# dwavebinarycsp

Release 0.1.2

**D-Wave Systems Inc** 

# Contents

1 Documentation	3
2 Indices and tables	35
Python Module Index	37
Index	30

Library to construct a binary quadratic model from a constraint satisfaction problem with small constraints over binary variables.

Below is an example usage:

```
import dwavebinarycsp
import dimod

csp = dwavebinarycsp.factories.random_2in4sat(8, 4)  # 8 variables, 4 clauses

bqm = dwavebinarycsp.stitch(csp)

resp = dimod.ExactSolver().sample(bqm)

for sample, energy in resp.data(['sample', 'energy']):
    print(sample, csp.check(sample), energy)
```

Contents 1

2 Contents

## CHAPTER 1

**Documentation** 

**Note:** This documentation is for the latest version of dwavebinarycsp. Documentation for the version currently installed by dwave-ocean-sdk is here: dwavebinarycsp.

## 1.1 Introduction

dwavebinarycsp is a library to construct a binary quadratic model from a constraint satisfaction problem (CSP) with small constraints over binary variables (represented as either binary values {0, 1} or spin values {-1, 1}).

For an introduction to CSPs, see Constraints.

## 1.1.1 Example: Solving a Map-Coloring CSP

The map-coloring CSP, for example, is to assign a color to each region of a map such that any two regions sharing a border have different colors.

The constraints for the map-coloring problem can be expressed as follows:

- Each region is assigned one color only, of C possible colors.
- The color assigned to one region cannot be assigned to adjacent regions.

Solving such problems as the map-coloring CSP on a sampler such as the D-Wave system necessitates that the mathematical formulation use binary variables because the solution is implemented physically with qubits, and so must translate to spins  $s_i \in \{-1, +1\}$  or equivalent binary values  $x_i \in \{0, 1\}$ . This means that in formulating the problem by stating it mathematically, you might use unary encoding to represent the C colors: each region is represented by C variables, one for each possible color, which is set to value 1 if selected, while the remaining C-1 variables are 0.

This example finds a solution to the map-coloring problem for a map of Canada using four colors. Canada's 13 provinces are denoted by postal codes:



Fig. 1: Coloring a map of Canada with four colors.

Code	Province	Code	Province
AB	Alberta	BC	British Columbia
MB	Manitoba	NB	New Brunswick
NL	Newfoundland and Labrador	NS	Nova Scotia
NT	Northwest Territories	NU	Nunavut
ON	Ontario	PE	Prince Edward Island
QC	Quebec	SK	Saskatchewan
YT	Yukon		

Table 1: Canadian Provinces' Postal Codes

#### The workflow for solution is as follows:

- 1. Formulate the problem as a graph, with provinces represented as nodes and shared borders as edges, using 4 binary variables (one per color) for each province.
- 2. Create a binary constraint satisfaction problem and add all the needed constraints.
- 3. Convert to a binary quadratic model.
- 4. Sample.
- 5. Plot a valid solution, if such was found.

The following sample code creates a graph of the map with provinces as nodes and shared borders between provinces as edges (e.g., "('AB', 'BC')" is an edge representing the shared border between British Columbia and Alberta). It creates a binary constraint satisfaction problem based on two types of constraints:

- csp.add\_constraint(one\_color\_configurations, variables) represents the constraint that each node (province) select a single color, as represented by valid configurations one\_color\_configurations = {(0, 0, 0, 1), (0, 0, 1, 0), (0, 1, 0, 0), (1, 0, 0, 0)}
- csp.add\_constraint(not\_both\_1, variables) represents the constraint that two nodes (provinces) with a shared edge (border) not both select the same color.

```
import dwavebinarycsp
from dwave.system.samplers import DWaveSampler
from dwave.system.composites import EmbeddingComposite
```

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```
import networkx as nx
import matplotlib.pyplot as plt
# Represent the map as the nodes and edges of a graph
provinces = ['AB', 'BC', 'MB', 'NB', 'NL', 'NS', 'NT', 'NU', 'ON', 'PE', 'QC', 'SK',
'YT']
neighbors = [('AB', 'BC'), ('AB', 'NT'), ('AB', 'SK'), ('BC', 'NT'), ('BC', 'YT'), (
\hookrightarrow 'MB', 'NU'),
             ('MB', 'ON'), ('MB', 'SK'), ('NB', 'NS'), ('NB', 'QC'), ('NL', 'QC'), (
\hookrightarrow 'NT', 'NU'),
             ('NT', 'SK'), ('NT', 'YT'), ('ON', 'QC')]
# Function for the constraint that two nodes with a shared edge not both select one,
⇔color
def not_both_1(v, u):
   return not (v and u)
# Function that plots a returned sample
def plot_map(sample):
   G = nx.Graph()
   G.add_nodes_from(provinces)
   G.add_edges_from(neighbors)
    # Translate from binary to integer color representation
   color_map = {}
    for province in provinces:
          for i in range(colors):
            if sample[province+str(i)]:
                color_map[province] = i
    # Plot the sample with color-coded nodes
   node_colors = [color_map.get(node) for node in G.nodes()]
   nx.draw_circular(G, with_labels=True, node_color=node_colors, node_size=3000,...
plt.show()
# Valid configurations for the constraint that each node select a single color
one_color_configurations = \{(0, 0, 0, 1), (0, 0, 1, 0), (0, 1, 0, 0), (1, 0, 0, 0)\}
colors = len(one_color_configurations)
# Create a binary constraint satisfaction problem
csp = dwavebinarycsp.ConstraintSatisfactionProblem(dwavebinarycsp.BINARY)
# Add constraint that each node (province) select a single color
for province in provinces:
    variables = [province+str(i) for i in range(colors)]
    csp.add_constraint(one_color_configurations, variables)
# Add constraint that each pair of nodes with a shared edge not both select one color
for neighbor in neighbors:
   v, u = neighbor
          for i in range(colors):
        variables = [v+str(i), u+str(i)]
                    csp.add_constraint(not_both_1, variables)
# Convert the binary constraint satisfaction problem to a binary quadratic model
bgm = dwavebinarycsp.stitch(csp)
# Set up a solver using the local system's default D-Wave Cloud Client configuration,
                                                                          (continues on next page)
→ file
```

1.1. Introduction 5

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```
# and sample 50 times
sampler = EmbeddingComposite(DWaveSampler())  # doctest: +SKIP
response = sampler.sample(bqm, num_reads=50)  # doctest: +SKIP

# Plot the lowest-energy sample if it meets the constraints
sample = next(response.samples())  # doctest: +SKIP
if not csp.check(sample):  # doctest: +SKIP
    print("Failed to color map")
else:
    plot_map(sample)
```

The plot shows a solution returned by the D-Wave solver. No provinces sharing a border have the same color.

