Control Theory Homework 4

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Introduction

This task is a continuation of the previous one, dedicated to linearization of the non-linear system. Now the goal is to check the observability of the system and to simulate the observer.

The considered system could be described by the system of equations:

$$(M+m)\ddot{x} - mlcos(\theta)\ddot{\theta} + mlsin(\theta)\dot{\theta}^2 = F(1)$$

$$-cos(\theta)\ddot{x} + l\ddot{\theta} - gsin(\theta) = 0(2)$$

The system dynamics can be written in state space form:

$$\dot{z} = f(z) + g(z)u$$

$$y = h(z) = \begin{bmatrix} x & \theta \end{bmatrix}^T$$

where $z = \begin{bmatrix} x & \theta & \dot{x} & \dot{\theta} \end{bmatrix}^T$ is the state vector of the system, y is the output vector,
which is equal to

$$\dot{z} = f(z) + g(z)u = \begin{bmatrix} \dot{x} \\ \dot{\theta} \\ \ddot{x} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} \dot{x} \\ \dot{\theta} \\ \frac{msin(\theta)(gcos(\theta) - l\dot{\theta}^2)}{M+msin^2(\theta)} \\ \frac{sin(\theta)(g(M+m) - lmcos(\theta)\dot{\theta}^2)}{l(M+msin^2(\theta))} \end{bmatrix} + F \begin{bmatrix} 0 \\ 0 \\ \frac{1}{M+msin^2(\theta)} \\ \frac{cos(\theta)}{l(M+msin^2(\theta))} \end{bmatrix}$$

$$y = h(z) = \begin{bmatrix} x \\ \theta \end{bmatrix}$$

(according to the calculations for the previous homework)

The dynamics of the system around unstable equilibrium of the pendulum ($\bar{z} = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}^T$) can be described by a linear system

$$\delta\dot{z} = A\delta z + B\delta u$$

$$\delta y = C\delta z$$

,

$$A = \frac{\delta f(z)}{\delta z} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{mg}{M} & 0 & 0 \\ 0 & \frac{g(M+m)}{lM} & 0 & 0 \end{bmatrix}$$

$$B = g(\bar{z}) = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{M} \\ \frac{1}{lM} \end{bmatrix}$$

$$C = \frac{\delta h(z)}{\delta z} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

that is obtained from linearization of the nonlinear dynamics around $\bar{z}$.

Main part

Task 1

Prove that it is possible to design state observer linearized system

System is observable, if matrix $S = \begin{bmatrix} C \\ CA \\ CA^2 \\ CA^3 \end{bmatrix}$ has rank 4

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$CA = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$CA^2 = \begin{bmatrix} 0 & \frac{mg}{M} & 0 & 0 \\ 0 & \frac{g(m+M)}{lM} & 0 & 0 \end{bmatrix}$$

$$CA^3 = \begin{bmatrix} 0 & 0 & 0 & \frac{mg}{M} \\ 0 & 0 & 0 & \frac{g(m+M)}{lM} \end{bmatrix}$$

$$S = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{mg}{M} & 0 & 0 \\ 0 & \frac{g(m+M)}{lM} & 0 & 0 \\ 0 & 0 & 0 & \frac{mg}{M} \\ 0 & 0 & 0 & \frac{g(m+M)}{lM} \end{bmatrix}$$

We can clearly see the Identity matrix 4x4 at the upper part of the S matrix, thus its rank is 4

Task 2

For open loop state observer, is the error dynamics stable?

Open-loop state observer has a form: $\dot{\hat{z}} = A\hat{z} + Bu$

Error dynamics:

$\epsilon = \hat{z} - z$, $\dot{z} = Az + Bu$, $\dot{\epsilon} = A\epsilon$

Thus, open loop state observe is stable, when A is negative definite, which is not the case, because:

$$Det(\begin{bmatrix} -\lambda & 0 & 1 & 0 \\ 0 & -\lambda & 0 & 1 \\ 0 & \frac{mg}{M} & -\lambda & 0 \\ 0 & \frac{g(M+m)}{lM} & 0 & -\lambda \end{bmatrix}) = -\lambda(-\lambda^3 + \lambda(\frac{g(M+m)}{lM})) = \lambda^2(\lambda^2 - \frac{g(M+m)}{lM})$$

if $\lambda = 0$ or $\lambda = \pm\sqrt{\frac{g(M+m)}{lM}}$

Task 3

Design Luenberger observer for linearized system using both pole placement and LQR methods

Theory

Luenberger observer has form:

$$\hat{z}_{k+1} = A\hat{z_k} + Bu_k + L(y_k - \hat{y_k})$$

$$\hat{y_k} = C\hat{z_k} + Du_k$$

In the given case, $D = 0$

Pole placement method:

it is usually used in case: $A - BL \prec 0$,

now the system $A - LC \prec 0$ is given, if it is transposed: $A^T - C^T L^T \prec 0$

$$L^T = poles(A^T, C^T, eigVals)$$

LQR method (possible, because, C is a part of Identity matrix):

$$L^T = lqr(A^T, C^T, Q, R)$$

Implementation details

The physical parameters were chosen physically-reasonable: greater then zero, also this set of parameters was used for the previous homework: $g = 9.81, M = 0.2, m = 1, l = 0.1$

