Detection of Gauss-Markov Random Fields under Routing Energy Constraint

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***Abstract*— The problem of maximizing detection performance subject to an energy constraint is analyzed in an asymptotic setting. The correlation between the sensor measurements is incorporated through the Gauss-Markov random field with Euclidean nearest-neighbor dependency graph. An average energy constraint is imposed on a routing scheme with an approximation factor of two, and the resulting Neyman-Pearson error exponent is optimized with respect to the density of the deployed sensors. It is shown that the behavior of this optimal density crucially depends on the ratio between the measurement variances under the two hypotheses and displays a threshold behavior. Below the threshold value of the variance ratio, the optimal density tends towards infinity for any feasible energy constraint. On the other hand, when the variance ratio is above the threshold, the optimal density is characterized by the energy constraint.**

Index Terms— **detection, Gauss-Markov random fields, Routing, error exponent.**

I. Introduction

The deployment of sensors is the first step in establishing a network. It influences the performance of the network, including the energy consumed in routing data. However, designing optimal deployment strategies requires finding optimal locations for all nodes and is not feasible for a large network. In contrast, deploying nodes randomly but according to an optimal density may be more tractable. In this paper, we assume that the nodes are placed IID according to the uniform or Poisson distribution with a fixed node density.

In a network of sensors measuring a correlated signal field, the node density influences the extent of correlation among the measurements and thereby detection performance. Moreover, the energy required to route data typically depends on the inter-node distance, and in turn, the node density. Hence, we can design an optimal density that achieves the best tradeoff between the energy consumed in routing data and the resulting detection performance. Of course, the optimal density depends on the routing protocols employed. In the classical approach, a layered architecture separates the design of routing from the application. For energy-constrained networks, application-specific routing may offer better tradeoffs, and we employ one such routing scheme.

To characterize the detection performance, we consider the Neyman-Pearson (NP) error exponent. Our objective is to find an optimal node density λ_* that maximizes the detection error exponent D, under a constraint $\bar{E}$ on the average energy consumption $\bar{\mathrm{C}}$ of routing per node,

$$\lambda_* \stackrel{\Delta}{=} \arg\max_{\lambda>0} D_\lambda \quad \text{subject to } \bar{\mathrm{C}} \leq \bar{E}. \tag{1}$$

Note that both the energy constraint and detection performance are asymptotic in the number of nodes.

We address the following questions: does an optimal density exist? And if so, what is its value? Is it one of the extremes, viz., zero or infinity? This is an important question, since if the optimal node density is either zero or infinity, then we can simply place the nodes in as small/large an observation area as possible. On the other hand, if this is not the case, then we need to deploy the nodes, based on the optimal density.

A. Summary of Results

We assume the presence of correlation under the alternative hypothesis $\mathcal{H}_1$ through the Gauss-Markov random field (GMRF) model. Again, assuming an uniform signal field model (same measurement variance at every node) and a correlation function decaying with distance, we study the effect of node density on the error exponent. To this end, we unify the results from our previous works [1], [2], which independently characterize the detection performance and routing for a GMRF.

Intuitively, when both the hypotheses have the same measurement variance, they can be distinguished only by the presence of correlation under $\mathcal{H}_1$. Correlation is maximized when all the nodes are clustered close to one another, since correlation decays with distance. Hence, ignoring the energy constraint, the optimal density should be infinite. We prove that this is indeed true.

For the general case, when the measurement variances are different, the behavior is decided by the ratio of the variances under $\mathcal{H}_1$ and $\mathcal{H}_0$. We show that the optimal density is infinite when the variance ratio is below a threshold value. Moreover, imposing any feasible routing-energy

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constraint does not change the optimal density. We provide the expression for the threshold on the variance ratio. On the other hand, above this threshold the optimal density is characterized by the energy constraint.

II. System Model

For the matrix $\mathbf{A} = [A(i,j)]$, $A(i,j)$ denotes the element in the i^{th} row and j^{th} column and $|\mathbf{A}|$ its determinant. For two sets A and B, let $A \backslash B = \{i : i \in A, i \notin B\}$.

