

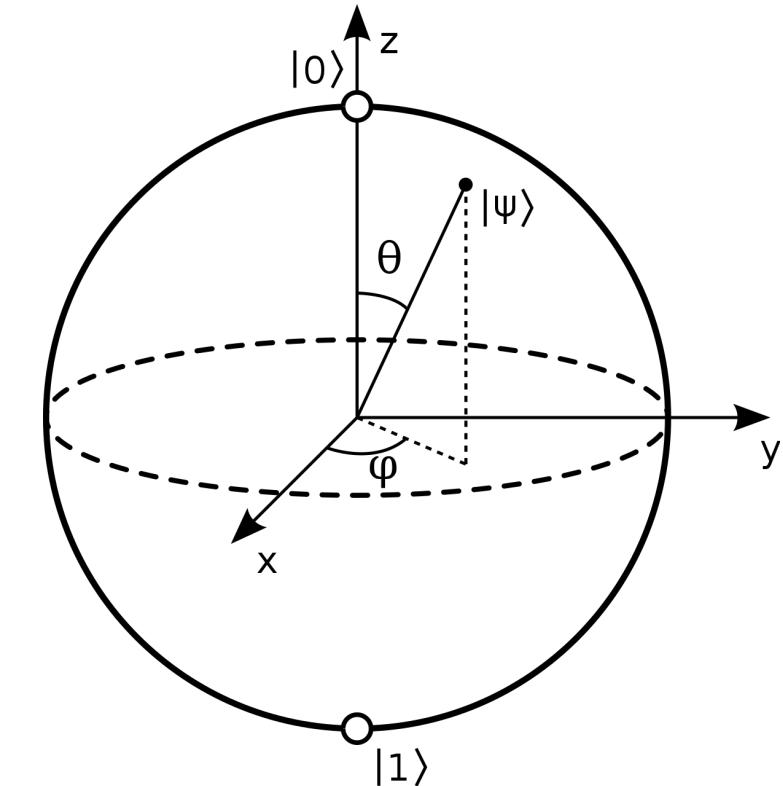
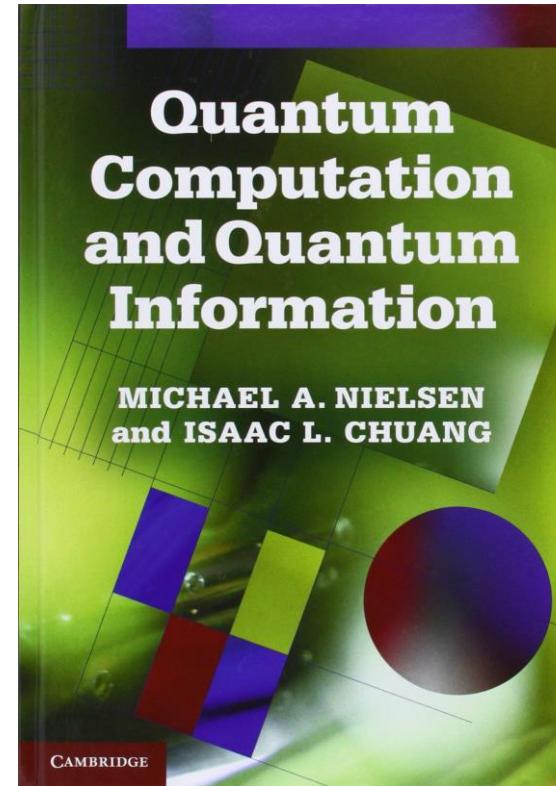
# INTRODUCTION TO QUANTUM COMPUTING

Quantum computing and Quantum computers

*Daniele Ottaviani*

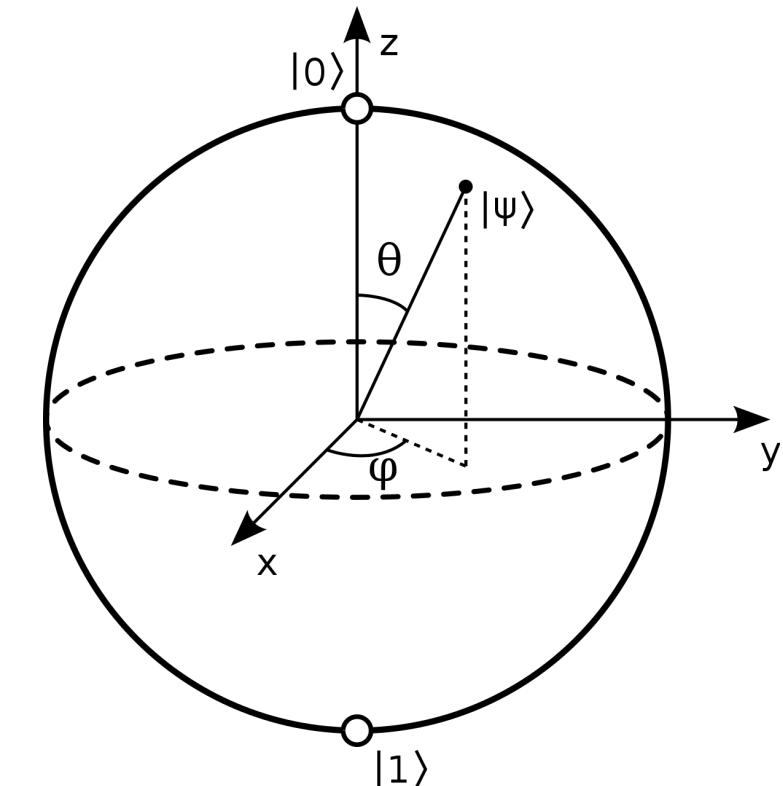
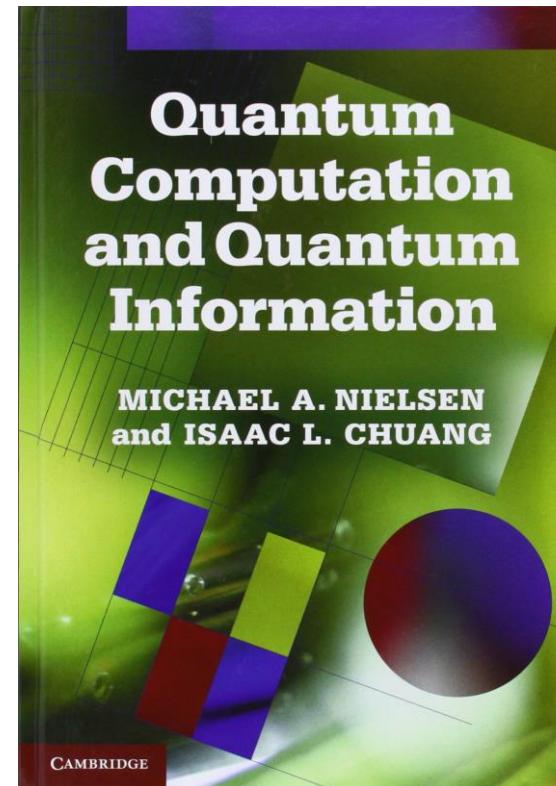
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- Quantum computing is a science based on principles of information theory and quantum mechanics



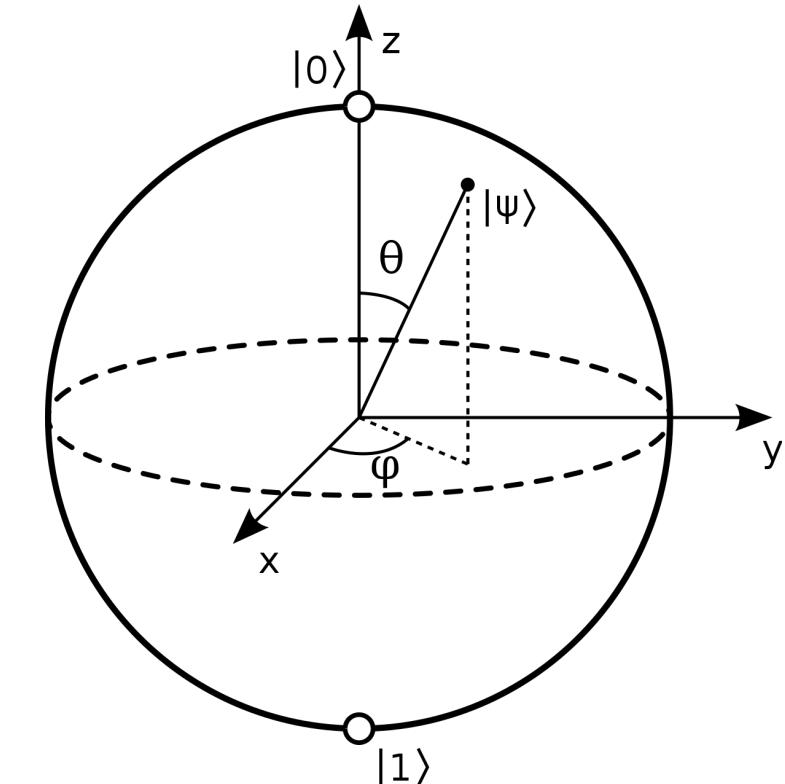
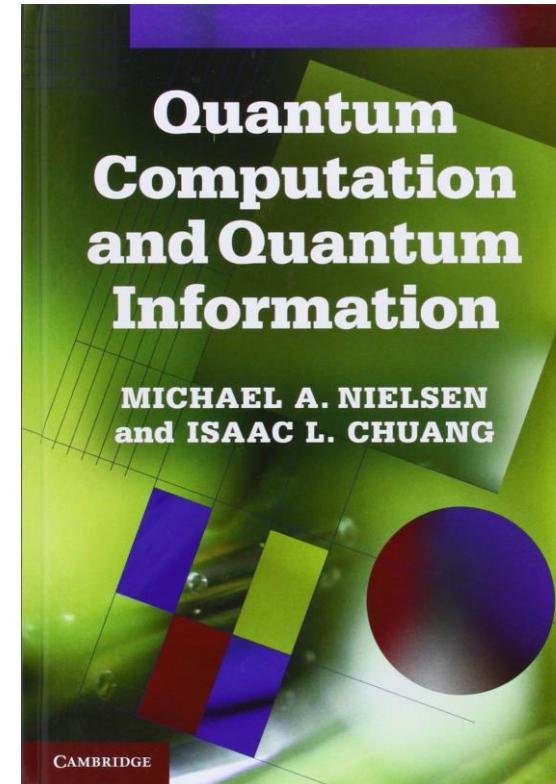
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- In its structure it defines the universal language to efficiently write quantum algorithms