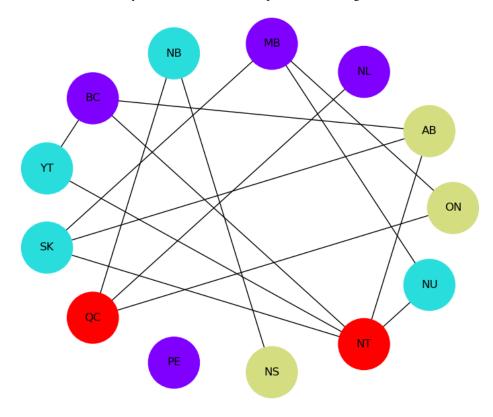


Fig. 2: Solution for a map of Canada with four colors. The graph comprises 13 nodes representing provinces connected by edges representing shared borders. No two nodes connected by an edge share a color.

## 1.2 Reference Documentation

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## 1.2.1 Defining Constraint Satisfaction Problems

Constraint satisfaction problems require that all a problem's variables be assigned values, out of a finite domain, that result in the satisfying of all constraints. The ConstraintSatisfactionProblem class aggregates all con-

straints and variables defined for a problem and provides functionality to assist in problem solution, such as verifying whether a candidate solution satisfies the constraints.

#### **Class**

## class ConstraintSatisfactionProblem(vartype)

A constraint satisfaction problem.

**Parameters vartype** (Vartype/str/set) – Variable type for the binary quadratic model. Supported values are:

```
SPIN, 'SPIN', {-1, 1}BINARY, 'BINARY', {0, 1}
```

#### constraints

Constraints that together constitute the constraint satisfaction problem. Valid solutions satisfy all of the constraints

```
Type list[Constraint]
```

#### variables

Variables of the constraint satisfaction problem as a dict, where keys are the variables and values a list of all of constraints associated with the variable.

```
Type dict[variable, list[Constraint]]
```

## vartype

Enumeration of valid variable types. Supported values are SPIN or BINARY. If *vartype* is SPIN, variables can be assigned -1 or 1; if BINARY, variables can be assigned 0 or 1.

```
Type dimod. Vartype
```

## **Example**

This example creates a binary-valued constraint satisfaction problem, adds two constraints, a=b and  $b\neq c$ , and tests a,b,c=1,1,0.

```
>>> import operator
>>> csp = dwavebinarycsp.ConstraintSatisfactionProblem('BINARY')
>>> csp.add_constraint(operator.eq, ['a', 'b'])
>>> csp.add_constraint(operator.ne, ['b', 'c'])
>>> csp.check({'a': 1, 'b': 1, 'c': 0})
True
```

#### **Methods**

#### Adding variables and constraints

ConstraintSatisfactionProblem.	Add a constraint.	
add_constraint()		
ConstraintSatisfactionProblem.	Add a variable.	
add_variable(v)		

## dwavebinarycsp.ConstraintSatisfactionProblem.add\_constraint

ConstraintSatisfactionProblem.add\_constraint(constraint, variables=())
Add a constraint.

#### **Parameters**

- **constraint** (function/iterable/*Constraint*) Constraint definition in one of the supported formats:
  - 1. Function, with input arguments matching the order and *vartype* type of the *variables* argument, that evaluates True when the constraint is satisfied.
- 2. List explicitly specifying each allowed configuration as a tuple.
- 3. Constraint object built either explicitly or by dwavebinarycsp.factories.
- variables (iterable) Variables associated with the constraint. Not required when constraint is a Constraint object.

## **Examples**

This example defines a function that evaluates True when the constraint is satisfied. The function's input arguments match the order and type of the *variables* argument.

This example explicitly lists allowed configurations.

```
>>> csp = dwavebinarycsp.ConstraintSatisfactionProblem(dwavebinarycsp.SPIN)
>>> eq_configurations = {(-1, -1), (1, 1)}
>>> csp.add_constraint(eq_configurations, ['v0', 'v1'])
>>> csp.check({'v0': -1, 'v1': +1})
False
>>> csp.check({'v0': -1, 'v1': -1})
True
```

This example uses a Constraint object built by dwavebinarycsp.factories.

```
>>> import dwavebinarycsp.factories.constraint.gates as gates
>>> csp = dwavebinarycsp.ConstraintSatisfactionProblem(dwavebinarycsp.BINARY)
>>> csp.add_constraint(gates.and_gate(['a', 'b', 'c'])) # add an AND gate
>>> csp.add_constraint(gates.xor_gate(['a', 'c', 'd'])) # add an XOR gate
>>> csp.check({'a': 1, 'b': 0, 'c': 0, 'd': 1})
True
```

## dwavebinarycsp.ConstraintSatisfactionProblem.add\_variable

ConstraintSatisfactionProblem.add\_variable(v) Add a variable.

**Parameters v** (*variable*) – Variable in the constraint satisfaction problem. May be of any type that can be a dict key.

## **Examples**

This example adds two variables, one of which is already used in a constraint of the constraint satisfaction problem.

```
>>> import operator
>>> csp = dwavebinarycsp.ConstraintSatisfactionProblem(dwavebinarycsp.SPIN)
>>> csp.add_constraint(operator.eq, ['a', 'b'])
>>> csp.add_variable('a') # does nothing, already added as part of the constraint
>>> csp.add_variable('c')
>>> csp.check({'a': -1, 'b': -1, 'c': 1})
True
>>> csp.check({'a': -1, 'b': -1, 'c': -1})
True
```

## Satisfiability

ConstraintSatisfactionProblem.
check(solution)

Check that a solution satisfies all of the constraints.

