A Large Deviation Analysis of Detection over Multi-Access Fading Channels with Random Number of Sensors

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Research Objectives

- Problem of distributed detection in a large Wireless Sensor Network.
- Presence of unreliable sensors, multi-access fading channel, limited Bandwidth and Energy : Classical approach inadequate.
- Propose a communication scheme.
- Establish performance guarantees for our scheme.

Our Approach

- Incorporate Poisson no. of transmitting sensors.
- Propose Type-Based Random Access scheme.
- Two scenarios : Fusion Center knows/does not know the number of transmitting sensors.
- Use Large deviation approach to derive Detection Error Exponents, as mean number of transmitting sensors goes to infinity.
- Apply Gärtner-Ellis Theorem to characterize the impact of Random number of sensors.
- Use a Minimum Rate Detector and prove that it yields Best Bayesian Error Exponent for TBRA.

Introduction

Classical Distributed Detection

- Sensors : Sense physical phenomenon and transmit their local decisions.
- Fusion Center: Make decision on the phenomenon.
- Sensor-Fusion Center Communication Perfect (Error - free) with rate constraints.
- Typically in Radar communication.

Key Issues

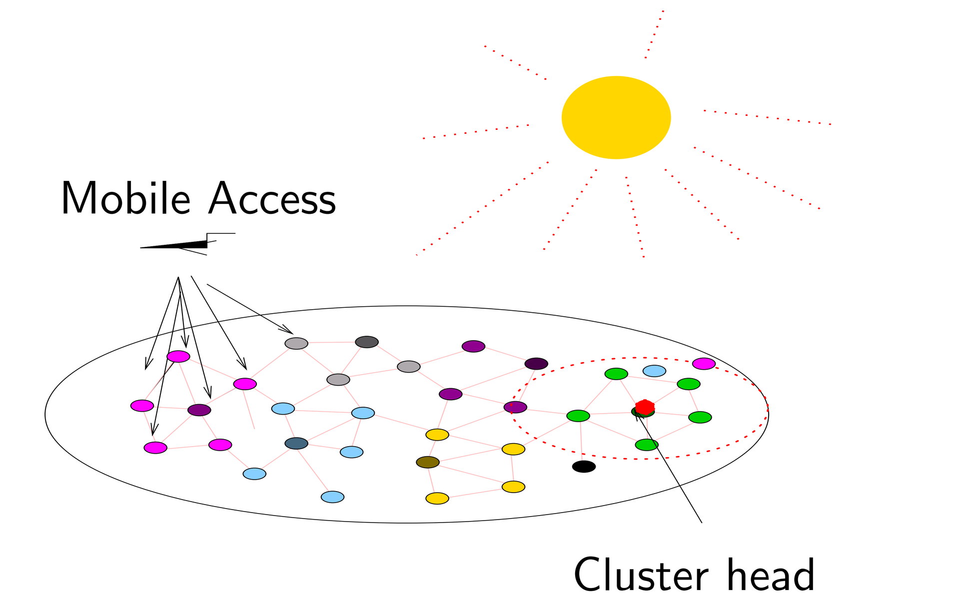
Quantization (Sensor rule) and Inference (FC rule).

Large Wireless Sensor Networks

- Low Power and Low Rate Transmissions.
- Bandwidth Allocation to large number of sensors.
- Multi-access Channel with Fading.
- Energy Efficiency to prolong network life-time.
- Faulty, sleeping or poorly placed sensors.
- Schemes requiring dedicated orthogonal channel for each sensor (TDMA, FDMA, CDMA) inefficient.