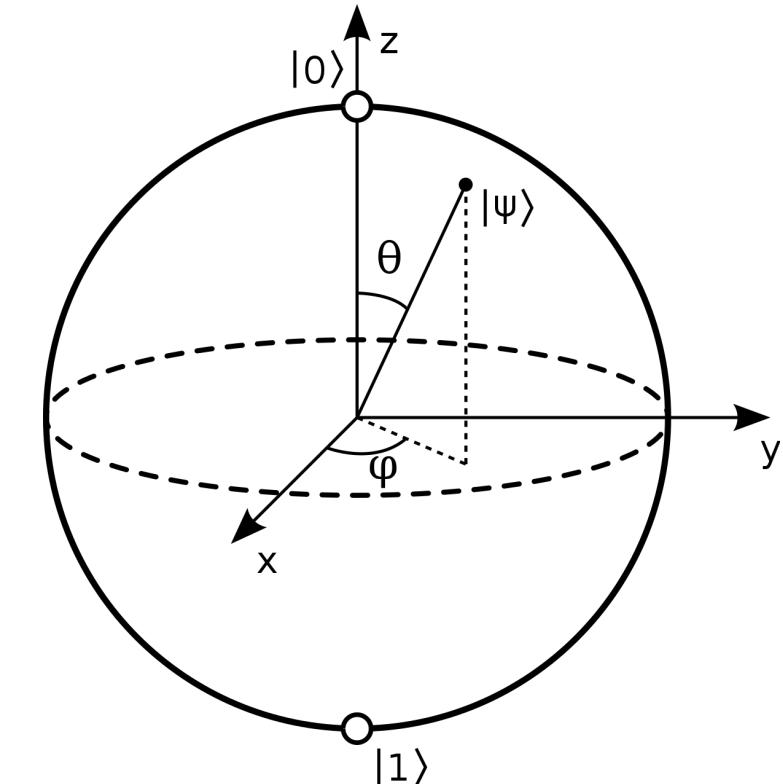
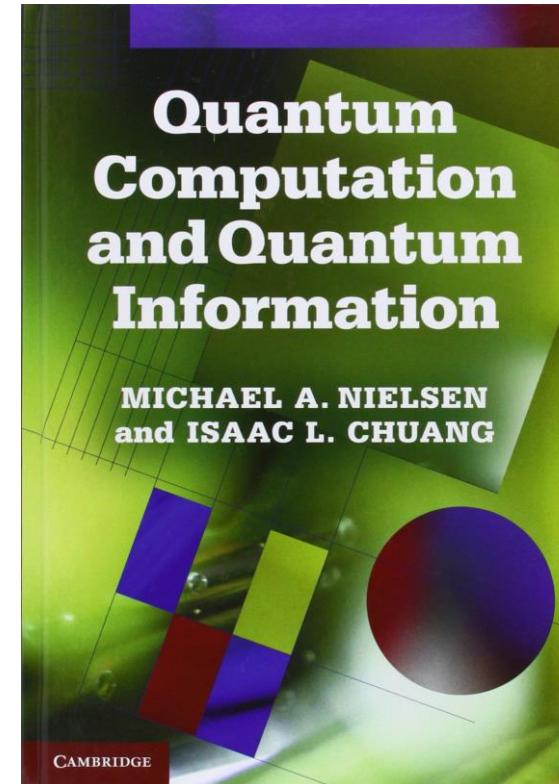
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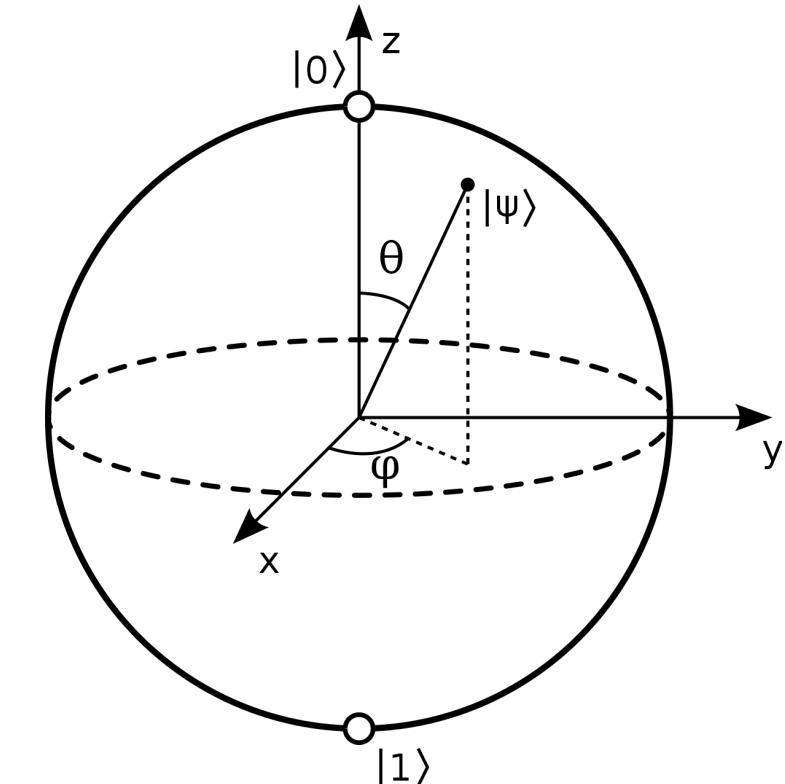
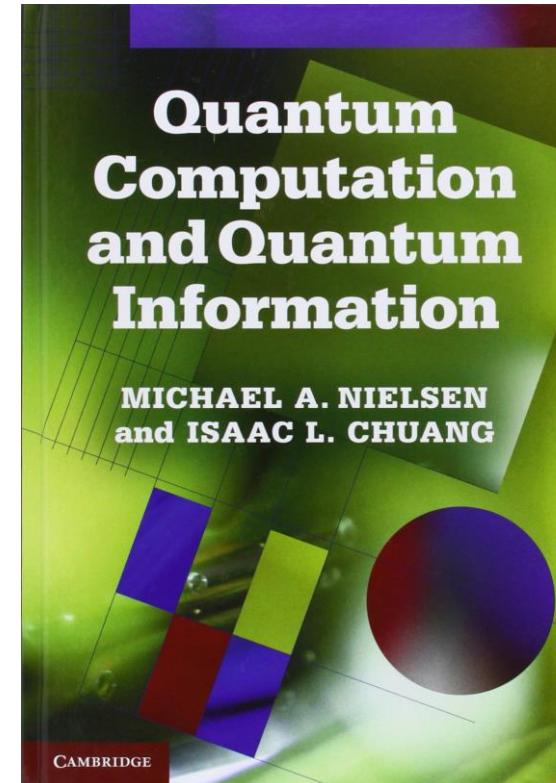
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- Such computers are called Quantum Computers



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## Quantum Superposition Principle

In the quantum mechanical regime, objects do not manifest only a single state, but all the possible states that they can assume at the same time (superposition state). If significantly disturbed (for example, with a measurement), the superposition state collapses and the object is forced to assume a classical state.



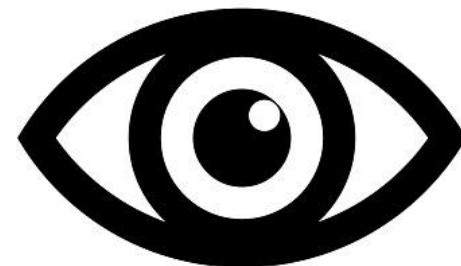
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## Measurement

With the term *observation* (or *measurement*) of a quantum state, we mean that process which, through special instruments, quantifies a certain quantity belonging to a microscopic object. This measurement can be seen as a sort of *intrusion* of the macroscopic world into the microscopic world: consequently, it inevitably perturbs the quantum state of matter, causing the state to collapse into a classical state, the only category of states that we, humans of the macroscopic world, can recognize.



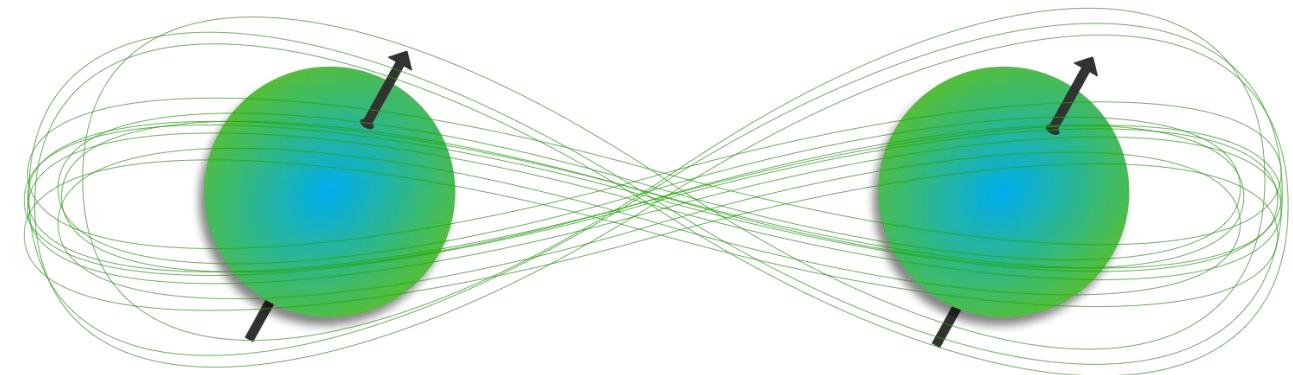
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- Second principle is quantum entanglement

## Quantum Entanglement

Phenomenon closely related to the superposition principle and the collapse of the quantum state into a classical state after a measurement, this principle states that there is a particular condition that can bind two quantum particles regardless of distance. This link is such that the observation of one of the two particles is also reflected on the other, conditioning the classical state in which it collapses (which can be the same or diametrically opposite)



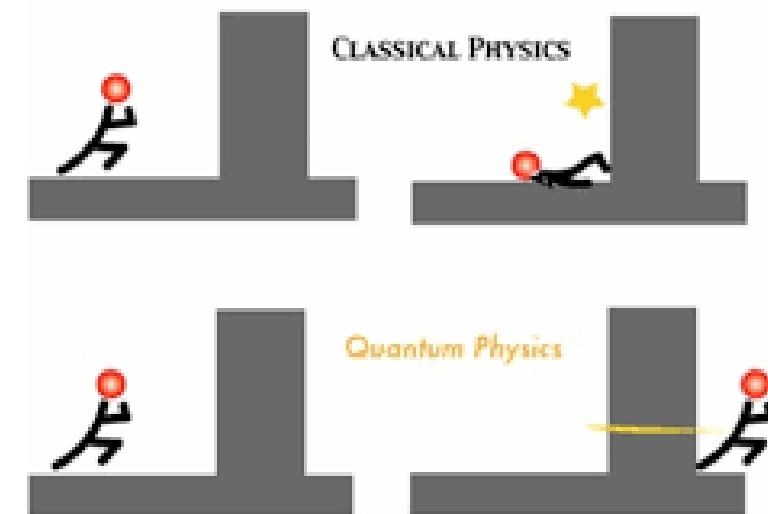
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- Second principle is quantum entanglement
- Third one is the **tunneling effect**

## Tunnelling Effect

By definition, we speak of tunneling effect when a particle manages to make a transition that is not normally possible according to the rules of classical mechanics.

In simple words, this principle states that, under certain circumstances, a quantum particle can cross energy barriers, as if a ball could be able to pass through a wall instead of bouncing against it!

