Cost-Performance Tradeoff in Multi-hop Aggregation for Statistical Inference

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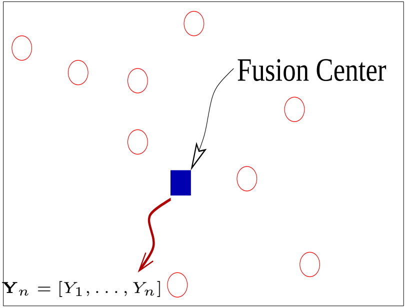
Distributed Statistical Inference

Sensor Network Applications: Statistical Inference

- Detection, e.g., Target, Pollutant
- Estimation, e.g., Temperature, Pressure



Classical Distributed Inference

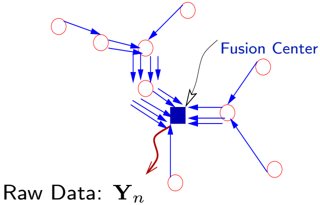
- Sensors: take measurements
- Fusion Center: Final decision

Design Issues

- Sensor Quantization
- Fusion Center Rule

Non-Scalable Nature of Layered Architecture

Routing: Shortest Path



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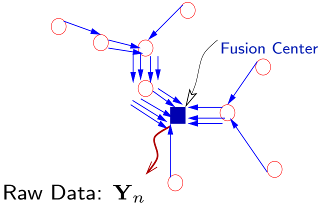
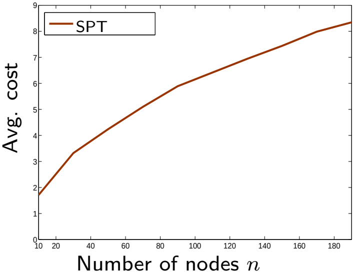


Routing Cost along link (i, j)= dist$(i, j)^2$



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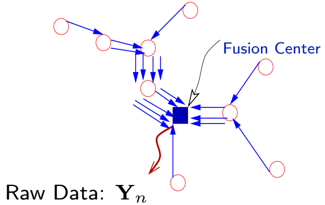

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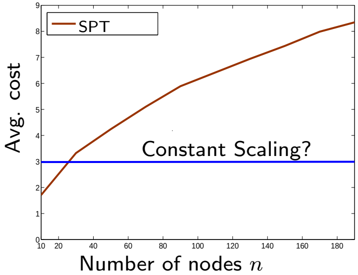

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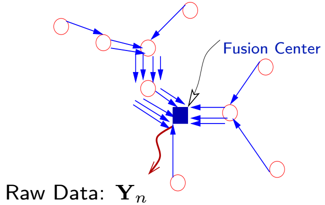

In-Network Data Fusion

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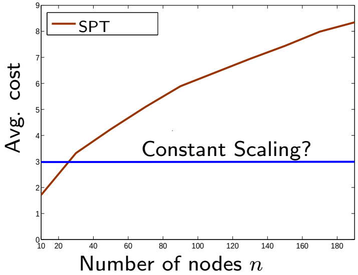

Sufficient Statistic For Inference

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- Function of raw data: same inference performance at fusion center
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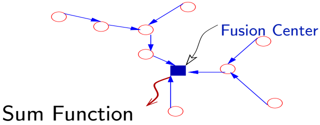

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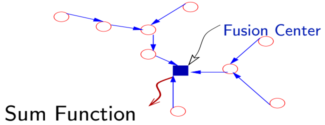

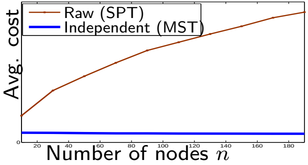

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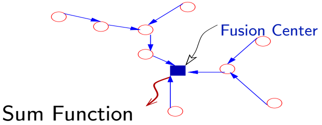

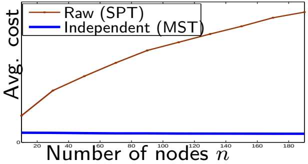

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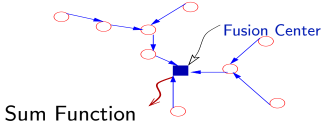

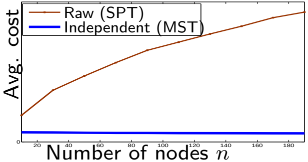

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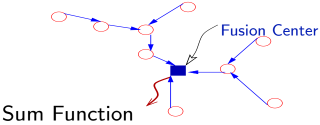

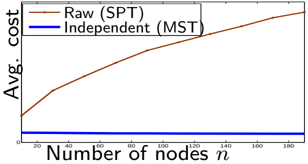

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- LLR$= \sum_i \log \frac{f(Y_i; \mathcal{H}_0)}{f(Y_i; \mathcal{H}_1)}$ minimally sufficient to decide hypothesis

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Definition

A statistic is minimally sufficient if every other sufficient statistic is a function of it: Maximum dimensionality reduction

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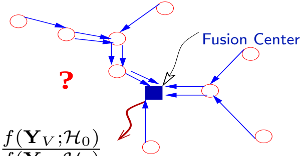

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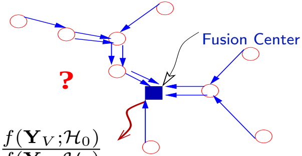

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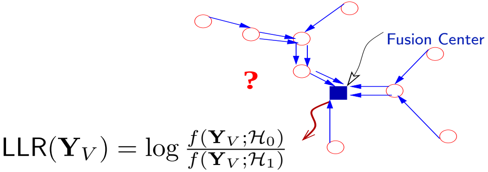

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Fusion Scheme?

