Homework 4

Assignment Date: Friday (03/23/2018)
Collection Date: Friday (04/20/2018) 11:59PM by email
Grade: **Total 100 points**

Do any two or all three problems for a max 100 points.

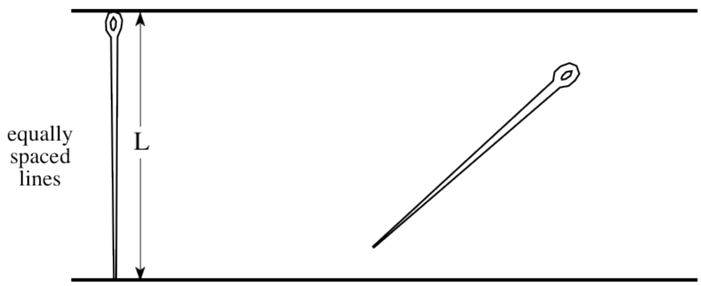
Problem 4.1 (50 Points): Mr. Poor made an investment of $1,000,000 precisely 260 trading days ago. The stock portfolio (the totality of all investments including stocks, bonds, foreign exchanges etc) change from the previous day follows the following normal distribution

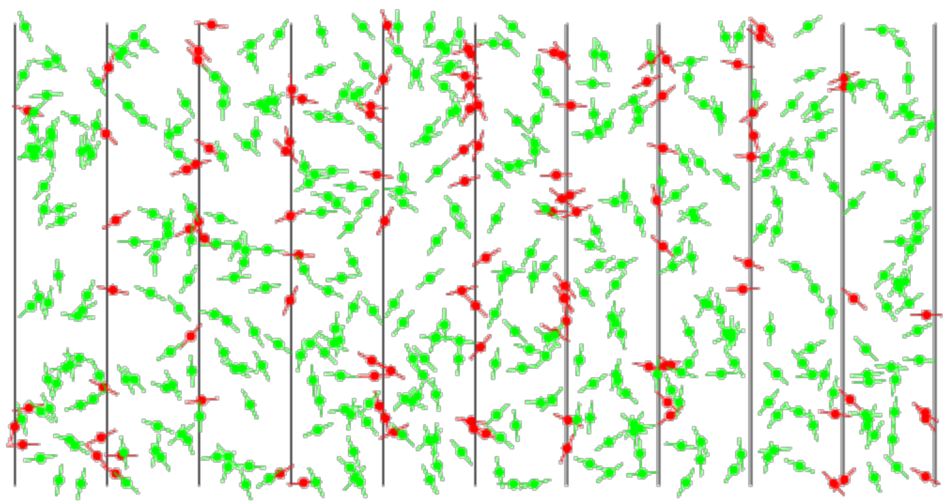
$$P(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

where μ is the mean, σ is the standard deviation, x is the change rate of portfolio from the previous day, and $P(x)$ is the probability of change x. Hypothetically, each day, Mr. Poor pays his trader(s) a performance fee of 3.333% of his total gain if $x > 0$. Otherwise, if $x \leq 0$, the trader(s) gets no fees or any deductions, *i.e.*, Mr. Poor takes the losses alone. *Is it this course's business to ask "Is this fair?"*

(1) If the mean $\mu = 0.1949\%$ and standard deviation mean $\sigma = 0.2018\%$, please compute, and plot a graph for, Mr. Poor's investment value (a time series) at the end of each of the 260 trading days.
(2) Do the same as above if the mean $\mu = -0.1989\%$ and standard deviation mean (unchanged) $\sigma = 0.6464\%$.
(3) Graph the two time series in one figure and briefly explain why they follow such trends.
(4) Report the total portfolio values after 260th trading day for both scenarios.

Problem 4-1 (50 points): Buffon's Needle refers to a simple Monte Carlo method for the estimation of the value of $\pi = 3.14159265$... The idea is very simple. Suppose you have a tabletop with a number of parallel lines drawn on it, which are equally spaced (say the spacing is 1 inch). Suppose you also have a pin or needle, which is also an inch long. If you drop the needle on the table, you will find that one of two things happens: (1) The needle crosses or touches one of the lines, or (2) the needle crosses no lines. The idea now is to keep dropping this needle over and over on the table, and to record the statistics: keeping track of both the total number of times N that the needle is randomly dropped on the table and the number of times N_C that it crosses a line.

