> **#задание 1 вар.3 : Упростите алгебраическое выражение**

> $c1 := \frac{x^3+3x^2-9x-27}{x^3-5x^2-15x-72};$

$c2 := \frac{49x^4-882x^2+3969}{x^4-8x^3-27x+216};$

simplify -

$simplify\left(\frac{c1}{c2}\right);$

restart;

$$c1 := \frac{x^3+3x^2-9x-27}{x^3-5x^2-15x-72}$$

$$c2 := \frac{49x^4-882x^2+3969}{x^4-8x^3-27x+216}$$

$$\frac{1}{49} \qquad \textbf{(1)}$$

> **#задание 2 вар.3 : Приведите выражение к многочлену стандартного вида**

> $c1 := (4x-3)\cdot(3x^2+1)\cdot(5x+2);$

e x p a n d -

$expand(c1);$

restart;

$$c1 := (4x-3)\ (3x^2+1)\ (5x+2)$$

$$60x^4-21x^3+2x^2-7x-6 \qquad \textbf{(2)}$$

> **#задание 3 вар.3 : Разложите многочлен на множители**

f a c t o r -

> $c1 := factor(4x^4-31x^3+33x^2\text{-}93x+63);$

restart;

$$c1 := (x-7)\ (4x-3)\ (x^2+3) \qquad \textbf{(3)}$$

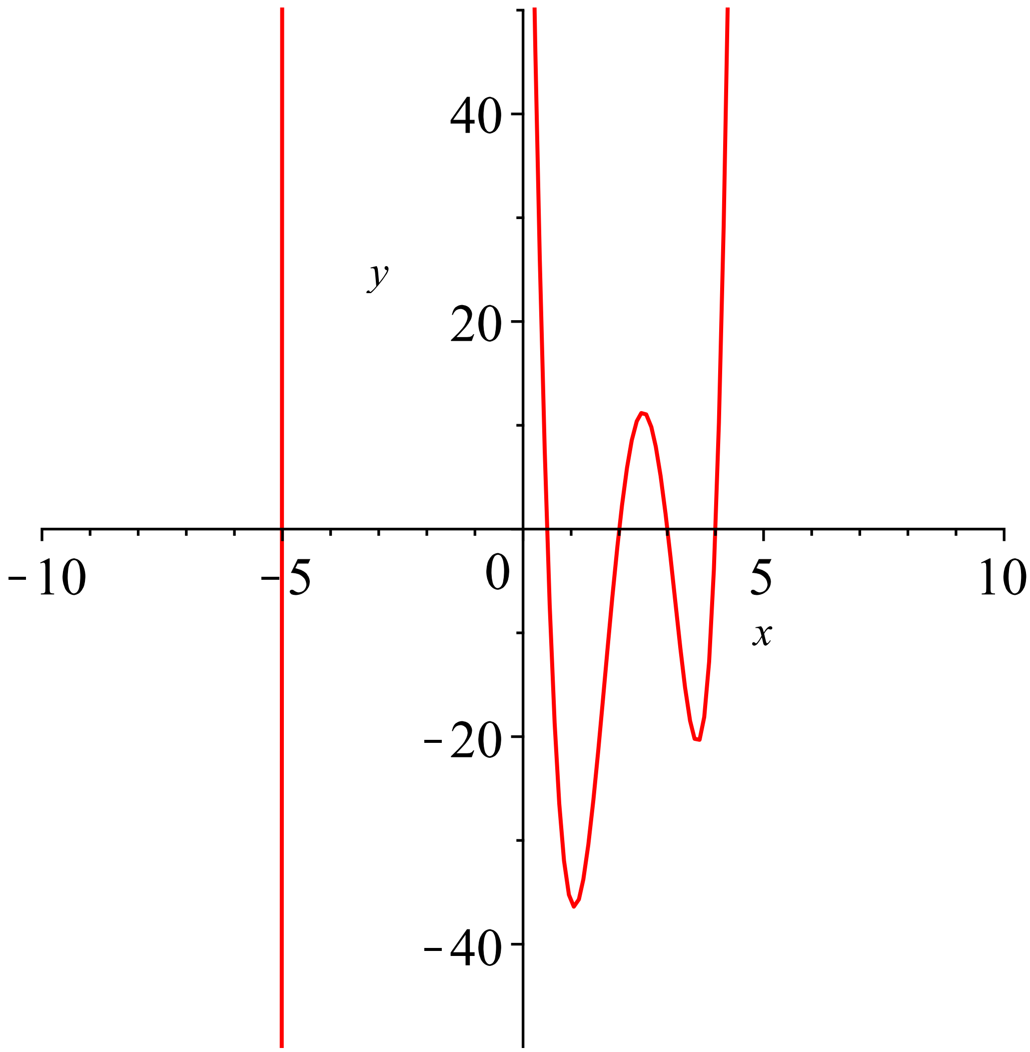
> ***#задание 4 вар.3 : Постройте график многочлена $P_5(x)$ и найдите все #его корни.***

> $P(x) := 2x^5 - 9x^4 - 34x^3 + 231x^2 - 346x + 120;$

plot - построение графика функции

plot(P(x), x = -10 .. 10 , y = -50 .. 50 , color = red,);

$$P := x \rightarrow 2x^5 - 9x^4 - 34x^3 + 231x^2 - 346x + 120$$



s o l v e -

> *solve(c1);*
restart

$$0 \quad (4)$$

> ***задание 5 вар.3 Разложите рациональную дробь на сумму простейших дробей***

$$c1 := \frac{5x^4+7x^3+3x-1}{\left(x^2+2\right)\cdot(x-1)^2\cdot\left(x^2-9\right)};$$

c o n v e r t -
parfrac - необходим для

convert(*c1*, *parfrac*);
restart;

$$c1 := \frac{5x^4+7x^3+3x-1}{\left(x^2+2\right)(x-1)^2\left(x^2-9\right)}$$

$$-\text{необходим для} + parfrac$$

$$-\frac{229}{144\,(x-1)} + \frac{-49x-25}{99\,\left(x^2+2\right)} - \frac{103}{528\,(x+3)} + \frac{301}{132\,(x-3)} - \frac{7}{12\,(x-1)^2} \qquad \textbf{(5)}$$

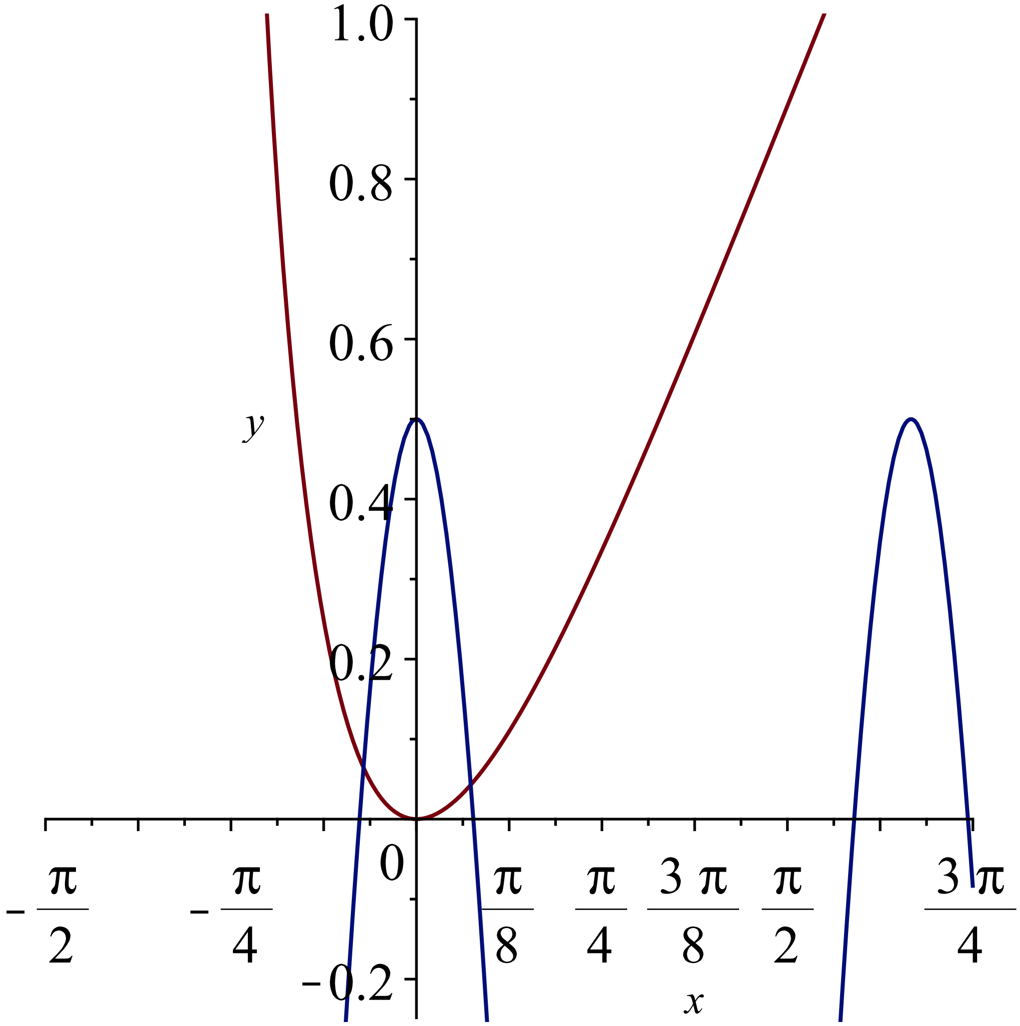
> ***#задание 6 вар.3. Решите графически уравнение и найдите его #приближенные корни с точностью до*** 10^{-5}

