

## **ME572: Phase #2**

Due on April 20, 2012

RR robot from homework #7. The analysis is the same as for HW 7. The peak joint velocity is in joint 1 at position 12.

Symbolic jacobian matrix for RR.

$$\begin{aligned}J(1,1) &= [-20 \cdot \sin(\theta_1 + \theta_2) - 20 \cdot \sin(\theta_1)] \\J(1,2) &= [-20 \cdot \sin(\theta_1 + \theta_2)] \\J(2,1) &= [20 \cdot \cos(\theta_1 + \theta_2) + 20 \cdot \cos(\theta_1)] \\J(2,2) &= [20 \cdot \cos(\theta_1 + \theta_2)] \\J(3,1) &= [0] \\J(3,2) &= [0] \\J(4,1) &= [0] \\J(4,2) &= [0] \\J(5,1) &= [0] \\J(5,2) &= [0] \\J(6,1) &= [1] \\J(6,2) &= [1]\end{aligned}$$

Joint position and velocities for trajectory1.dat.

```
21 % Number of positions
% t(s)   Theta1   Theta2   Omega1   Omega2
0.000    81.469   197.254   0.000    0.000
0.025    81.672   196.852   0.284   -0.562
0.050    82.282   195.646   0.568   -1.122
0.075    83.097   194.041   0.569   -1.120
0.100    83.913   192.439   0.571   -1.118
0.125    84.732   190.839   0.574   -1.116
0.150    85.557   189.241   0.579   -1.115
0.175    86.392   187.645   0.588   -1.114
0.200    87.246   186.051   0.606   -1.113
0.225    88.140   184.458   0.648   -1.112
0.250    89.140   182.865   0.778   -1.111
0.275    90.652   181.274   1.680   -1.111
0.300   264.697   180.320  17.302    1.107
0.325   268.186   181.910  -0.056    1.111
0.350   267.780   183.502  -0.407    1.112
0.375   267.130   185.095  -0.486    1.112
0.400   266.410   186.688  -0.516    1.113
0.425   265.660   188.283  -0.530    1.114
0.450   264.894   189.880  -0.539    1.115
0.475   264.312   191.079  -0.272    0.558
0.500   264.118   191.478   0.000    0.000
```

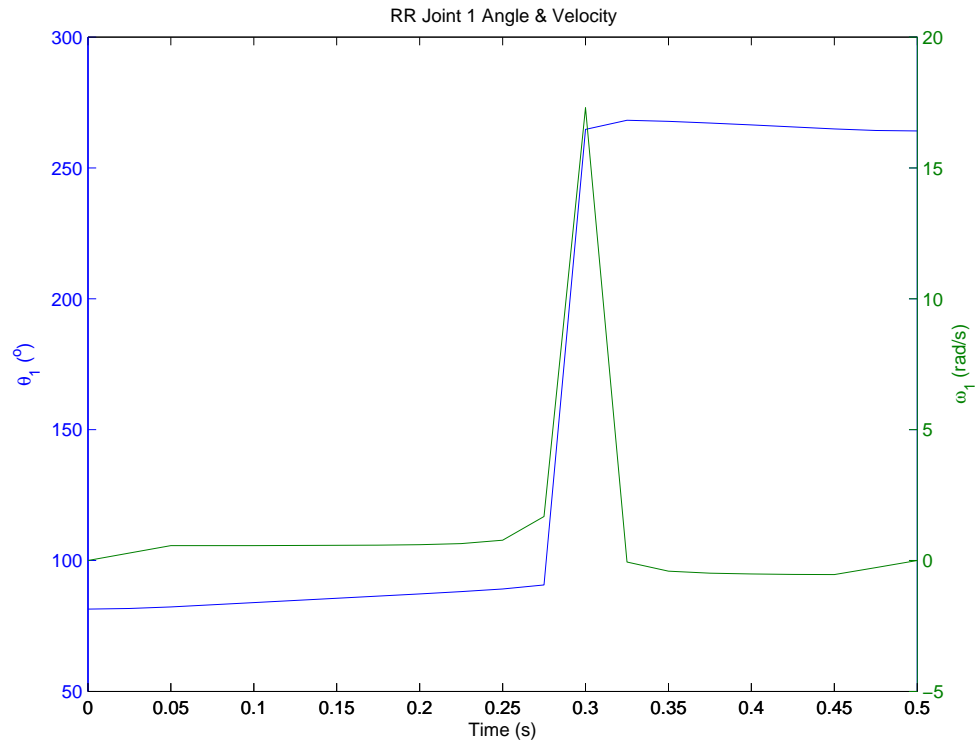


Figure 1: RR Joint 1 Angle and Velocity

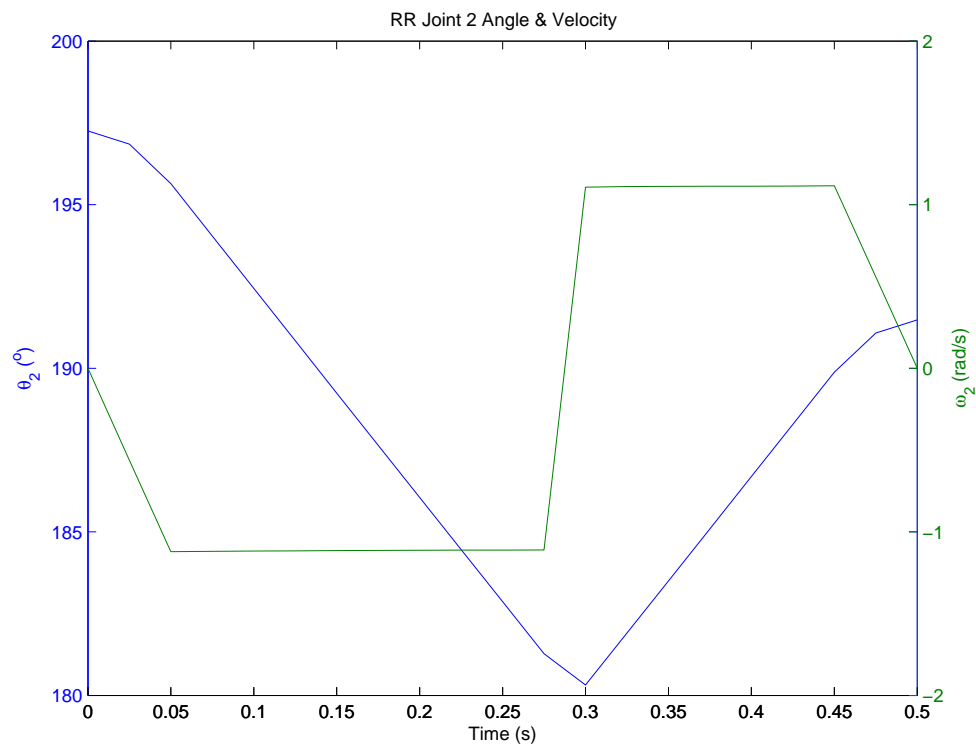


Figure 2: RR Joint 2 Angle and Velocity

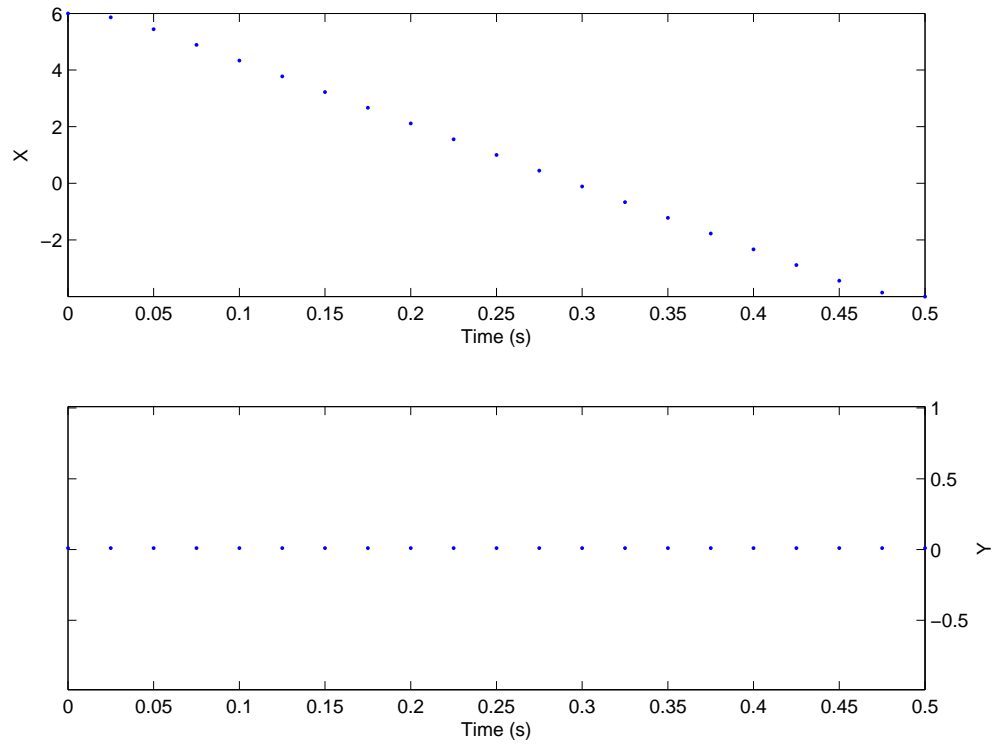


Figure 3: Position of the end effector for each of the coordinates vs time

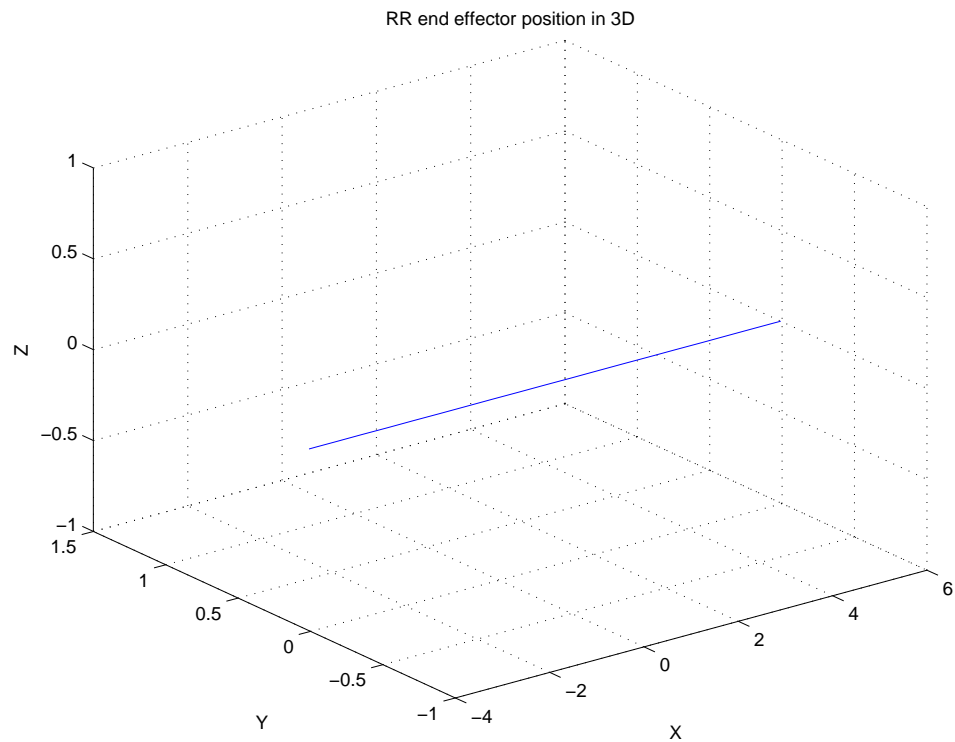


Figure 4: RR end effector position in 3D

World transforms of the robot at positions calculated for coordinate interpolated motion.

