

# P8160 - Project 3

## Bayesian modeling of hurricane

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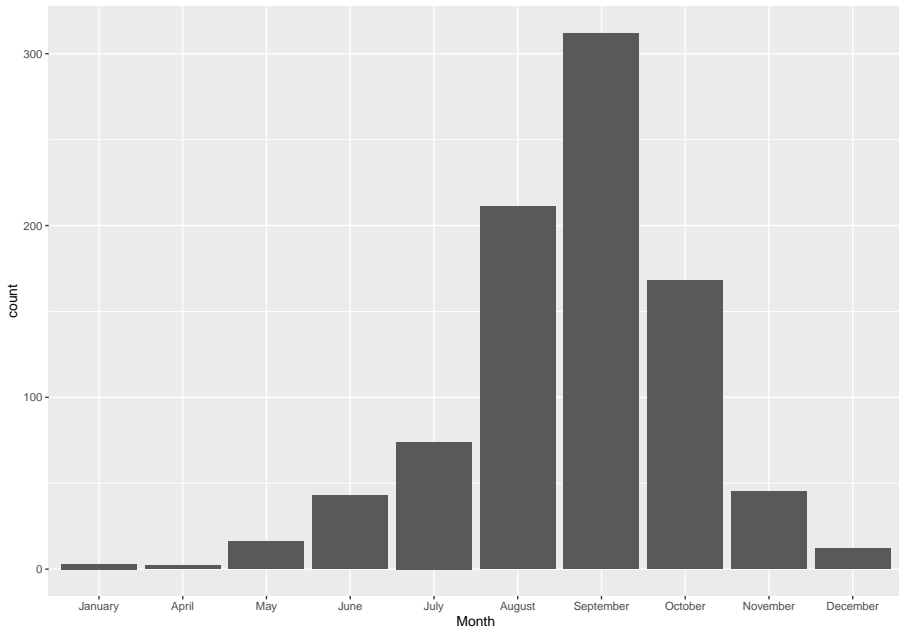
# Introduction

- Hurricanes can result in death and economical damage
- There is an increasing desire to predict the speed and damage of the hurricanes
- Use Bayesian Model and Markov Chain Monte Carlo algorithm
  - ▶ Predict the wind speed of hurricanes
  - ▶ Study how hurricanes is related to death and financial loss

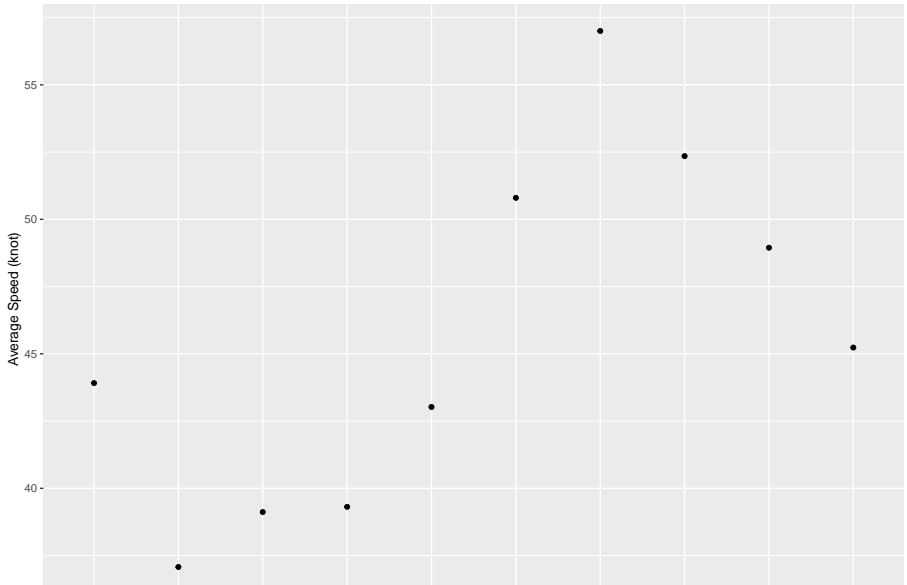
# Dataset

- Hurrican703 dataset: 22038 observations  $\times$  8 variables
  - ▶ 702 hurricanes in the North Atlantic area in year 1950-2013 with tra
- Processed dataset: add 5 more variables into hurrican703
- Hurricanoutcome2 dataset: 43 observations  $\times$  14 variables