$c1 := \ln^2(x+1);$

$c2 := 2\cdot\cos(3\cdot x) - 1.5;$

$plot\left([\ c1\ ,\ c2\ ], x = -\frac{\pi}{2} .. \frac{3\pi}{4}, y = -\frac{1}{4} .. 1\right);$

$$c1 := \ln(x+1)^2$$
$$c2 := 2\cos(3x) - 1.5$$



> $evalf(fsolve(c1 = c2, x = -\infty..0), 6);$

$$-0.224192 \tag{6}$$

> $evalf(fsolve(c1 = c2, x = 0..+\infty), 6);$

e v a l f -

$$0.229913 \tag{7}$$

> *restart;*

задание 7 вар.3 Докажите, что $\lim_{n \to \infty} a_n = a$***, определив номер*** n_ε***, начиная с которого все члены последовательности*** (a_n) ***попадут в*** **ε** ***— окресность точки a.***
Проиллюстрируйте полученный результат с помощью чертежа в Maple, положив $\varepsilon = 1$

$an := \frac{5 \cdot n + 2}{3 \cdot n - 1}:$

$a := \frac{5}{3}:$

$anSeq := seq\left(\left[n, \frac{5 \cdot n + 2}{3 \cdot n - 1}\right], n = 1 ..30\right):$

seq - ,

$assume(n, natural, \varepsilon, positive);$

assume, **n** , **ε**

, $\mathbf{lim_{n \to \infty}\, a_n = a}$, это значит для $\forall\ \boldsymbol{\varepsilon} > 0$ n_ε ,

, , *по модулю отличаются от* **a** **ε**, .

. $|a_n - a| < \boldsymbol{\varepsilon}$

$anSimplifyied := |simplify(an - a)|:$

$|an - a| = anSimplifyied;$

$$\frac{5\, n\sim + 2}{3\, n\sim - 1} - \frac{5}{3} = \frac{11}{9\, n\sim - 3} \tag{8}$$

ε

$\frac{11}{9\, n - 3} < \varepsilon$

> $assume(n, natural, \varepsilon, positive); solve(anSimplifyied < \varepsilon, [n], UseAssumptions)$

\# **UseAssumptions** m a p l e **assume**

$$\begin{cases} [[n\sim = n\sim]] & \frac{3\, \varepsilon\sim + 11}{9\, \varepsilon\sim} < n\sim \\ [\ ] & otherwise \end{cases} \tag{9}$$

, $n_\varepsilon > \frac{3\,\varepsilon + 11}{9\,\varepsilon}$. *Значит* $\mathbf{lim_{n \to \infty}\, a_n = a}$

$\varepsilon := \frac{1}{10}:$

$solve(anSimplifyied < \varepsilon, n, UseAssumptions)$

$$\begin{cases} [\ ] & n\sim \le \frac{113}{9} \\ [n\sim] & \frac{113}{9} < n\sim \end{cases} \tag{10}$$

первый номер, **n**, начиная с которого все элементы последовательности $\mathbf{a_n}$

ε

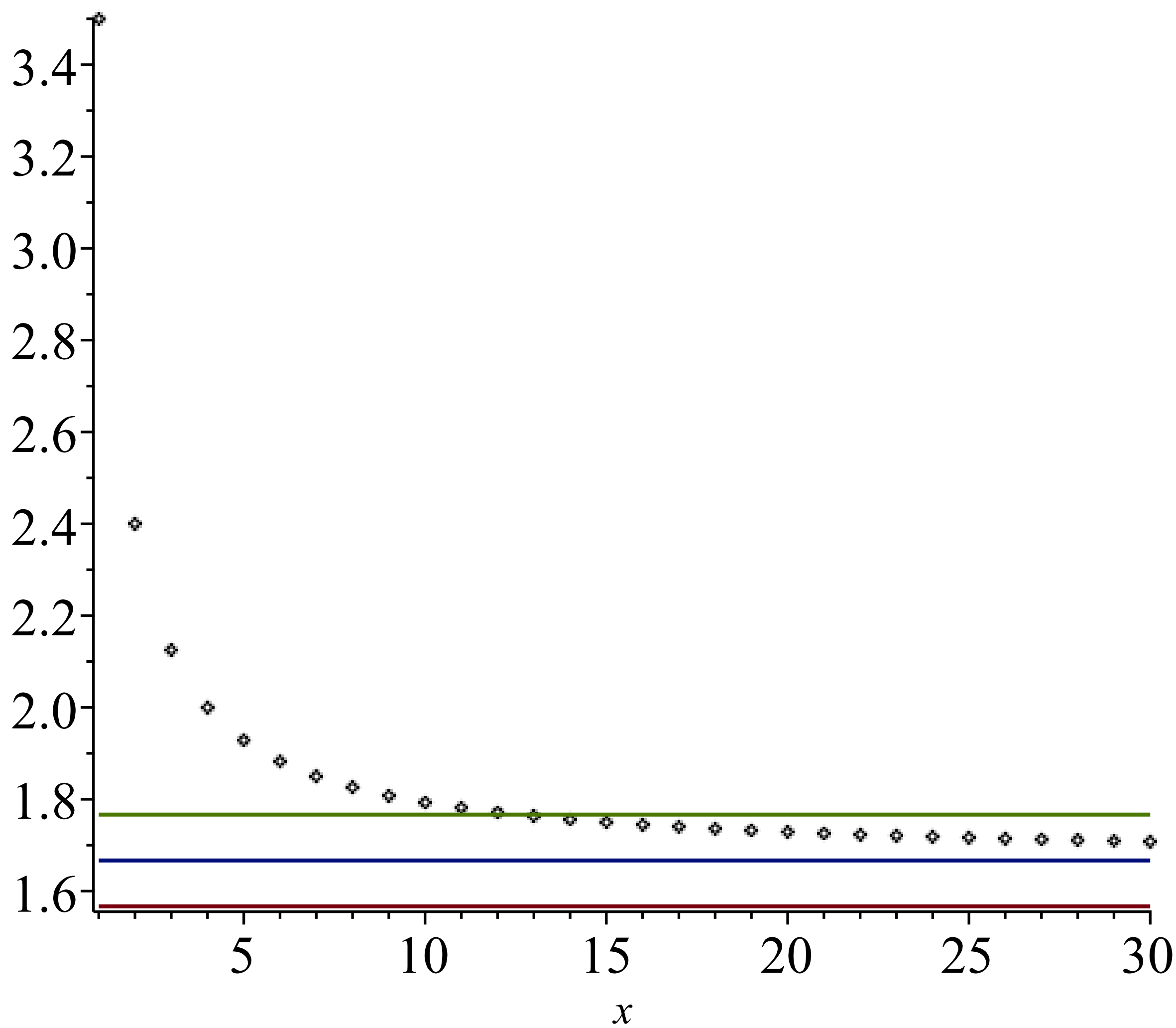
$\boldsymbol{n} = \mathrm{trunc}\left(simplify\left(\frac{113}{9} + 1\right)\right)$

$$n\sim = 13 \quad (11)$$

> $topl1 := plots[pointplot](\{anSeq\}) :$

> $topl2 := plot\left(\left[a - \frac{1}{10}, a, a + \frac{1}{10}\right], x = 1..30\right) :$

> $plots[display](topl1, topl2)$



> $restart;$

8 . . 3

$assume(n, natural) :$

$an := \mathrm{sqrt}(n \cdot (n+2)) - \mathrm{sqrt}(n^2 - 2 \cdot n + 3) :$

$Limit(an, n = \mathrm{infinity}) = limit(an, n = \mathrm{infinity});$

$a := limit(an, n = \mathrm{infinity}) :$

an - , -

.

$$\lim_{n\sim \to \infty}\left(\sqrt{n\sim\,(n\sim+2)} - \sqrt{n\sim^2 - 2\,n\sim + 3}\right) = 2 \tag{12}$$

. :

,

an.

