Learning Loopy Graphical Models with Latent Variables: Efficient Methods and Guarantees

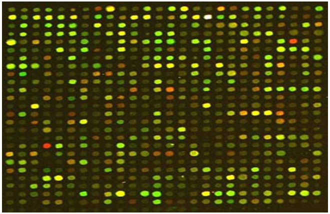
Anima Anandkumar

U.C. Irvine

Challenge: High-Dimensional Learning

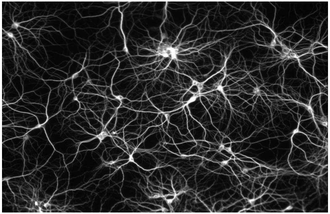
Social Networks

Genetic Analysis



Financial Modeling

Neural Activity

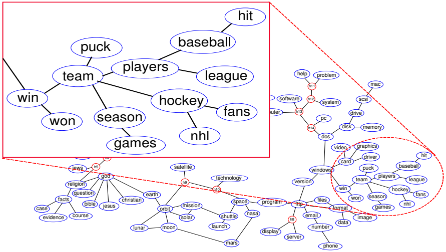


Examples for Graphical Approaches

Modeling High-Dimensional Data

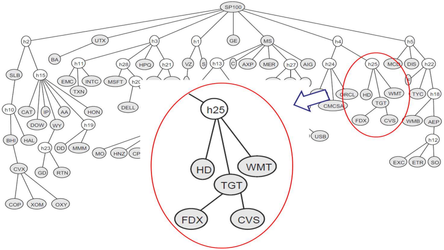
- Qualitative relationships: graph structure.
- Quantitative relationships: interaction strengths.

Topic Models



- Data: Word co-occurrences.
- Graph: Topic-word structure.

Financial Models



- Data: Stock returns.
- Graph: Company Classification.

Phylogenetics, Social Interactions, Computer Vision, . . .

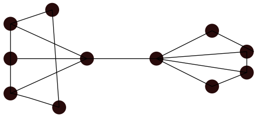
High-Dimensional Analysis

Steps Involved

- Estimate graph structure and strength of interactions.
- Employ the model to predict future behavior.

Focus on High-Dimensional Graph Estimation

- Graphical model on p (labeled) nodes
- n observations at the nodes



Challenges for High-Dimensional Estimation

- Computational Complexity: large p
- Sample Complexity: No. of samples n for consistency $(p \gg n)$
- Presence of Hidden or Latent Variables

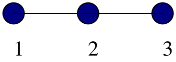
Goals

Tractable regimes, Novel methods, Provable guarantees

Walk-up: Learning Tree Models

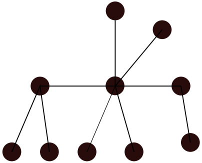
Data processing inequality for Markov chains

$$I(X_1; X_3) \leq I(X_1; X_2), I(X_2; X_3).$$



Tree Structure Estimation (Chow and Liu '68)

- MLE: Max-weight tree with estimated mutual information weights



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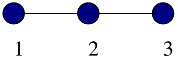

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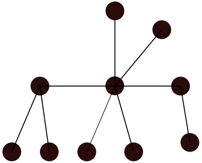

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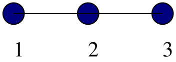

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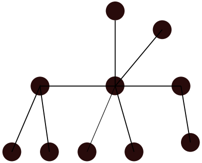
Sample complexity: $\dfrac{\log p}{n} = O(1).$



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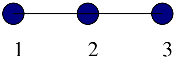

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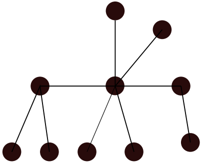
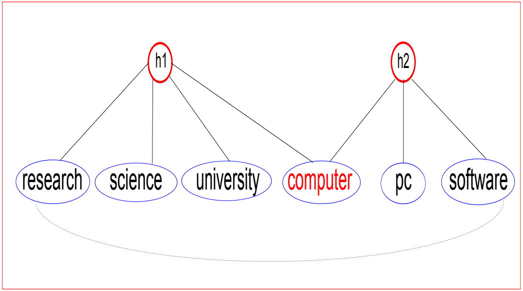


What other classes of graphical models are tractable for learning?

Beyond Tree Models: Motivation

Topic Models

- Common words in different topics.
- Presence of latent or hidden variables.



Efficient Methods for High-dimensional Graph Estimation.

State of Art Approaches

Approaches Employed

Combinatorial approaches, Convex relaxation.

Algorithms for Structure Estimation

- Chow and Liu (68): Tree estimation
- Meinshausen and Bühlmann (06): Convex relaxation
- Ravikumar, Wainwright, Lafferty (10): Convex relaxation
- Bresler, Mossel and Sly (09): Bounded-degree graphs ...

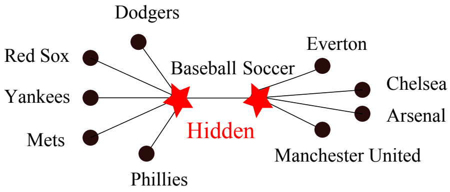
Learning with Hidden Variables

- Erdös, et. al. (99): Latent trees
- Daskalakis, Mossel and Roch (06): Latent trees
- Chandrasekaran, Parrilo and Willsky (11): Latent Gaussian models, ...

Summary of Results

Structure Estimation in Latent Variable Models

- Number of hidden variables and location unknown
- Estimate graph over all variables



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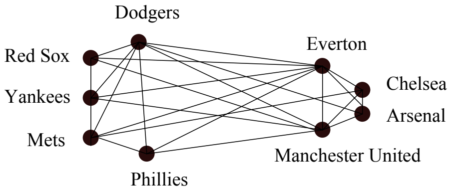

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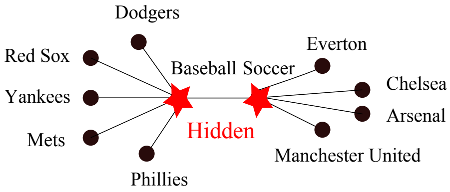

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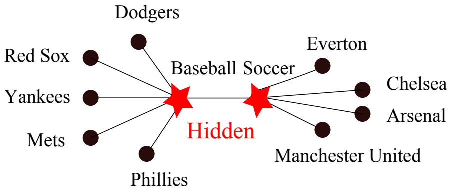


Contributions

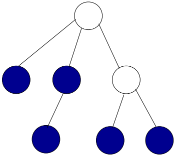
- Trees and girth-constrained graphs.
- Algorithms based on pairwise statistics.
  - Local tests to recover global structure.
- Low sample and computational requirements
- Applicable in topic, financial and social domains

Graph Estimation in Loopy Models with Latent Variables

Outline

Learning Latent Graphical Models

- Number and location of hidden variables unknown
- Estimate graph over all variables
- Trees and girth-constrained graphs



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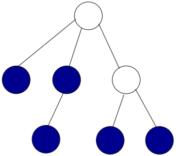

Information Distances $[d_{i,j}]$ for Tree Models

Gaussian: $d_{ij} := -\log|\rho_{ij}|$. Discrete: $d_{ij} := -\log|\,\mathrm{Det}(P_{i,j})|$.

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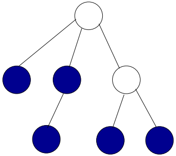

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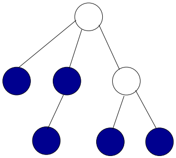

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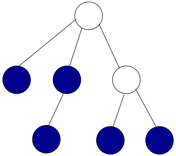

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Learning latent tree using $[\hat{d}_{i,j}]$

Siblings Test Based on Information Distances

Exact Statistics: Distances $[d_{i,j}]$

Let $\Phi_{ijk} := d_{i,k} - d_{j,k}$.

