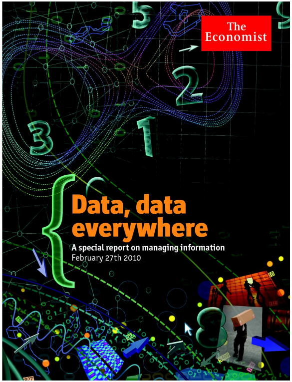
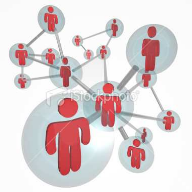
Beyond Sparse Graphical Models: Incorporating Mixtures and Residuals

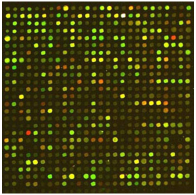
Anima Anandkumar

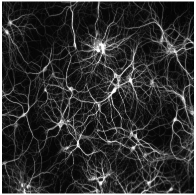
U.C. Irvine

Data Deluge and Data Desert





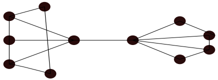




- Current technologies unable to handle data deluge.
- Current algorithms unable to handle data desert.

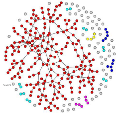
High-dimensional data: Many variables, few samples

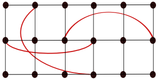
Graph-Based Models for High-Dimensional Data

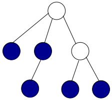


- Qualitative: Graph structure(s).
- Quantitative: Interaction strengths.

Parsimonious representation via sparse graphs







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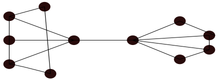

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Parsimonious representation via sparse graphs

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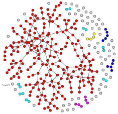
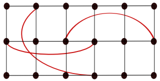
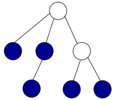


- Estimate structure(s) and parameters from samples.
- Employ model to predict future behavior.





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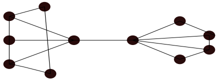

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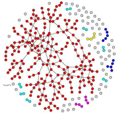

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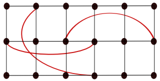
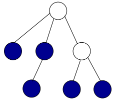
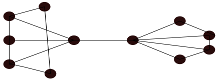


Challenges



- Computational complexity: Large no. of variables.
- Sample complexity: Fewer observations.
- Latent or Hidden Variables: Unobserved influences.
- Parsimony vs. Faithful representation

Graph-Based Models for High-Dimensional Data



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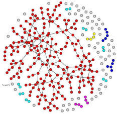

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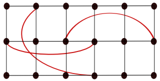

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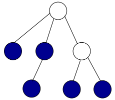

Goals

Tractable models, Novel algorithms, Provable guarantees, Applications.

Examples of Graph-based Representations

Motivating Example: Topic Modeling

Data: word counts in documents.

Graph: Topic-word relationships.

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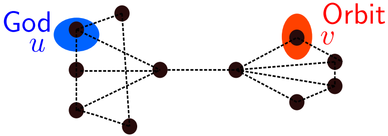
Independence models



Marginal Independence

$X_u \perp\!\!\!\perp X_v$

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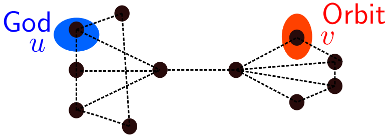

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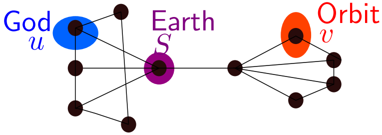

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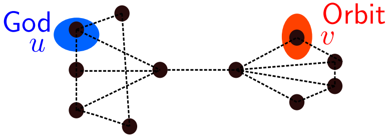

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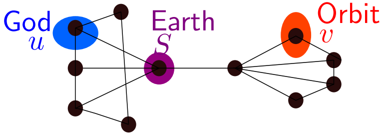

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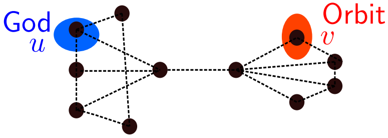

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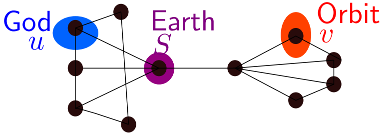

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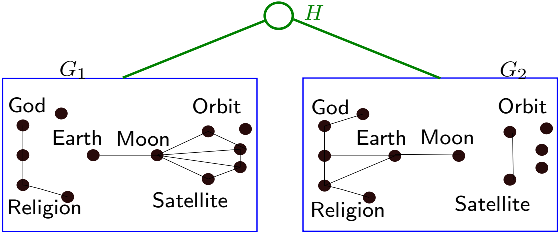
Solution: High-dimensional modeling via multiple graphs

