Tensor Contractions with Extended BLAS Kernels on CPU and GPU

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Joint work with U.N. Niranjan, Animashree Anandkumar and Cris Cecka

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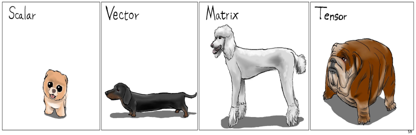
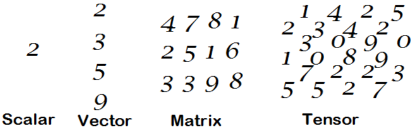
Tensor Contraction-Motivation





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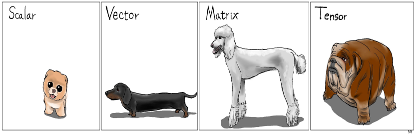

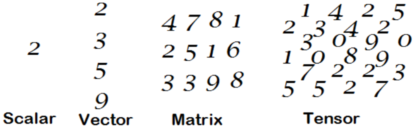

Why we need tensor?

1. Analysis of high dimension data

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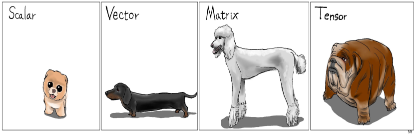

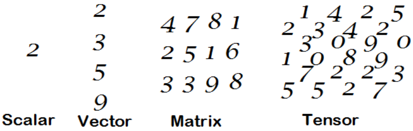

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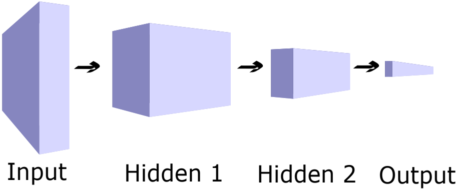

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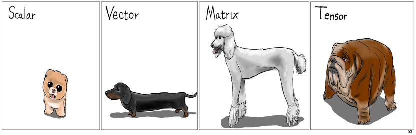

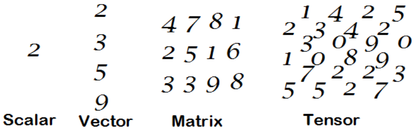

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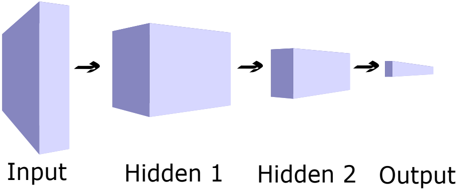

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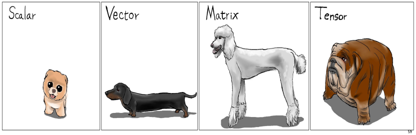

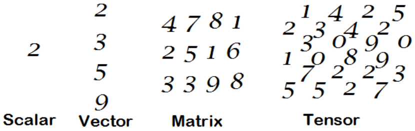

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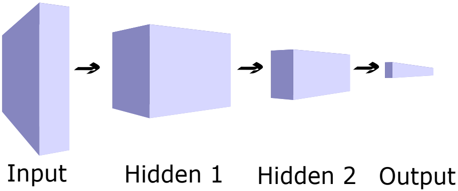

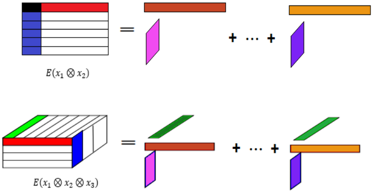

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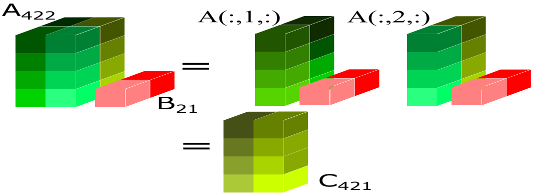
What is tensor contraction

$$C_{\mathcal{C}} = \alpha\, A_{\mathcal{A}}\, B_{\mathcal{B}} + \beta\, C_{\mathcal{C}}$$



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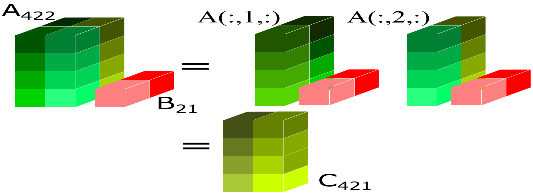

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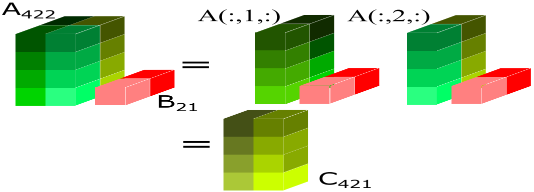

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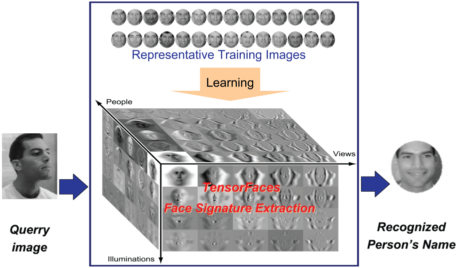

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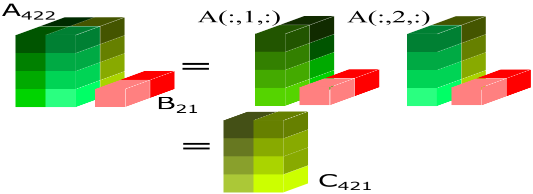

