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Лабoратрная работа 3.2.
Обыкновенные дифференциальные уравнения высших порядков

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Вариант 3

Задание 1.

1) $x = y''^2 + \ln y''$ 2) $arctgx(x^2+1)(yy''-y'^2) = yy'$

3) $y' = xy'' - \sqrt{y''}$ 4) $2y'' = \frac{y'}{x} - \frac{y}{x^2} + \frac{e^{\sqrt{x}}}{\sqrt{x}}$

> **1.1**

> $de1 := x = diff(y(x), x\$2)^2 + \ln(diff(y(x), x\$2));$

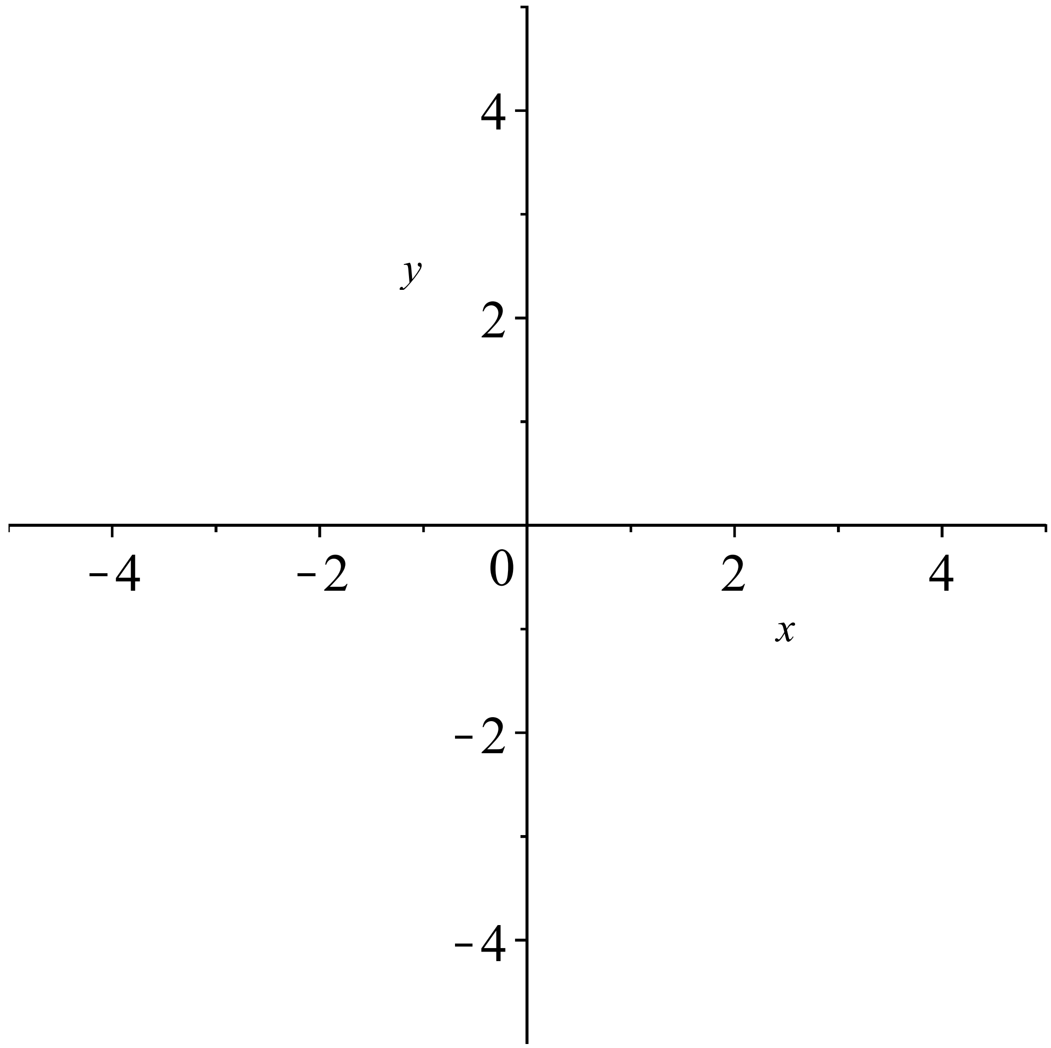
$$de1 := x = \left(\frac{d^2}{dx^2}\, y(x)\right)^2 + \ln\left(\frac{d^2}{dx^2}\, y(x)\right) \tag{1}$$

