THE STRUCTURE OF EXTREME EVENTS IN THE SECTORS OF THE S&P 500

Abstract

Extreme value statistics provides accurate estimates for the probabilities of rare events such as huge losses in the equity market. We are interested in the extremal dependence between the different sectors of the S&P 500. We perform an exploratory analysis of the S&P 500 sector indices daily losses, using twenty years of data provided by Standard & Poor's. We extract a structure of extreme events using several extremal dependence measures. We employ the framework of regular variation to construct a tail dependence matrix. We present a clustering approach to reveal extremal dependence and detect patterns in extremal observations. We study a conditional model describing the extremal dependence between the S&P 500 and its sectors.

Data

- 20 years of data: 11/02/2002 - 23/03/2022
- 12 indices: S&P 500 + 11 sectors
- Adjusted close prices transformed to scaled log daily losses
- 5064 observations of 12 variables

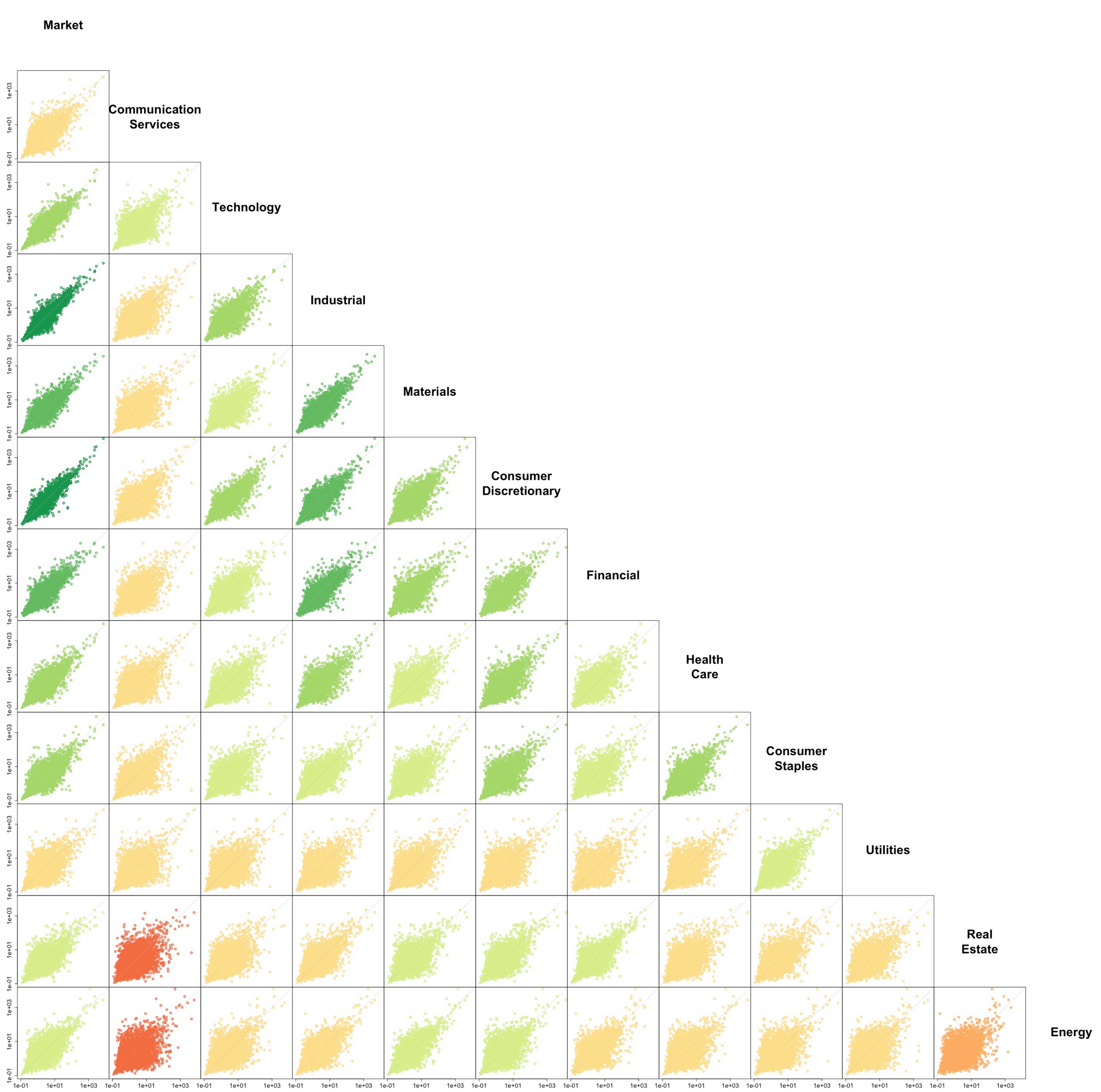

Figure 1: Data scatterplot after transformation to Fréchet scale. Color describes extremal dependence. Axes are on log scale.