- $-d_{i,j} < \Phi_{ijk} = \Phi_{ijk'} < d_{i,j} \ \forall\, k, k' \neq i, j,$ $\iff$ i, j leaves with common parent
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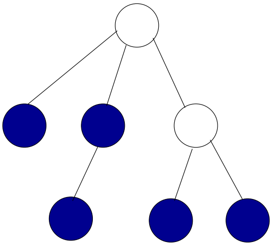

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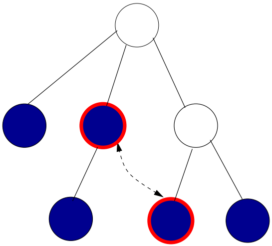

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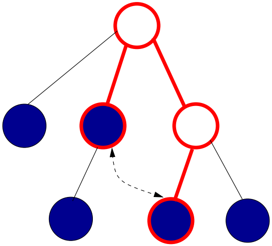

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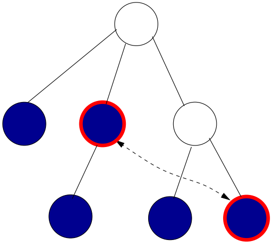

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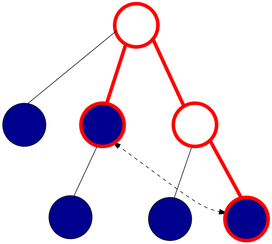

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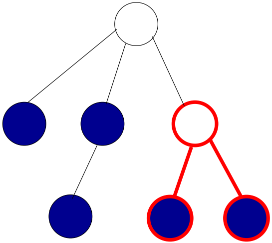

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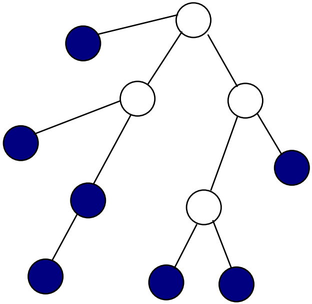
Recursive Grouping Algorithm (Choi, Tan, **A.**, Willsky)

- Sibling test and remove leaves
- Build tree from bottom up



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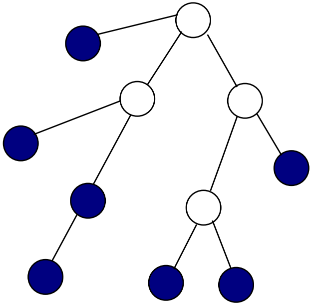

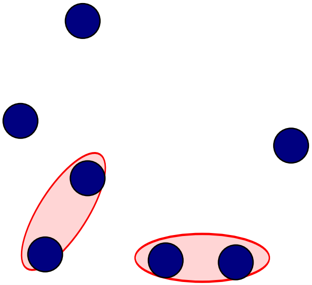

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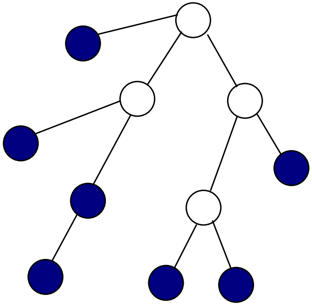

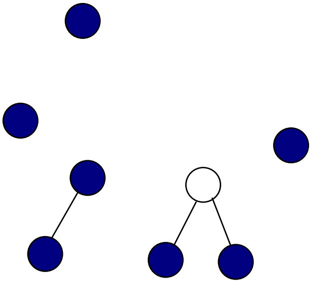

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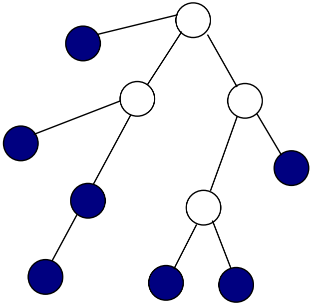

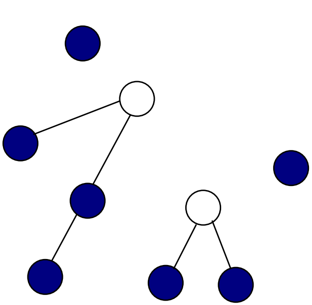

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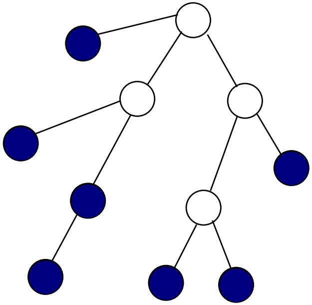

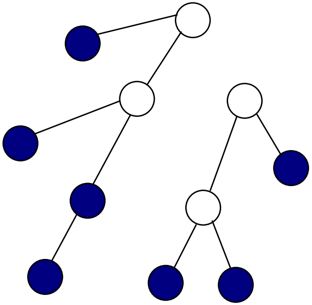

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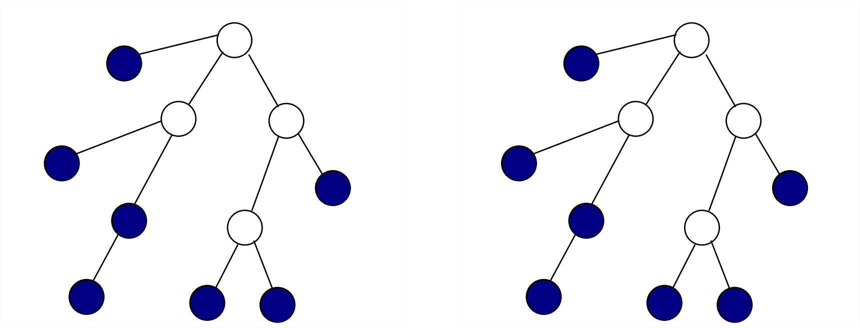
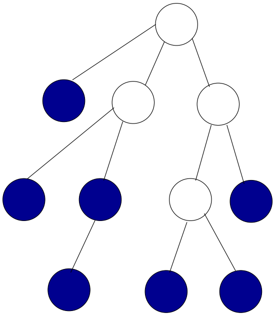
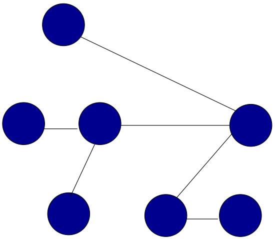


Chow-Liu Based Grouping Algorithm

Efficient Initial Tree on Observed Nodes (MST)

Minimum spanning tree using edge weights $[\hat{d}_{i,j}]$.





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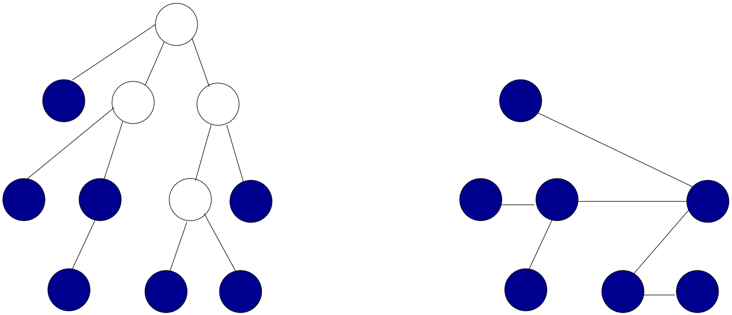

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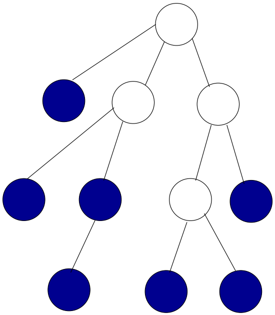

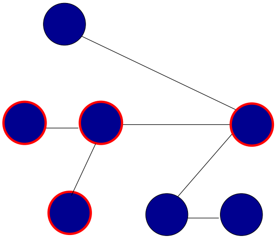

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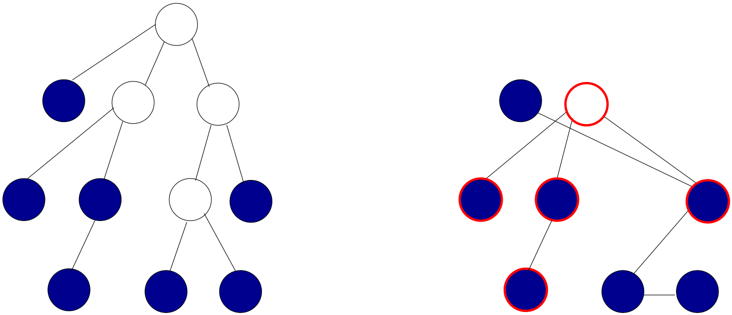

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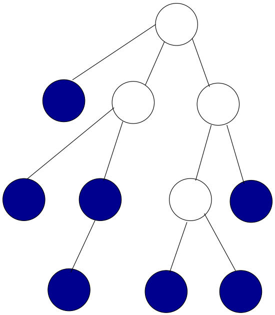