For pole placement algorithm this list of eigenvalues was chosen: $\begin{bmatrix} -1.1 & -1.2 & -1.3 & -1.4 \end{bmatrix}$ (it was also used in the previous homework)

For lqr method Q was set to 4x4 identity matrix and $R = \begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix}$

Task 4

Design state feedback controller for linearized system (or use the one from previous homework)

The controller was taken from the previous homework, the result of the simulation is presented on the figure 1

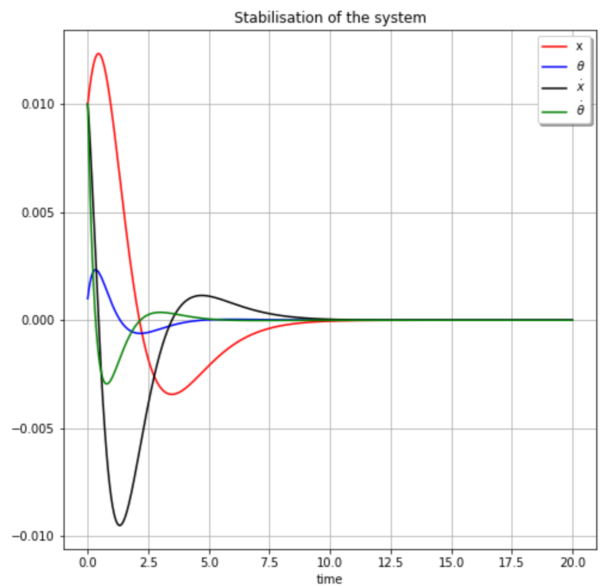

Figure 1: a nice plot

Task 5

Simulate nonlinear system with Luenberger observer and state feedback controller that uses estimated states ($u = K\hat{x}$). Make sure that the system is stabilized for various initial conditions around $\bar{z}$. For simulation use the strategy we discussed during the last lab

The strategy of simulation is such that u is fixed on short time interval (between two dots). The K parameter was evaluated using pole-placement method and the same eigenvalues as before.

The initial conditions foot the simulation:

$$x_0 = \begin{bmatrix} 0.01 & 0.001 & 0.01 & 0.01 \end{bmatrix}$$
$$\hat{x}_0 = \begin{bmatrix} 0.005 & 0.0005 & 0.005 & 0.005 \end{bmatrix}$$

The resulted simulation is displayed on the figure 2

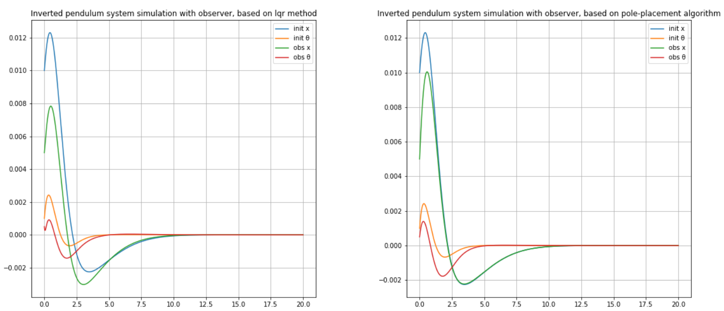

Figure 2: Simulation of the system with lqr and pole placement observer

At the figure 3 the comparison of LQR and pole placement algorithm for observer is shown.

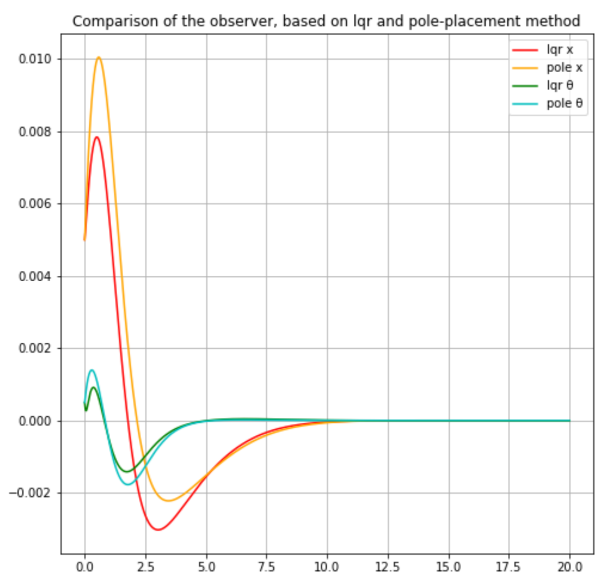

Figure 3: Comparison of LQR and pole-placement observer

It can be seen, that the systems converges approximately simultaneously, but have different amplitudes: the second algorithm has greater amplitude, then the LQR.

