P8160 - Project 3
Baysian modeling of hurricane

Renjie Wei, Hao Zheng, Xinran Sun
Wentong Liu, Shengzhi Luo

2022-05-09

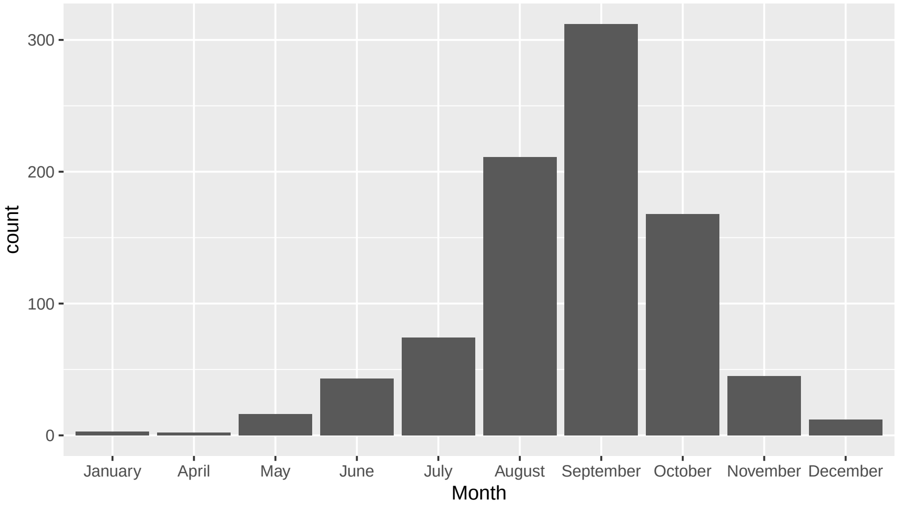
Introduction

- Hurricanes can result in death and economical damage
- There is an increasing desire to predict the speed and damage of the hurricanes
- Use Bayesian Model and Markov Chain Monte Carlo algorithm
  - Predict the wind speed of hurricanes
  - Study how hurricanes is related to death and financial loss

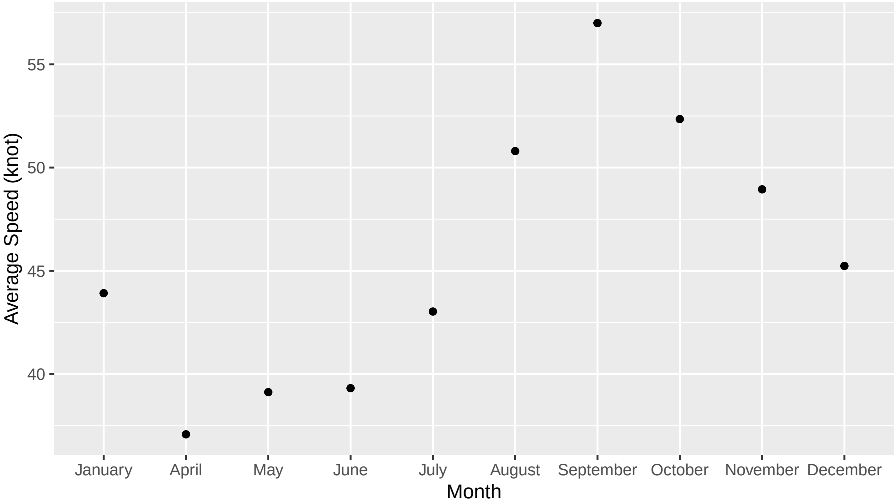
Dataset

- Hurrican703 dataset: 22038 observations $\times$ 8 variables
  - 702 hurricanes in the North Atlantic area in year 1950-2013
- Processed dataset: add 5 more variables into hurrican703
- Hurricanoutcome2 dataset: 43 observations $\times$ 14 variables