Advantages: Cross-Layer Design

- Raw data not needed at fusion center (sufficient statistic)
- Goal: to achieve optimal inference performance

Optimal Node Selection For Tradeoff

Cost-Performance Tradeoff

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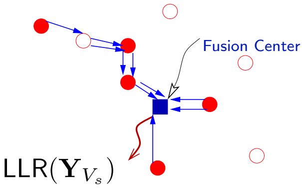

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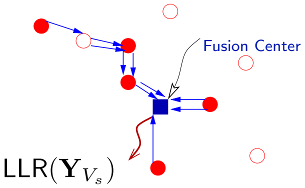

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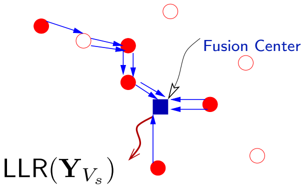

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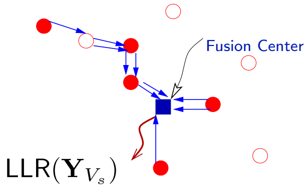

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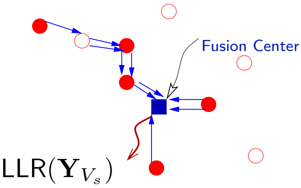

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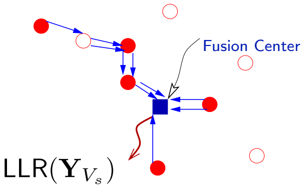

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Brute Force: $2^{|V|}$ Possible Subsets

Outline

Binary Hypothesis Testing

Null Hypothesis: $\mathcal{C}_0$

$$\mathcal{H}_0 : \prod_{c \in \mathcal{C}_0} f(\mathbf{Y}_c; \mathcal{H}_0)$$

Alternative: $\mathcal{C}_1$

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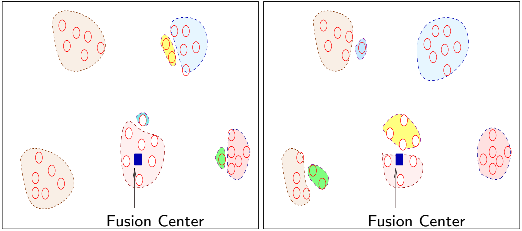

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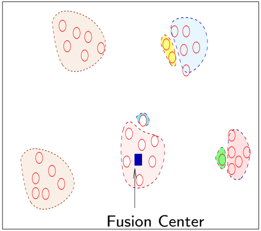

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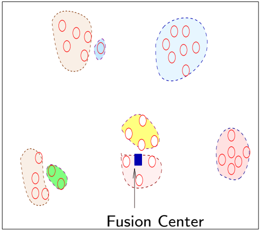

Spatial Correlation Model: Large Inter-Node, Small Intra-Node Distance

Conditionally independent node clusters

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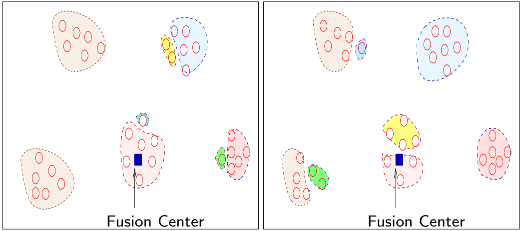

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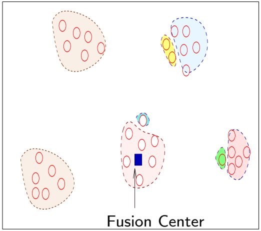

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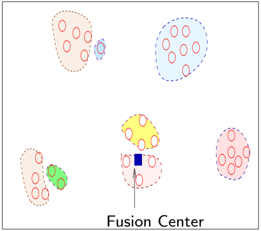

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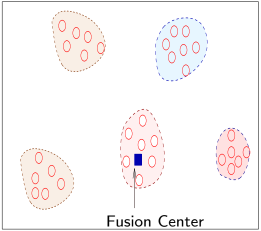

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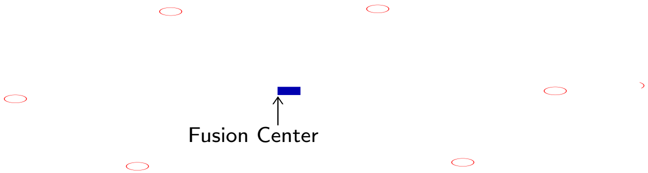

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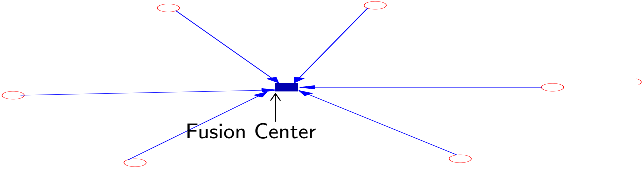
Single Hop with Independent Measurements



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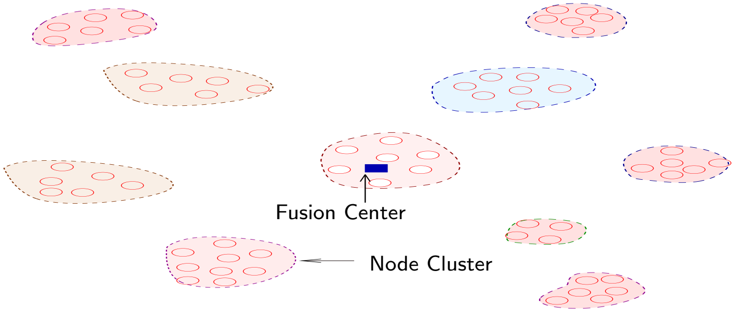
Multi Hop with Correlated Measurements



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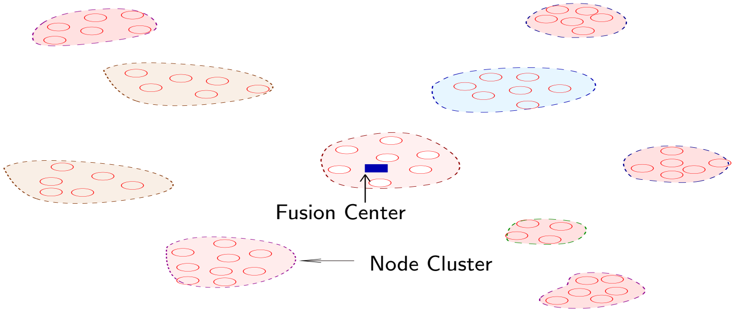

