ME572: Homework #6

Due on March 5, 2012

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Problem 1

For the 2-D manipulator used in homework set #3, set up a general form for the Jacobian matrix in terms of the joint variables.

From the previous homework T_w is shown below:

$$T_w = \begin{bmatrix} \cos(\theta_1+\theta_3) & 0 & \sin(\theta_1+\theta_3) & 3\cdot\sin(\theta_1+\theta_3)+3\cdot\cos(\theta_1)+s_2\cdot\sin(\theta_1) \\ \sin(\theta_1+\theta_3) & 0 & -\cos(\theta_1+\theta_3) & 3\cdot\sin(\theta_1)-3\cdot\cos(\theta_1+\theta_3)-s_2\cdot\cos(\theta_1) \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}$$

The Jacobian matrix is in the form of:

$$\begin{Bmatrix} \dot{P}_x \\ \dot{P}_y \\ \dot{P}_z \\ \omega_x \\ \omega_y \\ \omega_z \end{Bmatrix} = \begin{bmatrix} J \\ 6\times 3 \end{bmatrix} \begin{Bmatrix} \dot{\theta}_1 \\ \dot{S}_2 \\ \dot{\theta}_3 \end{Bmatrix} \tag{2}$$

Where the columns of the Jacobian for the revolute and prismatic joint i are given by Eqn. 3 and 4, respectively.

$$\begin{Bmatrix} \vec{a}_{i-1}\times\left(\vec{P}_E-\vec{P}_{i-1}\right) \\ \vec{a}_{i-1} \end{Bmatrix} \tag{3}$$

$$\begin{Bmatrix} \vec{a}_{i-1} \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{4}$$

So the Jacobian matrix for this problem should be:

$$J = \begin{Bmatrix} \vec{a}_0\times\left(\vec{P}_E-\vec{P}_0\right) & \vec{a}_1 & \vec{a}_2\times\left(\vec{P}_E-\vec{P}_2\right) \\ & 0 & \\ \vec{a}_0 & 0 & \vec{a}_2 \\ & 0 & \end{Bmatrix} \tag{5}$$

Substituting in the appropriate values, the Jacobian for this 2-D manipulator is:

$$J = \begin{bmatrix} 3\cdot\cos(\theta_1+\theta_3)-3\cdot\sin(\theta_1)+s_2\cdot\cos(\theta_1) & \sin(\theta_1) & 3\cdot\cos(\theta_1+\theta_3) \\ 3\cdot\sin(\theta_1+\theta_3)+3\cdot\cos(\theta_1)+s_2\cdot\sin(\theta_1) & -\cos(\theta_1) & 3\cdot\sin(\theta_1+\theta_3) \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix} \tag{6}$$

The velocity end effector equations can be verified by taking the derivative of the world transform T_m.

$$T_m = \phi_1 T_1 \phi_2 T_2 \phi_3 T_3 \tag{7}$$

$$\frac{dT_m}{dt} = \frac{d\phi_1}{dt} T_1 \phi_2 T_2 \phi_3 T_3 + \phi_1 T_1 \frac{d\phi_2}{dt} T_2 \phi_3 T_3 + \phi_1 T_1 \phi_2 T_2 \frac{\phi_3}{dt} T_3 \tag{8}$$

Where $\frac{d\phi_i}{dt} = \dot{\phi} \cdot Q_i \phi_i$ and Q depends on if the joint is revolute or prismatic. The 4^{th} column of $\frac{dT_m}{dt}$ will correspond to $\begin{Bmatrix} \dot{P}_x \\ \dot{P}_y \\ \dot{P}_z \end{Bmatrix}$ from the output of the Jacobian matrix, which it does.

Problem 2

If the joint variables for the 2-D manipulator are: $\theta_1 = 330^o$; $S_2 = 3$"; $\theta_3 = 80^o$, solve for the following using the Jacobian.

(a) $\dot{X}_p$, $\dot{Y}_p$, $\dot{\omega}_z$ for $\dot{\theta}_1 = 5$ rad/sec, $\dot{S}_2 = -4.0$"/sec, and $\dot{\theta}_3 = -2.0$ rad/sec.

(b) $\dot{\theta}_1$, $\dot{S}_2$, $\dot{\theta}_3$ for $\dot{X}_p = -2.0$"/sec, $\dot{Y}_p = 5.0$"/sec, and $\dot{\omega}_z = -3.0$ rad/sec.

To solve for the inverse Jacobian matrix the J matrix needs to be square. (Or at least there need to be at least as many knowns as unknowns to solve for). In the case of this matrix, there are only 3 parameters that we can tune so at most we can only solve for them if we are given 3 or less velocity parameters, otherwise it will be over constrained.