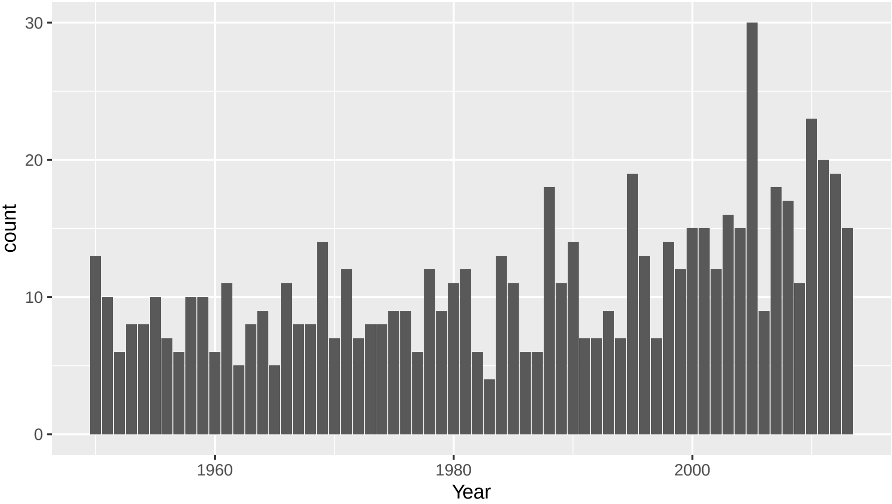
EDA - Count of Hurricanes in Each Month



EDA - Average Speed (knot) of Hurricanes in Each Month

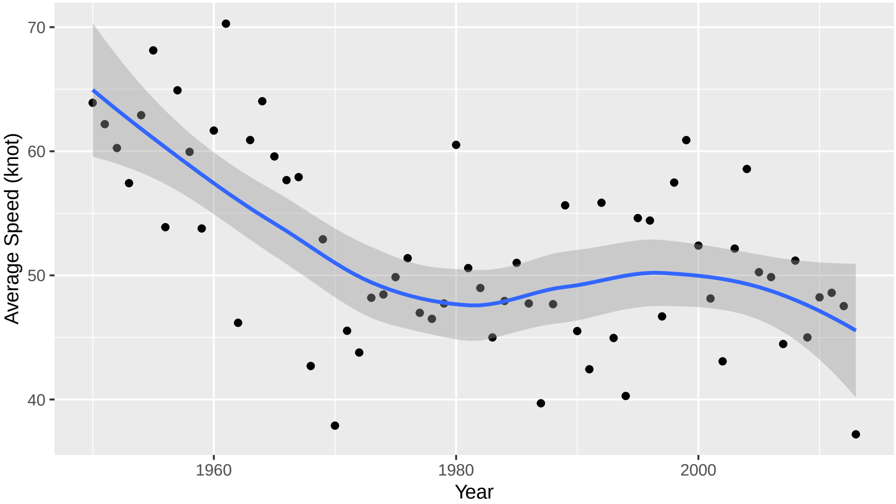


EDA - Count of Hurricanes in Each Year

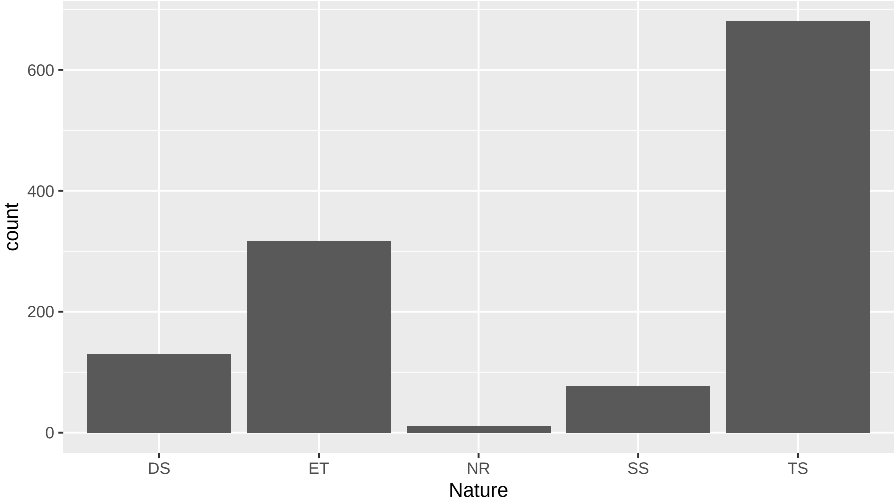


EDA - Average Speed (knot) of Hurricanes in Each Year

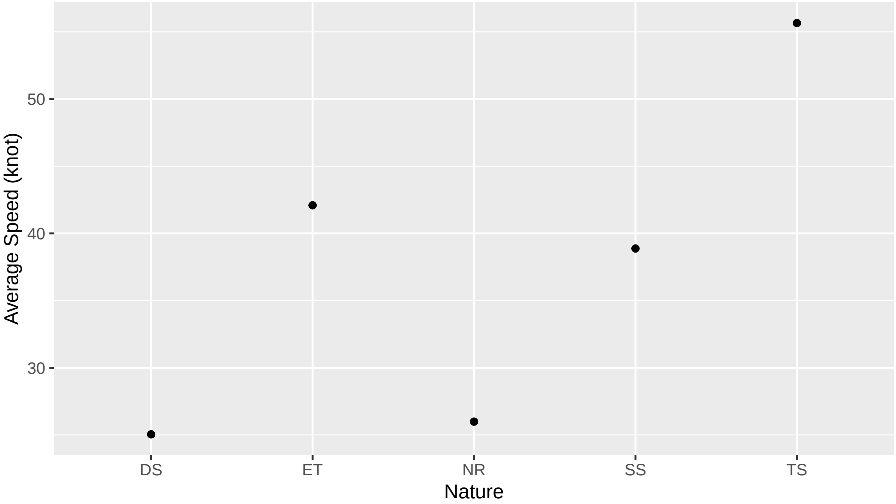
```
## `geom_smooth()` using formula 'y ~ x'
```



EDA - Count of Hurricanes in Each Nature



EDA - Average Speed (knot) of Hurricanes in Each Nature



Bayesian Model Setting

Model

The suggested Bayesian model is

$$Y_i(t+6) = \beta_{0,i} + \beta_{1,i}Y_i(t) + \beta_{2,i}\Delta_{i,1}(t) + \beta_{3,i}\Delta_{i,2}(t) + \beta_{4,i}\Delta_{i,3}(t) + \epsilon_i(t)$$

