Грунского Алексея

№1

$L=\mathcal{L}\left\{\begin{pmatrix}1\\1\\-5\\1\end{pmatrix}\begin{pmatrix}2\\-7\\1\\1\end{pmatrix}\right\}$ $\quad L^{\perp}:\begin{cases}x_1+x_2-5x_3+x_4=0\\2x_1-7x_2+x_3+x_4=0\end{cases}$

$$\begin{pmatrix}1&1&-5&1\\2&-7&1&1\end{pmatrix}\sim\begin{pmatrix}1&1&-5&1\\0&-9&11&-1\end{pmatrix}\sim\left(\begin{array}{cc|cc}1&1&5&-1\\0&-9&-11&1\end{array}\right)\sim\left(\begin{array}{cc|cc}1&1&5&-1\\0&1&\frac{11}{9}&-\frac{1}{9}\end{array}\right)\sim\left(\begin{array}{cc|cc}1&0&\frac{34}{9}&-\frac{8}{9}\\0&1&\frac{11}{9}&-\frac{1}{9}\end{array}\right)$$

$$\begin{pmatrix}x_1\\x_2\\x_3\\x_4\end{pmatrix}=t_1\begin{pmatrix}\frac{34}{9}\\\frac{11}{9}\\1\\0\end{pmatrix}+t_2\begin{pmatrix}-\frac{8}{9}\\-\frac{1}{9}\\0\\1\end{pmatrix}$$

$$L^{\perp}=\mathcal{L}\left\{\begin{pmatrix}\frac{34}{9}\\\frac{11}{9}\\1\\0\end{pmatrix}\begin{pmatrix}-\frac{8}{9}\\-\frac{1}{9}\\0\\1\end{pmatrix}\right\}=\mathcal{L}\left\{\begin{array}{cc}a_1&a_2\\\begin{pmatrix}34\\11\\9\\0\end{pmatrix}&\begin{pmatrix}-8\\-1\\0\\9\end{pmatrix}\end{array}\right\}$$

ортонормируем

ортогонализуем

$b_1=a_2 \quad b_2=a_1-c_1a_2 \quad c_1=\frac{\langle a_1,b_1\rangle}{\langle b_1,b_1\rangle}=\frac{-283}{146}$

$$b_2=\begin{pmatrix}34\\11\\9\\0\end{pmatrix}+\frac{283}{146}\begin{pmatrix}-8\\-1\\0\\9\end{pmatrix}=\begin{pmatrix}\frac{1350}{73}\\\frac{1323}{146}\\9\\\frac{2547}{146}\end{pmatrix}$$

b_1 ортогонален b_2

$$b_1\sim\begin{pmatrix}\frac{-8}{\sqrt{146}}\\-\frac{1}{\sqrt{146}}\\0\\\frac{9}{\sqrt{146}}\end{pmatrix}\quad b_2\sim\begin{pmatrix}\frac{300}{\sqrt{213014}}\\\frac{147}{\sqrt{213014}}\\\frac{146}{\sqrt{213014}}\\\frac{283}{\sqrt{213014}}\end{pmatrix}$$

№2

$$L^{\perp}:\begin{cases}x_1+2x_2-2x_3-3x_4+5x_5=0\\x_1+2x_2+3x_5=0\\2x_1+4x_2-x_3+7x_4=0\end{cases}$$

$$\begin{pmatrix}1&2&-2&-3&5\\1&2&0&0&3\\2&4&-1&7&0\end{pmatrix}\sim\begin{pmatrix}1&2&-2&-3&5\\0&0&2&3&-2\\0&0&3&13&-10\end{pmatrix}\sim\begin{pmatrix}1&2&-2&-3&5\\0&0&2&3&-2\\0&0&0&\frac{17}{2}&-7\end{pmatrix}\sim\begin{pmatrix}1&-2&-3&2&5\\0&2&3&0&-2\\0&0&\frac{17}{2}&0&-7\end{pmatrix}\sim$$

$$\sim\left(\begin{array}{ccc|cc}1&-2&-3&-2&-5\\0&2&3&0&2\\0&0&\frac{17}{2}&0&7\end{array}\right)\sim\left(\begin{array}{ccc|cc}1&-2&-3&-2&-5\\0&2&3&0&2\\0&0&1&0&\frac{14}{17}\end{array}\right)\sim\left(\begin{array}{ccc|cc}1&-2&-3&-2&-3\\0&1&0&0&-\frac{4}{17}\\0&0&1&0&\frac{14}{17}\end{array}\right)\sim\left(\begin{array}{ccc|cc}1&0&0&-2&-3\\0&1&0&0&-\frac{4}{17}\\0&0&1&0&\frac{14}{17}\end{array}\right)$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_6 \end{pmatrix} = t_1 \begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + t_2 \begin{pmatrix} -3 \\ 0 \\ -\frac{4}{17} \\ \frac{14}{17} \\ 1 \end{pmatrix}$$

$$L^\perp = \mathcal{L}\left\{ \begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} -3 \\ 0 \\ -\frac{4}{17} \\ \frac{14}{17} \\ 1 \end{pmatrix} \right\} = \mathcal{L}\left\{ \begin{array}{cc} a_1 & a_2 \\ \begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} -51 \\ 0 \\ -4 \\ 14 \\ 17 \end{pmatrix} \end{array} \right\}$$