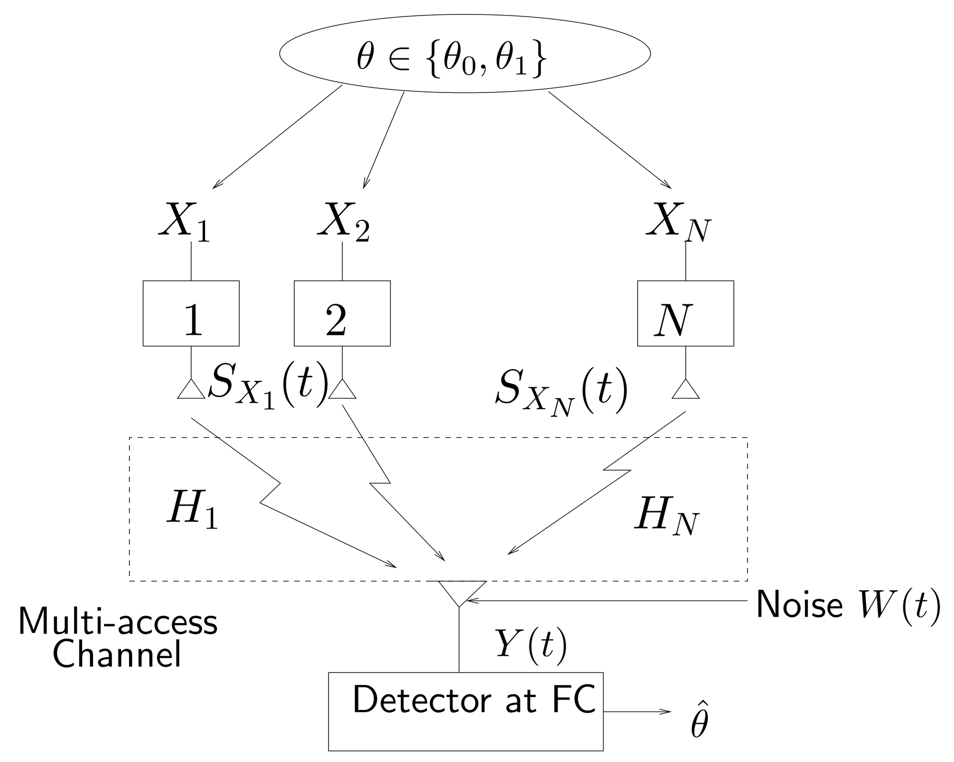
Medium Access Design is a key component.

Random Access



- Random Number of Sensors in a data collection.
- Probabilistic Wake-up : Sensors transmit based on a coin-flip.
- Sensors transmit only Significant Data.
- FC is a Mobile Access Point : collects data from different geographic regions.

System Model



Detection of Binary Hypothesis

$$\mathcal{H}_0 : \theta = \theta_0 \text{ vs. } \mathcal{H}_1 : \theta = \theta_1.$$

Poisson number of sensors N with mean λ.

Sensor Quantization: Sensor data X_j quantized to M levels and Conditionally IID given θ.

Multi-access model

- Flat IID fading: H_j with mean $\mu_H > 0$.
- Additive White Gaussian Noise $W(t)$.

Detector at FC : Minimum Rate Detector.

