Distributed Statistical Inference using Type Based Random Access over Multi-access Fading Channels

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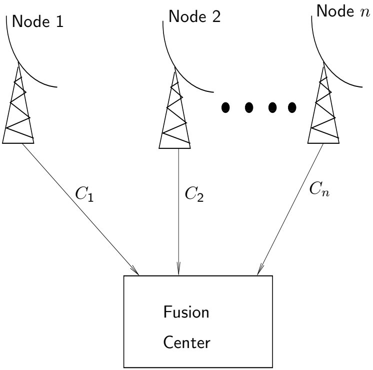
03/22/2006.

Classical Distributed Inference



- Sensors : Sense physical phenomenon and transmit their local decisions.
- Fusion Center: Make inference on the phenomenon.
- Sensor-Fusion Center Communication Perfect (Error - free) with rate constraints.
- Typically in Radar communication.

Key Issues

- Quantization @ sensors.
- Inference @ fusion center.

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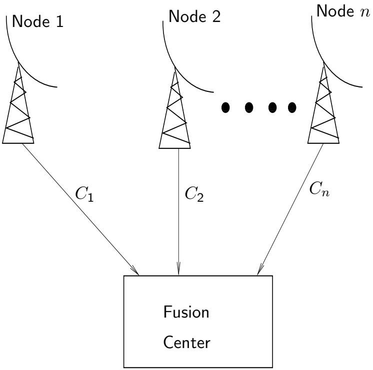

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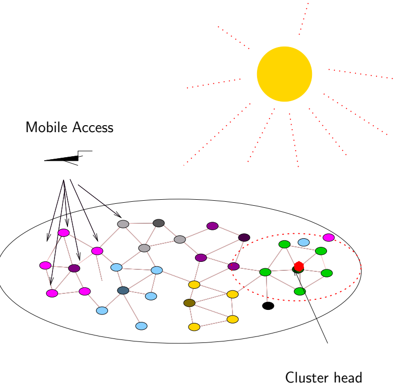
Inference in Large Wireless Sensor Networks



Characteristics

- Low Power and Low Rate Transmissions.
- Bandwidth Allocation.
- Multiaccess Channel with Fading.
- Energy Efficiency to prolong network life-time.
- Faulty, sleeping or poorly placed sensors.
- Deterministic scheduling (TDMA) may not be appropriate.

Medium Access Design is a key component.

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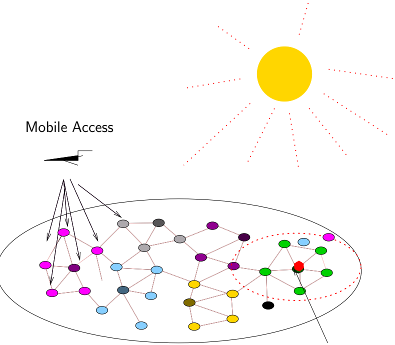

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Random Access

- Model : Random Number of Sensors in a data collection.
- Probabilistic Wake-up : Transmit based on a coin-flip.
- Transmit only Significant Data.
- Fusion center is a Mobile Access Point : collects data from different geographic regions.

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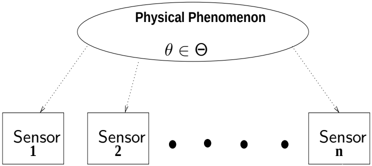
Distributed Inference over Multi-Access Channels

Detection (Binary Hypothesis) and Estimation.



Random number of sensors per collection
N_i are IID with mean λ.

Sensor Quantization: $X_{i,j}$ quantized to M levels and Conditionally IID given θ

$$X_{ij} \sim \mathbf{p}_\theta = (p_\theta(1), \cdots, p_\theta(M))$$

Multi-access model

- Flat IID fading: $H_{i,j}$
- AWGN $W(t)$ with PSD $=\sigma^2$.

Inference at Fusion Center

- Neyman Pearson or Bayesian Detection.
- Maximum Likelihood Estimation.

Multiple collections: i–time index, j–sensor index.