## dwavebinarycsp.ConstraintSatisfactionProblem.check

ConstraintSatisfactionProblem.check(solution)

Check that a solution satisfies all of the constraints.

**Parameters solution** (container) – An assignment of values for the variables in the constraint satisfaction problem.

**Returns** True if the solution satisfies all of the constraints; False otherwise.

Return type bool

## **Examples**

This example creates a binary-valued constraint satisfaction problem, adds two logic gates implementing Boolean constraints,  $c=a \wedge b$  and  $d=a \oplus c$ , and verifies that the combined problem is satisfied for a given assignment.

```
>>> import dwavebinarycsp.factories.constraint.gates as gates
>>> csp = dwavebinarycsp.ConstraintSatisfactionProblem(dwavebinarycsp.BINARY)
>>> csp.add_constraint(gates.and_gate(['a', 'b', 'c'])) # add an AND gate
>>> csp.add_constraint(gates.xor_gate(['a', 'c', 'd'])) # add an XOR gate
>>> csp.check({'a': 1, 'b': 0, 'c': 0, 'd': 1})
True
```

#### **Transformations**

ConstraintSatisfactionProblem.	Fix the value of a variable and remove it from the con-
fix_variable(v,)	straint satisfaction problem.

## dwavebinarycsp.ConstraintSatisfactionProblem.fix variable

```
ConstraintSatisfactionProblem.fix_variable(v, value)
```

Fix the value of a variable and remove it from the constraint satisfaction problem.

#### **Parameters**

- **v** (*variable*) Variable to be fixed in the constraint satisfaction problem.
- **value** (*int*) Value assigned to the variable. Values must match the *vartype* of the constraint satisfaction problem.

## **Examples**

This example creates a spin-valued constraint satisfaction problem, adds two constraints, a = b and  $b \neq c$ , and fixes variable b to +1.

```
>>> import operator
>>> csp = dwavebinarycsp.ConstraintSatisfactionProblem(dwavebinarycsp.SPIN)
>>> csp.add_constraint(operator.eq, ['a', 'b'])
>>> csp.add_constraint(operator.ne, ['b', 'c'])
>>> csp.check({'a': +1, 'b': +1, 'c': -1})
True
>>> csp.check({'a': -1, 'b': -1, 'c': +1})
True
>>> csp.fix_variable('b', +1)
>>> csp.check({'a': +1, 'b': +1, 'c': -1}) # 'b' is ignored
True
>>> csp.check({'a': -1, 'b': -1, 'c': +1})
False
>>> csp.check({'a': +1, 'c': -1})
True
>>> csp.check({'a': -1, 'c': +1})
False
```

## 1.2.2 Converting to a Binary Quadratic Model

Constraint satisfaction problems can be converted to binary quadratic models to be solved on samplers such as the D-Wave system.

## **Compilers**

Compilers accept a constraint satisfaction problem and return a dimod.BinaryQuadraticModel.

<pre>stitch(csp[, min_classical_gap, max_graph_size])</pre>	Build a binary quadratic model with minimal energy
	levels at solutions to the specified constraint satisfaction
	problem.

## dwavebinarycsp.stitch

```
stitch (csp, min_classical_gap=2.0, max_graph_size=8)
```

Build a binary quadratic model with minimal energy levels at solutions to the specified constraint satisfaction problem.

#### **Parameters**

- csp (ConstraintSatisfactionProblem) Constraint satisfaction problem.
- min\_classical\_gap (float, optional, default=2.0) Minimum energy gap from ground. Each constraint violated by the solution increases the energy level of the binary quadratic model by at least this much relative to ground energy.
- max\_graph\_size (int, optional, default=8) Maximum number of variables in the binary quadratic model that can be used to represent a single constraint.

Returns BinaryQuadraticModel

#### **Notes**

For a *min\_classical\_gap* > 2 or constraints with more than two variables, requires access to factories from the penaltymodel ecosystem to construct the binary quadratic model.

#### **Examples**

This example creates a binary-valued constraint satisfaction problem with two constraints, a = b and  $b \neq c$ , and builds a binary quadratic model such that each constraint violation by a solution adds the default minimum energy gap.

```
>>> import operator
>>> csp = dwavebinarycsp.ConstraintSatisfactionProblem(dwavebinarycsp.BINARY)
>>> csp.add_constraint(operator.eq, ['a', 'b']) # a == b
>>> csp.add_constraint(operator.ne, ['b', 'c']) # b != c
>>> bqm = dwavebinarycsp.stitch(csp)
```

Variable assignments that satisfy the CSP above, violate one, then two constraints, produce energy increases of the default minimum classical gap:

```
>>> bqm.energy({'a': 0, 'b': 0, 'c': 1}) # doctest: +SKIP
-2.0
>>> bqm.energy({'a': 0, 'b': 0, 'c': 0}) # doctest: +SKIP
0.0
>>> bqm.energy({'a': 1, 'b': 0, 'c': 0}) # doctest: +SKIP
2.0
```

This example creates a binary-valued constraint satisfaction problem with two constraints, a = b and  $b \neq c$ , and builds a binary quadratic model with a minimum energy gap of 4. Note that in this case the conversion to binary quadratic model adds two ancillary variables that must be minimized over when solving.

```
>>> import operator
>>> import itertools
>>> csp = dwavebinarycsp.ConstraintSatisfactionProblem(dwavebinarycsp.BINARY)
>>> csp.add_constraint(operator.eq, ['a', 'b']) # a == b
>>> csp.add_constraint(operator.ne, ['b', 'c']) # b != c
>>> bqm = dwavebinarycsp.stitch(csp, min_classical_gap=4.0)
>>> list(bqm) # doctest: +SKIP
['a', 'aux1', 'aux0', 'b', 'c']
```

Variable assignments that satisfy the CSP above, violate one, then two constraints, produce energy increases of the specified minimum classical gap:

```
>>> min([bqm.energy({'a': 0, 'b': 0, 'c': 1, 'aux0': aux0, 'aux1': aux1}) for
... aux0, aux1 in list(itertools.product([0, 1], repeat=2))]) # doctest: +SKIP
-6.0
>>> min([bqm.energy({'a': 0, 'b': 0, 'c': 0, 'aux0': aux0, 'aux1': aux1}) for
... aux0, aux1 in list(itertools.product([0, 1], repeat=2))]) # doctest: +SKIP
-2.0
>>> min([bqm.energy({'a': 1, 'b': 0, 'c': 0, 'aux0': aux0, 'aux1': aux1}) for
... aux0, aux1 in list(itertools.product([0, 1], repeat=2))]) # doctest: +SKIP
2.0
```

This example finds for the previous example the minimum graph size.