, : $\boldsymbol{n}_{\boldsymbol{\varepsilon}}$,

, ,

ε

$assume(\varepsilon, positive)$:
$|an - a| < \varepsilon$;

$$2 - \sqrt{n\sim\,(n\sim+2)} + \sqrt{n\sim^2 - 2\,n\sim + 3} < \varepsilon\sim \tag{13}$$

$assume(n, natural, \varepsilon, positive)$; $solve(|an - a| < \varepsilon, n, UseAssumptions)$;

$$\begin{cases} [n\sim] & 2 - \sqrt{n\sim\,(n\sim+2)} + \sqrt{n\sim^2 - 2\,n\sim + 3} < \varepsilon\sim \\ [\,] & otherwise \end{cases} \tag{14}$$

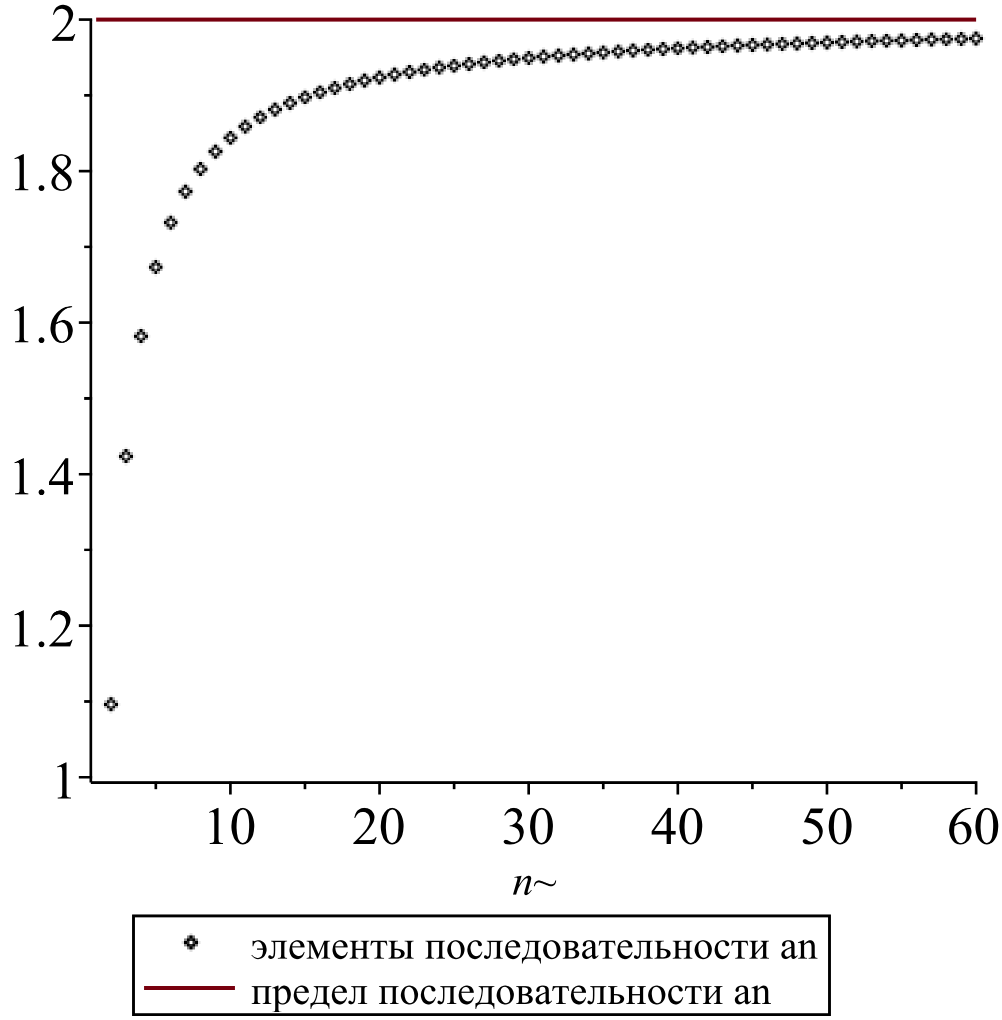
n - .

, **n** m a p l e .

$anSeq := seq\left(\left[n, \mathrm{sqrt}(n\cdot(n+2)) - \mathrm{sqrt}\left(n^2 - 2\cdot n + 3\right)\right], n = 1..60\right)$:

,

```
> toPlot1 := plots[pointplot]({anSeq}) :
> toPlot2 := plot(a, n = 1..60) :
> plots[display](toPlot1, toPlot2, view = [1..60, 1..2], legend
      = [typeset("элементы последовательности an"),
      typeset("предел последовательности an")]);
```



,

a **an**

> *restart*;

.

$assume(n, natural, \varepsilon, positive);$

$$an := \left(\frac{3\,n^2 - 5\,n}{3\,n^2 - 5\,n + 7}\right)^{n+1}:$$

> $Limit(an, n = \text{infinity}) = limit(an, n = \text{infinity});$

$$\lim_{n\sim \to \infty} \left(\frac{3\,n\sim^2 - 5\,n\sim}{3\,n\sim^2 - 5\,n\sim + 7}\right)^{n\sim + 1} = 1 \quad \textbf{(15)}$$

> $a := limit(an, n = \text{infinity}):$

> $assume(n, natural, \varepsilon, positive);\ \ solve(|an - a| < \varepsilon, n, UseAssumptions);$

(16)

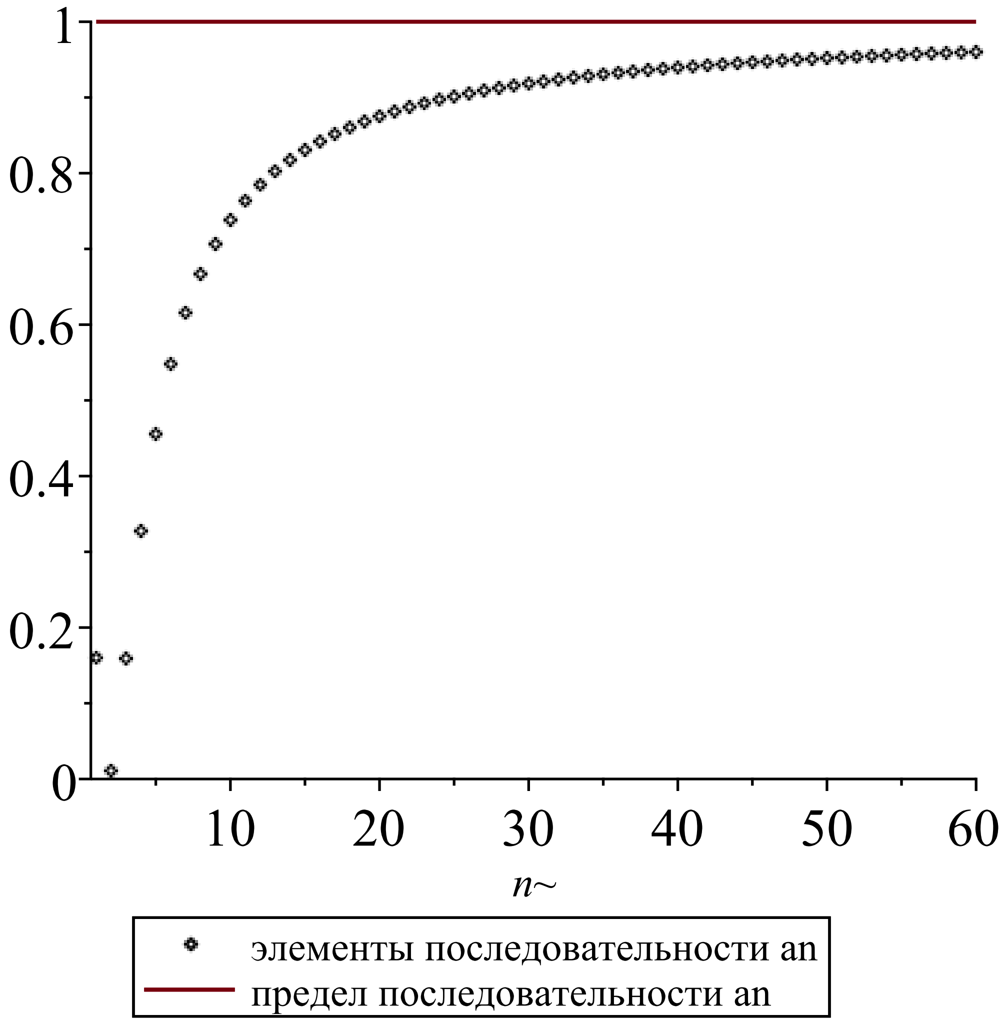
$$\begin{cases} [n\sim] & \left|\left(\dfrac{3\,n\sim^2-5\,n\sim}{3\,n\sim^2-5\,n\sim+7}\right)^{n\sim+1}-1\right| < \varepsilon\sim \\ [\,] & otherwise \end{cases} \tag{16}$$