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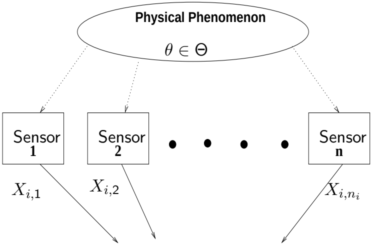

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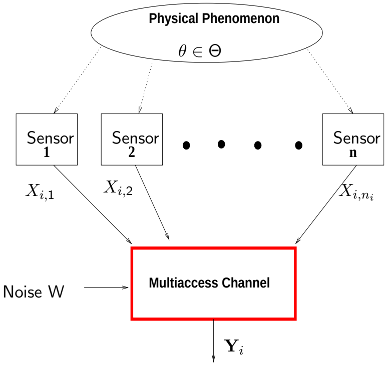

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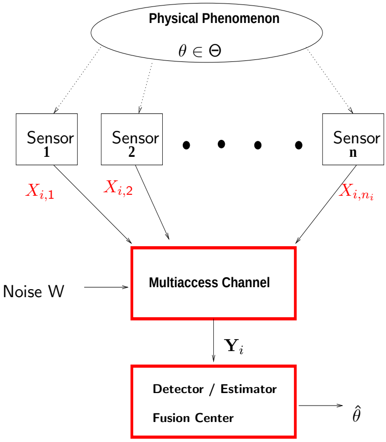
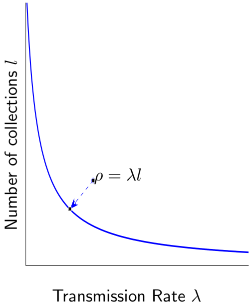


Multiple collections: i–time index, j–sensor index.

Spatio-Temporal Tradeoff

- Mean Transmitting Rate λ and Number of Data Collections l.
- Suppose we fix mean number of transmissions is $\rho \stackrel{\Delta}{=} \lambda l$, (proportional to energy budget).
- Should energy be allocated to simultaneous transmissions : large λ ?
- Or should we collect more data : large l ?
- Small λ : not enough sensors transmit , cannot counter noise.
- But, large λ : Less Observations as l is small.
- Role of multi access channel : Coherence or Cancelation ?



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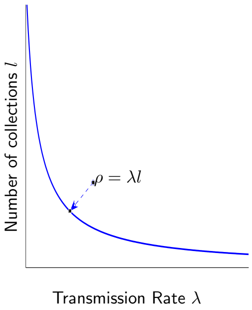

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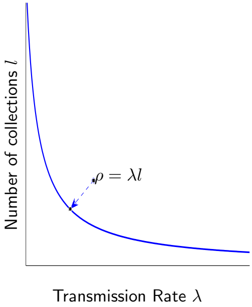


Fading Coherence Index γ

- Define Fading Coherence index as

$$\gamma = \frac{|\mathbb{E}(H)|^2}{\mathsf{Var}(H)}.$$

- Non Coherent ($\gamma = 0$) : Uniform phase uncertainty (*e.g.*,Rayleigh.)
- Perfectly Coherent ($\gamma = \infty$) : Deterministic Channel or no fading.

Outline

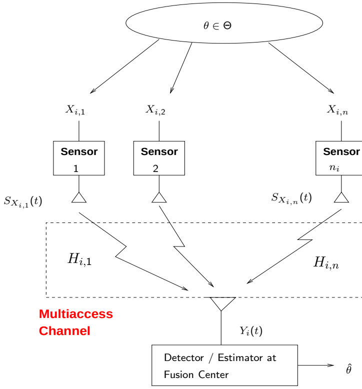
Outline

Type Based Random Access

- Signal Waveform
  $S_1(t), \ldots, S_M(t)$—a pre-determined set of M orthogonal waveforms with energy constraint $\mathcal{E}$.

- Sensor Encoding
  Quantized Data $X_{i,j} = x$ is encoded to waveform $S_x(t)$

$$S_{i,j}(t; x) = S_x(t)$$



- Observation @ FC:

$$Y_i(t) = \sum_{j=1}^{N_i} H_{i,j} \sqrt{\mathcal{E}} S_{X_{i,j}}(t - \tau_{i,j}) + W_i(t).$$

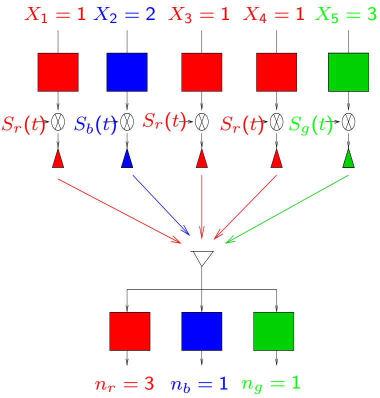
- Narrow band signal assumption :
  $S_{i,j}(t - \tau_{i,j}) \approx S_{i,j}(t)$.

- Data-centric in contrast to user-centric schemes (TDMA, FDMA or CDMA).