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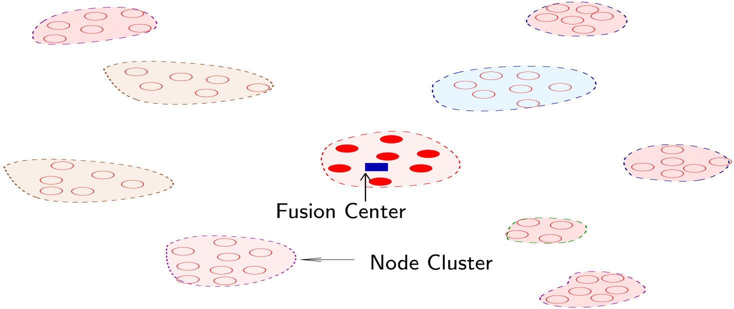

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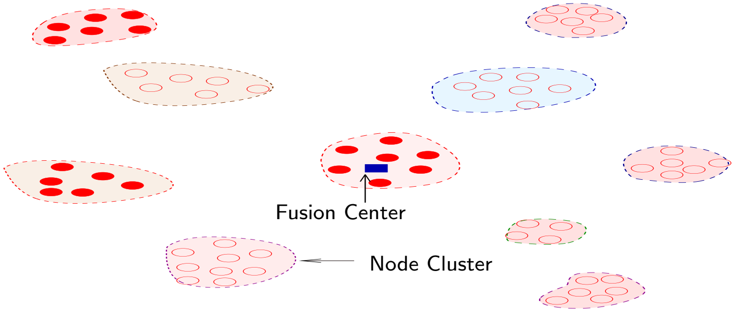
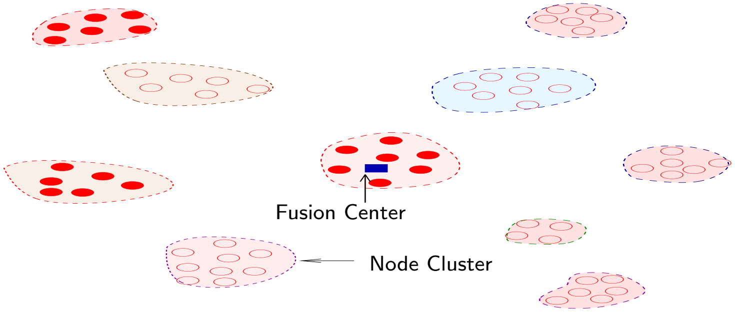


Subset of Node Selection Policies & Fusion Schemes

- Cluster selection : Select all or none of nodes in a clusters

Sensor Selection & Data Fusion

Multi Hop with Correlated Measurements



Subset of Node Selection Policies & Fusion Schemes

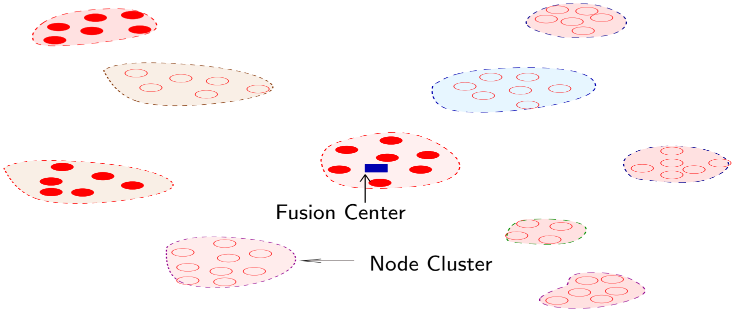
- Cluster selection : Select all or none of nodes in a clusters

Distributed Processing of Cluster : LLR($\mathbf{Y}_c$) for $c \in \mathcal{C}$

Designate site of computation: Processor of cluster c

Sensor Selection & Data Fusion

Multi Hop with Correlated Measurements



Subset of Node Selection Policies & Fusion Schemes

- Cluster selection : Select all or none of nodes in a clusters
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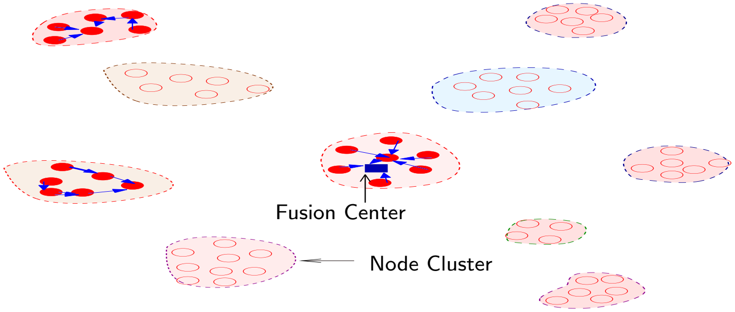
Multi Hop with Correlated Measurements



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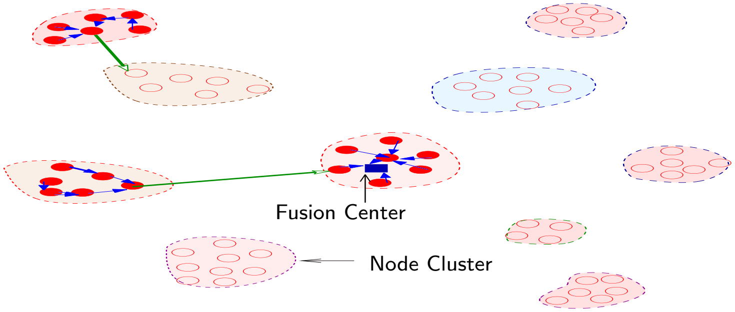

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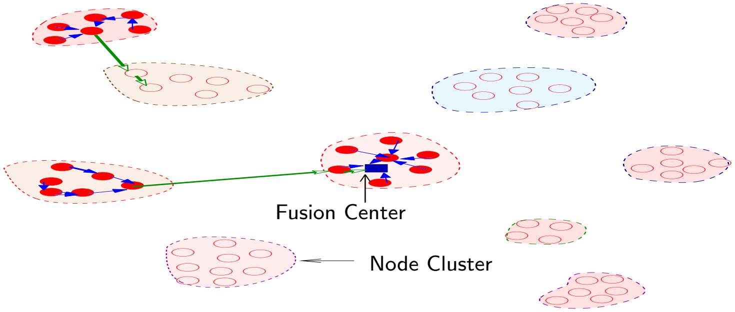

