Control Theory Homework 1

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1 Introduction

In this assignment there were tasks on the topics: state-space model, transfer function and points of equilibrium.

2 Task 1

Consider the system

$$3.2316y^{(6)} + 2.2279y^{(5)} + 2.4488y^{(4)} + 0.9344y^{(3)} + 3.9760y^{(2)} + 1.8276y = b_0$$

For $b_0 = 0$ and $b_0 = 0.7094$ answer the following questions:
- is the ODE stable? Does its solution converge or diverge?
- How can you explain it?
- How would you analyse eigenvalues of an LTV system? What does that even mean?

This system can be represented as a *state-space model*:

$$\dot{X} = \begin{bmatrix} -0.689 & -0.758 & -0.289 & -1.230 & -0. & -0.566 \\ 1. & 0. & 0. & 0. & 0. & 0. \\ 0. & 1. & 0. & 0. & 0. & 0. \\ 0. & 0. & 1. & 0. & 0. & 0. \\ 0. & 0. & 0. & 1. & 0. & 0. \\ 0. & 0. & 0. & 0. & 1. & 0. \end{bmatrix} X + \begin{bmatrix} \frac{b_0}{3.2316} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

where $X = \begin{bmatrix} y^{(5)} \\ y^{(4)} \\ y^{(3)} \\ \ddot{y} \\ \dot{y} \\ y \end{bmatrix}$

The system can be solved numerically using such tools as Python's library scipy.

The simulation of this system was made with the initial parameters in range from -1 to 1

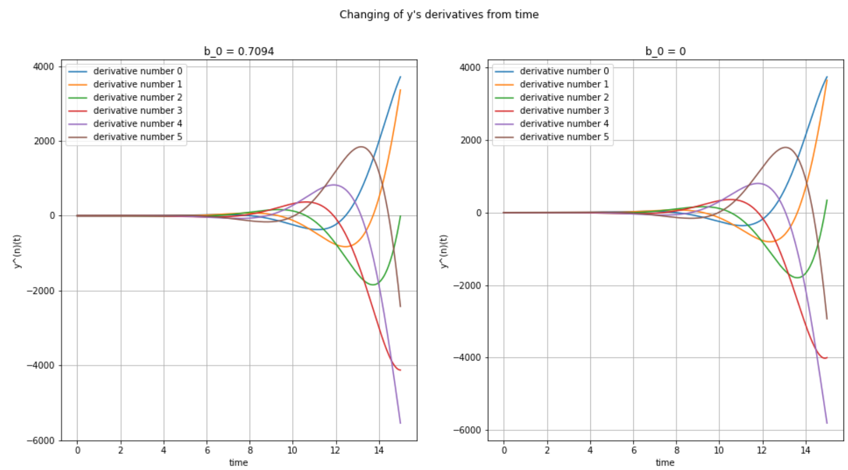

Figure 1: The simulation of a given system

On the figure 1 we can see the graphs of the given system with $b_0 = 0.7094(1)$ and $b_0 = 0$ (2). As it can be observed, in both cases $\lim_{time \to \infty} y = \infty$, thus out system diverge and thus is not stable.

In case, when $b_0 = 0$ this fact can be reached using the eigenvalues of the resulted matrix. They are:
$-0.88310149 + 0.76221959j$,
$-0.88310149 - 0.76221959j$,
$0.55078504 + 0.71738984j$,
$0.55078504 - 0.71738984j$,
$-0.01238896 + 0.7126587j$,
$-0.01238896 - 0.7126587j$.
There is a pair of eigenvalues with positive real part ($0.55078504+0.71738984j, 0.55078504-0.71738984j$), thus our system diverge.

3 Task 2

For spring-mass-damper system derive state-space model and transfer-function. Moreover, for physically consistent parameters ($m \geq 0$, $b \geq 0$, $k \geq 0$) obtain as many types of equilibrium points as you can (center, saddle point, etc.). Draw phase portrait for each case.

3.1 State-space model

The equation of spring-mass-damper system looks like:

$$m\ddot{x} + b\dot{x} + kx = F$$

In our case $F = 0$ The state-space model of the system is:

$$\begin{cases} \dot{X} = \begin{bmatrix} -b/m & -k/m \\ 1 & 0 \end{bmatrix} X, where X = \begin{bmatrix} \dot{x} \\ x \end{bmatrix} \\ y = X \end{cases}$$

3.2 Transfer function

The transfer function of the system is:

$$x = \frac{F}{mp^2 + bp + k}$$

3.3 Types of equilibrium points

There are generally 4 types of equilibrium points: node, saddle, center and focus.