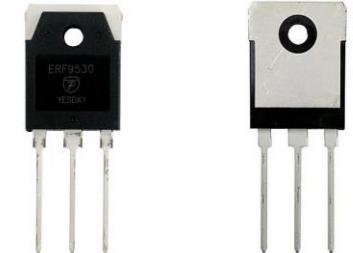


# The First Quantum Revolution

- Despite their strange nature, over the years more and more experiments have validated the theories of quantum mechanics



Laser



Transistor



GPS



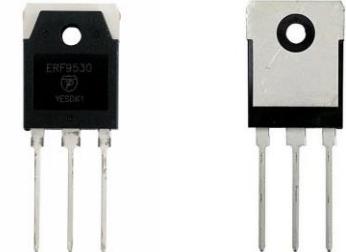
Touch Screen

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- The exploitation phase of quantum physics starts between 1960 and 1970, in the period that we could define as the First Quantum Revolution, and continues to this day.



Laser



Transistor



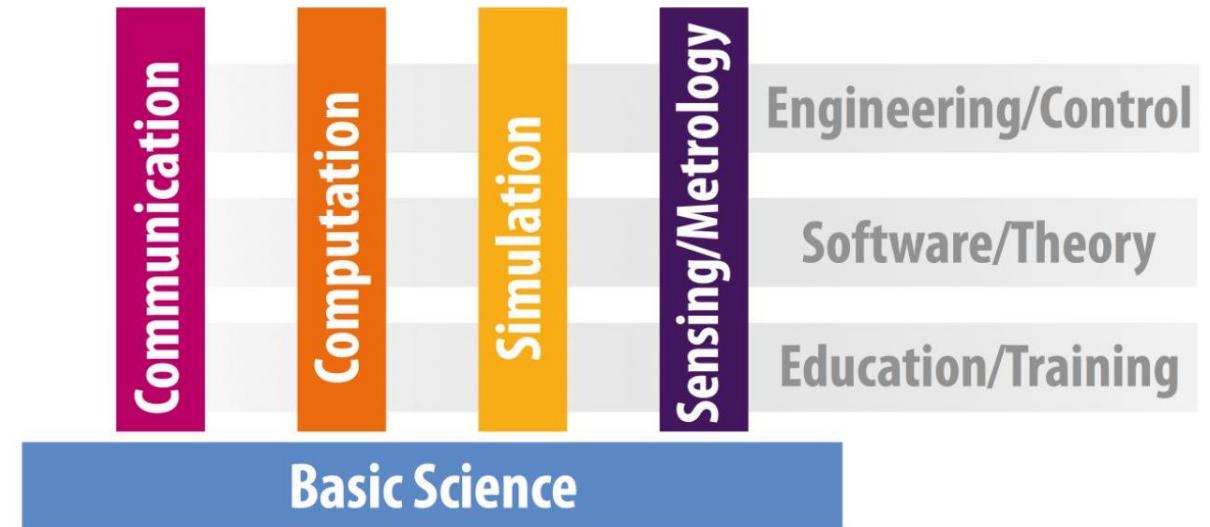
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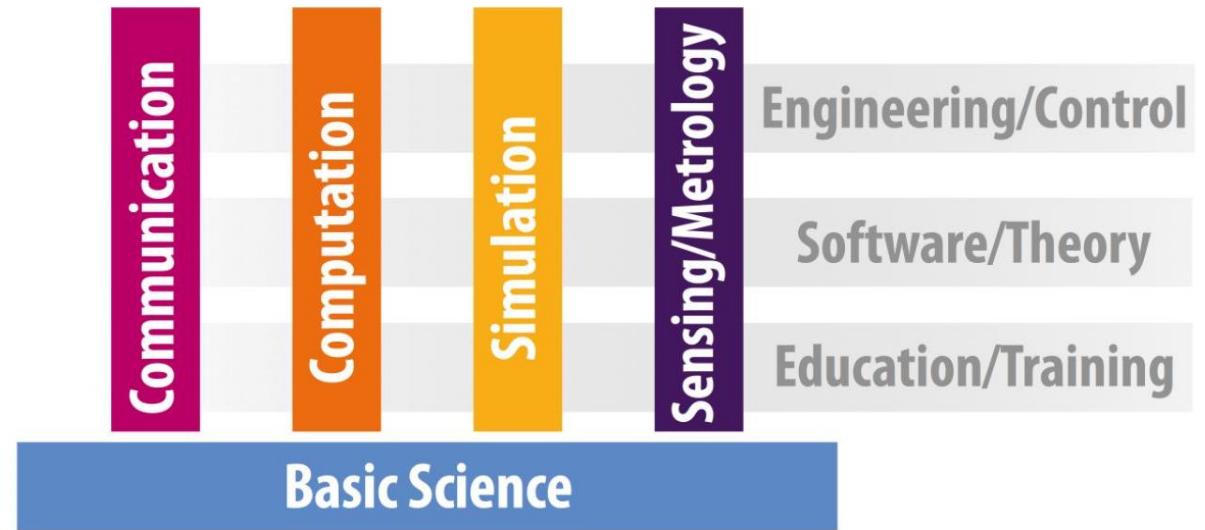
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*We are able to exploit quantum mechanics to dominate the behavior of a system of atoms*



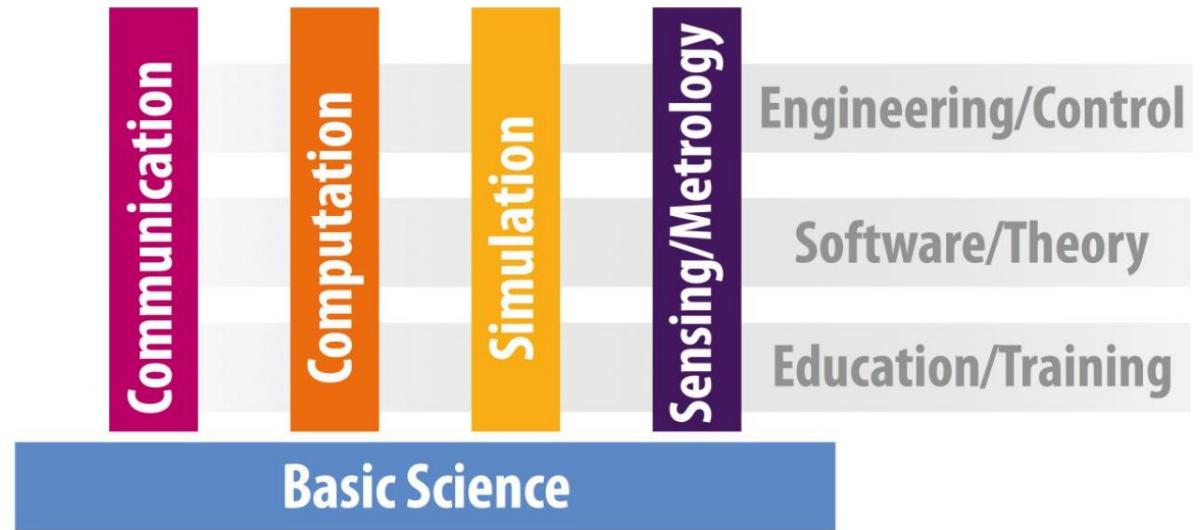
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- In the context of the second quantum revolution, however, the key concept becomes:  
*We are able to exploit quantum mechanics to dominate the behavior of a **single** quantum object*
- Quantum Technologies are born: Quantum Computing is one of them



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# The Idea of a Quantum Computer



Nature isn't classical, dammit, and if you want to make a simulation of nature, you'd better make it quantum mechanical, and by golly it's a wonderful problem, because it doesn't look so easy.

— *Richard P. Feynman* —

AZ QUOTES

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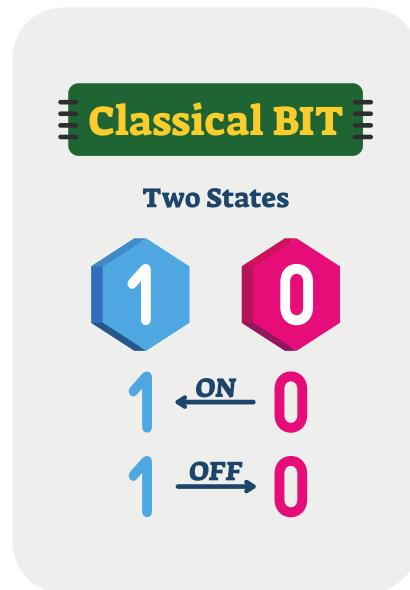
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- With the sentence reported in the previous slide, Feynman has attracted the attention of the scientific community to a non-trivial problem
- In parallel, many other scientists (Deutsch, Benioff) also began to develop the concept of quantum computer: quantum computing was born

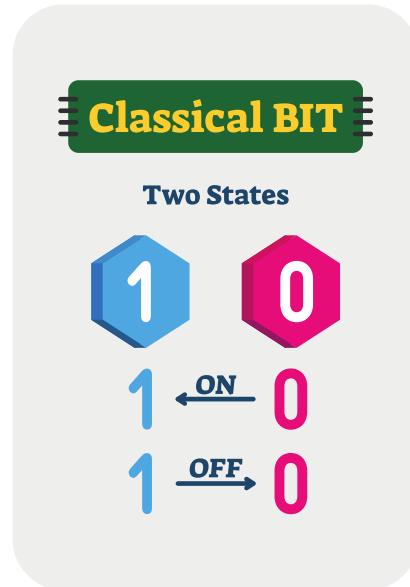
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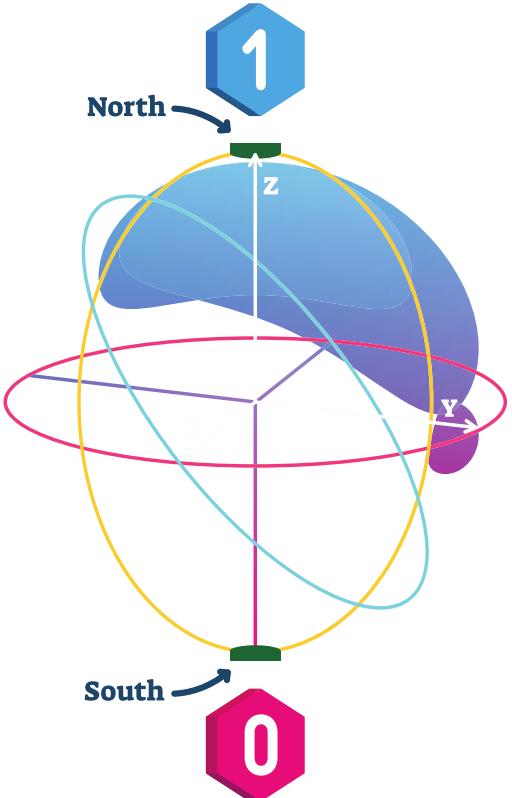
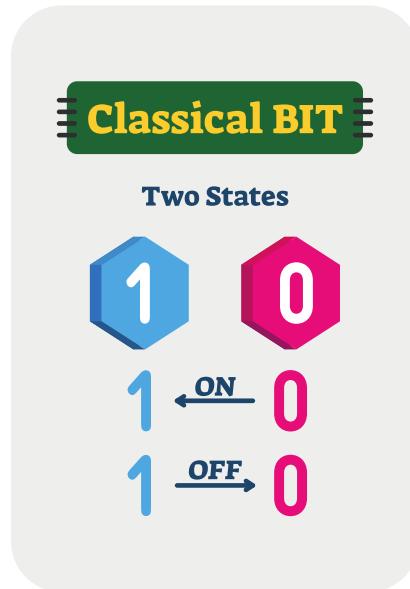


