Evenness-controllable point cloud simplification via graph filter

Anonymous VCIP Submission
Paper ID:

***Abstract*—keep sharp features like edges while keep the evenness of points**

***Index Terms*—Include at least 5 keywords or phrases**

I. Introduction

what's simplification and what's the meaning of it

II. Related Work

A. Point Cloud Simplification

what's done
advantages and disadvantages

- Top = 19mm (0.75")
- Bottom = 43mm (1.69")
- Left = Right = 14.32mm (0.56")

B. Graph Signal Processing

what's GSP and how to construct graph

III. Problem Formulation

Here, we describe point cloud simplification as a process of resampling of the point cloud: given a point cloud P with $|P| = N$, find a point cloud $P' \subset P$ with $|P'| = M < N$. We define the simplification rate $\alpha = M/N$.

For convenience, we represent the point cloud with N points and K attributes as $X \in \mathbb{R}^{N\times K}$, where ith row represents the ith point, denoted as x_i^T. Attributes can be coordinates, colors and others, $K \geq 3$. To represent the simplified point cloud, we consider the diagonal matrix Ψ, called resampling matrix with $\Psi_{ii} = 1$ if x_i in the simplified point cloud and $\Psi_{ii} = 0$ if not. Thus, the simplified point cloud can be represented as ΨX.

Our goal is to find the optimal resampling matrix Ψ to keep most geometry features of the point cloud while keep the evenness. Inspired by Chen[], we use graph filter to extract features of point cloud and select points with higher features. We use the random walk Laplacian

$$L_0 = D^{-1}L = I - D^{-1}W$$

to extract features, which is a high-pass graph filter keeping sharp features. Thus, we can represent features of point cloud X as L_0X and the remaining features (of the simplified point cloud) as $\Psi L_0 X$. Now we define the feature loss of simplification as

$$\mathcal{L}_f = \|\Psi L_0 X - L_0 X\|_F^2.$$

Here, we use the F-norm of matrix and set F as 2.

However, merely using random walk Laplacian will cause the unevenness of point cloud because edges with sharp features will be saved sound while the surfaces will be neglected, which will cause extreme unevenness. To avoid this extreme unevenness, we define a evenness term to control the evenness of the simplified point cloud.

When constructing the graph, we select points within radius r as point's neighbour. If we suppose the point cloud is even, the number of neighbours of each point should be approximately equal and thus we can use k-nearest neighbours – when the point cloud is even, the degree of each node is approximately equal.

We use binary matrix A to represent the adjacency of graph i.e $A_{ij} = 1$ if and only if x_j is one of the neighbours of x_i. Each line of A represent the relation of the point with its neighbours and the sum of each line should be k. By means of the definition of Ψ, the adjacency matrix of the simplified point cloud graph can be represented as $A\Psi$. Given the simplification rate α, the number of neighbours in the simplified point cloud graph should be approximately equal to $alphak$ if simplified evenly. So we define the evenness term as

$$\mathcal{L}_e = \|A\Psi\mathbf{1} - \alpha k\mathbf{1}\|_F^2,$$

where $\mathbf{1}$ represent the column vector with every element equal to 1.

Now we can formulate the point cloud simplification problem as an optimization problem:

$$\min_{\Psi} \mathcal{L} = \mathcal{L}_f + \lambda\mathcal{L}_e = \|\Psi L_0 X - L_0 X\|_F^2 + \lambda\|A\Psi\mathbf{1} - \alpha k\mathbf{1}\|_F^2,$$

s.t. $\Psi_{ii} \in 0, 1, i = 1, 2, ..., N$; $\Psi_{ij} = 0, i \neq j$; $tr(\Psi) = \alpha N$, where λ is a hyper-parameter to keep balance of feature and evenness.

IV. Formulation Optimization

The optimization problem we put forward before is a combinatorial optimization problem, which is NP-hard. To simplify the algorithm, we relax the constraints to approximate diagonal matrix Ψ and $0 \le \Psi_{ij} \le 1$. Then the optimization problem can be represented as

$$\min_{\Psi} \mathcal{L} = \|\Psi L_0 X - L_0 X\|_F^2 + \lambda \|A\Psi\mathbf{1} - \alpha k \mathbf{1}\|_F^2, \quad \text{s.t. } tr(\Psi) = \alpha N,\ \|\Psi\|_F^2 = \alpha N.$$

Now we can use the method of Lagrange multiplier to solve this optimization problem. Suppose the Lagrange multipliers for the two constraints are β and γ respectively, the solution will be

$$\Psi = (\gamma I + I + \lambda A^2)^{-1}(L_0 X X^T L_0^T + \lambda k \alpha A J - 1/2\beta I)(L_0 X X^T L_0^T + \gamma I + J)^{-1},$$

where $I \in \mathbb{R}^{N\times N}$ represent the identify matrix and $J = \mathbf{1}\mathbf{1}^T \in \mathbb{R}^{N\times N}$ represent the matrix with every element equal to 1. And we heuristically set γ and β as $1/\alpha - 1 - 2k\lambda$ and $-\alpha\gamma(\gamma+1)$ respectively.

Each line of relaxed Ψ, denoted as Ψ_i^T can be regarded as weights of point x_j to be selected relevant to other points. We sum each element of Ψ_i and define it as the priority of point x_j. Then we sort the points according to their priority and select the top α points as the simplified point cloud.

V. Experiment Results

Our algorithm depends on matrix operations, which is time and storage expensively. To solve this problem, we divide the point cloud into small grids first, and then simplify each grid respectively. For better performance, there is small overlapping between adjacent grid.

A. Results compared to previous algorithms

bunny??? Alice??? dragon??? monster???

time???????? performance: visually(manifold) and quantitatively(error)

B. Results on excessively large point clouds

landscape???

VI. Conclusion

advantage: avoidance of normals graph filter, local & global sharp features(edges) while even

disadvantage: small holes!

Recommended font sizes are shown in Table I.

/

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All title and author details must be in single-column format and must be centered.

TABLE I
FONT SIZES FOR PAPERS

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10	level-1 heading (in Small Caps), paragraph		level-2 heading, level-3 heading, author affiliation
11	author name		
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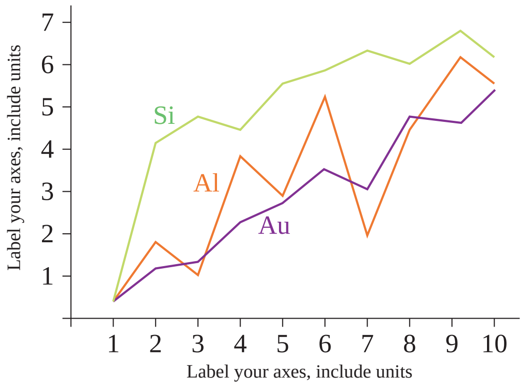

Fig. 1. A sample line graph using colors which contrast well both on screen and on a black-and-white hardcopy

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Fig. 2. Example of an unacceptable low-resolution image

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Fig. 3. Example of an image with acceptable resolution

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VII. Conclusion

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