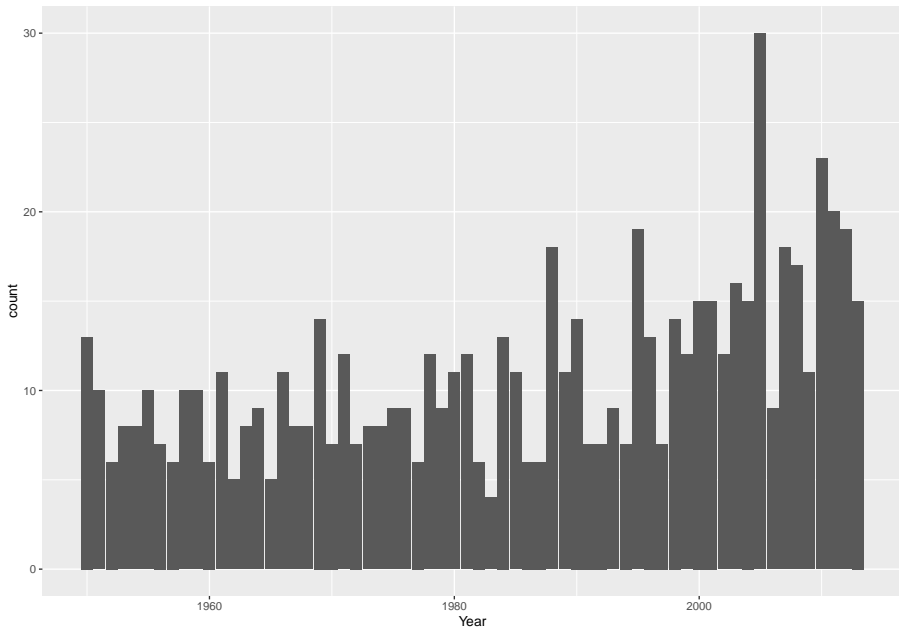
# EDA - Count of Hurricanes in Each Month



