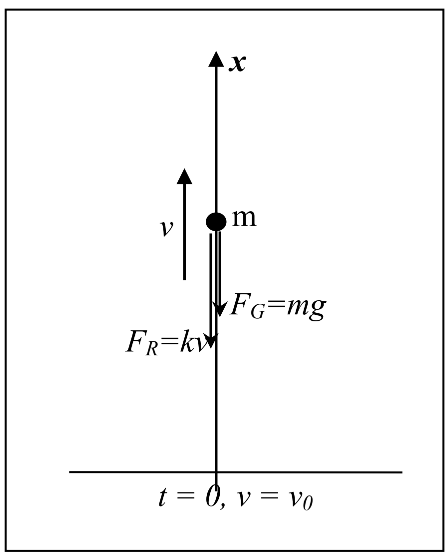
Homework 3

Assignment Date: Friday (01/26/2018)
Collection Date: Friday (03/23/2018) 11:59PM by email
Grade: **Total 100 points**

Do any two of three problems for a max total of 100 points.
If all three are attempted, the best two will be credited.

Problem 3.1 (50 Points): Shooting a bullet of mass $m = 0.1\ kg$ straight up from ground to the sky at an initial speed of $v_0 = 300.0\ m/s$. FYI: Sound speed is 343.2 m/s. There are two forces acting on the bullet: $F_G = mg$ (the gravity) and $F_R = kv$ (the air resistance). The bullet will reach the highest point and then reverse its direction to fall down. Naturally, when the bullet goes up, both F_G and F_R are in the opposite direction of the bullet motion. When the bullet falls down, F_G still points toward the earth while F_R points up (opposite direction of the bullet motion). You are given the resistance constant $\mathrm{k} = 9.11 \times 10^{-3}$ Newton×s/m and the gravitational constant $\mathrm{g} = 9.8\ \mathrm{m/s^2}$.

Write a program to compute the total amount of time (and the total distance traveled by) the bullet is motion in the air.



Notes: For computational convenience, one may consider the following two differential equations for the two different motions (up & down):
(a) When going up (the velocity points up):

$$m\frac{dv}{dt} = -F_G - F_R = -mg - kv$$

(b) When going down (the velocity points down):

$$m\frac{dv}{dt} = F_G - F_R = mg - kv$$

Problem 3.2 (50 Points): In a refrigerated warehouse that's kept at a constant temperature of 33.33F, a corpse was found at midnight with a temperature of 91.11F. Two hours (120 minutes) later, its temperature dropped to 66.66F. At the time of death, the person's body temperature was at the normal 98.88F. Write a program to estimate when the person die? Also, compute the time when the corpse's temperature will reach 44.44F.

The corpse's temperature changes according to the Newton's Law of Cooling invented by I. Newton in 1701:

$$\begin{cases} \dfrac{dT(t)}{dt} = k(A - T) \\ T(t = 0) = T_0 \end{cases}$$

where A is the temperature of the warehouse, $T(t)$ is the temperature of the corpse at time t, and $T(t = 0) = T_0$ is the initial temperature of the corpse (the instance when the person died.)

Problem 3.3 (50 Points)

A prey-predator model involving two competing (and yet, sometimes, cooperating) stocks can be expressed by the following system of two coupled ODEs:

$$\begin{cases} x' = ax + bxy \\ y' = cy + exy \\ x(t=0) = 199 \\ y(t=0) = 21 \\ a = 0.222 \\ b = -0.0011 \\ c = -1.999 \\ e = 0.010 \end{cases}$$

where "x" and "y" denote the stock values for both stocks (although you might see negative stock values) and a, b, c, e are given constants or parameters. Please use your favorite method to

(1) Compute the stock values at $t = 11$ with time step size $\Delta t = 0.0001$;

(2) Draw a phase diagram for the stock values for $t \in [0, 11]$;

(3) Calculate the first instance when $y \geq x$.

Of course, the following chickens and foxes also compete (and sometimes, cooperate). For that matter, you may find thousands of examples.

They cooperate.

They compete.