P8131_hw7_rw2844

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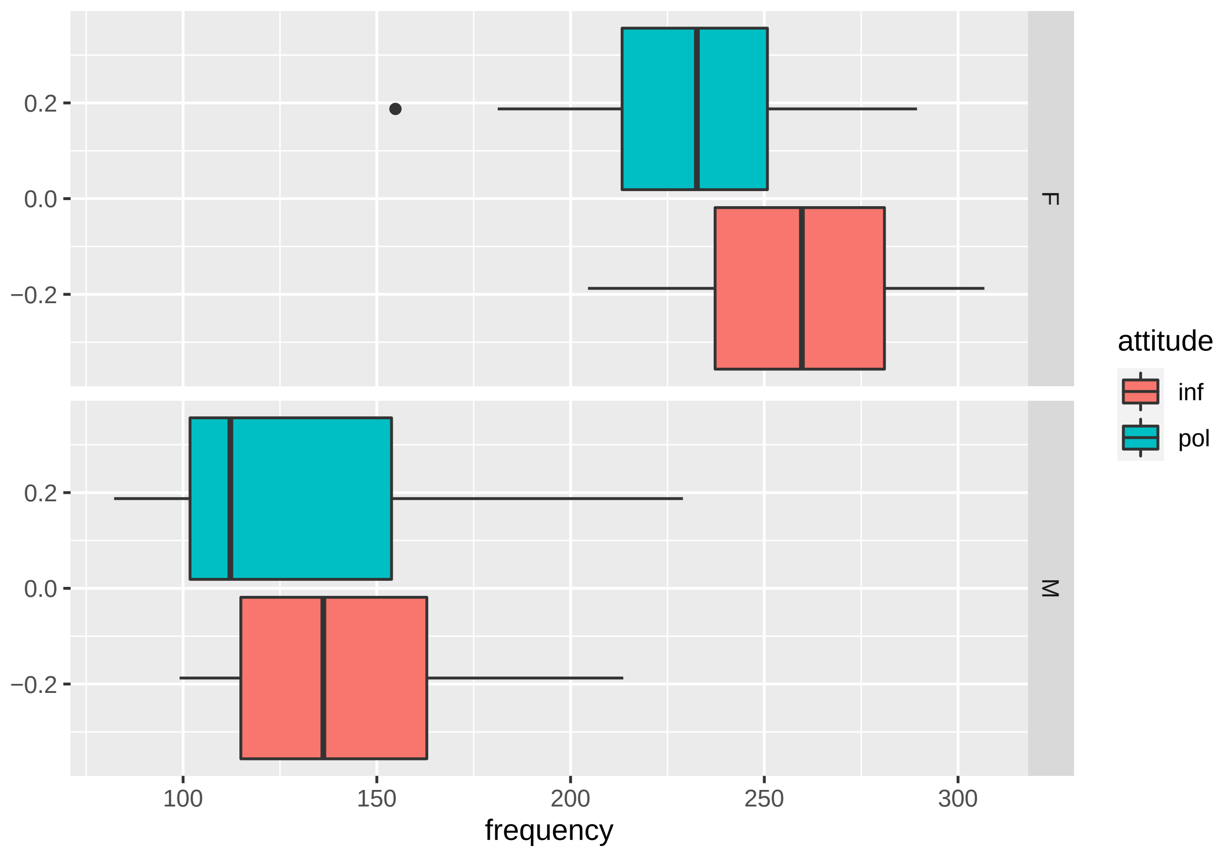
2022-04-07

Problem a

Exploratory analysis: provide boxplots to show the relation between gender/attitude and pitch (ignoring different scenarios).

Answer



From the boxplot, we can see that:

- Considering gender, the pitch of male is lower than the pitch of female
- Considering attitude, the pitch of polite attitude is lower than the pitch of informal attitude.
- The pitch difference between genders are more obvious than that between different attitudes.

Problem b

Fit a mixed effects model with random intercepts for different subjects (gender and attitude being the fixed effects). What is the covariance matrix for a subject Y_i? What is the covariance matrix for the estimates of fixed effects (Hint: 3×3 matrix for intercept, gender and attitude)? What are the BLUPs for subject-specific intercepts? What are the residuals?

