Assignment 4

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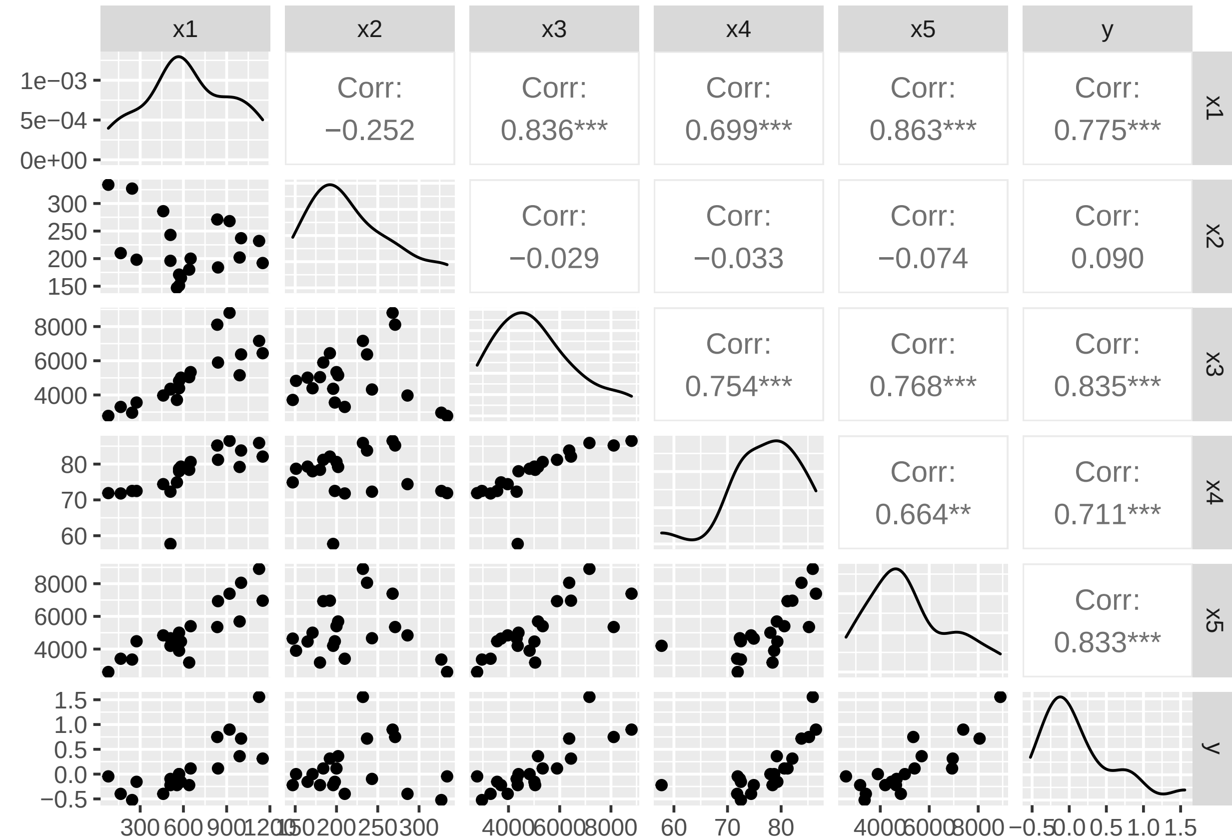
Problem 1

This is an experimental data of five laboratory measurements conducted to interpret total oxygen demand in dairy waste. Data were collected on samples kept in suspension in water in a laboratory for 220 days, and we assume that all observations are independent.

Table 1: Total oxygen demand in dairy wastes

Item	Variable	Description	Unit
1	y	$log(oxygen\ demand)$	mg oxygen per unit
2	x1	$biological\ oxygen\ demand$	mg/liter
3	x2	$Total\ Kjeldahl\ nitrogen$	mg/liter
4	x3	$Total\ solids$	mg/liter
5	x4	$Total\ volatile\ solids$	mg/liter
6	x5	$Chemical\ oxygen\ demand$	mg/liter

```
##       Day              x1               x2               x3
##  Min.   :  0.0   Min.   :  79.0   Min.   :147.0   Min.   :2777
##  1st Qu.: 35.0   1st Qu.: 497.5   1st Qu.:183.0   1st Qu.:3904
##  Median : 76.0   Median : 576.5   Median :201.0   Median :4918
##  Mean   : 80.4   Mean   : 633.5   Mean   :219.7   Mean   :5075
##  3rd Qu.:110.8   3rd Qu.: 860.0   3rd Qu.:249.2   3rd Qu.:6014
##  Max.   :220.0   Max.   :1150.0   Max.   :334.0   Max.   :8804
##        x4              x5              y
##  Min.   :57.70   Min.   :2599   Min.   :-0.5229
##  1st Qu.:72.50   1st Qu.:4125   1st Qu.:-0.2218
##  Median :78.55   Median :4752   Median :-0.0229
##  Mean   :77.34   Mean   :5171   Mean   : 0.1192
##  3rd Qu.:81.42   3rd Qu.:6000   3rd Qu.: 0.3252
##  Max.   :86.50   Max.   :8905   Max.   : 1.5563
```