### 1.2.3 Other CSP Formats

#### **DIMACS**

The DIMACS format is used to encode boolean satisfiability problems in conjunctive normal form.

#### **CNF**

load\_cnf(fp)

Load a constraint satisfaction problem from a .cnf file.

#### dwavebinarycsp.io.cnf.load cnf

#### load\_cnf (fp)

Load a constraint satisfaction problem from a .cnf file.

Parameters fp (file, optional) - .write()-supporting file object DIMACS CNF formatted

file.

**Returns** ConstraintSatisfactionProblem a binary-valued SAT problem.

## **Examples**

```
>>> import dwavebinarycsp as dbcsp
...
>>> with open('test.cnf', 'r') as fp: # doctest: +SKIP
... csp = dbcsp.cnf.load_cnf(fp)
```

## 1.2.4 Reducing Constraints

Constraints can sometimes be reduced into several smaller constraints.

#### **Functions**

irreducible\_components(constraint)

Determine the sets of variables that are irreducible.

## dwavebinarycsp.irreducible\_components

## irreducible\_components(constraint)

Determine the sets of variables that are irreducible.

Let V(C) denote the variables of constraint C. For a configuration x, let x|A denote the restriction of the configuration to the variables of A. Constraint C is reducible if there is a partition of V(C) into nonempty subsets A and B, and two constraints  $C_A$  and  $C_B$ , with  $V(C_A) = A$  and  $C_B$  and  $C_B$  such that a configuration x is feasible in C if and only if x|A is feasible in  $C_A$  and x|B is feasible in  $C_B$ . A constraint is irreducible if it is not reducible.

**Parameters** constraint (Constraint) – Constraint to attempt to reduce.

**Returns** List of tuples in which each tuple is a set of variables that is irreducible.

**Return type** list[tuple]

#### **Examples**

This example reduces a constraint, created by specifying its valid configurations, to two constraints. The original constraint, that valid configurations for a,b,c are 0,0,1 and 1,1,1, can be represented by two reduced constraints, for example, (c=1) & (a=b). For comparison, an attempt to reduce a constraint representing an AND gate fails to find a valid reduction.

```
>>> const = dwavebinarycsp.Constraint.from_configurations([(0, 0, 1), (1, 1, 1)],
...
| dwavebinarycsp.BINARY)
>>> dwavebinarycsp.irreducible_components(const)
[('c',), ('a', 'b')]
>>> const_and = dwavebinarycsp.Constraint.from_configurations([(0, 0, 0), (0, 1, 40), (1, 0, 0), (1, 1, 1)],
...
| dwavebinarycsp.BINARY)
```

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```
>>> dwavebinarycsp.irreducible_components(const_and)
[('a', 'b', 'c')]
```

## 1.2.5 Defining Constraints

Solutions to a constraint satisfaction problem must satisfy certains conditions, the constraints of the problem, such as equality and inequality constraints. The Constraint class defines constraints and provides functionality to assist in constraint definition, such as verifying whether a candidate solution satisfies a constraint.

#### **Class**

class Constraint (func, configurations, variables, vartype, name=None)

A constraint.

#### variables

Variables associated with the constraint.

Type tuple

#### func

Function that returns True for configurations of variables that satisfy the constraint. Inputs to the function are ordered by *variables*.

Type function

#### configurations

Valid configurations of the variables. Each configuration is a tuple of variable assignments ordered by variables.

**Type** frozenset[tuple]

#### vartype

Variable type for the constraint. Accepted input values:

```
SPIN, 'SPIN', {-1, 1}
BINARY, 'BINARY', {0, 1}
Type dimod. Vartype
```

#### name

Name for the constraint. If not provided on construction, defaults to 'Constraint'.

Type str

#### **Examples**

This example defines a constraint, named "plus1", based on a function that is True for (y1, y0) = (x1, x0) + 1 on binary variables, and demonstrates some of the constraint's functionality.

```
>>> def plus_one(y1, y0, x1, x0): # y=x++ for two bit binary numbers
... return (y1, y0, x1, x0) in [(0, 1, 0, 0), (1, 0, 0, 1), (1, 1, 1, 0)]
...
>>> const = dwavebinarycsp.Constraint.from_func(
... plus_one,
```

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```
['out1', 'out0', 'in1', 'in0'],
... dwavebinarycsp.BINARY,
... name='plus1')
>>> print(const.name) # Check constraint defined as intended
plus1
>>> len(const)
4
>>> in0, in1, out0, out1 = 0, 0, 1, 0
>>> const.func(out1, out0, in1, in0) # Order matches variables
True
```

This example defines a constraint based on specified valid configurations that represents an AND gate for spin variables, and demonstrates some of the constraint's functionality.

#### **Methods**

### Construction

```
Constraint.from_configurations(...[, Construct a constraint from valid configurations. name])

Constraint.from_func(func, variables, vartype) Construct a constraint from a validation function.
```

## dwavebinarycsp.Constraint.from\_configurations

#### **Parameters**

- **configurations** (*iterable[tuple]*) Valid configurations of the variables. Each configuration is a tuple of variable assignments ordered by *variables*.
- variables (iterable) Iterable of variable labels.
- vartype (Vartype/str/set) Variable type for the constraint. Accepted input values:

```
- SPIN, 'SPIN', {-1, 1}
- BINARY, 'BINARY', {0, 1}
```

• name (string, optional, default='Constraint') - Name for the constraint.

## **Examples**

This example creates a constraint that variables a and b are not equal.

This example creates a constraint based on specified valid configurations that represents an OR gate for spin variables.