> $anSeq := seq\left(\left[n, \left(\dfrac{3\,n^2-5\,n}{3\,n^2-5\,n+7}\right)^{n+1}\right], n=1..60\right):$

> $toPlot1 := plots[pointplot](\{anSeq\}):$

> $toPlot2 := plot(a, n=1..60):$

> $plots[display](toPlot1, toPlot2, view=[1..60, 0..1], legend$
= [*typeset*("элементы последовательности an"),
typeset("предел последовательности an")]);



> *restart*;

9 . . 3 -

:

1 . .

2 . **.**
3 .
.
4 . **,**
- **.**
5 . **,**

=1 ,=5 ,=0

1) **.**

piecewise,

$$f := (x1,\ldots,xn) \rightarrow (expr)$$

expr,

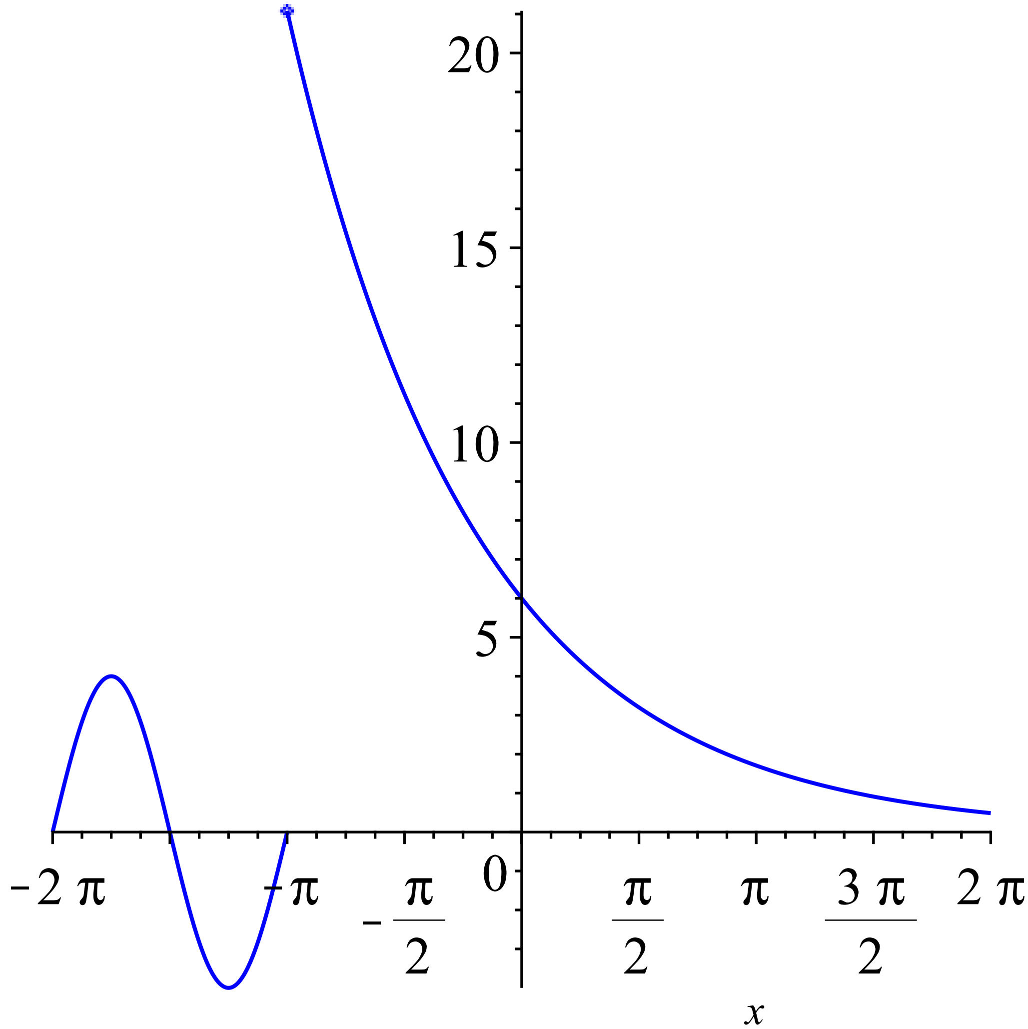
$f := x \rightarrow piecewise\left(x < -\pi, 4\sin(2x), x \geq -\pi, 6e^{-\frac{2}{5}x}\right);$

Error, invalid sequence

piecewise,

$$f := x \mapsto \begin{cases} 4\sin(2x) & x < -\pi \\ 6\,e^{-\frac{2x}{5}} & -\pi \leq x \end{cases} \tag{17}$$

> $toPlot3 := plot(f(x), color = blue, discont = true);$



2) .

> $limit(f(x), x = -\pi, left);$

$$0 \tag{18}$$

> $limit(f(x), x = -\pi, right);$

$$6\,(e^{\pi})^{2/5} \tag{19}$$

> $limit(f(x), x = +\infty);$

предел на +∞

$$0 \tag{20}$$

> $limit(f(x), x = -\infty);$

предел на -∞

$$-4..4 \tag{21}$$

> **3**

)***Найдите производную* и *неопределенный интеграл на каждом из промежутков непрерывности.***

$df := diff(f(x), x);$

$$df := \begin{cases} 8\cos(2x) & x < -\pi \\ undefined & x = -\pi \\ -\frac{12\,e^{-\frac{2x}{5}}}{5} & -\pi < x \end{cases} \quad (22)$$