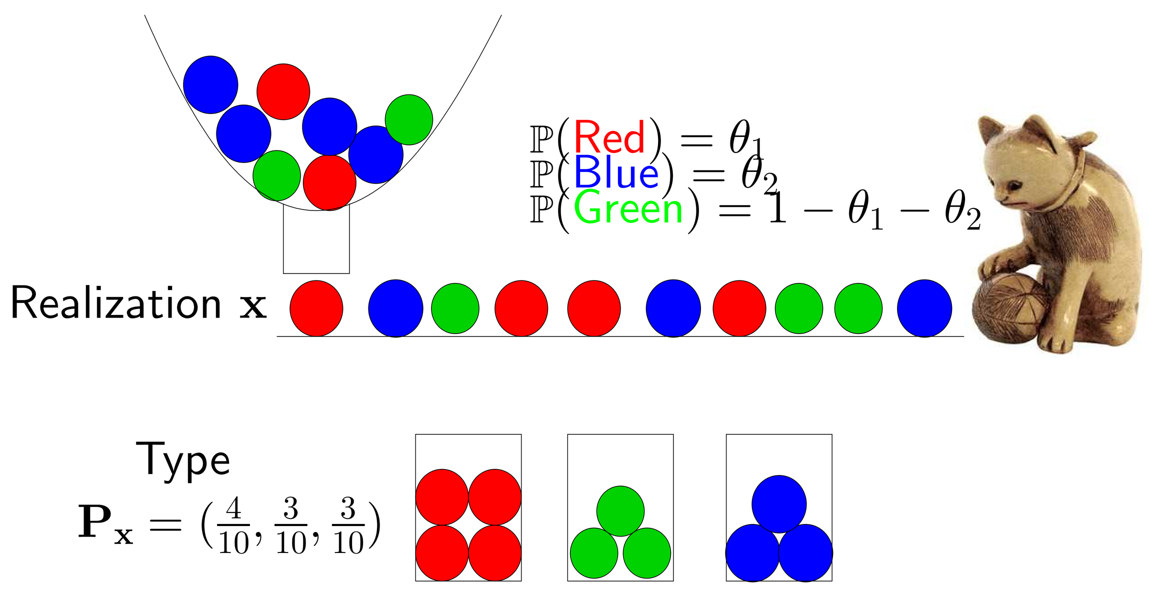
Error Exponents

Type-I/II Errors : $\alpha \triangleq \mathbb{P}\{\mathcal{H}_0 \to \mathcal{H}_1\}$, $\beta \triangleq \mathbb{P}\{\mathcal{H}_1 \to \mathcal{H}_0\}$.

$$\eta_1 \triangleq -\lim_{\lambda\to\infty} \frac{1}{\lambda} \log \alpha, \quad \eta_2 \triangleq -\lim_{\lambda\to\infty} \frac{1}{\lambda} \log \beta.$$

Worst Exponent Wins : Bayesian Error Exponent is $\min(\eta_1, \eta_2)$.

Type Based Random Access



Type $\mathbf{P}_\mathbf{x}$ gives sufficient statistics. Thus it suffices to deliver $\mathbf{P}_\mathbf{x}$ at FC.

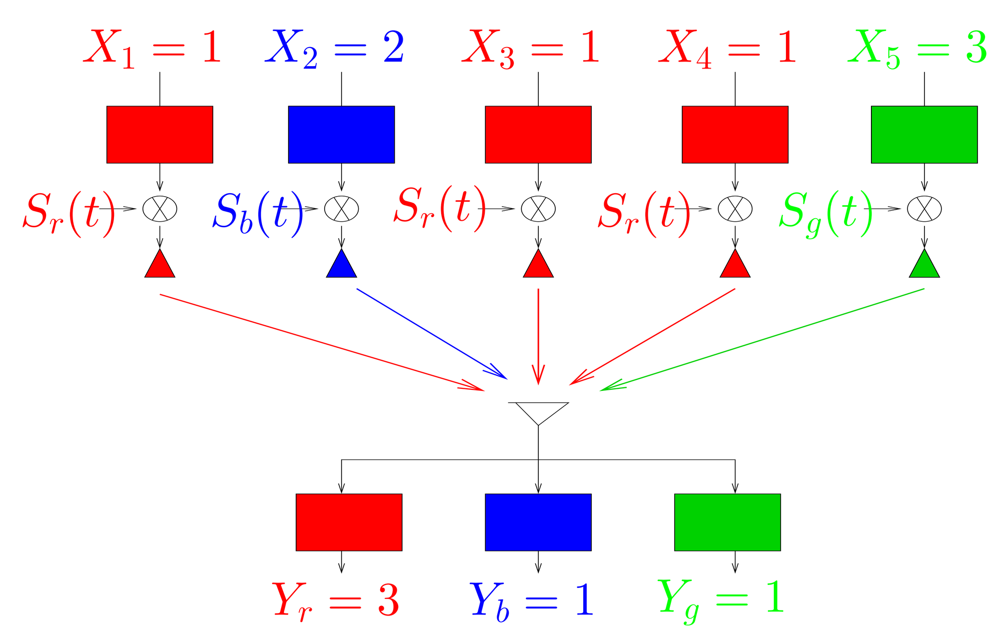
Signal Waveform : $S_1(t), \ldots, S_M(t)$—a pre-determined set of M orthogonal waveforms with energy $\mathcal{E}$.

Sensor Encoding : Quantized Data $X_j = x$ is encoded to waveform $S_x(t)$.

Waveform @ FC assuming synchronization:

$$Y(t) = \sum_{j=1}^{N} H_j \sqrt{\mathcal{E}} S_{X_j}(t) + W(t).$$

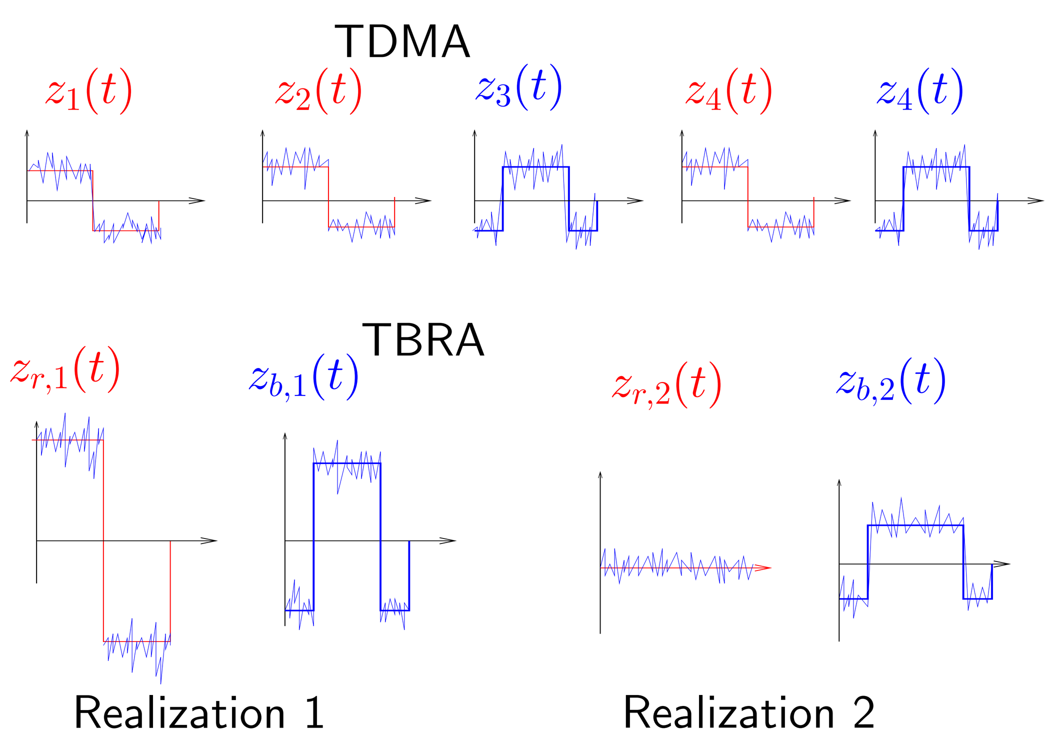
TBRA is a Data-centric in contrast to user-centric schemes (TDMA, FDMA or CDMA).



Key: MAC adds transmissions with same data level.

Under Ideal Conditions (Deterministic N, no fading and noise), $\frac{\mathbf{Y}}{\lambda}$ gives Type or Empirical Distribution of X_j.

Intuitions, Advantages, and Caveat



Advantages

- Noise attenuated by $1/\lambda^2$.
- Bandwidth $\propto M$ (not λ).

Caveats

- Effect of Fading.
- Random no. of sensors.

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Minimum Rate Detector

- Optimal (Likelihood Ratio) Detector intractable.

$$L(\mathbf{Y}) \triangleq \frac{\mathbb{P}(\mathbf{Y};\theta_1)}{\mathbb{P}(\mathbf{Y};\theta_0)} \underset{1}{\overset{0}{\lessgtr}} \tau.$$

- Asymptotic nature of output $\mathbf{Y}$ using Large Deviations Principle (LDP).
- Probability of a Rare event from its "mean" behavior by quantifying the Rate function $I(\mathbf{Y})$

$$I:\mathbb{R}^M \to \mathbb{R}_+ \cup \{\infty\}.$$

- Characterization through Asymptotically "Tight" Exponential Upper and Lower bounds.