At this particular orientation with $\theta_1 = 330^o$, $S_2 = 3$", and $\theta_3 = 80^o$ the Jacobian matrix reduces to Eqn. 9. Since we are only interested in $\dot{X}_p$, $\dot{Y}_p$, $\dot{\omega}_z$ (Since it is a 2D manipulator) it can be further reduced to Eqn. **??** with the input and output relationships shown in Eqns. 12 and 13.

$$J = \begin{bmatrix} 6.026 & -0.500 & 1.928 \\ 3.396 & -0.866 & 2.298 \\ 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 \\ 1.000 & 0.000 & 1.000 \end{bmatrix} \tag{9}$$

$$J = \begin{bmatrix} 6.026 & -0.500 & 1.928 \\ 3.396 & -0.866 & 2.298 \\ 1.000 & 0.000 & 1.000 \end{bmatrix} \tag{10}$$

$$J^{-1} = \begin{bmatrix} 0.289 & -0.167 & -0.174 \\ 0.366 & -1.366 & 2.433 \\ -0.289 & 0.167 & 1.174 \end{bmatrix} \tag{11}$$

$$\begin{Bmatrix} \dot{X}_p \\ \dot{Y}_p \\ \dot{\omega}_z \end{Bmatrix} = [J] \begin{Bmatrix} \dot{\theta}_1 \\ \dot{S}_2 \\ \dot{\theta}_3 \end{Bmatrix} \tag{12}$$

$$\begin{Bmatrix} \dot{\theta}_1 \\ \dot{S}_2 \\ \dot{\theta}_3 \end{Bmatrix} = [J]^{-1} \begin{Bmatrix} \dot{X}_p \\ \dot{Y}_p \\ \dot{\omega}_z \end{Bmatrix} \tag{13}$$

Substituting in the given parameters the solutions for part a and b are in Eqns. 14 and 15, respectively.

Angle velocities are given in rad/sec and linear velocities are in inches/second.

$$\begin{Bmatrix} \dot{X}_p \\ \dot{Y}_p \\ \dot{\omega_z} \end{Bmatrix} = \begin{bmatrix} 28.275 \\ 15.849 \\ 3.000 \end{bmatrix} \tag{14}$$

$$\begin{Bmatrix} \dot{\theta}_1 \\ \dot{S}_2 \\ \dot{\theta}_3 \end{Bmatrix} = \begin{bmatrix} -0.890 \\ -14.863 \\ -2.110 \end{bmatrix} \tag{15}$$

$$\tag{16}$$

Problem 3

Using Cramer's rule to solve for each of the equations you get the general solution below:

$$\dot{\theta}_1 = \frac{\mathrm{X_p}\ \cos(\theta_1) + \mathrm{Y_p}\ \sin(\theta_1) - 3\,\omega_\mathrm{z}\ \cos(\theta_3)}{\mathrm{S_2}} \tag{17}$$

$$\dot{S}_2 = \frac{3\,\mathrm{X_p}\ \cos(\theta_1) + 3\,\mathrm{Y_p}\ \sin(\theta_1) - 9\,\omega_\mathrm{z}\ \cos(\theta_3)}{\mathrm{S_2}} - \mathrm{Y_p}\ \cos(\theta_1) + \mathrm{X_p}\ \sin(\theta_1) + 3\,\omega_\mathrm{z}\ \sin(\theta_3) \tag{18}$$

$$\dot{\theta}_3 = \omega_\mathrm{z} - \frac{\mathrm{X_p}\ \cos(\theta_1) + \mathrm{Y_p}\ \sin(\theta_1) - 3\,\omega_\mathrm{z}\ \cos(\theta_3)}{\mathrm{S_2}} \tag{19}$$

The equation will divide by zero when S_2 is 0. This can also be found by just looking at the determinant of the Jacobian matrix, $-S_2$. The Jacobian for $S_2 = 0$ with the angles remaining the same is shown below. Attempting to take the determinant of it with Matlab results in very large numbers and Matlab complaining about a singularity. The Jacobian will not work when $S_2 = 0$.

$$J = \begin{bmatrix} 3.43 & -0.50 & 1.93 \\ 4.90 & -0.87 & 2.30 \\ 1.00 & 0.00 & 1.00 \end{bmatrix} \tag{20}$$

$$J^{-1} = \begin{bmatrix} -1501199875790165.25 & 866718152394885.75 & 903024166285322.75 \\ -4503599627370496.00 & 2600154457184656.00 & 2709072498855971.00 \\ 1501199875790165.25 & -866718152394885.75 & -903024166285321.75 \end{bmatrix} \tag{21}$$