> $sol1 := dsolve(de1, y(x));$

$$sol1 := y(x) = \int\!\!\int \frac{e^x \sqrt{2}}{2\sqrt{\frac{e^{2x}}{\mathrm{LambertW}(2\,e^{2x})}}}\,dx\,dx + \_C1\,x + \_C2 \tag{2}$$

> $plot(subs(\_C1 = 0, \_C2 = 1, rhs(sol1)), x = -5..5, y = -5..5, thickness = 2);$

Warning, unable to evaluate the function to numeric values in the region; see the plotting command's help page to ensure the calling sequence is correct



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поскольку maple не решает данное дифференциальное уравнение, то мы подумаем что с этим можно сделать.
Данное уравнение является дифференциальным уравнением допускающим понижение порядка вида: $F\left(x, y^{(n)}\right) = 0$
Значит надо проинтегрировать данное уравнение. Однако мы этого сделать не можем, так как не удастся разрешить данное уравнение
относительно y''. Значит введем замену $y''= t$. Получим:

> $xequ := x = t^2 + \ln(t)$

$$xequ := x = t^2 + \ln(t) \quad (3)$$

из $\frac{dy'}{dx} = t$, выразим dy': $dy = dx \cdot t$

> $difx := diff(rhs(xequ), t);$

$$difx := 2\,t + \frac{1}{t} \quad (4)$$

составим следующее ДУ:

> $dushtrih := diff(y(t), t) = t \cdot \frac{\mathrm{d}}{\mathrm{d}\,t}(rhs(xequ));$