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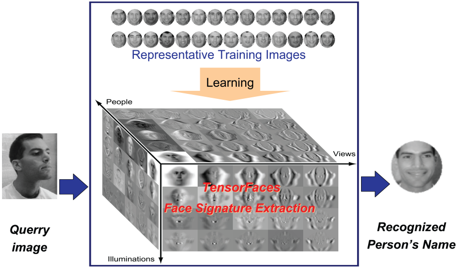
1. Machine learning: Tensor decomposition
2. Physics
3. Chemistry



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Efficient computing frame

1. Static analysis solutions: loop reorganization/fusion
2. Parallel and distributed computing system: BatchedGEMM functions in MKL 11.3 β, CuBLAS v4.1

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   - $C_{mnp} \rightarrow C_{mpn}$.
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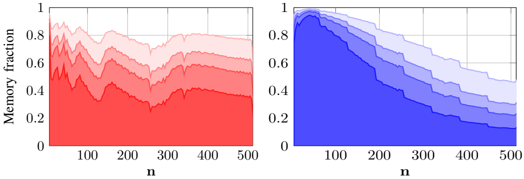

Figure: The fraction of time spent in copies/transpositions when computing $C_{mnp} = A_{mk}B_{pkn}$. Lines are shown with 1, 2, 3, and 6 total transpositions performed on either the input or output. (Left) CPU. (Right) GPU.

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- **Provided evaluation strategies for tensor contractions**
- **Applied to tensor decomposition**

BLAS Operators

BLAS(Basic Linear Algebra Subprograms): Low-level routines for performing common linear algebra operations.

Level 1	**Level 2**	**Level 3**
$y \leftarrow \alpha x + y$	$y \leftarrow \alpha op(A)x + \beta y$	$C \leftarrow \alpha op(A)op(B) + \beta C$

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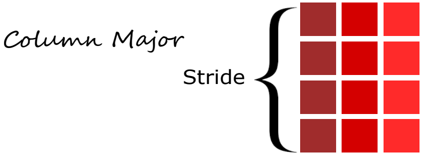

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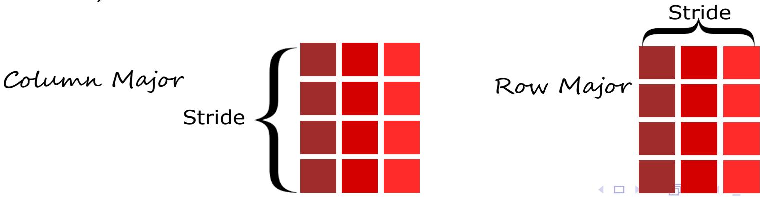

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Extended BLAS Kernel for tensor one-index contraction

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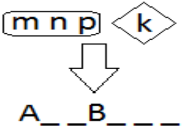
$$C_{mnp} = A_{**} \times B_{***}$$



If fixing C, there are total $3 \times 2 \times 3 \times 2 \times 1 = 36$ cases.

Tensor Contraction-Extended BLAS Operators

Case	Contraction	Kernel1	Kernel2	Kernel3
1.1	$A_{mk}B_{knp}$	$C_{m(np)} = A_{mk}B_{k(np)}$	$C_{mn[p]} = A_{mk}B_{kn[p]}$	$C_{m[n]p} = A_{mk}B_{k[n]p}$

Table: Case 1.1 single index contraction operations between a second-order tensor and a third-order tensor and possible mappings to Level 3 BLAS routines

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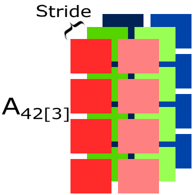

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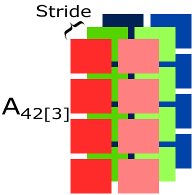

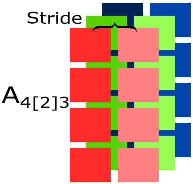

Analysis

Flatten V.S. SBGEMM

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Mixed mode batching on input/output tensors

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Mixed mode batching on input/output tensors

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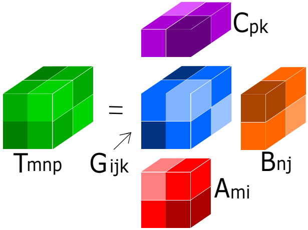
Applications:Tucker Decomposition

$$T_{mnp} = G_{ijk} A_{mi} B_{nj} C_{pk}$$



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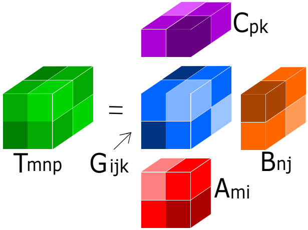

Main steps in the algorithm

- $Y_{mjk} = T_{mnp} B^{t}_{nj} C^{t}_{pk}$
- $Y_{ink} = T_{mnp} A^{t+1}_{mi} C^{t}_{pk}$
- $Y_{ijp} = T_{mnp} B^{t+1}_{nj} A^{t+1}_{mi}$

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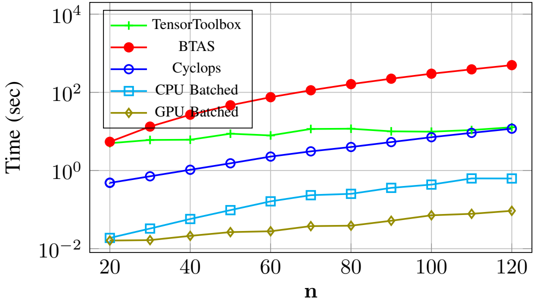

Figure: Performance on Tucker decomposition.

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- Available in CuBLAS 8.0
- Future works: Multi-index contractions or sparse tensor algebra.

Thank you!

More information: http://shiyangdaisy1.wixsite.com/ucieecs