A. Gauss-Markov Random Field

A GMRF, in addition to being a Gaussian random field, satisfies special conditional independence properties. A simple example is the first-order auto-regressive process, where the conditional independence of the observations is based on causality. However, a spatial random field has a far richer set of conditional independencies, requiring a more general definition [3, p. 21].

Definition 1 (GMRF): Given a point set $\mathcal{V} = \{1, \ldots, n\}$, $\mathbf{Y}_{\mathcal{V}} = \{Y_i : i \in \mathcal{V}\}$ is a GMRF with an (undirected) dependency graph $\mathcal{G}_d(\mathcal{V}, \mathcal{E})$ if $\mathbf{Y}_{\mathcal{V}}$ is a Gaussian random field, and $\forall i, j \in \mathcal{V}$, Y_i and Y_j are conditionally independent given observations at all other nodes if i and j are not neighbors,

$$Y_i \perp Y_j | \mathbf{Y}_{-ij} \iff (i,j) \notin \mathcal{E}_d, \ \forall i, j \in \mathcal{V}, i \neq j, \quad (2)$$

where $\perp$ denotes conditional independence and $\mathbf{Y}_{-ij} \triangleq (Y_k : k \in \mathcal{V}, k \neq i, j)$.

B. Network and Energy Model

We assume that the energy consumed by a node i can be represented by the sum of a constant processing energy $C_p > 0$ and the transmission energy[1] [4]. For details, see [2]. The energy required for the transmission of a real number from i to j is denoted by $C_{i,j}$. We assume that

$$C_{i,j} = C_t |\text{dist}(i,j)|^{\nu}, 2 \leq \nu \leq 4, \quad (3)$$

where C_t is a constant, $\text{dist}(i,j)$ is the inter-node distance and ν is the channel path-loss exponent. Further, we assume that the processing energy C_p at every node is constant and independent of the number of transmissions from the node.

The set of transmissions in a node set $\mathcal{V}$ can be represented as a digraph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$. Note that this transmission digraph is different from the dependency graph of the signal field. Let $\text{C}(\mathcal{G})$ denote the total transmission energy given by

$$\text{C}(\mathcal{G}) \triangleq C_t \sum_{e \in \mathcal{E}} R_e^{\nu}, \quad (4)$$

where r_e is the Euclidean edge length. We further require that $\mathcal{G} \subset \text{UDG}(\mathcal{V})$, the unit-disk graph, defined as the set of edges between any two nodes within an unit Euclidean distance of each other.

[1]We ignore the energy consumed at the receiver.

III. Hypothesis-testing problem

Let $\mathcal{V} = \{1, \ldots, n\}$ be a set of n nodes on the plane and let $\mathbf{Y}_n$ be the random vector of observation samples $Y_i, i \in \mathcal{V}$,

$$\mathbf{Y}_n \triangleq [Y_1, \ldots, Y_n]^T. \quad (5)$$

The hypothesis-testing problem is

$$\mathcal{H}_0 : \mathbf{Y}_n \sim \mathcal{N}(\mathbf{0}, \sigma_0^2 \mathbf{I}) \quad \text{vs.} \quad \mathcal{H}_1 : \mathbf{Y}_n \sim \mathcal{N}(\mathbf{0}, \mathbf{\Sigma}_{1,\mathcal{V}}), \quad (6)$$

where the covariance matrix $\mathbf{\Sigma}_{1,\mathcal{V}}$ of a GMRF depends on the configuration of nodes in $\mathcal{V}$.