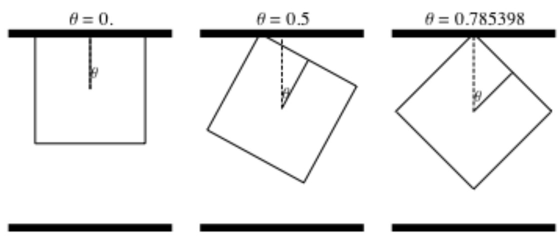




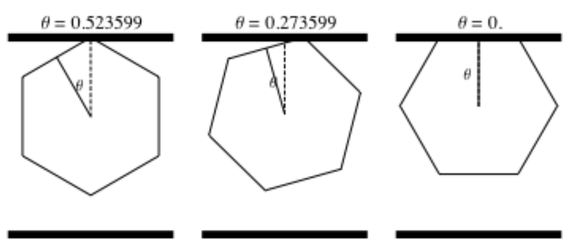
Buffon needle problem has many variations. For tossing n-gons of diameter D=1 to lines of distance L=1, the following is the probability of hits:

n	p	$N[p]$
2	$\frac{2}{\pi}$	0.63662
3	$\frac{3\sqrt{3}}{\pi}$	0.82699
4	$\frac{2\sqrt{2}}{\pi}$	0.90032
5	$\frac{5\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}}{\pi}$	0.93549
6	$\frac{3}{\pi}$	0.95493
8	$\frac{4\sqrt{2-\sqrt{2}}}{\pi}$	0.97450
12	$\frac{3\sqrt{2}\left(\sqrt{3}-1\right)}{\pi}$	0.98862
∞	1	1.00000

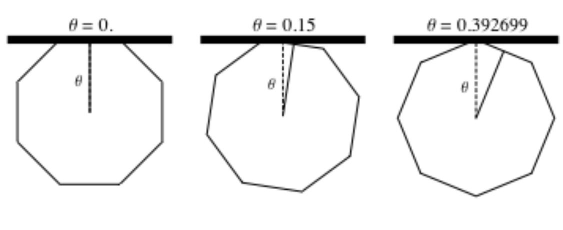
Tossing Squares:



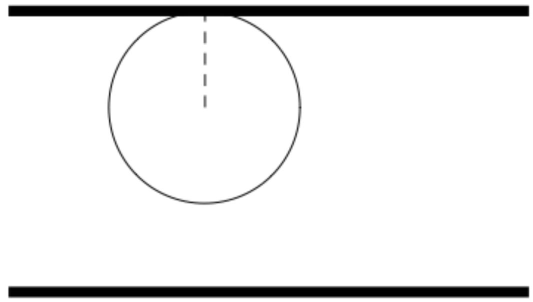
Tossing Hexagons:



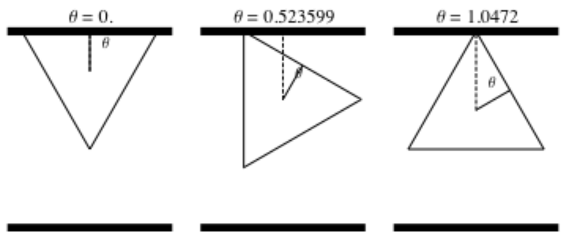
Tossing Octagons:



Tossing Disks:



Tossing Triangles:



Your HW requires you to perform numerical experiments for tossing equilateral triangles of side length 1/2 to parallel lines of distance 1. Estimate the probability when any part of the triangle crosses a parallel line for 300,000,000 tosses.

Problem 4.3 (50 Points): We set a 2-dimensional Cartesian coordinate system for a section of a river such that the *y*-axis lies at the river's west bank and points north and the *x*-axis runs from west to east. The water is assumed to flow strictly northward at all times and its speed, denoted by $w(x)$, varies depending on the x-coordinate of the location. The water flow will drag the boat at full efficiency and you may ignore the water resistance (*Many simplifications were made to make HW life less painful!*)

A ferryboat tries to cross the river from a point $(a, 0)$ to dock at $(0,0)$. It can do a lot of good things to cross optimally in time, energy, and safety but it does two simple things: moving at a constant speed v_B relative to water and keeping its head toward the docking point $(0, 0)$ at all times.

Please compute the boat's trajectories for three cases of boat speeds: $v_B = 5, 10, 15$

In this problem, we assume the river width $a = 1000$ (any unit) and $v_0 = 10$.