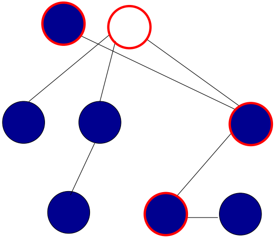

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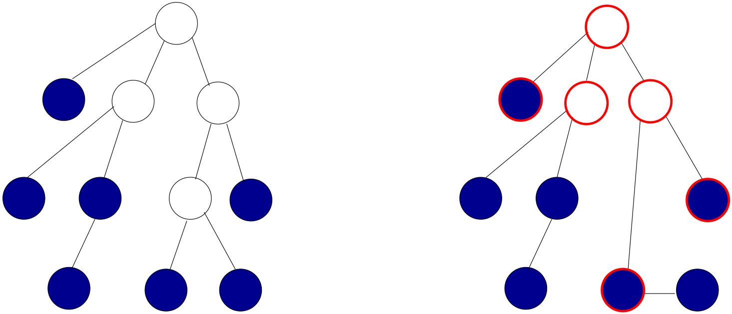

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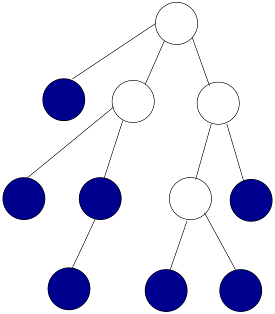

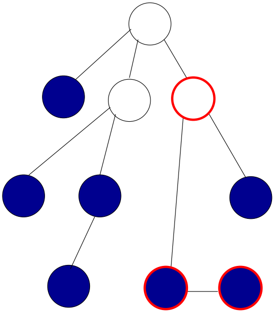

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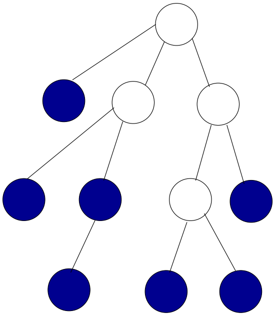
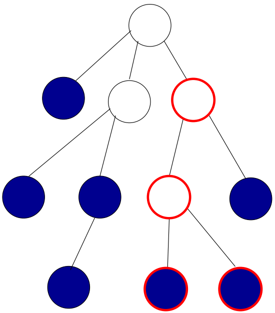




Proof Ideas

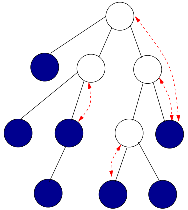
Relating Chow-Liu Tree with Latent Tree

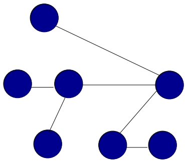
- Surrogate $\mathrm{Sg}(i)$ for node i: observed node with strongest correlation

$$\boxed{\mathrm{Sg}(i) := \operatorname*{argmin}_{j \in V} d_{i,j}}$$

- Neighborhood preservation

$$\boxed{(i,j) \in T \Rightarrow (\mathrm{Sg}(i), \mathrm{Sg}(j)) \in T_{\mathrm{ML}}.}$$





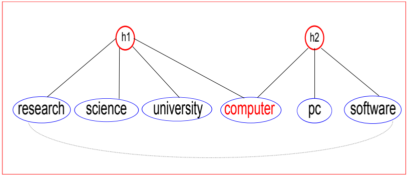
Chow-Liu grouping reverses edge contractions

Proof by induction

Loopy Graphical Models with Latent Nodes

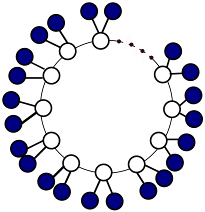
Motivation: Topic Models

- Common words among topics.
- Latent or hidden nodes.
- Typically long cycles: Locally tree-like.



Latent Models on Large Girth Graphs

- Pairwise statistics not related to trees in general



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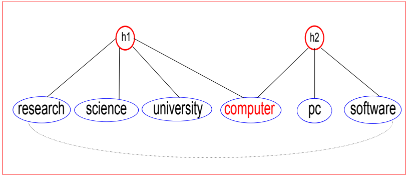

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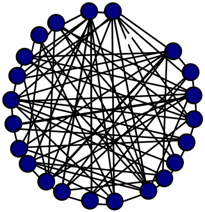

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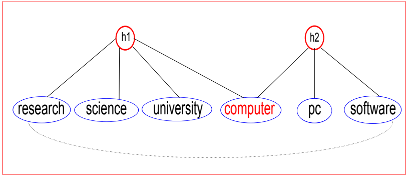

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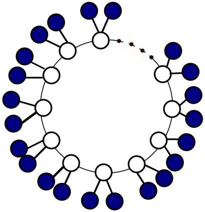

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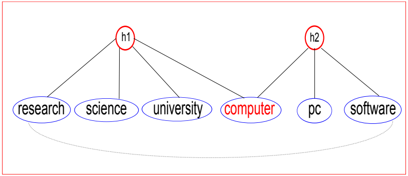

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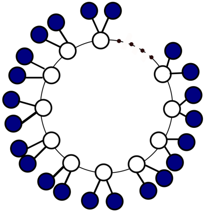

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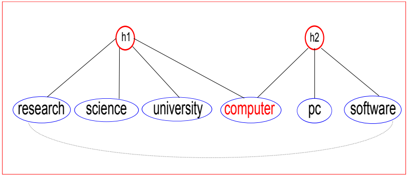

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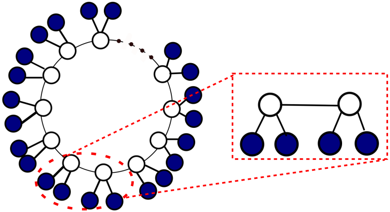

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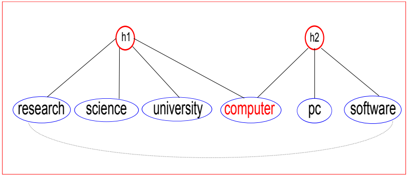

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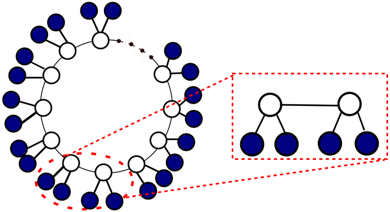
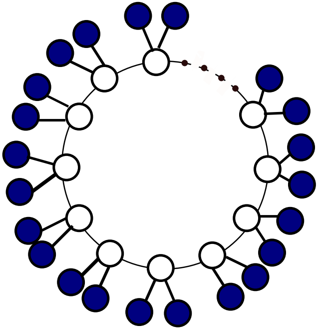


Local additivity $\boxed{d_{k,l} \approx \sum_{(i,j) \in \mathrm{Path}(k,l;E)} d_{i,j}.}$

Overview of Proposed Method

- Consider local neighborhoods for building local MST
- Merge the MSTs to obtain a loopy graph
- Run latent tree routine on different local neighborhoods



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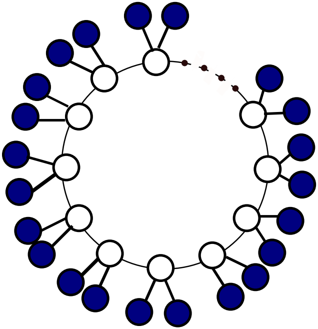

Local CL Grouping

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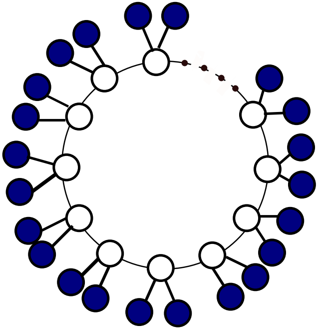

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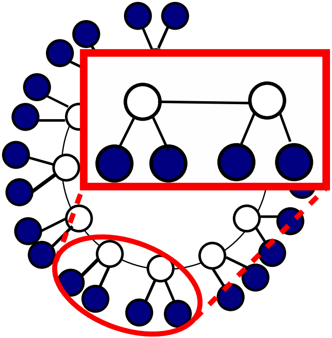

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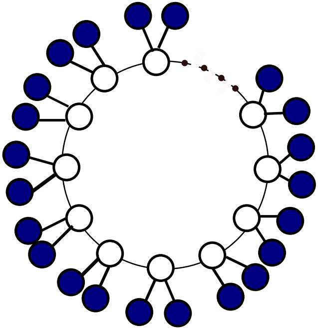

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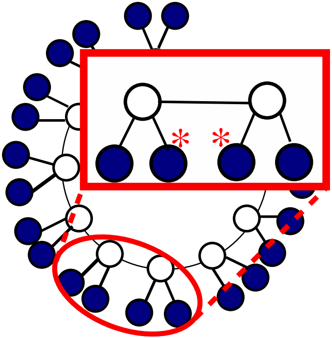