POSITION 1: INPUT: JOINT VARIABLES ( 81.469, 197.254,)

|         |        |       |       |       |
|---------|--------|-------|-------|-------|
| OUTPUT: | 0.152  | 0.988 | 0.000 | 6.000 |
|         | -0.988 | 0.152 | 0.000 | 0.010 |
|         | 0.000  | 0.000 | 1.000 | 0.000 |
|         | 0.000  | 0.000 | 0.000 | 1.000 |

POSITION 2: INPUT: JOINT VARIABLES ( 81.672, 196.852,)

|         |        |       |       |       |
|---------|--------|-------|-------|-------|
| OUTPUT: | 0.148  | 0.989 | 0.000 | 5.861 |
|         | -0.989 | 0.148 | 0.000 | 0.010 |
|         | 0.000  | 0.000 | 1.000 | 0.000 |
|         | 0.000  | 0.000 | 0.000 | 1.000 |

POSITION 3: INPUT: JOINT VARIABLES ( 82.282, 195.646,)

|         |        |       |       |       |
|---------|--------|-------|-------|-------|
| OUTPUT: | 0.138  | 0.990 | 0.000 | 5.444 |
|         | -0.990 | 0.138 | 0.000 | 0.010 |
|         | 0.000  | 0.000 | 1.000 | 0.000 |
|         | 0.000  | 0.000 | 0.000 | 1.000 |

POSITION 4: INPUT: JOINT VARIABLES ( 83.097, 194.041,)

|         |        |       |       |       |
|---------|--------|-------|-------|-------|
| OUTPUT: | 0.124  | 0.992 | 0.000 | 4.889 |
|         | -0.992 | 0.124 | 0.000 | 0.010 |
|         | 0.000  | 0.000 | 1.000 | 0.000 |
|         | 0.000  | 0.000 | 0.000 | 1.000 |

POSITION 5: INPUT: JOINT VARIABLES ( 83.913, 192.439,)

|         |        |       |       |       |
|---------|--------|-------|-------|-------|
| OUTPUT: | 0.111  | 0.994 | 0.000 | 4.333 |
|         | -0.994 | 0.111 | 0.000 | 0.010 |
|         | 0.000  | 0.000 | 1.000 | 0.000 |
|         | 0.000  | 0.000 | 0.000 | 1.000 |

POSITION 6: INPUT: JOINT VARIABLES ( 84.732, 190.839,)

|         |        |       |       |       |
|---------|--------|-------|-------|-------|
| OUTPUT: | 0.097  | 0.995 | 0.000 | 3.778 |
|         | -0.995 | 0.097 | 0.000 | 0.010 |
|         | 0.000  | 0.000 | 1.000 | 0.000 |
|         | 0.000  | 0.000 | 0.000 | 1.000 |

POSITION 7: INPUT: JOINT VARIABLES ( 85.557, 189.241,)

|         |        |       |       |       |
|---------|--------|-------|-------|-------|
| OUTPUT: | 0.084  | 0.996 | 0.000 | 3.222 |
|         | -0.996 | 0.084 | 0.000 | 0.010 |
|         | 0.000  | 0.000 | 1.000 | 0.000 |
|         | 0.000  | 0.000 | 0.000 | 1.000 |

POSITION 8: INPUT: JOINT VARIABLES ( 86.392, 187.645,)

|         |        |       |       |       |
|---------|--------|-------|-------|-------|
| OUTPUT: | 0.070  | 0.998 | 0.000 | 2.667 |
|         | -0.998 | 0.070 | 0.000 | 0.010 |
|         | 0.000  | 0.000 | 1.000 | 0.000 |
|         | 0.000  | 0.000 | 0.000 | 1.000 |

POSITION 9: INPUT: JOINT VARIABLES ( 87.246, 186.051,)

|         |        |       |       |       |
|---------|--------|-------|-------|-------|
| OUTPUT: | 0.058  | 0.998 | 0.000 | 2.111 |
|         | -0.998 | 0.058 | 0.000 | 0.010 |
|         | 0.000  | 0.000 | 1.000 | 0.000 |
|         | 0.000  | 0.000 | 0.000 | 1.000 |

POSITION 10: INPUT: JOINT VARIABLES ( 88.140, 184.458,)

|         |        |       |       |       |
|---------|--------|-------|-------|-------|
| OUTPUT: | 0.045  | 0.999 | 0.000 | 1.556 |
|         | -0.999 | 0.045 | 0.000 | 0.010 |
|         | 0.000  | 0.000 | 1.000 | 0.000 |
|         | 0.000  | 0.000 | 0.000 | 1.000 |

POSITION 11: INPUT: JOINT VARIABLES ( 89.140, 182.865,)

|         |        |       |       |       |
|---------|--------|-------|-------|-------|
| OUTPUT: | 0.035  | 0.999 | 0.000 | 1.000 |
|         | -0.999 | 0.035 | 0.000 | 0.010 |
|         | 0.000  | 0.000 | 1.000 | 0.000 |
|         | 0.000  | 0.000 | 0.000 | 1.000 |

POSITION 12: INPUT: JOINT VARIABLES ( 90.652, 181.274,)

|         |        |       |       |       |
|---------|--------|-------|-------|-------|
| OUTPUT: | 0.034  | 0.999 | 0.000 | 0.444 |
|         | -0.999 | 0.034 | 0.000 | 0.010 |
|         | 0.000  | 0.000 | 1.000 | 0.000 |
|         | 0.000  | 0.000 | 0.000 | 1.000 |

POSITION 13: INPUT: JOINT VARIABLES ( 264.697, 180.320,)

|         |       |        |       |        |
|---------|-------|--------|-------|--------|
| OUTPUT: | 0.087 | -0.996 | 0.000 | -0.111 |
|         | 0.996 | 0.087  | 0.000 | 0.010  |
|         | 0.000 | 0.000  | 1.000 | 0.000  |
|         | 0.000 | 0.000  | 0.000 | 1.000  |

POSITION 14: INPUT: JOINT VARIABLES ( 268.186, 181.910,)

|         |        |        |       |        |
|---------|--------|--------|-------|--------|
| OUTPUT: | -0.002 | -1.000 | 0.000 | -0.667 |
|         | 1.000  | -0.002 | 0.000 | 0.010  |
|         | 0.000  | 0.000  | 1.000 | 0.000  |
|         | 0.000  | 0.000  | 0.000 | 1.000  |

POSITION 15: INPUT: JOINT VARIABLES ( 267.780, 183.502,)

|         |        |        |       |        |
|---------|--------|--------|-------|--------|
| OUTPUT: | -0.022 | -1.000 | 0.000 | -1.222 |
|         | 1.000  | -0.022 | 0.000 | 0.010  |
|         | 0.000  | 0.000  | 1.000 | 0.000  |
|         | 0.000  | 0.000  | 0.000 | 1.000  |

POSITION 16: INPUT: JOINT VARIABLES ( 267.130, 185.095,)

|         |        |        |       |        |
|---------|--------|--------|-------|--------|
| OUTPUT: | -0.039 | -0.999 | 0.000 | -1.778 |
|         | 0.999  | -0.039 | 0.000 | 0.010  |
|         | 0.000  | 0.000  | 1.000 | 0.000  |
|         | 0.000  | 0.000  | 0.000 | 1.000  |

POSITION 17: INPUT: JOINT VARIABLES ( 266.410, 186.688,)

|         |        |        |       |        |
|---------|--------|--------|-------|--------|
| OUTPUT: | -0.054 | -0.999 | 0.000 | -2.333 |
|         | 0.999  | -0.054 | 0.000 | 0.010  |
|         | 0.000  | 0.000  | 1.000 | 0.000  |
|         | 0.000  | 0.000  | 0.000 | 1.000  |

POSITION 18: INPUT: JOINT VARIABLES ( 265.660, 188.283,)

|         |        |        |       |        |
|---------|--------|--------|-------|--------|
| OUTPUT: | -0.069 | -0.998 | 0.000 | -2.889 |
|         | 0.998  | -0.069 | 0.000 | 0.010  |
|         | 0.000  | 0.000  | 1.000 | 0.000  |
|         | 0.000  | 0.000  | 0.000 | 1.000  |

POSITION 19: INPUT: JOINT VARIABLES ( 264.894, 189.880,)

|         |        |        |       |        |
|---------|--------|--------|-------|--------|
| OUTPUT: | -0.083 | -0.997 | 0.000 | -3.444 |
|         | 0.997  | -0.083 | 0.000 | 0.010  |
|         | 0.000  | 0.000  | 1.000 | 0.000  |
|         | 0.000  | 0.000  | 0.000 | 1.000  |

POSITION 20: INPUT: JOINT VARIABLES ( 264.312, 191.079,)

|         |        |        |       |        |
|---------|--------|--------|-------|--------|
| OUTPUT: | -0.094 | -0.996 | 0.000 | -3.861 |
|         | 0.996  | -0.094 | 0.000 | 0.010  |
|         | 0.000  | 0.000  | 1.000 | 0.000  |
|         | 0.000  | 0.000  | 0.000 | 1.000  |

POSITION 21: INPUT: JOINT VARIABLES ( 264.118, 191.478,)

|         |        |        |       |        |
|---------|--------|--------|-------|--------|
| OUTPUT: | -0.098 | -0.995 | 0.000 | -4.000 |
|         | 0.995  | -0.098 | 0.000 | 0.010  |
|         | 0.000  | 0.000  | 1.000 | 0.000  |
|         | 0.000  | 0.000  | 0.000 | 1.000  |

Manipulability of the robot.

| % | t(s)  | ax    | by     |
|---|-------|-------|--------|
|   | 0.000 | 5.926 | 20.022 |
|   | 0.025 | 5.792 | 20.020 |
|   | 0.050 | 5.390 | 20.015 |
|   | 0.075 | 4.850 | 20.009 |
|   | 0.100 | 4.307 | 20.006 |
|   | 0.125 | 3.760 | 20.003 |
|   | 0.150 | 3.211 | 20.002 |
|   | 0.175 | 2.661 | 20.001 |
|   | 0.200 | 2.108 | 20.000 |
|   | 0.225 | 1.554 | 20.000 |
|   | 0.250 | 1.000 | 20.000 |
|   | 0.275 | 0.445 | 20.000 |
|   | 0.300 | 0.112 | 20.000 |
|   | 0.325 | 0.667 | 20.000 |
|   | 0.350 | 1.222 | 20.000 |

|       |       |        |
|-------|-------|--------|
| 0.375 | 1.776 | 20.000 |
| 0.400 | 2.329 | 20.000 |
| 0.425 | 2.881 | 20.001 |
| 0.450 | 3.431 | 20.002 |
| 0.475 | 3.842 | 20.004 |
| 0.500 | 3.979 | 20.004 |

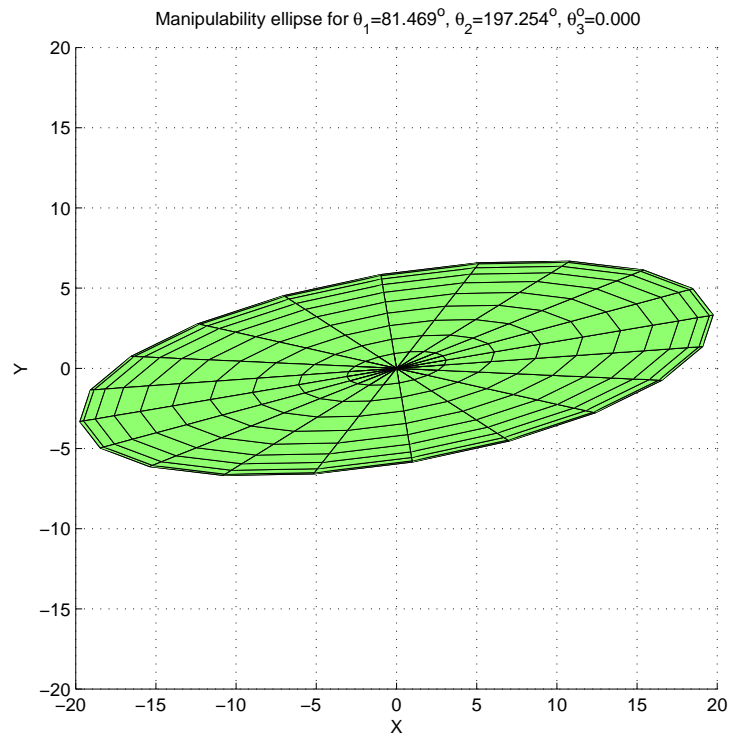


Figure 5: Manipulability of RR at position 0.