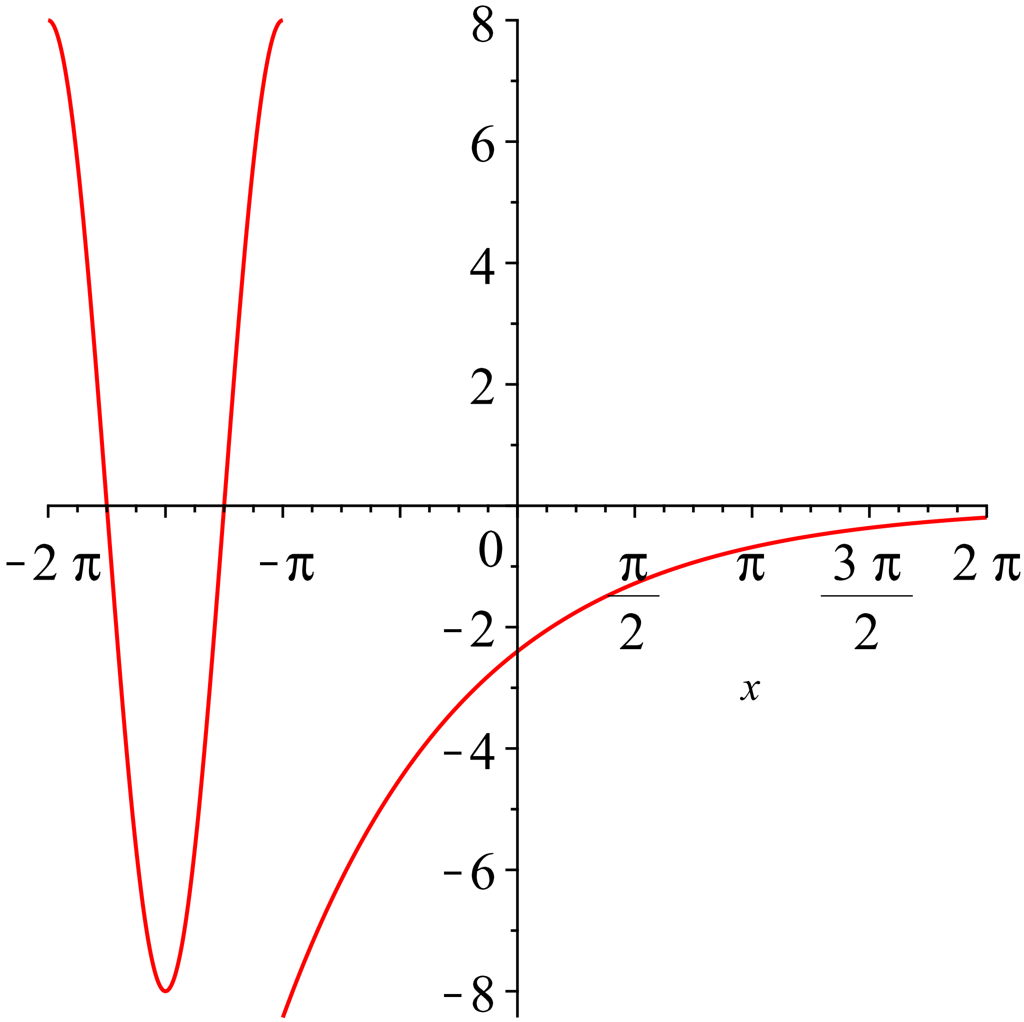
> $intf := int(f(x), x);$

$$intf := \begin{cases} -2\cos(2x) & x \le -\pi \\ -15\,e^{-\frac{2x}{5}} - 2 + 15\left(e^{\pi}\right)^{2/5} & -\pi < x \end{cases} \quad (23)$$

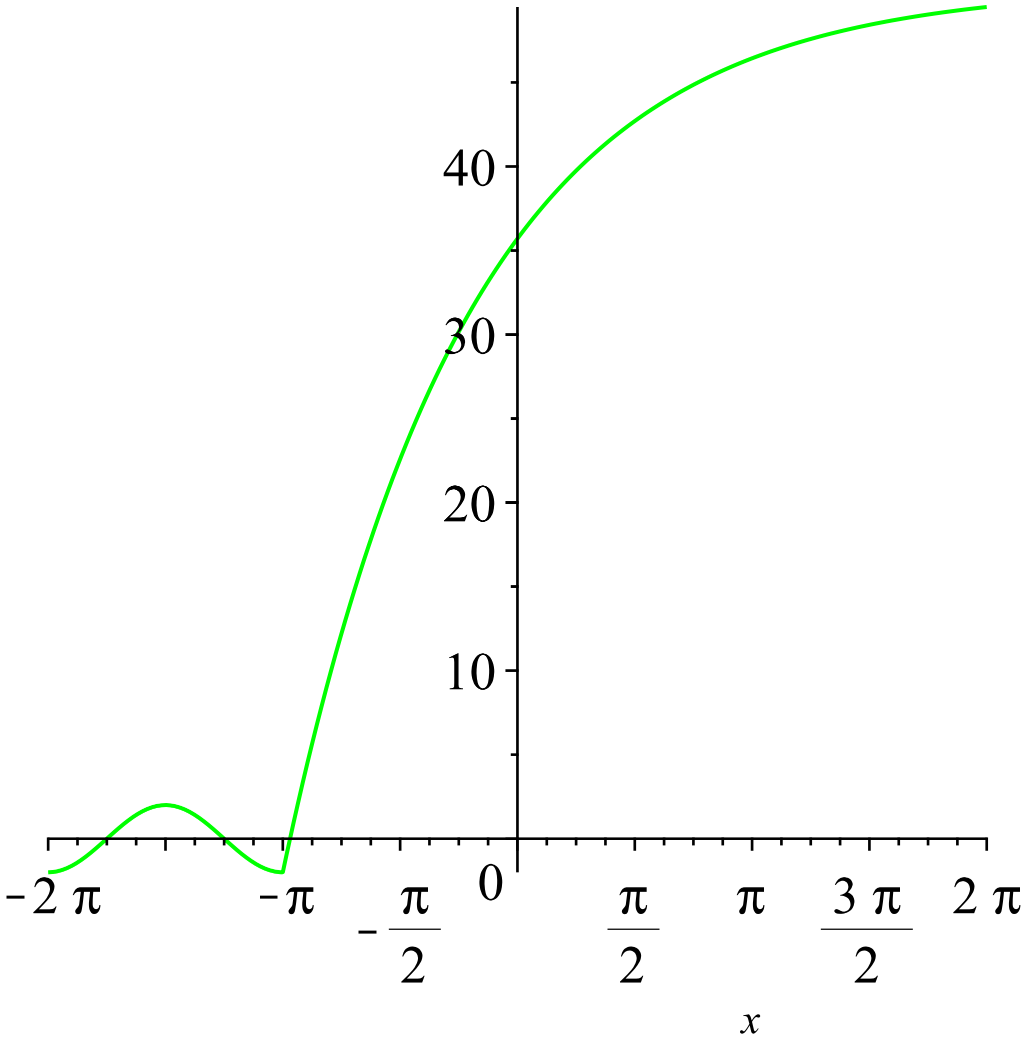
> **4**) ,

- .

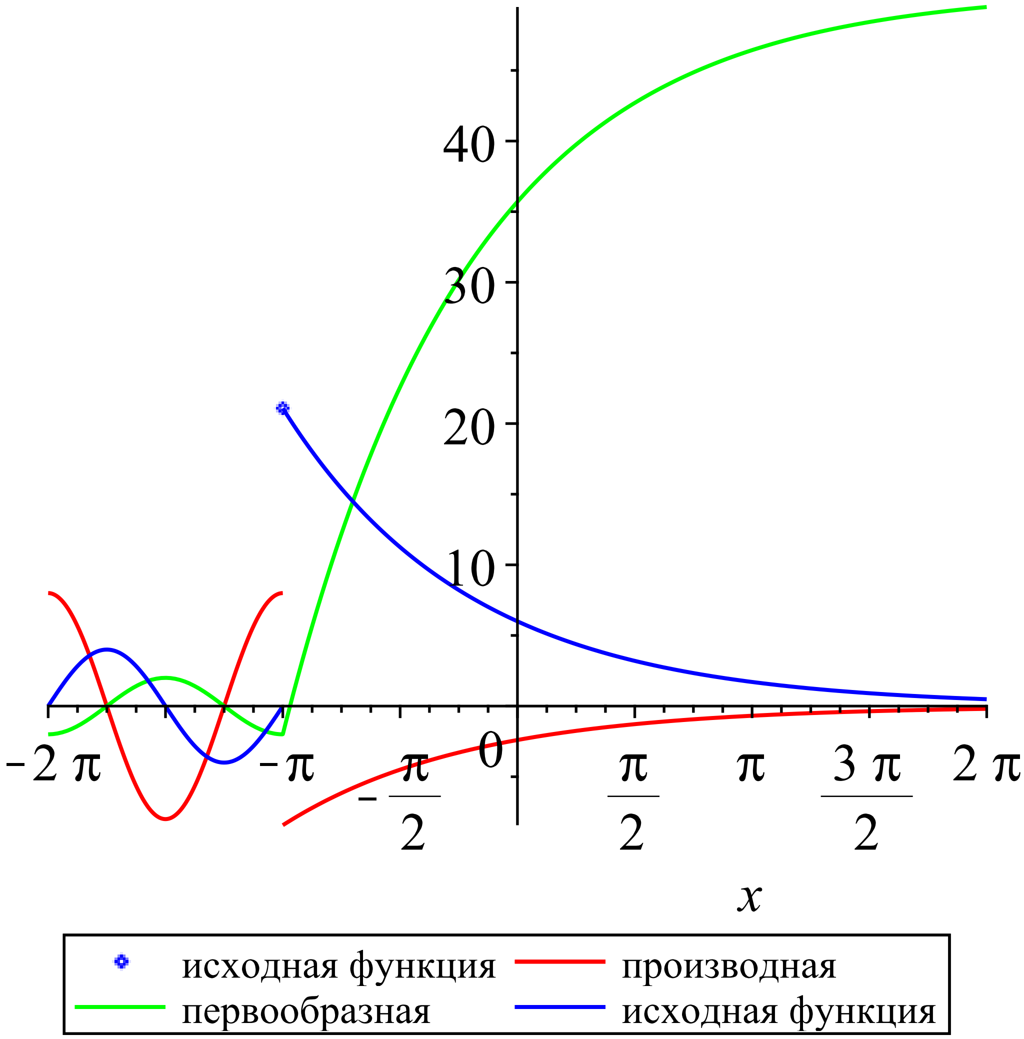
$toPlot1 := plot(df, color = red, discont = true);$