> $dsolve(dushtrih, y(t))$

$$de := \frac{d}{dt}\, y(t) = \left(2\,t + \frac{1}{t}\right) t$$

$$y(t) = \frac{2}{3}\,t^3 + t + \_C1 \qquad \textbf{(5)}$$

т.е. имеем $y' = \frac{2}{3}t^3 + t + \_C1$

На основании этого составим ДУ

$dy = \left(\frac{2}{3}t^3 + t + \_C1\right)\cdot dx$

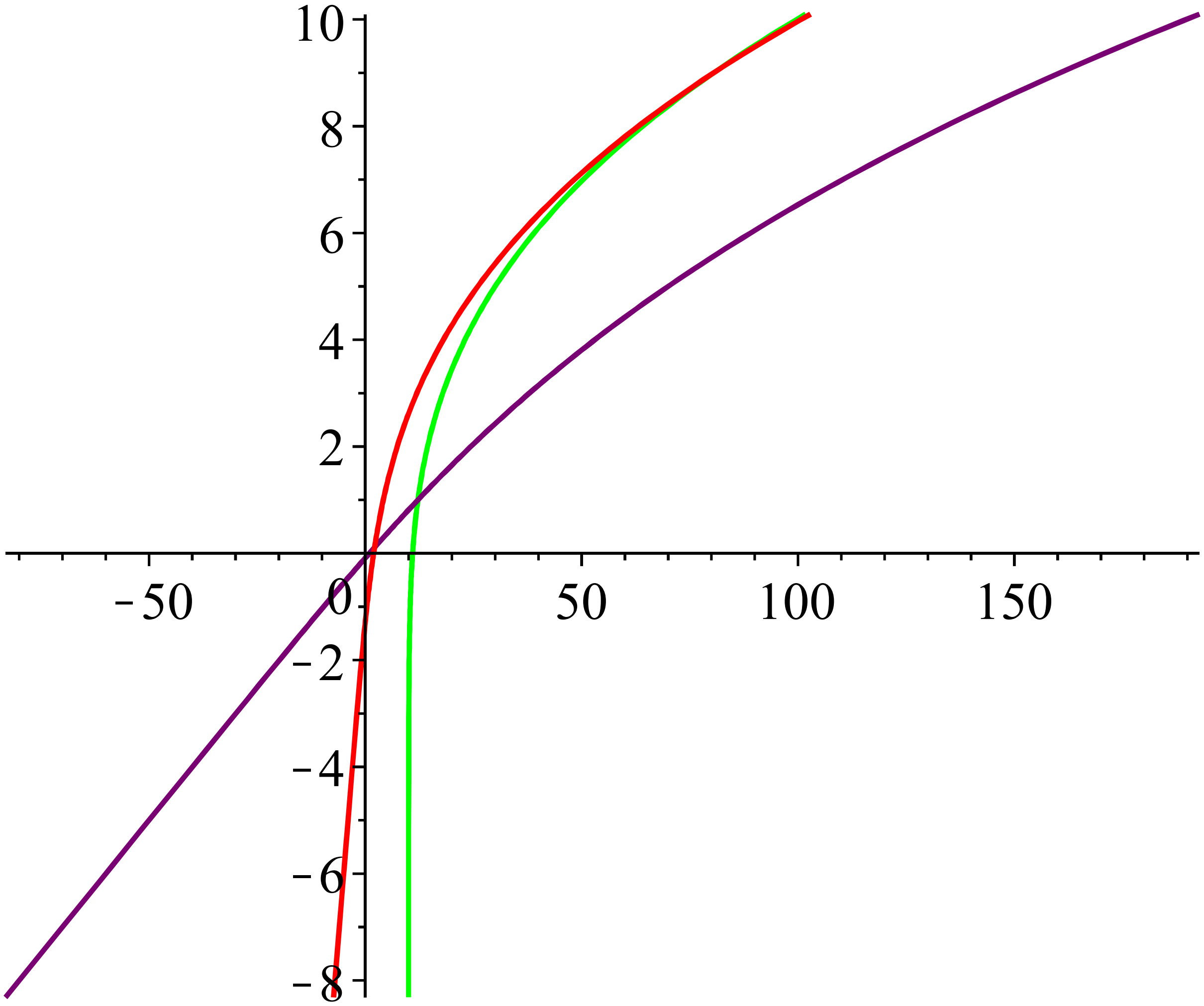
> $du := \frac{d}{d\,t}y(t) = \left(\frac{2}{3}t^3 + t + \_C1\right)\cdot\frac{d}{d\,t}(rhs(xequ));$

$$du := \frac{d}{dt}\, y(t) = \left(\frac{2}{3}\,t^3 + t + \_C1\right)\left(2\,t + \frac{1}{t}\right) \qquad \textbf{(6)}$$

> $sol := rhs(dsolve(du, y(t)));$

$$sol := \frac{4\,t^5}{15} + \frac{8\,t^3}{9} + \_C1\,t^2 + t + \_C1\,\ln(t) + \_C2 \qquad \textbf{(7)}$$

> $x := t \rightarrow \ln(t) + t^2:$
$line1 := plot([subs(\_C1 = 0,\ \_C2 = 10,\ sol(t)),\ x(t),\ t = -1\,..3],\ color = green,\ thickness = 2):$
$line2 := plot([subs(\_C1 = 10,\ \_C2 = 0,\ sol(t)),\ x(t),\ t = -1\,..3],\ color = purple,\ thickness = 2):$
$line3 := plot([subs(\_C1 = 1,\ \_C2 = 1,\ sol(t)),\ x(t),\ t = -1\,..3],\ color = red,\ thickness = 2):$
$plots[display](line1, line2, line3);$



> *restart*;