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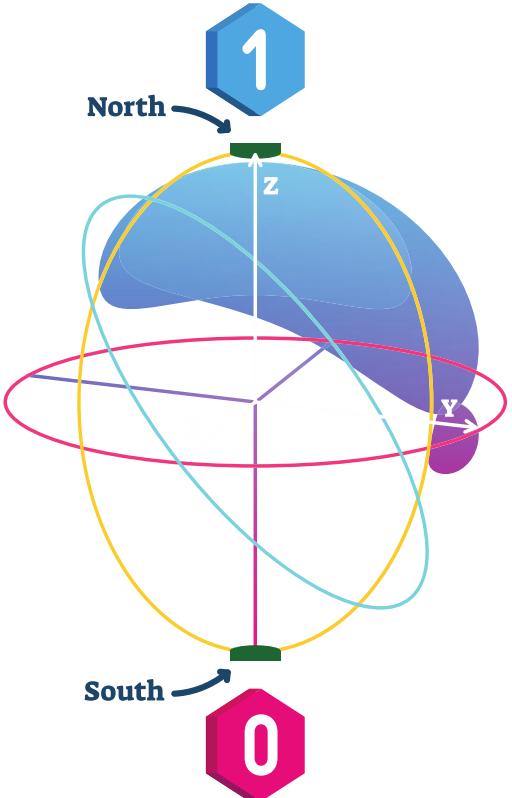
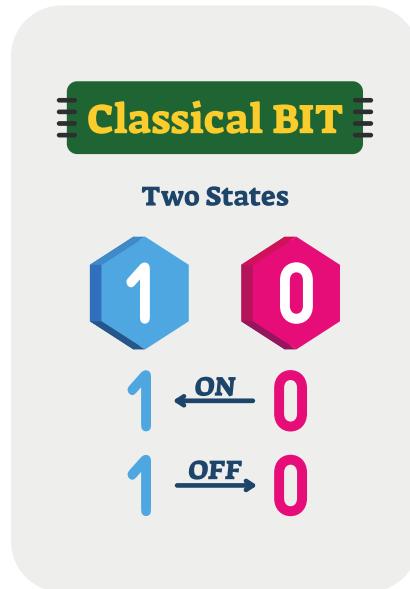
- In classical computing, the basic logical unit for a computer is the **BIT**
- The **BIT** is an object capable of assuming **only two states**, on and off (commonly 0 and 1)
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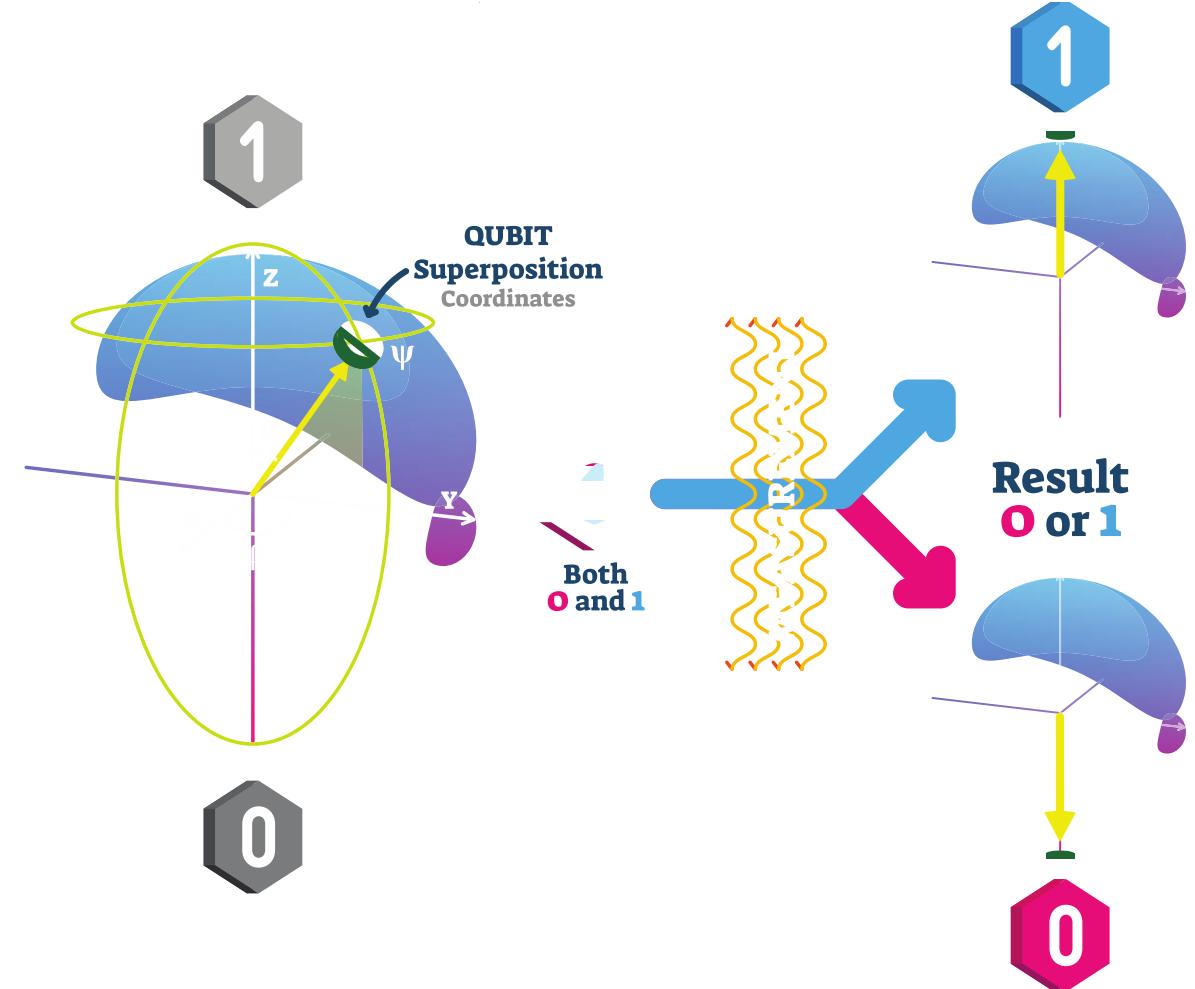
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- This *something* is called **Qubit**