Key: MAC adds transmissions with same data level

Matched Filter Output

Schematic of TBRA



Ideal Conditions with $\lambda = 5$

Matched Filtering

$$\begin{aligned} \mathbf{Y}_i &\triangleq \frac{1}{\sqrt{\mathcal{E}}}\Big[\langle Y_i(t), S_1(t)\rangle, \cdots, \langle Y_i(t), S_M(t)\rangle\Big] \\ &= \sum_{j=1}^{N_i} H_{i,j}\mathbf{e}_{X_{i,j}} + \mathbf{W}_i, \quad \mathbf{W}_i \overset{iid}{\sim} \mathcal{N}(0, \frac{\sigma^2}{\mathcal{E}}\mathbf{I}) \end{aligned}$$

where $\mathbf{e}_{X_1}, \cdots, \mathbf{e}_{X_M}$ are basis vectors.

Ideal Conditions (Deterministic $N_i \equiv \lambda$, $H_{i,j} \equiv 1$ or $\gamma = \infty$ and $\mathbf{W}_i \equiv 0$)

$$\mathbf{Y}_i = \sum_{j=1}^{\lambda} \mathbf{e}_{X_{i,j}}.$$

jth entry of $\mathbf{Y}_i$: no. of sensors quantizing to level j.

$\frac{\mathbf{Y}_i}{\lambda}$ gives Type or Empirical Distribution of $X_{i,j}$.

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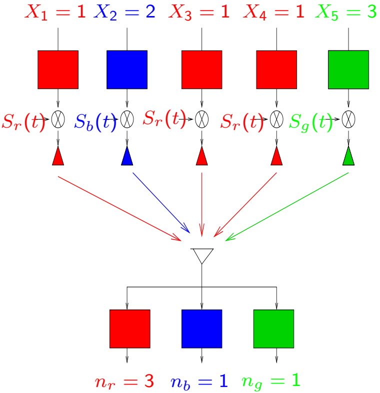

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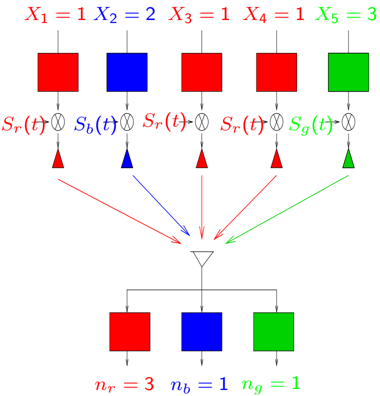

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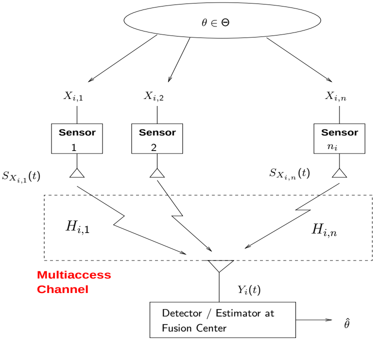
Role of Coherence Index γ for TBRA

- MAC adds transmissions with same data level
- Large γ : Better addition of signals in the mean since

$$\mathbb{E}\mathbf{Y}_i = \lambda \mathbb{E}(H)\mathbf{p}_\theta.$$

- Small γ : Effect of sensor data is only a second order effect (through the Channel Variance).



$$\gamma = \frac{|\mathbb{E}(H)|^2}{\mathrm{Var}(H)}.$$

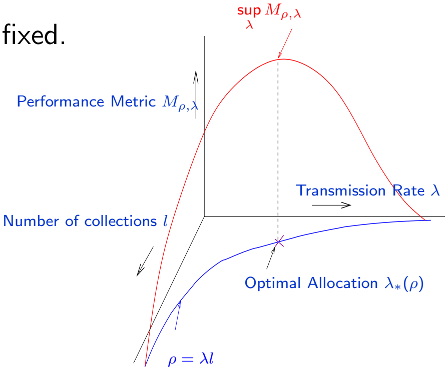
Outline

Spatio-Temporal Tradeoff

- Mean no. of transmissions $\rho = \lambda l$ fixed.
- Performance Metric $M_{\rho,\lambda}$.
- Optimal allocation in λ and l

$$\lambda_*(\rho) = \arg\sup_{\lambda} M_{\rho,\lambda}.$$

- For finite ρ, $M_{\rho,\lambda}$ intractable in our setup.