- $x2, x3, x5$ *and* y are right skewed, while $x4$ is left skewed.
- From the correlation coefficient table we found that the estimator of $x4$, β_4 has a very high negative correlation associate with the intercept (β_0), which might result from the range of $x4$, which compared to other variables, is relatively small.
- As we can observe from the plot, the values of correlation coefficient between $x1$ *and* $x3$, $x1$ *and* $x4$, $x1$ *and* $x5$, $x3$ *and* $x5$, $x4$ *and* $x5$ are very high, in addition, the response variable y seemed to have be positively correlated with $x1, x3, x4$ *and* $x5$.

Fit a multiple regression model $y = \beta_0 + \beta_1 * x1 + \beta_2 * x2 + \beta_3 * x3 + \beta_4 * x4 + \beta_5 * x5 + \epsilon$, using y as the dependent variable and all $x_j's$ as the independent variables.

Below is the result of fitting a multiple regression model.

```
##
## Call:
## lm(formula = y ~ x1 + x2 + x3 + x4 + x5)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.39447 -0.11847  0.00053  0.08313  0.56232
##
## Coefficients:
##                Estimate Std. Error t value Pr(>|t|)
```

```
## (Intercept) -2.156e+00  9.135e-01  -2.360   0.0333 *
## x1          -9.012e-06  5.184e-04  -0.017   0.9864
## x2           1.316e-03  1.263e-03   1.041   0.3153
## x3           1.278e-04  7.690e-05   1.662   0.1188
## x4           7.899e-03  1.400e-02   0.564   0.5815
## x5           1.417e-04  7.375e-05   1.921   0.0754 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.2618 on 14 degrees of freedom
## Multiple R-squared:  0.8107, Adjusted R-squared:  0.743
## F-statistic: 11.99 on 5 and 14 DF,  p-value: 0.0001184
##
## Correlation of Coefficients:
##    (Intercept) x1    x2    x3    x4
## x1  0.09
## x2 -0.20        0.47
## x3  0.31       -0.51 -0.27
## x4 -0.93       -0.09 -0.04 -0.39
## x5  0.01       -0.64 -0.25 -0.03 -0.09
```

a. Form a 95% confidence interval for β_3 and again for β_5.

- 95% Critical value, $df_\Omega = n - p = 20 - 6 = 14$

```
## [1] 2.144787
```

- 95% confidence interval for β_3 *and* β_5.

$confidence\ interval = estimate \pm critical\ value * s.e\ of\ estimate$

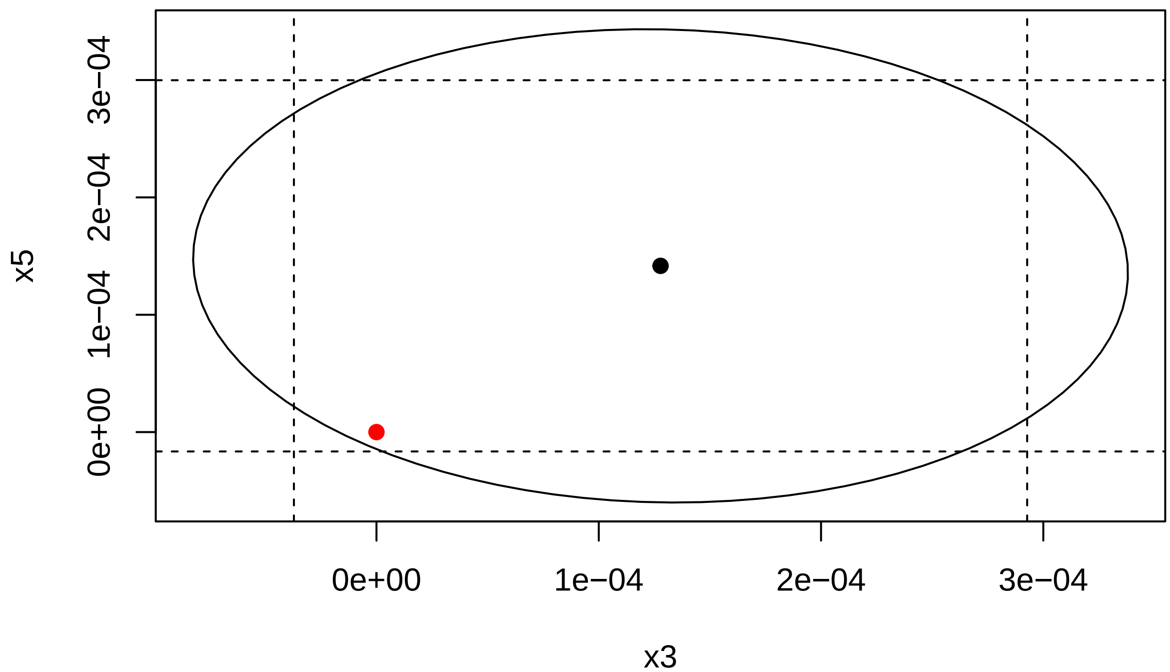
	2.5 %	97.5 %
x3	-3.71e-05	0.0002927
x5	-1.65e-05	0.0002998