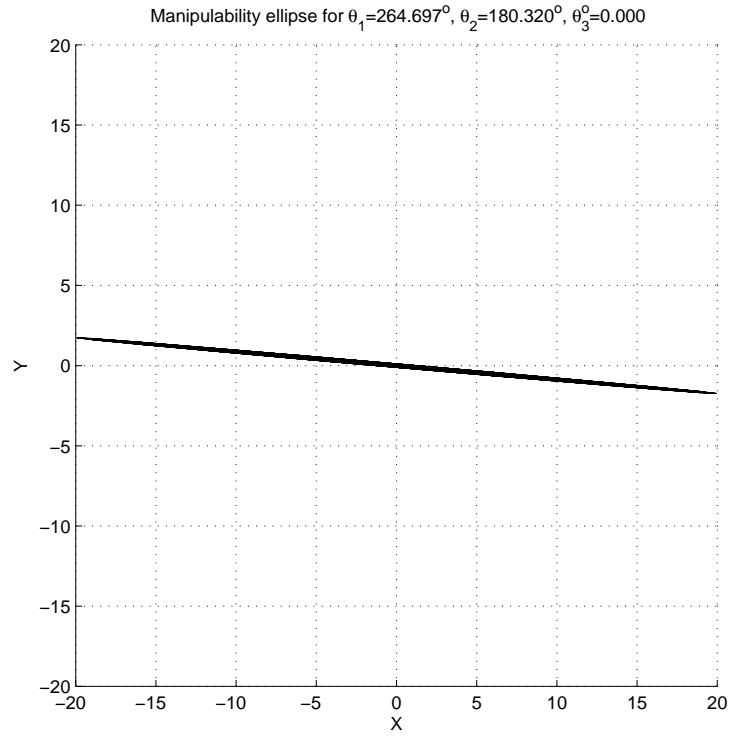


Figure 6: Manipulability of RR at position 12.

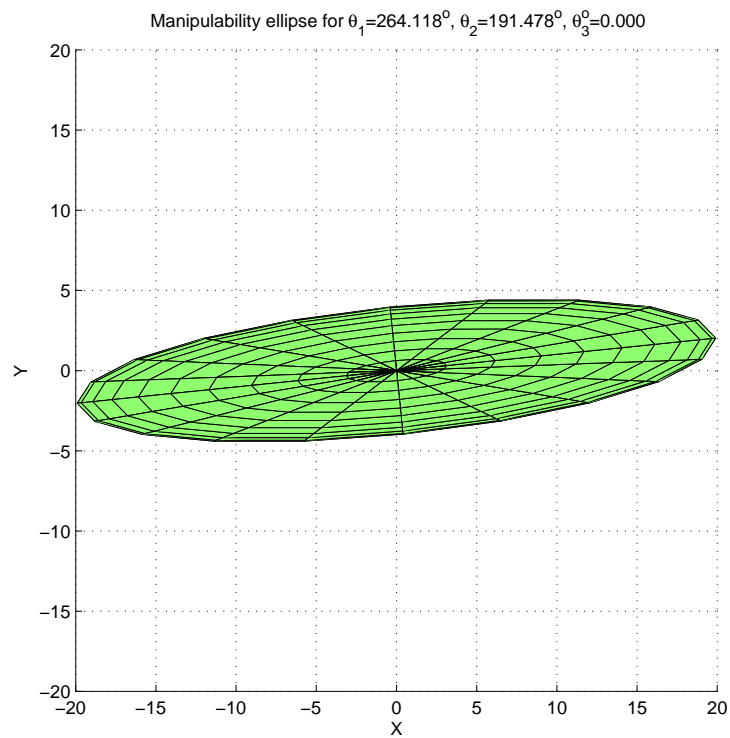


Figure 7: Manipulability of RR at position 20.

RRR robot from homework #4. The peak joint velocity is in joint 2 at position 10.

Symbolic jacobian matrix for RRR.

$$\begin{aligned}
 J(1,1) &= [5 \cdot \cos(\theta_1) \cdot \sin(\theta_2) - 4 \cdot \sin(\theta_1) + 4 \cdot \cos(\theta_1) \cdot \cos(\theta_2) \cdot \sin(\theta_3) + 4 \cdot \cos(\theta_1) \cdot \cos(\theta_3) \cdot \sin(\theta_2)] \\
 J(1,2) &= [\sin(\theta_1) \cdot (4 \cdot \cos(\theta_2 + \theta_3) + 5 \cdot \cos(\theta_2))] \\
 J(1,3) &= [4 \cdot \cos(\theta_2 + \theta_3) \cdot \sin(\theta_1)] \\
 J(2,1) &= [4 \cdot \cos(\theta_1) + 5 \cdot \sin(\theta_1) \cdot \sin(\theta_2) + 4 \cdot \cos(\theta_2) \cdot \sin(\theta_1) \cdot \sin(\theta_3) + 4 \cdot \cos(\theta_3) \cdot \sin(\theta_1) \cdot \sin(\theta_2)] \\
 J(2,2) &= [-\cos(\theta_1) \cdot (4 \cdot \cos(\theta_2 + \theta_3) + 5 \cdot \cos(\theta_2))] \\
 J(2,3) &= [-4 \cdot \cos(\theta_2 + \theta_3) \cdot \cos(\theta_1)] \\
 J(3,1) &= [0] \\
 J(3,2) &= [-4 \cdot \sin(\theta_2 + \theta_3) - 5 \cdot \sin(\theta_2)] \\
 J(3,3) &= [-4 \cdot \sin(\theta_2 + \theta_3)] \\
 J(4,1) &= [0] \\
 J(4,2) &= [\cos(\theta_1)] \\
 J(4,3) &= [\cos(\theta_1)] \\
 J(5,1) &= [0] \\
 J(5,2) &= [\sin(\theta_1)] \\
 J(5,3) &= [\sin(\theta_1)] \\
 J(6,1) &= [1] \\
 J(6,2) &= [0] \\
 J(6,3) &= [0]
 \end{aligned}$$

Joint position and velocities for trajectory2.dat.

| 17 % Number of positions |       |         |         |         |        |         |         |
|--------------------------|-------|---------|---------|---------|--------|---------|---------|
| %                        | t(s)  | Theta1  | Theta2  | Theta3  | Omega1 | Omega2  | Omega3  |
|                          | 0.000 | 0.000   | 326.976 | 50.858  | 0.000  | 0.000   | 0.000   |
|                          | 0.015 | 359.861 | 324.008 | 57.465  | -0.331 | -6.535  | 14.560  |
|                          | 0.030 | 359.406 | 316.558 | 74.095  | -0.751 | -10.385 | 23.232  |
|                          | 0.045 | 358.701 | 308.475 | 92.237  | -0.895 | -8.619  | 19.415  |
|                          | 0.060 | 357.859 | 301.530 | 107.937 | -1.070 | -7.626  | 17.298  |
|                          | 0.075 | 356.851 | 295.261 | 122.191 | -1.282 | -7.008  | 15.969  |
|                          | 0.090 | 355.645 | 289.414 | 135.512 | -1.531 | -6.634  | 15.086  |
|                          | 0.105 | 354.209 | 283.791 | 148.200 | -1.814 | -6.498  | 14.477  |
|                          | 0.120 | 352.519 | 278.115 | 160.433 | -2.122 | -6.853  | 13.991  |
|                          | 0.135 | 350.560 | 271.129 | 172.081 | -2.436 | -11.350 | 12.651  |
|                          | 0.150 | 348.336 | 228.896 | 174.008 | -2.736 | -89.233 | -11.495 |
|                          | 0.165 | 345.866 | 174.785 | 162.643 | -3.006 | -37.545 | -13.887 |
|                          | 0.180 | 343.182 | 154.756 | 150.488 | -3.233 | -13.147 | -14.384 |
|                          | 0.195 | 340.323 | 148.097 | 137.893 | -3.412 | -3.653  | -14.956 |
|                          | 0.210 | 337.331 | 147.121 | 124.708 | -3.544 | 0.919   | -15.779 |
|                          | 0.225 | 335.022 | 148.445 | 114.271 | -1.808 | 1.539   | -8.325  |
|                          | 0.240 | 334.244 | 149.172 | 110.656 | 0.000  | 0.000   | 0.000   |