# EDA - Average Speed (knot) of Hurricanes in Each Month

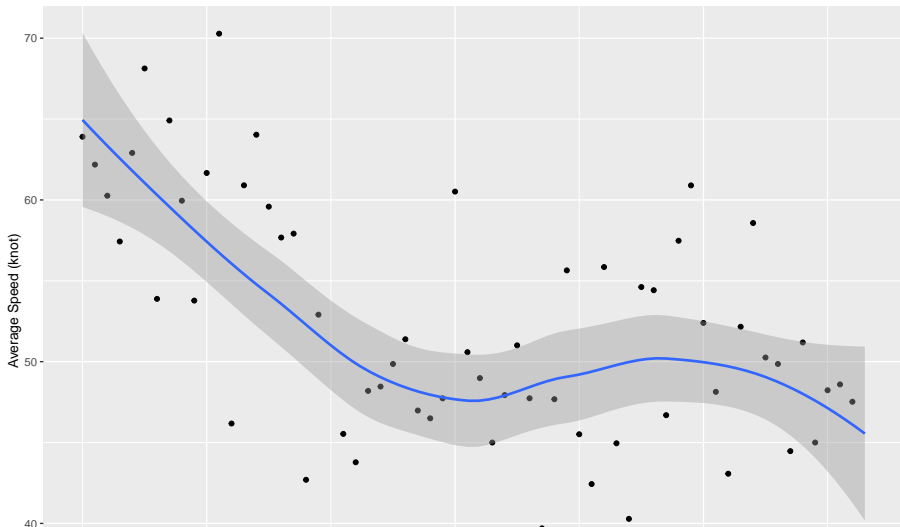


# EDA - Count of Hurricanes in Each Year

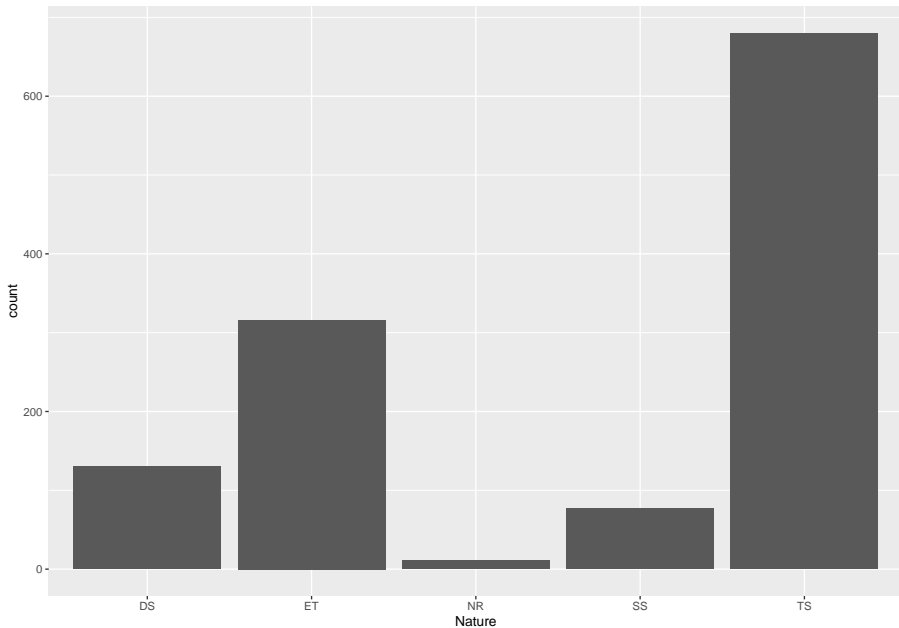


# EDA - Average Speed (knot) of Hurricanes in Each Year

```
## `geom_smooth()` using formula 'y ~ x'
```

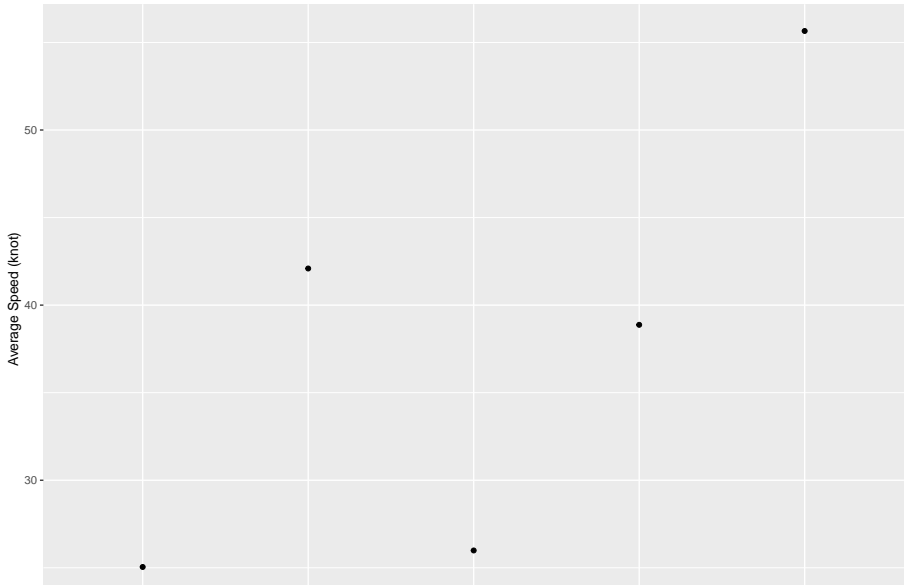


# EDA - Count of Hurricanes in Each Nature





# EDA - Average Speed (knot) of Hurricanes in Each Nature



# Bayesian Model Setting

## Model

The suggested Bayesian model is

$$Y_i(t+6) = \beta_{0,i} + \beta_{1,i}Y_i(t) + \beta_{2,i}\Delta_{i,1}(t) + \beta_{3,i}\Delta_{i,2}(t) + \beta_{4,i}\Delta_{i,3}(t) + \epsilon_i(t)$$

where

- where  $Y_i(t)$  the wind speed at time  $t$  (i.e. 6 hours earlier),  $\Delta_{i,1}(t)$ ,  $\Delta_{i,2}(t)$  and  $\Delta_{i,3}(t)$  are the changes of latitude, longitude and wind speed between  $t$  and  $t - 6$ , and  $\epsilon_{i,t}$  follows a normal distributions with mean zero and variance  $\sigma^2$ , independent across  $t$ .
- $\beta_i = (\beta_{0,i}, \beta_{1,i}, \dots, \beta_{5,i})$ , we assume that  $\beta_i \sim N(\mu, \Sigma_{d \times d})$ , where  $d$  is dimension of  $\beta_i$ .

## Priors

$$P(\sigma^2) \propto \frac{1}{\sigma^2}; \quad P(\mu) \propto 1; \quad P(\Sigma^{-1}) \propto |\Sigma|^{-(d+1)} \exp(-\frac{1}{2}\Sigma^{-1})$$

## Posterior

# Joint posterior

## Notations

- $X_i(t)\beta_i^\top = \beta_{0,i} + \beta_{1,i}Y_i(t) + \beta_{2,i}\Delta_{i,1}(t) + \beta_{3,i}\Delta_{i,2}(t) + \beta_{4,i}\Delta_{i,3}(t)$
- For  $i^{th}$  hurricane, there may be  $m_i$  times of record (excluding the first and second observation), let

$$Y_i = \begin{pmatrix} Y_i(t_0 + 6) \\ Y_i(t_1 + 6) \\ \vdots \\ Y_i(t_{m_i-1} + 6) \end{pmatrix}_{m_i \times 1}$$

- Hence,  $Y_i | X_i, \beta_i, \sigma^2 \sim N(X_i\beta_i^\top, \sigma^2 I)$
- Where,  $X_i$  is a  $m_i \times d$  dimensional matrix

$$X_i = \begin{pmatrix} 1 & Y_i(t_0) & \Delta_{i,1}(t_0) & \Delta_{i,2}(t_0) & \Delta_{i,3}(t_0) \\ 1 & Y_i(t_1) & \Delta_{i,1}(t_1) & \Delta_{i,2}(t_1) & \Delta_{i,3}(t_1) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & Y_i(t_{m_i-1}) & \Delta_{i,1}(t_{m_i-1}) & \Delta_{i,2}(t_{m_i-1}) & \Delta_{i,3}(t_{m_i-1}) \end{pmatrix}$$