## dwavebinarycsp.Constraint.from\_func

classmethod Constraint.from\_func (func, variables, vartype, name=None)
Construct a constraint from a validation function.

#### **Parameters**

- **func** (function) Function that evaluates True when the variables satisfy the constraint.
- variables (iterable) Iterable of variable labels.
- vartype (Vartype/str/set) Variable type for the constraint. Accepted input values:

```
- SPIN, 'SPIN', {-1, 1}
- BINARY, 'BINARY', {0, 1}
```

• name (string, optional, default='Constraint') - Name for the constraint.

## **Examples**

This example creates a constraint that binary variables a and b are not equal.

```
>>> import operator
>>> const = dwavebinarycsp.Constraint.from_func(operator.ne, ['a', 'b'], 'BINARY')
>>> print(const.name)
Constraint
>>> (0, 1) in const.configurations
True
```

This example creates a constraint that out = NOT(x) for spin variables.

## **Satisfiability**

|--|

## dwavebinarycsp.Constraint.check

Constraint.check (solution)

Check that a solution satisfies the constraint.

**Parameters** solution (container) – An assignment for the variables in the constraint.

**Returns** True if the solution satisfies the constraint; otherwise False.

Return type bool

### **Examples**

This example creates a constraint that  $a \neq b$  on binary variables and tests it for two candidate solutions, with additional unconstrained variable c.

#### **Transformations**

Constraint.fix_variable(v, value)	Fix the value of a variable and remove it from the constraint.
Constraint.flip_variable(v)	Flip a variable in the constraint.

## dwavebinarycsp.Constraint.fix variable

```
Constraint.fix_variable(v, value)
```

Fix the value of a variable and remove it from the constraint.

#### **Parameters**

- v (variable) Variable in the constraint to be set to a constant value.
- val (int) Value assigned to the variable. Values must match the Vartype of the constraint.

## **Examples**

This example creates a constraint that  $a \neq b$  on binary variables, fixes variable a to 0, and tests two candidate solutions.

```
>>> const = dwavebinarycsp.Constraint.from_func(operator.ne,
... ['a', 'b'], dwavebinarycsp.BINARY)
>>> const.fix_variable('a', 0)
>>> const.check({'b': 1})
True
>>> const.check({'b': 0})
False
```

## dwavebinarycsp.Constraint.flip variable

```
Constraint.flip_variable(v)
```

Flip a variable in the constraint.

**Parameters v** (*variable*) – Variable in the constraint to take the complementary value of its construction value.

## **Examples**

This example creates a constraint that a = b on binary variables and flips variable a.

## Copies and projections

Constraint.copy()

Create a copy.

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## Table 11 – continued from previous page

Constraint.projection(variables)

Create a new constraint that is the projection onto a subset of the variables.

## dwavebinarycsp.Constraint.copy

```
Constraint.copy()
Create a copy.
```

## **Examples**

This example copies constraint  $a \neq b$  and tests a solution on the copied constraint.

```
>>> import operator
>>> const = dwavebinarycsp.Constraint.from_func(operator.ne,
... ['a', 'b'], 'BINARY')
>>> const2 = const.copy()
>>> const2 is const
False
>>> const2.check({'a': 1, 'b': 1})
False
```

## dwavebinarycsp.Constraint.projection

```
Constraint.projection(variables)
```

Create a new constraint that is the projection onto a subset of the variables.

**Parameters variables** (*iterable*) – Subset of the constraint's variables.

**Returns** A new constraint over a subset of the variables.

Return type Constraint

#### **Examples**

#### 1.2.6 Factories

dwavebinarycsp currently provides factories for constraints representing Boolean gates and satisfiability problems and CSPs for circuits and satisfiability problems.

#### **Constraints**

#### Gates

<pre>gates.and_gate(variables[, vartype, name])</pre>		AND gate.
<pre>gates.or_gate(variables[, vartype, name])</pre>		OR gate.
gates.xor_gate(variables[, vartype, nan	ne])	XOR gate.
gates.halfadder_gate(variables[,	vartype,	Half adder.
name])		
<pre>gates.fulladder_gate(variables[,</pre>	vartype,	Full adder.
name])		

## dwavebinarycsp.factories.constraint.gates.and\_gate

```
and_gate (variables, vartype=<Vartype.BINARY: frozenset({0, 1})>, name='AND')
AND gate.
```

#### **Parameters**

- **variables** (list) Variable labels for the and gate as [in1, in2, out], where in1, in2 are inputs and out the gate's output.
- **vartype**(*Vartype*, *optional*, *default='BINARY'*) **Variable type**. Accepted input values:
  - Vartype.SPIN, 'SPIN', {-1, 1}
  - Vartype.BINARY, 'BINARY', {0, 1}
- name (str, optional, default='AND') Name for the constraint.

**Returns** Constraint that is satisfied when its variables are assigned values that match the valid states of an AND gate.

**Return type** Constraint(Constraint)

## **Examples**

```
>>> import dwavebinarycsp.factories.constraint.gates as gates
>>> csp = dwavebinarycsp.ConstraintSatisfactionProblem(dwavebinarycsp.BINARY)
>>> csp.add_constraint(gates.and_gate(['a', 'b', 'c'], name='AND1'))
>>> csp.check({'a': 1, 'b': 0, 'c': 0})
True
```

#### dwavebinarycsp.factories.constraint.gates.or gate

```
\label{eq:cor_gate} \begin{split} \text{or\_gate} \ (\textit{variables}, \textit{vartype} = <\!\textit{Vartype}. \textit{BINARY: frozenset}(\{0, 1\}) >\!, \textit{name} = '\textit{OR'}) \\ \text{OR gate}. \end{split}
```

## **Parameters**

• **variables** (list) – Variable labels for the and gate as [in1, in2, out], where in1, in2 are inputs and out the gate's output.

- **vartype**(*Vartype*, *optional*, *default='BINARY'*) **Variable** type. Accepted input values:
  - Vartype.SPIN, 'SPIN', {-1, 1}
  - Vartype.BINARY, 'BINARY', {0, 1}
- name (str, optional, default='OR') Name for the constraint.