Asymptotic Performance Metric $M(\lambda)$

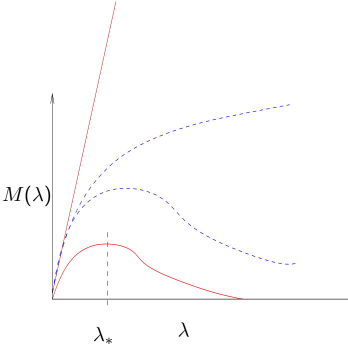
- Metric : Asymptotic (in ρ) Performance.

$$M(\lambda) \overset{\Delta}{=} \lim_{\rho \to \infty} M_{\rho,\lambda}.$$

- Goal : Existence of a optimal λ_* such that

$$\lambda_* = \arg\max_{\lambda} M(\lambda).$$

- λ_* is bounded : Cancelation, Avoid Interference.
- λ_* is unbounded : Coherence, Simultaneous Transmissions.



Performance Metric $M(\lambda)$

Detection

- Performance Metric is Detection error exponent

$$M(\lambda) \triangleq -\lim_{\rho\to\infty} \frac{1}{\rho} \log P_e(\rho,\lambda),$$

where $P_e(\rho,\lambda)$ is detection error probability.

- Under Neyman-Pearson or Bayesian setting,

$$\begin{aligned} M^{\mathrm{NP}}(\lambda) &= \frac{1}{\lambda} D_\lambda(f_0 \| f_1), \\ M^{\mathrm{B}}(\lambda) &= \frac{1}{\lambda} C_\lambda(f_0, f_1), \end{aligned}$$

$D_\lambda(f_0\|f_1)$: Kullback-Leibler distance.
$C_\lambda(f_0, f_1)$: Chernoff information.

Estimation

- Define Performance Metric

$$M(\lambda) \triangleq \frac{I_\lambda(\theta)}{\lambda},$$

$I_\lambda(\theta)$: Fisher Information of $\mathbf{Y}_i$.

- Cramer Rao Bound for any unbiased estimator $\hat\theta$ based on ρ mean no. of transmissions and transmission rate λ,

$$\mathrm{Var}(\hat\theta - \theta) \geq \frac{\lambda}{\rho I_\lambda(\theta)}.$$

- Asymptotic Efficiency of ML estimator

$$\sqrt{\rho}(\hat\theta_{\mathrm{MLE}} - \theta) \xrightarrow{d} \mathcal{N}(0, \frac{\lambda}{I_\lambda(\theta)}).$$

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Performance Metric

Optimal TBRA

Conclusion

Theorem on Existence of optimal λ

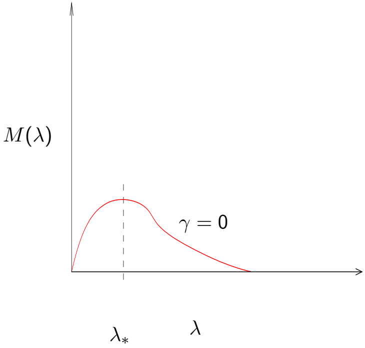
Under Regularity Conditions in the paper,

Non Coherent Channels ($\mathbb{E}(H) = 0$ or $\gamma = 0$) :
Existence of Bounded optimal λ_*

$$\lim_{\lambda \to 0} M(\lambda) = \lim_{\lambda \to \infty} M(\lambda) = 0,$$

which implies that there exists $0 < \lambda_* < \infty$ such that

$$\sup_{\lambda} M(\lambda) = \frac{1}{\lambda_*} I_{\lambda_*}(\theta).$$



Deterministic Channels ($\text{Var}(H) = 0$ or $\gamma = \infty$) : No Bounded optimizing λ_*,

$$M(\lambda) = \Theta(\lambda) \quad , \lambda \to \infty,$$

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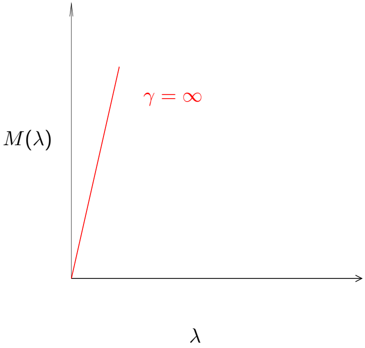

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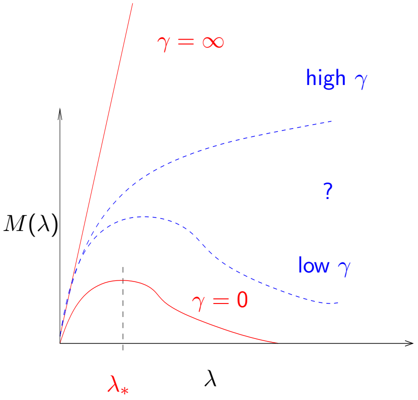
$M(\lambda)$ for different Coherence Indices.