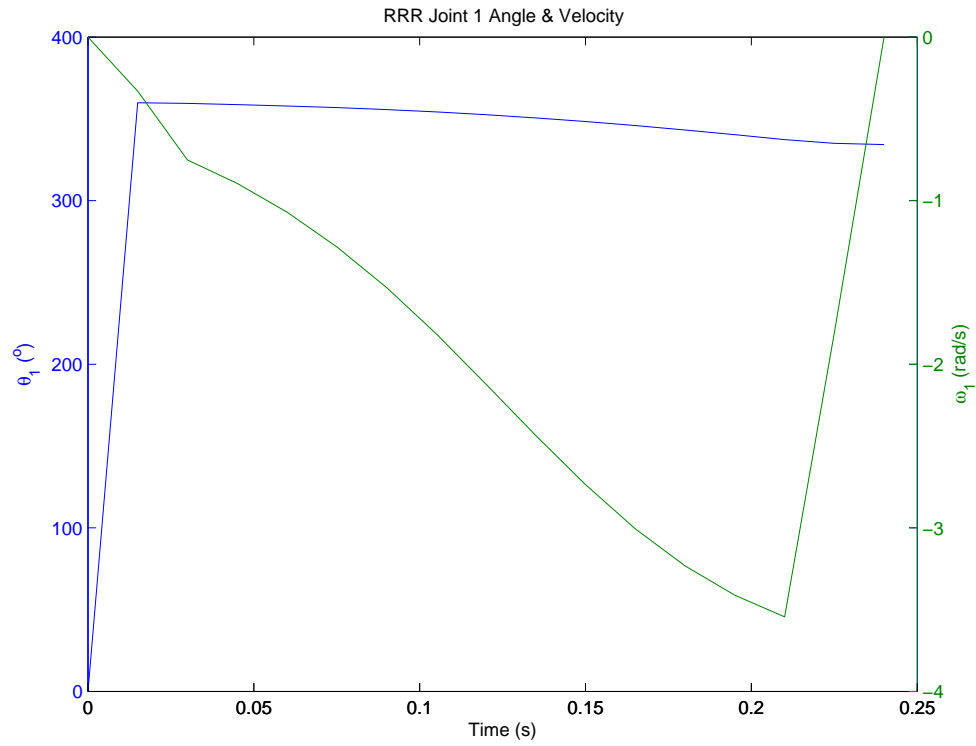


Figure 8: RRR Joint 1 Angle and Velocity

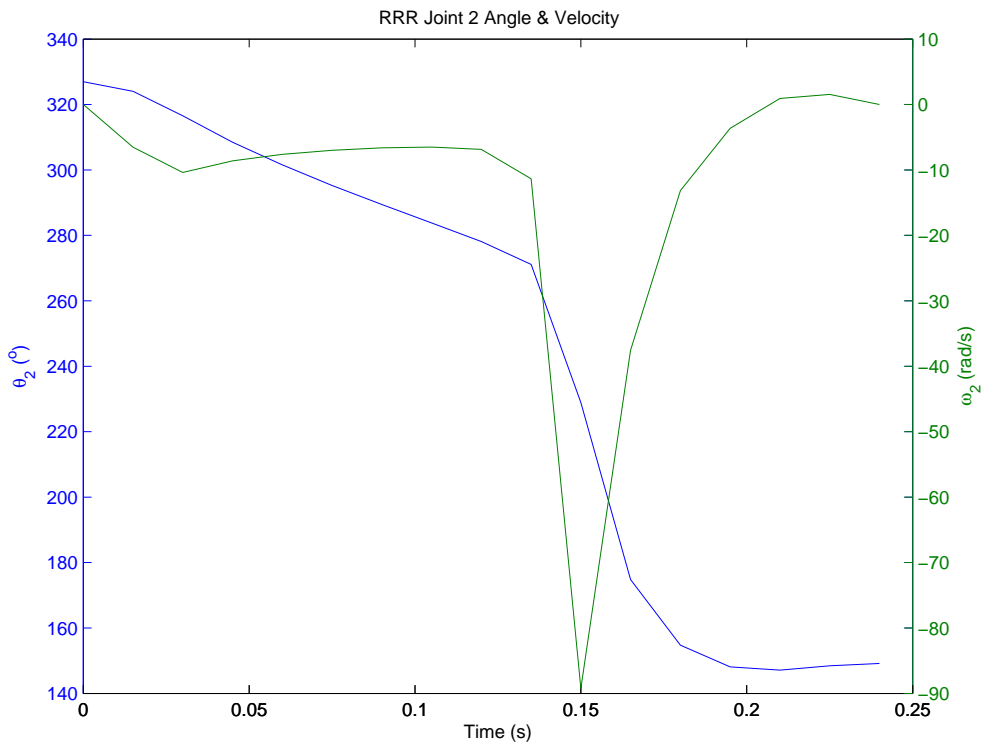


Figure 9: RRR Joint 2 Angle and Velocity

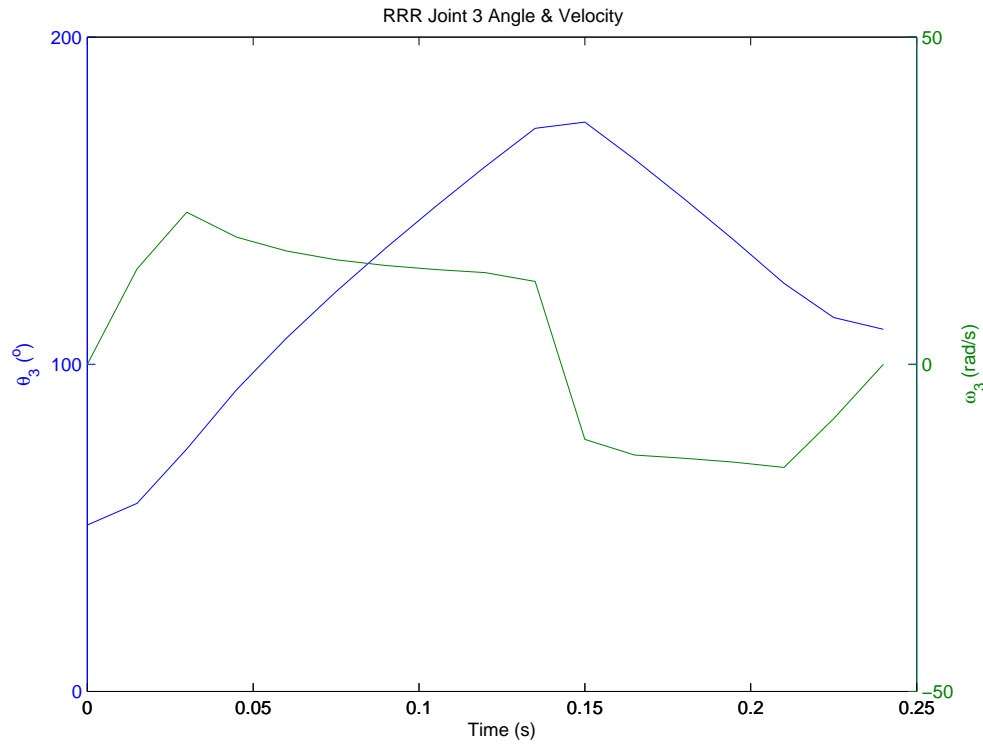


Figure 10: RRR Joint 3 Angle and Velocity

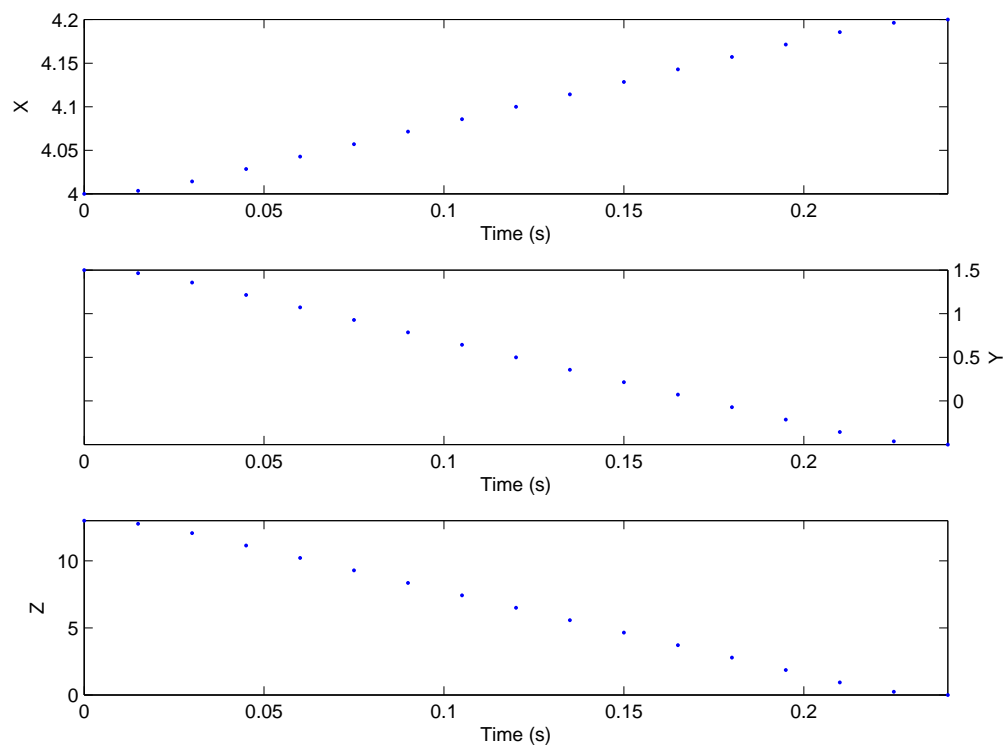


Figure 11: Position of the end effector for each of the coordinates vs time

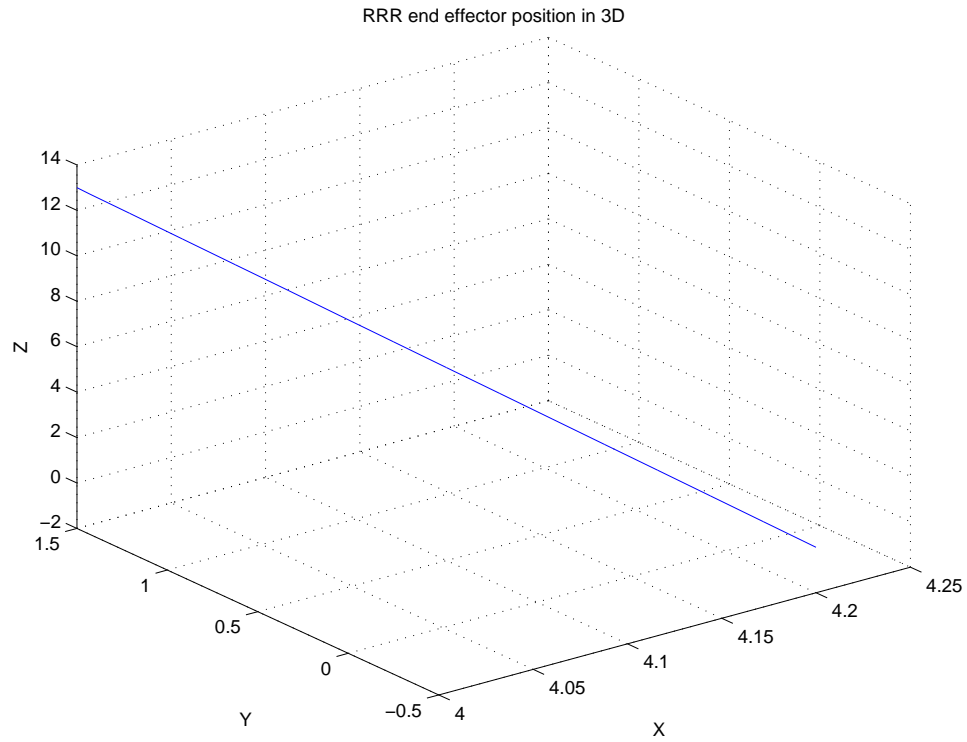


Figure 12: RRR end effector position in 3D

World transforms of the robot at positions calculated for coordinate interpolated motion.

POSITION 1: INPUT: JOINT VARIABLES ( 0.000, 326.976, 50.858,)

OUTPUT:     1.000     0.000     0.000     4.000  
             0.000     0.952     -0.306     1.500  
             0.000     0.306     0.952     13.000  
             0.000     0.000     0.000     1.000

POSITION 2: INPUT: JOINT VARIABLES ( 359.861, 324.008, 57.465,)

OUTPUT:     1.000     0.002     -0.001     4.004  
             -0.002     0.931     -0.366     1.464  
             0.000     0.366     0.931     12.768  
             0.000     0.000     0.000     1.000

POSITION 3: INPUT: JOINT VARIABLES ( 359.406, 316.558, 74.095,)

OUTPUT:     1.000     0.009     -0.005     4.014  
             -0.010     0.860     -0.510     1.357  
             0.000     0.510     0.860     12.071  
             0.000     0.000     0.000     1.000

POSITION 4: INPUT: JOINT VARIABLES ( 358.701, 308.475, 92.237,)

OUTPUT:     1.000     0.017     -0.015     4.029  
             -0.023     0.758     -0.652     1.214  
             0.000     0.652     0.758     11.143  
             0.000     0.000     0.000     1.000

POSITION 5: INPUT: JOINT VARIABLES ( 357.859, 301.530, 107.937,)

OUTPUT:     0.999     0.024     -0.028     4.043  
             -0.037     0.649     -0.759     1.071  
             0.000     0.760     0.650     10.214  
             0.000     0.000     0.000     1.000

POSITION 6: INPUT: JOINT VARIABLES ( 356.851, 295.261, 122.191,)

OUTPUT:     0.998     0.030     -0.046     4.057  
             -0.055     0.537     -0.842     0.929  
             0.000     0.843     0.538     9.286  
             0.000     0.000     0.000     1.000

POSITION 7: INPUT: JOINT VARIABLES ( 355.645, 289.414, 135.512,)

OUTPUT:     0.997     0.032     -0.069     4.071  
             -0.076     0.423     -0.903     0.786  
             0.000     0.906     0.424     8.357  
             0.000     0.000     0.000     1.000

POSITION 8: INPUT: JOINT VARIABLES ( 354.209, 283.791, 148.200,)

OUTPUT:     0.995     0.031     -0.096     4.086  
             -0.101     0.308     -0.946     0.643  
             0.000     0.951     0.309     7.429  
             0.000     0.000     0.000     1.000

POSITION 9: INPUT: JOINT VARIABLES ( 352.519, 278.115, 160.433,)

|         |        |       |        |       |
|---------|--------|-------|--------|-------|
| OUTPUT: | 0.991  | 0.026 | -0.128 | 4.100 |
|         | -0.130 | 0.197 | -0.972 | 0.500 |
|         | 0.000  | 0.980 | 0.199  | 6.500 |
|         | 0.000  | 0.000 | 0.000  | 1.000 |