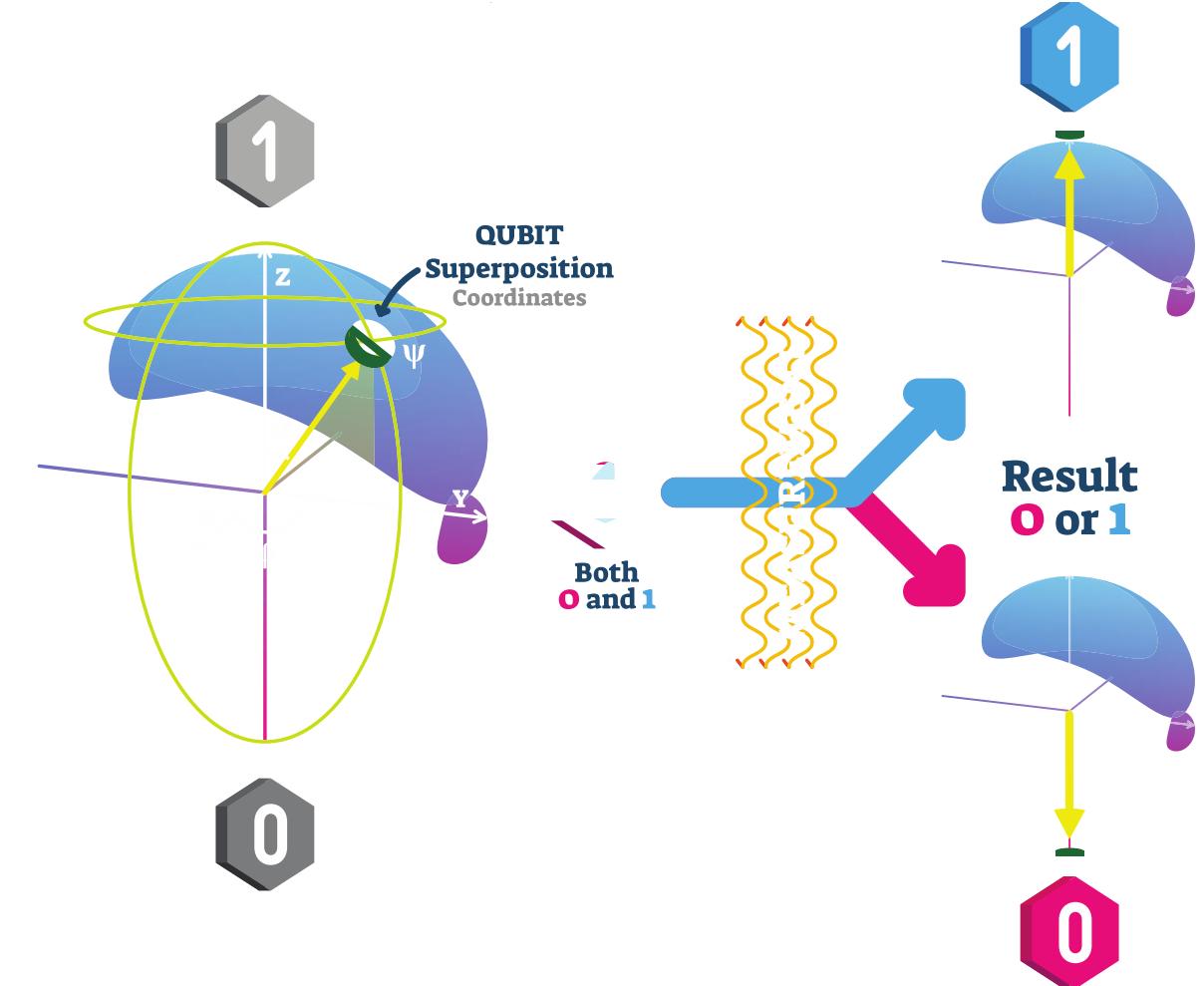
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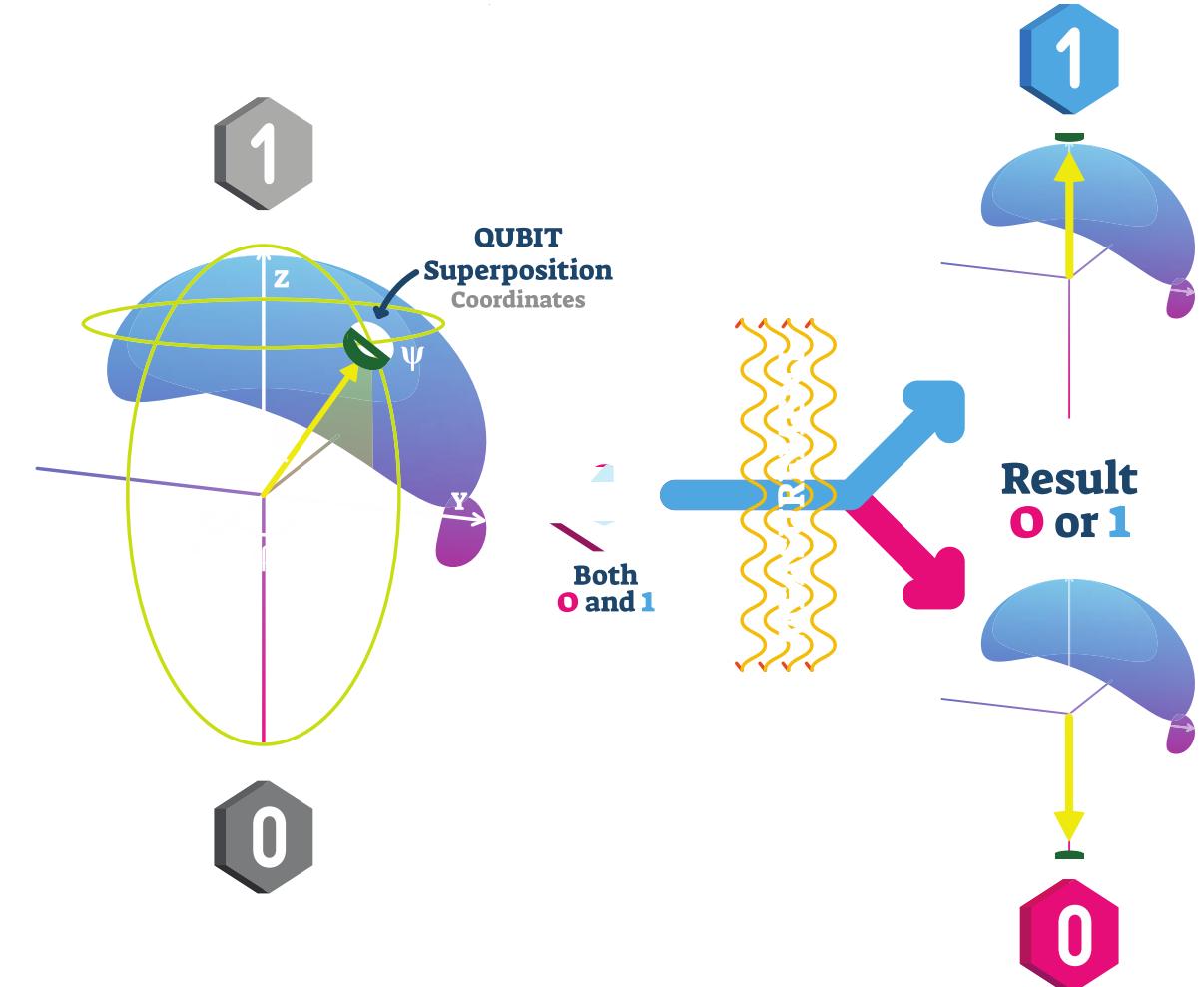
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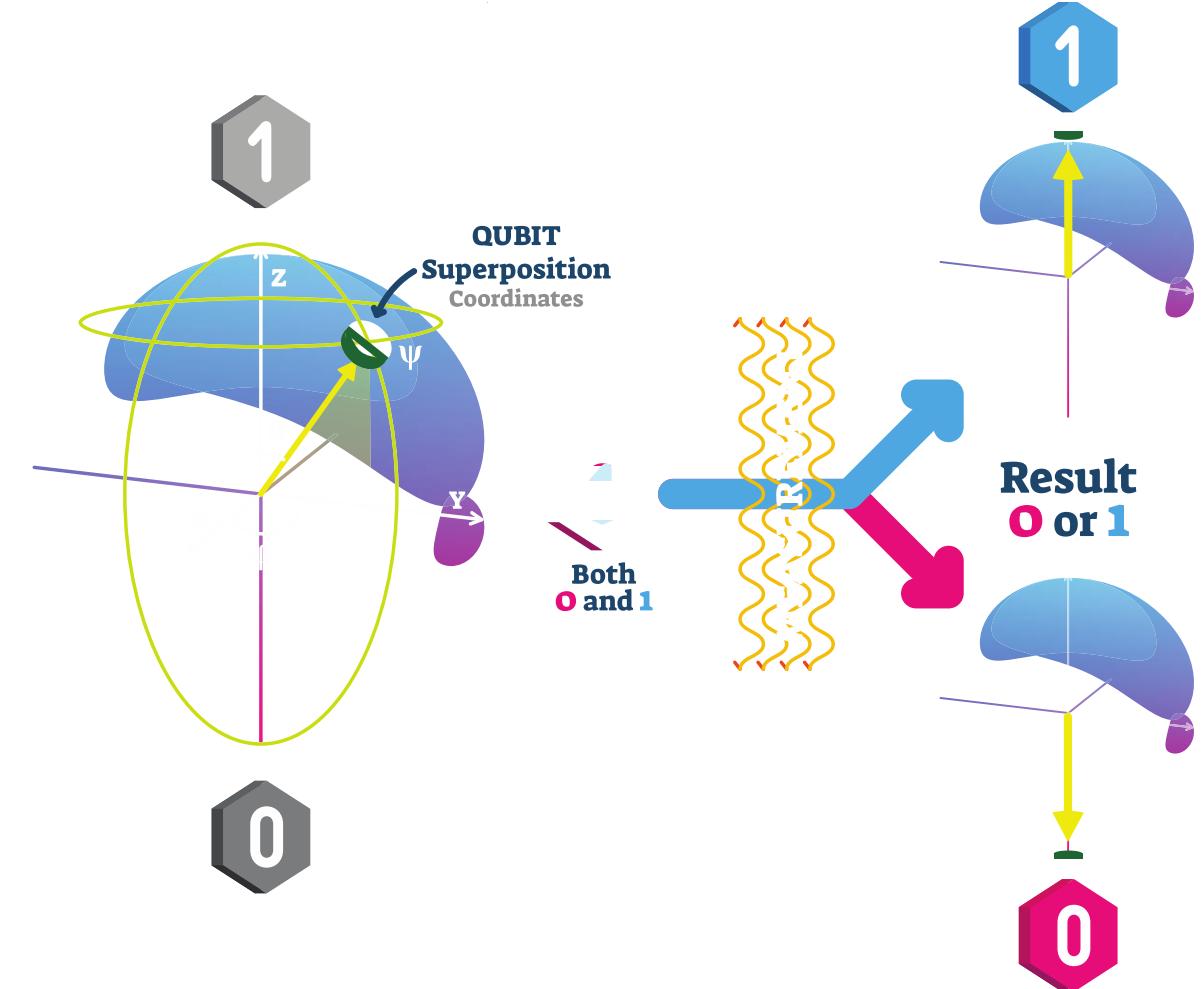
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- In this way, the qubit allows us to take quantum effects into account in its manipulation



