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AMS326 HW4 Report

For this homework, I have done questions one and three.

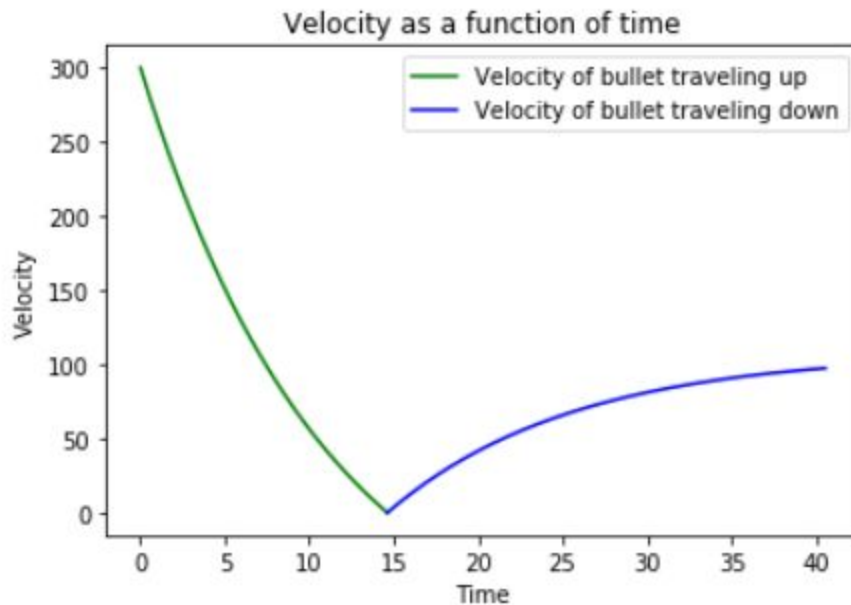
Question 1

This problem is split into 2 parts to solve. The first part is solving the differential equation of the motion of the bullet going up, and the second part is solving the differential equation of the motion of the bullet coming down. Euler's method is used for both parts.

For the first part, the differential equation to be solved is $\frac{dv}{dt} = -g - \frac{kv}{m}$ from $v = 300$ to $v = 0$ since the velocity of the bullet has to be 0 when changing directions from up to down. Euler's method, which is $v_1 = v_0 + h * \frac{dv}{dt}$, where h is the length of the interval from the first point to the second point, is done repeatedly until $v_1 = 0$. The distance traveled by the bullet, which is the area under the graph of the solved equation, is then calculated using the midpoint method by adding $\frac{v_0 + v_1}{2} * h$, where v_0 is the current velocity calculated and v_1 is the velocity of the next step, for each iteration of Euler's method. **It is found that the time taken for the bullet to reach the top of its path is 14.622 seconds, and the distance the bullet traveled upwards is 1720.16 meters.**

For the second part, the differential equation to be solved is $\frac{dv}{dt} = -g - \frac{kv}{m}$. Because the final velocity of the bullet is unknown, Euler's method is repeated until the distance the bullet traveled downwards, or the area under the solved equation, is equal to the distance the bullet traveled upward, which is 1720.16 meters. **It is found that the total time taken for the bullet to travel downwards is 25.934 seconds.**