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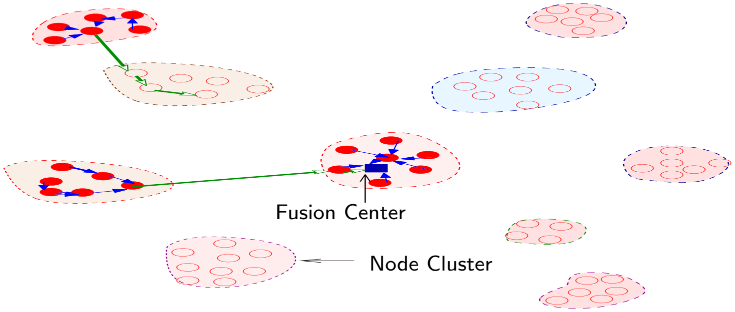

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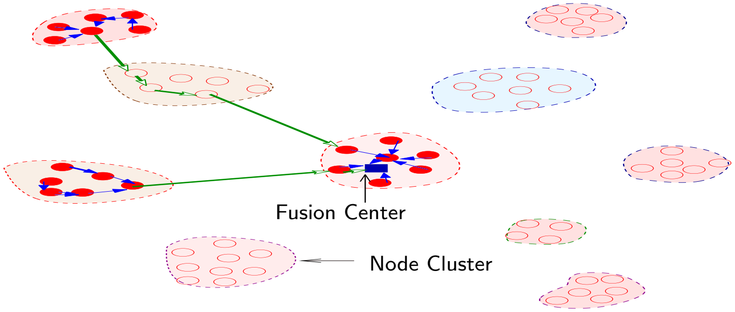

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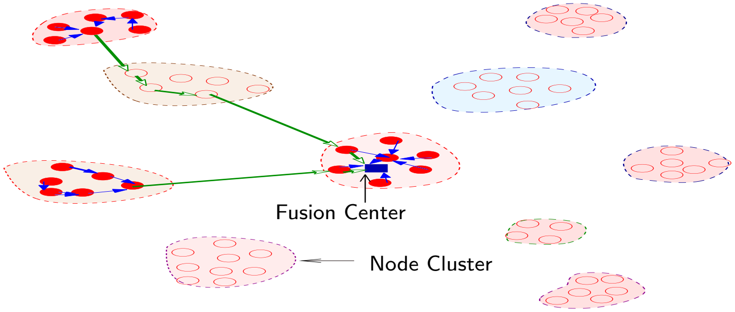

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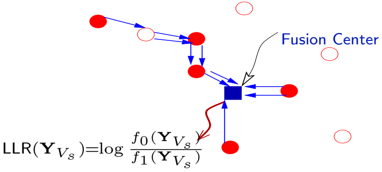
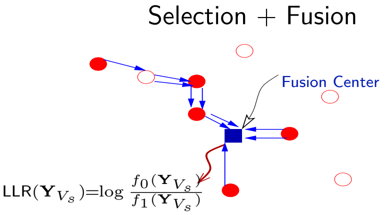
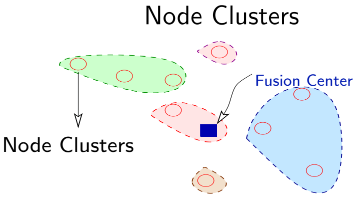
Problem Formulation of In-network Data Fusion

Selection + Fusion



Problem Formulation of In-network Data Fusion





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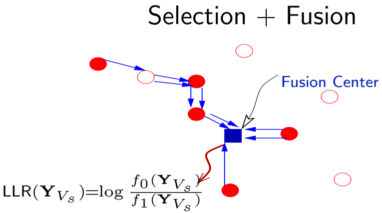

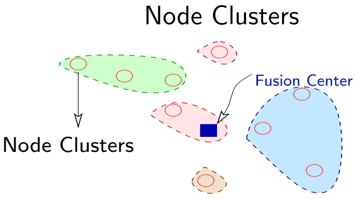

Cost-Performance Tradeoff

Select subset of node clusters for processing s.t. optimal tradeoff between routing costs and fraction of Neyman-Pearson mis-detection probability

$$\min_{V_s \subset V} \left[\mathsf{C}(G(V_s)) + \mu \log \frac{P_M(V_s)}{P_M(V)} \right], \quad \mu > 0.$$

Summary of Results for Cost-Performance Tradeoff

$$\min_{V_s \subset V} \left[\mathsf{C}(G(V_s)) + \qquad \mu \log \frac{P_M(V_s)}{P_M(V)} \qquad \right], \quad \mu > 0,$$

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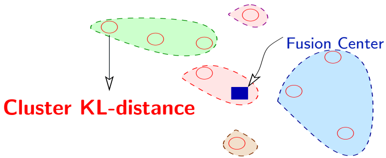
Neyman Pearson Error Exponent for Node Clusters

$$\mathcal{D} \overset{\Delta}{=} -\,\mathsf{p} \lim_{|V| \to \infty} \frac{1}{|V|} \log P_M(V) = \mathsf{p} \lim_{|V| \to \infty} \frac{1}{|V|} \sum_{c \in \mathcal{C}} D(f(\mathbf{Y}_c; \mathcal{H}_0) || f(\mathbf{Y}_c; \mathcal{H}_1)).$$

Summary of Results for Cost-Performance Tradeoff

Performance Measure



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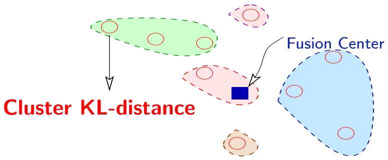
Summary of Results for Cost-Performance Tradeoff

Performance Measure



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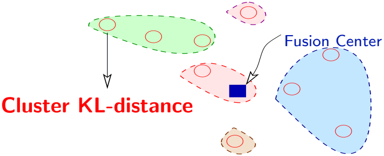
Decentralized Performance Measure: KL-Distance of Cluster