High-dimensional Modeling via Multiple Graphs

Graphical Model Mixtures

- Multiple graphs: context specific dependencies
- Hidden context
- Learning guarantees



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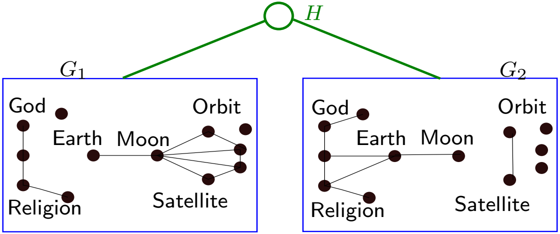
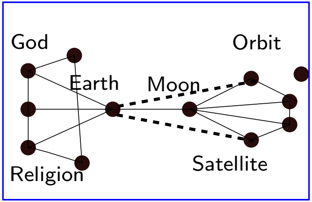


Markov+Independence Models

- Multiple graphs: different statistical relationships
- Markov and Independence Graphs
- Efficient decomposition



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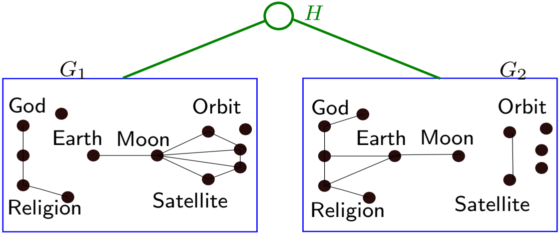

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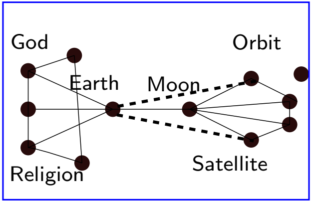

Novel Approaches Beyond Sparse Graphical Modeling

State of Art Approaches

Learning Sparse Graphical Models

- Combinatorial: Bresler, Mossel & Sly. **A***, Tan & Willsky.
- Convex: Meinshausen & Bühlmann. Ravikumar, Wainwright & Lafferty.

Learning with Latent Variables

- Trees: Erdös, et. al., Daskalakis, Mossel & Roch. Choi, Tan, **A*** & Willsky.
- Loopy models: Chandrasekaran, Parrilo & Willsky. **A*** & Valluvan.

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*Special EECS Seminar, March 12, 2012.

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Learning Mixture Models

- Gaussian Mixtures: Dasgupta. Kannan et al. Chaudhuri et al.
  - Separation condition for mixture components
- Method of Moments: Prony, Belkin & Sinha. Moitra & Valiant.
  - Comp. & sample complexities exponential in no. of components
- Latent Class Models: Chang. Hsu, Kakade & Zhang. Mossel & Roch.
  - Mixtures of discrete product distributions.

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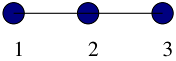
Outline

Warm-up: Learning Tree Models

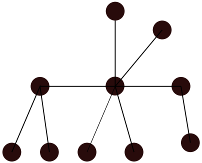
Data processing inequality for Markov chains

$$I(X_1; X_3) \leq I(X_1; X_2), I(X_2; X_3).$$



Tree Structure Estimation (Chow and Liu '68)

- MLE: Max-weight tree with estimated mutual information weights



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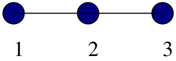

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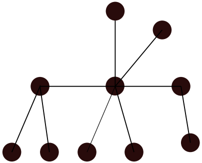

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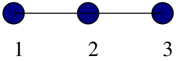

Tree Structure Estimation (Chow and Liu '68)

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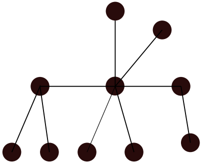
- n samples and p nodes



Sample complexity: $\dfrac{\log p}{n} = O(1).$

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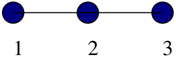

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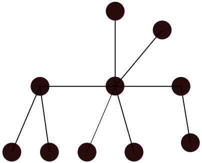

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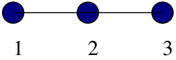

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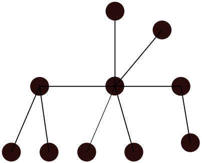

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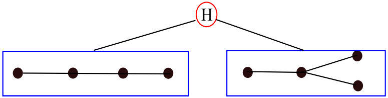
What other models are tractable for learning and inference?

Beyond Trees: Tree Mixture Models

Tree Mixture Model

- Each component is a tree model
- Class variable is latent or hidden



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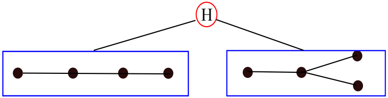

Why use tree mixtures?