- *Node points* could be obtained if both eigenvalues are real and have the same sign.
- *Saddle points* could be obtained if both eigenvalues are real, but have a different sign
- *Center points* could be obtained if both eigenvalues are purely imaginary numbers (their real pat equals to 0)
- *Focus points* could be obtained if both eigenvalues are complex and have non-zero real part

Lets consider our matrix from state-space model:

$$A = \begin{bmatrix} -b/m & -k/m \\ 1 & 0 \end{bmatrix}$$

Let $a = b/m$, $b = k/m$
Its eigenvalues could be found from the equation:

$$\lambda^2 + a\lambda + b = 0$$

Discriminant defines if our eigenvalues are complex:

$$D = a^2 - 4b$$

$$\lambda = \frac{1}{2}(-a \pm \sqrt{a^2 - 4b})$$

If our mass and coefficients are greater or equal then zero, then $a \leq \sqrt{a^2 - 4b}$, thus in case, when $D > 0$, all lambdas are less then zero (cannot have different signs). *So, the saddle equilibrium point is not reachable for this system*

Node equilibrium points Such equilibrium was reached using parameters:

$$m = 0.6, b = 0.7, k = 0.1$$

Th resulted eigenvalues are: -1. and -0.16666667

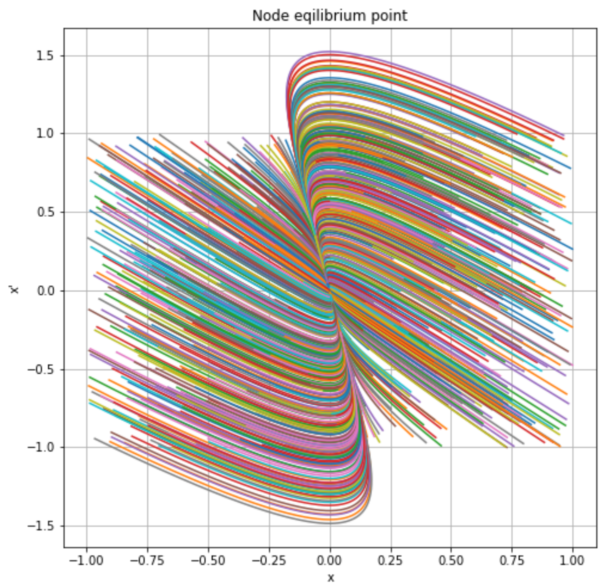

Figure 2: Node equilibrium with parameters: m = 0.6, b = 0.7, k = 0.1

As it can be seen on the 2, the graph looks like a functions that approach to some line.

Center equilibrium points Such equilibrium was reached using parameters:

$$m = 1, b = 0, k = 0.3$$

Th resulted eigenvalues are: -0.+0.54772256j and 0.-0.54772256j As it can be seen on the 3, the graph looks like a lot of circles.

Focus equilibrium points Such equilibrium was reached using parameters:

$$m = 0.6, b = 0.05, k = 0.8$$

Th resulted eigenvalues are: -0.04166667+1.15394854j and -0.04166667-1.15394854j

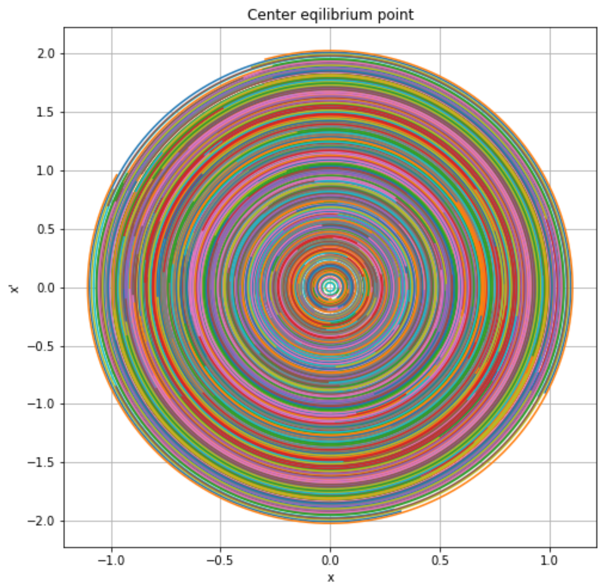

Figure 3: Center equilibrium with parameters: m = 1, b = 0, k = 0.3

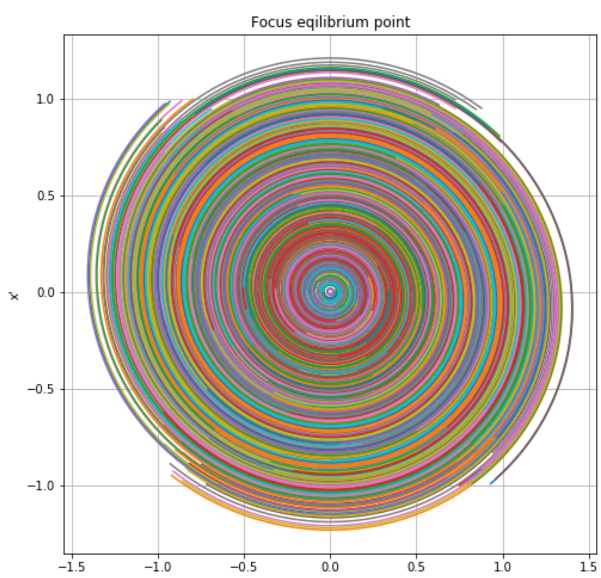

Figure 4: Focus equilibrium with parameters: m = 0.6, b = 0.05, k = 0.8

As it can be seen on the 4, the graph looks like a lot of spirals.

4 Conclusion

To sum up for spring-mass-dumper system it is impossible to reach saddle stable points.