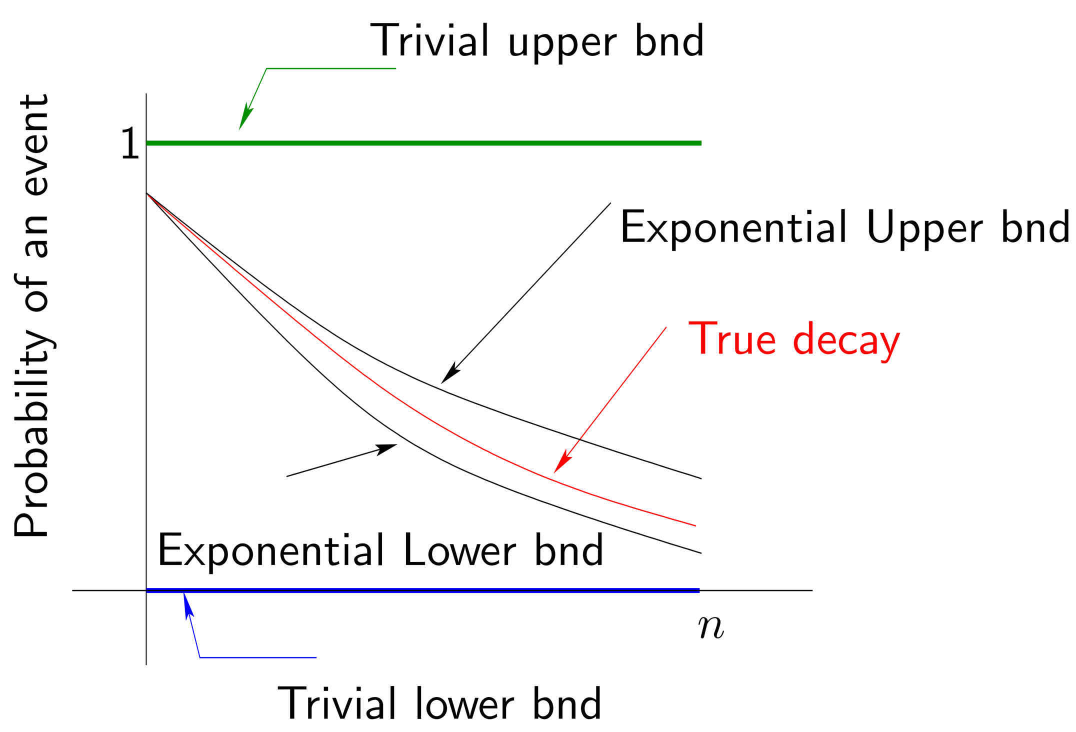
Motivating Example

$X_i \overset{IID}{\sim} \mathcal{N}(0,1)$. Then $\hat{S}_n \triangleq \frac{1}{n}\sum_{i=1}^{n} X_i$ satisfies LDP

$$\frac{\delta\sqrt{n}}{n\delta^2+1} e^{-\frac{n\delta^2}{2}} \leq \mathbb{P}(|\hat{S}_n| > \delta) = 2Q(\delta\sqrt{n}) \leq \frac{1}{\delta\sqrt{n}} e^{-\frac{n\delta^2}{2}}$$

Rate Fn. : $I(\delta) = \lim_{n\to\infty} -\frac{1}{n}\log \Pr(|\hat{S}_n| > \delta) = \frac{\delta^2}{2}$

Interpretation of LDP



For large λ, Probability of output $\mathbf{Y}$ being near $\mathbf{y}$ behaves as $\exp(-\lambda I(\mathbf{y}))$.

Decision Regions of MRD

Decide $\mathcal{H}_0$ if asymptotic likelihood under $\mathcal{H}_0$ is higher $(\exp(-\lambda I_0(\mathbf{y})) \geq \exp(-\lambda I_1(\mathbf{y})))$ *i.e.*,

$$\Gamma_0 = \{I_0(\mathbf{y}) \leq I_1(\mathbf{y})\}, \quad \Gamma_1 = \mathbb{R}^k \setminus \Gamma_0.$$

Asymptotic optimality of MRD

MRD achieves best Bayesian Error exponent for TBRA.

Impact of Random Access

Two Scenarios under Random Access

- FC knows number of transmitting sensors.
- FC does not know number of transmitting sensors.

We prove LDP for both the scenarios and characterize the Rate function.

Let $\phi(\mathbf{t})$ be Moment Generating Function of a single "faded" observation.

Nature of N	Rate Function $I(\cdot)$
Deterministic	$I^d(\mathbf{y}) = \sup_{\mathbf{t}}(\langle \mathbf{y},\mathbf{t}\rangle - \log\phi(\mathbf{t}))$
Random, Known	$I^{rk}(\mathbf{y}) = 1 - \exp(-I^d(\mathbf{y}))$
Random, Unknown	$I^{ru}(\mathbf{y}) = \sup_{\mathbf{t}}\big(\langle \mathbf{y},\mathbf{t}\rangle - \phi(\mathbf{t}) + 1\big)$

