ME572: Homework #5

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Problem 1

See attached page for drawings on XYZ coordinate frames on each link.

Θ_1 Rotation about Z_0 from X_0 to X_1. θ_1 on the manipulator.

d_1 Distance from O_0 to the intersection of Z_0 and Z_1 (O_1) along Z_0. Defined as L1.

a_1 Distance along X_1 to get to O_1. 0, there is no common normal because Z_0 and Z_1 intersect.

α_1 Rotation about X_1 to align Z_0 to Z_1. Immediately appears to be -90^o, but since X_1 is defined as into the page it is 90^o.

Θ_2 Rotation about Z_1 from X_1 to X_2 (ccw). θ_2 on the manipulator.

d_2 Distance from O_1 to the intersection of Z_1 and Z_2 (O_2) along Z_1.

a_2 Distance along X_2 to get to O_2. 0, there is no common normal because Z_1 and Z_2 intersect.

α_2 Rotation about X_2 to align Z_1 to Z_2. Immediately appears to be 90^o, but since X_2 is defined as into the page it is -90^o.

Θ_3 Rotation about Z_2 from X_2 to X_3 (ccw). -90^o to align X_2 to X_3 (n).

d_3 Distance from O_2 to the intersection of Z_2 and Z_3 (O_3). S_3 since this is the prismatic joint.

a_3 Distance along X_3 to get to O_3. 0, there is no common normal because Z_2 and Z_3 lie on top of each other.

α_3 Rotation about X_3 to align Z_2 to Z_3. 0. Z_2 and Z_3 point in the same direction.

All together the parameters for each of the the different joints are as follows:

	Θ_i	d_i	a_i	α_i
1	θ_1	L_1	0	90^o
2	θ_2	d_2	0	-90^o
3	-90^o	S_3	0	0

Substituting these into the DH matrix given in the notes the individual transformation matrices can be generated.

$$A_1 = \begin{bmatrix} \cos(\theta_1) & 0 & \sin(\theta_1) & 0 \\ \sin(\theta_1) & 0 & -\cos(\theta_1) & 0 \\ 0 & 1 & 0 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}$$

$$A_2 = \begin{bmatrix} \cos(\theta_2) & 0 & -\sin(\theta_2) & 0 \\ \sin(\theta_2) & 0 & \cos(\theta_2) & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}$$

$$A_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & S_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3}$$

$$T = A_1 A_2 A_3 \tag{4}$$

$$= \begin{bmatrix} \sin(\theta_1) & \cos(\theta_1)\cdot\cos(\theta_2) & -\cos(\theta_1)\cdot\sin(\theta_2) & d_2\cdot\sin(\theta_1) - S_3\cdot\cos(\theta_1)\cdot\sin(\theta_2) \\ -\cos(\theta_1) & \cos(\theta_2)\cdot\sin(\theta_1) & -\sin(\theta_1)\cdot\sin(\theta_2) & -d_2\cdot\cos(\theta_1) - S_3\cdot\sin(\theta_1)\cdot\sin(\theta_2) \\ 0 & \sin(\theta_2) & \cos(\theta_2) & L_1 + S_3\cdot\cos(\theta_2) \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{5}$$

Problem 1 continued on next page...

This manipulator transform is not the same as the one as found from the other method because the variables are defined slightly different and since the link coordinate axes are not the exact same.

As drawn θ_1 from the DH matrix is actually $\theta_1' + 90^o$ where θ_1' is the angle as defined in the previous homework. This is because for the Θ_1 rotation (as drawn) X_1 actually points into the page. So if the manipulator is shown with all angles in the zero position, the DH θ_1 is actually at 90^0.

This can be verified by substituting in the below variables in. The correct orientation should be an identity matrix (identical to X_0, Y_0, and Z_0). The position of the end effector should be $P_x = d_2$, $P_y = 0$, $P_z = L_1 + S_3$, which is what is shown below

$$\theta_1 = 0 \tag{6}$$

$$L_1 = 5 \tag{7}$$

$$\theta_2 = 0 \tag{8}$$

$$d_2 = 4 \tag{9}$$

$$S_3 = 3 \tag{10}$$

$$A_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{11}$$

$$A_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{12}$$

$$A_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{13}$$

$$T = \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 8 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{14}$$