> **1.2**

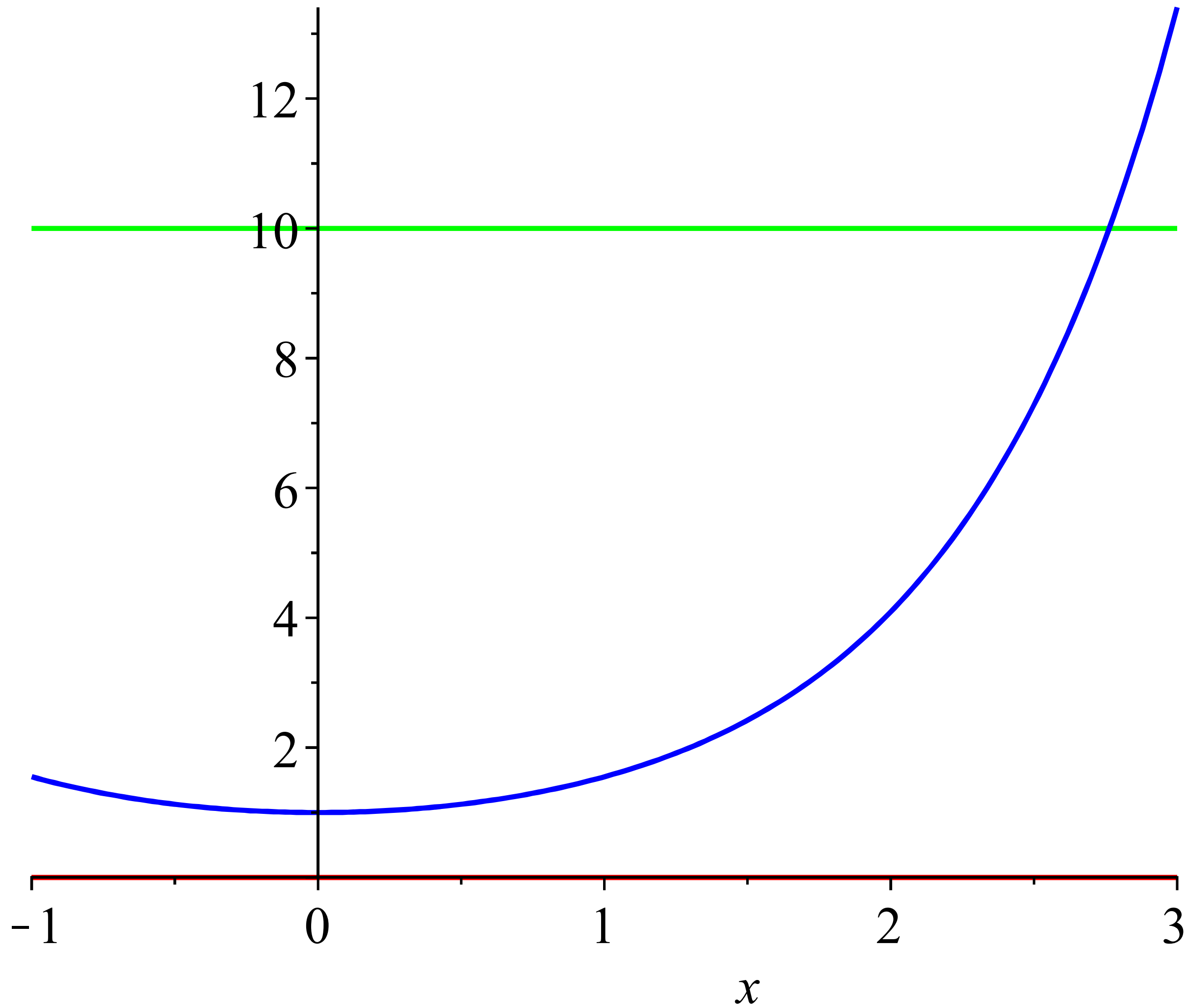
> $de := \arctan(x) \cdot (x^2+1) \cdot (y(x) \cdot diff(y(x), x\$2) - diff(y(x), x)^2) = y(x) \cdot diff(y(x), x);$

$$de := \arctan(x)\,(x^2+1)\left(y(x)\left(\frac{d^2}{dx^2}\,y(x)\right) - \left(\frac{d}{dx}\,y(x)\right)^2\right) = y(x)\left(\frac{d}{dx}\,y(x)\right) \quad \textbf{(8)}$$

> $sol := rhs(dsolve(de, y(x)));$

$$sol := e^{\_C1\,x \arctan(x)}\,(x^2+1)^{-\frac{\_C1}{2}}\,\_C2 \quad \textbf{(9)}$$