- Efficient Inference: BP on component trees and combining them.
- Similarly marginalization and sampling also efficient.

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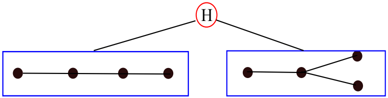

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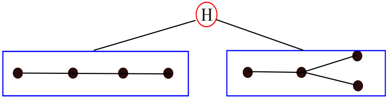

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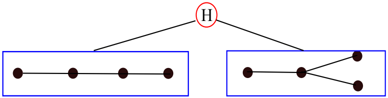

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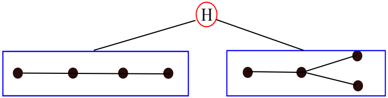

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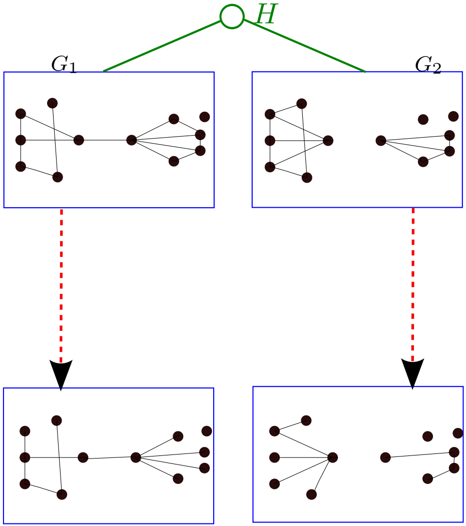
Novel approach to learning tree mixture approximations

Mixtures of Graphical Models: Our Approach

Our Approach

- Consider data from graphical model mixture
- Output tree mixture: best tree approx. of each component



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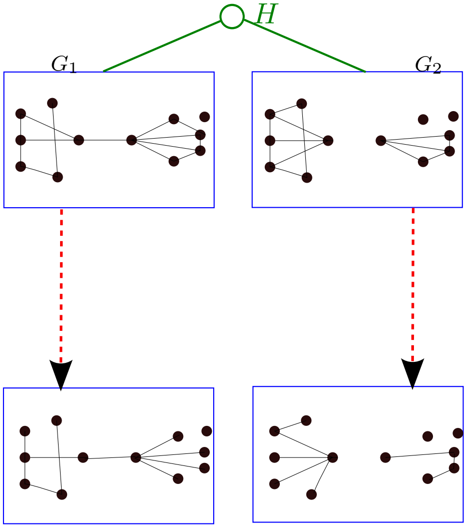

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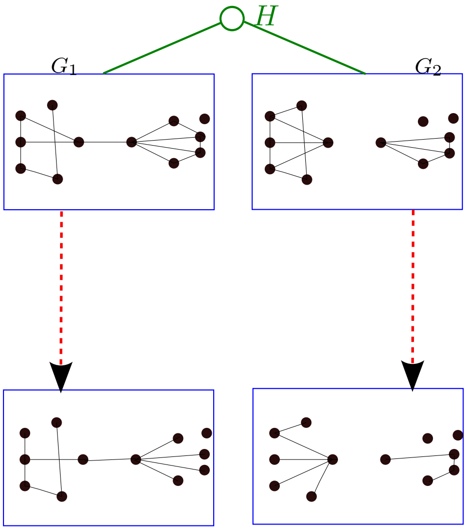

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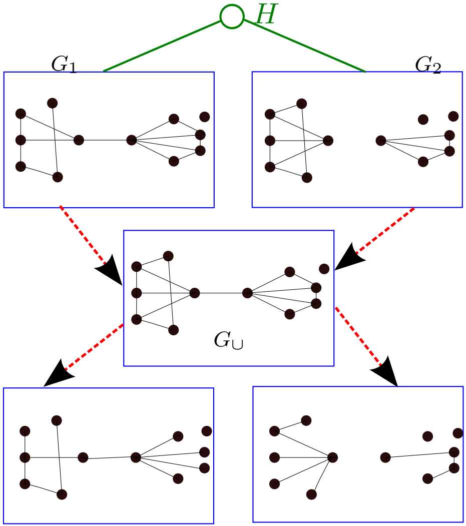

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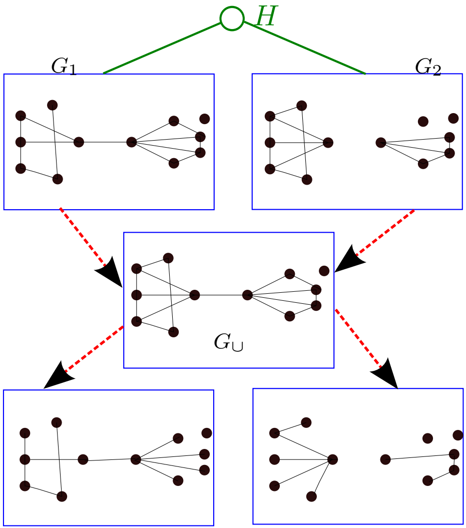