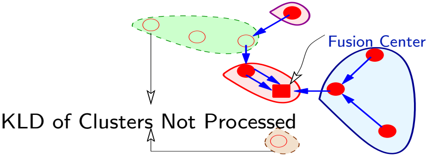
$$\log \frac{P_M(V_s)}{P_M(V)} \approx \sum_{c \in \mathcal{C} \setminus \mathcal{C}_s} D(f(\mathbf{Y}_c; \mathcal{H}_0) || f(\mathbf{Y}_c; \mathcal{H}_1)).$$

Summary of Results for Cost-Performance Tradeoff

Performance Measure



Optimal Tradeoff



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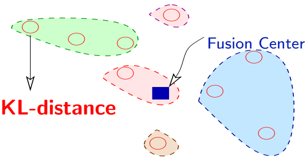

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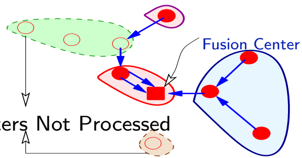

$$\min_{V_s \subset V} \left[\mathsf{C}(G(V_s)) + \mu \sum_{c \in \mathcal{C} \setminus \mathcal{C}_s} D(f_0(\mathbf{Y}_c) || f_1(\mathbf{Y}_c)) \right], \quad \mu > 0,$$

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Results for Cost-Performance Tradeoff (cont.)

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Cost-Performance Tradeoff

Select subset of clusters for processing such that optimal sum of routing costs plus KLD of clusters not selected

$$\min_{\mathcal{C}_s \subset \mathcal{C}} [\mathsf{C}(G) + \mu \sum_{c \notin \mathcal{C}_s} D(f(\mathbf{Y}_c; \mathcal{H}_0) || f(\mathbf{Y}_c; \mathcal{H}_1))].$$

Results for Cost-Performance Tradeoff (cont.)

Cost-Performance Tradeoff

Select subset of clusters for processing such that optimal sum of routing costs plus KLD of clusters not selected

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Prize-Collecting Steiner tree (PCST) Reduction

- PCST expanded graph with scaled cluster KL distance as cluster representative node penalty
- Approximation factor preserving reduction
- Goemans-Williamson approximation algorithm applicable
- Approximation factor of $2 - \dfrac{1}{\#\text{ of profitable clusters} - 1}$

Outline

Neyman Pearson Error Exponent

Neyman-Pearson Detection

Minimize $P_M = P[\text{Decision} = \mathcal{H}_0 | \mathcal{H}_1]$ s.t. $P_F = P[\text{Decision} = \mathcal{H}_1 | \mathcal{H}_0] \leq \alpha$

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Neyman-Pearson Detection

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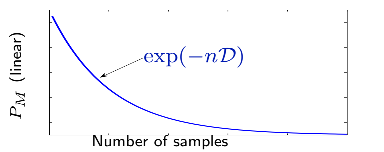
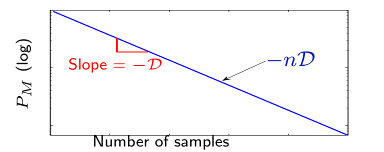
Error Exponent: Rate of Decay of Error Probability





$$P_M \approx e^{-n\mathcal{D}}$$

$$\log P_M \approx -n\mathcal{D}$$

Neyman Pearson Error Exponent

Neyman-Pearson Detection

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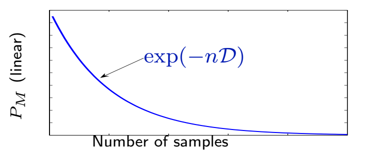

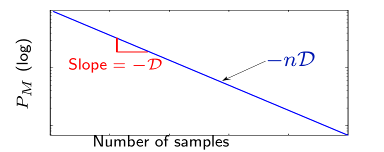

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NP Error Exponent for Clusters under Uniform Convergence

$$\mathcal{D} = \text{p} \lim_{|V|\to\infty} \frac{1}{|V|} \text{LLR}(\mathbf{Y}_V), \quad \mathbf{Y}_V \sim \mathcal{H}_0 \qquad \text{(Chen '96)}$$

$$\mathcal{D} = \text{p} \lim_{|V|\to\infty} \frac{1}{|V|} \sum_{c\in\mathcal{C}} D(f(\mathbf{Y}_c;\mathcal{H}_0) || f(\mathbf{Y}_c;\mathcal{H}_1))$$

Neyman Pearson Error Exponent

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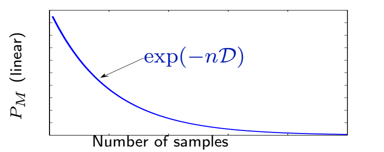

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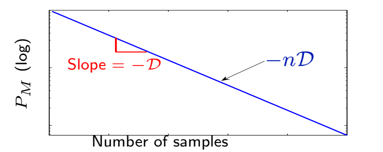

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KLD of each cluster is a decentralized performance measure

Cost-Performance Tradeoff for Inference

Problem Statement

Select subset of clusters for processing s.t. optimal tradeoff between routing costs and detection performance

$$\min_{\mathcal{C}_s \subset \mathcal{C}} [\mathsf{C}(G) + \mu \sum_{c \notin \mathcal{C}_s} D(f(\mathbf{Y}_c; \mathcal{H}_0) || f(\mathbf{Y}_c; \mathcal{H}_1))]$$

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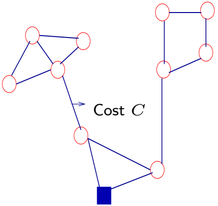
Network & Communication Model

Connected Network, Bidirectional Links, Unicast Mode

Comm. Graph with Link Costs



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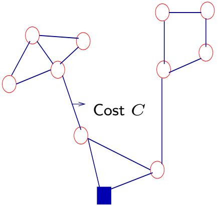
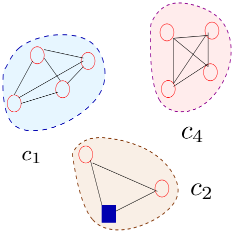