POSITION 10: INPUT: JOINT VARIABLES ( 350.560, 271.129, 172.081,)

|         |        |       |        |       |
|---------|--------|-------|--------|-------|
| OUTPUT: | 0.986  | 0.019 | -0.163 | 4.114 |
|         | -0.164 | 0.117 | -0.980 | 0.357 |
|         | 0.000  | 0.993 | 0.118  | 5.571 |
|         | 0.000  | 0.000 | 0.000  | 1.000 |

POSITION 11: INPUT: JOINT VARIABLES ( 348.336, 228.896, 174.008,)

|         |        |       |        |       |
|---------|--------|-------|--------|-------|
| OUTPUT: | 0.979  | 0.148 | -0.138 | 4.129 |
|         | -0.202 | 0.717 | -0.667 | 0.214 |
|         | 0.000  | 0.681 | 0.733  | 4.643 |
|         | 0.000  | 0.000 | 0.000  | 1.000 |

POSITION 12: INPUT: JOINT VARIABLES ( 345.866, 174.785, 162.643,)

|         |        |        |       |       |
|---------|--------|--------|-------|-------|
| OUTPUT: | 0.970  | 0.225  | 0.094 | 4.143 |
|         | -0.244 | 0.895  | 0.372 | 0.071 |
|         | 0.000  | -0.384 | 0.923 | 3.714 |
|         | 0.000  | 0.000  | 0.000 | 1.000 |

POSITION 13: INPUT: JOINT VARIABLES ( 343.182, 154.756, 150.488,)

|         |        |        |       |        |
|---------|--------|--------|-------|--------|
| OUTPUT: | 0.957  | 0.167  | 0.236 | 4.157  |
|         | -0.289 | 0.552  | 0.782 | -0.071 |
|         | 0.000  | -0.817 | 0.577 | 2.786  |
|         | 0.000  | 0.000  | 0.000 | 1.000  |

POSITION 14: INPUT: JOINT VARIABLES ( 340.323, 148.097, 137.893,)

|         |        |        |       |        |
|---------|--------|--------|-------|--------|
| OUTPUT: | 0.942  | 0.093  | 0.324 | 4.171  |
|         | -0.337 | 0.259  | 0.905 | -0.214 |
|         | 0.000  | -0.961 | 0.275 | 1.857  |
|         | 0.000  | 0.000  | 0.000 | 1.000  |

POSITION 15: INPUT: JOINT VARIABLES ( 337.331, 147.121, 124.708,)

|         |        |        |       |        |
|---------|--------|--------|-------|--------|
| OUTPUT: | 0.923  | 0.012  | 0.385 | 4.186  |
|         | -0.385 | 0.029  | 0.922 | -0.357 |
|         | 0.000  | -0.999 | 0.032 | 0.929  |
|         | 0.000  | 0.000  | 0.000 | 1.000  |

POSITION 16: INPUT: JOINT VARIABLES ( 335.022, 148.445, 114.271,)

|         |        |        |        |        |
|---------|--------|--------|--------|--------|
| OUTPUT: | 0.906  | -0.054 | 0.419  | 4.196  |
|         | -0.422 | -0.115 | 0.899  | -0.464 |
|         | 0.000  | -0.992 | -0.127 | 0.232  |
|         | 0.000  | 0.000  | 0.000  | 1.000  |

POSITION 17: INPUT: JOINT VARIABLES ( 334.244, 149.172, 110.656,)

OUTPUT:      0.901      -0.077      0.428      4.200  
              -0.435      -0.159      0.886      -0.500  
              0.000      -0.984      -0.177      0.000  
              0.000      0.000      0.000      1.000

Manipulability of the robot.

| % | t(s)  | ax    | by    | cz    |
|---|-------|-------|-------|-------|
|   | 0.000 | 1.307 | 1.822 | 9.771 |
|   | 0.015 | 1.307 | 1.994 | 9.539 |
|   | 0.030 | 1.239 | 2.452 | 8.853 |
|   | 0.045 | 1.123 | 2.917 | 7.961 |
|   | 0.060 | 1.001 | 3.268 | 7.106 |
|   | 0.075 | 0.873 | 3.536 | 6.305 |
|   | 0.090 | 0.734 | 3.738 | 5.581 |
|   | 0.105 | 0.574 | 3.885 | 4.968 |
|   | 0.120 | 0.384 | 3.980 | 4.515 |
|   | 0.135 | 0.165 | 3.987 | 4.302 |
|   | 0.150 | 0.173 | 2.345 | 5.365 |
|   | 0.165 | 0.648 | 1.745 | 5.703 |
|   | 0.180 | 0.644 | 3.248 | 5.340 |
|   | 0.195 | 0.757 | 4.027 | 5.290 |
|   | 0.210 | 0.885 | 4.049 | 5.890 |
|   | 0.225 | 0.980 | 3.855 | 6.524 |
|   | 0.240 | 1.010 | 3.775 | 6.746 |

Manipulability ellipse for  $\theta_1=0.000^\circ$ ,  $\theta_2=326.976^\circ$ ,  $\theta_3=50.858^\circ$

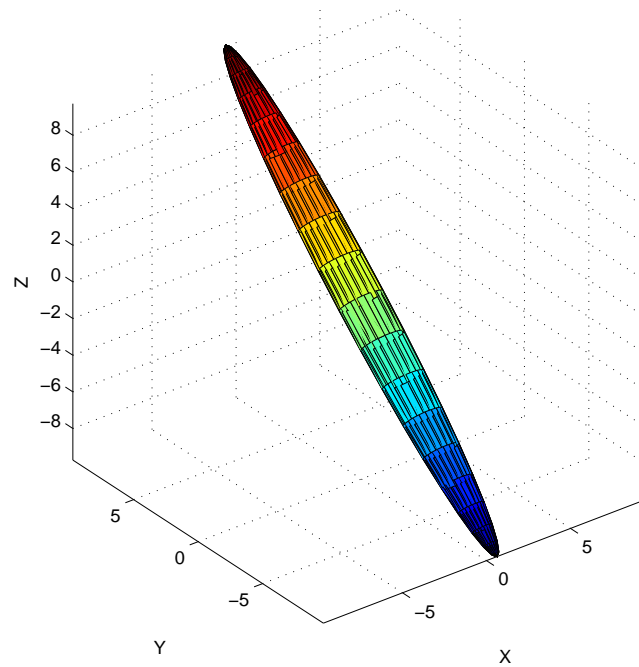


Figure 13: Manipulability of RRR at position 0.



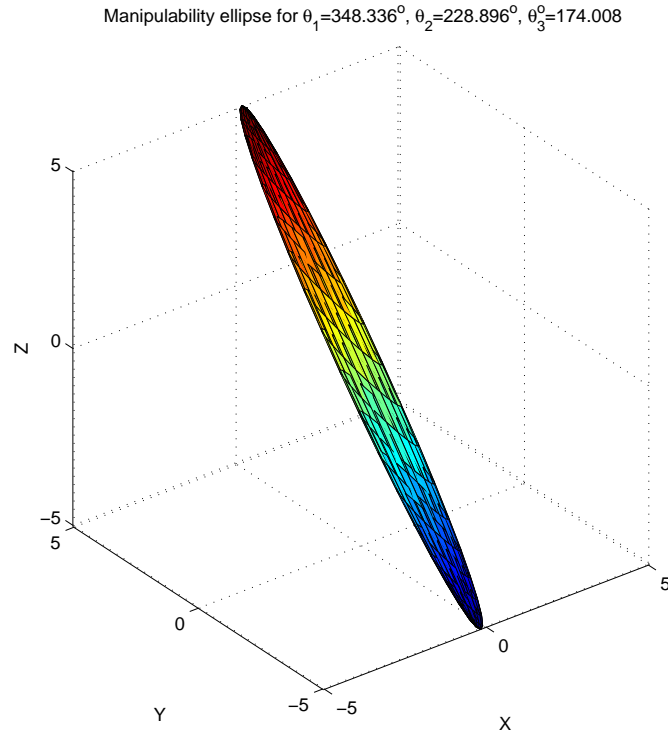


Figure 14: Manipulability of RRR at position 10.

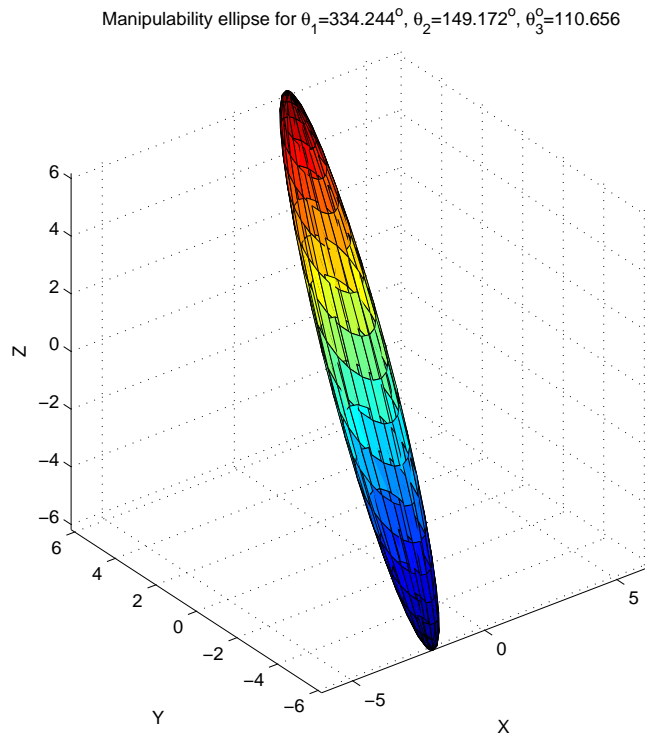


Figure 15: Manipulability of RRR at position 16.

Changing the time step to 0.0001 s gives a finer step of the X, Y and Z positions. It also shows where the actual peak angular joint rate is as it approaches the singularity.

RR robot for  $\Delta T=0.0001$ s.