> $line1 := plot([subs(\_C1 = 0, \_C2 = 10, sol(t))], x = -1..3, color = green, thickness = 2):$
$line2 := plot([subs(\_C1 = 10, \_C2 = 0, sol(t))], x = -1..3, color = red, thickness = 2):$
$line3 := plot([subs(\_C1 = 1, \_C2 = 1, sol(t))], x = -1..3, color = blue, thickness = 2):$
$plots[display](line1, line2, line3);$



> *restart*;

> **1.3**

> $de := diff(y(x), x) = x \cdot diff(y(x), x\$2) - \mathrm{sqrt}(diff(y(x), x\$2));$

$$de := \frac{d}{dx}\, y(x) = x\left(\frac{d^2}{dx^2}\, y(x)\right) - \sqrt{\frac{d^2}{dx^2}\, y(x)} \tag{10}$$

> $sol := dsolve(de, y(x));$

как можно заметить dsolve дал нам два решения.

$\frac{1}{2}\, \_C1^2 x^2 + \_C1\, x + \_C2$ - это общее решение данного ДУ. Его мы получим в ходе вычислений.

$-\frac{\ln(x)}{4} + \_C1$ - это решение, получается из-за того, что при замене y' = t, мы получим уравнение Клеро.

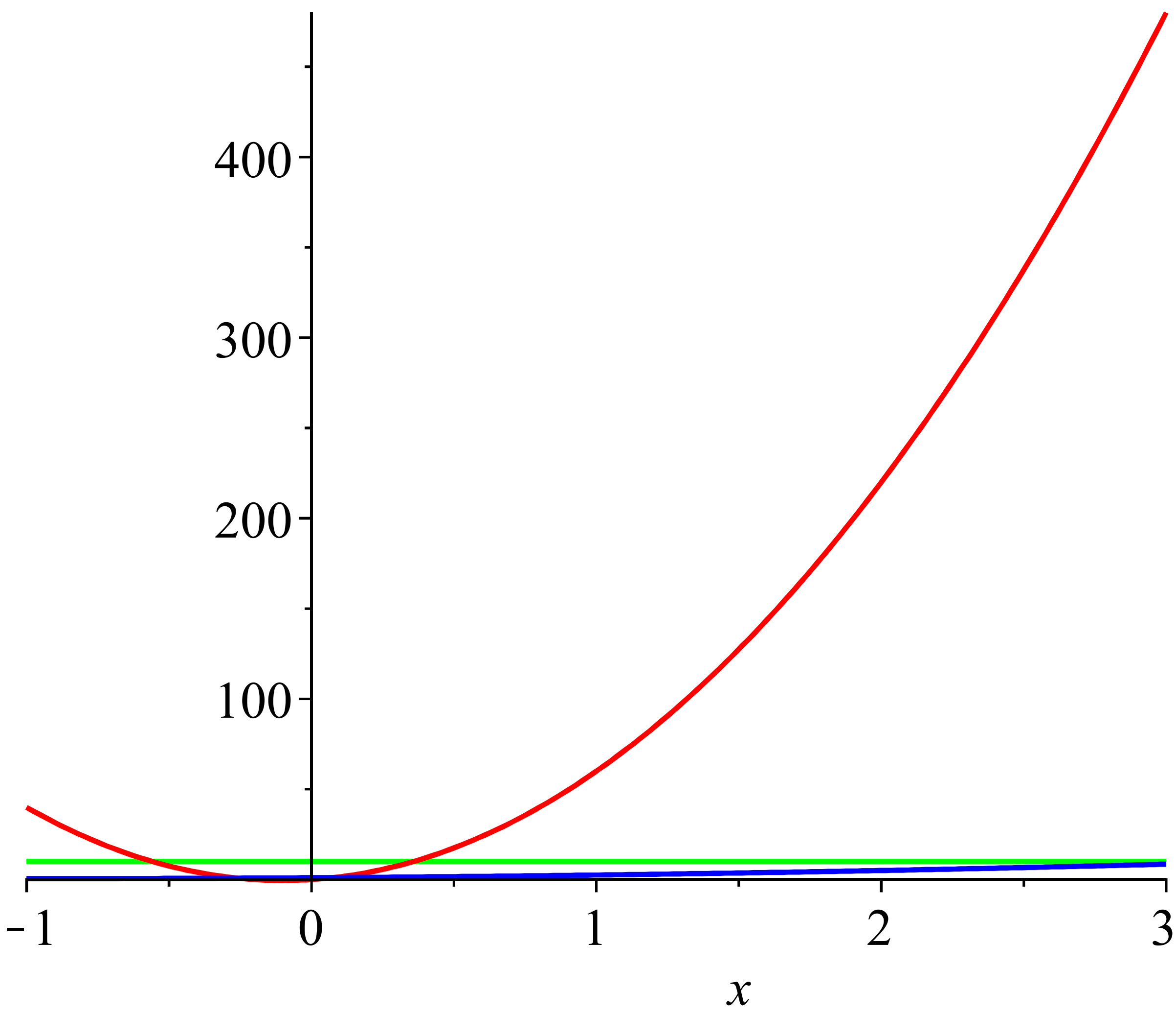
Данное уравнение обладает общим и особым решением. Так вот наше решение $y(x) = -\frac{\ln(x)}{4}$ получается как раз из этого особого решения.