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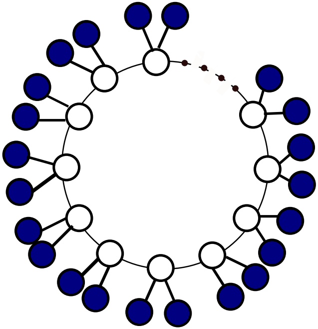

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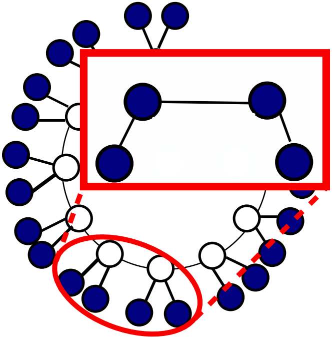

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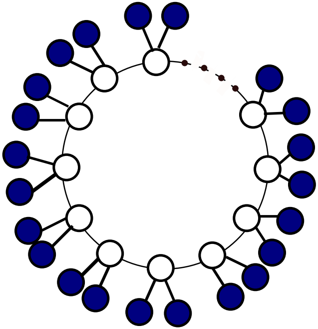

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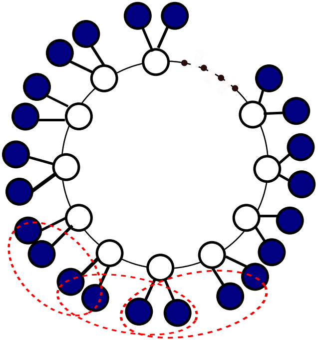

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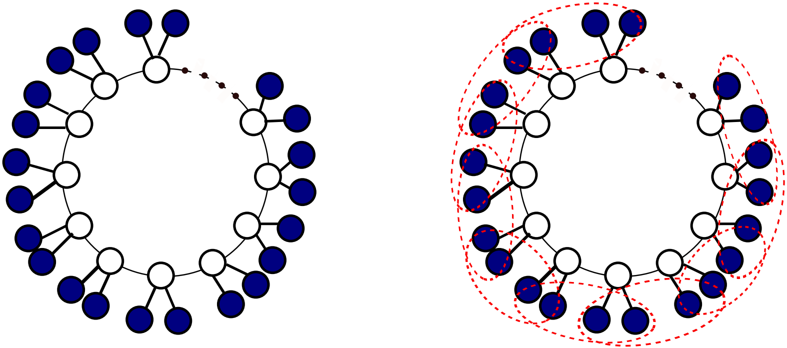

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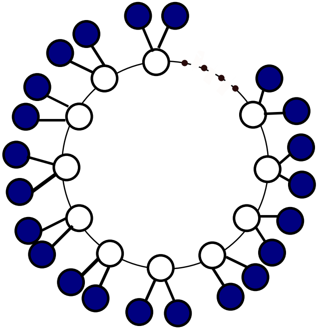

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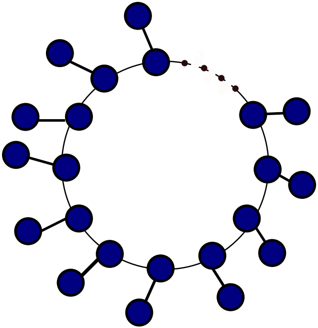

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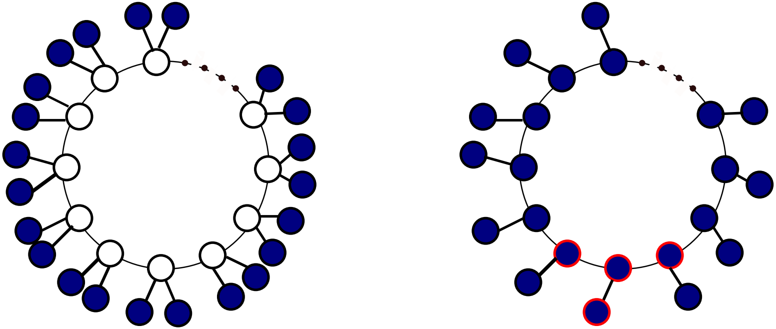

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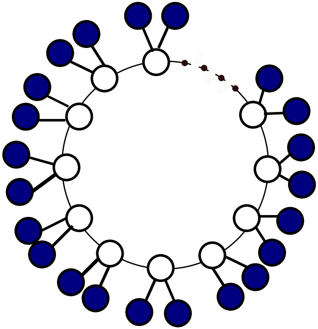

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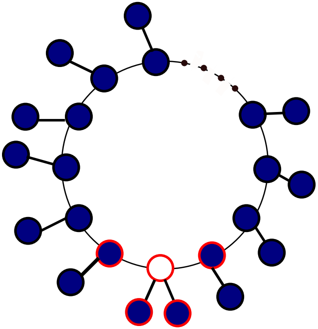

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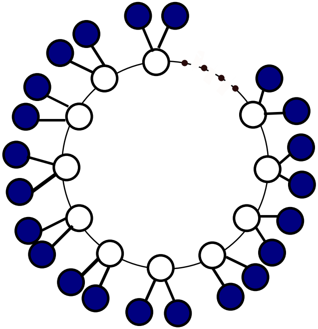

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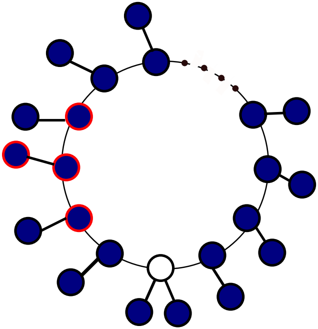

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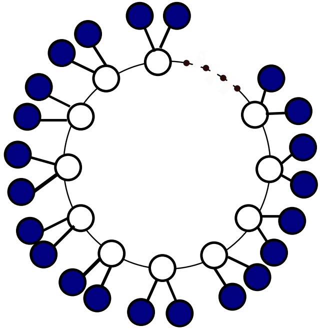

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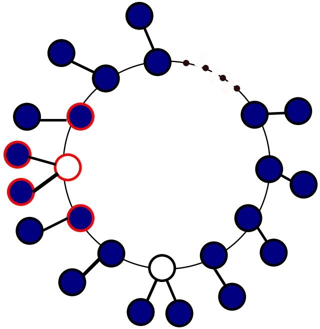

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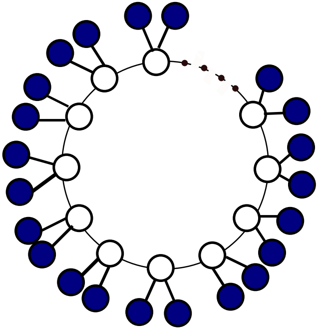

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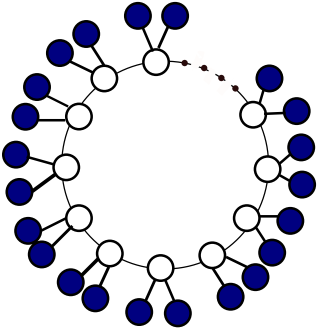

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Guarantees for Latent Structure Learning

- Depth δ: worst-case distance between hidden and observed nodes.
- Parameter β: depends on min. and max. node and edge potentials
  - $\beta = 1$ for homogeneous models.

Theorem (A. , Valluvan '12)

Proposed method correctly recovers graph structure w.h.p. on p observed nodes and n samples when

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- Fully observed case $\delta = 0$: $\boxed{n = \Omega(J_{\min}^{-2}\log p)}$.

Latent Models on Large Girth Graphs Akin to Latent Trees

Insights and Implications

Tradeoff between depth δ and girth g

Roughly require: $\delta < g/4$.

Tradeoff between max. edge strength $J_{\max}$ and degree Δ

Require $J_{\max} < \operatorname{atanh}(\Delta^{-1})$.