- where $Y_i(t)$ the wind speed at time t (i.e. 6 hours earlier), $\Delta_{i,1}(t)$, $\Delta_{i,2}(t)$ and $\Delta_{i,3}(t)$ are the changes of latitude, longitude and wind speed between t and $t-6$, and $\epsilon_{i,t}$ follows a normal distributions with mean zero and variance σ^2, independent across t.
- $\beta_i = (\beta_{0,i}, \beta_{1,i}, ..., \beta_{5,i})$, we assume that $\beta_i \sim N(\mu, \Sigma_{d\times d})$, where d is dimension of β_i.

Priors

$$P(\sigma^2) \propto \frac{1}{\sigma^2}; \quad P(\mu) \propto 1; \quad P(\Sigma^{-1}) \propto |\Sigma|^{-(d+1)} \exp(-\frac{1}{2}\Sigma^{-1})$$

Posterior

- Derive $\pi(\Theta|Y)$, where $\Theta = (\mathbf{B}^\top, \mu^\top, \sigma^2, \Sigma)$, $\mathbf{B} = (\beta_1^\top, ..., \beta_n^\top)^\top$

Joint posterior

Notations

- $X_i(t)\beta_i^\top = \beta_{0,i} + \beta_{1,i}Y_i(t) + \beta_{2,i}\Delta_{i,1}(t) + \beta_{3,i}\Delta_{i,2}(t) + \beta_{4,i}\Delta_{i,3}(t)$
- For i^{th} hurricane, there may be m_i times of record (excluding the first and second observation), let

$$Y_i = \begin{pmatrix} Y_i(t_0+6) \\ Y_i(t_1+6) \\ \vdots \\ Y_i(t_{m_i-1}+6) \end{pmatrix}_{m_i \times 1}$$

- Hence, $Y_i \mid X_i, \beta_i, \sigma^2 \sim N(X_i\beta_i^\top, \sigma^2 I)$
- Where, X_i is a $m_i \times d$ dimensional matrix

$$X_i = \begin{pmatrix} 1 & Y_i(t_0) & \Delta_{i,1}(t_0) & \Delta_{i,2}(t_0) & \Delta_{i,3}(t_0) \\ 1 & Y_i(t_1) & \Delta_{i,1}(t_1) & \Delta_{i,2}(t_1) & \Delta_{i,3}(t_1) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & Y_i(t_{m_i-1}) & \Delta_{i,1}(t_{m_i-1}) & \Delta_{i,2}(t_{m_i-1}) & \Delta_{i,3}(t_{m_i-1}) \end{pmatrix}$$

Joint posterior

Posterior

$$
\begin{aligned}
\pi(\Theta|Y) &= \pi(\mathbf{B}^\top, \mu^\top, \sigma^2, \Sigma \mid Y) \\
&\propto \underbrace{\prod_{i=1}^{n} f(Y_i \mid \beta_i, \sigma^2)}_{\text{likelihood of } Y} \underbrace{\prod_{i=1}^{n} \pi(\beta_i \mid \mu, \Sigma)}_{\text{distribution of } \mathbf{B}} \underbrace{P(\sigma^2)P(\mu)P(\Sigma^{-1})}_{\text{priors}} \\
&\propto \prod_{i=1}^{n} \Big\{ (2\pi\sigma^2)^{-m_i/2} \exp\big\{ -\frac{1}{2}(Y_i - X_i\beta_i^\top)^\top(\sigma^2 I)^{-1}(Y_i - X_i\beta_i^\top) \\
&\times \prod_{i=1}^{n} \Big\{ \det(2\pi\Sigma)^{-\frac{1}{2}} \exp\big\{ -\frac{1}{2}(\beta_i - \mu)\Sigma^{-1}(\beta_i - \mu)^\top \big\} \Big\} \\
&\times \frac{1}{\sigma^2} \times \det(\Sigma)^{-(d+1)} \exp\big\{ -\frac{1}{2}\Sigma^{-1} \big\}
\end{aligned}
$$

MCMC Algorithm

- Monte Carlo Method
  - Random sampling method to estimate quantity
- Markov Chain
  - Generates a sequence of random variables where the current state only depends on the nearest past
- Example: Gibbs Sampler
  - MCMC approaches with known conditional distributions
  - Samples from each random variables in turn given the value of all the others in the distribution

Conditional Posterior

- To apply MCMC using Gibbs sampling, we need to find conditional posterior distribution of each parameter, then we can implement Gibbs sampling on these conditional posterior distributions.
  - $\pi(\mathbf{B}|Y, \mu^\top, \sigma^2, \Sigma)$
  - $\pi(\sigma^2|Y, \mathbf{B}^\top, \mu^\top, \Sigma)$
  - $\pi(\Sigma|Y, \mathbf{B}^\top, \mu^\top, \sigma^2)$
  - $\pi(\mu|Y, \mathbf{B}^\top, \sigma^2, \Sigma)$