**Returns** Constraint that is satisfied when its variables are assigned values that match the valid states of an OR gate.

Return type Constraint(Constraint)

## **Examples**

```
>>> import dwavebinarycsp.factories.constraint.gates as gates
>>> csp = dwavebinarycsp.ConstraintSatisfactionProblem(dwavebinarycsp.SPIN)
>>> csp.add_constraint(gates.or_gate(['x', 'y', 'z'], {-1,1}, name='OR1'))
>>> csp.check({'x': 1, 'y': -1, 'z': 1})
True
```

## dwavebinarycsp.factories.constraint.gates.xor gate

 $xor_gate(variables, vartype = < Vartype.BINARY: frozenset({0, 1})>, name = 'XOR')$  XOR gate.

#### **Parameters**

- **variables** (list) Variable labels for the and gate as [in1, in2, out], where in1, in2 are inputs and out the gate's output.
- **vartype**(*Vartype*, *optional*, *default='BINARY'*) **Variable type**. Accepted input values:
  - Vartype.SPIN, 'SPIN', {-1, 1}
  - Vartype.BINARY, 'BINARY', {0, 1}
- name (str, optional, default='XOR') Name for the constraint.

**Returns** Constraint that is satisfied when its variables are assigned values that match the valid states of an XOR gate.

Return type Constraint(Constraint)

## **Examples**

```
>>> import dwavebinarycsp.factories.constraint.gates as gates
>>> csp = dwavebinarycsp.ConstraintSatisfactionProblem(dwavebinarycsp.BINARY)
>>> csp.add_constraint(gates.xor_gate(['x', 'y', 'z'], name='XOR1'))
>>> csp.check({'x': 1, 'y': 1, 'z': 1})
False
```

## dwavebinarycsp.factories.constraint.gates.halfadder\_gate

halfadder\_gate (variables, vartype=<Vartype.BINARY: frozenset({0, 1})>, name='HALF\_ADDER')
Half adder.

## **Parameters**

- **variables** (list) Variable labels for the and gate as [in1, in2, sum, carry], where in1, in2 are inputs to be added and sum and 'carry' the resultant outputs.
- **vartype**(*Vartype*, *optional*, *default='BINARY'*) **Variable type**. Accepted input values:
  - Vartype.SPIN, 'SPIN', {-1, 1}
  - Vartype.BINARY, 'BINARY', {0, 1}
- name (str, optional, default='HALF\_ADDER') Name for the constraint.

**Returns** Constraint that is satisfied when its variables are assigned values that match the valid states of a Boolean half adder.

Return type Constraint(Constraint)

## **Examples**

#### dwavebinarycsp.factories.constraint.gates.fulladder gate

**fulladder\_gate** (variables, vartype=<Vartype.BINARY: frozenset({0, 1})>, name='FULL\_ADDER') Full adder.

### **Parameters**

- **variables** (list) Variable labels for the and gate as [in1, in2, in3, sum, carry], where in1, in2, in3 are inputs to be added and sum and 'carry' the resultant outputs.
- **vartype**(*Vartype*, *optional*, *default='BINARY'*) **Variable type**. Accepted input values:
  - Vartype.SPIN, 'SPIN', {-1, 1}
  - Vartype.BINARY, 'BINARY', {0, 1}
- name (str, optional, default='FULL\_ADDER') Name for the constraint.

**Returns** Constraint that is satisfied when its variables are assigned values that match the valid states of a Boolean full adder.

**Return type** Constraint(Constraint)

## **Examples**

## **Satisfiability Problems**

```
sat.sat2in4(pos[, neg, vartype, name])
```

Two-in-four (2-in-4) satisfiability.

## dwavebinarycsp.factories.constraint.sat.sat2in4

```
sat2in4 (pos, neg=(), vartype=<Vartype.BINARY: frozenset({0, 1})>, name='2-in-4') Two-in-four (2-in-4) satisfiability.
```

#### **Parameters**

- **pos** (*iterable*) Variable labels, as an iterable, for non-negated variables of the constraint. Exactly four variables are specified by *pos* and *neg* together.
- **neg** (tuple) Variable labels, as an iterable, for negated variables of the constraint. Exactly four variables are specified by *pos* and *neg* together.
- **vartype**(*Vartype*, *optional*, *default='BINARY'*) **Variable type**. Accepted input values:
  - Vartype.SPIN, 'SPIN', {-1, 1}
  - Vartype.BINARY, 'BINARY', {0, 1}
- name (str, optional, default='2-in-4') Name for the constraint.

**Returns** Constraint that is satisfied when its variables are assigned values that satisfy a two-in-four satisfiability problem.

**Return type** Constraint(Constraint)

## **Examples**

#### **CSPs**

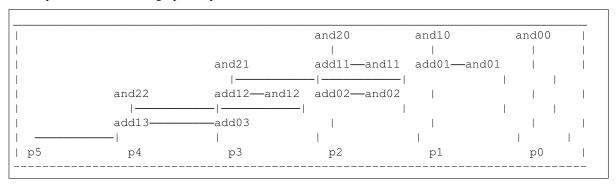
circuits.multiplication_circuit(nbit[,	Multiplication circuit constraint satisfaction problem.
vartype])	
sat.random_2in4sat(num_variables,	Random two-in-four (2-in-4) constraint satisfaction
num_clauses)	problem.
sat.random_xorsat(num_variables,	Random XOR constraint satisfaction problem.
num_clauses)	

## dwavebinarycsp.factories.csp.circuits.multiplication\_circuit

```
multiplication_circuit (nbit, vartype=<Vartype.BINARY: frozenset({0, 1})>) Multiplication circuit constraint satisfaction problem.
```

A constraint satisfaction problem that represents the binary multiplication ab = p, where the multiplicands are binary variables of length *nbit*; for example,  $2^m a_{nbit} + ... + 4a_2 + 2a_1 + a_0$ .

The square below shows a graphic representation of the circuit:



#### **Parameters**

- **nbit** (*int*) Number of bits in the multiplicands.
- **vartype**(*Vartype*, *optional*, *default='BINARY'*) **Variable type**. Accepted input values:
  - Vartype.SPIN, 'SPIN', {-1, 1}
  - Vartype.BINARY, 'BINARY', {0, 1}

**Returns** CSP that is satisfied when variables a, b, p are assigned values that correctly solve binary multiplication ab = p.