$$sol := y(x) = -\frac{\ln(x)}{4} + \_C1, y(x) = \frac{1}{2}\, \_C1^2 x^2 + \_C1\, x + \_C2 \tag{11}$$

> $sol := \frac{1}{2}\,\_C1^2\,x^2 + \_C1\,x + \_C2;$

$$sol := \frac{1}{2}\,\_C1^2\,x^2 + \_C1\,x + \_C2 \quad \textbf{(12)}$$

> $line1 := plot([subs(\_C1 = 0,\ \_C2 = 10,\ sol(t))],\ x = -1\,..3,\ color = green,\ thickness = 2):$
> $line2 := plot([subs(\_C1 = 10,\ \_C2 = 0,\ sol(t))],\ x = -1\,..3,\ color = red,\ thickness = 2):$
> $line3 := plot([subs(\_C1 = 1,\ \_C2 = 1,\ sol(t))],\ x = -1\,..3,\ color = blue,\ thickness = 2):$
> $plots[display](line1,\ line2,\ line3);$

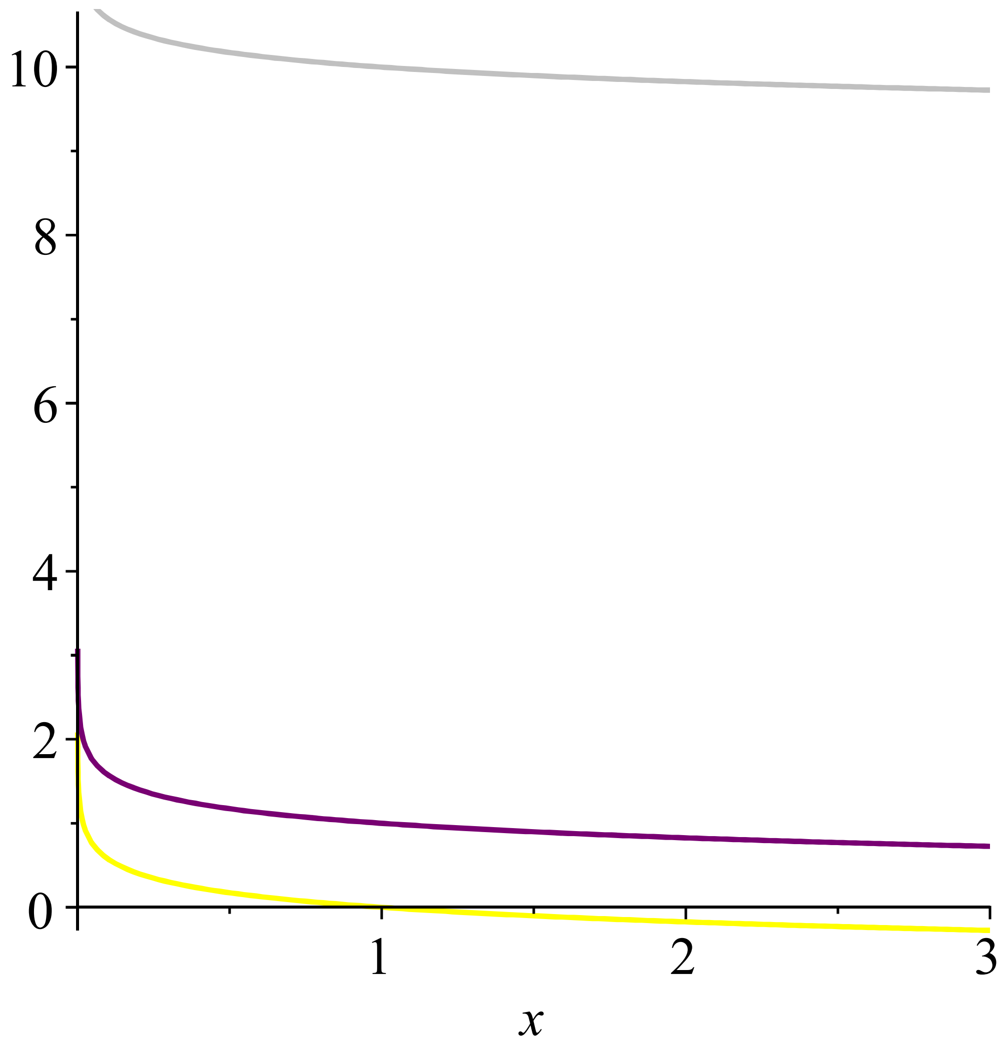


> $soln := -\frac{\ln(x)}{4} + \_C1;$

$$soln := -\frac{\ln(x)}{4} + \_C1 \quad \textbf{(13)}$$

> $line4 := plot([subs(\_C1 = 0,\ soln(t))],\ x = -1\,..3,\ color = yellow,$

$thickness = 2)$:
$line5 := plot([subs(\_C1 = 10, soln(t))], x = -1 ..3, color = grey,$
$thickness = 2)$:
$line6 := plot([subs(\_C1 = 1, soln(t))], x = -1 ..3, color = purple,$
$thickness = 2)$:
$plots[display](line4, line5, line6);$



> **1.4**

> $du := 2 \cdot diff(y(x), x\$2) = \frac{diff(y(x), x)}{x} - \frac{y(x)}{x^2} + \frac{e^{\mathrm{sqrt}(x)}}{\mathrm{sqrt}(x)};$

$$du := 2 \frac{d^2}{dx^2} y(x) = \frac{\frac{d}{dx} y(x)}{x} - \frac{y(x)}{x^2} + \frac{e^{\sqrt{x}}}{\sqrt{x}} \quad (14)$$