> $toPlot2 := plot(intf, color = green);$



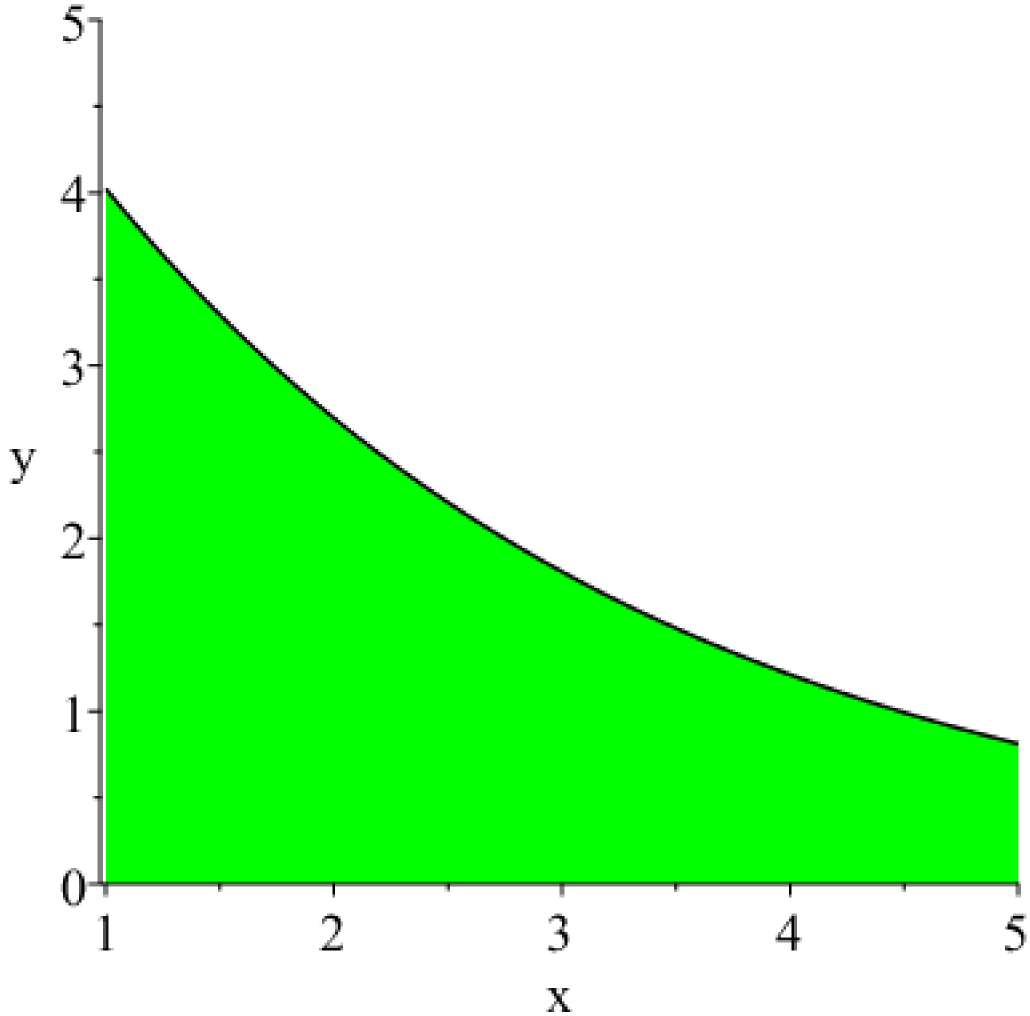
> *plots*[*display*](*toPlot1*, *toPlot2*, *toPlot3*, *legend* = [*typeset*("производная"), *typeset*("первообразная"), *typeset*("исходная функция")]);



> **5**) ***Найдите площадь криволинейной трапеции, ограниченной графиком функции* и *прямых* = 1 , = 5 , = 0 .**

функции и прямых **(24)**

> $plots[inequal]\left(\left\{y - 6e^{-\frac{2}{5}x} \le 0\right\}, x = 1..5, y = 0..5, optionsfeasible = (color = green)\right);$

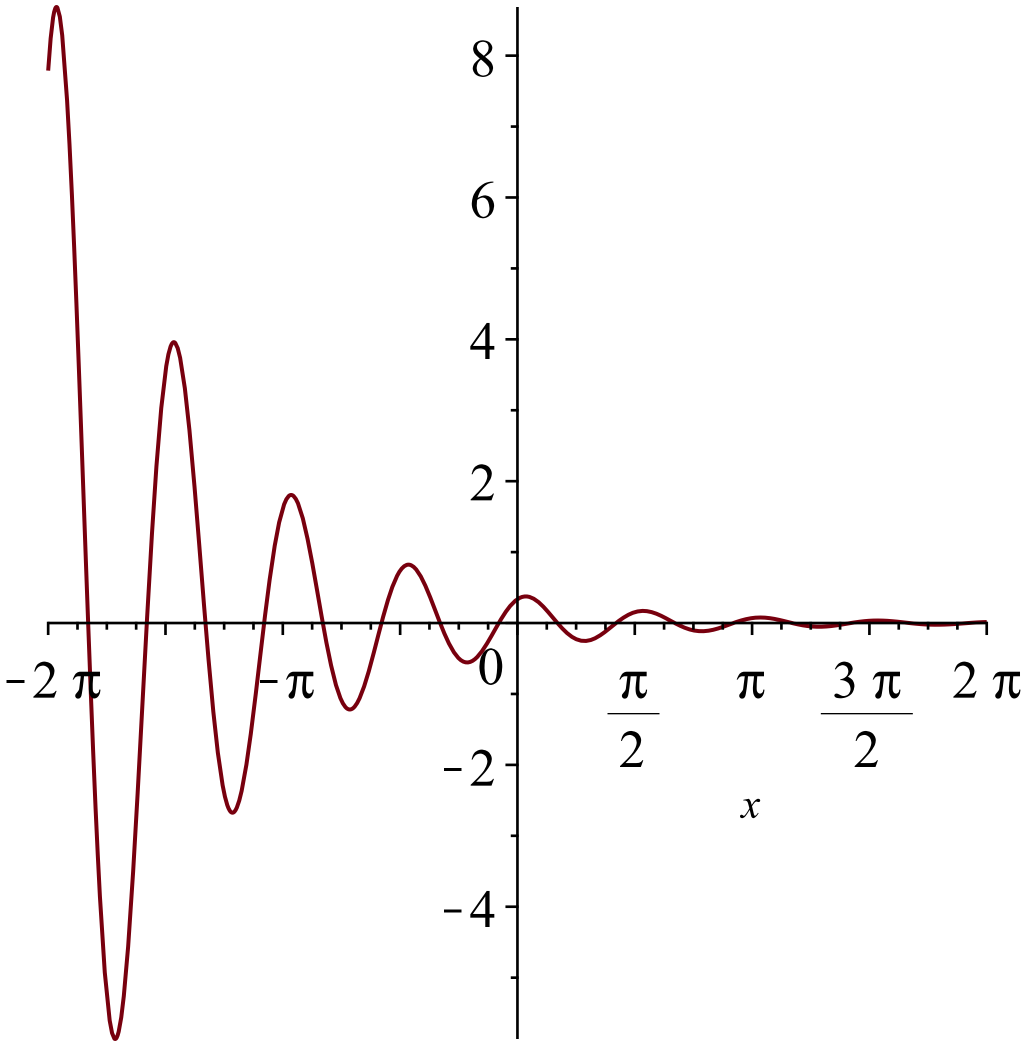


> $S := evalf(int(f(x), x = 1..5), 10);$
restart;

$$S := 8.024771442 \qquad (25)$$

1 0 . . 2 - (
2)