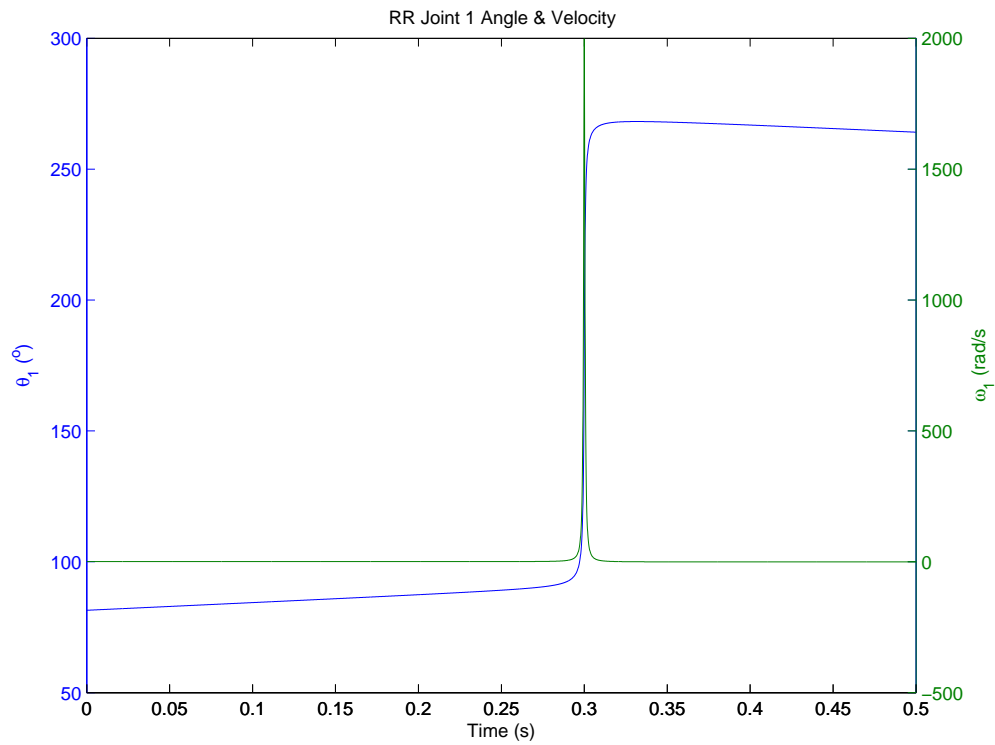


Figure 16: RR Joint 1 Angle and Velocity

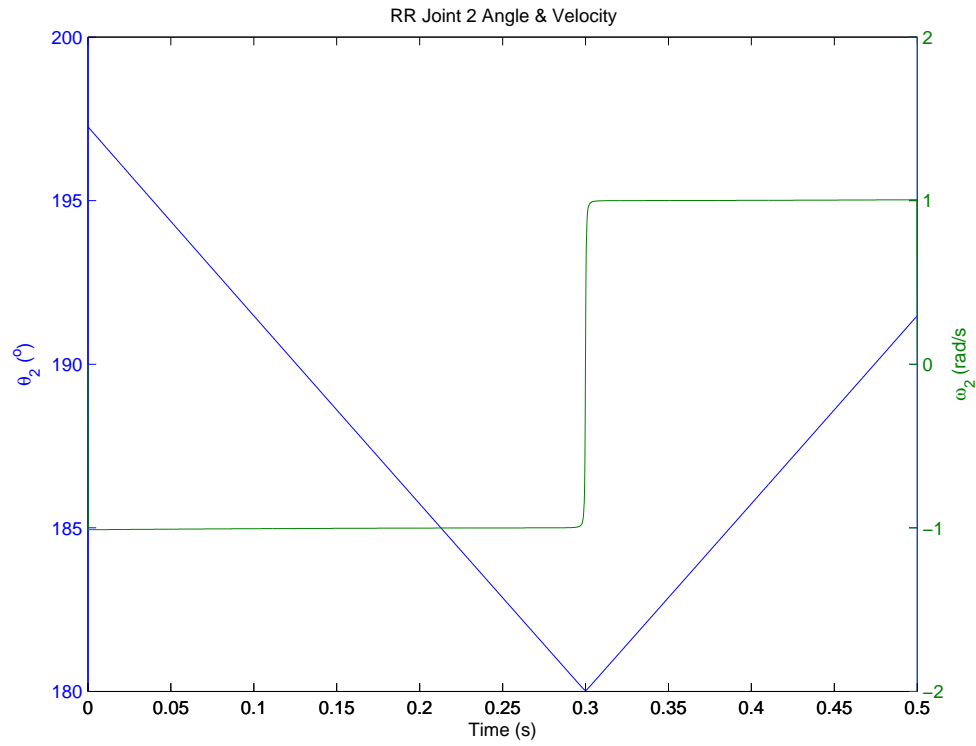


Figure 17: RR Joint 2 Angle and Velocity

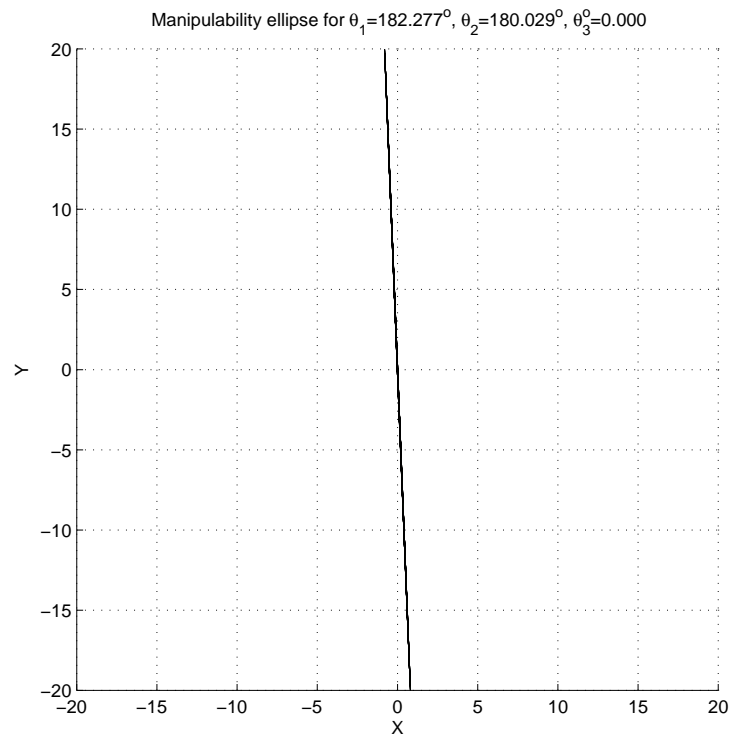


Figure 18: Manipulability of RR at position 3000.

RRR robot for  $dT=0.0001s$ .

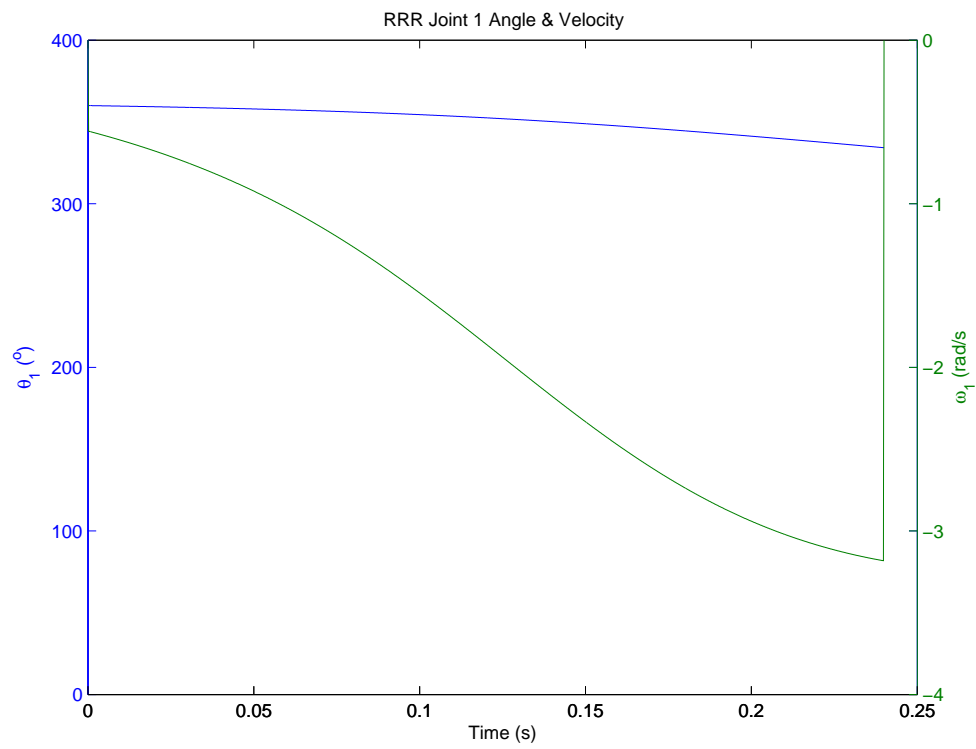


Figure 19: RRR Joint 1 Angle and Velocity

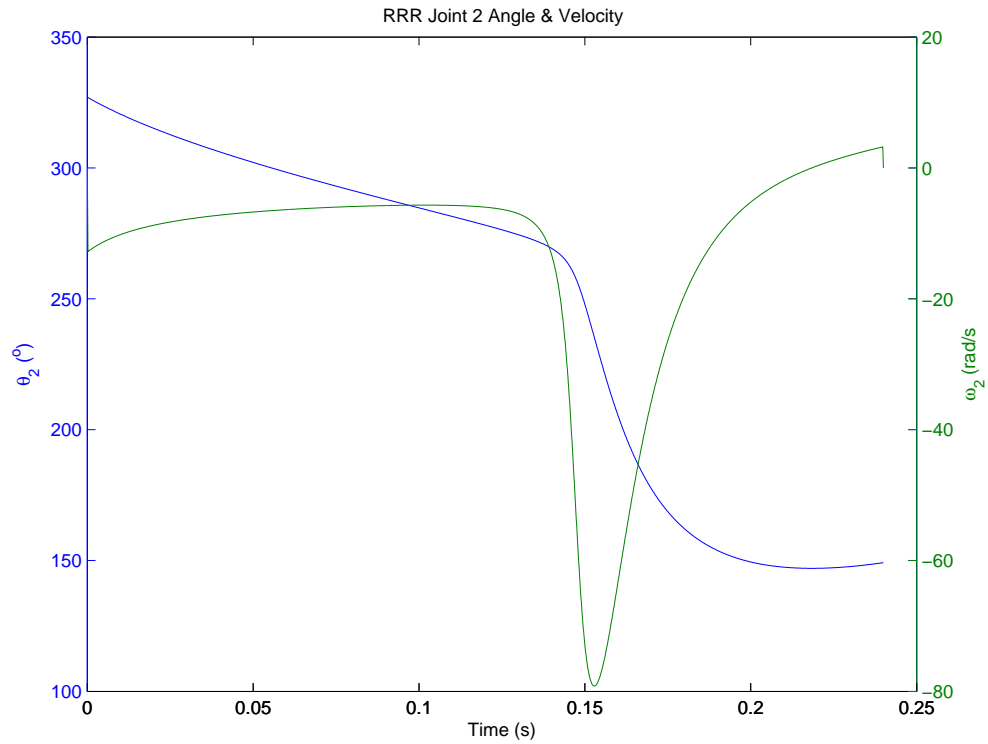


Figure 20: RRR Joint 2 Angle and Velocity

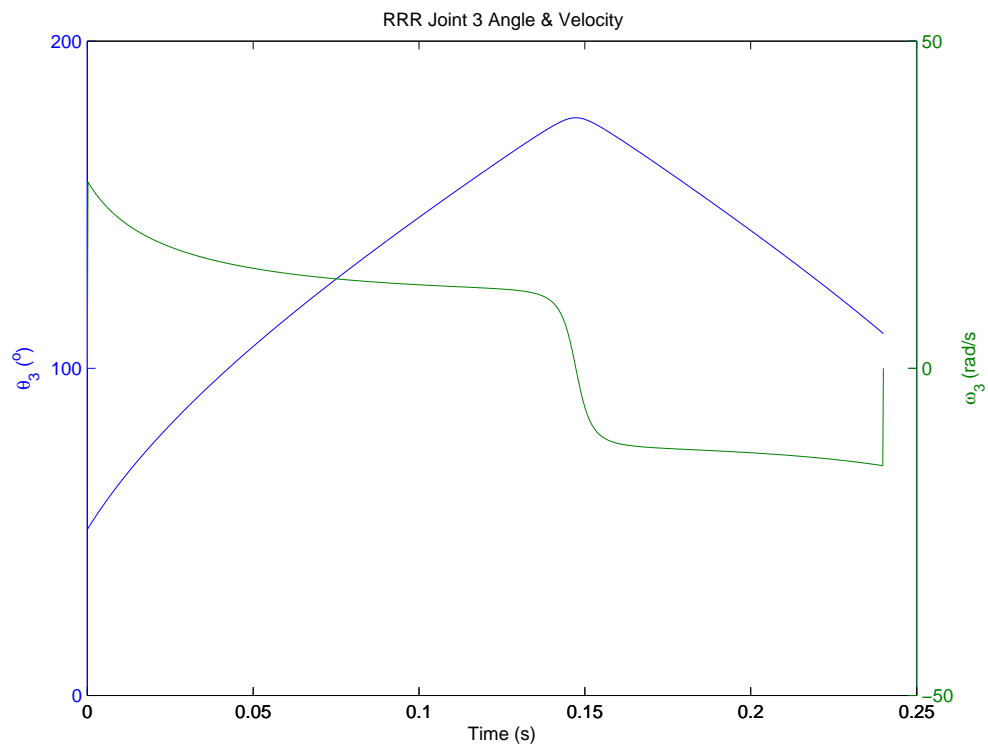


Figure 21: RRR Joint 3 Angle and Velocity

Manipulability ellipse for  $\theta_1=348.535^\circ$ ,  $\theta_2=235.477^\circ$ ,  $\theta_3^0=174.814$