- Both intervals contain 0, this indicates that the null hypotheses $H_0 : \beta_3 = 0$ *and* $H_0 : \beta_5 = 0$ would not be rejected at the 5% significance level. We can see from the summary where tha p-value for β_3 *and* β_5 are 11.88% and 7.54% respectively, confirming the point.

b. Form a confidence interval for $\beta_3 + 2 * \beta_5$.

	x
2.5 %	-0.0000702
97.5 %	0.0008924

c. Show a 95% C.R. graph for β_3 *and* β_5. Plot the origin. State the hypothesis test and its out-



come.

```
## [1] -0.03156963
```

- The seemly parallelism of the major axis of the ellipse to the y-axis and that of the semi-minor axis to the x-axis suggests that the two estimators, β_3, β_5 are uncorrelated.
- We can deduced from the plot that the joint hypothesis $H_0 : \beta_3 = \beta_5 = 0$ is not reject because the origin lies right inside the ellipse
- Both of the hypotheses $H_0 : \beta_3 = 0$ *and* $H_0 : \beta_5 = 0$ are not rejected either because 0 does lie within the vertical and horizontal dashed lines which represents as the C.I. of $x3$ *and* $x5$ respectively.
- We can further conduct some experiments testing on the significance of β_3 where x5 is included in the model and the significance of β_5 where $x3$ is included in the model to ensure that both variables $x3, x5$ are not significant when the true model is assumed to be $y = \beta_0 + \beta_1 * x1 + \beta_2 * x2 + \beta_3 * x3 + \beta_4 * x4 + \beta_5 * x5 + \epsilon$.

```
## Analysis of Variance Table
##
## Model 1: y ~ x1 + x2 + x4 + x5
## Model 2: y ~ x1 + x2 + x4
##   Res.Df    RSS Df Sum of Sq      F  Pr(>F)
## 1     15 1.1488
## 2     16 1.4159 -1  -0.26713 3.4878 0.08149 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
## Analysis of Variance Table
##
## Model 1: y ~ x1 + x2 + x3 + x4
## Model 2: y ~ x1 + x2 + x4
##   Res.Df    RSS Df Sum of Sq      F Pr(>F)
## 1     15 1.2124
## 2     16 1.4159 -1  -0.20355 2.5184 0.1334
```

- In conclusion, as we can observe from the above tests and visualization, all of our hypotheses are not rejected, which indicate that neither of the estimators, β_3, β_5 are significant enough to be indispensable for response variable y under the 95% confidence level.

d. If a 95% joint confidence region was computed for $\beta_1, \beta_2, \beta_3, \beta_4, \beta_5$, would the origin lie inside or outside the region? Explain.

	2.5 %	97.5 %
(Intercept)	-4.1153842	-0.1969077
x1	-0.0011208	0.0011027
x2	-0.0013940	0.0040258
x3	-0.0000371	0.0002927
x4	-0.0221278	0.0379255
x5	-0.0000165	0.0002998

	x
(Intercept)	FALSE
x1	TRUE
x2	TRUE
x3	TRUE
x4	TRUE
x5	TRUE

- As we can observe from the table, if measured individually, none of the predictors are significant; however, combining univariate tests to test on joint relationship may come up with a biased result, thus we need to conduct a test to compute the boundaries of the joint confidence region.