ортогонализируем

$b_1 = a_1 \quad b_2 = a_2 - c_1 b_1 \quad c_1 = \dfrac{\langle a_2, b_1 \rangle}{\langle b_1, b_1 \rangle} = \dfrac{102}{5}$

$$b_2 = \begin{pmatrix} -51 \\ 0 \\ -4 \\ 14 \\ 17 \end{pmatrix} - \frac{102}{5} \begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -\frac{51}{5} \\ -\frac{102}{5} \\ -4 \\ 14 \\ 17 \end{pmatrix} \sim \begin{pmatrix} -51 \\ -102 \\ -20 \\ 70 \\ 85 \end{pmatrix}$$

$$\mathcal{L}\left\{ \begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} -51 \\ -102 \\ -20 \\ 70 \\ 85 \end{pmatrix} \right\} = L^\perp$$

← ортог. базис.

№3.

$$L^\perp: \begin{cases} x_1 + x_2 = 0 \\ x_1 - x_2 + x_4 = 0 \\ x_1 + x_4 = 0 \\ 2x_2 - x_4 = 0 \end{cases} \leftarrow \text{решение} \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

$$L^\perp = \mathcal{L}\left\{ \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \right\}$$

$$L = \mathcal{L}\left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} 0 \\ 2 \\ 0 \\ -1 \end{pmatrix} \right\}$$

$$rg \begin{pmatrix} 1 & 1 & 1 & 0 \\ 1 & -1 & 0 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & -1 \end{pmatrix} = rg \begin{pmatrix} 1 & 1 & 1 & 0 \\ 1 & -1 & 0 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} =$$

$$= 1 + rg \begin{pmatrix} 1 & -1 & 0 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = 1 + rg \begin{pmatrix} 1 & -1 & 0 & 2 \\ 0 & 0 & 0 & -1 \end{pmatrix} =$$

$$= 1 + 1 + rg(1 \ \ -1) = 3$$

$$\begin{array}{ccc} a_1 & a_2 & a_3 \\ \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \end{pmatrix} & \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} \end{array} \leftarrow \text{базис } L$$

$b_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} \quad b_2 = a_2 - c_1 b_1 \quad c_1 = \dfrac{\langle a_2, b_1 \rangle}{\langle b_1, b_1 \rangle} = 0$

$b_2 = a_2 \quad b_3 = a_3 - \tilde{c}_1 b_1 - \tilde{c}_2 b_2$

$\tilde{c}_1 = \dfrac{\langle a_3, b_1 \rangle}{\langle b_1, b_1 \rangle} = \dfrac{1}{2}, \quad \tilde{c}_2 = \dfrac{\langle a_3, b_2 \rangle}{\langle b_2, b_2 \rangle} = \dfrac{2}{3}$

$$b_3 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} - \frac{1}{2}\begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} - \frac{2}{3}\begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -\frac{1}{6} \\ \frac{1}{6} \\ 0 \\ \frac{1}{3} \end{pmatrix} \sim \begin{pmatrix} -1 \\ 1 \\ 0 \\ 2 \end{pmatrix}$$

Нормируем

$$b_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} \quad b_2 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \end{pmatrix} \quad b_3 = \frac{1}{\sqrt{6}}\begin{pmatrix} -1 \\ 1 \\ 0 \\ 2 \end{pmatrix}$$

$$\alpha = \langle b_1, y\rangle = \sqrt{2} \quad \beta = \langle b_2, y\rangle = \frac{1}{\sqrt{3}} \quad \gamma = \langle b_3, y\rangle = \frac{2}{\sqrt{6}}$$

$$x = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} + \frac{1}{3}\begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \end{pmatrix} + \frac{1}{3}\begin{pmatrix} -1 \\ 1 \\ 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix}$$

$$\cos\varphi = \frac{\langle x, y\rangle}{\sqrt{\langle x,x\rangle \cdot \langle y,y\rangle}} = \frac{3}{\sqrt{3\cdot 7}} = \frac{3}{\sqrt{21}}$$