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Sample complexity for uniform node sampling

Given ρ fraction of nodes as observed nodes,

$$\boxed{n = \Omega\left(\Delta^2 \rho^{-4} (\log p)^5\right).}$$

Necessary conditions for structure recovery

For any deterministic algorithm, the number of samples n needs to be

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2. Structure Estimation in Latent Graphical Models
   - Latent Tree Models
   - Loopy Latent Models
3. Experiments
4. Conclusion and Extensions

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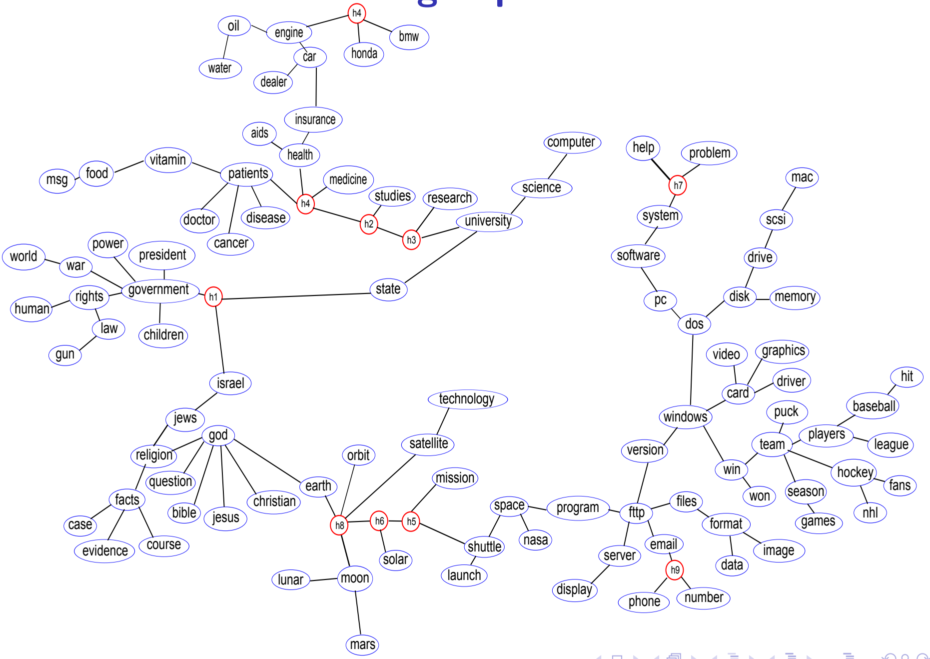

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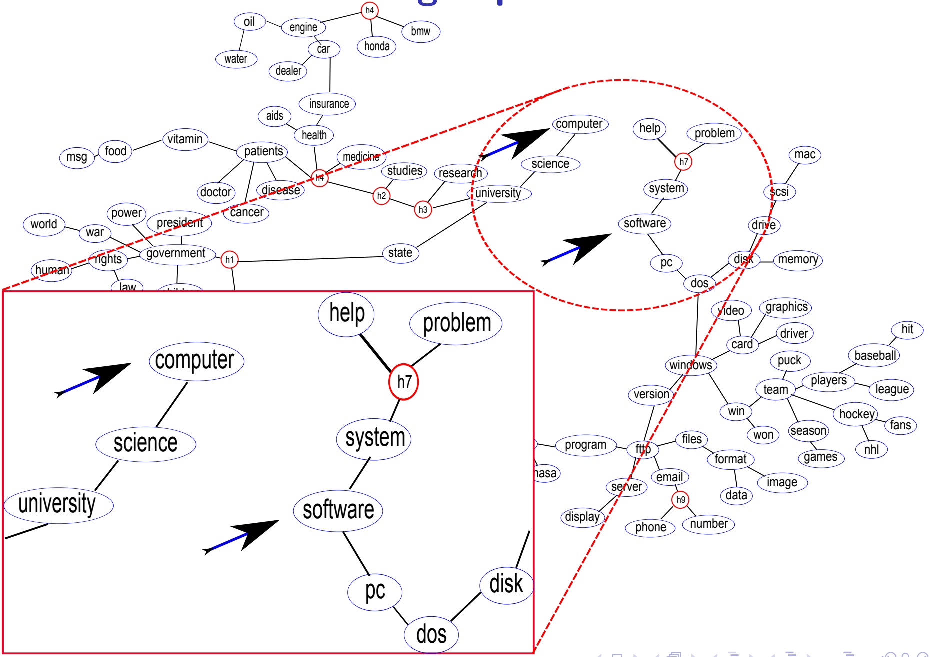

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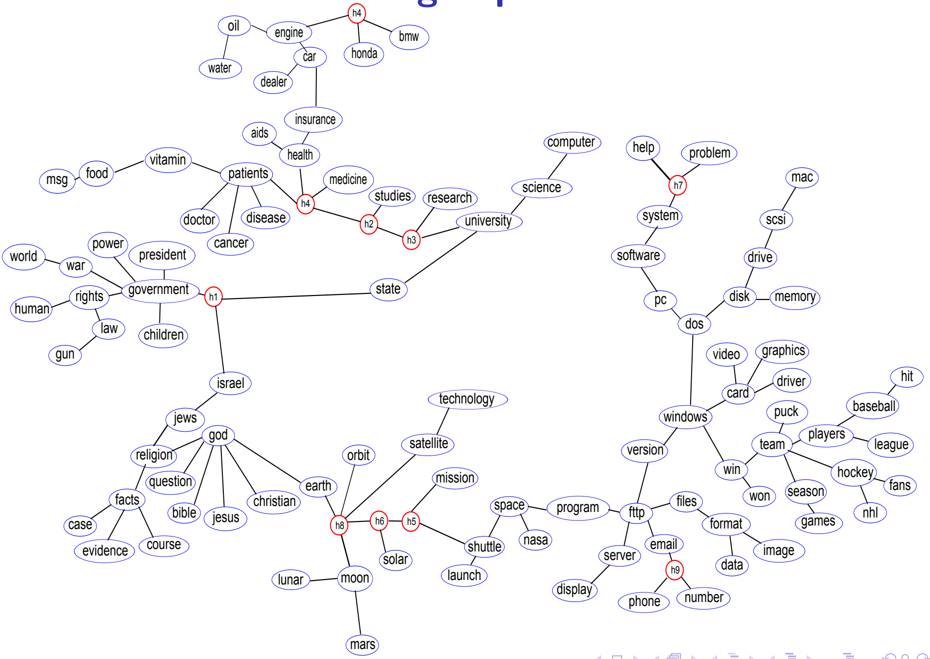

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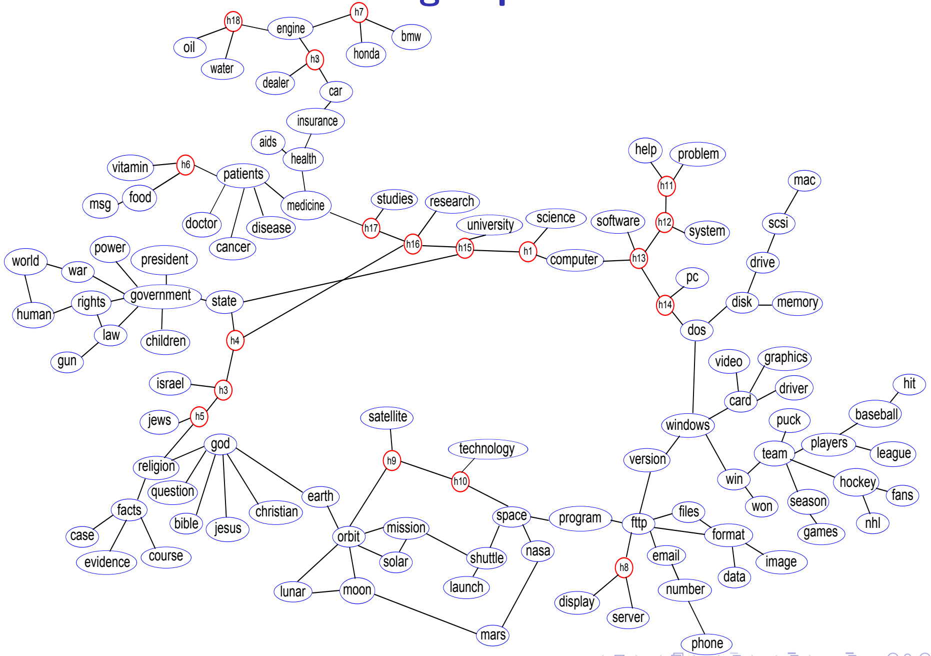

