Up or Down report

Leo Venn ID: 3430125

Valentin Kiselev, ID: 5844920

Our method for solving the puzzle is as shown in the image below with the instructions for following the image on the next page.

LEGEND:

? represents an unknown state of a cup

?? represents being unsure which two possible board states you could be in after one move

U represents a cup facing up

D represents a cup facing down

D:$^{X}_{X}$ (D followed by a colon and two small letters) represents a diagonal check with the small letters indicating the state of the cups revealed

S:$^{X}_{X}$ (S followed by colon and two small letters) represents a side check with the small letters indicating the state of the cups revealed

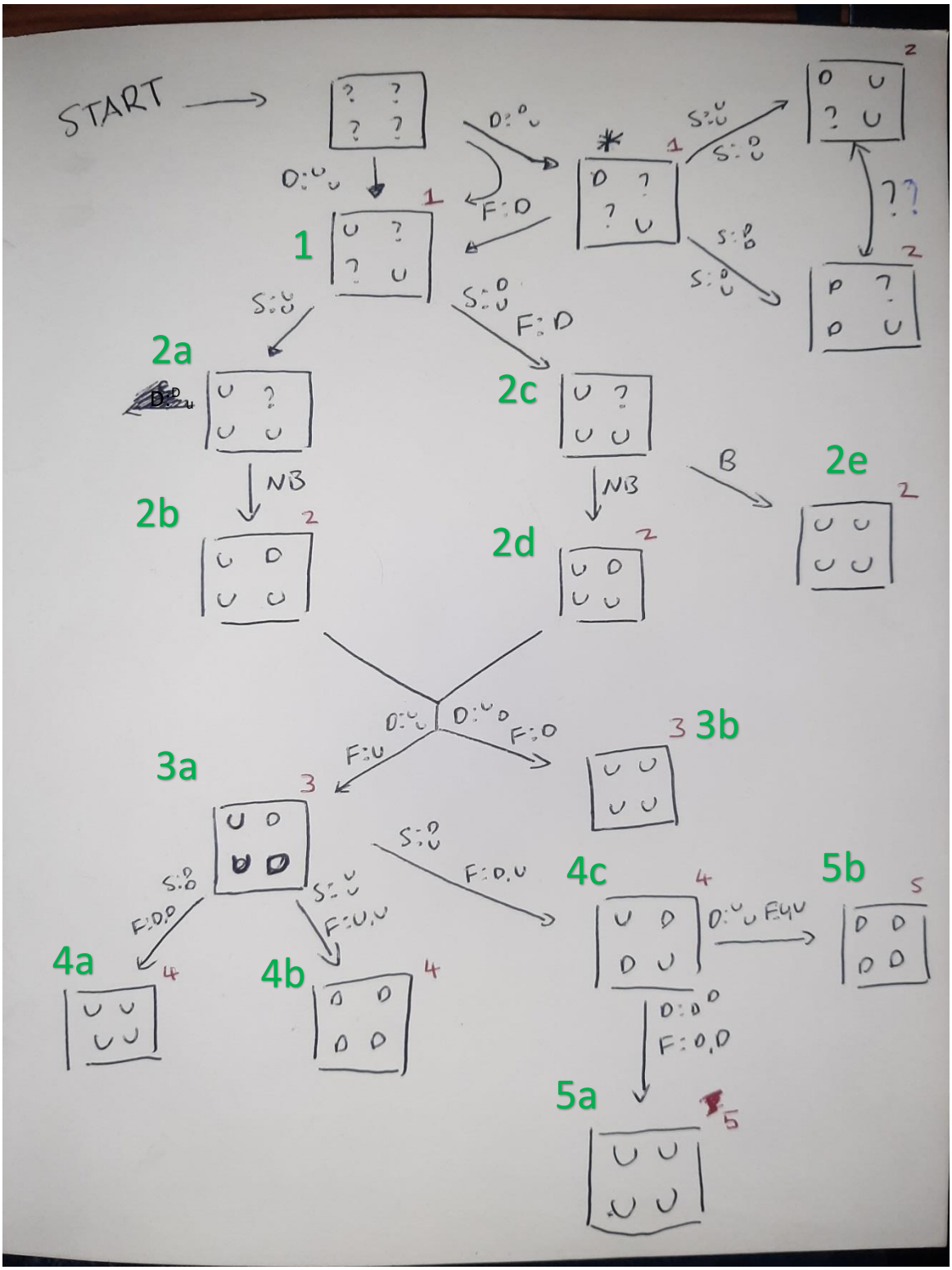
F:XX represents a flip action with the letters after indicating the state of the cup to be flipped

B represents a Bell being run to signal the game is over

NB represents no bell rung

The red numbers indicate how many moves it takes to get to that board state

The green numbers/letters are the names of the board states, with the numbers representing the amount of moves to get there.



* Always do the flip move associated with revealing D, U after a diagonal check. The other moves branching from this state leave you in one of two states which you cannot deduce immediately. These states require 2 moves and so it is always better to do the flip move and be in a state in which you know all cup states after 2 moves. This keeps the max number of steps to solve the puzzle at 5.

