# P8160 - Project 3 Baysian modeling of hurricane

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#### **EDA**

### Joint posterior

$$\pi(\Theta|Y)$$

$$= \pi(\mathbf{B}^{\top}, \mu^{\top}, \sigma^{2}, \Sigma \mid$$

$$\propto \prod_{i=1}^{n} f(Y_{i} \mid \beta_{i}, \sigma^{2})$$

$$\begin{split} &= \pi(\mathbf{B}^{\top}, \mu^{\top}, \sigma^2, \Sigma \mid Y) \\ &\propto \prod_{i=1}^n f(\boldsymbol{Y}_i \mid \boldsymbol{\beta}_i, \sigma^2) \prod_{i=1}^n \pi(\boldsymbol{\beta}_i \mid \boldsymbol{\mu}, \Sigma) P(\sigma^2) P(\boldsymbol{\mu}) P(\Sigma^{-1}) \end{split}$$

$$\propto \prod_{i=1}^{n} f(Y_i \mid \boldsymbol{\beta}_i, \sigma^2) \prod_{i=1}^{n} \pi(\boldsymbol{\beta}_i \mid \boldsymbol{\mu}, \boldsymbol{\Sigma}) P(\sigma^2) P(\boldsymbol{\mu}) P(\boldsymbol{\Sigma}^{-1})$$

$$\approx \prod_{i=1}^{n} \int (2\pi\sigma^2)^{-m_i/2} \exp\left(-\frac{1}{2}(\boldsymbol{V} - \boldsymbol{V}, \boldsymbol{\beta}^\top)^\top (\sigma^2 \boldsymbol{I})^{-1}) \boldsymbol{V} \right)$$

$$\propto \prod_{i=1}^n \left\{ (2\pi\sigma^2)^{-m_i/2} \exp\big\{ -\frac{1}{2} (\boldsymbol{Y}_i - \boldsymbol{X}_i \boldsymbol{\beta}_i^\top)^\top (\sigma^2 I)^{-1} (\boldsymbol{Y}_i - \boldsymbol{X}_i \boldsymbol{\beta}_i^\top) (\boldsymbol{Y}_i$$

$$\times \prod_{i=1}^{n} \left\{ \det(2\pi\Sigma)^{-\frac{1}{2}} \exp\left\{ -\frac{1}{2} (\boldsymbol{\beta}_i - \boldsymbol{\mu}) \boldsymbol{\Sigma}^{-1} (\boldsymbol{\beta}_i - \boldsymbol{\mu})^\top \right\} \right\} \times \frac{1}{\sigma^2} > 0$$

$$\times \prod_{i=1}^{n} \left\{ \det(2\pi\Sigma)^{-\frac{1}{2}} \exp\big\{ -\frac{1}{2} (\boldsymbol{\beta}_i - \boldsymbol{\mu}) \boldsymbol{\Sigma}^{-1} (\boldsymbol{\beta}_i - \boldsymbol{\mu})^\top \big\} \right\} \times \frac{1}{\sigma^2}$$

## MCMC algorithm

#### **Conditional Posterior**

➤ To apply MCMC using Gibbs sampling, we need to find conditional posterior distribution of each parameter, then we can implement Gibbs sampling on these conditional posterior distributions.

$$\blacktriangleright \ \pi(\mathbf{B}|Y,\mu^\top,\sigma^2,\Sigma)$$

$$\blacktriangleright \ \pi(\sigma^2|Y,\mathbf{B}^\top,\mu^\top,\Sigma)$$