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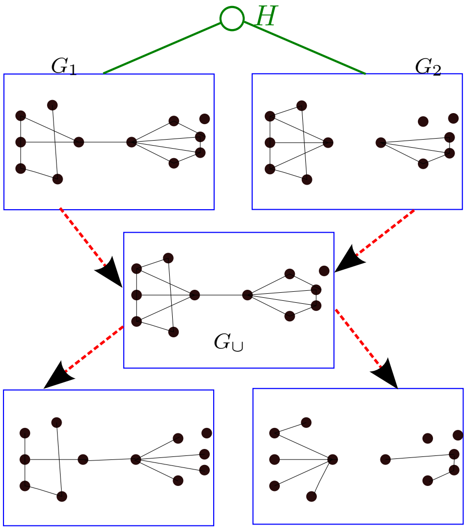

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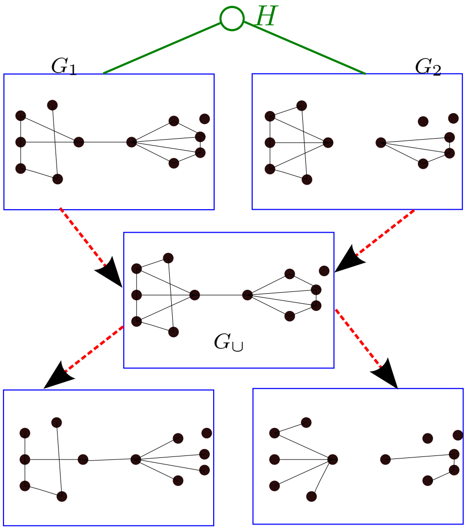

Outline

Sparse Graphical Model Selection: Intuitions

- First consider a graphical model with no latent variables

Markov Property of Graphical Models

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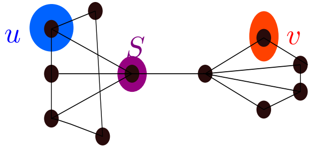

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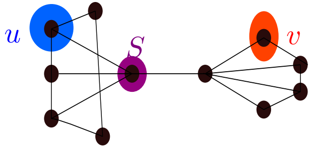

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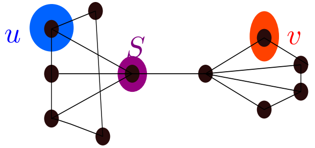

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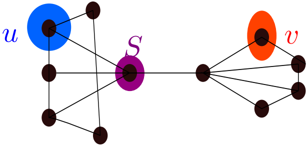

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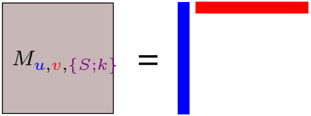

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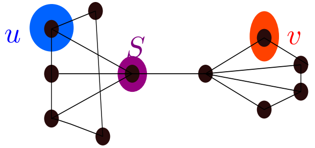

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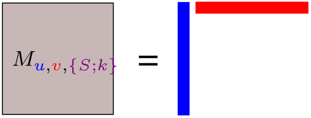

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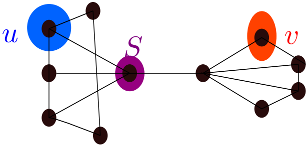

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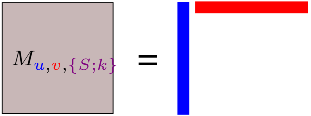

Rank Test on Pairwise Probability Matrices

Extending Rank Tests to Mixtures

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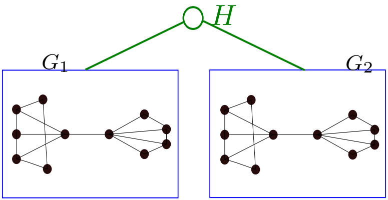

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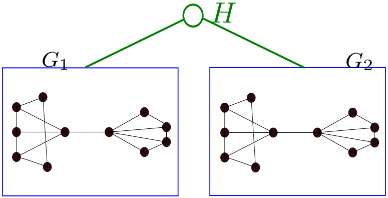

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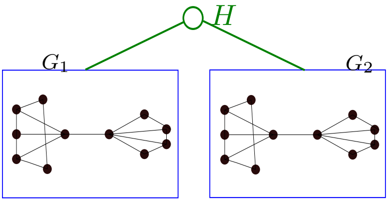

