ME572: Homework #7, part 2

Due on March 21, 2012

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Dexterity ellipsoid.
The Jacobian matrix is in the form of:

$$\begin{Bmatrix} \dot{P}_x \\ \dot{P}_y \\ \dot{P}_z \\ \omega_x \\ \omega_y \\ \omega_z \end{Bmatrix} = \begin{bmatrix} J \\ 6 \times 2 \end{bmatrix} \begin{Bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{Bmatrix} \tag{1}$$

The Jacobian matrix for this problem is:

$$J = \begin{Bmatrix} \vec{a}_0 \times \left(\vec{P}_E - \vec{P}_0\right) & \vec{a}_1 \times \left(\vec{P}_E - \vec{P}_1\right) \\ \vec{a}_0 & \vec{a}_1 \end{Bmatrix} \tag{2}$$

Where:

$$P_e = \begin{bmatrix} 20 \cdot \cos(\theta_1 + \theta_2) + 20 \cdot \cos(\theta_1) \\ 20 \cdot \sin(\theta_1 + \theta_2) + 20 \cdot \sin(\theta_1) \\ 0 \end{bmatrix} \tag{3}$$

$$a_0 = \begin{bmatrix} 0.00 \\ 0.00 \\ 1.00 \end{bmatrix} \tag{4}$$

$$P_0 = \begin{bmatrix} 20 \cdot \cos(\theta_1 + \theta_2) + 20 \cdot \cos(\theta_1) \\ 20 \cdot \sin(\theta_1 + \theta_2) + 20 \cdot \sin(\theta_1) \\ 0 \end{bmatrix} \tag{5}$$

$$a_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{6}$$

$$P_1 = \begin{bmatrix} 20 \cdot \cos(\theta_1 + \theta_2) \\ 20 \cdot \sin(\theta_1 + \theta_2) \\ 0 \end{bmatrix} \tag{7}$$

$$J = \begin{bmatrix} -20 \cdot \sin(\theta_1 + \theta_2) - 20 \cdot \sin(\theta_1) & -20 \cdot \sin(\theta_1 + \theta_2) \\ 20 \cdot \cos(\theta_1 + \theta_2) + 20 \cdot \cos(\theta_1) & 20 \cdot \cos(\theta_1 + \theta_2) \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix} \tag{8}$$

The manipulability of linear velocity is given by $\dot{\vec{x}} = J_{top}\dot{q}$ where J_{top} is square. For the manipulator:

$$J_{top} = \begin{bmatrix} -20\cdot\sin(\theta_1+\theta_2) - 20\cdot\sin(\theta_1) & -20\cdot\sin(\theta_1+\theta_2) \\ 20\cdot\cos(\theta_1+\theta_2) + 20\cdot\cos(\theta_1) & 20\cdot\cos(\theta_1+\theta_2) \end{bmatrix} \quad (9)$$

$$B^{-1} = (J_{top}J_{top}^T)^{-1} \quad (10)$$

$$B^{-1}_{(1,1)} = \left[-(\cos(2\cdot\theta_1+\theta_2) + \cos(2\cdot\theta_1)/2 + \cos(2\cdot\theta_1+2\cdot\theta_2) + \cos(\theta_2) + 3/2)/(400\cdot(\cos(\theta_2)^2 - 1))\right] \quad (11)$$

$$B^{-1}_{(1,2)} = \left[(\sin(2\cdot\theta_1+\theta_2) + \sin(2\cdot\theta_1)/2 + \sin(2\cdot\theta_1+2\cdot\theta_2))/(400\cdot\sin(\theta_2)^2)\right] \quad (12)$$

$$B^{-1}_{(2,1)} = \left[(\sin(2\cdot\theta_1+\theta_2) + \sin(2\cdot\theta_1)/2 + \sin(2\cdot\theta_1+2\cdot\theta_2))/(400\cdot\sin(\theta_2)^2)\right] \quad (13)$$

$$B^{-1}_{(2,2)} = \left[(\cos(2\cdot\theta_1+\theta_2) + \cos(2\cdot\theta_1)/2 + \cos(2\cdot\theta_1+2\cdot\theta_2) - \cos(\theta_2) - 3/2)/(400\cdot(\cos(\theta_2)^2 - 1))\right] \quad (14)$$

The ellipsoid can then be determined from the eigenvalues and eigenvectors of B^{-1} where:

$$a_x = \frac{1}{\sqrt{\lambda_1}} \quad (15)$$

$$b_y = \frac{1}{\sqrt{\lambda_2}} \quad (16)$$

$$(17)$$

The rotation through β can me found by comparing B^{-1} to the rotation matrix.