Joint Effect

```
##          [,1]
## [1,] 3.215305
```

Critical Value

```
## [1] 0.02956926
```

- After testing the joint effect of $\beta_1, \beta_2, \beta_3, \beta_4, \beta_5$, we found the calculated result is significantly larger than the corresponding critical value $2 * \hat{\sigma}^2 * F_{5,14} = 0.02957$, which implies that the origin $(0, 0, 0, 0, 0)$ is lying outside the region.
- Thus, we can reject the null hypothesis that $H_0 : \beta_1 = \beta_2 = \beta_3 = \beta_4 = \beta_5 = 0$.

e. Suppose non-volatile solids have no linear effect on the response. State a hypothesis that reflects this suspicion, and test it using a C.I. in your answer to one of the above questions. Explain why the chosen confidence interval can be used to do this work.

- Arrange our previous model: $y = \beta_0 + \beta_1 * x1 + \beta_2 * x2 + \beta_3 * x3 + \beta_4 * x4 + \beta_5 * x5 + \epsilon$
- Total solids - volatile solids = non-volatile solids: $y = \beta_0 + \beta_1 * x1 + \beta_2 * x2 + \beta_3 * (x3 - x4) + \beta_4 * x4 + \beta_5 * x5 + \epsilon$
- $y = \beta_0 + \beta_1 * x1 + \beta_2 * x2 + \beta_3 * x3 + (\beta_4 - \beta_3) * x4 + \beta_5 * x5 + \epsilon$
- $y = \beta_0 + \beta_1 * x1 + \beta_2 * x2 + \beta_3 * x3 + \beta_4^{'} * x4 + \beta_5 * x5 + \epsilon,\ where\ \beta_4^{'} = \beta_4 - \beta_3$
- As we can observe from the model equation, originally, $x4 \subset x3$, where β_3 is an estimator for both volatile and non-volatile solids while β_4 is an estimator for volatile solids, and therefore β_4 explains both regular parts from $x3$ *and* $x4$; yet with the arrangement, now β_3 becomes the estimator for non-volatile solids which is uncorrelated to predictor $x4$.
- Hence, we hypothesized that $H_0 : \beta_3 = 0$, Since the arrangement is only a linear transformation performed on variable $x3$ which does not change the value of β_3, I would apply the C.I. value calculated from question **a** to test this hypothesis.
  - Set $\beta_3 = 0$
  - Introduce C.I. for β_3 from model1

    ```
    ##         2.5 %       97.5 %
    ## -3.713929e-05  2.927368e-04
    ```

  - Whether we accept the null hypothesis

    ```
    ## [1] TRUE
    ```

- Since 0 lies in the C.I. of β_3, we do not reject the null hypothesis, that is, the non-volatile solids have no linear effect on the response.

Problem 2

This data are a random sample of home sales from Spring 1993 in Albuquerque.

Table 6: Home sales data

Item	Variable	Description	Unit
1	Price	Selling price	\$100
2	SQFT	living space	$feet^2$
3	Age	Age of home	$year$
4	Features	Number out of 11 features	(dish washer, refrigerator, microwave, disposer, washer, intercom, skylight(s), compactor, dryer, handicap fit, cable TV access)

Item	Variable	Description	Unit
5	NE	Located in northeast sector of city	1 *or* 0
6	Corner	Corner location	1 *or* 0
7	Taxes	Annual taxes	$

a. There are a large number of missing values in the *Age* variable. We could either exclude *Age* from our models for the selling price or we could keep *Age* and exclude the cases that have missing values for *Age*. Which choice is better for this data? Explain your reasoning.

- Check how many missing values are in the data.

```
##      Price           SQFT            Age           Features          NE        
##  Min.   : 540   Min.   : 837   Min.   : 1.00   Min.   :0.00   Min.   :0.0000  
##  1st Qu.: 780   1st Qu.:1280   1st Qu.: 5.75   1st Qu.:3.00   1st Qu.:0.0000  
##  Median : 960   Median :1549   Median :13.00   Median :4.00   Median :1.0000  
##  Mean   :1063   Mean   :1654   Mean   :14.97   Mean   :3.53   Mean   :0.6667  
##  3rd Qu.:1200   3rd Qu.:1894   3rd Qu.:19.25   3rd Qu.:4.00   3rd Qu.:1.0000  
##  Max.   :2150   Max.   :3750   Max.   :53.00   Max.   :8.00   Max.   :1.0000  
##                                NA's   :49                                     
##      Corner          Tax        
##  Min.   :0.000   Min.   : 223.0  
##  1st Qu.:0.000   1st Qu.: 600.0  
##  Median :0.000   Median : 731.0  
##  Mean   :0.188   Mean   : 793.5  
##  3rd Qu.:0.000   3rd Qu.: 919.0  
##  Max.   :1.000   Max.   :1765.0  
##                  NA's   :10      
```

- As we can observe from the result of the summary, there are 49 na values in variable *Age* and 10 in variable Tax.
- Furthermore, although $Features$ variable description indicated there are 11 different features in the data, yet as we can see from the table, there are only 8 features at most including in our sample data.

