1 Buildings

Definition 1 A Polyhedral complex is a certain finite dimensional CW-complex. Each n-cell of the polyhedral complex is

Definition 2 Suppose P is a simple convex polytope in X^n . Let F_i be the codimension-one faces of P. Suppose that, for any two faces F_i and F_j , if their intersection is non-empty, then the dihedral angle between the faces is p_i/m_i , for some m_i in 2, 3, 4, ... Now set $m_i = 1, m_i$ in f_i if f_i if f_i if f_i intersection. Let f_i be the reflection of f_i across f_i and let f_i be the group generated by the set of f_i . Then f_i is the Coxeter group with generators f_i and Coxeter matrix f_i in f_i is a discrete subgroup of f_i is a strict fundamental domain for the f_i across f_i and f_i it is a strict fundamental domain for the f_i and f_i it is f_i .

Definition 3 Let (W, S) be a Coxeter group generated by a simple convex polytope P. A building of type (W, S) is a polyhedral complex, which is a union of subcomplexs, called apartments. An apartment is isometric to the tiling of X^n derived from P, and each copy of P in the tiling is called a chamber. Now the apartments and chambers must satisfy

- 1. Given any two chambers, there exists an apriment containing both of them.
- 2. Given any two apartments A and B, there exists an isometry from A to B which fixes $A \cap B$ pointwise.

Example 1 Let us consider a single copy of X^n . We can tile this copy by P, and we get a thin building. This means that we only have one apartment. Clearly this satisfies the first condition - any two chambers immediately lie in the only apartment.

Now let us look at the second condition. If the two chambers have no intersection, then, as each chamber is a copy of P, they are clearly isometric, and we are done. Now if the two chambers have a non-empty intersection, we have two cases:

- 1. If they share a common edge, then reflection along this edge gives us our isometry.
- 2. If they only share a common point