The optimal decision-rule under the Neyman-Pearson formulation is a threshold test based on the log-likelihood ratio (LLR). Let $p[\mathbf{Y}_n | \mathcal{V}; \mathcal{H}_j]$ be the conditional PDF of the observations given the point set $\mathcal{V}$ under hypothesis j.

$$\text{LLR}(\mathbf{Y}_n, \mathcal{G}_d) \triangleq \log \frac{p[\mathbf{Y}_n, \mathcal{V}; \mathcal{H}_0]}{p[\mathbf{Y}_n, \mathcal{V}; \mathcal{H}_1]}. \quad (7)$$

A. Covariance matrix of GMRF

We make additional assumption on the structure of the covariance matrix $\mathbf{\Sigma}_{1,\mathcal{V}}$ of the GMRF under $\mathcal{H}_1$ viz., that the nodes have the same measurement variance for any node configuration $\mathcal{V}$, i.e.,

$$\Sigma_{1,\mathcal{V}}(i,i) \triangleq \sigma_1^2, \ i = 1, \ldots, n. \quad (8)$$

We denote the ratio between the variances under the alternative and the null hypothesis at each node by

$$K \triangleq \frac{\sigma_1^2}{\sigma_0^2}. \quad (9)$$

We also assume that under $\mathcal{H}_1$, the amount of correlation between the neighbors i, j of the dependency graph is specified by an arbitrary function g, which has the Euclidean edge length R_{ij} as its argument. From (8), we have

$$g(R_{ij}) \triangleq \frac{\Sigma_{1,\mathcal{V}}(i,j)}{\sigma_1^2}, \ \forall \ (i,j) \in \mathcal{E}. \quad (10)$$

The correlation function g is required to satisfy some regularity conditions. We assume that

$$g(0) \triangleq M < 1, \quad g(\infty) = 0, \quad (11)$$

and that g is a monotonically decreasing convex function of the edge length. This is a reasonable assumption, since the amount of correlation usually decays as nodes become farther apart. Moreover, partial correlation or $g(0) = M < 1$ arises due to signal imperfections. Note that this has the same effect on correlation as imposing an exclusion region on how near two nodes can be placed. Some examples of the correlation functions, satisfying our assumptions, are

$$g(r) = M \text{e}^{-ar}, \ g(r) = \frac{M}{1 + r^a}, \ 0 \leq a, M < 1.$$

Note that we do not assume the presence of additive Gaussian measurement noise in the signal model, as considered in the literature before. This is because with the additive noise, the resulting GMRF no longer has a sparse dependency graph. As an alternative formulation, we model imperfect measurements by assuming partially-correlated nodes.

B. Expression for log-likelihood ratio

We incorporate the assumptions (8-10) in the theorem below to obtain the LLR for detection.

Theorem 1 (Log-likelihood ratio): Under the assumptions (8-10), the log-likelihood ratio in (7) for the hypothesis-testing problem in (6), given point set $\mathcal{V} = \{1, \ldots, n\}$, is

$$\begin{aligned} \mathrm{LLR}(\mathbf{Y}_n, \mathcal{G}_d) &= n \log \frac{\sigma_1}{\sigma_0} + \frac{1}{2}\Big[\sum_{i\in\mathcal{V}} \Big(\frac{1}{\sigma_1^2} - \frac{1}{\sigma_0^2}\Big) Y_i^2 \\ &+ \sum_{(i,j)\in\mathcal{E}} \Big\{ \log[1 - g^2(R_{ij})] \\ &+ \frac{g^2(R_{ij})}{1 - g^2(R_{ij})} \frac{Y_i^2 + Y_j^2}{\sigma_1^2} \\ &- \frac{2g(R_{ij})}{1 - g^2(R_{ij})} \frac{Y_i Y_j}{\sigma_1^2} \Big\}\Big], \end{aligned} \tag{12}$$

where R_{ij} is the Euclidean edge length of $(i,j) \in \mathcal{E}$.