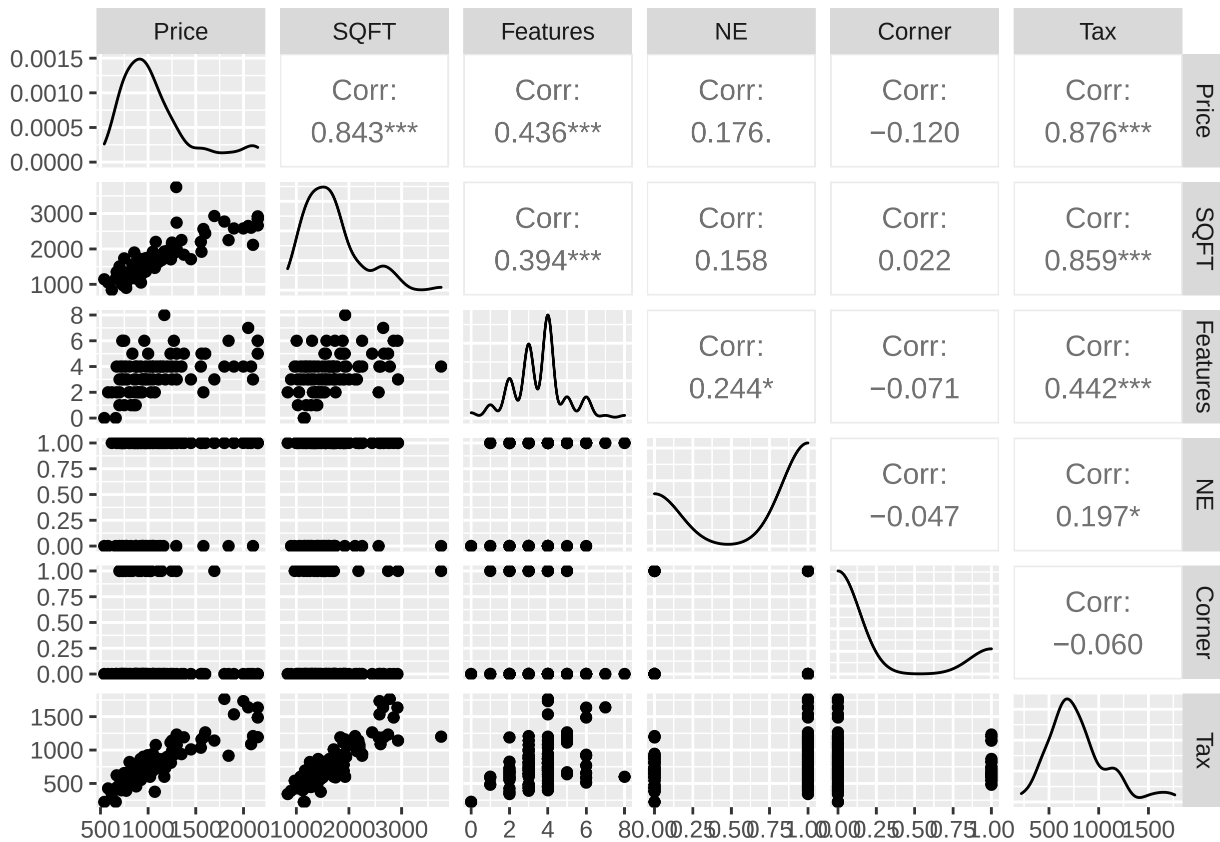
```
## [1] 117
```

- However, there are only 117 samples in this data, removing all the missing values in *Age* would indicate dropping nearly half the amount of the observations, which is irrational to give up so much observations; therefore, choosing to exclude variable *Age* from the dataset is a more economical and reasonable option.