Answer

The model is:

$$Y_{ij} = \beta_0 + \beta_1 \times \text{genderM}_{ij} + \beta_2 \times \text{attitudePol}_{ij} + b_i + \epsilon_{ij}$$

where $i = 1, \ldots, 6, \; j = 1, \ldots, 14$

The covariance matrix of Y_i is:

$$\text{cov}(\mathbf{Y}_i) = \begin{pmatrix} \sigma_b^2 + \sigma^2 & \sigma_b^2 & \ldots & \sigma_b^2 \\ \sigma_b^2 & \sigma_b^2 + \sigma^2 & \ldots & \sigma_b^2 \\ \ldots & & & \ldots \\ \sigma_b^2 & \sigma_b^2 & \ldots & \sigma_b^2 + \sigma^2 \end{pmatrix} = \begin{pmatrix} 1445.9 & 598.2 & \ldots & 598.2 \\ 598.2 & 1445.9 & \ldots & 598.2 \\ \ldots & & & \ldots \\ 598.2 & 598.2 & \ldots & 1445.9 \end{pmatrix}$$

The covariance matrix for the estimates of fixed effects is:

$$\text{cov}(\beta) = \begin{pmatrix} \text{var}(\beta_0) & \text{cov}(\beta_0, \beta_1) & \text{cov}(\beta_0, \beta_2) \\ \text{cov}(\beta_0, \beta_1) & \text{var}(\beta_1) & \text{cov}(\beta_1, \beta_2) \\ \text{cov}(\beta_0, \beta_2) & \text{cov}(\beta_1, \beta_2) & \text{var}(\beta_2) \end{pmatrix} = \begin{pmatrix} 229.7 & -219.6 & -20.2 \\ -219.6 & 439.16 & 6.5 \times 10^{-15} \\ -20.2 & 6.5 \times 10^{-15} & 40.4 \end{pmatrix}$$

The BLUPs for subject-specific intercepts are:

Table 1: BLUPs for subject-specific intercepts

	BLUP
F1	-13.576
F2	10.171
F3	3.405
M3	27.960
M4	4.739
M7	-32.700

The residuals are:

```
##          F1          F1          F1          F1          F1          F1
## -10.1086926 -38.9110735  61.6913074  16.2889265 -19.5086926  43.4889265
##          F1          F1          F1          F1          F1          F1
##  27.3913074  33.3889265   8.4913074   8.9889265 -42.2086926 -12.7110735
##          F1          F1          F3          F3          F3          F3
## -26.9110735 -68.6086926 -10.6898326 -23.0922136  -3.5898326  -9.3922136
##          F3          F3          F3          F3          F3          F3
##  26.6101674   5.6077864  35.0101674  46.4077864  -7.7898326  -7.8922136
##          F3          F3          F3          F3          M4          M4
## -13.8898326  18.4077864   4.0077864 -54.8898326 -22.2262298 -29.3286108
##          M4          M4          M4          M4          M4          M4
##  96.0737702 -38.0286108 -20.7262298  60.6713892  60.4737702   9.9713892
```

```
##          M4          M4          M4          M4          M4          M4
## -31.1262298 -26.0286108 -22.9262298 -16.7286108  -6.9286108  -6.4262298
##          M7          M7          M7          M7          M7          M7
##  -9.3872916 -16.3896725 -13.2872916 -11.1896725  -9.5872916  -5.2896725
##          M7          M7          M7          M7          M7          M7
##   1.6127084   4.5103275  -1.7872916 -12.5896725  13.3127084  -7.2896725
##          M7          M7          F2          F2          F2          F2
##   8.9103275  12.1127084 -14.4550462 -35.8574271  -0.8550462  -7.4574271
##          F2          F2          F2          F2          F2          F2
##  42.2449538  34.6425729  -3.9550462  29.0425729  30.5449538  27.0425729
##          F2          F2          F2          F2          M3          M3
## -39.1550462 -41.2574271  13.8425729 -19.9550462  -2.3471929  12.6504261
##          M3          M3          M3          M3          M3          M3
## -13.7471929  23.5504261   4.0528071   9.9504261  51.3528071  14.7504261
##          M3          M3          M3          M3          M3          M3
##   4.5528071 -19.6495739  -9.4471929 -18.1495739 -15.0495739  -2.8471929
## attr(,"label")
## [1] "Fitted values"
```

Problem c

Fit a mixed effects model with intercepts for different subjects (gender, attitude and their interaction being the fixed effects). Use likelihood ratio test to compare this model with the model in part (b) to determine whether the interaction term is significantly associated with pitch.