If the initial conditions will be changed (the second set is $x_0 = \begin{bmatrix}0.5 & 0.01, 0.1 & 0.01\end{bmatrix}$, $\hat{x}_0 = \begin{bmatrix}0.02 & 0.002 & 0.02 & 0.02\end{bmatrix}$, the third is $x_0 = \begin{bmatrix}0.5 & 0.1 & 0.03 & 0.01\end{bmatrix}$, $\hat{x}_0 = \begin{bmatrix}0.4 & 0.007 & 0.9 & 0.1\end{bmatrix}$), the system will also converge, as illustrated on figure 4

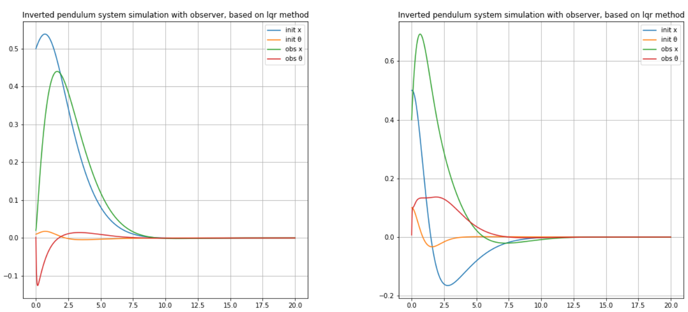

Figure 4: Simulation with second and third set of initial values

Task 6

Add white Gaussian noise to the output ($\delta y = C\delta z + v$). What happens to the state estimation?

The Gaussian nose, added go the output results in the noise in observer and harder stabilization of the system. The result of the simulation can be seen on the figure 5

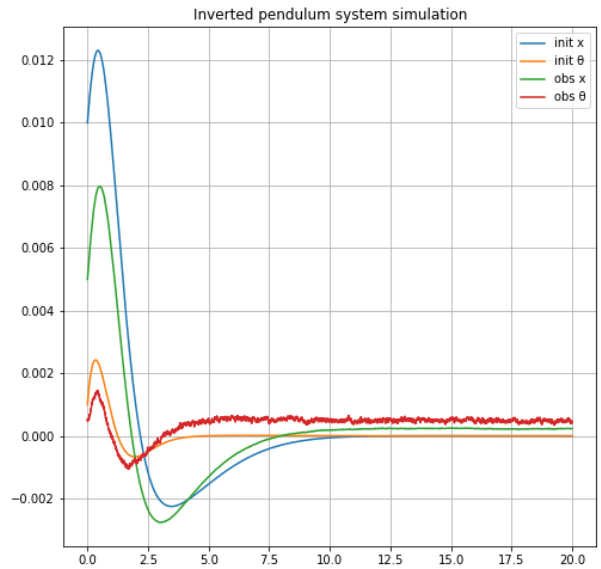

Figure 5: Addition of white Gaussian noise to the output of the system

On the given figure the absolute value of noise on the output is less then 0.0005.

Task 7

Add white Gaussian noise to the dynamics ($\delta\dot{z} = A\delta z + B\delta u + w$). What happens to the state estimation and control system?

Addition of the white Gaussian noise to the dynamics of the system results in unpredictability or noise in the behaviour of the initial system and thus increases the noise on the observer.

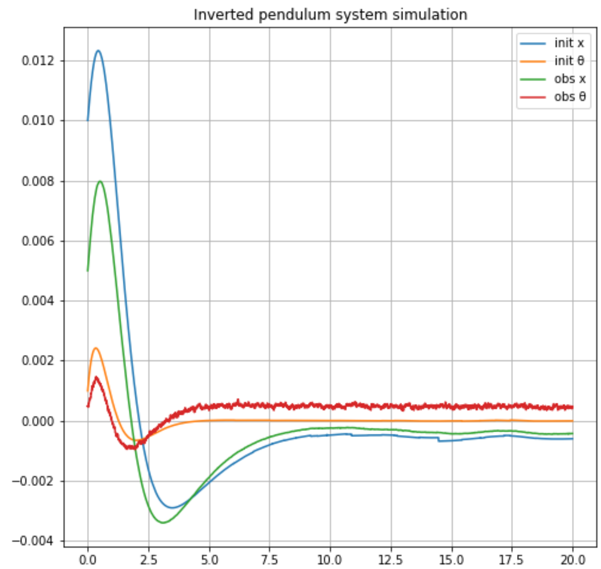

Figure 6: Addition of white Gaussian noise to the dynamics of the system

On the given figure the absolute value of noise on the output is less then 0.0005, and the noise on system's dynamics is less then 0.00005 by absolute value.

Conclusion

Usual methods as usual observer does not perform well, in case of absence of some noise in the system, that is the reason, why Kalman filter is often used.