Theorem 1 gives a closed-form expression for the log-likelihood ratio, in terms of the edges of the nearest-neighbor dependency graph of the GMRF. Note in (12), the cross-terms are only between the neighbors of the dependency graph, which is exploited to yield explicit data-fusion and routing schemes [2], summarized in section IV. Note that (12) contains two kinds of terms. The *edge potentials*, denoted by $\phi_{i,j}(Y_i, Y_j)$, that depend on a pair of observations, given by

$$\begin{aligned} \phi_{i,j}(i,j) &\triangleq \frac{1}{2}\log[1 - g^2(R_{ij})] - \frac{g(R_{ij})}{1 - g^2(R_{ij})} \frac{Y_i Y_j}{\sigma_1^2} \\ &+ \frac{g^2(R_{ij})}{1 - g^2(R_{ij})} \frac{Y_i^2 + Y_j^2}{2\sigma_1^2}, \end{aligned} \tag{13}$$

and the *node potentials* $\phi_i(Y_i)$, depending on a single observation, given by

$$\phi_i(Y_i) \triangleq \log \frac{\sigma_1}{\sigma_0} + \frac{1}{2}\Big(\frac{1}{\sigma_1^2} - \frac{1}{\sigma_0^2}\Big) Y_i^2. \tag{14}$$

Hence, the LLR can be written in a compact form

$$\mathrm{LLR}(\mathbf{Y}_n; \mathcal{G}_d) = \sum_{i\in\mathcal{V}} \phi_i(Y_i) + \sum_{(i,j)\in\mathcal{E}_d} \phi_{i,j}(Y_i, Y_j). \tag{15}$$

IV. Minimum Energy Routing

The aim of optimal routing for detection is to ensure the delivery of the LLR, given by (15), to the designated fusion center v_0, while minimizing the total energy consumption. However, this is NP-hard and we instead employ a 2-approximation routing algorithm for data-fusion of a Markov random field (DFMRF). See [2]. We define two types of transmissions for routing.

Definition 2: The *data-transmission graph* $\mathrm{DTG}(\mathcal{V})$ is defined as the transmission subgraph that consists of transmissions of the raw sensor data. The *likelihood-aggregation graph* $\mathrm{AG}(\mathcal{V})$ is defined as the transmission subgraph that consists of transmissions of the aggregates of the log-likelihood ratio. The nodes that process data from other nodes i.e., those in the likelihood-aggregation graph, are known as the set of *aggregators*, denoted by $\mathcal{V}_{\mathrm{AG}}$.

The data-transmission graph $\mathrm{DTG}(\mathcal{V})$ needs to compute all the edge potentials of the LLR in (15) of the form $\phi_{i,j}(Y_i, Y_j)$. These edge potentials exist only between the neighbors in the dependency graph $\mathcal{G}_d$. Therefore, we consider $\mathrm{DTG}(\mathcal{V})$ as $\mathcal{G}_d$ itself, with arbitrarily assigned directions. The data-fusion mechanism has two phases. During the data-transmission phase, transmissions of raw data occurs along the $\mathrm{DTG}(\mathcal{V})$. At the end of this phase, every internal node i computes

$$\begin{aligned} m(i) &= \phi_i(Y_i) + \sum_{<j,i>\in \mathrm{DTG}(\mathcal{V})} \phi_{i,j}(Y_i, Y_j) \\ &+ \sum_{<j,i>\in \mathrm{DTG}(\mathcal{V}), j\notin\mathcal{V}_{\mathrm{AG}}} \phi_i(Y_j). \end{aligned} \tag{16}$$

This local contribution is then aggregated and delivered to the fusion center, along the aggregation graph $\mathrm{AG}(\mathcal{V})$, given by $\mathrm{DMST}(\mathcal{V})$, the minimum spanning tree, directed towards the fusion center. On receiving the aggregates from all its direct predecessors in $\mathrm{AG}(\mathcal{V})$, each node $i \neq v_0$ combines them with its local contribution and transmits $l(i)$ to its direct successor,

$$l(i) = \sum_{<j,i>\mathrm{AG}(\mathcal{V})} l(j) + m(i). \tag{17}$$

At the end of the aggregation process, the LLR is available at the fusion center and is given by $l(v_0)$.