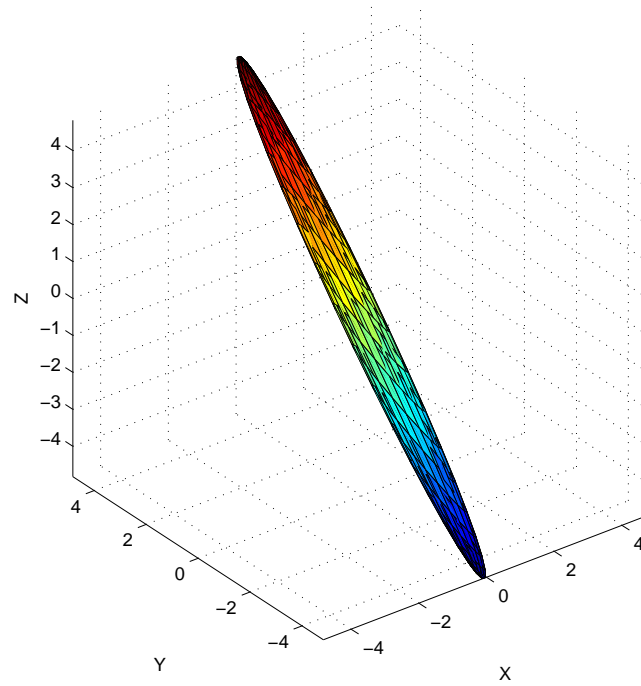


Figure 22: Manipulability of RRR at position 1528.

# Appendices

## M-Code

### phase1.m

This has been slightly edited since last time to work better in the phase2 workflow.

```

1 function varargout=phase1(file)
2 error(nargchk(0, 1, nargin, 'struct'))
3 error(nargchk(0, 4, nargout, 'struct'))
4 if nargin<1||~exist(file,'file')
5     file='trajectory1.robot.dat';
6 else
7     if strcmpi(file,'prompt')
8         [filename, pathname] = uigetfile( ...
9             {'*.dat'; '*.txt'; '*.*'}, ...
10            'Pick a robot datafile');
11         feval(mfilename,fullfile(pathname,filename));
12         return;
13     end
14 end
15 fid=fopen(file,'r');
16 n=fscanf(fid,'%d',1);
17 m=fscanf(fid,'%d',1);
18 robotType=fscanf(fid,'%s',1);
19 Q=fscanf(fid,'%f',[n,m]);
20 fclose(fid);
21 Tm=cell(1,m);
22 for i=1:m
23     Tm.tmp=Tworld(robotType,Q(:,i));
24     Tm{i}=Tm.tmp{end};
25 end
26 if nargout==3
27     for i=1:m
28         P(i,:)=Tm{i}(1:3,4)';
29     end
30     varargout{1}=P(:,1);
31     varargout{2}=P(:,2);
32     varargout{3}=P(:,3);
33 else
34     [p,n]=fileparts(file);
35     phase1Tex=fullfile(p,[n '_tm.tex']);
36     if nargout==1
37         varargout{1}=phase1Tex;
38     end
39     fid=fopen(phase1Tex,'w');
40     fprintf(fid,'\\newpage\nWorld transforms of the robot at positions calculated for ...
41         coordinate interpolated motion.\n');
42     for i=1:m
43         fprintf(fid,'\\begin{verbatim}\n');
44         fprintf(fid,'POSITION %d: INPUT: JOINT VARIABLES (' ,i);
45         fprintf(fid,'% 8.3f',Q(:,i));
46         fprintf(fid,')\nOUTPUT:');
47         fprintf(fid,'% 10.3f',Tm{i}(1,:));
48         fprintf(fid,'\n');
49         fprintf(fid,'% 10.3f',Tm{i}(2,:));
50         fprintf(fid,'\n');
51         fprintf(fid,'%10.3f',Tm{i}(3,:));
52         fprintf(fid,'\n');
53         fprintf(fid,'%10.3f',Tm{i}(4,:));
54         fprintf(fid,'\n');

```



```

54     fprintf(fid, '\\end{verbatim} \\pagebreak[1]');
55     end
56     fclose(fid);
57 end
58 end
59 function m=phiR(theta)
60 theta=theta.*pi./180;
61 m=[cos(theta) -sin(theta) 0 0
62     sin(theta)  cos(theta) 0 0
63     0           0         1 0
64     0           0         0 1];
65 end
66 function m=phiP(X)
67 m=[1 0 0 0
68     0 1 0 0
69     0 0 1 X
70     0 0 0 1];
71 end

```

## phase2.m

Main program for phase 2.

```

1  function phase2(file)
2  close all;
3  error(nargchk(0, 1, nargin, 'struct'))
4  error(nargchk(0, 0, nargout, 'struct'))
5  if nargin<1||~exist(file, 'file')
6      file='trajectory1.dat';
7      feval(mfilename, file);
8      file='trajectory2.dat';
9      feval(mfilename, file);
10 %     file='trajectory1_slow.dat';
11 %     feval(mfilename, file);
12 %     file='trajectory2_slow.dat';
13 %     feval(mfilename, file);
14     return;
15 else
16     if strcmpi(file, 'prompt')
17         [filename, pathname] = uigetfile( ...
18             {'*.dat'; '*.txt'; '*.*'}, ...
19             'Pick a robot datafile');
20         feval(mfilename, fullfile(pathname, filename));
21         return;
22     end
23 end
24 %% Data Input
25 % Open the data file.
26 fid=fopen(file, 'r');
27 % Read in times.
28 Tf=fscanf(fid, '%f', 1);
29 dT=fscanf(fid, '%f', 1);
30 % Read in Initial Position.
31 Px(1)=fscanf(fid, '%f', 1);
32 Py(1)=fscanf(fid, '%f', 1);
33 Pz(1)=fscanf(fid, '%f', 1);
34 % Read in Final Position.
35 Px(2)=fscanf(fid, '%f', 1);
36 Py(2)=fscanf(fid, '%f', 1);

```

```

37 Pz(2)=fscanf(fid,'%f',1);
38 % Open the TexFile that is used to write the outputs to.
39 [p,n]=fileparts(file);
40 texFid=fopen(fullfile(p,[n '.tex']), 'w');
41 %% Calculations
42 t=[0:dT:Tf]';
43 [X,Vx,-]=trajPlanner(Px(1),Px(2),Tf,dT,2,2); % Plan the X Path
44 [Y,Vy,-]=trajPlanner(Py(1),Py(2),Tf,dT,2,2); % Plan the Y Path
45 [Z,Vz,-]=trajPlanner(Pz(1),Pz(2),Tf,dT,2,2); % Plan the Z Path
46 % Determine robot type based on Z. If it's all 0 then it's just an RR.
47 if all(Z==0)
48     robotType='RR';
49 else
50     robotType='RRR';
51 end
52 %%
53 theta=invKin(robotType,[X Y Z]); % Do inverse kinematics.
54 dTheta=zeros(size(theta,1),3); % Preallocate for speed.
55 % Calculate the Jacobian symbolically.
56 if strcmpi(robotType,'RRR')
57     syms theta1 theta2 theta3 real;
58     J=Jcalc(robotType,[theta1 theta2 theta3]);
59 else
60     syms theta1 theta2 real;
61     J=Jcalc(robotType,[theta1 theta2]);
62 end
63 dataOut=fullfile(p,[n '_jacobian.tex']);
64 fid=fopen(dataOut,'w');
65 fprintf(fid,'\\begin{eqnarray}\\n');
66 for i=1:size(J,1)
67     for j=1:size(J,2)
68         if i==size(J,1) && j==size(J,2)
69             fprintf(fid,'J(%d,%d)&=\\s\\n \\nonumber',i,j,mat2latex(simplify(J(i,j))));
70         else
71             fprintf(fid,'J(%d,%d)&=\\s\\n \\nonumber \\\\',i,j,mat2latex(simplify(J(i,j))));
72         end
73     end
74 end
75 fprintf(fid,'\\end{eqnarray}\\n');
76 fprintf(texFid,'Symbolic jacobian matrix for %s.\\n',robotType);
77 fprintf(texFid,'\\input{\\s}\\n',dataOut); % Write it to the main file to include.
78 % Calculate numerically.
79 for i=1:size(theta,1) % For each position.
80     J=Jcalc(robotType,theta(i,:)); % Calculate the Jacobian.
81     switch robotType
82         % Get the top Jacobian and calculate joint rates.
83         case 'RR',
84             JTop=J(1:2,1:2);
85             J_inv=JTop^-1;
86             dTheta(i,1:2)=[J_inv*[Vx(i) Vy(i)]]';
87         case 'RRR',
88             JTop=J(1:3,1:3);
89             J_inv=JTop^-1;
90             dTheta(i,:)=[J_inv*[Vx(i) Vy(i) Vz(i)]]';
91     end
92     % Get the manipulability.
93     [ax(i),by(i),cz(i),Beta(i),R{i}]=manipulability(J);
94 end
95 % dTheta=dTheta.*180/pi; % Just to check answers against HW7
96 %% Position of the Joints
97 dataOut=fullfile(p,[n '_trajectory.tex']);
98 fid=fopen(dataOut,'w');

```

```

99 fprintf(fid, '\\begin{verbatim}\\n');
100 fprintf(fid, '%d %% Number of positions\\n', length(X));
101 if strcmpi(robotType, 'RRR')
102     fprintf(fid, '%s t(s) ...
        %9s%9s%9s%9s%9s\\n', 'Theta1', 'Theta2', 'Theta3', 'Omega1', 'Omega2', 'Omega3');
103     fprintf(fid, '% 8.3f % 8.3f % 8.3f % 8.3f % 8.3f % 8.3f % ...
        8.3f\\n', [t, theta(:,1), theta(:,2), theta(:,3), dTheta(:,1), dTheta(:,2), dTheta(:,3)]');
104 else
105     fprintf(fid, '%s t(s) %9s%9s%9s%9s\\n', 'Theta1', 'Theta2', 'Omega1', 'Omega2');
106     fprintf(fid, '% 8.3f % 8.3f % 8.3f % 8.3f % ...
        8.3f\\n', [t, theta(:,1), theta(:,2), dTheta(:,1), dTheta(:,2)]');
107 end
108 fprintf(fid, '\\end{verbatim}\\n');
109 fclose(fid);
110 fprintf(texFid, 'Joint position and velocities for %s.\\n', [n '.dat']);
111 fprintf(texFid, '\\input{%s}\\n', dataOut); % Write it to the main file to include.
112
113 %% Joint positions & velocities.
114 [n, n] = fileparts(file);
115
116 % Determine the number of joints to print out.
117 if strcmpi(robotType, 'RR')
118     q=2;
119 else
120     q=3;
121 end
122 for i=1:q
123     close all;
124     h=plotyy(t, theta(:,i), t, dTheta(:,i));
125     xlabel('Time (s)');
126     ylabel(h(1), sprintf('\\theta-%d (^o)', i));
127     ylabel(h(2), sprintf('\\omega-%d (rad/s)', i));
128     title(sprintf('%s Joint %d Angle & Velocity', robotType, i));
129     jointFile=sprintf('%s.joint%d', n, i);
130     % print('--depsc2', jointFile);
131     fprintf(texFid, '\\fig{%s}{%s Joint %d Angle and Velocity}\\n', jointFile, robotType, i);
132 end
133
134 %% Plot the EECS
135 close all; % New figure & increment.
136 subplot(q,1,1); % New sub plot with the number of axes
137 plot(t, X, '.'); axis('tight'); % Plot X
138 ylabel('X'); % Label
139 xlabel('Time (s)');
140 subplot(q,1,2);
141 plot(t, Y, '.'); axis('tight'); % Plot Y
142 ylabel('Y'); % Label
143 xlabel('Time (s)');
144 set(gca, 'YAxisLocation', 'Right'); % Flip the axes to the other side for readability.
145 if strcmpi(robotType, 'RRR'); % If it has a 3rd linkage, print the 3rd dimension.
146     subplot(3,1,3);
147     plot(t, Z, '.'); axis('tight');
148     ylabel('Z');
149     xlabel('Time (s)');
150 end
151 eeecsFile=sprintf('%s.EECS', n);
152 % print('--depsc2', eeecsFile);
153 fprintf(texFid, '\\fig{%s}{Position of the end effector for each of the coordinates vs ...
        time}\\n', eeecsFile);
154 % Now in 3D
155 close all;
156 plot3(X, Y, Z);