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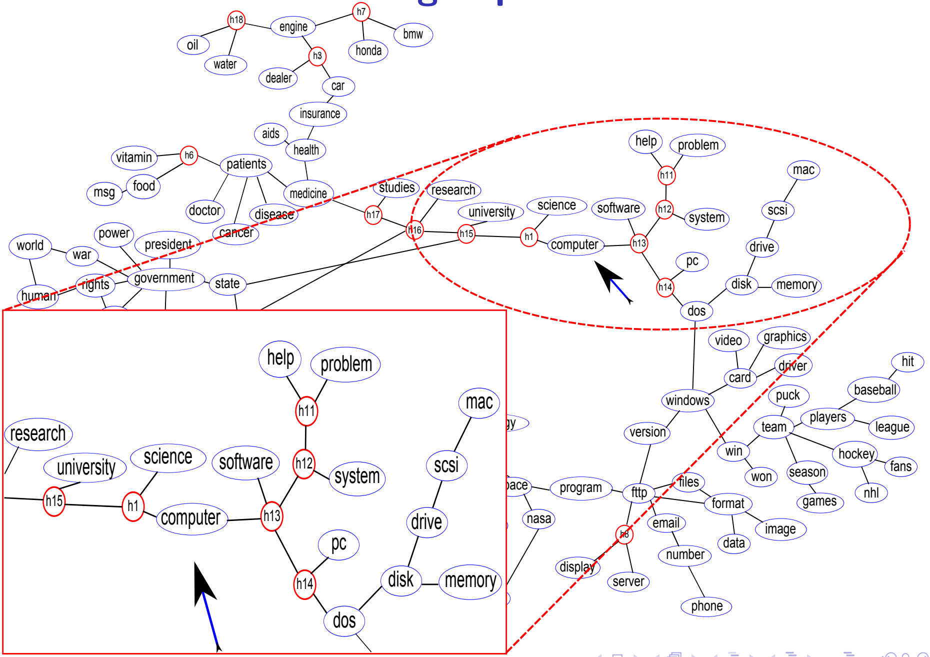

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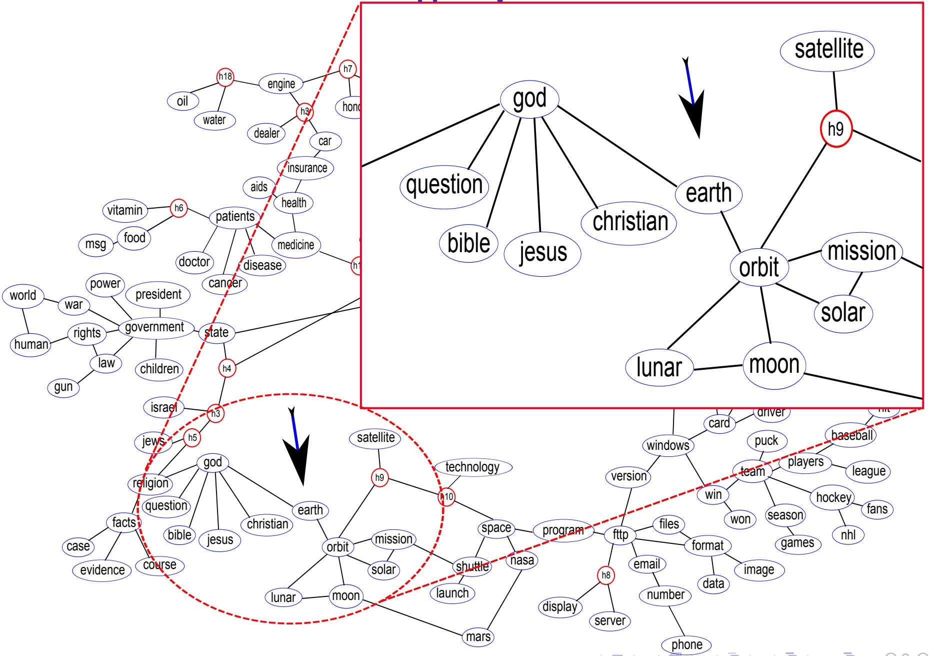

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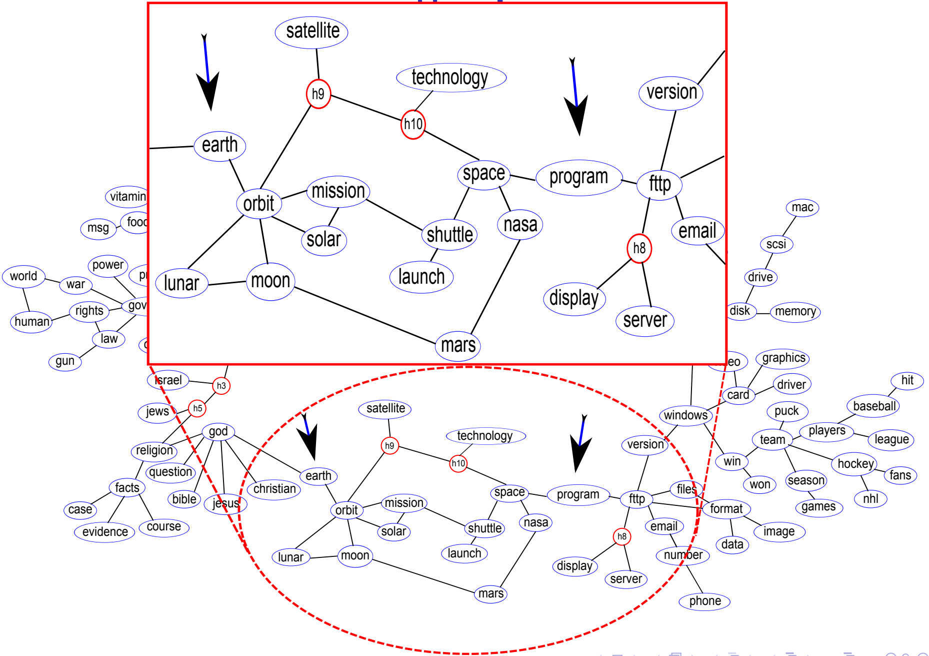

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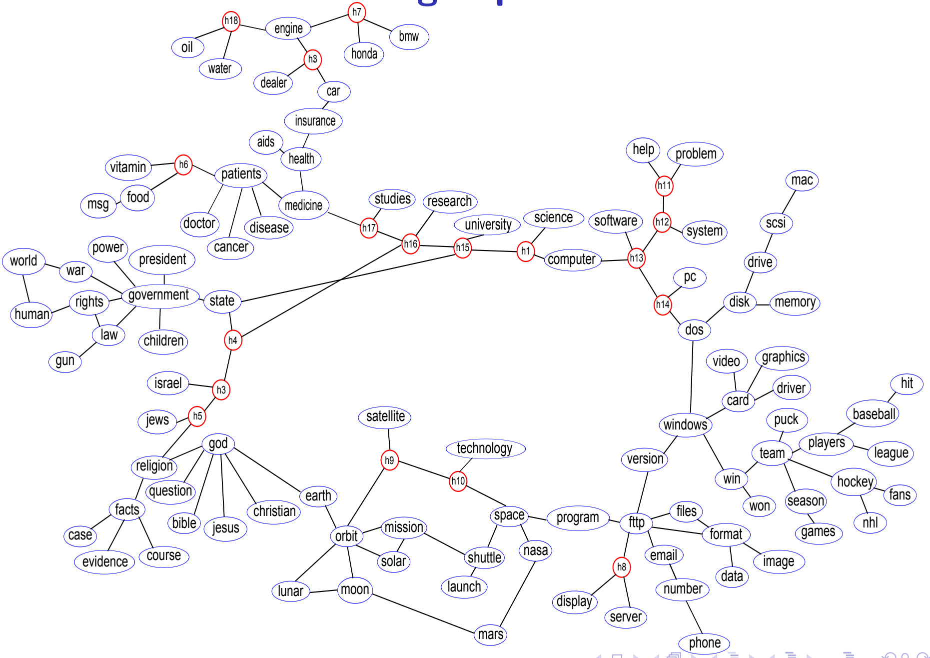
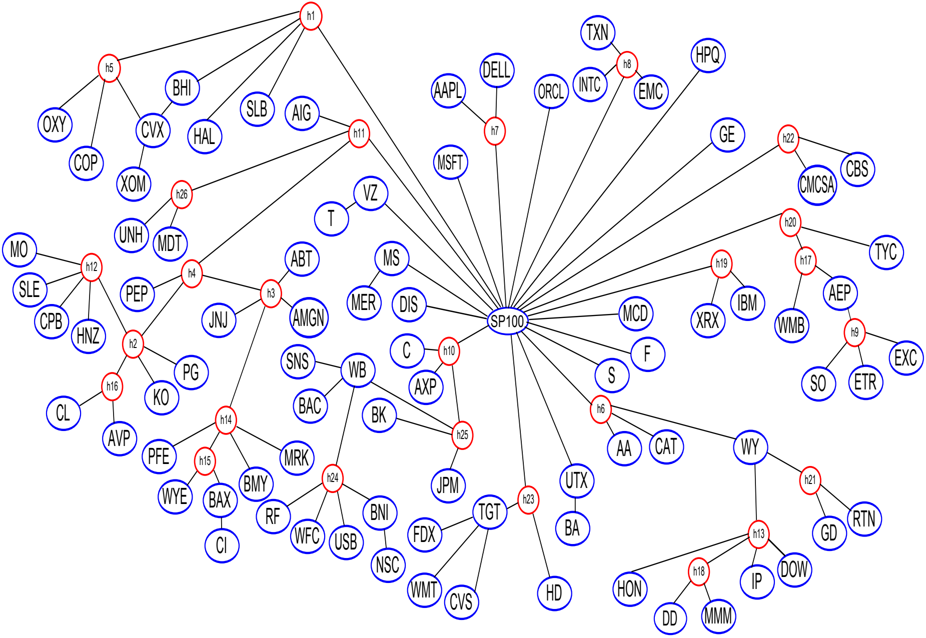