Theorem 2 (Approx. for Arbitrary Placement): DFMRF is a 2-approximation algorithm for any arbitrary placement.

$$\frac{\mathrm{C}(\mathrm{DFMRF}(\mathcal{V}))}{\mathrm{C}(\mathcal{G}_*(\mathcal{V}))} \leq 2. \tag{18}$$

V. Detection Error Exponent

It is intractable to evaluate the routing energy consumption or the detection performance for an arbitrary point set. Therefore, we assume that the nodes are placed randomly, according to a point process defined on expanding regions.

Let $(\mathcal{B}_n)_{n\geq 1}$ denote a sequence of "regular" Borel sets (such as squares or circles) of area $\frac{n}{\lambda}$, centered at the origin, for any positive constant λ. Let $\mathcal{U}_{n,\lambda}$ be a binomial point process on $\mathcal{B}_n$ with intensity λ, i.e., n nodes distributed i.i.d uniform on the region $\mathcal{B}_n$ with node density λ. Let $\mathcal{P}_{n,\lambda}$ be the homogeneous Poisson process on $\mathcal{B}_n$ with constant intensity λ. The number of nodes n goes to infinity, with fixed node density λ.

In this section, we characterize the detection error exponent. Under the Neyman-Pearson detection, for a fixed Type-I error bound, the exponent of the Type-II error is independent of the type-I error bound [6], and given by

$$D \stackrel{\Delta}{=} \lim_{n\to\infty} \frac{1}{n} \log \frac{p[\mathbf{Y}_n;\mathcal{H}_0]}{p[\mathbf{Y}_n|\mathcal{V};\mathcal{H}_1]}, \quad \text{under } \mathcal{H}_0. \tag{19}$$

In the theorem below, we give a closed form for the error exponent, derived[2] in [1], in terms of the variables and functions defined below.

$$f_1(x) \stackrel{\Delta}{=} \log[1-x^2], f_2(x) \stackrel{\Delta}{=} \frac{2x^2}{[1-x^2]}, \tag{20}$$

$$h_i(x;g) \stackrel{\Delta}{=} f_i(g(x)) - \frac{\pi}{2\omega} f_i(g(\sqrt{\frac{\pi}{\omega}}x)), \tag{21}$$

$$h(x,K;g) \stackrel{\Delta}{=} h_1(x;g) + \frac{1}{K} h_2(x;g). \tag{22}$$

Let Z denote the Rayleigh random variable with variance $(2\pi)^{-1}$. Let ω be the area of the union of two unit- radii circles with centers unit distant apart, given by

$$\omega = \frac{4\pi}{3} + \frac{\sqrt{3}}{2} \approx 5.06. \tag{23}$$

Theorem 3 (Expression for D): For a GMRF on NNG with correlation function g, with the nodes drawn from the binomial or the Poisson process with node density λ and region area $\frac{n}{\lambda}$, the error exponent D for Neyman-Pearson detection is

$$D(\lambda,K;g) = \frac{1}{2}\Big[\mathbb{E}_\lambda\, h\big(Z\lambda^{-0.5},K;g\big) + \log K + \frac{1}{K} - 1\Big], \tag{24}$$

where $\mathbb{E}_\lambda$ is the expectation over the random variable Z.
Proof : Due to lack of space, we omit the derivation which employs a special law of large numbers for graph functionals [5] and can be found in [1]. □

Note that in (25), the expectation term captures the correlation structure of the GMRF and the remaining terms represent the detection error exponent for two IID Gaussian processes with variance ratio K, denoted by $D_{IID}(K)$, i.e.,

$$D(\lambda,K;g) = \frac{1}{2}\mathbb{E}_\lambda\, h\big(Z\lambda^{-0.5},K;g\big) + D_{IID}(K). \tag{25}$$

[2]The expression given in [1] is in a different form, but reduces to (25).