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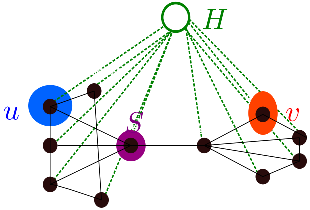

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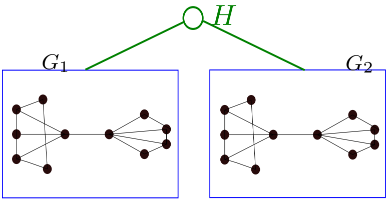

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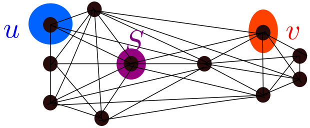

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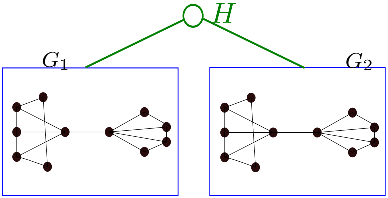

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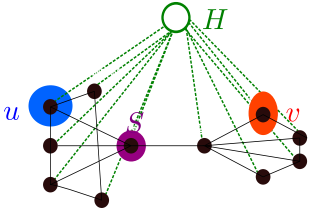

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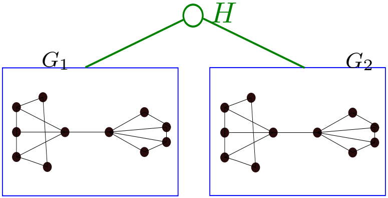

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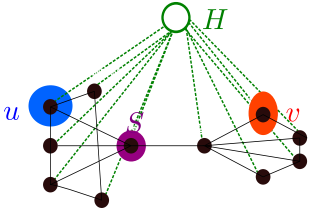

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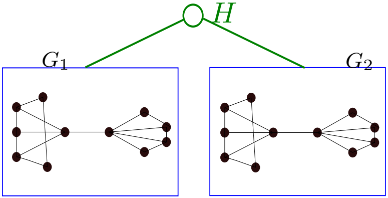

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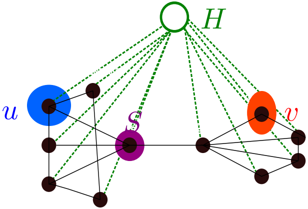

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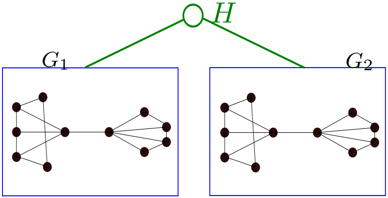

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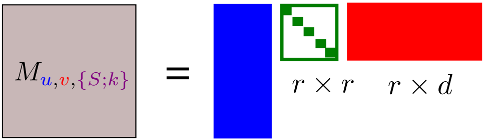
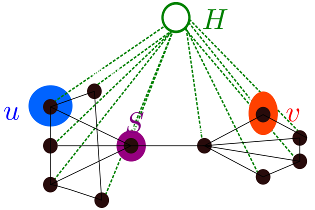


$d \times d$ $\quad$ $d \times r$



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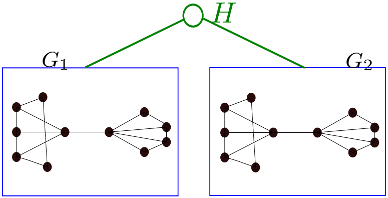

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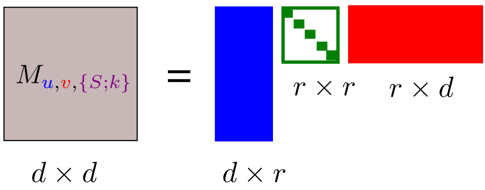

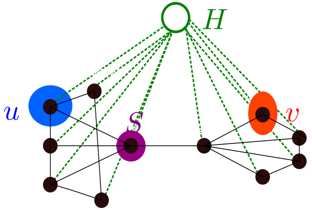

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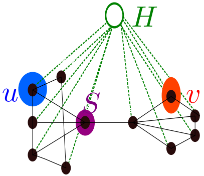

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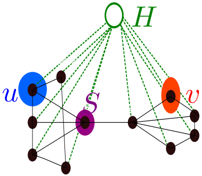

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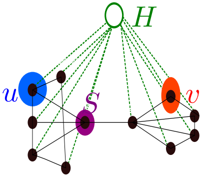

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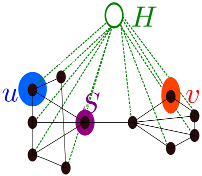