Price	SQFT	Features	NE	Corner	Tax
2050	2650	7	1	0	1639
2080	2600	4	1	0	1088
2150	2664	5	1	0	1193
2150	2921	6	1	0	1635
1999	2580	4	1	0	1732
1900	2580	4	1	0	1534

Price	SQFT	Features	NE	Corner	Tax
Min. : 540	Min. : 837	Min. :0.000	Min. :0.0000	Min. :0.0000	Min. : 223.0
1st Qu.: 815	1st Qu.:1290	1st Qu.:3.000	1st Qu.:0.0000	1st Qu.:0.0000	1st Qu.: 600.0
Median : 975	Median :1565	Median :4.000	Median :1.0000	Median :0.0000	Median : 731.0
Mean :1077	Mean :1667	Mean :3.533	Mean :0.6636	Mean :0.1963	Mean : 793.5
3rd Qu.:1190	3rd Qu.:1897	3rd Qu.:4.000	3rd Qu.:1.0000	3rd Qu.:0.0000	3rd Qu.: 919.0
Max. :2150	Max. :3750	Max. :8.000	Max. :1.0000	Max. :1.0000	Max. :1765.0

- New data after excluding Age from the dataset and removing na values from variable Tax.



- House $Price$ seems to be positively correlated with $SQFT, Tax$ which maps to our intuition. While the relationship between $Price$ and other variables are unclear.

b. Fit a model with $Price$ as the response and $SQFT, Features, NE, Corner, Tax$ as predictors. Form 95% and 99% C.I. for their coefficients. Explain how the p-value for the parameter for $Corner$ relates to whether zero falls in the two corresponding C.I.. $Price = \beta_0 + \beta_1 * SQFT + \beta_2 * Features + \beta_3 * NE + \beta_4 * Corner + \beta_5 * Tax + \epsilon$

```
##
## Call:
## lm(formula = Price ~ SQFT + factor(Features) + factor(NE) + factor(Corner) +
##     Tax)
##
```

```
## Residuals:
##     Min      1Q  Median      3Q     Max
## -507.41  -72.61  -14.43   64.13  607.27
##
## Coefficients:
##                    Estimate Std. Error t value Pr(>|t|)
## (Intercept)       154.54826  136.56039   1.132 0.260631
## SQFT                0.25275    0.06704   3.770 0.000285 ***
## factor(Features)1 -34.35628  153.76729  -0.223 0.823685
## factor(Features)2 -44.18014  140.34877  -0.315 0.753620
## factor(Features)3 -31.40899  138.77495  -0.226 0.821436
## factor(Features)4 -44.75297  139.38047  -0.321 0.748859
## factor(Features)5   8.07140  156.79580   0.051 0.959055
## factor(Features)6  35.90463  151.67262   0.237 0.813386
## factor(Features)7  98.54855  242.25824   0.407 0.685086
## factor(Features)8 118.43407  220.81098   0.536 0.592977
## factor(NE)1        -4.56309   37.89202  -0.120 0.904405
## factor(Corner)1   -83.75216   44.94169  -1.864 0.065503 .
## Tax                 0.69047    0.12462   5.540 2.74e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 176.4 on 94 degrees of freedom
## Multiple R-squared:  0.8128, Adjusted R-squared:  0.7889
## F-statistic: 34.01 on 12 and 94 DF,  p-value: < 2.2e-16
##
## Correlation of Coefficients:
##                   (Intercept) SQFT  factor(Features)1 factor(Features)2
## SQFT              -0.39
## factor(Features)1 -0.80        0.19
## factor(Features)2 -0.87        0.19  0.81
## factor(Features)3 -0.87        0.20  0.83              0.92
## factor(Features)4 -0.87        0.20  0.83              0.92
## factor(Features)5 -0.76        0.18  0.76              0.85
## factor(Features)6 -0.78        0.14  0.77              0.85
## factor(Features)7 -0.51        0.20  0.53              0.59
## factor(Features)8 -0.48       -0.06  0.49              0.54
## factor(NE)1        0.01        0.01 -0.14             -0.15
## factor(Corner)1    0.06       -0.18 -0.11             -0.14
## Tax                0.27       -0.85 -0.23             -0.26
##                   factor(Features)3 factor(Features)4 factor(Features)5
## SQFT
## factor(Features)1
## factor(Features)2
## factor(Features)3
## factor(Features)4  0.95
## factor(Features)5  0.88              0.89
## factor(Features)6  0.88              0.89              0.83
## factor(Features)7  0.62              0.64              0.61
## factor(Features)8  0.55              0.55              0.51
## factor(NE)1       -0.14             -0.16             -0.18
## factor(Corner)1   -0.13             -0.12             -0.17
## Tax               -0.30             -0.32             -0.34
##                   factor(Features)6 factor(Features)7 factor(Features)8
```

```
## SQFT
## factor(Features)1
## factor(Features)2
## factor(Features)3
## factor(Features)4
## factor(Features)5
## factor(Features)6
## factor(Features)7  0.60
## factor(Features)8  0.52              0.34
## factor(NE)1       -0.19             -0.12              -0.16
## factor(Corner)1   -0.06             -0.07               0.00
## Tax               -0.28             -0.37               0.00
##                   factor(NE)1 factor(Corner)1
## SQFT
## factor(Features)1
## factor(Features)2
## factor(Features)3
## factor(Features)4
## factor(Features)5
## factor(Features)6
## factor(Features)7
## factor(Features)8
## factor(NE)1
## factor(Corner)1    0.04
## Tax               -0.05        0.19
```