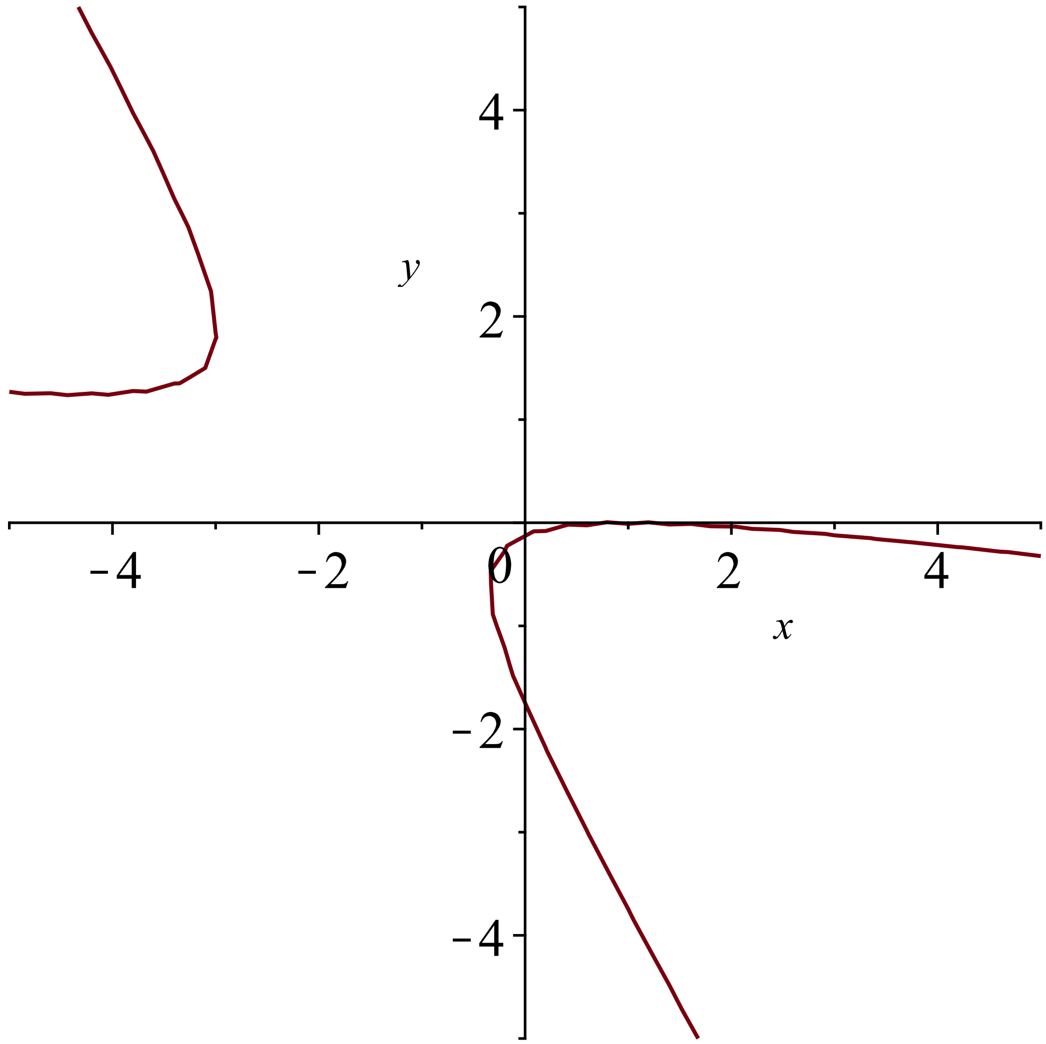
1) $f := x \rightarrow \frac{4}{10} \cdot e^{-\frac{1}{2}x} \cdot \sin(4x + 1) :$
$plot(f(x));$



2)

$with(plots) : with(LinearAlgebra) :$

$plots[implicitplot](7 \cdot x^2 + 60 \cdot x \cdot y + 32 \cdot y^2 - 14 \cdot x + 60 \cdot y + 7 = 0, x = -5 ..5, y = -5 ..5);$



($7x^2 + 60xy + 32y^2$)

$Q := x \rightarrow 7x^2 + 60xy + 32y^2;$
$dQx := diff(Q(x), x);$
$dQy := diff(Q(x), y);$

$$Q := x \mapsto 7x^2 + 60xy + 32y^2$$
$$dQx := 14x + 60y$$
$$dQy := 60x + 64y \quad \textbf{(26)}$$

:
$M := Matrix([[14, 60], [60, 64]]);$

$$M := \begin{bmatrix} 14 & 60 \\ 60 & 64 \end{bmatrix} \quad \textbf{(27)}$$

:
$v := LinearAlgebra[Eigenvectors](M);$

(28)

$$v := \left[\begin{array}{c} -26 \\ 104 \end{array}\right], \left[\begin{array}{cc} -\frac{3}{2} & \frac{2}{3} \\ 1 & 1 \end{array}\right] \quad \textbf{(28)}$$

> $e1 := Normalize(Column(v[2], [1]), Euclidean);$

$$e1 := \left[\begin{array}{c} -\frac{3\sqrt{13}}{13} \\ \frac{2\sqrt{13}}{13} \end{array}\right] \quad \textbf{(29)}$$

> $e2 := Normalize(Column(v[2], [2]), Euclidean);$

$$e2 := \left[\begin{array}{c} \frac{2\sqrt{13}}{13} \\ \frac{3\sqrt{13}}{13} \end{array}\right] \quad \textbf{(30)}$$

> $subs\left(x = e1[1] \cdot x1 + e2[1] \cdot y1, y = e1[2] \cdot x1 + e2[2] \cdot y1, 7 \cdot x^2 + 60 \cdot x \cdot y + 32 \cdot y^2 - 14 \cdot x + 60 \cdot y + 7\right);$

$$7\left(-\frac{3\,x1\sqrt{13}}{13} + \frac{2\,y1\sqrt{13}}{13}\right)^2 + 60\left(-\frac{3\,x1\sqrt{13}}{13} + \frac{2\,y1\sqrt{13}}{13}\right)\left(\frac{2\,x1\sqrt{13}}{13} + \frac{3\,y1\sqrt{13}}{13}\right) + 32\left(\frac{2\,x1\sqrt{13}}{13} + \frac{3\,y1\sqrt{13}}{13}\right)^2 + \frac{162\,x1\sqrt{13}}{13} + \frac{152\,y1\sqrt{13}}{13} + 7 \quad \textbf{(31)}$$

> $expr := simplify(\%);$

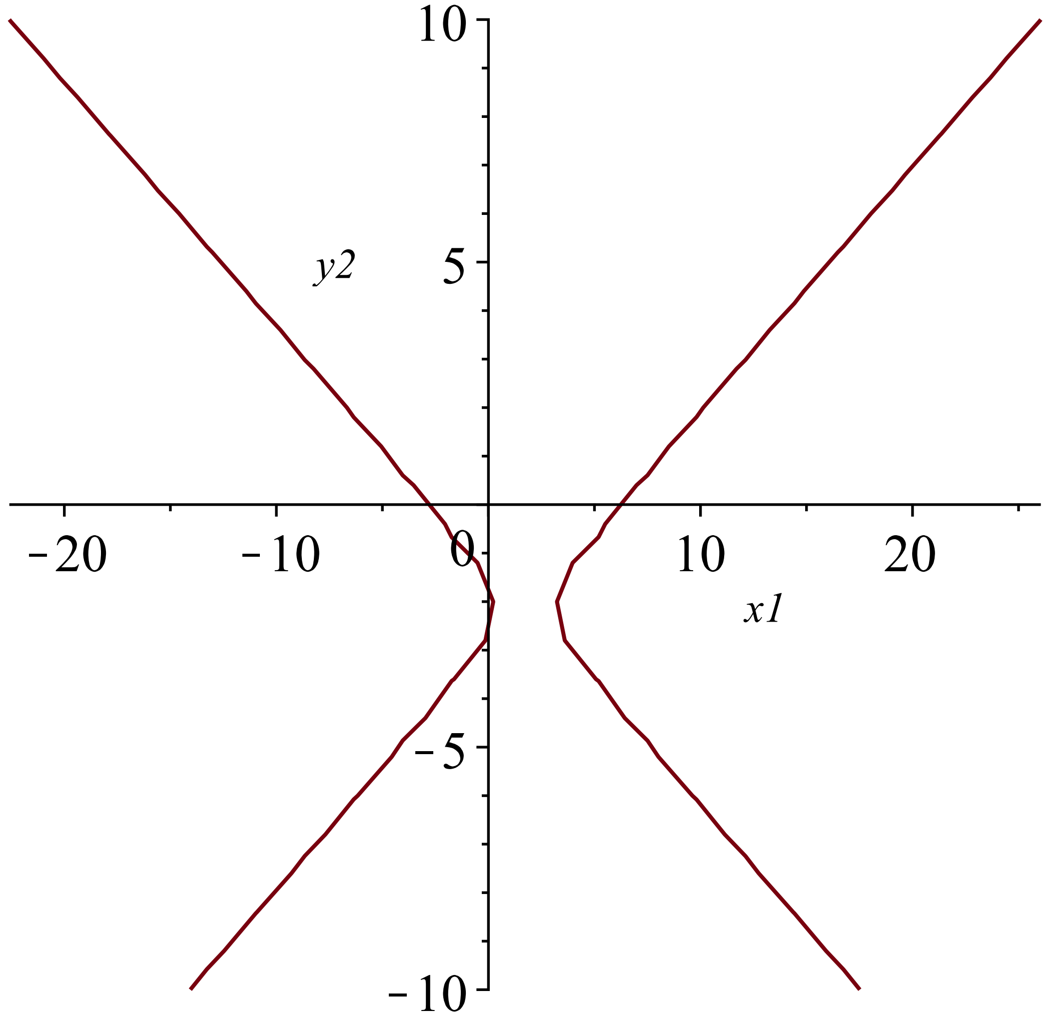
$$expr := \frac{(162\,x1 + 152\,y1)\sqrt{13}}{13} - 13\,x1^2 + 52\,y1^2 + 7 \quad \textbf{(32)}$$