Return type CSP (ConstraintSatisfactionProblem)

## **Examples**

This example creates a multiplication circuit CSP that multiplies two 3-bit numbers, which is then formulated as a binary quadratic model (BQM). It fixes the multiplacands as a=5, b=3 (101 and 011) and uses a simulated annealing sampler to find the product, p=15 (001111).

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## dwavebinarycsp.factories.csp.sat.random\_2in4sat

random\_2in4sat (num\_variables, num\_clauses, vartype=<Vartype.BINARY: frozenset({0, 1})>, satisfi-able=True)
Random two-in-four (2-in-4) constraint satisfaction problem.

## Parameters

- num\_variables (integer) Number of variables (at least four).
- num\_clauses (integer) Number of constraints that together constitute the constraint satisfaction problem.
- **vartype**(*Vartype*, *optional*, *default='BINARY'*) **Variable** type. Accepted input values:
  - Vartype.SPIN, 'SPIN', {-1, 1}
  - Vartype.BINARY, 'BINARY', {0, 1}
- **satisfiable** (bool, optional, default=True) True if the CSP can be satisfied.

**Returns** CSP that is satisfied when its variables are assigned values that satisfy a two-in-four satisfiability problem.

**Return type** CSP (ConstraintSatisfactionProblem)

## **Examples**

This example creates a CSP with 6 variables and two random constraints and checks whether a particular assignment of variables satisifies it.

```
>>> import dwavebinarycsp.factories as sat
>>> csp = sat.random_2in4sat(6, 2)
>>> csp.constraints # doctest: +SKIP
[Constraint.from_configurations(frozenset({(1, 0, 1, 0), (1, 0, 0, 1), (1, 1, 1, 1, 1), (0, 1, 1, 0), (0, 0, 0, 0), (0, 1, 0, 1)}), (2, 4, 0, 1), Vartype.BINARY, name='2-in-4'),
Constraint.from_configurations(frozenset({(1, 0, 1, 1), (1, 1, 0, 1), (1, 1, 1, 1, 1, 1, 1), (0, 1, 0, 0), (0, 0, 1, 0)}), (1, 2, 4, 5), Vartype.BINARY, name='2-in-4')]
>>> csp.check({0: 1, 1: 0, 2: 1, 3: 1, 4: 0, 5: 0}) # doctest: +SKIP
True
```

## dwavebinarycsp.factories.csp.sat.random\_xorsat

 $\begin{tabular}{ll} {\bf random\_xorsat} & (num\_variables, & num\_clauses, & vartype=<Vartype.BINARY: & frozenset(\{0, 1\})>, & satisfiable=True) \\ & {\bf Random XOR constraint satisfaction problem.} \end{tabular}$ 

#### **Parameters**

- num\_variables (integer) Number of variables (at least three).
- num\_clauses (integer) Number of constraints that together constitute the constraint satisfaction problem.
- **vartype**(*Vartype*, *optional*, *default='BINARY'*) **Variable type**. Accepted input values:
  - Vartype.SPIN, 'SPIN', {-1, 1}
  - Vartype.BINARY, 'BINARY', {0, 1}
- **satisfiable** (bool, optional, default=True) True if the CSP can be satisfied.

**Returns** CSP that is satisfied when its variables are assigned values that satisfy a XOR satisfiability problem.

Return type CSP (ConstraintSatisfactionProblem)

## **Examples**

This example creates a CSP with 5 variables and two random constraints and checks whether a particular assignment of variables satisifies it.

```
>>> import dwavebinarycsp.factories as sat
>>> csp = sat.random_xorsat(5, 2)
>>> csp.constraints  # doctest: +SKIP
[Constraint.from_configurations(frozenset({(1, 0, 0), (1, 1, 1), (0, 1, 0), (0, 0, 0, 0)}), (4, 3, 0),
Vartype.BINARY, name='XOR (0 flipped)'),
Constraint.from_configurations(frozenset({(1, 1, 0), (0, 1, 1), (0, 0, 0), (1, 0, 0, 0)}), (2, 0, 4),
Vartype.BINARY, name='XOR (2 flipped) (0 flipped)')]
>>> csp.check({0: 1, 1: 0, 2: 0, 3: 1, 4: 1})  # doctest: +SKIP
True
```

## 1.3 Examples

**Release** 0.1.2 **Date** May 14, 2021

## 1.3.1 Circuit Fault Diagnosis

Fault diagnosis is the combinational problem of quickly localizing failures as soon as they are detected in systems. Circuit fault diagnosis (CFD) is the problem of identifying a minimum-sized set of gates that, if faulty, explains an observation of incorrect outputs given a set of inputs.

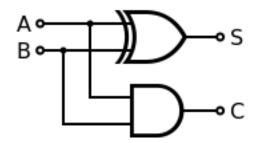


Fig. 3: A Half Adder made up of an XOR gate and an AND gate.

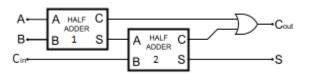


Fig. 4: A Full Adder made up of two Half Adders.

The following example demonstrates some of the techniques available to formulate a given problem so it can be solved on the D-Wave system.

## **Circuit Fault Diagnosis with Explicit Fault Variables**

We can construct the constraints for the circuit fault diagnosis in the following way:

- Each input/output/wire in the circuit is represented by a binary variable in the problem.
- Each gate can either be:
  - Healthy, in which case it behaves according to it's normal truth table.
  - Faulty, in which case it does not.

To build these constraints, we start with the truth table for the gate we wish to encode, say an AND gate:

Α	В	Output
0	0	0
1	0	0
0	1	0
1	1	1

We then add a new explicit fault variable, which encodes whether the gate is faulty or now.

Α	В	Output	Faulty
0	0	0	0
1	0	0	0
0	1	0	0
1	1	1	0
0	0	1	1
1	0	1	1
0	1	1	1
1	1	0	1

1.3. Examples 27

This new truth table with the explicit fault variable allows the CSP to be satisfied even when the gate is not healthy.