# Joint posterior

## Posterior

$$\begin{aligned}\pi(\Theta|Y) &= \pi(\mathbf{B}^\top, \mu^\top, \sigma^2, \Sigma | Y) \\ &\propto \underbrace{\prod_{i=1}^n f(Y_i | \beta_i, \sigma^2)}_{\text{likelihood of } Y} \underbrace{\prod_{i=1}^n \pi(\beta_i | \mu, \Sigma)}_{\text{distribution of } \mathbf{B}} \underbrace{P(\sigma^2)P(\mu)P(\Sigma^{-1})}_{\text{priors}} \\ &\propto \prod_{i=1}^n \left\{ (2\pi\sigma^2)^{-m_i/2} \exp \left\{ -\frac{1}{2}(Y_i - X_i\beta_i^\top)^\top (\sigma^2 I)^{-1} (Y_i - X_i\beta_i^\top) \right\} \right\} \\ &\quad \times \prod_{i=1}^n \left\{ \det(2\pi\Sigma)^{-\frac{1}{2}} \exp \left\{ -\frac{1}{2}(\beta_i - \mu)\Sigma^{-1}(\beta_i - \mu)^\top \right\} \right\} \\ &\quad \times \frac{1}{\sigma^2} \times \det(\Sigma)^{-(d+1)} \exp \left\{ -\frac{1}{2}\Sigma^{-1} \right\}\end{aligned}$$

# MCMC Algorithm

## Conditional Posterior

- To apply MCMC using Gibbs sampling, we need to find conditional posterior distribution of each parameter, then we can implement Gibbs sampling on these conditional posterior distributions.
  - ▶  $\pi(\mathbf{B}|Y, \mu^\top, \sigma^2, \Sigma)$
  - ▶  $\pi(\sigma^2|Y, \mathbf{B}^\top, \mu^\top, \Sigma)$
  - ▶  $\pi(\Sigma|Y, \mathbf{B}^\top, \mu^\top, \sigma^2)$
  - ▶  $\pi(\mu|Y, \mathbf{B}^\top, \sigma^2, \Sigma)$

# MCMC Algorithm - Conditional Posterior

- $\beta_i$ :  $\pi(\beta_i|Y, \mu^\top, \sigma^2, \Sigma) \sim \mathcal{N}(\hat{\beta}_i, \hat{\Sigma}_{\beta_i})$ 
  - ▶ where  $\hat{\beta}_i = (\Sigma^{-1} + X_i^\top (\sigma^2 I)^{-1} X_i)^{-1} Y_i^\top (\sigma^2 I)^{-1} X_i + \mu \Sigma^{-1}$ ,  $\hat{\Sigma}_{\beta_i} = (\Sigma^{-1} + X_i^\top (\sigma^2 I)^{-1} X_i)^{-1}$
- $\sigma^2$ :
$$\pi(\sigma^2|Y, \mathbf{B}^\top, \mu^\top, \Sigma) \sim IG(\frac{1}{2} \sum_{i=1}^n m_i, \frac{1}{2} \sum_{i=1}^n (Y_i - X_i \beta_i^\top)^\top (Y_i - X_i \beta_i^\top))$$
- $\Sigma$ :  $\pi(\Sigma|Y, \mathbf{B}^\top, \mu^\top, \sigma^2) \sim IW(n + d + 1, I + \sum_{i=1}^n (\beta_i - \mu)(\beta_i - \mu)^\top)$
- $\mu$ :  $\pi(\mu|Y, \mathbf{B}^\top, \sigma^2, \Sigma) \sim \mathcal{N}(\frac{1}{n} \sum_{i=1}^n \beta_i, \frac{1}{n} \Sigma)$

# MCMC Algorithm - Parameter Updates

The update of parameters is component wise, at  $(t + 1)^{\text{th}}$  step, updating parameters in the following the order:

❶ Sample  $\mathbf{B}^{(t+1)}$ , i.e., sample each  $\beta_i^{(t+1)}$  from  $\mathcal{N}(\hat{\beta}_i^{(t)}, \hat{\Sigma}_{\beta_i}^{(t)})$

❷ Then, sample  $\sigma^2$  from

$$IG(\frac{1}{2} \sum_{i=1}^n m_i, \frac{1}{2} \sum_{i=1}^n (Y_i - X_i \beta_i^{(t+1)})^\top (Y_i - X_i \beta_i^{(t+1)}))$$

❸ Next, sample  $\Sigma^{(t+1)}$  from

$$IW(n + d + 1, I + \sum_{i=1}^n (\beta_i^{(t+1)} - \mu^{(t)})(\beta_i^{(t+1)} - \mu^{(t)})^\top)$$

❹ Finally, sample  $\mu^{(t+1)}$  from  $\mathcal{N}(\frac{1}{n} \sum_{i=1}^n \beta_i^{(t+1)}, \frac{1}{n} \Sigma^{(t+1)})$