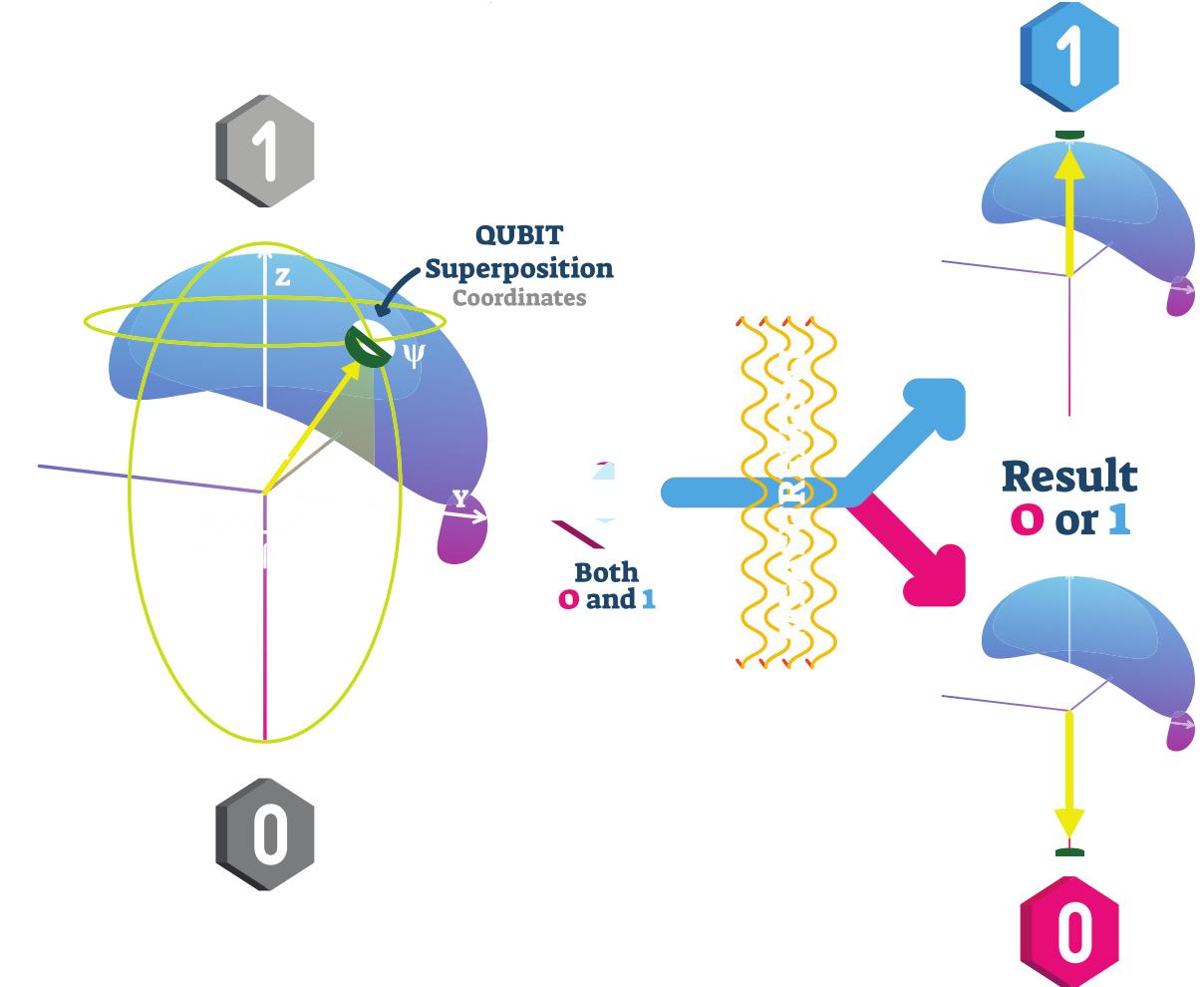
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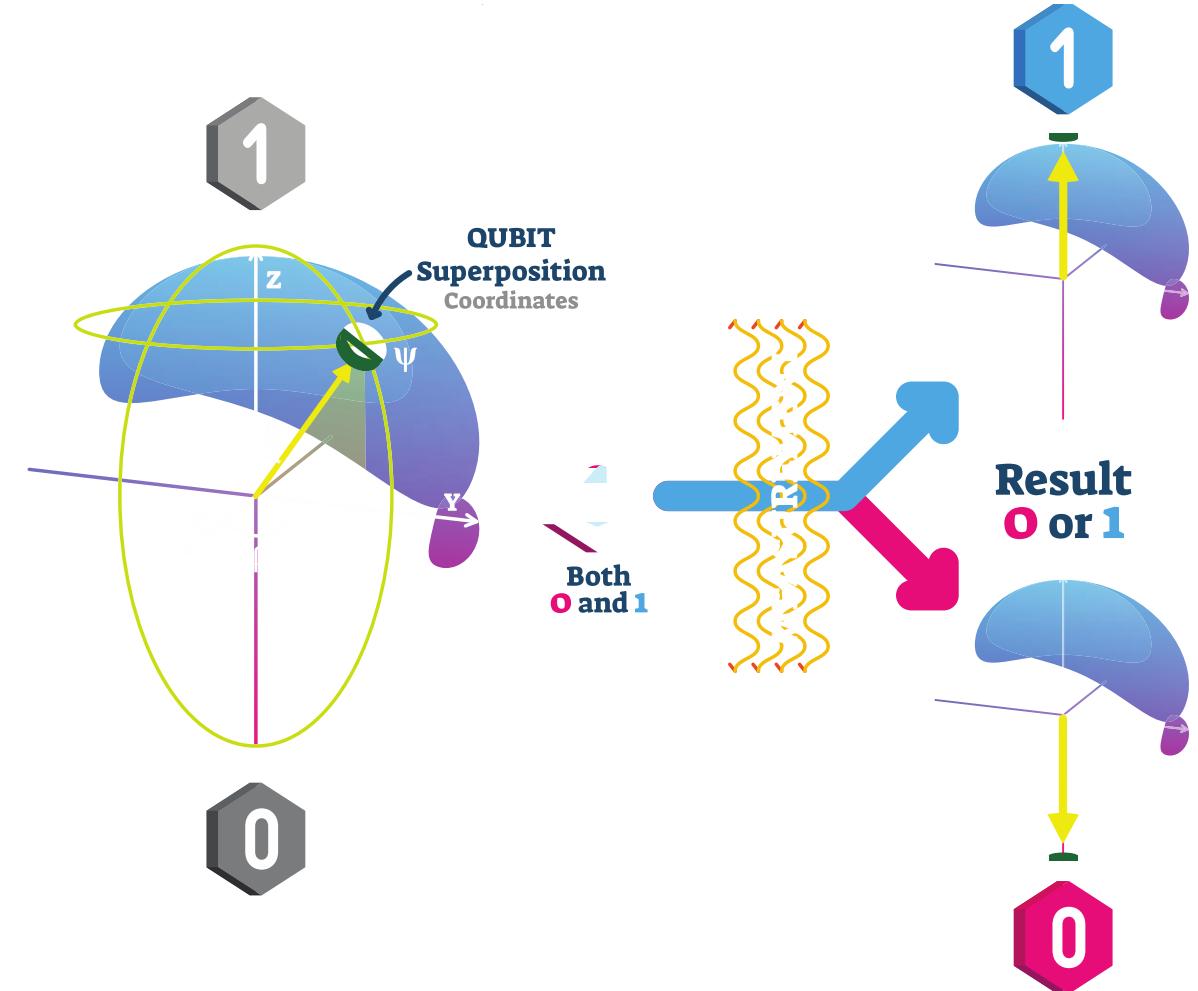
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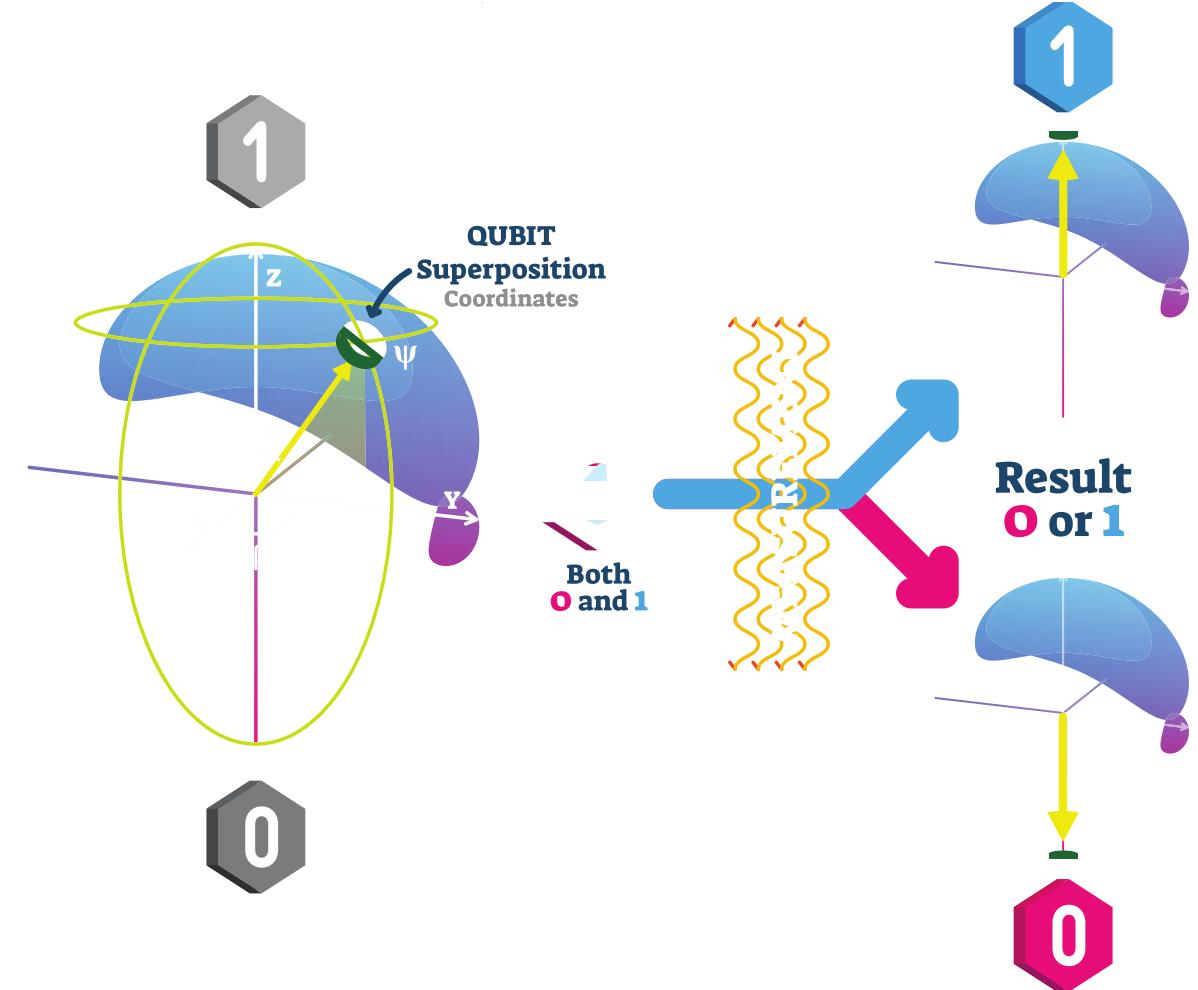
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- Let's start with a quick review of linear algebra and quantum mechanics mathematical formalism



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$$\begin{pmatrix} 1 & 3 \\ 2 & 1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \cdot 2 + 3 \cdot (-1) \\ 2 \cdot 2 + 1 \cdot (-1) \end{pmatrix}$$
$$= \begin{pmatrix} 2 - 3 \\ 4 - 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$$

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Unitary matrices have significant importance in quantum mechanics because they preserve norms.

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} t \\ z \end{pmatrix}$$

$$|(x, y)| = c \Rightarrow |(t, z)| = c$$

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a complex square matrix  $U$  is unitary if its conjugate transpose  $U^*$  is also its inverse, that is, if

$$UU^* = U^*U = I$$

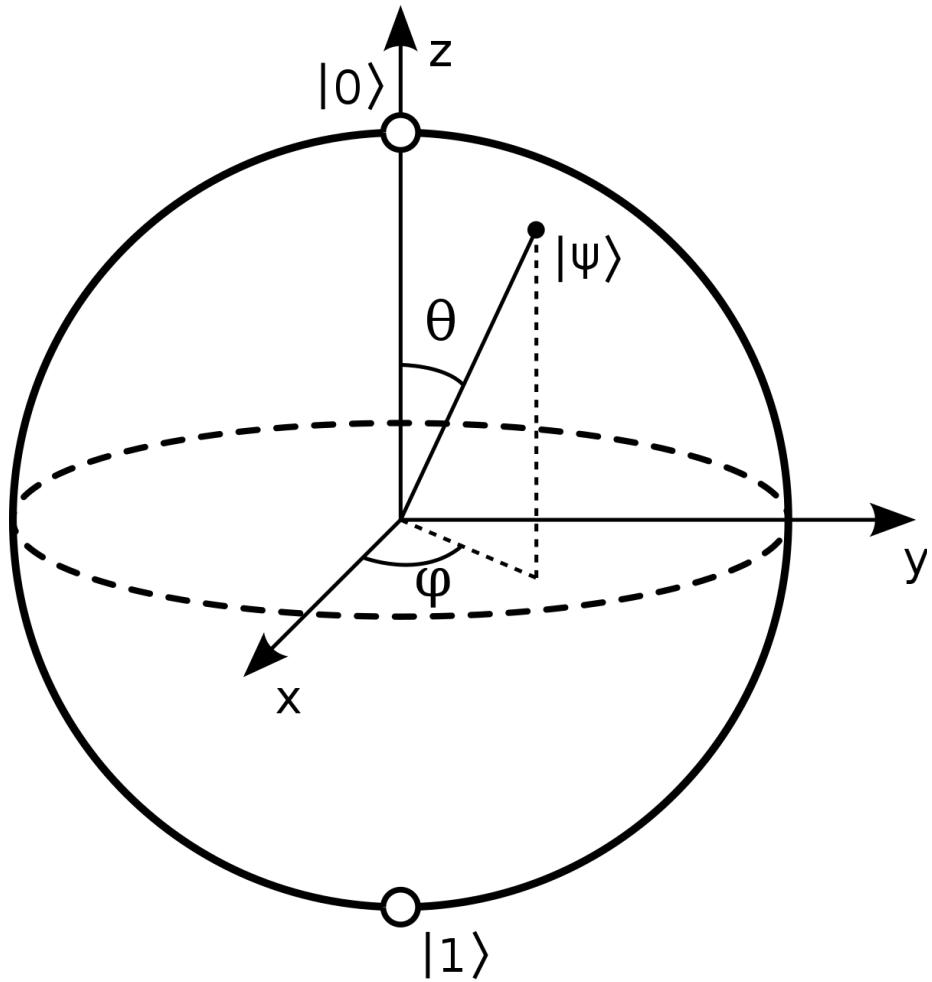
Unitary matrices have significant importance in quantum mechanics because they preserve norms.