```

```

157 title(sprintf('%s end effector position in 3D',robotType));
158 xlabel('X');
159 ylabel('Y');
160 zlabel('Z');
161 grid('on');
162 eecsFile3=sprintf('%s_EECS_3D',n);
163 % print('--depsc2',eecsFile3);
164 fprintf(texFid, '\\fig{%s}{%s end effector position in 3D}\\n',eecsFile3,robotType);
165
166 %% Do Phase 1 & Forward Kinematics.
167 [p,n] = fileparts(file);
168 phasel_file=fullfile(p,[n '_robot.dat']); % Create the .dat file to write for phasel.
169 fid=fopen(phasel_file,'w'); % Open it.
170 fprintf(fid,'%d %d %s\\n',2+~all(Z==0),length(X),robotType); % Write the first line.
171 if strcmpi('RRR',robotType)
172     fprintf(fid,'%10f\\t%10f\\t%10f\\n',[theta(:,1),theta(:,2),theta(:,3)]'); % Print each ...
        of the joint values.
173 else
174     fprintf(fid,'%10f\\t%10f\\n',[theta(:,1),theta(:,2)]'); % Print each of the joint values.
175 end
176 fclose(fid); % Close the file.
177 [X2,Y2,Z2]=feval('phasel',phasel_file); % Get the EECS position based on forward kinematics.
178 phaselTex= feval('phasel',phasel_file); % Print phasel to a .tex file.
179 fprintf(texFid, '\\input{%s}\\n',phaselTex);% Write it to the main file to include.
180
181 %% Manipulability.
182 [p,n] = fileparts(file);
183 dataOut=fullfile(p,[n '_manip.tex']);
184 fid=fopen(dataOut,'w');
185 fprintf(fid,'Manipulability of the robot.\\n \\pagebreak[1]');
186 fprintf(fid, '\\begin{verbatim}\\n');
187 if strcmpi(robotType,'RRR')
188     fprintf(fid,'%s %s %s\\n','ax','by','cz');
189     fprintf(fid,'% 8.3f% 8.3f % 8.3f % 8.3f\\n',[t';ax;by;cz]);
190 else
191     fprintf(fid,'%s %s %s\\n','ax','by');
192     fprintf(fid,'% 8.3f% 8.3f % 8.3f\\n',[t';ax;by]);
193 end
194 fprintf(fid, '\\end{verbatim}\\n');
195 fprintf(texFid, '\\input{%s}\\n',dataOut);% Write it to the main file to include.
196
197 %% Plot the ellipsoid.
198 [ix,iy]=find(abs(dTheta)==max(max(abs(dTheta)))); % Find the point where joint velocity is ...
        near the maximum.
199
200 if strcmpi('RRR',robotType)
201     for i=1:ix:length(ax)
202         [Ex,Ey,Ez]=ellipsoid(0,0,0,ax(i),by(i),cz(i));
203         % Rotate each Point
204         for j=1:size(Ex,1)
205             for k=1:size(Ex,2)
206                 E=R{i}*[Ex(j,k) Ey(j,k) Ez(j,k)]';
207                 Ex(j,k)=E(1);
208                 Ey(j,k)=E(2);
209                 Ez(j,k)=E(3);
210             end
211         end
212     end
213     close all;
214     surf(Ex,Ey,Ez)
215     title(sprintf('Manipulability ellipse for \\theta_1=%3f^o, \\theta_2=%3f^o, ...
        \\theta_3^o=%3f', theta(i,1), theta(i,2),theta(i,3)))
216     axis equal

```

```

216     axis([-1 1 -1 1 -1 1]*max(axis));
217     xlabel('X');
218     ylabel('Y');
219     zlabel('Z');
220     eFile=sprintf('%s_ell%d',n,i);
221     % print('---depsc2',eFile);
222     fprintf(texFid, '\\fig{%s}{Manipulability of %s at position ...
        %d.}\\n',eFile,robotType,i-1);
223 end
224 else
225     for i=[1 ix length(ax)]
226         % 2D Way
227         [ex,ey] = calculateEllipse(0, 0, ax(i), by(i), Beta(i));
228         % close all;
229         % plot(ex,ey);
230         % title(sprintf('Manipulability ellipse for \\theta_1=%.3f^o, \\theta_2=%.3f^o', ...
        theta(i,1), theta(i,2)))
231         % axis equal
232         % axis([-1 1 -1 1]*max(axis));
233         % xlabel('X');
234         % ylabel('Y');
235
236         % 3D Way
237         [Ex,Ey,Ez]=ellipsoid(0,0,0,ax(i),by(i),0);
238         % Rotate each Point
239         for j=1:size(Ex,1)
240             for k=1:size(Ex,2)
241                 E=R{i}* [Ex(j,k) Ey(j,k)]';
242                 Ex(j,k)=E(1);
243                 Ey(j,k)=E(2);
244             end
245         end
246         close all;
247         surf(Ex,Ey,Ez);
248         title(sprintf('Manipulability ellipse for \\theta_1=%.3f^o, \\theta_2=%.3f^o, ...
        \\theta_3^o=%.3f', theta(i,1), theta(i,2),theta(i,3)))
249         axis equal
250         axis([-1 1 -1 1 -1 1]*ceil(max(axis)));
251         xlabel('X');
252         ylabel('Y');
253         zlabel('Z');
254         eFile=sprintf('%s_ell%d',n,i);
255         % View from the top.
256         set(gca,'CameraPosition',[0 0 1])
257         % print('---depsc2',eFile);
258         fprintf(texFid, '\\fig{%s}{Manipulability of %s at position ...
        %d.}\\n',eFile,robotType,i-1);
259     end
260 end
261 %% Sanity check the CIM trajectory planning against the output from phasel
262 if max(abs(X-X2))>.1
263     error('Something went wrong with forward kinematics. Check phasel.m and shape.m');
264 else
265     disp('EECS X-axis validated');
266 end
267 if max(abs(Y-Y2))>.1
268     error('Something went wrong with forward kinematics. Check phasel.m and shape.m');
269 else
270     disp('EECS Y-axis validated');
271 end
272 if max(abs(Z-Z2))>.1
273     error('Something went wrong with forward kinematics. Check phasel.m and shape.m');

```

```

274 else
275     disp('EECS Z-axis validated');
276 end
277 %%
278 fclose('all');

```

## JCalc.m

Jacobian calculator.

```

1 function J=Jcalc(robotType,Q)
2 [jointType,n]=shape(robotType);
3 [Tm,n]=Tworld(robotType,Q);
4 % Get the final P position.
5 Pe=Tm{end}(1:3,4);
6 % For each link find out the Jacobian column.
7 for i=1:n
8     switch jointType(i)
9         case 0, % Rotation joint
10             an=Tm{i}(1:3,3);
11             Pn=Tm{i}(1:3,4);
12             J(:,i)=[cross(an,Pe-Pn);an];
13         case 1, % Prismatic joint
14             an=Tm{i}(1:3,3);
15             J(:,i)=[an;0;0;0];
16         otherwise,
17             error('Unknown joint type %d',jointType(i))
18     end
19 end
20 % Clean up symbolic stuff to make it more readable.
21 if ~isnumeric(J)
22     J=simplify(J);
23 end
24 end

```

## Tworld.m

Calculate  $T_w$  and return it. This was pulled out of phase1 and made its own function because it was used multiple times in multiple different functions. (Such as in the Jacobian)

```

1 function [Tm,n]=Tworld(robotType,Q)
2 % This us used so often, make it a new function.
3
4 % Get data from the shape matrix.
5 [jointType,T]=shape(robotType);
6 n=length(jointType);
7 if length(Q)<n % Number of input variables has to be the same as the number of transforms.
8     error('Incorrect number of input parameters. There are %d joints but only %d variables ...
9         specified',n,length(Q));
10 end
11 Tm=cell(1,n+1); % Empty cell array for speed.
12 Tm{1}=eye(4); % Initial one is an identity. Because of Matlab's 1 indexing everything is ...
13     actually n-1.
14 for j=1:length(jointType)
15     switch jointType(j)
16         case 0,
17             phi=phiR(Q(j));

```

```

16         case 1,
17             phi=phiP(Q(j));
18         otherwise,
19             error('Unknown joint type %d', jointType(j))
20     end
21     % Find each intermediate transform matrix.
22     Tm{j+1}=Tm{j}*phi*T{j};
23 end
24 end
25 % Sub functions
26 function m=phiR(theta)
27 if isnumeric(theta)
28     theta=theta*pi/180;
29 end
30 m=[cos(theta) -sin(theta) 0 0
31     sin(theta)  cos(theta) 0 0
32     0           0         1 0
33     0           0         0 1];
34 end
35 function m=phiP(X)
36 m=[1 0 0 0
37     0 1 0 0
38     0 0 1 X
39     0 0 0 1];
40 end

```

## shape.m

New shape.m function with the added RR.

```

1 function [jointType,T]=shape(robotType)
2 switch robotType
3     case 'RR', % From homework 7
4         jointType=[0 0];
5         L1=20;
6         L2=20;
7         % T1
8         T{1}=[1 0 0 L1
9             0 1 0 0
10            0 0 1 0
11            0 0 0 1];
12        % T2
13        T{2}=[1 0 0 L2
14            0 1 0 0
15            0 0 1 0
16            0 0 0 1];
17        return;
18    case 'RRP', % From homework 4
19        jointType=[0 0 1];
20        % Joint parameters
21        L1=5;
22        d1=4;
23        % T1
24        T{1}=[0 0 1 d1
25            1 0 0 0
26            0 1 0 L1
27            0 0 0 1];
28        % T2
29        T{2}=[0 1 0 0

```

```

30         0 0 1 0
31         1 0 0 0
32         0 0 0 1];
33     % T3
34     T{3}=eye(4);
35     return
36     case 'RRR', % From homework 4
37         d1=4;
38         L3=4;
39         L1=5;
40         L2=5;
41         jointType=[0 0 0];
42         T{1}=[0 0 1 d1
43             1 0 0 0
44             0 1 0 L1
45             0 0 0 1];
46         T{2}=[1 0 0 0
47             0 1 0 L2
48             0 0 1 0
49             0 0 0 1];
50         T{3}=[0 1 0 0
51             0 0 1 L3
52             1 0 0 0
53             0 0 0 1];
54     otherwise,
55         error('Unknown robot type %s',robotType)
56 end
57
58 end

```

## manipulability.m

Manipulability calculator.

```

1 function [ax,by,cz,Beta,R]=manipulability(J)
2 n=min([size(J,2) 3]);
3 Jtop=J(1:n,1:n);
4 B=Jtop*Jtop';
5 [R,L1]=eigs(B^-1);
6 E=diag(L1);
7 R1=R*sign(R(1,1));
8 ax=1/sqrt(E(1));
9 by=1/sqrt(E(2));
10 Beta=acos(R1(1,1));
11 if n==3
12     cz=1/sqrt(E(3));
13 else
14     cz=NaN;
15 end

```