- **Note.** Features, Corner and NE are qualitative data, the numeric value are categorical.

	2.5 %	97.5 %
(Intercept)	-116.5955928	425.6921070
SQFT	0.1196449	0.3858495
factor(Features)1	-339.6648385	270.9522849
factor(Features)2	-322.8459148	234.4856306
factor(Features)3	-306.9499127	244.1319319
factor(Features)4	-321.4961537	231.9902206
factor(Features)5	-303.2503320	319.3931363
factor(Features)6	-265.2449074	337.0541723
factor(Features)7	-382.4608585	579.5579628
factor(Features)8	-319.9913118	556.8594541
factor(NE)1	-79.7985862	70.6724125
factor(Corner)1	-172.9849346	5.4806216
Tax	0.4430308	0.9379164

	0.5 %	99.5 %
(Intercept)	-204.4891622	513.5856764
SQFT	0.0764987	0.4289958
factor(Features)1	-438.6331879	369.9206342
factor(Features)2	-413.1777790	324.8174948
factor(Features)3	-396.2688303	333.4508495
factor(Features)4	-411.2047956	321.6988625

	0.5 %	99.5 %
factor(Features)5	-404.1679021	420.3107063
factor(Features)6	-362.8650747	434.6743396
factor(Features)7	-538.3841223	735.4812266
factor(Features)8	-462.1106032	698.9787456
factor(NE)1	-104.1868089	95.0606352
factor(Corner)1	-201.9104934	34.4061804
Tax	0.3628201	1.0181270

- p-value for $Corner$ is not significant whether the significance level is 95% or 99%. Since zero falls in both $\alpha = 5\%, \alpha = 1\%$ confidence intervals.

c. Predict the $Price$ of a specific house with $SQFT = 2500, Features = 5, NE = 1, Corner = 1, Tax = 120$. Give an appropriate 95% C.I..

$Price = 154.548 + 0.253 * SQFT - 34.356 * F_1 - 44.180 * F_2 - 31.409 * F_3 - 44.753 * F_4 + 8.071 * F_5 + 35.905 * F_6 + 98.549 * F_7 + 118.434 * F$

- **Note.** $If\ Featurs = x, F_x = 1, x \in [1, 8]$

```
##        fit      lwr      upr
## 1 1534.741 1396.064 1673.417
```

- This is an interpolation prediction computing for the C.I. of mean response.