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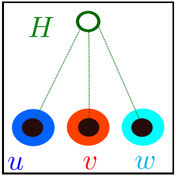

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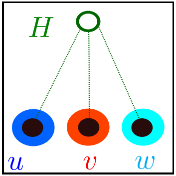

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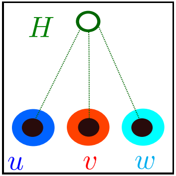

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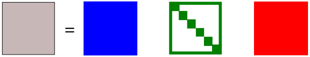

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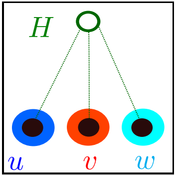

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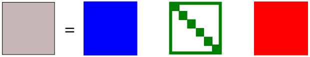

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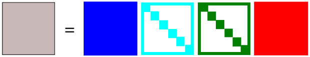
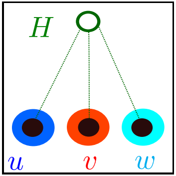


Mixture of Product Distributions: Intuitions

- $X_u \perp\!\!\!\perp X_v \perp\!\!\!\perp X_w | H$
- Union graph of mixture components is trivial
- Transition matrices full rank (non-singular)
- Consider special case when $d = r$



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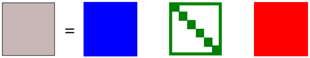

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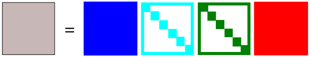

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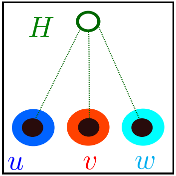

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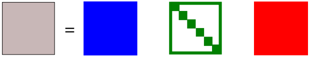

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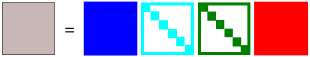

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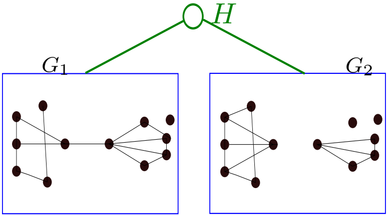
Efficient estimation of non-singular product mixtures

Learning Graphical Model Mixtures

Adapt Eigenvalue Method for Graphical Model Mixtures?

Challenges

- Not a mixture of product distributions: Eigenvalue method not valid.



Solutions

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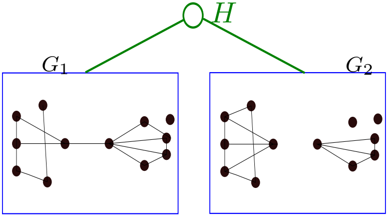

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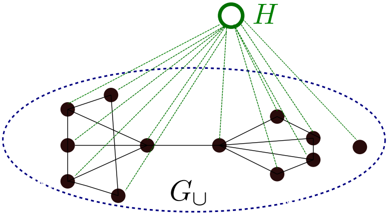
- $G_\cup$: union graph learnt from rank test



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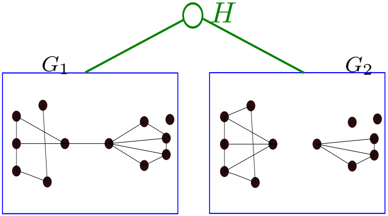

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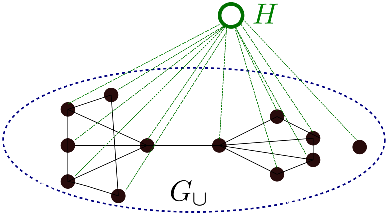

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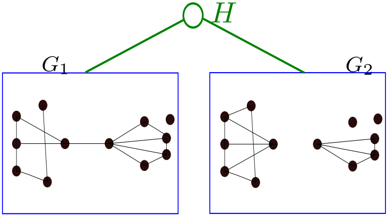

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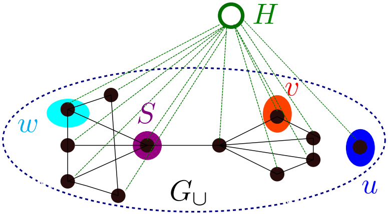

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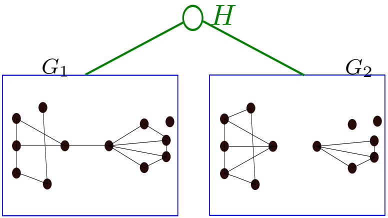

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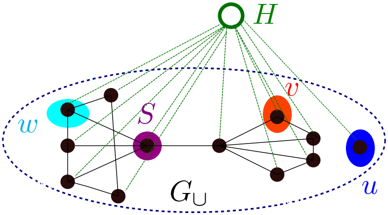