MCMC Algorithm - Conditional Posterior

- β_i: $\pi(\beta_i|Y, \mu^\top, \sigma^2, \Sigma) \sim \mathcal{N}(\hat{\beta}_i, \hat{\Sigma}_{\beta_i})$
  - where $\hat{\beta}_i = (\Sigma^{-1} + X_i^\top(\sigma^2 I)^{-1}X_i)^{-1}Y_i^\top(\sigma^2 I)^{-1}X_i + \mu\Sigma^{-1}, \hat{\Sigma}_{\beta_i} = (\Sigma^{-1} + X_i^\top(\sigma^2 I)^{-1}X_i)^{-1}$
- σ^2: $\pi(\sigma^2|Y, \mathbf{B}^\top, \mu^\top, \Sigma) \sim IG(\frac{1}{2}\sum_{i=1}^n m_i, \frac{1}{2}\sum_{i=1}^n (Y_i - X_i\beta_i^\top)^\top(Y_i - X_i\beta_i^\top))$
- Σ: $\pi(\Sigma|Y, \mathbf{B}^\top, \mu^\top, \sigma^2) \sim IW(n+d+1,\ I + \sum_{i=1}^n (\beta_i - \mu)(\beta_i - \mu)^\top)$
- μ: $\pi(\mu|Y, \mathbf{B}^\top, \sigma^2, \Sigma) \sim \mathcal{N}(\frac{1}{n}\sum_{i=1}^n \beta_i, \frac{1}{n}\Sigma)$

MCMC Algorithm - Parameter Updates

The update of parameters is component wise, at $(t+1)^{\text{th}}$ step, updating parameters in the following the order:

1. Sample $\mathbf{B}^{(t+1)}$, i.e., sample each $\beta_i^{(t+1)}$ from $\mathcal{N}(\hat{\beta}_i^{(t)}, \hat{\Sigma}_{\beta_i}^{(t)})$
2. Then, sample σ^2 from
   $IG(\frac{1}{2}\sum_{i=1}^{n} m_i, \frac{1}{2}\sum_{i=1}^{n}(Y_i - X_i\beta_i^{(t+1)^\top})^\top(Y_i - X_i\beta_i^{(t+1)^\top}))$
3. Next, sample $\Sigma^{(t+1)}$ from
   $IW(n+d+1,\ I + \sum_{i=1}^{n}(\beta_i^{(t+1)} - \mu^{(t)})(\beta_i^{(t+1)} - \mu^{(t)})^\top)$
4. Finally, sample $\mu^{(t+1)}$ from $\mathcal{N}(\frac{1}{n}\sum_{i=1}^{n}\beta_i^{(t+1)}, \frac{1}{n}\Sigma^{(t+1)})$

MCMC Algorithm - Train-Test split and Inital Values

Train-test split

- Drop the data of hurricane with less than 3 observations. Results in 697 hurricanes
- Within each hurricane's data, randomly 80% train, 20% test

Initial Values

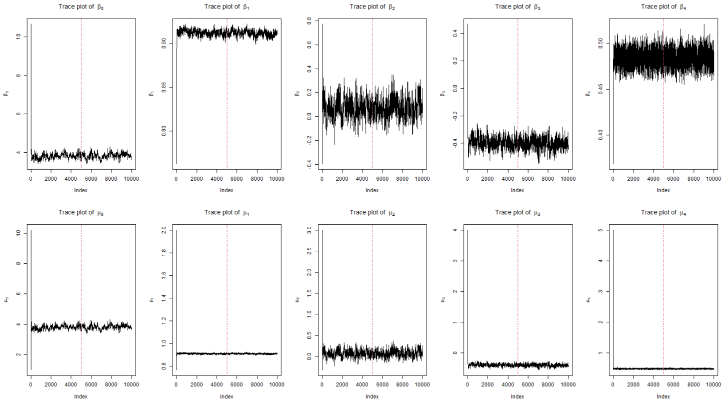
1. For initial value of $\mathbf{B}$, we run multivariate linear regressions for each hurricane and use the regression coefficients β_i^{MLR} as the initial value for β_i. Then, the initial value of $\mathbf{B}$ can be represented as $\mathbf{B}_{init} = (\beta_1^{MLR^\top}, \dots, \beta_n^{MLR^\top})^\top$.
2. For initial value of μ, we take the average of β_i^{MLR}, that is $\mu_{init} = \frac{1}{n} \sum_{i=1}^{n} \beta_n^{MLR}$
3. For initial value of σ^2, we take the average of the MSE for i hurricanes.
4. For initial value of Σ, we just set it to a simple diagonal matrix.