d. Suppose you are only told that $SQFT = 2500$. Predict the $Price$ and 95% C.I..

```
## Warning in predict.lm(model2, interval = "prediction"): predictions on current data refer to _future_
```

```
##             fit         lwr       upr
## 1   1418.1319766  889.451658 1946.8123
## 2    750.9011812  369.767088 1132.0353
## 3    916.2190550  517.773971 1314.6641
## 4   1450.6336437 1042.185047 1859.0822
## 5   1451.5846011 1052.637448 1850.5318
## 6   1236.1570900  844.829949 1627.4842
## 7   1487.4891863 1087.097484 1887.8809
## 8    881.4024876  483.109554 1279.6954
## 9    680.6166847  297.140836 1064.0925
## 10   914.0430196  515.608935 1312.4771
## 11   683.5811421  286.941138 1080.2211
## 12   652.3282236  269.134669 1035.5218
## 13   828.0896187  429.931014 1226.2482
## 14   538.1319766    9.451658 1066.8123
## 15   379.2357727   -3.027859  761.4994
## 16   431.0239678   51.045826  811.0021
## 17   472.2370781   82.793975  861.6802
## 18   330.3581168  -57.213176  717.9294
## 19   585.9591419  203.291300  968.6270
## 20   375.5350634   -4.700886  755.7710
## 21   399.9081096   17.688237  782.1280
```

```
## 22   324.3982300  -56.195060  704.9915
## 23   317.1780540  -76.839875  711.1960
## 24   507.3222694  109.928864  904.7157
## 25   285.1057416 -103.012937  673.2244
## 26   771.4419546  378.307078 1164.5768
## 27   693.2362414  312.642598 1073.8299
## 28   752.4258551  368.077808 1136.7739
## 29   210.0248045 -179.330925  599.3805
## 30   167.1554568 -221.510233  555.8211
## 31   293.9337335  -86.926907  674.7944
## 32   123.5108976 -270.297535  517.3193
## 33   190.4481989 -197.922031  578.8184
## 34   228.2363509 -182.888202  639.3609
## 35   217.6486422 -170.609774  605.9071
## 36   227.1483332 -183.975238  638.2719
## 37   100.7032331 -283.966817  485.3733
## 38     0.1689456 -385.333355  385.6712
## 39   385.4642833  -13.212435  784.1410
## 40   308.2150243  -91.595158  708.0252
## 41    62.0543929 -328.554534  452.6633
## 42    45.8656904 -338.671634  430.4030
## 43  1289.6070192  885.625374 1693.5887
## 44   994.5563318  595.583153 1393.5295
## 45   588.7865390  208.800685  968.7724
## 46   905.8586758  503.909939 1307.8074
## 47   755.7730503  365.660036 1145.8861
## 48   448.4322515   68.507073  828.3574
## 49   545.2658297  165.391616  925.1400
## 50   437.5520742   57.595357  817.5088
## 51   508.2732268  128.428310  888.1181
## 52   737.8449684  356.845967 1118.8440
## 53   519.1534041  139.306129  899.0007
## 54   332.4510102  -49.980702  714.8827
## 55   286.7542654  -95.933973  669.4425
## 56   473.8933155   91.676730  856.1099
## 57    63.2739741 -320.919032  447.4670
## 58   353.7747088  -26.599278  734.1487
## 59   430.3726062   48.183117  812.5621
## 60   259.5761426 -145.256293  664.4086
## 61   306.8660963  -81.249358  694.9816
## 62   407.6073758   20.153778  795.0610
## 63   304.6900608  -83.424797  692.8049
## 64   235.0492124 -154.187266  624.2857
## 65    68.7140628 -315.373989  452.8021
## 66   104.2023294 -307.120622  515.5253
## 67    46.3023465 -339.229296  431.8340
## 68   229.8777475 -171.332893  631.0884
## 69   115.4988253 -267.738787  498.7364
## 70   251.3771919 -136.786836  639.5412
## 71   -53.2677733 -444.054920  337.5194
## 72   826.0599038  431.958547 1220.1613
## 73   296.6319241  -93.407992  686.6718
## 74   343.4244001  -46.766065  733.6149
## 75   218.0098923 -194.908986  630.9288
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## 76   240.6232216 -144.055931  625.3024
## 77    13.2275154 -374.057613  400.5126
## 78   170.9493787 -223.692661  565.5914
## 79   -32.9560411 -490.806649  424.8946
## 80   -10.1813630 -401.675582  381.3129
## 81   903.2260208  513.371954 1293.0801
## 82    74.1565085 -312.212471  460.5255
## 83   -30.7800057 -488.630614  427.0706
## 84    44.2195236 -346.726134  435.1652
## 85   673.3546834  270.703061 1076.0063
## 86   874.3770537  476.157259 1272.5968
## 87    52.0629773 -366.540520  470.6665
## 88   516.5430696  133.661554  899.4246
## 89   475.6350519   91.294346  859.9758
## 90   362.0445515  -20.990918  745.0800
## 91   443.6458815   60.821729  826.4700
## 92   389.2449949    6.312287  772.1777
## 93   600.3204350  217.086277  983.5546
## 94   455.6140766   72.796446  838.4317
## 95   298.3104819 -110.484701  707.1057
## 96   426.6335459   30.361733  822.9054
## 97   545.2681868  160.566084  929.9703
## 98   567.0285414  182.170325  951.8868
## 99   291.7600550  -92.640833  676.1609
## 100  515.4550519  132.576100  898.3340
## 101  469.1069455   84.789312  853.4246
## 102  342.4602324  -40.669223  725.5897
## 103  372.2733672  -11.919943  756.4667
## 104  267.3870088 -116.257551  651.0316
## 105  328.3160018  -54.891736  711.5237
## 106  210.2418671 -179.636866  600.1206
## 107  109.0562179 -281.681236  499.7937
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