# MCMC Algorithm - Initial Values

- 1 For initial value of  $\mathbf{B}$ , we run multivariate linear regressions for each hurricane and use the regression coefficients  $\beta_i^{MLR}$  as the initial value for  $\beta_i$ . Then, the initial value of  $\mathbf{B}$  can be represented as

$$\mathbf{B}_{init} = (\beta_1^{MLR^\top}, \dots, \beta_n^{MLR^\top})^\top.$$

- 2 For initial value of  $\mu$ , we take the average of  $\beta_i^{MLR}$ , that is

$$\mu_{init} = \frac{1}{n} \sum_{i=1}^n \beta_i^{MLR}$$

- 3 For initial value of  $\sigma^2$ , we take the average of the MSE for  $i$  hurricanes.
- 4 For initial value of  $\Sigma$ , we just set it to a simple diagonal matrix, i.e.  $\Sigma_{init} = \text{diag}(1, 2, 3, 4, 5)$

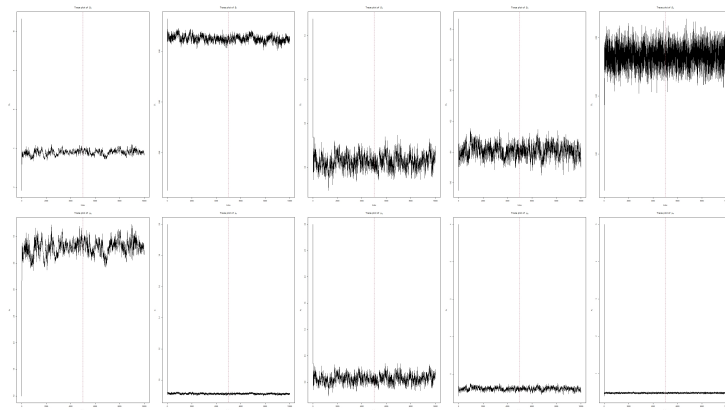


# MCMC Results

## Details

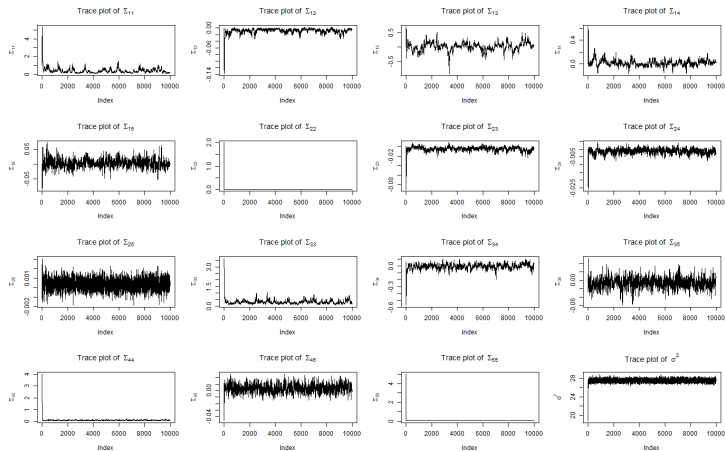
- 10000 iterations
- First 5000 iterations as burn-in period
- Estimates and inferences based on last 5000 MCMC samples

# MCMC Results - Trace Plots 1



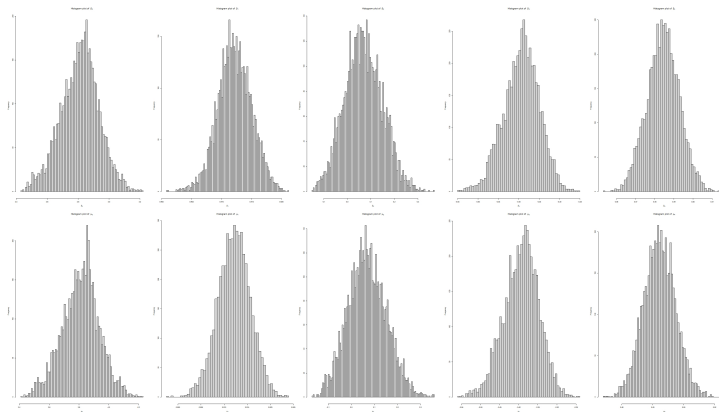
Trace plots of model parameters, based on 10000 MCMC sample