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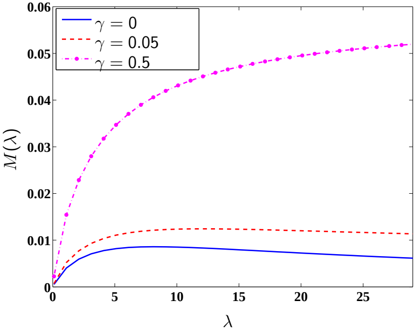
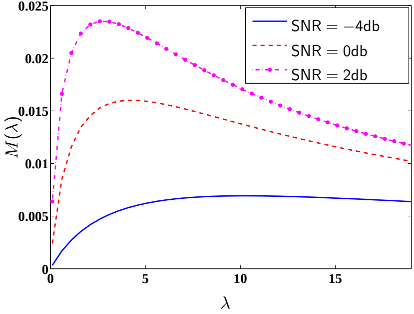
Numerical Plots of Performance Metric



(SNR = 7db, $\sigma_H^2 = 0.1$)



($\gamma = 0$, $\sigma_H^2 = 1$)

Normalized Chernoff Information vs. Transmission Rate.

Asymptotic Normal Distribution

- To analyze asymptotic behavior of $M(\lambda)$: compute Performance metric for limiting distribtution $\tilde{M}(\lambda)$.
- Since by continuity $\lim_{\lambda\to\infty} M(\lambda) = \lim_{\lambda\to\infty} \tilde{M}(\lambda)$.
- CLT for Random Number of Summands :

$$\frac{\mathbf{Y} - \lambda\mathbb{E}(H)\mathbf{p}_\theta}{\sqrt{\lambda}} \xrightarrow{d} \mathcal{N}\Big(0, \mathrm{Var}(H)\mathrm{Diag}(\mathbf{p}_\theta)\Big) \quad \text{as} \quad \lambda \to \infty.$$

- Gaussian Metric $\tilde{M}(\lambda)$: closed form expressions.
- For Large λ, approximate actual $M(\lambda)$ by Gaussian $\tilde{M}(\lambda)$.

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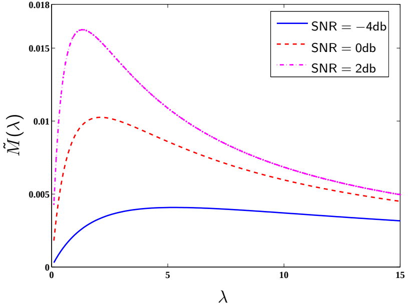
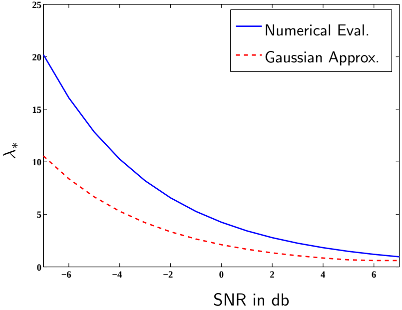
Gaussian Approximation



Gaussian Metric vs. Transmission Rate. ($\gamma = 0$, $\sigma_H^2 = 1$)



Optimal λ_* vs. SNR in db. ($\gamma = 0$, $\sigma_H^2 = 1$)

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Summary

- Introduced TBRA : removes requirement of channel coherency and handles random number of sensors.
- Provided a general characterization of Performance metric of estimation and provided approximate solutions.
- Proved the existence of optimal spatio-temporal allocation scheme dependent on Channel Coherence Index.

Related Publication

- A. Anandkumar and L. Tong, A Large Deviation Analysis of Detection over Multi-Access Channels with Random Number of Sensors, *accepted to Proc. of ICASSP 06, Toulouse, France, May 2006.*

- A. Anandkumar and L. Tong, Type-Based Random Access for Distributed Detection over Multiaccess Fading Channels, *Submitted to IEEE Trans. Signal Proc., Dec. 2005.*

- A. Anandkumar, L. Tong and A. Swami, Large deviation analysis of Sequential distributed detection using Type based Random Access, *To be submitted to Proc. of EUSIPCO, Sep. 2006.*

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- G. Mergen, V. Naware, and L. Tong, Asymptotic Detection Performance of Type-Based Multiple Access Over Multiaccess Fading Channels, submitted to IEEE Trans. on Signal Processing, May 2005.

- Ke Liu and A. M. Sayed, Optimal distributed detection strategies for wireless sensor networks,, in 42nd Annual Allerton Conf. on Commun., Control and Comp., Oct. 2004.

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Thank You !