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} t \\ z \end{pmatrix}$$

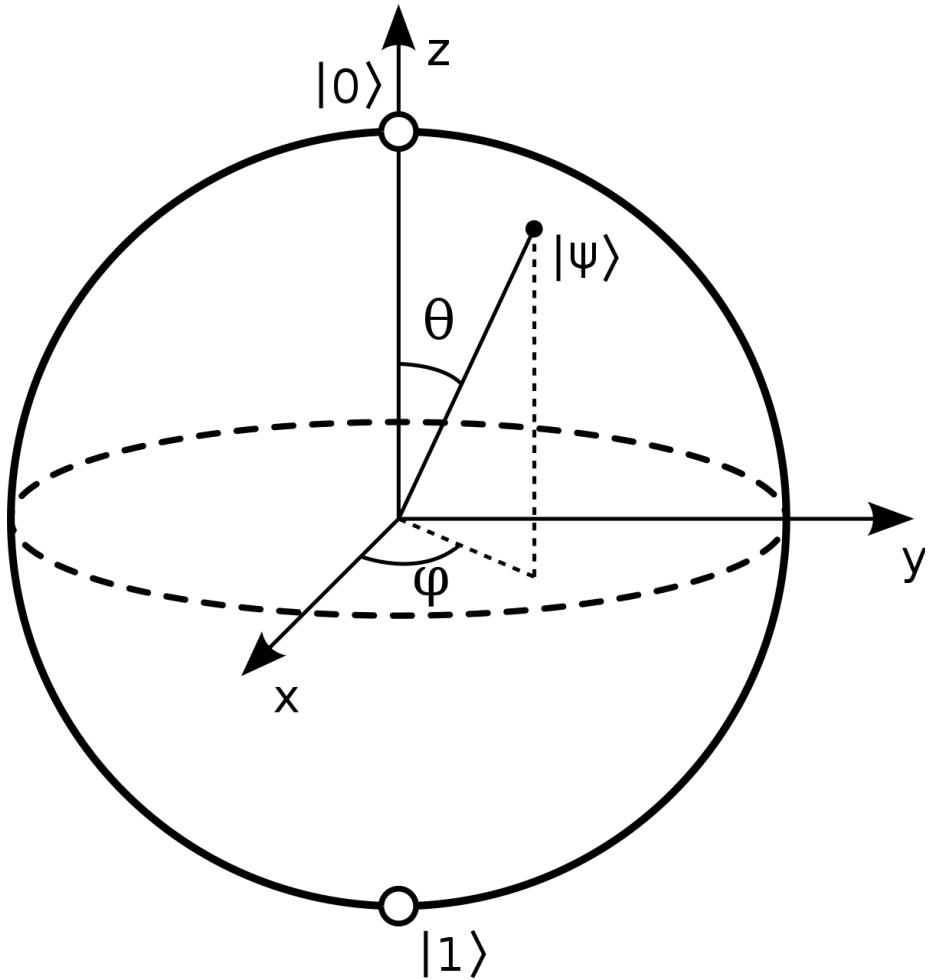
$$|(x, y)| = 1 \Rightarrow |(t, z)| = 1$$

# Qubits and Bloch Sphere

- A qubit is defined as a radius of the Bloch sphere.



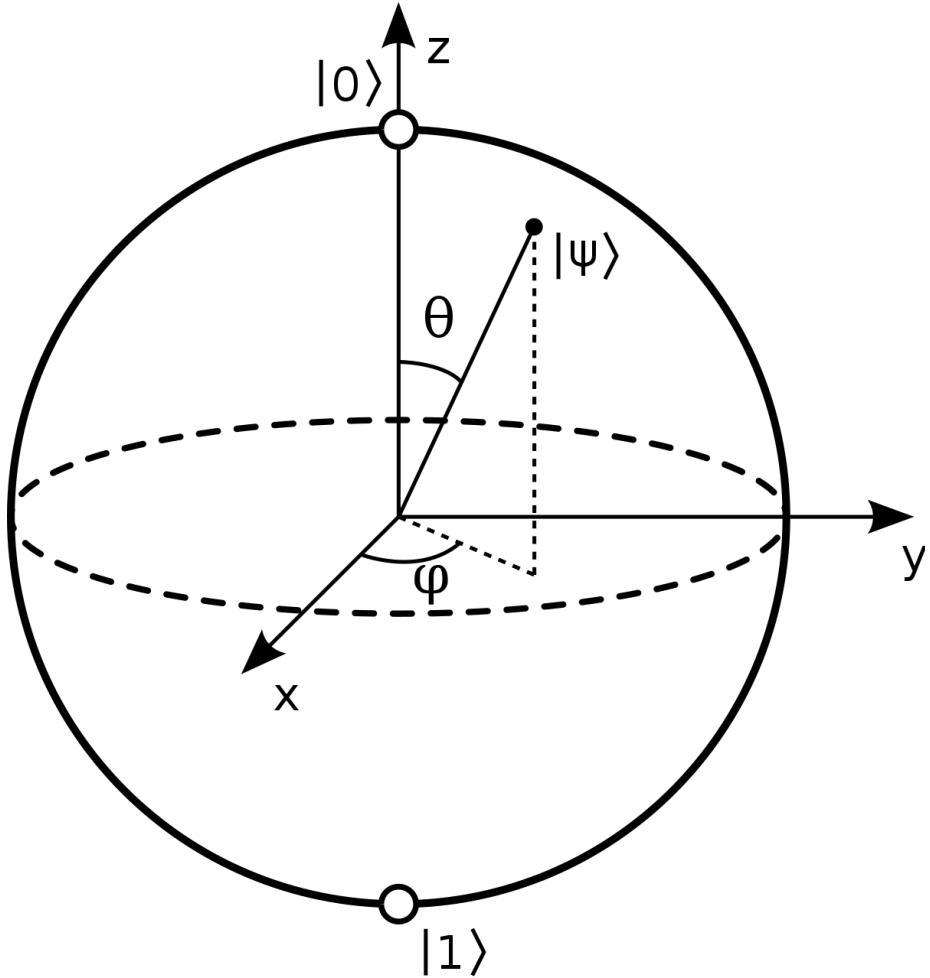
# Qubits and Bloch Sphere



- A qubit is defined as a radius of the Bloch sphere.
- The Bloch sphere is a mathematical object defined in  $\mathbb{C}^2$ , whose elements on the surface can be identified by the coordinates

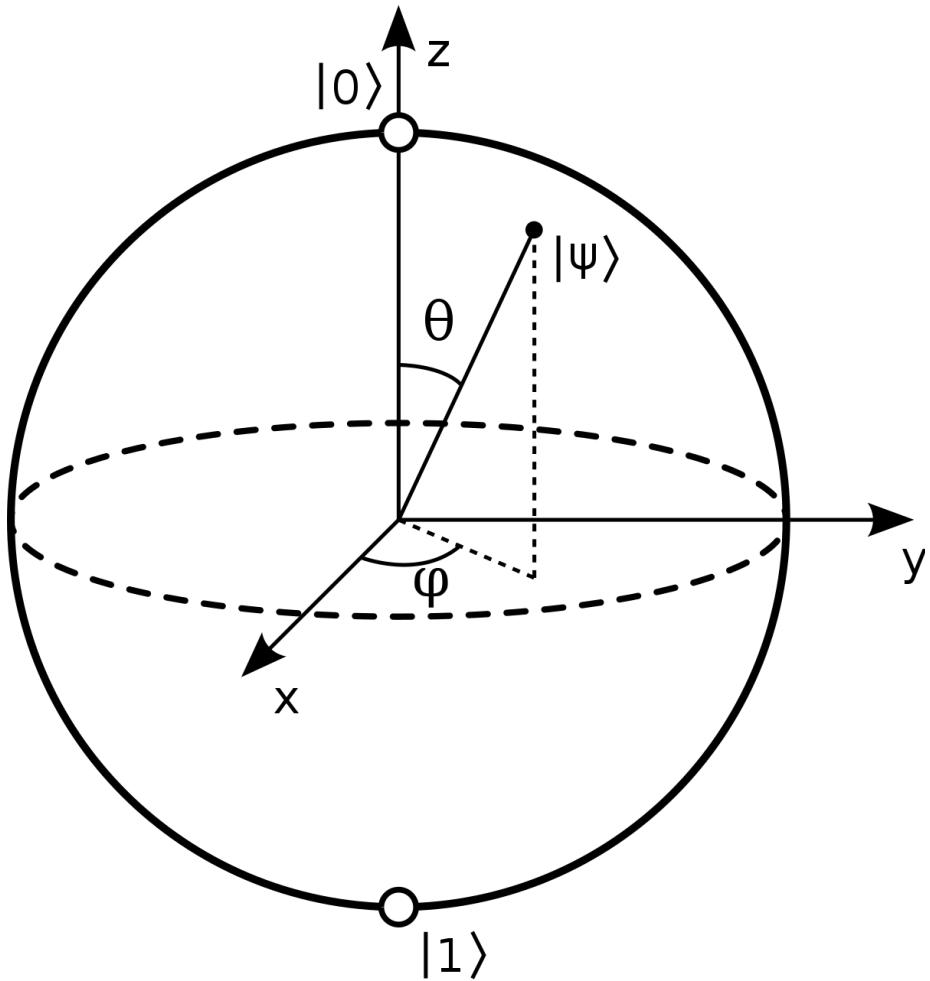
$$\left( \cos\left(\frac{\theta}{2}\right), e^{i\varphi} \sin\left(\frac{\theta}{2}\right) \right)$$

# Qubits and Bloch Sphere



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$$\left( \cos\left(\frac{\theta}{2}\right), e^{i\varphi} \sin\left(\frac{\theta}{2}\right) \right)$$
- The poles of the Bloch sphere represent the classical states 0 and 1. As we can see, they are highlighted in Dirac notation since they are defined by two vectors