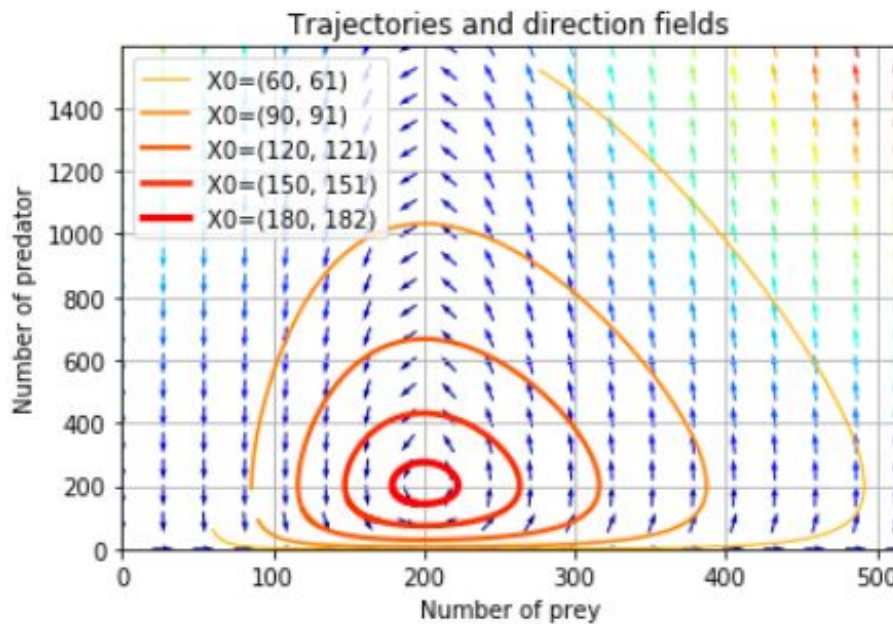
In total, the bullet took 40.555 seconds to go both upwards and downwards, and traveled 3440.32 meters. A graph is drawn in the program to verify and graphically understand the results.



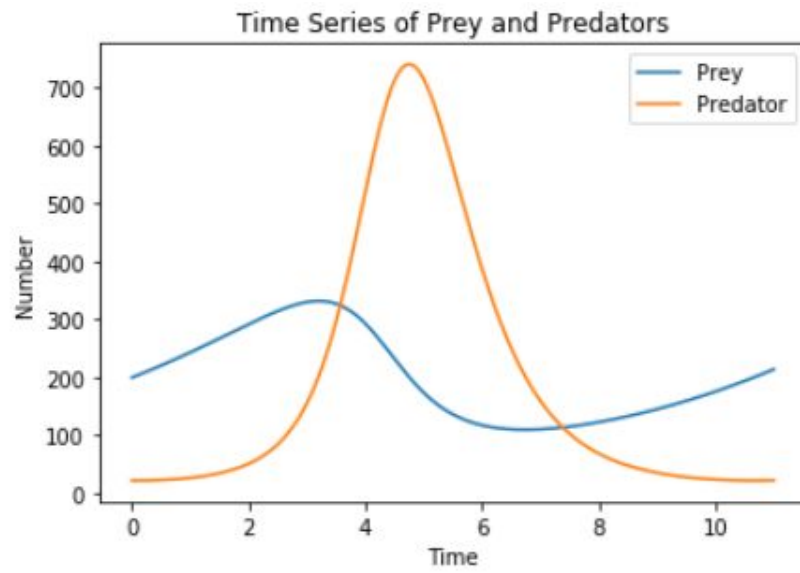
Question 2

- 1) To find the the next value of each differential equation, Euler's method is used in the function findNext. $x_1 = x + h * x'$ and $y_1 = y + h * y'$, where h is the length of the interval from the current step to the next step, or Δt , and x' and y' are the given functions for the predator-prey equation. The function findstep then finds all the values from time 0, with the given initial prey and predators values, to time 11 can be found, with $\Delta t = 0.0001$. This is done by calling the findNext $\frac{11}{\Delta t}$ times. **It is found that at time 11, the number of prey is 213.2067, rounded down to 213, and the number of predators is 21.4363, rounded down to 21.**
- 2) The phase graph is graphed with the help of code from the scipy-cookbook website. First, the equilibrium is found when the number of prey = c/e and the number of

predators = a/b . The values for the start numbers are then established, as are the values for the colors. The given functions x' and y' are then integrated and plotted. Finally, the direction arrows are drawn. The graph is shown below.



- 3) The findNext function is called repeatedly until the the number of predators is greater or equal to the number of prey starting with the given initial values of predators and prey. The number of iterations, given by the variable counter, is counted. The first instance when the number of predators is greater or equal to the number of prey is found by the number of iterations multiplied by the length of each step, or $counter * h$. **It is found that the first instance occurs at $t=3.5696$** , with the number of prey is 323.6620 rounded down to 323, and the number of predators is 323.6730 rounded down to 323.
- 4) The prey-predator equations are graphed in a time series graph to be graphically analyzed.



Sources

<http://scipy-cookbook.readthedocs.io/items/LotkaVolterraTutorial.html>