# ME572: Homework #2

Due on Jan 27, 2012

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# Problem 1

1) What is the rotation matrix for a rotation of 60 degrees about the OU axis, followed by a rotation of 300 degrees about the OW axis, followed by a rotation of 45 degrees about the OY axis?

The axes before any rotations are shown in Eqn. 1, 2, & 3.

$$x = u \tag{1}$$
$$y = v \tag{2}$$
$$z = w \tag{3}$$

$R_{u,\theta_1}$ is the initial rotation matrix. $R_{w,\theta_2}$ is a rotation about the body so is post multiplied. $R_{y,\theta_3}$ is with respect to the origin and is premultiplied. Test

$$\begin{Bmatrix} p_x \\ p_y \\ p_z \end{Bmatrix} = [R_{y,\theta_3}][R_{u,\theta_1}][R_{w,\theta_2}] \begin{Bmatrix} P_u \\ P_v \\ P_w \end{Bmatrix} \tag{4}$$

$$R_{u,\theta_1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta_1) & -\sin(\theta_1) \\ 0 & \sin(\theta_1) & \cos(\theta) \end{bmatrix} \tag{5}$$

$$R_{w,\theta_2} = \begin{bmatrix} \cos(\theta_2) & -\sin(\theta_2) & 0 \\ \sin(\theta_2) & \cos(\theta_2) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{6}$$

$$R_{y,\theta_3} = \begin{bmatrix} \cos(\theta_3) & 0 & \sin(\theta_3) \\ 0 & 1 & 0 \\ -\sin(\theta_3) & 0 & \cos(\theta_3) \end{bmatrix} \tag{7}$$

$$R = \begin{bmatrix} \cos(\theta_2)\cos(\theta_3) + \sin(\theta_1)\sin(\theta_2)\sin(\theta_3) & \cos(\theta_2)\sin(\theta_1)\sin(\theta_3) - \cos(\theta_3)\sin(\theta_2) & 0 \\ \cos(\theta_1)\sin(\theta_2) & \cos(\theta_1)\cos(\theta_2) & 0 \\ \cos(\theta_3)\sin(\theta_1)\sin(\theta_2) - \cos(\theta_2)\sin(\theta_3) & \sin(\theta_2)\sin(\theta_3) + \cos(\theta_2)\cos(\theta_3)\sin(\theta_1) & 0 \end{bmatrix} \tag{8}$$

$$\begin{Bmatrix} P_x \\ P_y \\ P_z \end{Bmatrix} = \begin{bmatrix} -0.1768 & 0.9186 & 0 \\ -0.4330 & 0.2500 & 0 \\ -0.8839 & -0.3062 & 0 \end{bmatrix} \begin{Bmatrix} P_u \\ P_v \\ P_w \end{Bmatrix} \tag{9}$$

# Problem 2

List all other sequences of rotations which result in the same net rotation as shown in Eqn. 10.

$$R = R_\phi R_\alpha R_\beta R_\theta \tag{10}$$

For the purposes of notation a rotation about u is an x rotation w.r.t. the body, v is a y rotation w.r.t. the body, & w is a z rotation w.r.t. the body. Rotations x, y, z are in reference to the fixed frame. The following are all other sequences which result in the same rotation 10.

Problem 2 continued on next page... 

$$\begin{aligned}
&R_{x,\phi}, R_{w,\alpha}, R_{u,\beta}, R_{v,\theta} \\
&R_{z,\alpha}, R_{x,\phi}, R_{u,\beta}, R_{v,\theta} \\
&R_{z,\alpha}, R_{u,\beta}, R_{x,\phi}, R_{v,\theta} \\
&R_{x,\beta}, R_{v,\theta}, R_{z,\alpha}, R_{x,\phi} \\
&R_{x,\beta}, R_{z,\alpha}, R_{v,\theta}, R_{x,\phi} \\
&R_{x,\beta}, R_{z,\alpha}, R_{x,\phi}, R_{v,\theta} \\
&R_{y,\theta}, R_{x,\beta}, R_{z,\alpha}, R_{x,\phi}
\end{aligned} \tag{11}$$

The number of possibly combinations is $2^{n-1}$ where n is nthe number of rotational matricies. The number of combinations possible when starting from the final position follows the corresponding row of the Pascal's triangle.

Example: For 4 rotations with the final rotation matrix of [A][B][C][D]. Capital letters will represent a fixed rotation, lowercase a body rotation.

There is 1 possible combination when starting with the $1^{st}$ final position:

A [b][c][d]

There are 3 possible combinations when starting with the $2^{nd}$ final position:

B [A][c][d]

B [c][A][d]

B [c][d][A]

There are 3 possible combinations when starting with the $3^{rd}$ final position:

C [B][A][d]

C [B][d][A]

C [d][B][A]

There is 1 possible combination when starting with the $4^{th}$ final position:

D [C][B][A]

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $n_r = 1$: | | | | | 1 | | | | | |
| $n_r = 2$: | | | | 1 | | 1 | | | | |
| $n_r = 3$: | | | 1 | | 2 | | 1 | | | |
| $n_r = 4$: | | 1 | | 3 | | 3 | | 1 | | |
| $n_r = 5$: | 1 | | 4 | | 6 | | 4 | | 1 | |

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# Problem 3

Net rotation matrix formula is shown in Eqn. 12. The calculations for angles $\alpha$ and $\beta$ are show in Eqn. 14 and 15, respectively. The complete rotation matrix is shown in Eqn. 16

$$
\begin{aligned}
[R_{\alpha,\beta,\phi}] &= [R_{x,-\alpha}][R_{y,\beta}][R_{z,\phi}][R_{y,-\beta}][R_{x,\alpha}] && (12)\\
\phi &= 30^\circ && (13)\\
\alpha &= \cos^{-1}\left(\frac{r_z}{\sqrt{r_y^2+r_z^2}}\right) = \sin^{-1}\left(\frac{r_y}{\sqrt{r_y^2+r_z^2}}\right) = \tan^{-1}\left(\frac{r_y}{r_z}\right) && (14)\\
\beta &= \cos^{-1}\left(\frac{\sqrt{r_y^2+r_z^2}}{\sqrt{r_x^2+r_y^2+r_z^2}}\right) = \sin^{-1}\left(\frac{r_x}{\sqrt{r_x^2+r_y^2+r_z^2}}\right) = \tan^{-1}\left(\frac{r_x}{\sqrt{r_y^2+r_z^2}}\right) && (15)\\
[R_{\alpha,\beta,\phi}] &= \begin{bmatrix} 0.2887 & -0.0846 & 0.4928 \\ -0.4928 & 0.7217 & -0.2639 \\ 0.0846 & 0.5526 & 0.7217 \end{bmatrix} && (16)
\end{aligned}
$$