Node Clusters



Cluster Selection

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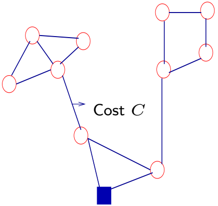

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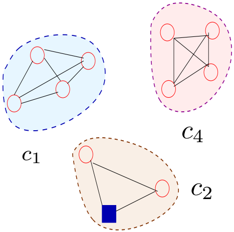

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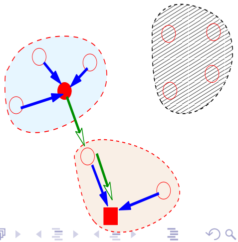
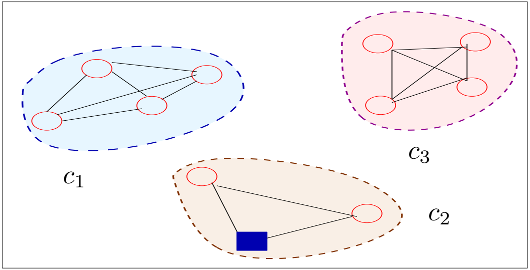
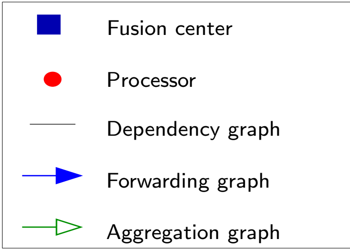


Overview of Fusion Schemes





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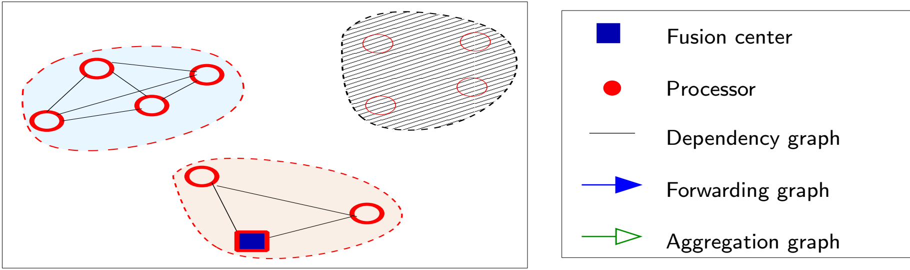

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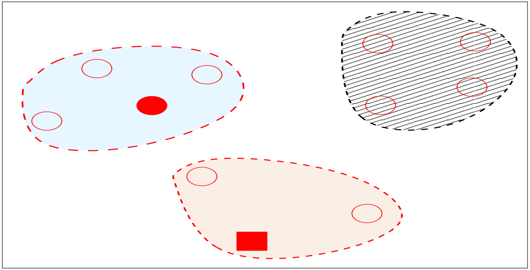

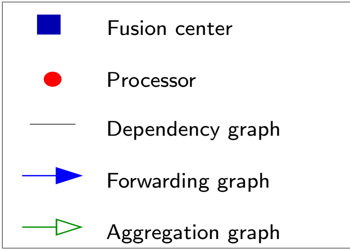

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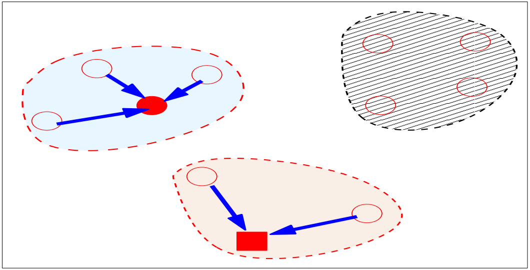

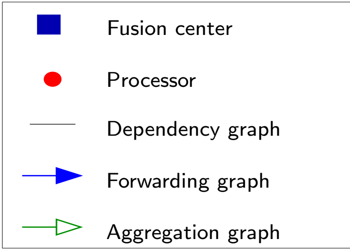

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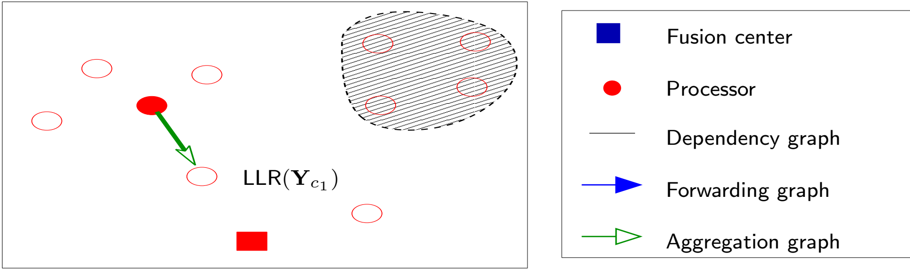
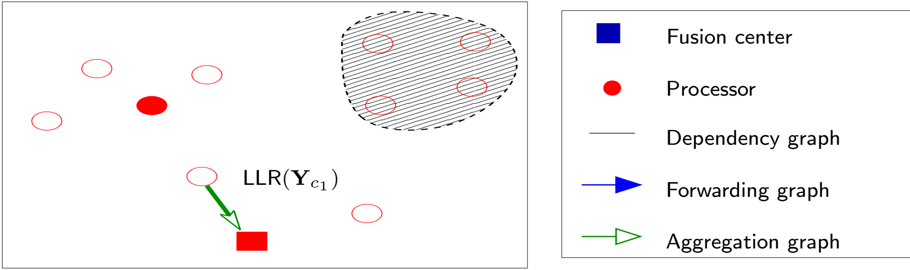


Overview of Fusion Schemes



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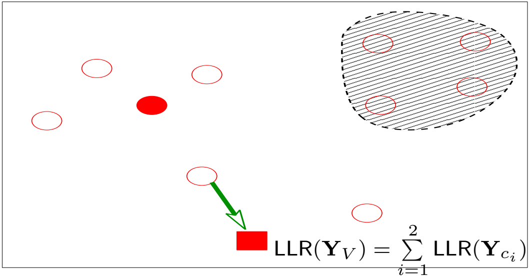
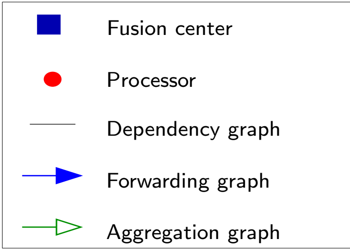
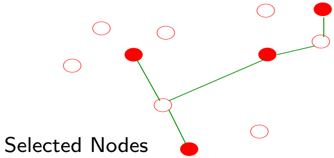