Common features between Fixed and Random N

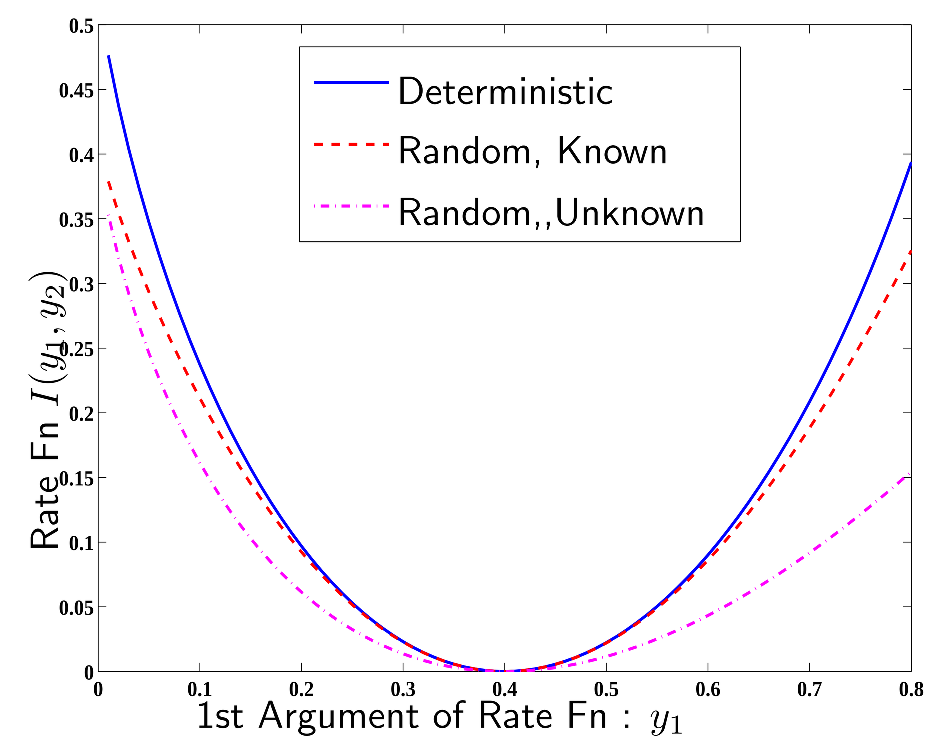
- Noise does not affect the rate function.
- Minimum Rate Detector is Asymptotically Optimal, in terms of Error Exponents

Advantages of Random Access

- Easier Computation of Rate Function : Optimization reduced to M independent 1-dimensional equations.
- Easier Scalability : Just need to increase the wake-up probability of sensors.

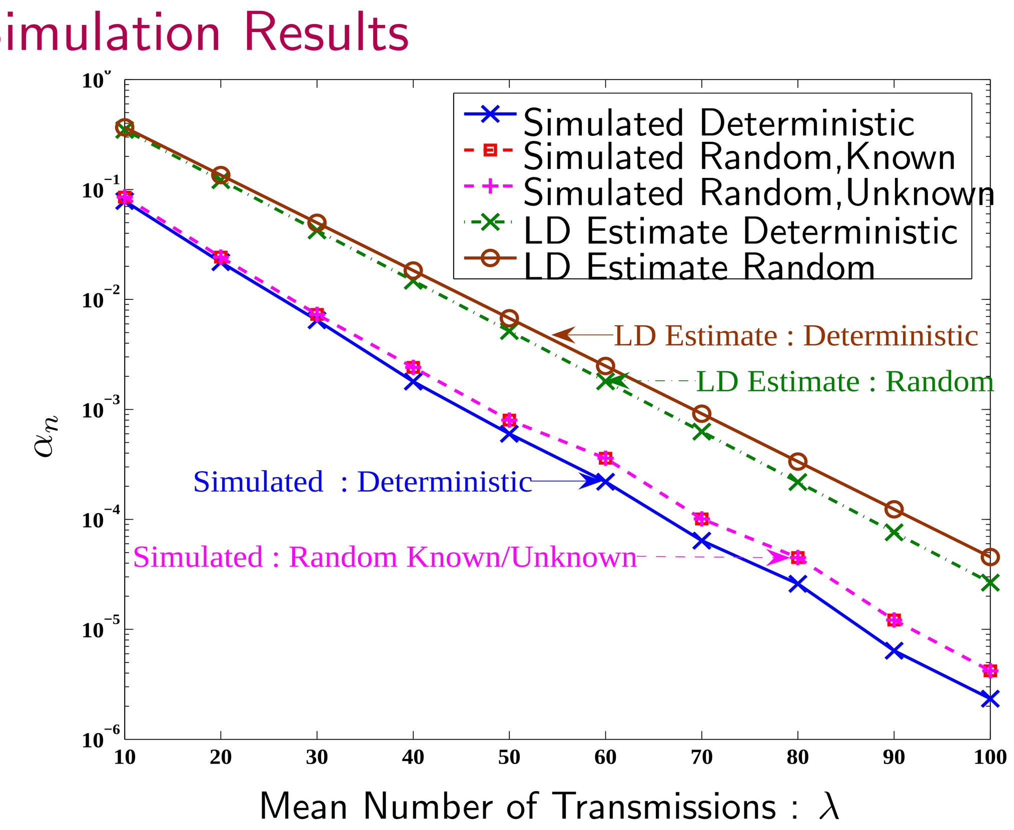
Caveat : Slower decay of Error Probabilities .