MCMC Results

Details

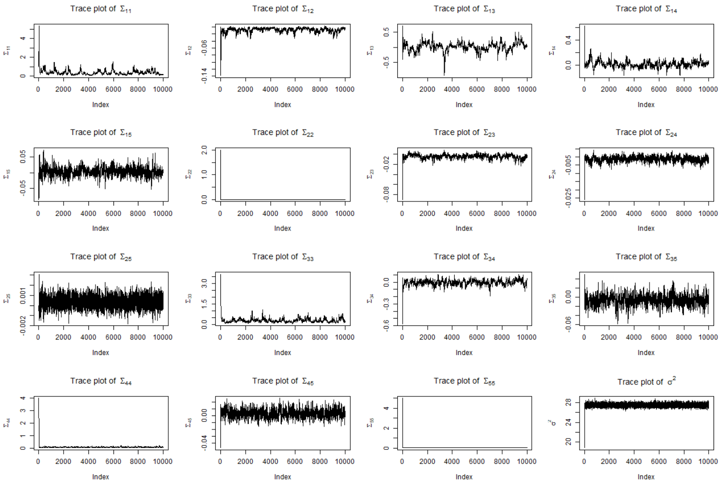
- 10000 iterations
- First 5000 iterations as burn-in period
- Estimates and inferences based on last 5000 MCMC samples

MCMC Results - Trace Plots 1



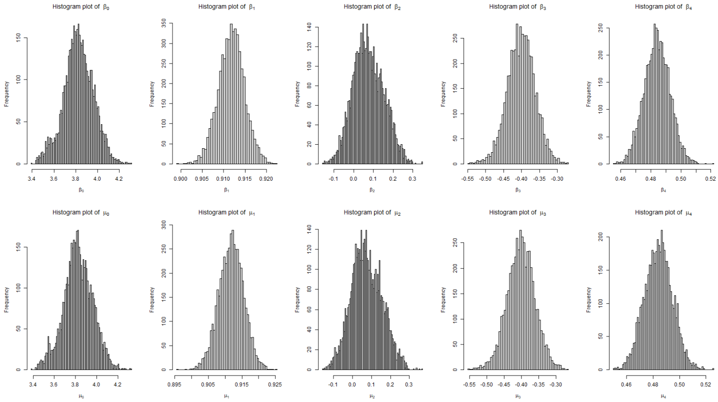
Trace plots of model parameters, based on 10000 MCMC sample

MCMC Results - Trace Plots 2



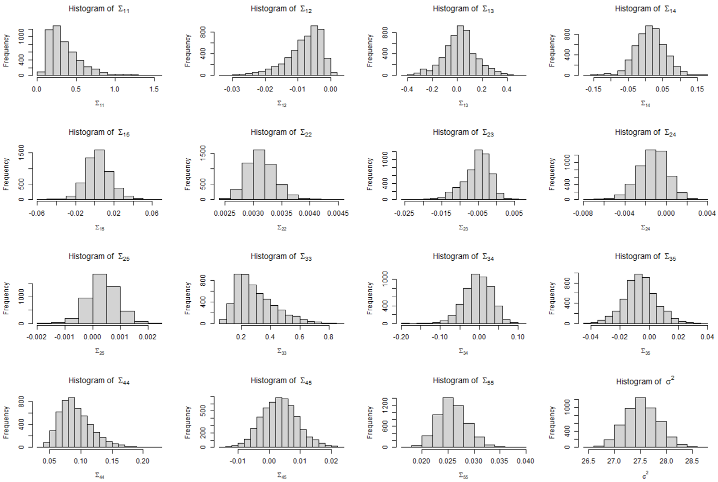
Trace plots of variance parameters, based on 10000 MCMC sample

MCMC Results - Histograms 1



Histograms of model parameters, based on last 5000 MCMC sample

MCMC Results - Histograms 2



Histograms of variance parameters, based on last 5000 MCMC sample

MCMC Results - Model Parameter Estimations and Inferences

Variables	$\bar{\beta}_i$	$\text{Var}(\bar{\beta}_i)$	95% CI of $\bar{\beta}_i$	$\bar{\mu}$	$\text{Var}(\bar{\mu})$	95% CI of $\bar{\mu}$
intercept	3.8252	0.0185	(3.5588,4.0916)	3.8166	0.0190	(3.5468,4.0865)
Wind_prev	0.9118	0.0000	(0.9059,0.9177)	0.9121	0.0000	(0.9049,0.9194)
Lat_change	0.0744	0.0060	(-0.0776,0.2264)	0.0720	0.0065	(-0.0857,0.2298)
Long_change	-0.4014	0.0015	(-0.4771,-0.3257)	-0.3968	0.0016	(-0.4759,-0.3177)
Wind_change	0.4841	0.0001	(0.4674,0.5009)	0.4847	0.0001	(0.464,0.5053)

Bayesian posterior estimates for model parameters

MCMC Results - Variance Parameter Estimations and Inferences

$$\Sigma = \begin{pmatrix} 0.349 & -0.008 & 0.020 & 0.013 & 0.004 \\ -0.008 & 0.003 & -0.005 & -0.001 & 0.0004 \\ 0.020 & -0.005 & 0.296 & -0.003 & -0.006 \\ 0.013 & -0.001 & -0.003 & 0.092 & 0.003 \\ 0.004 & 0.0004 & -0.006 & 0.003 & 0.026 \end{pmatrix}$$