Answer

The fitted model is:

$$Y_{ij} = \beta_0 + \beta_1 \times \text{genderM}_{ij} + \beta_2 \times \text{attitudePol}_{ij} + \beta_3 \times \text{genderM}_{ij} \times \text{attitudePol}_{ij} + b_i + \epsilon_{ij}$$

The hypotheses are $\text{H}_0\text{: } \beta_3 = 0$ vs. $\text{H}_1\text{: } \beta_3 \neq 0$

```
##         Model df      AIC      BIC    logLik   Test  L.Ratio p-value
## LMM1.ML     1  5 825.6363 837.7904 -407.8182
## LMM2        2  6 826.2508 840.8357 -407.1254 1 vs 2 1.385523  0.2392
```

The likelihood ratio test shows that the Likelihood Ratio = 1.386 and the p-value = 0.2392 > 0.05, we fail to reject the null hypothesis and conclude that, at 0.05 significance level, the interaction term is not significantly associated with pitch.

Problem d

Write out the mixed effects model with random intercepts for both subjects and scenarios (gender and attitude being the fixed effects). Fit the model using `lmer` in the `lme4` package. Write out the covariance matrix for a subject Y_i. What is the interpretation of the coefficient for the fixed effect term attitude?

Answer

The fitted model is

$$Y_{ijk} = \beta_0 + \beta_1 \times \text{genderM}_{ij} + \beta_2 \times \text{attitudePol}_{ij} + \beta_3 \times \text{genderM}_{ij} \times \text{attitudePol}_{ij} + b_{1i} + b_{2k} + \epsilon_{ijk}$$

where $i = 1, \ldots, 6$, $j = 1, \ldots, 2$, and $k = 1, \ldots, 7$.

In this model, the variance of each response is given by:

$$\begin{aligned} \operatorname{var}(Y_{ijk}) &= \operatorname{var}(b_{1i} + b_{2k} + \epsilon_{ijk}) \\ &= \sigma_{b_1}^2 + \sigma_{b_2}^2 + \sigma^2 \\ &= 14.983^2 + 24.763^2 + 25.254^2 \\ &= 1475.461 \end{aligned} \tag{1}$$

The marginal covariance between any pair of response are:

$\operatorname{cov}(Y_{ij_1k_1}, Y_{ij_2k_2}) = \operatorname{cov}(b_{1i}+b_{2k_1}+\epsilon_{ij_1k_1}, b_{1i}+b_{2k_2}+\epsilon_{ij_2k_2}) = \operatorname{var}(b_{1i})+\operatorname{cov}(b_{1i}, b_{2k_1})+\operatorname{cov}(b_{1i}, b_{2k_2})+\operatorname{cov}(b_{2k_1}, b_{2k_2}) = \operatorname{var}(b_{1i})$

$\operatorname{cov}(Y_{ij_1k}, Y_{ij_2k}) = \operatorname{cov}(b_{1i}+b_{2k}+\epsilon_{ij_1k}, b_{1i}+b_{2k}+\epsilon_{ij_2k}) = \operatorname{var}(b_{1i})+\operatorname{cov}(b_{1i}, b_{2k})+\operatorname{cov}(b_{1i}, b_{2k})+\operatorname{var}(b_{2k}) = \operatorname{var}(b_{1i})+\operatorname{var}(b_{2k})$

$\operatorname{cov}(Y_{ijk_1}, Y_{ijk_2}) = \operatorname{cov}(b_{1i}+b_{2k_1}+\epsilon_{ij_1k}, b_{1i}+b_{2k_2}+\epsilon_{ij_2k}) = \operatorname{var}(b_{1i})+\operatorname{cov}(b_{1i}, b_{2k_1})+\operatorname{cov}(b_{1i}, b_{2k_2})+\operatorname{cov}(b_{2k_1}, b_{2k_2}) = \operatorname{var}(b_{1i})$

Wherevar$(b_{1i}) = 24.763^2 = 613.2062$, $\operatorname{var}(b_{1i}) + \operatorname{var}(b_{2k}) = 24.763^2 + 14.983^2 = 837.6965$.

Hence, the covariance matrix for a subject Y_i is:

$$\operatorname{cov}(\mathbf{Y}_i) = \begin{pmatrix} 1475.46 & \ldots & 613.21 & 837.70 & \ldots & 613.21 \\ \ldots & \ddots & \ldots & \ldots & \ddots & \ldots \\ 613.21 & \ldots & 1475.46 & 613.21 & \ldots & 837.70 \\ 837.70 & \ldots & 613.21 & 1475.46 & \ldots & 613.21 \\ \ldots & \ddots & \ldots & \ldots & \ddots & \ldots \\ 613.21 & \ldots & 837.70 & 613.21 & \ldots & 1475.46 \end{pmatrix}$$

Table 2: Fixed effect for double random intercept model

	estimate
(Intercept)	256.987
genderM	-108.798
attitudepol	-20.002

From the table above, we know that pitch is expected to be 20.002 Hz lower for polite attitude comparing with informal attitude on average holding everything else fixed.