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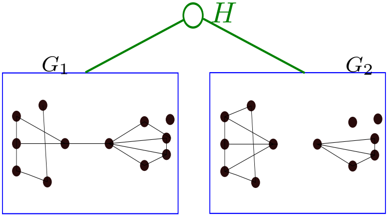

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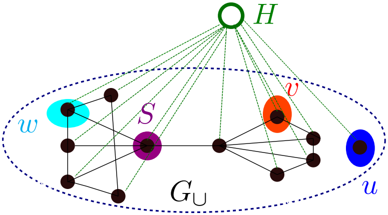

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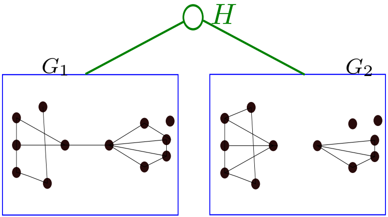

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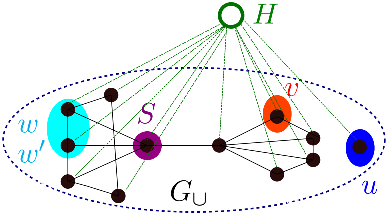

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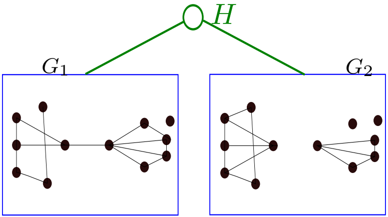

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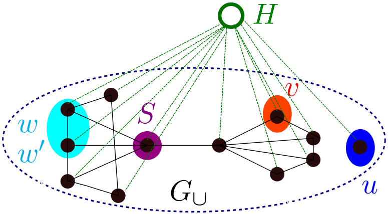

Efficient Estimation of Tree Mixture Approximations

Guarantees for Learning Graphical Model Mixtures

Steps Involved in Tree Mixture Approximation

- Rank tests for structure estimation of union graph $G_\cup$
- Eigenvalue decomposition for estimation of pairwise moments of mixture components
- Chow-Liu algorithm to estimate mixture component trees

Computationally Efficient Algorithm for Learning Graphical Model Mixtures

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Theorem (A. , Hsu, Kakade '12)

The above method recovers correct tree mixture approximation correctly w.h.p on p nodes of r component mixture under n samples when

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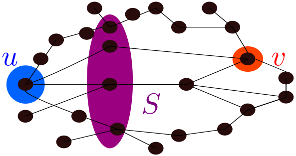
Efficient Learning of Multiple Graphs and Models in High Dimensions

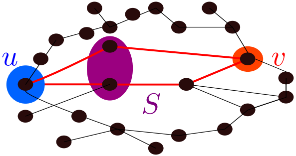
Extensions and Connections



Exact vs. Local Separators in Union Graph

η is bound on local separators of $G_\cup$ and mixture components satisfy correlation decay.

Extensions and Connections



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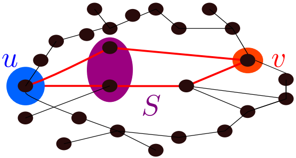
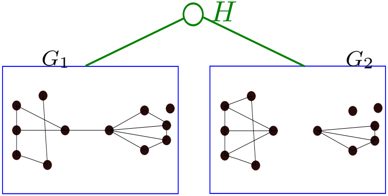


Estimation of component graphs

- Estimate joint moments in nbd. of union graph for each component.
- Neighborhood selection for each mixture component.
- Efficient for low degree union graphs.



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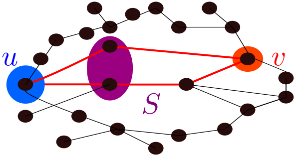

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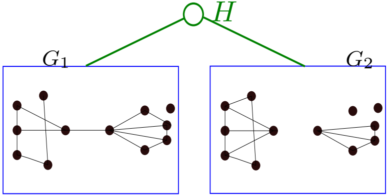

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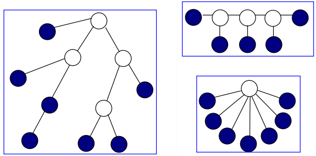
Estimation in other models



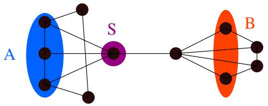
- HMM, latent trees and general multiview mixtures
- Improvement for product mixtures

Outline

Beyond Graphical Modeling: Incorporating Residuals

Recall notion of graphical models...