Numerical Example

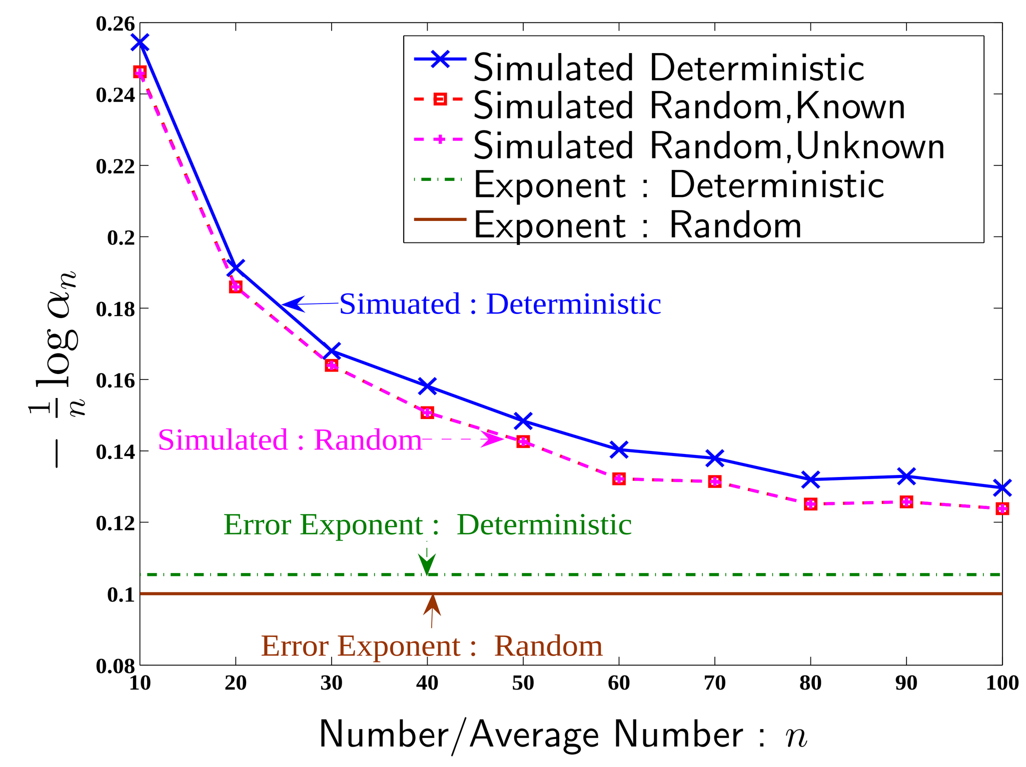


Shadow fading : H_j are Bernoulli $\{0,1\}$.

Simulation Results



Error Probability vs. Mean no. of sensors



Error Exponent vs. Mean no. of sensors

- Performance gap between the random and deterministic N at large λ as predicted.
- As λ grows, exponent in simulations approaches the theoretical value.

Conclusion

- Introduced Type-Based Random Access with attractive features : Constant Bandwidth requirement, easy scalability, reduced computation etc.,
- Studied impact of Random Access on detection through Large Deviations Theory.
- Characterized Cost of Random Access through Error Exponents.

Outlook

- Assumption of non-zero mean fading difficult to achieve in practice. Removal of requirement in [1].
- Design of sensor quantization rule not considered here.
- "Cross-Layer" optimization of local quantization, communications and global inference desired.

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