Hence, the effect of correlation on the error exponent is quantified in a compact form. It can be easily verified that the expectation term is zero, when $M = 0$ (no correlation). Note that the factor $\lambda^{-0.5}$ appears in the expectation term, since on an average, the edge-lengths scale by that amount. It is easy to see that $D_{IID}(K)$ is independent of node density.

A. Infinite Node Density

We first analyze the error exponent D, when the node density λ goes to infinity. This will provide insights to finding the optimal density. As $\lambda \to \infty$, we have

$$D(\infty,K;g) = D_{IID}(K) + \frac{1}{2}h(0,K,M),$$

where $h(0,K;g)$ depends on g only through $g(0) = M$, and

$$h(0,K,M) = (1-\frac{\pi}{2\omega})\Big(\log[1-M^2] + \frac{2M^2}{K[1-M^2]}\Big). \tag{26}$$

In the theorem below, we prove that the presence of correlation can either improve or degrade the error exponent, depending on the variance ratio K. We establish a threshold on K that determines the transition.

Lemma 1 (Behavior at Infinite Density ($\lambda = \infty$)): At $\lambda = \infty$, the correlation term $h(0,K,M)$ in (27) is positive, if the variance ratio K is below a threshold value $K_t(M)$,

$$h(0,K,M) \begin{cases} \geq 0, & \text{for } K < K_t(M), \quad (27a) \\ < 0, & \text{for } K > K_t(M). \quad (27b) \end{cases}$$

For a fixed $g(0) = M < 1$, the threshold $K_t(M)$ is

$$K_t(M) = -\frac{1}{\log(1-M^2)} \frac{2M^2}{1-M^2}, \tag{28}$$

and $2 < K_t(M) < \frac{2}{1-M^2}$.
Proof: From (27) and $(1-\frac{\pi}{2\omega}) > 0$. □

VI. Optimal Node Density

Our objective is to find an optimal node density λ_* maximizing the detection error exponent under a constraint on the average energy consumption of routing. In this section, we provide the results for optimal density. In the theorem below, we show that the nature of optimal density is determined by the threshold K_t on K.

Theorem 4 (Result on $\lambda_*(K,\bar{E})$): The optimal density in (1) that maximizes the error exponent, for correlation function with $g(0) = M < 1$, feasible constraint $\bar{E}$, is

$$\lambda_*(K,\bar{E}) = \infty, \quad \forall\, K < K_t(M), \tag{29}$$

where threshold K_t is given by (29), and

$$\lambda_*(K,\bar{E}) = \lambda_E, \quad \forall\, K > K_t(M), \tag{30}$$

where λ_E is a function of energy constraint $\bar{E}$.
Proof: See [7]. □

The above theorem states that when the variance ratio K is below the threshold K_t, for any feasible energy constraint, optimality is attained at infinite density. On the other hand, above the threshold, the optimal value depends on the energy constraint.

VII. Conclusions

The tradeoff between the energy consumption in routing data and the resulting detection performance at the fusion center, is an important problem in the context of sensor networks. In this paper, we aimed to gain a perspective on this problem and additionally incorporated correlation between the measurements through the Gauss-Markov random field model. We characterized the density of node deployment that maximizes the detection error exponent subject to a constraint on the average energy consumption. The measurement variance is crucial in determining whether the optimal node density is limited by the routing energy constraint and displays a threshold behavior. We have derived the threshold analytically and verified it with simulations.

The results on the optimal density are possible due to exploitation of the Markovian structure, in deriving the error exponent and the routing schemes. While acknowledging the limitations of its validity, we have made an attempt to characterize detection-energy tradeoffs for correlated measurements in two (and higher) dimensions, which to the best of our knowledge, has not been dealt before. Alternative formulations, not dealt in this paper, include selection of nodes with "useful" data, and incorporating node and link failures. We have also not considered the issue of quantization of measurements at various nodes. Another scenario is mobile nodes having regions of coverage, instead of static nodes, as assumed here, with observations at single points. Extensions to such scenarios would be of interest.

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