## **Example Code**

The following code demonstrates how to find both fault diagnoses and minimum fault diagnoses for the above Full Adder.

```
import dwavebinarycsp
from dimod import ExactSolver
def xor_fault(a, b, out, fault):
   """Returns True if XOR(a, b) == out and fault == 0 or XOR(a, b) != out and fault.
→== 1."""
   if (a != b) == out:
       return fault == 0
    else:
       return fault == 1
def and_fault(a, b, out, fault):
    """Returns True if AND(a, b) == out and fault == 0 or AND(a, b) != out and fault_
→== 1."""
   if (a and b) == out:
       return fault == 0
   else:
       return fault == 1
def or_fault(a, b, out, fault):
   """Returns True if OR(a, b) == out and fault == 0 or OR(a, b) != out and fault == 0
→1."""
   if (a or b) == out:
       return fault == 0
   else:
       return fault == 1
csp = dwavebinarycsp.ConstraintSatisfactionProblem(dwavebinarycsp.BINARY)
# the first half adder
csp.add_constraint(xor_fault, ['A1', 'B1', 'S1/A2', 'xor_fault_1'])
csp.add_constraint(and_fault, ['A1', 'B1', 'C1', 'and_fault_1'])
# the second half adder
csp.add_constraint(xor_fault, ['S1/A2', 'B2', 'S2', 'xor_fault_2'])
csp.add_constraint(and_fault, ['S1/A2', 'B2', 'C2', 'and_fault_2'])
# finally the AND gate
csp.add_constraint(or_fault, ['C1', 'C2', 'ORout', 'or_fault'])
# now, say that the behaviour we witnessed was HA(0, 1, 0) \rightarrow 1, 1.
# The 'A' input to the circuit is 'A1'
csp.fix_variable('A1', 0)
# The 'B' input to the circuit is 'B1'
```

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```
csp.fix_variable('B1', 1)
# the 'Cin' input to the circuit is 'B2'
csp.fix_variable('B2', 0)
# the sum output of the circuit is 'S2'
csp.fix_variable('S2', 1)
# the carry output of the circuit is 'ORout'
csp.fix_variable('ORout', 1)
# convert the csp to a bqm. We specify that the energy gap between the valid.
→configurations and
# the invalid ones must be at least 2.0
bqm = dwavebinarycsp.stitch(csp, min_classical_gap=2.0)
# set up any dimod solver. In this case we use the ExactSolver but any unstructured.
⇒solver would
# work
sampler = ExactSolver()
# we can determine the minimum and maximum number of faults that will induce this...
→behavior
response = sampler.sample(bqm)
min_energy = min(response.data_vectors['energy'])
fault_counts = []
for sample, energy in response.data(['sample', 'energy']):
    if csp.check(sample):
        n_faults = sum(sample[v] for v in sample if 'fault' in v)
        fault_counts.append(n_faults)
    else:
        # if the CSP is not satisfied, the energy should be above ground
        assert energy > min_energy
print('Minimum number of faults: ', min(fault_counts))
print('Maximum number of faults: ', max(fault_counts))
# If, instead of the ground states corresponding to all possible fault configurations,
# instead only wanted to sample from minimum fault configurations, we need to bias,
⇔against
# higher fault cardinalities. To do this, we add a small linear bias to the fault.
→variables.
# We also make sure that the bias we add is less than 2.0, or else we would affect,
→the energy
# levels.
bqm.add_variable('xor_fault_1', .5) # if the variable is present, add_variable adds.
→to the linear bias
bqm.add_variable('and_fault_1', .5)
bqm.add_variable('xor_fault_2', .5)
bgm.add_variable('and_fault_2', .5)
bqm.add_variable('or_fault', .5)
# now the samples that satisfy the csp and are minimum energy should be exactly the..
\hookrightarrow fault.
# diagnosis with only a single fault
                                                                          (continues on next page)
```

1.3. Examples 29

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```
response = sampler.sample(bqm)
min_energy = min(response.data_vectors['energy'])

min_fault_diagnoses = []

for sample, energy in response.data(['sample', 'energy']):
    if csp.check(sample) and energy == min_energy:
        min_fault_diagnoses.append([v for v in sample if ('fault' in v and_ sample[v])])
    else:
        # if the CSP is not satisfied, the energy should be above ground
        assert energy > min_energy

print('min fault diagnoses: ', min_fault_diagnoses)
```

## 1.4 Bibliography

## 1.5 Installation

To install:

```
pip install dwavebinarycsp
```

#### To build from source:

```
pip install -r requirements.txt
python setup.py install
```

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1.6. License 33

# CHAPTER 2

# Indices and tables

- genindex
- modindex
- search
- Glossary

# Python Module Index

```
C
dwavebinarycsp.core.constraint, 14
dwavebinarycsp.core.csp, 6

f
dwavebinarycsp.factories.constraint, 20
dwavebinarycsp.factories.csp, 23
i
dwavebinarycsp.io.cnf, 12

r
dwavebinarycsp.reduction, 13
```

38 Python Module Index

# Index

A  add_constraint() (ConstraintSatisfactionProblem	<pre>fulladder_gate() (in module dwavebina-</pre>
C check() (Constraint method), 17 check() (ConstraintSatisfactionProblem method), 9 configurations (Constraint attribute), 14 Constraint (class in dwavebinarycsp), 14 constraints (ConstraintSatisfactionProblem attribute), 7 ConstraintSatisfactionProblem (class in dwavebinarycsp), 7 copy() (Constraint method), 19	<pre>irreducible_components() (in module dwavebi-</pre>
D  dwavebinarycsp.core.constraint (module),	N name (Constraint attribute), 14  O or_gate() (in module dwavebina- rycsp.factories.constraint.gates), 20  P projection() (Constraint method), 19
fix_variable() (Constraint method), 18 fix_variable() (ConstraintSatisfactionProblem	R random_2in4sat() (in module dwavebina- rycsp.factories.csp.sat), 25 random_xorsat() (in module dwavebina- rycsp.factories.csp.sat), 26  S sat2in4() (in module dwavebina- rycsp.factories.constraint.sat), 23

40 Index