# MCMC Results - Trace Plots 2



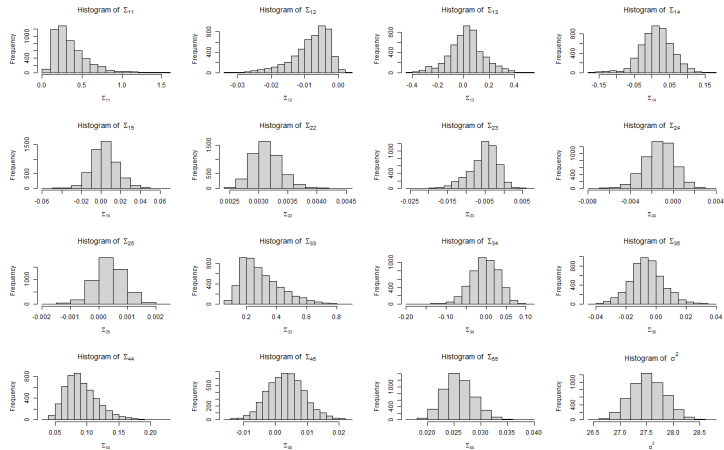
Trace plots of variance parameters, based on 10000 MCMC sample

# MCMC Results - Histograms 1



Histograms of model parameters, based on last 5000 MCMC sample

# MCMC Results - Histograms 2



Histograms of variance parameters, based on last 5000 MCMC sample

# MCMC Results - Model Parameter Estimations and Inferences

Variables	$\bar{\beta}_i$	$\text{Var}(\bar{\beta}_i)$	95% CI of $\bar{\beta}_i$	$\bar{\mu}$	$\text{Var}(\bar{\mu})$	95% CI of $\bar{\mu}$
intercept	3.8252	0.0185	(3.5588,4.0916)	3.8166	0.0190	(3.5468,4.0865)
Wind_prev	0.9118	0.0000	(0.9059,0.9177)	0.9121	0.0000	(0.9049,0.9194)
Lat_change	0.0744	0.0060	(-0.0776,0.2264)	0.0720	0.0065	(-0.0857,0.2298)
Long_change	-0.4014	0.0015	(-0.4771,-0.3257)	-0.3968	0.0016	(-0.4759,-0.3177)
Wind_change	0.4841	0.0001	(0.4674,0.5009)	0.4847	0.0001	(0.464,0.5053)

Bayesian posterior estimates for model parameters

# MCMC Results - Variance Parameter Estimations and Inferences

Parameters	Estimates	Variance	95% CI
$\Sigma_{11}$	0.3493	0.0435	(-0.0595,0.7581)
$\Sigma_{12}$	-0.0081	0.0000	(-0.0189,0.0027)
$\Sigma_{13}$	0.0201	0.0176	(-0.2399,0.2801)
$\Sigma_{14}$	0.0131	0.0019	(-0.0725,0.0987)
$\Sigma_{15}$	0.0035	0.0002	(-0.0215,0.0285)
$\Sigma_{22}$	0.0031	0.0000	(0.0026,0.0036)
$\Sigma_{23}$	-0.0053	0.0000	(-0.0125,0.0019)
$\Sigma_{24}$	-0.0013	0.0000	(-0.0041,0.0014)
$\Sigma_{25}$	0.0004	0.0000	(-7e-04,0.0014)
$\Sigma_{33}$	0.2960	0.0176	(0.0362,0.5558)
$\Sigma_{34}$	-0.0031	0.0012	(-0.0716,0.0653)
$\Sigma_{35}$	-0.0060	0.0001	(-0.0276,0.0156)
$\Sigma_{44}$	0.0918	0.0007	(0.0412,0.1424)
$\Sigma_{45}$	0.0034	0.0000	(-0.008,0.0148)
$\Sigma_{55}$	0.0258	0.0000	(0.0203,0.0313)

# Seasonal Difference Exploration

	Beta 0		Beta 1		Beta 2		Beta 3		Beta 4	
	Estimate	Pr(> t )	Estimate	Pr(> t )	Estimate	Pr(> t )	Estimate	Pr(> t )	Estimate	Pr(> t )
(Intercept)	4.5142875	0.0000000	1.3448481	0.0000000	-0.1056332	0.8629385	-1.0267628	0.001781	0.3051312	0.3817170
season	-0.0003543	0.0497902	-0.0002178	0.0001332	0.0000878	0.7757368	0.0003188	0.053474	0.0000902	0.6072986