NOTE 1: This board/cup states are invertible. For example, board 1 is U?/U? but could be D?/D?. Inverting the states does not change the outcome so if you are at board 1 with D?/D?, then simply follow the bellow instructions but with U's as D's and D's as U's.

NOTE 2: The image does not show all rotations or mirror symmetries of the boards shown. However, the outcomes of moves will always result in some symmetry of the shown boards and it is trivial to see which one your board will correspond to.

INSTRUCTIONS:

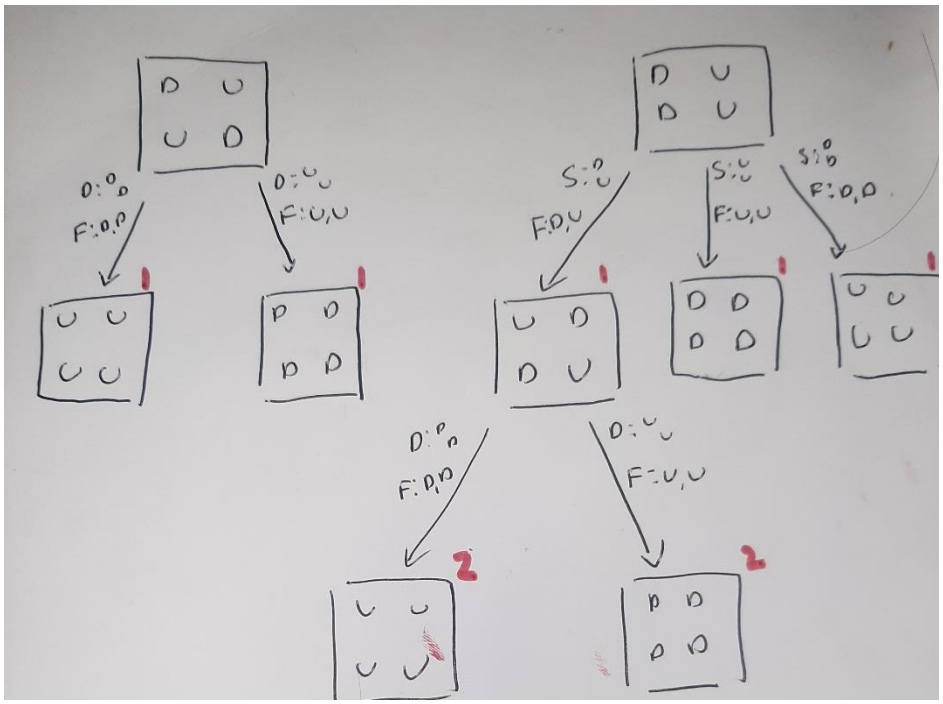
The goal of the game is to have all 4 cups in the same state. A bell will ring to signify we are done.

Referring to the image, we start at the start square. If the bell rings, we are done. If not, then:

1) We do a diagonal check. If we see D and U, we immediately do a flip of D to U to arrive at board 1. If we see U and U, then we are already there.
2) We do a side check. If we see a U and U then we are at board 2a, but we can deduce we must be at board 2b as we have not made any flip moves and the board was not solved at the very beginning. If the side check reveals D and U, then we are at 2c. From here if the bell rings we are at board 2e and are finished. If the there is no bell, then we must be at board 2d.
3) As board 2b and 2d are actually the same, we do a diagonal check. If we see a U and U, then we flip one U to get to board 3a. If we see a U and D, then we flip the D to get board 3b and are done.
4) From board 3a, we do a side check, if we see D and D, we flip both D's and arrive at board 4a where we are finished. If we see U and U, we flip both U's and arrive at board 4b and are finished. If we see a D and U, then we flip the D to a U and the U to a D and arrive at 4c.
5) from board 4c, we do a diagonal check. If we see a D and D, we flip both and are at board 5a where we are finished. If we see a U and U, then we flip both and are at board 5b and are finished.

MAX MOVES AND OUR METHOD

Our method requires at most 5 steps to solve the puzzle which is the smallest maximum number of steps of any method. To prove this first we look at our method. In our method the board cups state can become completely known by the second move in which the state of the board is always in a 3 of a kind 1 of a kind state (e.g., 3 up, 1 down). From here the puzzle can always be solved in at most 3 moves. To find a method that can beat this, we first look at the states of the board in which all cup states are known that are not 3:1. The only other kind is 2:2 with 2 configurations. The next image shows these board states and how they can be solved.



As we can see, the max number of steps to solve these states is 2. This means that the state of the board must be known in at least 2 moves to be less than 5. It must be 2 as a worst case (max move) scenario, the board cannot be solved by chance and must therefore have its state completely known. However, this board state cannot be completely known in 2 moves. The first move will reveal 2 cup states, and the second move must reveal one more cup state for a total of 3. As with 3:1 the final cup state can be known as a bell must ring if the board is solved, if not then there is only 1 board state left. However, in all other cases, there is one state left that cannot be deduced, therefore you cannot know the board is in either of the two 2:2 states in 2 moves. Therefore, to know the board state the moves made must be greater than 2 and therefore cannot be less than 5. Therefore, no method can have a maximum number of moves to solve the puzzle less than ours.