Stock Returns Data



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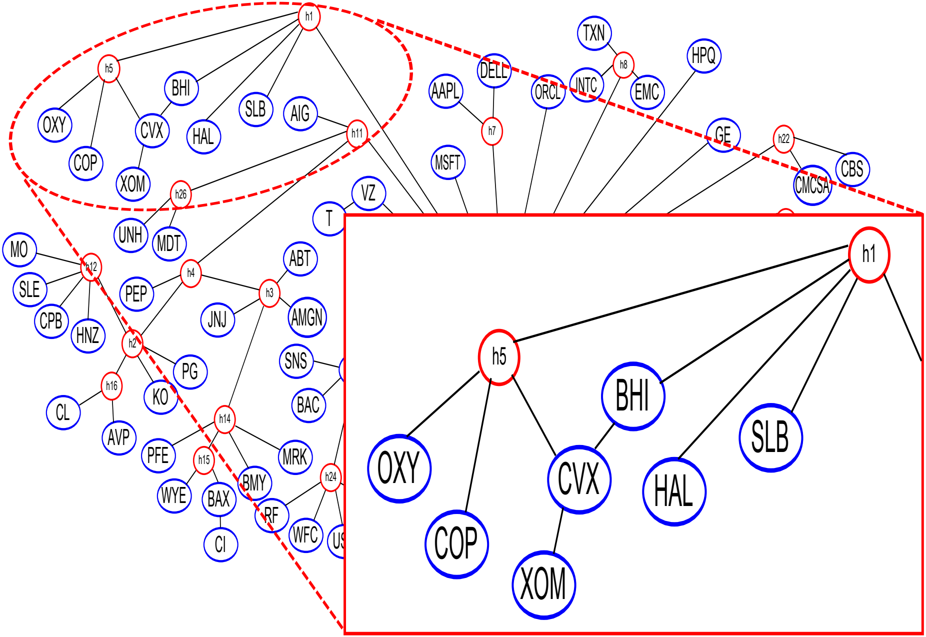

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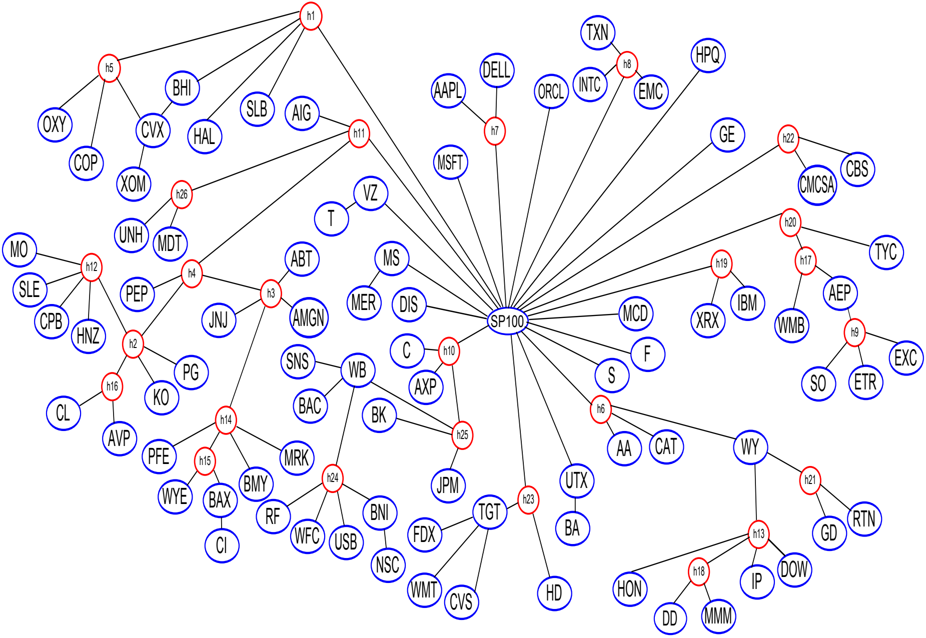

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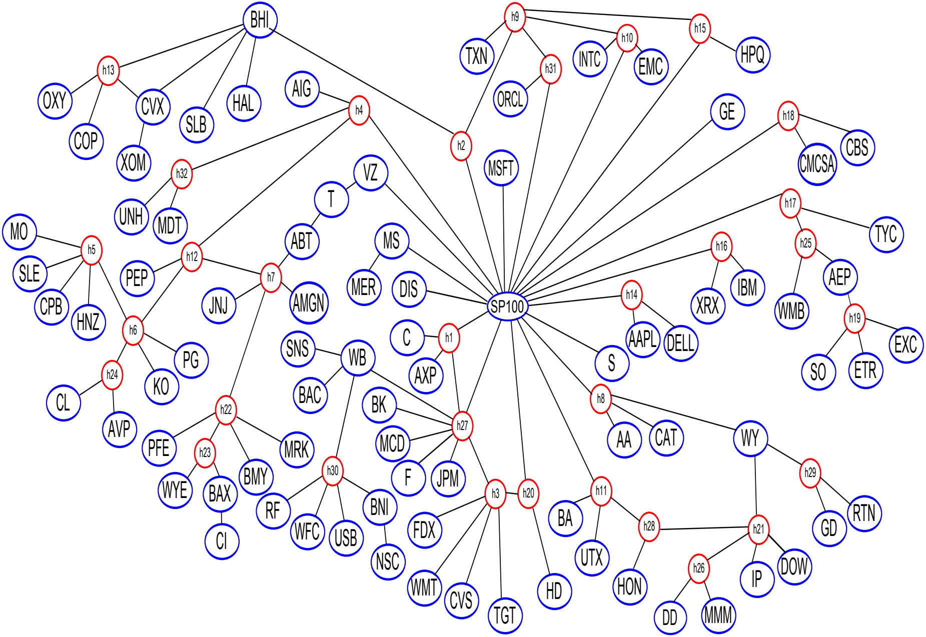

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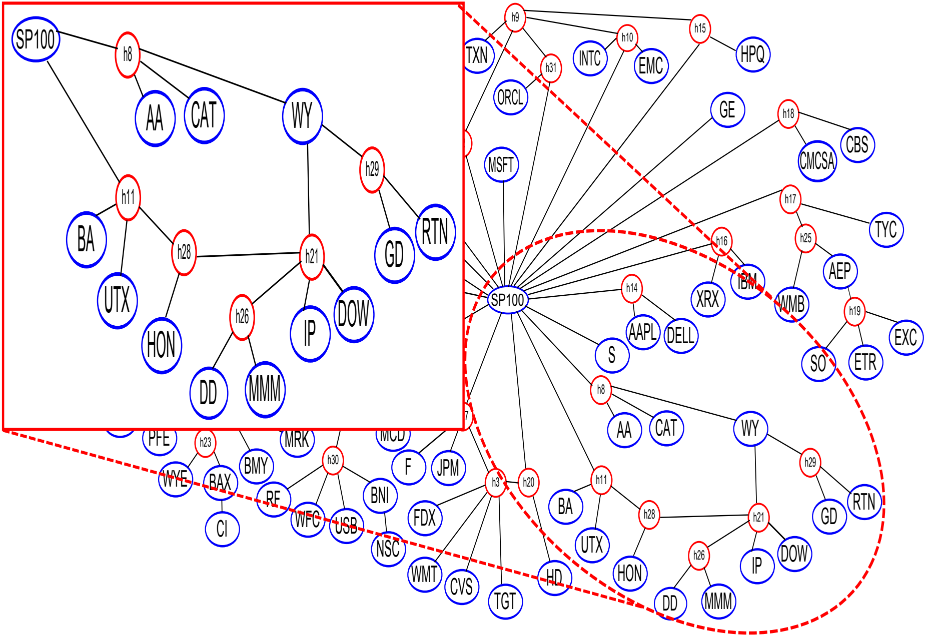