> $expr\_pseudocanon := Student[Precalculus][CompleteSquare](expr):$

> $expr\_canon := subs\left(y1 = y2 + \frac{81\,\mathrm{sqrt}(13)}{169}, expr\_pseudocanon\right);$

$$expr\_canon := 52\left(y2 + \frac{100\sqrt{13}}{169}\right)^2 - 13\left(x1 - \frac{81\sqrt{13}}{169}\right)^2 + \frac{6300}{169} \quad \textbf{(33)}$$

> $implicitplot(expr\_canon = 0, x1 = -100..100, y2 = -10..10);$

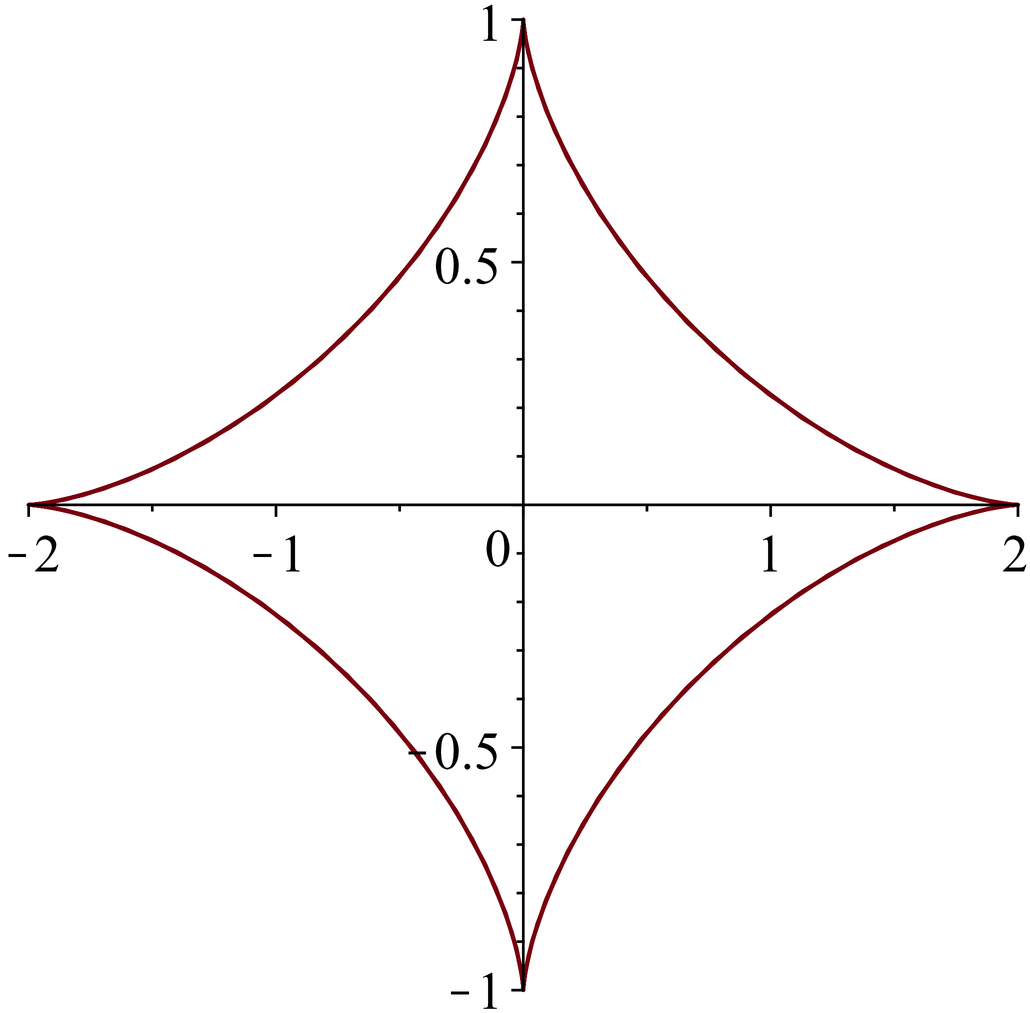


> **3**)

> $x := t \rightarrow 2\cos^3(t)$:

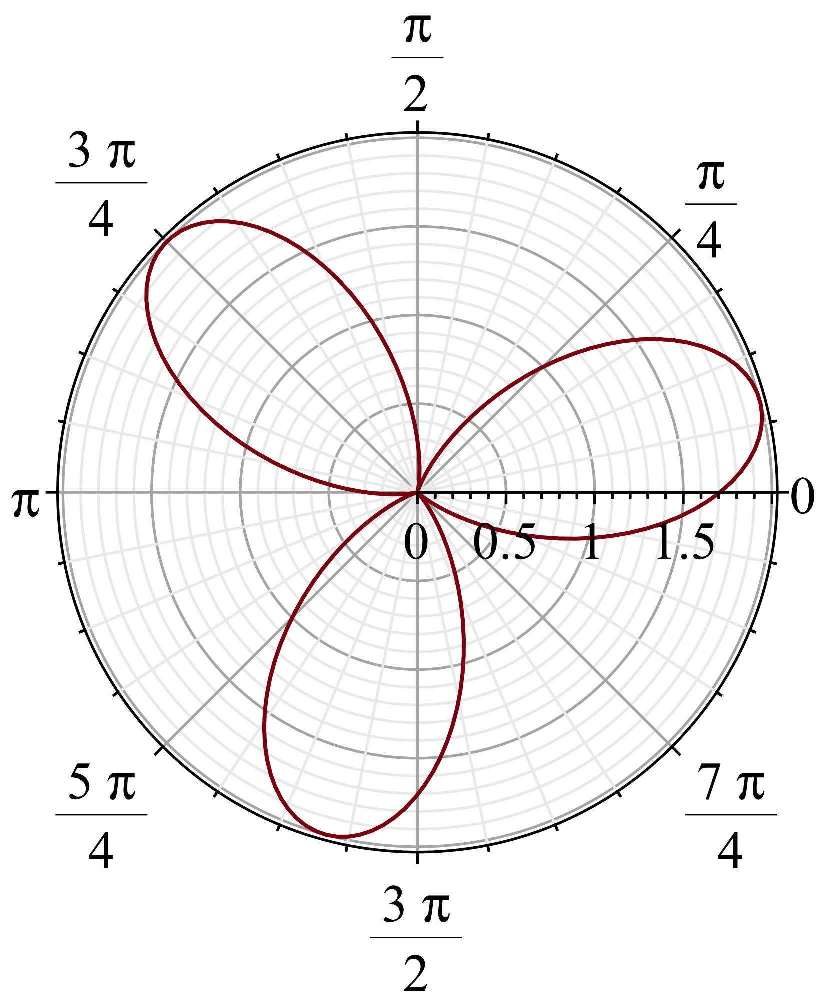
> $y := t \rightarrow \sin^3(t)$:

> $plot([x(t), y(t), t=-10..10]);$



> **4**)

> $plot\left(1+\sin\left(3\cdot\varphi+\frac{\pi}{4}\right), \varphi=0..2\cdot\pi, coords=\text{polar}, axiscoordinates=\text{polar}\right);$



$\frac{\pi}{2}$
$\frac{3\pi}{4}$
$\frac{\pi}{4}$
π
0
0
0.5
1
1.5
$\frac{5\pi}{4}$
$\frac{7\pi}{4}$
$\frac{3\pi}{2}$

>
>
>
>