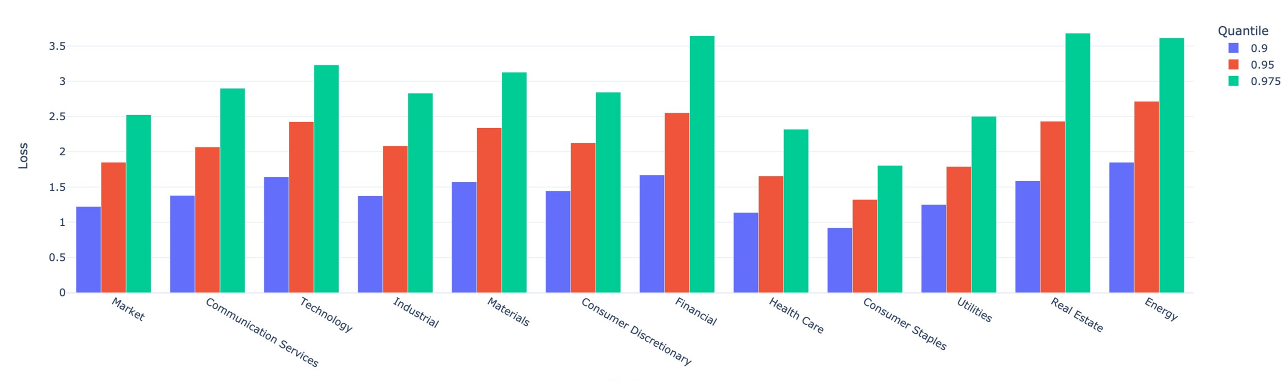

Figure 2: Sectors log daily losses quantiles estimates.

Polar Transformation

Let X be a random vector in $\mathbb{R}^d$. The polar transformation of X is the projection of X into the set $\mathbb{R}_+ \times \mathbb{S}_+^{d-1}$, by defining the radial and angular components

$$R = \|X\| \text{ and } W = X/R$$

Where $\mathbb{S}_+^{d-1} = \{x \in [0,\infty)^d : \|x\| = 1\}$.

Methods & Results

- Several dependence measures were used for this purpose such as the extremal correlation χ, the coefficient of tail dependence η and Pickands dependence function A. The measures are applied on pairs of sectors, as in Figure 1, and they all share the same results. Extremal correlation χ is used to build the graph in Figure 4, which describes the dependence structure for monthly maximum losses.
- We employ the framework of regular variation and angular measures to create a positive semi-definite tail dependence matrix, then perform an eigen-decomposition on it, similar to principal component analysis in a non-extreme case. If we suppose that the angular components have a joint Gaussian distribution, then the partial correlation can be extracted from the tail dependence matrix, and a graphical model describes the dependence structure of the sectors. Graphical lasso is used to introduce sparsity in the graph. The results are in Figure 5 for a penalisation parameter 0.8.
- Spherical k-means, a variant of k-means clustering algorithm, is applied in the analysis of extreme observations from the data, after performing a polar transformation. This allows to find clusters of extremal dependent sectors, as seen in Figure 3.