Prize Collecting Steiner Tree (PCST)



Definition

- Tree with minimum sum edge costs plus node penalties not spanned

$$T_* = \arg \min_{T=(V',E')} \Big[\sum_{e \in E'} c_e + \sum_{i \notin V'} \pi_i \Big].$$

- NP-hard, Goemans-Williamson algorithm has approx. ratio of $2 - \frac{1}{n-1}$

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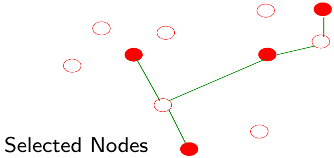

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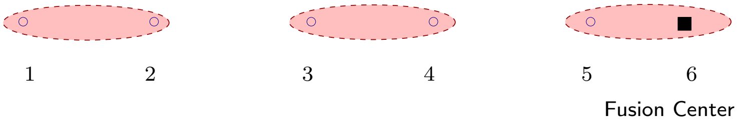
PCST Reduction

PCST on expanded graph with scaled cluster KLD as penalty

Example : Binary Clusters



Graph transformation and building prize-collecting Steiner tree.

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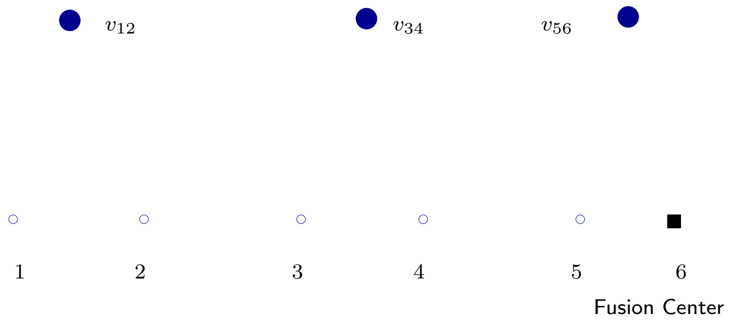

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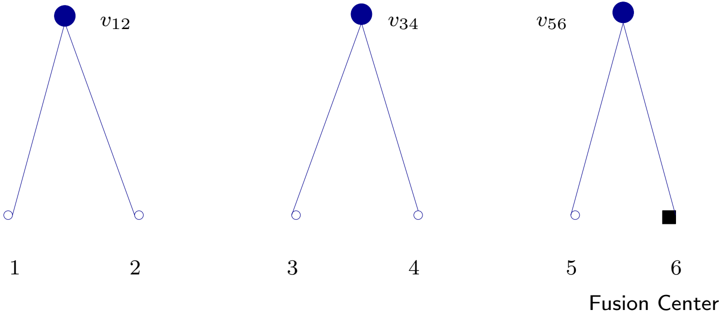

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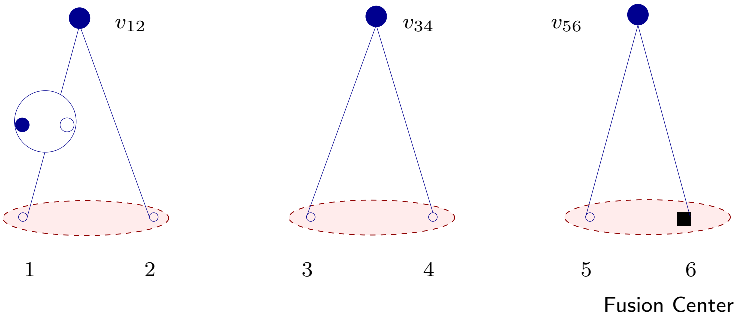

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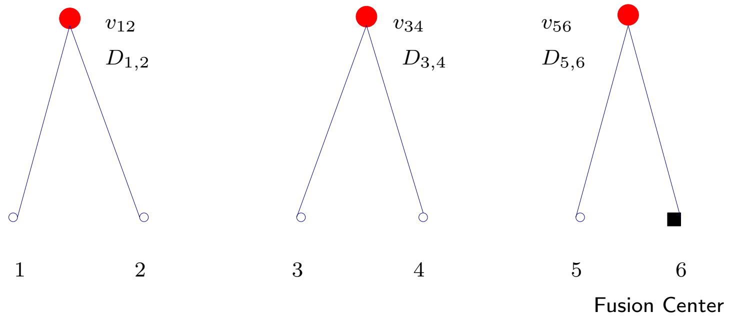

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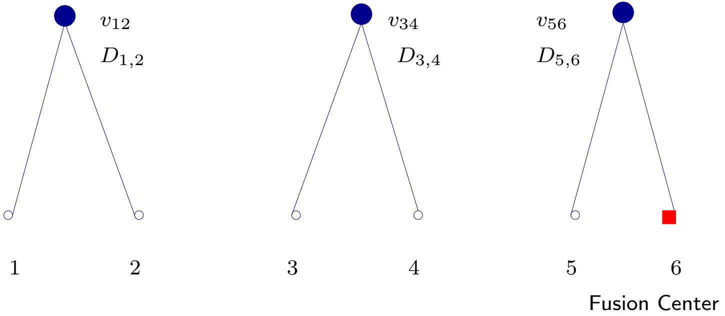

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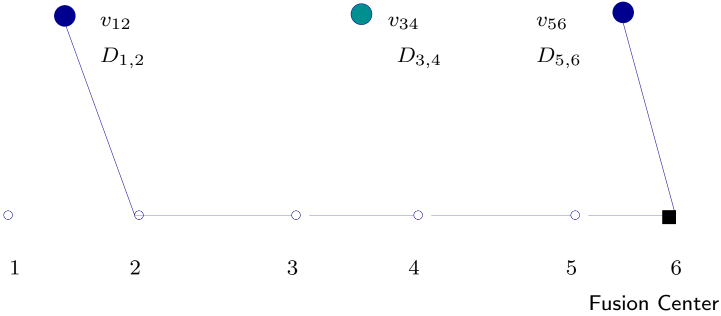