$$\theta_1 = 264.697437 \quad (18)$$

$$\theta_2 = 180.319597 \quad (19)$$

$$a_x = 0.111560 \quad (20)$$

$$b_y = 20.000000 \quad (21)$$

$$\beta = 85.017039 \quad (22)$$

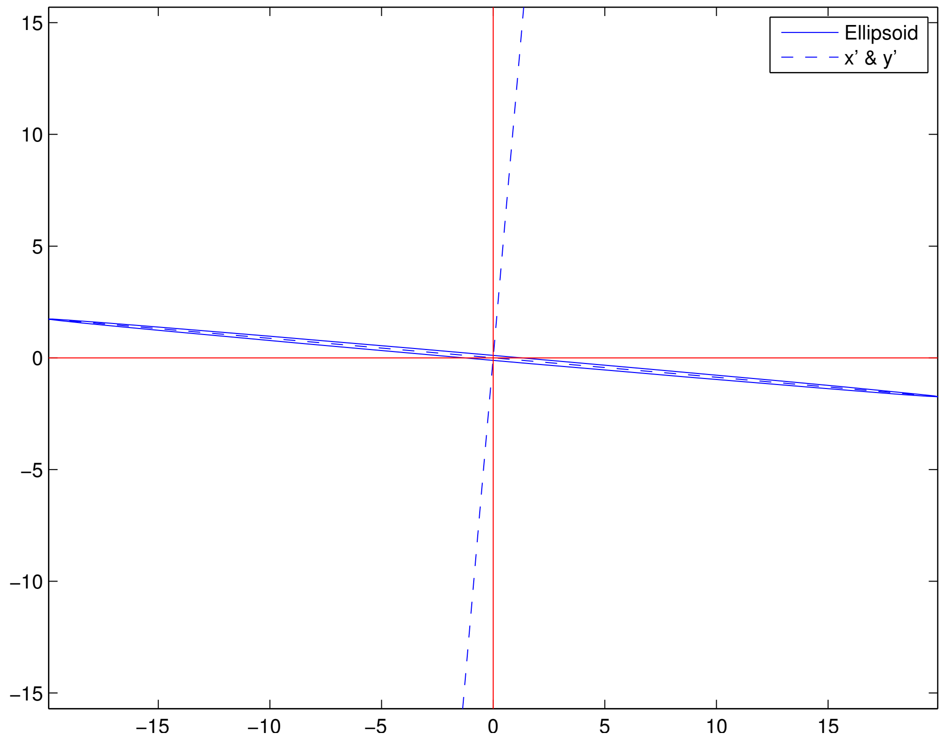

Figure 1: Ellipsoid immediately after ω_1 reaches its peak

The ellipsoids approaching ω_1 max (when linkage 1 switches quadrants).

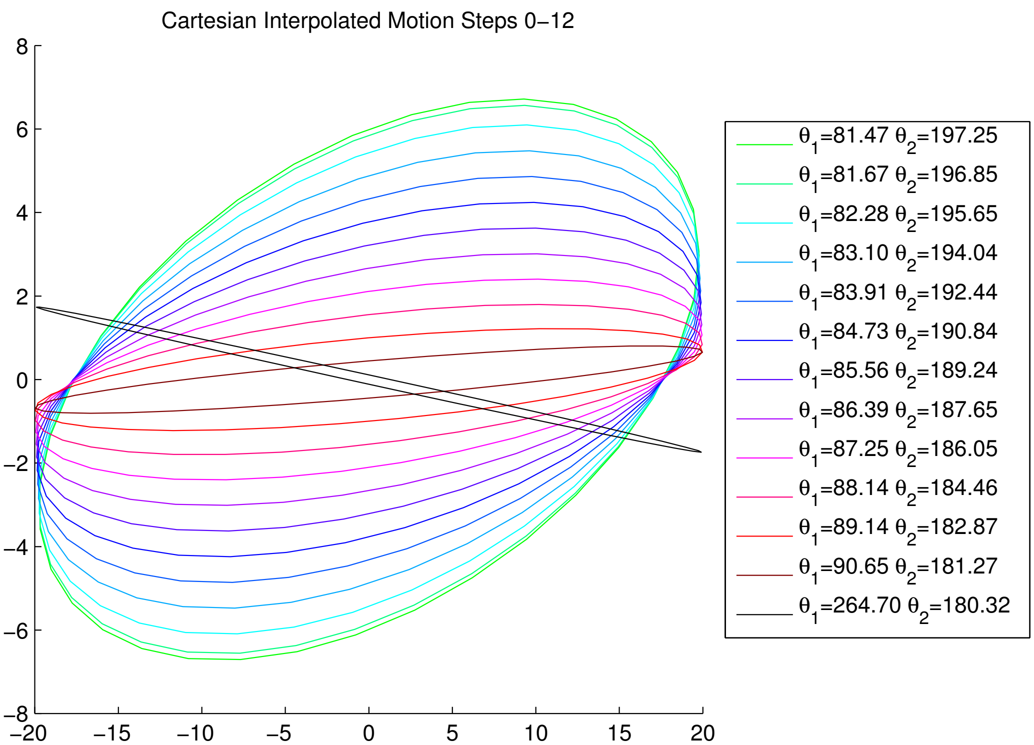

Figure 2: Ellipsoid for points 0-12