Fitted results of beta models



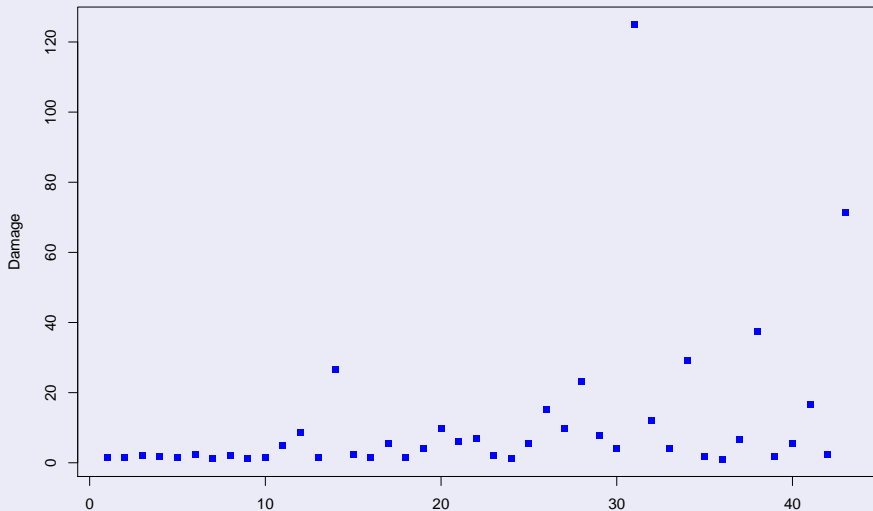
# Seasonal Difference Exploration

	Beta 0		Beta 1		Beta 2		Beta 3		Beta 4	
	Estimate	Pr(> t )	Estimate	Pr(> t )	Estimate	Pr(> t )	Estimate	Pr(> t )	Estimate	Pr(> t )
(Intercept)	4.4810021	0.0000000	1.3431063	0.0000000	0.0413063	0.9506172	-0.8336700	0.0185275	0.2890273	0.4482640
monthApril	0.0232609	0.8346449	0.0147943	0.6696787	0.0165579	0.9306863	0.0416468	0.6796126	0.0361823	0.7393892
monthMay	0.0259813	0.7827813	-0.0001180	0.9967888	0.0708822	0.6597505	0.0632772	0.4581672	-0.0162907	0.8594231
monthJune	0.0275693	0.7650618	0.0053935	0.8509869	-0.0070875	0.9641298	0.0556884	0.5047909	0.0237694	0.7918014
monthJuly	0.0125400	0.8914489	0.0154032	0.5901741	-0.0090910	0.9538180	0.0361214	0.6640154	0.0130817	0.8840332
monthAugust	-0.0198034	0.8284715	0.0233206	0.4124181	-0.0522548	0.7378961	0.0123691	0.8811234	0.0312427	0.7261962
monthSeptember	-0.0070528	0.9384385	0.0261005	0.3585599	-0.0361073	0.8169707	0.0212965	0.7966351	0.0444835	0.6177631
monthOctober	0.0093435	0.9185853	0.0210829	0.4587183	-0.0286163	0.8546050	0.0341549	0.6796975	0.0350505	0.6944480
monthNovember	0.0145692	0.8748155	0.0246144	0.3925264	0.0239972	0.8792681	0.0263450	0.7529105	0.0209069	0.8168323
monthDecember	0.0057977	0.9526542	0.0088244	0.7715305	-0.0543131	0.7447475	0.0422468	0.6326060	0.0114196	0.9046290
season	-0.0003419	0.0717253	-0.0002252	0.0001471	0.0000365	0.9101708	0.0002184	0.2032812	0.0000905	0.6249586
natureET	0.0008449	0.9774141	0.0037334	0.6877086	-0.0702038	0.1687975	-0.0263888	0.3286540	-0.0209217	0.4726774
natureNR	0.0008122	0.9866387	-0.0146142	0.3331114	0.0058967	0.9432660	0.0030556	0.9444979	-0.0217275	0.6462854
natureSS	0.0141564	0.4904257	-0.0033299	0.6021721	-0.0013517	0.9692484	0.0126339	0.4964264	-0.0238538	0.2339965
natureTS	0.0118370	0.4785102	-0.0059979	0.2486925	-0.0154533	0.5880814	-0.0231521	0.1258337	-0.0174987	0.2832214