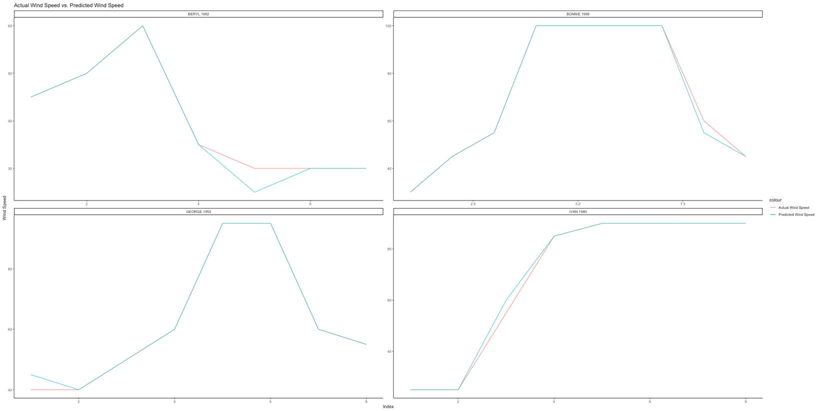
$$\rho = \begin{pmatrix} 1 & -0.245 & 0.063 & 0.073 & 0.037 \\ -0.245 & 1 & -0.174 & -0.078 & 0.041 \\ 0.063 & -0.174 & 1 & -0.019 & -0.069 \\ 0.073 & -0.078 & -0.019 & 1 & 0.070 \\ 0.037 & 0.041 & -0.069 & 0.070 & 1 \end{pmatrix}$$

Bayesian Model Performance

	ID	r_square	rmse	n_obs
1	BONNIE.1998	0.996	1.667	9
2	IVAN.1980	0.996	1.767	8
3	GEORGE.1950	0.993	1.768	8
4	MARIA.2011	0.964	1.768	8
5	BERYL.1982	0.971	1.889	7
6	FLORENCE.1960	0.927	1.890	7
7	LOIS.1966	0.990	1.890	7
8	ERIN.1989	0.991	1.890	7
9	GRETA.1970	0.893	2.041	6
10	HILDA.1964	0.995	2.236	5

R Squared and RMSE

Bayesian Model Performance



Actual Wind Speed vs. Predicted Wind Speed

Seasonal Difference Exploration

	Beta 0		Beta 1		Beta 2		Beta 3		Beta 4	
	Estimate	Pr(>|t|)	Estimate	Pr(>|t|)	Estimate	Pr(>|t|)	Estimate	Pr(>|t|)	Estimate	Pr(>|t|)
(Intercept)	4.4810021	0.0000000	1.3431063	0.0000000	0.0413063	0.9506172	-0.8336700	0.0185275	0.2890273	0.4482640
monthApril	0.0232609	0.8346449	0.0147943	0.6696787	0.0165579	0.9306863	0.0416468	0.6796126	0.0361823	0.7393892
monthMay	0.0259813	0.7827813	-0.0001180	0.9967888	0.0708822	0.6597505	0.0632772	0.4581672	-0.0162907	0.8594231
monthJune	0.0275693	0.7650618	0.0053935	0.8509869	-0.0070875	0.9641298	0.0556884	0.5047909	0.0237694	0.7918014
monthJuly	0.0125400	0.8914489	0.0154032	0.5901741	-0.0090910	0.9538180	0.0361214	0.6640154	0.0130817	0.8840332
monthAugust	-0.0198034	0.8284715	0.0233206	0.4124181	-0.0522548	0.7378961	0.0123691	0.8811234	0.0312427	0.7261962
monthSeptember	-0.0070528	0.9384385	0.0261005	0.3585599	-0.0361073	0.8169707	0.0212965	0.7966351	0.0444835	0.6177631
monthOctober	0.0093435	0.9185853	0.0210829	0.4587183	-0.0286163	0.8546050	0.0341549	0.6796975	0.0350505	0.6944480
monthNovember	0.0145692	0.8748155	0.0246144	0.3925264	0.0239972	0.8792681	0.0263450	0.7529105	0.0209069	0.8168323
monthDecember	0.0057977	0.9526542	0.0088244	0.7715305	-0.0543131	0.7447475	0.0422468	0.6326060	0.0114196	0.9046290
year	-0.0003419	0.0717253	-0.0002252	0.0001471	0.0000365	0.9101708	0.0002184	0.2032812	0.0000905	0.6249586
natureET	0.0008449	0.9774141	0.0037334	0.6877086	-0.0702038	0.1687975	-0.0263888	0.3286540	-0.0209217	0.4726774
natureNR	0.0008122	0.9866387	-0.0146142	0.3331114	0.0058967	0.9432660	0.0030556	0.9444979	-0.0217275	0.6462854
natureSS	0.0141564	0.4904257	-0.0033299	0.6021721	-0.0013517	0.9692484	0.0126339	0.4964264	-0.0238538	0.2339965
natureTS	0.0118370	0.4785102	-0.0059979	0.2486925	-0.0154533	0.5880814	-0.0231521	0.1258337	-0.0174987	0.2832214