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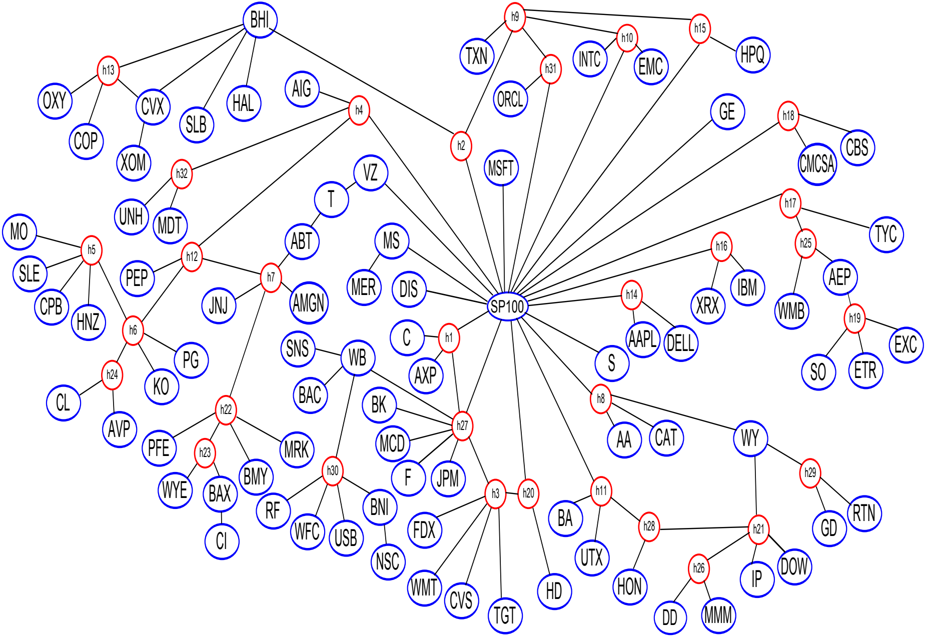

Outline

Summary and Outlook

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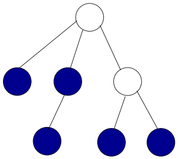
- High-dimensional estimation via graphical approaches
- Model classes where learning is tractable
- Efficient methods for learning
- Guarantees on sample and computational complexities



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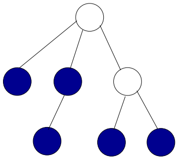


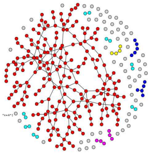
Outlook

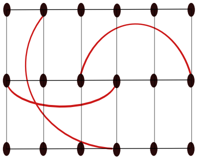
- Removing girth constraint on latent models
- Characterizing criterion for tractable learning
- Learning beyond regime of correlation decay

Extensions

Structure Estimation in Random Graph Models

- Fully observed models (no latent nodes)
- Random graph models such as Erdős-Rényi and small world





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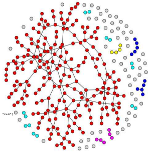

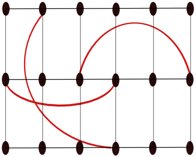

Graph Estimation Through Search for Vertex Separators

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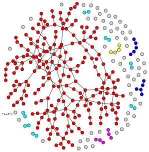

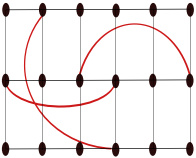

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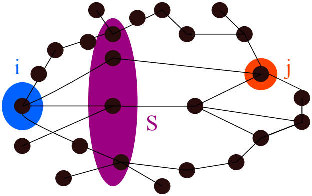

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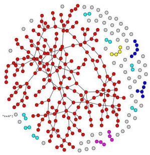

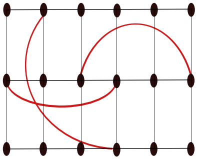

Graph Estimation Through Search for Vertex Separators

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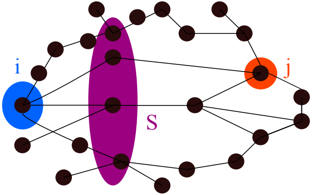

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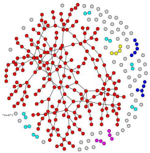

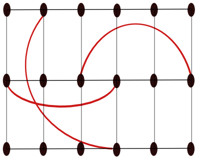

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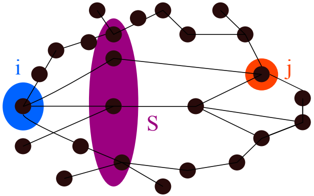
- Naïve search for separators is exponential in maximum node degree
- Novel criterion: sparse local separation



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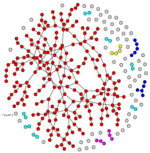

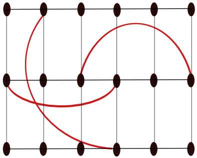

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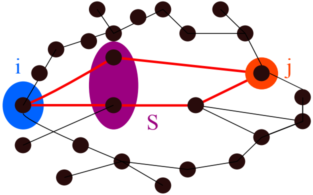

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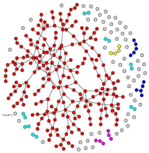

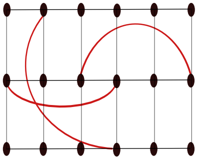

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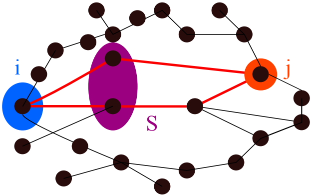

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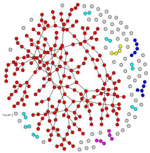

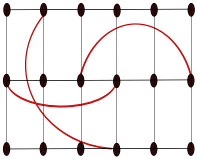

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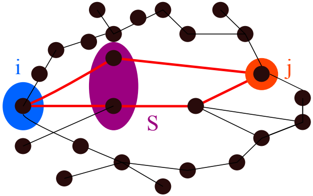

Novel Criteria for High-Dimensional Estimation

Extensions and Connections

Topology Discovery With Few Participants (A. , Hassidim, Kelner '11)

- End-to-end delays between participants in Erdős-Rényi random graph
- Edit distance guarantees with vanishing fraction of participants

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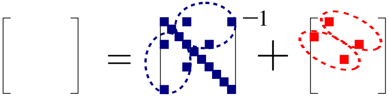
Covariance Decomposition



- Multiple graphs: combination of statistical relationships
- Markov and Independence Domains

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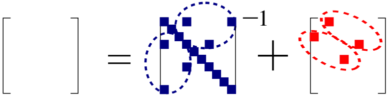

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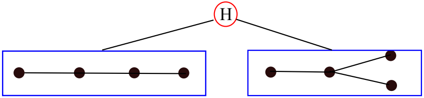
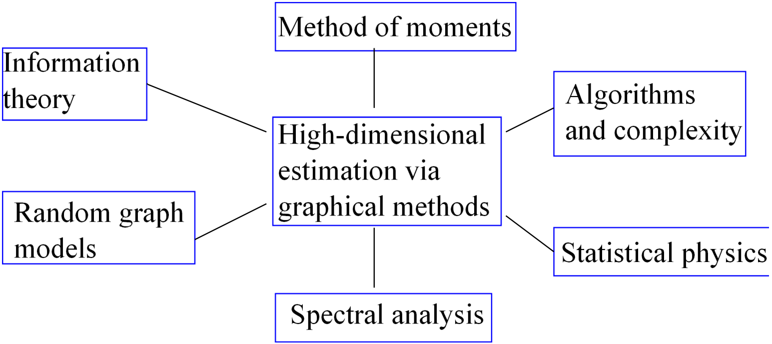
Graphical Model Mixtures



- Multiple graphs: context specific dependencies
- Hidden context
- Learning guarantees

The Big Picture



http://newport.eecs.uci.edu/anandkumar