# P8108 Homework 5

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2022-10-12

## Problem 3

### Proof

Since $T_i \sim Exp(h_i), T_j \sim Exp(h_j)$, we can get

$$f_{T_i}(t) = h_i \exp(-h_i t)$$
$$f_{T_j}(t) = h_i \exp(-h_j t)$$

Hence

$$\begin{aligned}
P(T_i \geq T_j) &= 1 - P(T_i < T_j) \\
&= 1 - \left(\int_0^\infty P(T_i < T_j \mid T_j = v) f_{T_j}(v) dv\right) \\
&= 1 - \left(\int_0^\infty \int_0^v f_{T_i}(u) f_{T_j}(v) du dv\right) && \text{(since } T_i \perp T_j\text{)} \\
&= 1 - \left(\int_0^\infty (1 - \exp(-h_i v)) h_j \exp(-h_j v) dv\right) \\
&= 1 - \left(\int_0^\infty h_j \exp(-h_j v) dv - \frac{h_j}{h_i + h_j} \int_0^\infty (h_i + h_j) \exp(-(h_i + h_j) v) dv\right) \\
&= 1 - \left(1 - \frac{h_j}{h_i + h_j}\right) \\
&= \frac{h_j}{h_i + h_j}
\end{aligned}$$

□