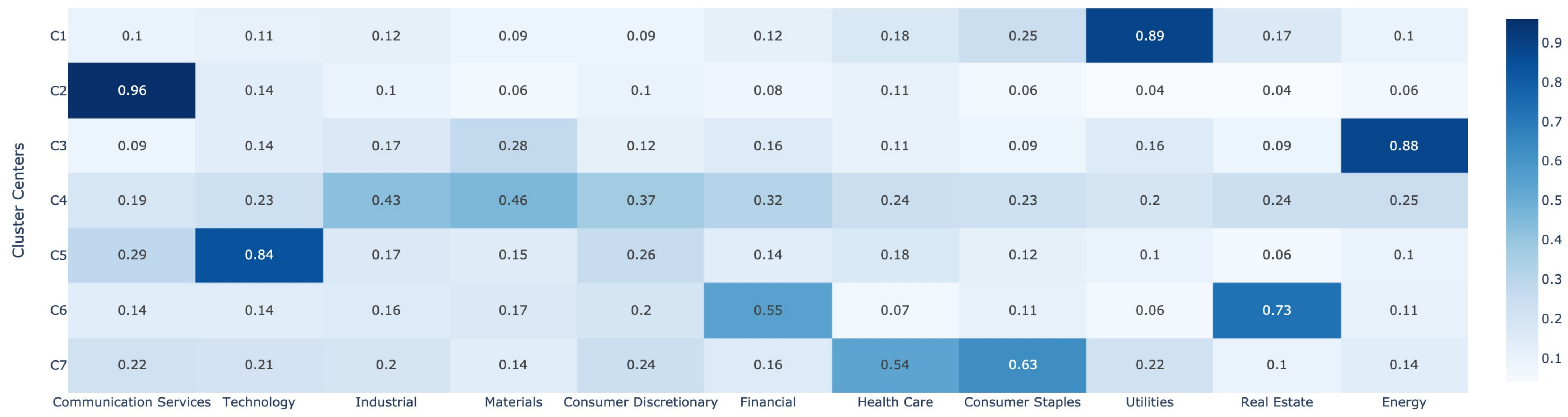

Figure 3: Estimated spherical k-means cluster centers.

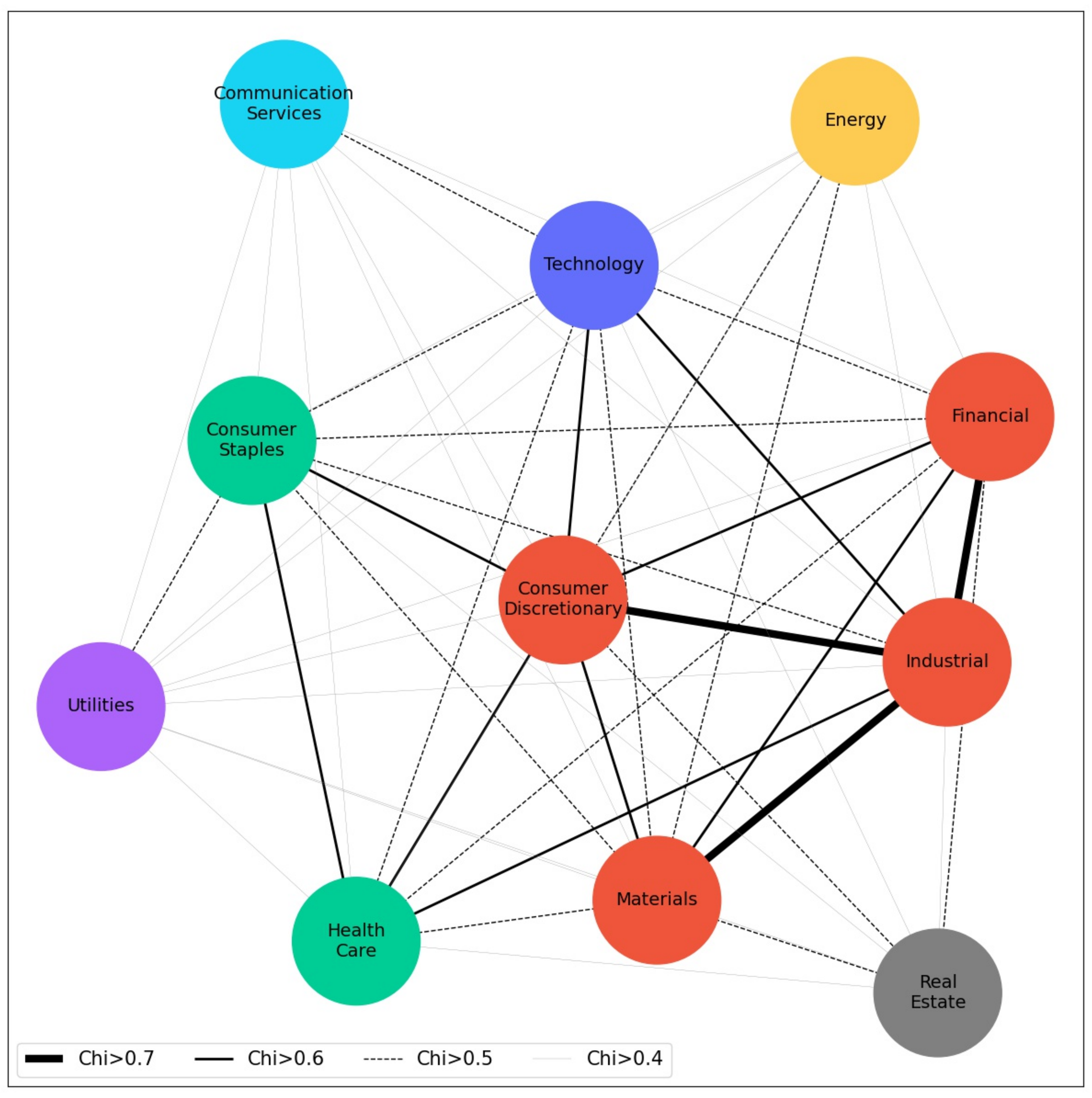

Figure 4: Chi network.