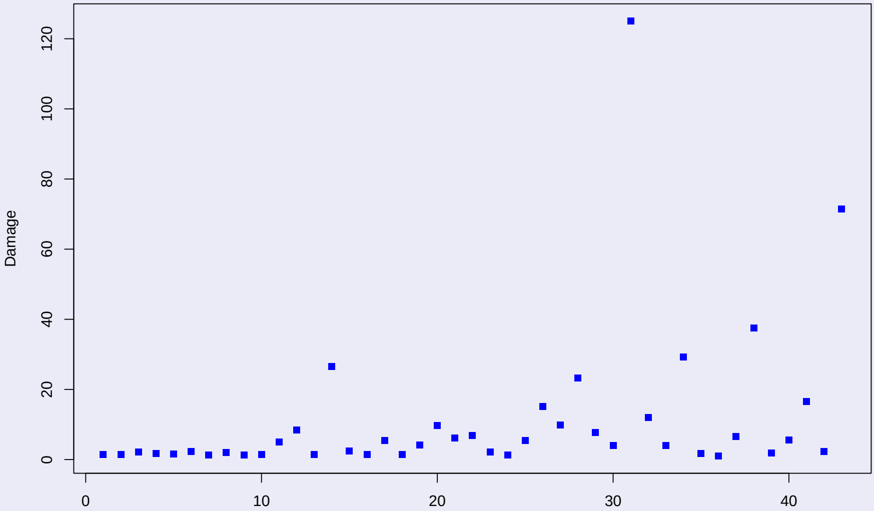
Seasonal Difference Exploration

	Beta 0		Beta 1		Beta 2		Beta 3		Beta 4	
	Estimate	Pr(>|t|)	Estimate	Pr(>|t|)	Estimate	Pr(>|t|)	Estimate	Pr(>|t|)	Estimate	Pr(>|t|)
(Intercept)	3.8365500	0.0000000	0.8942250	0.0000000	0.1606506	0.0000610	-0.3500900	0.0000000	0.4422452	0.0000000
seasonSummer	-0.0305003	0.2048954	0.0152377	0.0440074	-0.0979486	0.0167511	-0.0466127	0.0338037	0.0361669	0.1203099
seasonAutumn	-0.0235346	0.3248438	0.0209616	0.0053662	-0.0909590	0.0253577	-0.0434764	0.0463302	0.0487052	0.0354139
seasonWinter	-0.0186542	0.6535827	0.0034158	0.7936540	-0.0984181	0.1637856	-0.0094850	0.8023902	0.0149135	0.7107131

	Beta 0		Beta 1		Beta 2		Beta 3		Beta 4	
	Estimate	Pr(>|t|)	Estimate	Pr(>|t|)	Estimate	Pr(>|t|)	Estimate	Pr(>|t|)	Estimate	Pr(>|t|)
(Intercept)	4.5142875	0.0000000	1.3448481	0.0000000	-0.1056332	0.8629385	-1.0267628	0.001781	0.3051312	0.3817170
year	-0.0003543	0.0497902	-0.0002178	0.0001332	0.0000878	0.7757368	0.0003188	0.053474	0.0000902	0.6072986

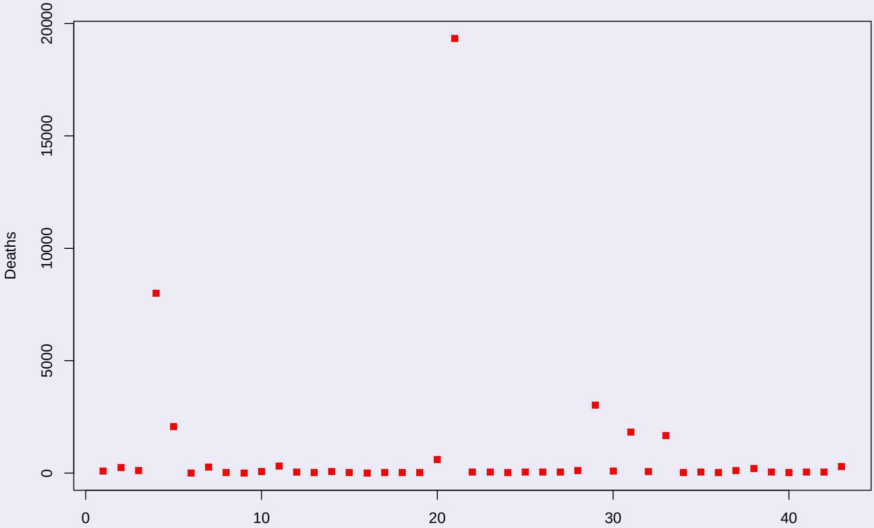
Predictions of Damage and Deaths

Basic plot of Damage and Deaths



Predictions of Damage and Deaths

Basic plot of Damage and Deaths



Generalized Linear Model - Poisson

The poisson model used in predicting deaths and damage is:

$$log(Damage * 1000 or Deaths) = \beta_i X_i$$

- where X_i includes $\beta_0 \sim \beta_4$ and the predictors in new data
- convert Damage units from billions to millions to get integer data

```
## Import Data
dat_hur = read.csv("hurrican703.csv") %>%
  janitor::clean_names() %>%
  group_by(id) %>%
  mutate(id = tolower(id),
         wind_early = lag(wind_kt, 1),
         lat_change = lag(latitude, 0) - lag(latitude, 1),
         long_change = lag(longitude, 0) - lag(longitude, 1),
         wind_change = lag(wind_kt, 0) - lag(wind_kt, 1)) %>%
  na.omit() %>%
  as.data.frame()
```