- Conditional Independence: $\mathbf{X}_A \perp\!\!\!\perp \mathbf{X}_B | \mathbf{X}_S$
- $P(\mathbf{x}) \propto \exp\left[\sum_{(i,j)\in G} \Psi_{i,j}(x_i, x_j)\right].$



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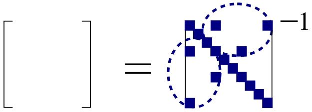

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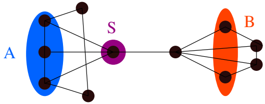

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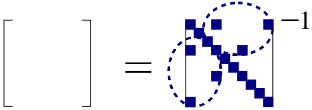

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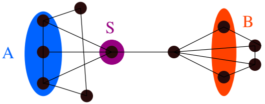

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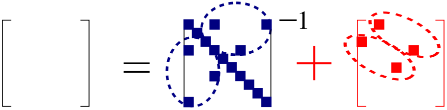

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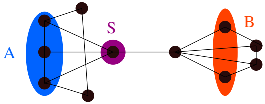

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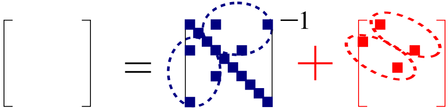

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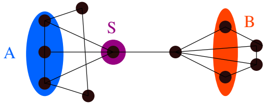

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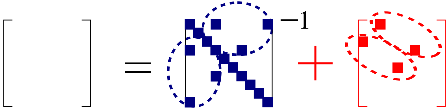

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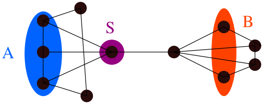

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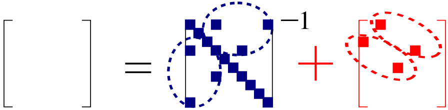

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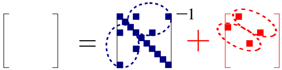

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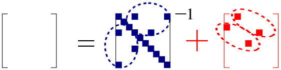

ℓ_1 penalized MLE for Graphical Models (Ravikumar et. al. ‘08)

- $\widehat{\Sigma}^n$: sample covariance using n i.i.d. samples

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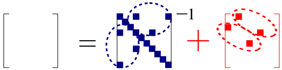

ℓ_1 penalized MLE for Graphical Models (Ravikumar et. al. '08)

- $\widehat{\Sigma}^n$: sample covariance using n i.i.d. samples

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Algorithm for Covariance Decomposition

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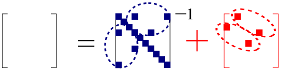

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Max-entropy Formulation for Graphical Models

- Lagrangian dual of ℓ_1-penalized MLE

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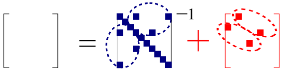

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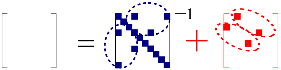

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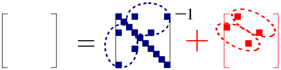

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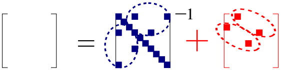

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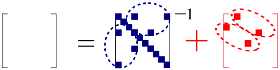

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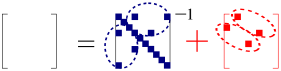
Efficient Method for Covariance Decomposition and Estimation

Guarantees for High-Dimensional Estimation

$$\boxed{\Sigma^* = {J_M^*}^{-1} + \Sigma_R^*.}$$



Theorem (Janzamin and A. '12)

When the number of samples n, number of nodes p and maximum degree Δ in the Markov graph (corresponding to J_M^*) satisfy

$$\boxed{\frac{\Delta^2 \log p}{n} = O(1),}$$

- $(\widehat{J}_M, \widehat{\Sigma}_R)$ are sparsistent and sign consistent
- satisfy norm guarantees

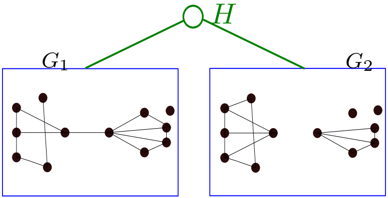
$$\boxed{\|\widehat{J}_M - J_M^*\|_\infty, \|\widehat{\Sigma}_R - \Sigma_R^*\|_\infty = O\left(\sqrt{\frac{\log p}{n}}\right).}$$

Guarantee Sparsistency and Efficient Estimation in Both Domains

Outline

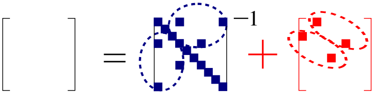
Summary and Outlook

Learning Graphical Model Mixtures



- Tree mixture approximations
- Combinatorial search + spectral decomposition
- Computational and sample guarantees

Markov/Independence Decomposition



- Efficient convex program for decomposition
- Similar requirements as graphical model selection

Outlook

- Converse results for learning graphical mixtures
- Mixed variables, latent models etc.