Fitted results of beta models for only the year variable

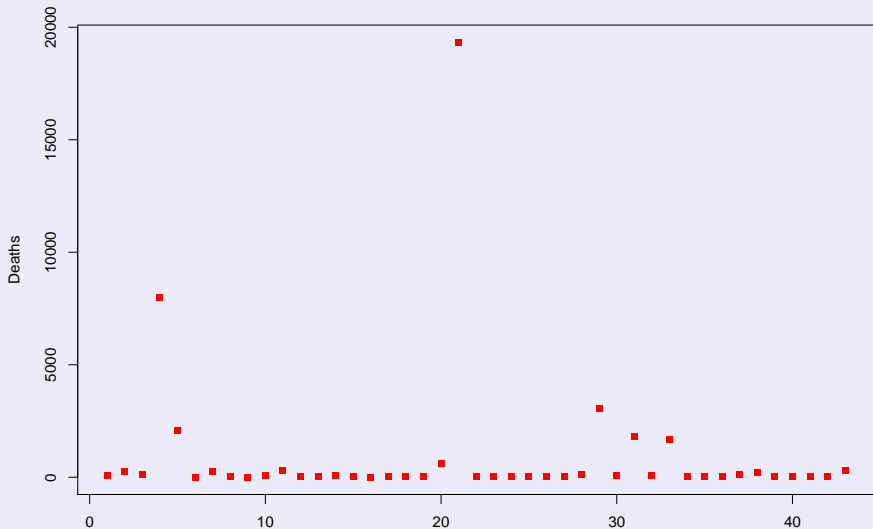
# Predictions of Damage and Deaths

## Basic plot of Damage and Deaths



# Predictions of Damage and Deaths

## Basic plot of Damage and Deaths



# Coefficient Table

[1]	" id	intercept	beta1	beta2	beta3	beta4 "
[2]	" :-----	-----:	-----:	-----:	-----:	-----: "
[3]	" agnes.1972	3.950974	0.9224097	0.0059532	-0.3103372	0.5453543 "
[4]	" alex.2010	3.798737	0.9370333	0.0698849	-0.3937358	0.5400187 "
[5]	" alicia.1983	3.897408	0.9036878	-0.0748341	-0.3994486	0.5477718 "
[6]	" allen.1980	3.687070	0.9655304	0.1306393	-0.5460144	0.5466129 "
[7]	" andrew.1992	3.676279	0.9375384	-0.2843257	-0.5782973	0.5370158 "
[8]	" betsy.1965	3.808396	0.9513766	-0.4500720	-0.3890718	0.4244575 "
[9]	" bob.1991	3.629466	0.9232143	0.0279527	-0.5751636	0.4382048 "
[10]	" camille.1969	3.994355	0.9355674	0.0729188	-0.5734830	0.6703910 "

Fitted results of beta models

# Predict Damage

	term	estimate	std.error	statistic	p.value
1	(Intercept)	-2.179428e+02	63.786161983	-3.416772	6.336828e-04
2	intercept	5.044916e+00	0.872632934	5.781258	7.414400e-09
3	beta1	6.283543e+01	14.027126920	4.479565	7.479523e-06
4	beta2	-1.095810e+00	0.424325439	-2.582476	9.809426e-03
5	beta3	3.378223e+00	0.816050104	4.139725	3.477231e-05
6	nobs	4.921117e-02	0.008036275	6.123630	9.146733e-10
7	Season	7.497698e-02	0.012627373	5.937655	2.891284e-09
8	MonthJune	-3.416174e+00	0.762110791	-4.482516	7.376795e-06
9	MonthNovember	-1.902107e+00	0.789148853	-2.410327	1.593822e-02
10	MonthOctober	-1.290673e+00	0.298201079	-4.328198	1.503344e-05
11	MonthSeptember	-1.764116e+00	0.243173467	-7.254558	4.029764e-13
12	NatureNR	-4.317468e+00	1.126675716	-3.832042	1.270843e-04
13	NatureTS	-2.038481e+00	0.452900892	-4.500942	6.765302e-06
14	Maxspeed	5.044572e-02	0.006764325	7.457613	8.810369e-14
15	Meanspeed	-6.565465e-02	0.015403789	-4.262240	2.023877e-05
16	Percent.Poor	-3.819578e-02	0.005858677	-6.519522	7.053169e-11

# Predict Deaths

	term	estimate	std.error	statistic	p.value
1	(Intercept)	1.164978e+02	1.257956e+01	9.260883	2.027487e-20
2	intercept	1.167475e+01	2.564192e-01	45.529931	0.000000e+00
3	beta1	1.141195e+02	2.200144e+00	51.869091	0.000000e+00
4	beta2	5.528798e+00	1.226329e-01	45.084128	0.000000e+00
5	beta3	8.561691e+00	2.853214e-01	30.007184	7.908823e-198
6	beta4	-1.049211e+01	3.058279e-01	-34.307225	6.123346e-258
7	nobs	3.430943e-03	1.116605e-03	3.072657	2.121619e-03
8	Season	6.102077e-03	2.093747e-03	2.914429	3.563401e-03
9	MonthJuly	-1.183782e+00	1.448847e-01	-8.170505	3.071002e-16
10	MonthJune	-1.291597e+00	8.968191e-02	-14.401980	5.028215e-47
11	MonthNovember	-2.533192e+00	1.551869e-01	-16.323490	6.718278e-60
12	MonthOctober	-1.546676e+00	6.466487e-02	-23.918335	1.974205e-126
13	MonthSeptember	-2.751167e-01	4.588850e-02	-5.995331	2.030720e-09
14	NatureNR	2.348783e+00	1.290216e-01	18.204563	4.748263e-74
15	NatureTS	3.563406e+00	1.209962e-01	29.450564	1.238185e-190
16	Meanspeed	-3.676417e-02	3.143216e-03	-11.696356	1.330451e-31
17	Maxpressure	-2.686076e-01	9.670821e-03	-27.775052	8.684053e-170
18	Meanpressure	5.377225e-03	2.009523e-04	26.758717	9.775966e-158
19	Total.Pop	9.410461e-07	2.587520e-08	36.368659	1.332659e-289
20	Percent.Poor	3.599824e-02	8.024514e-04	44.860342	0.000000e+00
21	Percent.USA	-7.214139e-03	5.570867e-04	-12.949761	2.356879e-38