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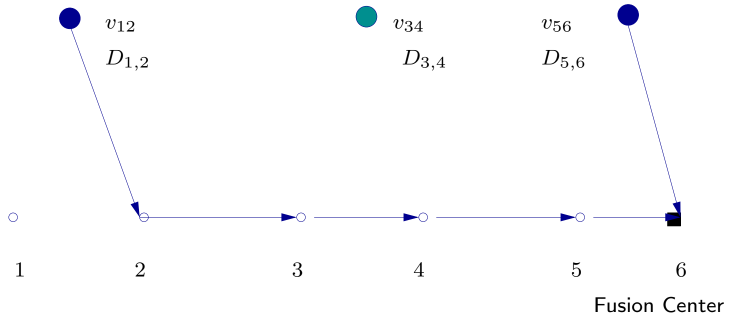

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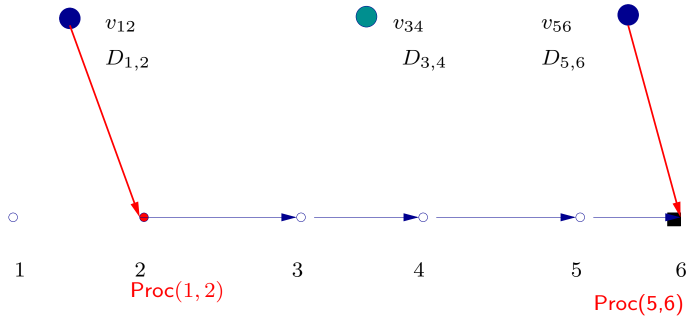

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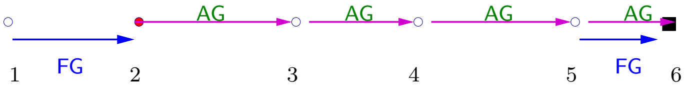

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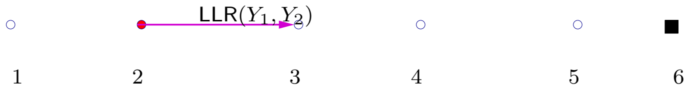

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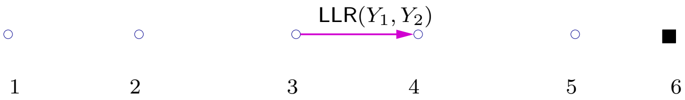

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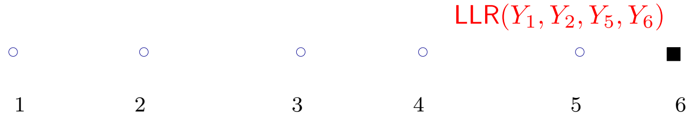

Graph transformation and building prize-collecting Steiner tree.

Outline

Conclusion

Summary of Cluster Based Data Fusion

- Exploit correlation structure for sensor selection and data fusion
- KL-distance of each node cluster as the performance measure
- Prize collecting Steiner tree reduction for cost-performance tradeoff
    - Approximation factor preserving reduction
    - Goemans-Williamson approximation algorithm applicable

Outlook

- Incorporating physical layer issues
    - Effect of interference, Broadcast nature of wireless medium
- General node selection policies

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Results for Minimum Cost Fusion

Minimum Cost Data Fusion for Inference (Infocom '08)

Min total routing costs s.t. likelihood ratio is delivered to fusion center

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Markov random field with dependency graph

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Optimal Node Density for Inference

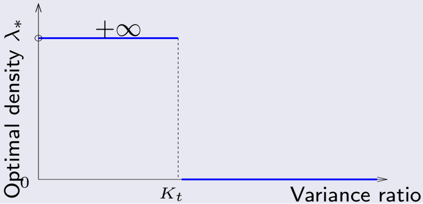
Optimal Node Density of Random Placement

Maximize error exponent subject to average routing cost constraint

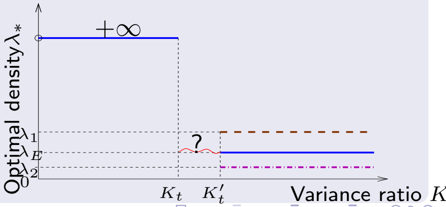
Large random networks (ITsub '06, SP '08)

- Closed-form error exponent and optimal node density
- Threshold effect on optimal node density
- Law of large numbers for graph functionals

No Energy constraint



Feasible Average Energy



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- A. Anandkumar, A. Ephremides, L. Tong, and A. Swami, "*Minimum Cost Routing with Local Processing for Distributed Statistical Inference*," in handbook on Array Processing and Sensor Networks, (S. Haykin and R. Liu, eds.), 2008.

Aggregation and Error Exponents in Large Random Networks

- A. Anandkumar, L. Tong, and A. Swami, "*Detection of Gauss-Markov random fields with nearest-neighbor dependency*," sub. to IEEE Tran. Information Theory, Jan.07
- A. Anandkumar, L. Tong, and A. Swami, "*Optimal Node Density for Detection in Energy Constrained Random Networks*," accepted to IEEE Tran. Signal Processing, Oct. 2007

Related Work

Correlated Data Gathering (Cristescu et al. 06, Scaglione&Servetto 02)

- Raw data not needed at fusion center
- Only the likelihood function for optimal inference

In-network Function Computation (Giridar & Kumar 06)

- Valid for symmetric functions
- LLR has this form only for independent data

Routing for Inference: For Special Correlation Models

- Independent Measurements (Yang & Blum, Yu & Ephremides)
- 1-D Gauss-Markov process (Sung et al. 06, Chamberland & Veeravalli)

Routing for Belief Propagation (Kreidl & Willsky 06, Williams et al. 05)

Local MAP estimate of raw data at each node: not global decision at FC