> $sol := rhs(dsolve(du, y(x)));$

$$sol := \_C1\, x + 2\, e^{\sqrt{x}} \sqrt{x} + \sqrt{x}\, \_C2 \quad (15)$$

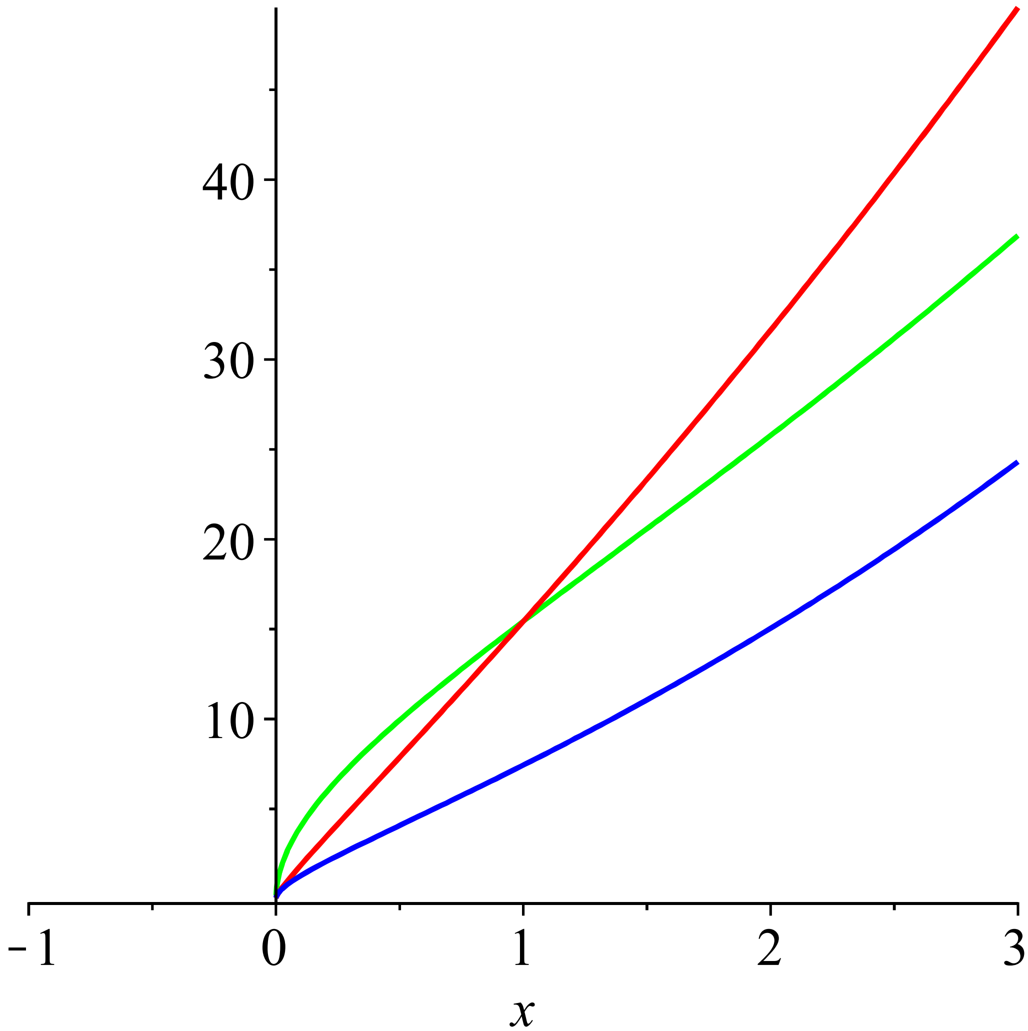
> $line1 := plot([subs(\_C1 = 0, \_C2 = 10, sol(t))], x = -1 ..3, color$

$= green, thickness = 2)$:

$line2 := plot([subs(\_C1 = 10, \_C2 = 0, sol(t))], x = -1 ..3, color = red, thickness = 2)$:

$line3 := plot([subs(\_C1 = 1, \_C2 = 1, sol(t))], x = -1 ..3, color = blue, thickness = 2)$:

$plots[display](line1, line2, line3);$



> **2.**

> $du := 2 \cdot x \cdot diff(y(x), x\$3) = diff(y(x), x\$2);$

$$du := 2\,x \left(\frac{d^3}{dx^3}\,y(x)\right) = \frac{d^2}{dx^2}\,y(x) \quad \textbf{(16)}$$

> $sol := rhs(dsolve(du, y(x)));$

$$sol := \_C1 + \_C2\,x + \_C3\,x^{5/2} \quad \textbf{(17)}$$

> **3.**

> $du := diff(y(x), x\$2) + 2 \cdot diff(y(x), x) = -2 \cdot e^x \cdot (\sin(x) + \cos(x));$

$$du := \frac{d^2}{dx^2}\,y(x) + 2\,\frac{d}{dx}\,y(x) = -2\,e^x\,(\sin(x) + \cos(x)) \quad \textbf{(18)}$$

> $sol := dsolve(du, y(x));$

$$sol := y(x) = \frac{e^{x}\cos(x)}{5} - \frac{3\,e^{x}\sin(x)}{5} - \frac{\_C1}{2\,(e^{x})^{2}} + \_C2 \tag{19}$$

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