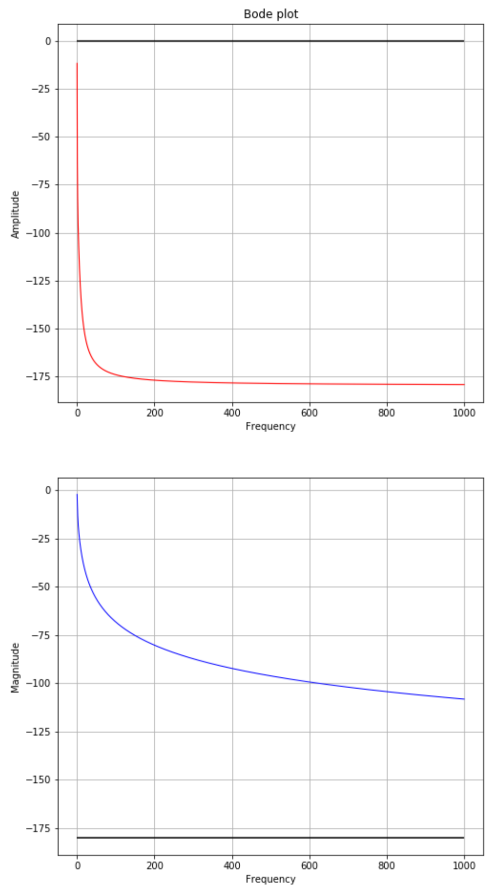

Figure 5: Bode plot of the given spring-mass-dumper system

The plot was made using the transfer function.

$$m\ddot{x} + b\dot{x} + kx = u$$

$$\ddot{x} + \frac{b}{m}\dot{x} + \frac{k}{m}x = \frac{k_d}{m}(\dot{x^*} - \dot{x}) + \frac{k_p}{m}(x^* - x)$$

$$p^2 x + px(\frac{b}{m} + \frac{k_d}{m}) + (\frac{k}{m} + \frac{k_p}{m})x = \frac{k_d}{m}\dot{x^*} + \frac{k_p}{m}x^*$$

$$x = \frac{k_d \dot{x^*} + k_p x^*}{mp^2 + p(b + k_d) + (k + k_p)}$$

4 Task 4

Task: implement PI/PID controller and compare it to PD controller. What are the differences between PI, PID and PD controller? The general form of input is $u = k_p e + k_d \dot{e} + k_i \sum e_i dt$, which is PID controller. Let's change the parameters a bit and look how will our system behave: For PID controller the parameters are: $k_i = 0.001, k_p = 20, k_d = 4$, for PI controller the parameters are the same as for PID, but $k_d = 0$, and for PD the $k_i = 0$.

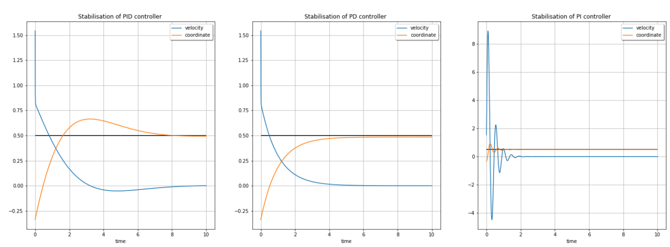

Figure 6: Different type of controllers, applied to the same initial conditions, k_i = 10, $k_p = k_d = 20$

As it can be seen, from figure 6, the D component tends to remove oscillations, while I component leads to the changes in the trajectory.

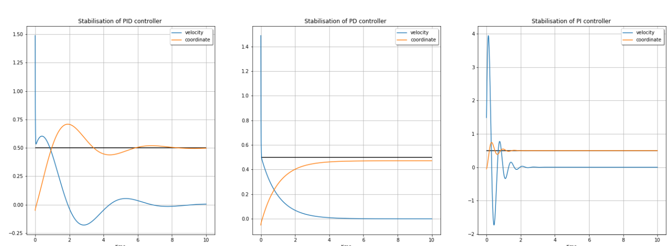

Figure 7: Different type of controllers, applied to the same initial conditions, k_i = 20, $k_p = k_d = 10$

If another set of parameters will be applied ($k_i = 20$, $k_p = k_d = 10$) (figure 7), then it can be seen, that I component also impacts on the closure of our trajectory to the desired values.

5 Conclusion

PID is a general controller, which unites P, PD, PI and a few other controllers. Each of its thee components makes some impact on the system and the only thing, that should be done is to tune the parameters.