Coefficient Table

```
 [1] "|id              | intercept|     beta1|      beta2|      beta3|     beta4|"
 [2] "|:---------------|---------:|---------:|----------:|----------:|---------:|"
 [3] "|agnes.1972      |  3.950974| 0.9224097|  0.0059532| -0.3103372| 0.5453543|"
 [4] "|alex.2010       |  3.798737| 0.9370333|  0.0698849| -0.3937358| 0.5400187|"
 [5] "|alicia.1983     |  3.897408| 0.9036878| -0.0748341| -0.3994486| 0.5477718|"
 [6] "|allen.1980      |  3.687070| 0.9655304|  0.1306393| -0.5460144| 0.5466129|"
 [7] "|andrew.1992     |  3.676279| 0.9375384| -0.2843257| -0.5782973| 0.5370158|"
 [8] "|betsy.1965      |  3.808396| 0.9513766| -0.4500720| -0.3890718| 0.4244575|"
 [9] "|bob.1991        |  3.629466| 0.9232143|  0.0279527| -0.5751636| 0.4382048|"
[10] "|camille.1969    |  3.994355| 0.9355674|  0.0729188| -0.5734830| 0.6703910|"
```

Fitted results of beta models

Predict Damage

	term	estimate	std.error	statistic	p.value
1	(Intercept)	-2.179428e+02	63.786161983	-3.416772	6.336828e-04
2	intercept	5.044916e+00	0.872632934	5.781258	7.414400e-09
3	beta1	6.283543e+01	14.027126920	4.479565	7.479523e-06
4	beta2	-1.095810e+00	0.424325439	-2.582476	9.809426e-03
5	beta3	3.378223e+00	0.816050104	4.139725	3.477231e-05
6	nobs	4.921117e-02	0.008036275	6.123630	9.146733e-10
7	Season	7.497698e-02	0.012627373	5.937655	2.891284e-09
8	MonthJune	-3.416174e+00	0.762110791	-4.482516	7.376795e-06
9	MonthNovember	-1.902107e+00	0.789148853	-2.410327	1.593822e-02
10	MonthOctober	-1.290673e+00	0.298201079	-4.328198	1.503344e-05
11	MonthSeptember	-1.764116e+00	0.243173467	-7.254558	4.029764e-13
12	NatureNR	-4.317468e+00	1.126675716	-3.832042	1.270843e-04
13	NatureTS	-2.038481e+00	0.452900892	-4.500942	6.765302e-06
14	Maxspeed	5.044572e-02	0.006764325	7.457613	8.810369e-14
15	Meanspeed	-6.565465e-02	0.015403789	-4.262240	2.023877e-05
16	Percent.Poor	-3.819578e-02	0.005858677	-6.519522	7.053169e-11

Predict Deaths

	term	estimate	std.error	statistic	p.value
1	(Intercept)	1.164978e+02	1.257956e+01	9.260883	2.027487e-20
2	intercept	1.167475e+01	2.564192e-01	45.529931	0.000000e+00
3	beta1	1.141195e+02	2.200144e+00	51.869091	0.000000e+00
4	beta2	5.528798e+00	1.226329e-01	45.084128	0.000000e+00
5	beta3	8.561691e+00	2.853214e-01	30.007184	7.908823e-198
6	beta4	-1.049211e+01	3.058279e-01	-34.307225	6.123346e-258
7	nobs	3.430943e-03	1.116605e-03	3.072657	2.121619e-03
8	Season	6.102077e-03	2.093747e-03	2.914429	3.563401e-03
9	MonthJuly	-1.183782e+00	1.448847e-01	-8.170505	3.071002e-16
10	MonthJune	-1.291597e+00	8.968191e-02	-14.401980	5.028215e-47
11	MonthNovember	-2.533192e+00	1.551869e-01	-16.323490	6.718278e-60
12	MonthOctober	-1.546676e+00	6.466487e-02	-23.918335	1.974205e-126
13	MonthSeptember	-2.751167e-01	4.588850e-02	-5.995331	2.030720e-09
14	NatureNR	2.348783e+00	1.290216e-01	18.204563	4.748263e-74
15	NatureTS	3.563406e+00	1.209962e-01	29.450564	1.238185e-190
16	Meanspeed	-3.676417e-02	3.143216e-03	-11.696356	1.330451e-31
17	Maxpressure	-2.686076e-01	9.670821e-03	-27.775052	8.684053e-170
18	Meanpressure	5.377225e-03	2.009523e-04	26.758717	9.775966e-158
19	Total.Pop	9.410461e-07	2.587520e-08	36.368659	1.332659e-289
20	Percent.Poor	3.599824e-02	8.024514e-04	44.860342	0.000000e+00
21	Percent.USA	-7.214139e-03	5.570867e-04	-12.949761	2.356879e-38