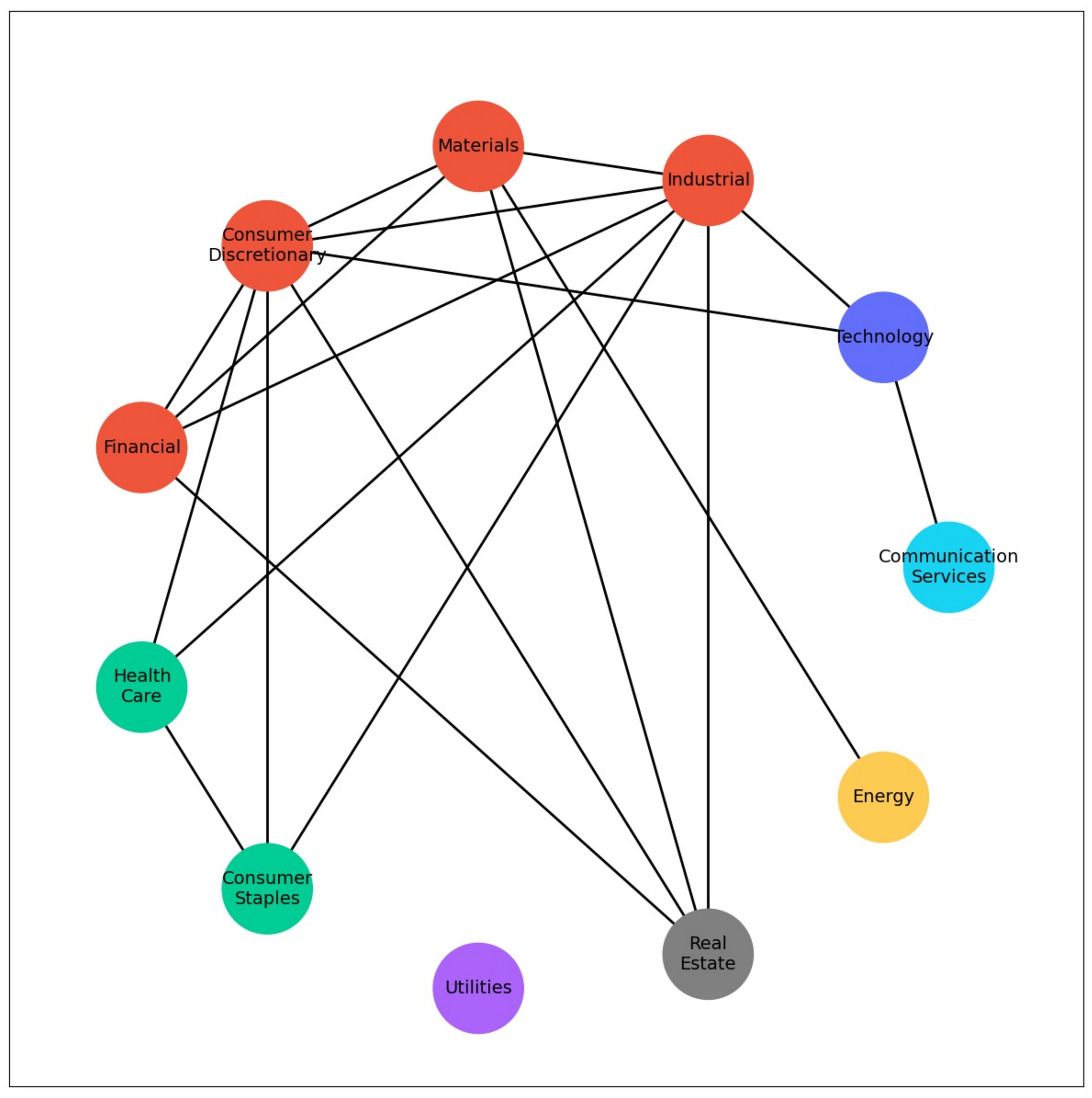

Figure 5: Graphical lasso model.

Conclusion

The dependence structure of the S&P 500 market can be summarised as follows:

- The most dependent sectors to the market are Industrial, Consumer Discretionary and Financial. The least dependent sectors to the market are Communication Services and Utilities.
- The most dependent pairs of sectors are Industrial with Materials, Consumer Discretionary and Financial based on all measures. We add to that Financial/Real Estate and Energy/Materials pairs, based on the multivariate analysis.
- The least dependent pairs of sectors are Communication Services and Utilities with nearly all the other sectors. We add to that Real Estate/Energy pair.

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