# ME572: Homework #3a

Due on Feb 3, 2012

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# Problem 1

Solve problem 2.6 in the reference text. Determine ${}^{i-1}A_i$ by inspection. These are listed in Eqn. 1 - 5. ${}^5A_0$ will be used to check the validity of the entire transform as it should be an identity matrix.

$$ {}^0A_1 = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & c+e \\ 0 & -1 & 0 & a-d \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1} $$

$$ {}^1A_2 = \begin{bmatrix} 0 & -1 & 0 & b \\ 0 & 0 & -1 & a-d \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2} $$

$$ {}^2A_3 = \begin{bmatrix} 0 & 0 & 1 & e \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & a \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3} $$

$$ {}^3A_4 = \begin{bmatrix} 0 & 0 & -1 & d \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & c \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4} $$

$$ {}^4A_5 = \begin{bmatrix} 0 & 0 & -1 & b \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & d \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{5} $$

$$ {}^5A_0 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & a \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{6} $$

${}^0A_i$ were all calculated by matrix multiplication and listed in Eqn. 7 - 10. Eqn. 11 was used check that all of the matricies were correct. ${}^0A_0$ is the identity matrix as shown below.

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$$ {}^0A_2 = \begin{bmatrix} 0 & 1 & 0 & -b \\ -1 & 0 & 0 & c+e \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{7} $$

$$ {}^0A_3 = \begin{bmatrix} 0 & 1 & 0 & -b \\ 0 & 0 & -1 & c \\ -1 & 0 & 0 & a \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{8} $$

$$ {}^0A_4 = \begin{bmatrix} 1 & 0 & 0 & -b \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & a-d \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{9} $$

$$ {}^0A_5 = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & a \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{10} $$

$$ {}^0A_0 = {}^0A_1 {}^1A_2 {}^2A_3 {}^3A_4 {}^4A_5 {}^5A_0 \tag{11} $$

$$ {}^0A_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{12} $$

# Problem 2

The joint constraint matrices are listed in Eqn. 13, 15, and 17 and the shape matrices Eqn. 14, 16, and 18.

$$\phi_1 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{13}$$

$$T_1 = \begin{bmatrix} 0.00 & 1.00 & 0.00 & 3.00 \\ 0.00 & 0.00 & -1.00 & 0.00 \\ -1.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 1.00 \end{bmatrix} \tag{14}$$

$$\phi_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & s2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{15}$$

$$T_2 = \begin{bmatrix} 0.00 & 0.00 & -1.00 & 0.00 \\ 0.00 & 1.00 & 0.00 & 0.00 \\ 1.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 1.00 \end{bmatrix} \tag{16}$$

$$\phi_3 = \begin{bmatrix} \cos\theta_3 & -\sin\theta_3 & 0 & 0 \\ \sin\theta_3 & \cos\theta_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{17}$$

$$T_3 = \begin{bmatrix} 0.00 & 0.00 & 1.00 & 3.00 \\ 1.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 1.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 1.00 \end{bmatrix} \tag{18}$$

To validate each of the joint constraight and shape matrices let $\theta_1 = 90^o$, $S_2 = 2$", and $\theta_3 = 0^o$ (as shown at the bottom of the homework page). By inspection we can verify that the above equations are correct.

$${}^0A_1 = \phi_1 T_1 = \begin{bmatrix} 0.00 & 0.00 & 1.00 & 0.00 \\ 0.00 & 1.00 & 0.00 & 3.00 \\ -1.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 1.00 \end{bmatrix} \tag{19}$$

$${}^0A_2 = \phi_1 T_1 \phi_2 T_2 = \begin{bmatrix} 1.00 & 0.00 & 0.00 & 2.00 \\ 0.00 & 1.00 & 0.00 & 3.00 \\ 0.00 & 0.00 & 1.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 1.00 \end{bmatrix} \tag{20}$$

$${}^0A_3 = T_w = \begin{bmatrix} 0.00 & 0.00 & 1.00 & 5.00 \\ 1.00 & 0.00 & 0.00 & 3.00 \\ 0.00 & 1.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 1.00 \end{bmatrix} \tag{21}$$

# Problem 3

The complete manipulator transform $[T_M]$ is shown in Eqn. 22 and reduced with trig identities to Eqn. 23.

$$T_w = \begin{bmatrix} \cos\theta_1\cos\theta_3 - \sin\theta_1\sin\theta_3 & 0 & \cos\theta_1\sin\theta_3 + \cos\theta_3\sin\theta_1 & 3\cos\theta_1 + 3\cos\theta_1\sin\theta_3 + 3\cos\theta_3\sin\theta_1 + s2\sin\theta_1 \\ \cos\theta_1\sin\theta_3 + \cos\theta_3\sin\theta_1 & 0 & \sin\theta_1\sin\theta_3 - \cos\theta_1\cos\theta_3 & 3\sin\theta_1 - 3\cos\theta_1\cos\theta_3 + 3\sin\theta_1\sin\theta_3 - s2\cos\theta_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (22)$$

$$T_w = \begin{bmatrix} cos(theta1 + theta3) & 0 & sin(theta1 + theta3) & 3 * sin(theta1 + theta3) + 3 * cos(theta1) + s2 * sin(theta1) \\ sin(theta1 + theta3) & 0 & -cos(theta1 + theta3) & 3 * sin(theta1) - 3 * cos(theta1 + theta3) - s2 * cos(theta1) \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (23)$$