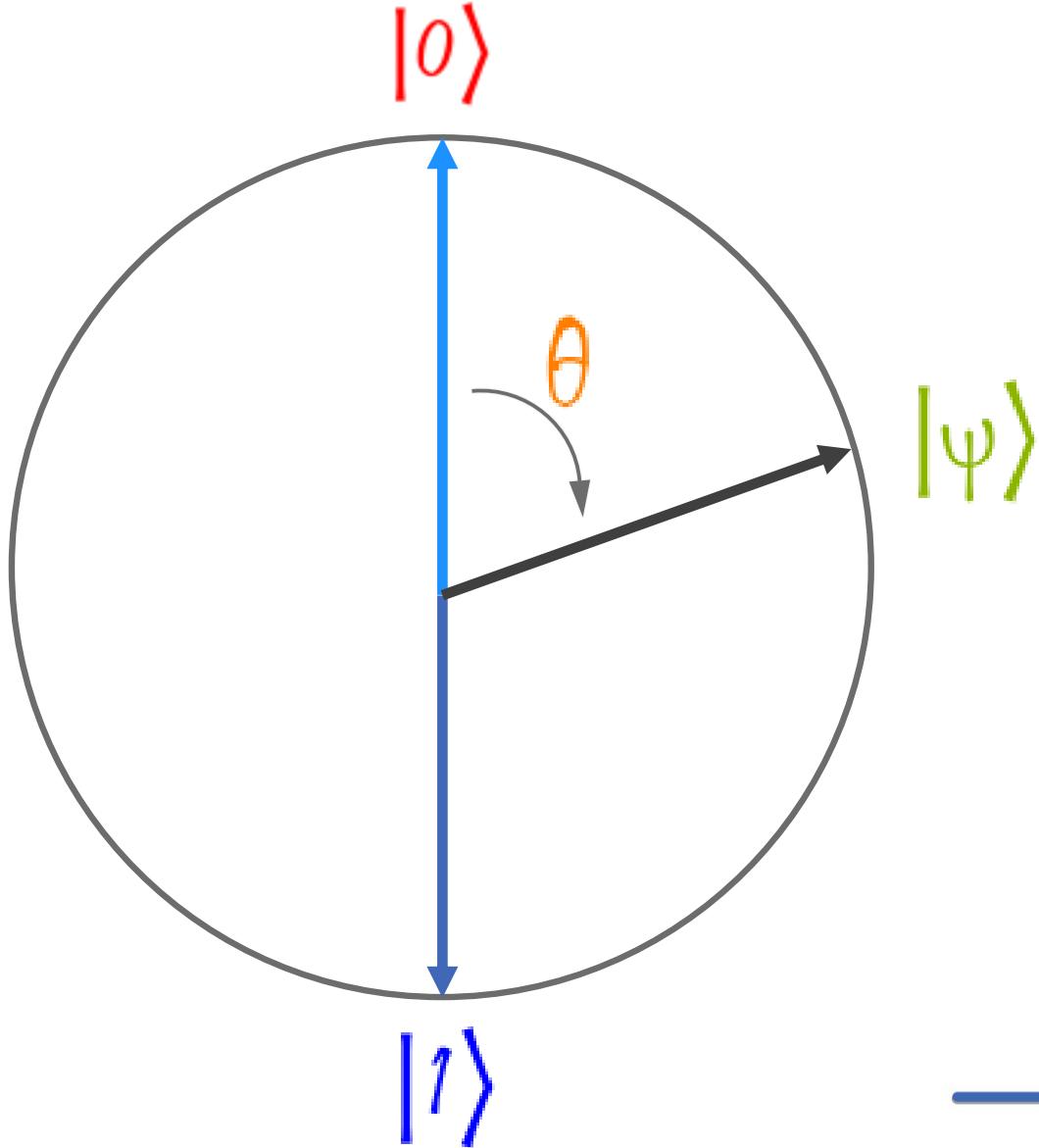
# Qubits and Bloch Sphere



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- Without losing generality, we can analyze the Bloch sphere only in the real field, ignoring the variable phi and bringing us back to a circumference

# Qubits and Bloch Sphere

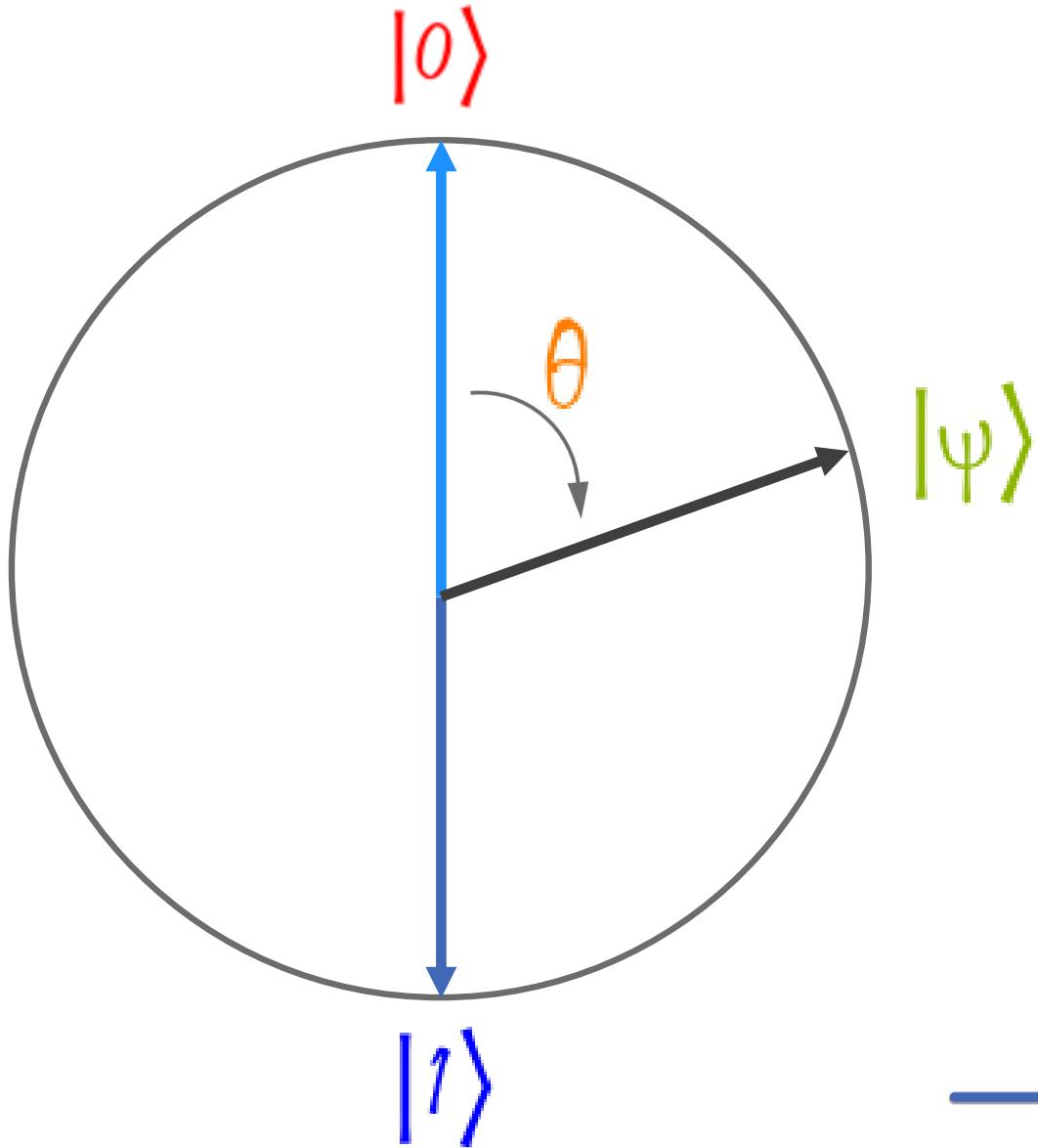
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$$|\psi\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \left( \cos\left(\frac{\theta}{2}\right), \sin\left(\frac{\theta}{2}\right) \right)$$

# Qubits and Bloch Sphere

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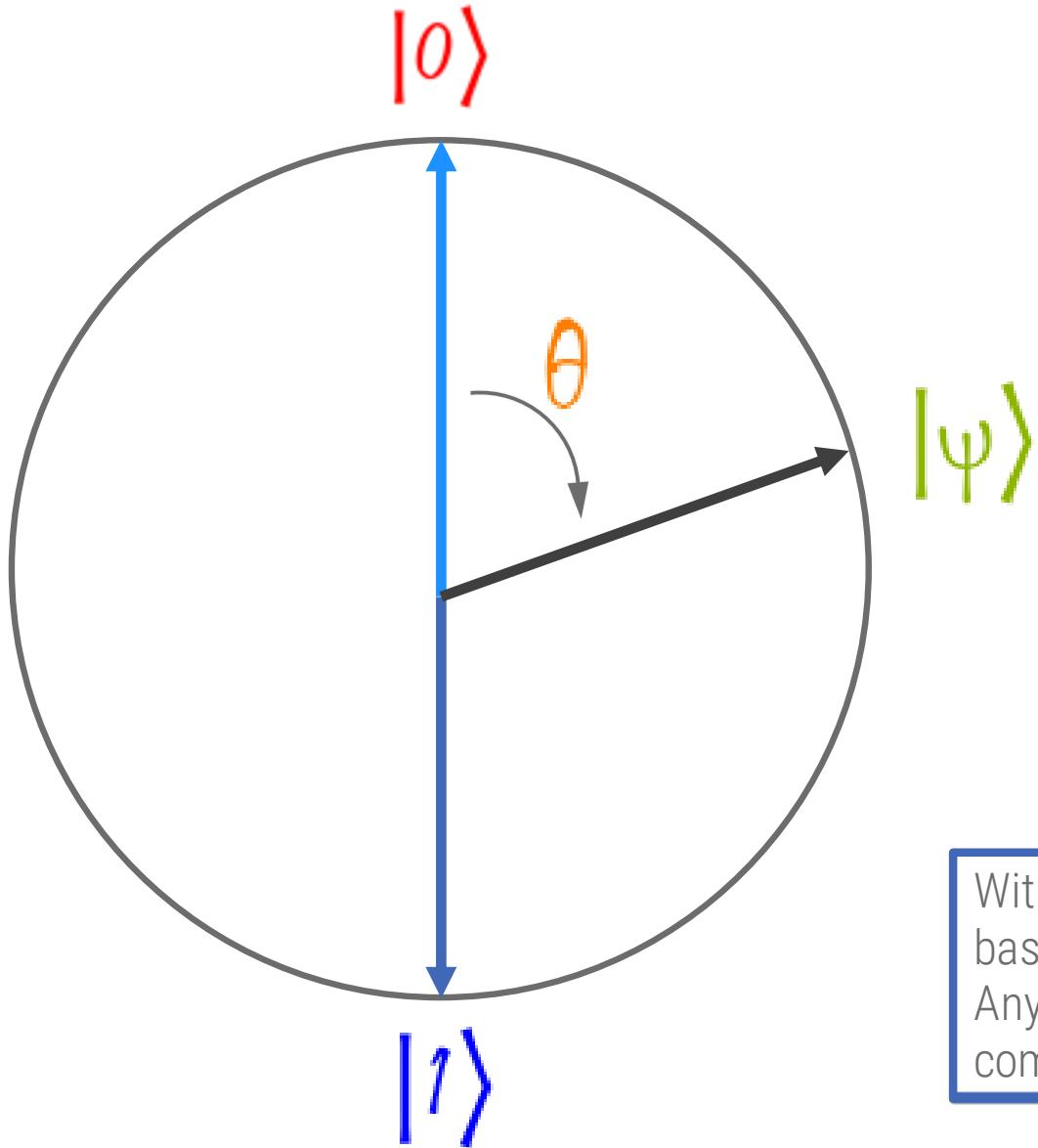


$$|\psi\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \left( \cos\left(\frac{\theta}{2}\right), \sin\left(\frac{\theta}{2}\right) \right)$$

$$|0\rangle = \left( \cos\left(\frac{0}{2}\right), \sin\left(\frac{0}{2}\right) \right) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$|1\rangle = \left( \cos\left(\frac{\pi}{2}\right), \sin\left(\frac{\pi}{2}\right) \right) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

# Qubits and Bloch Sphere



$$|\psi\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \left( \cos\left(\frac{\theta}{2}\right), \sin\left(\frac{\theta}{2}\right) \right)$$

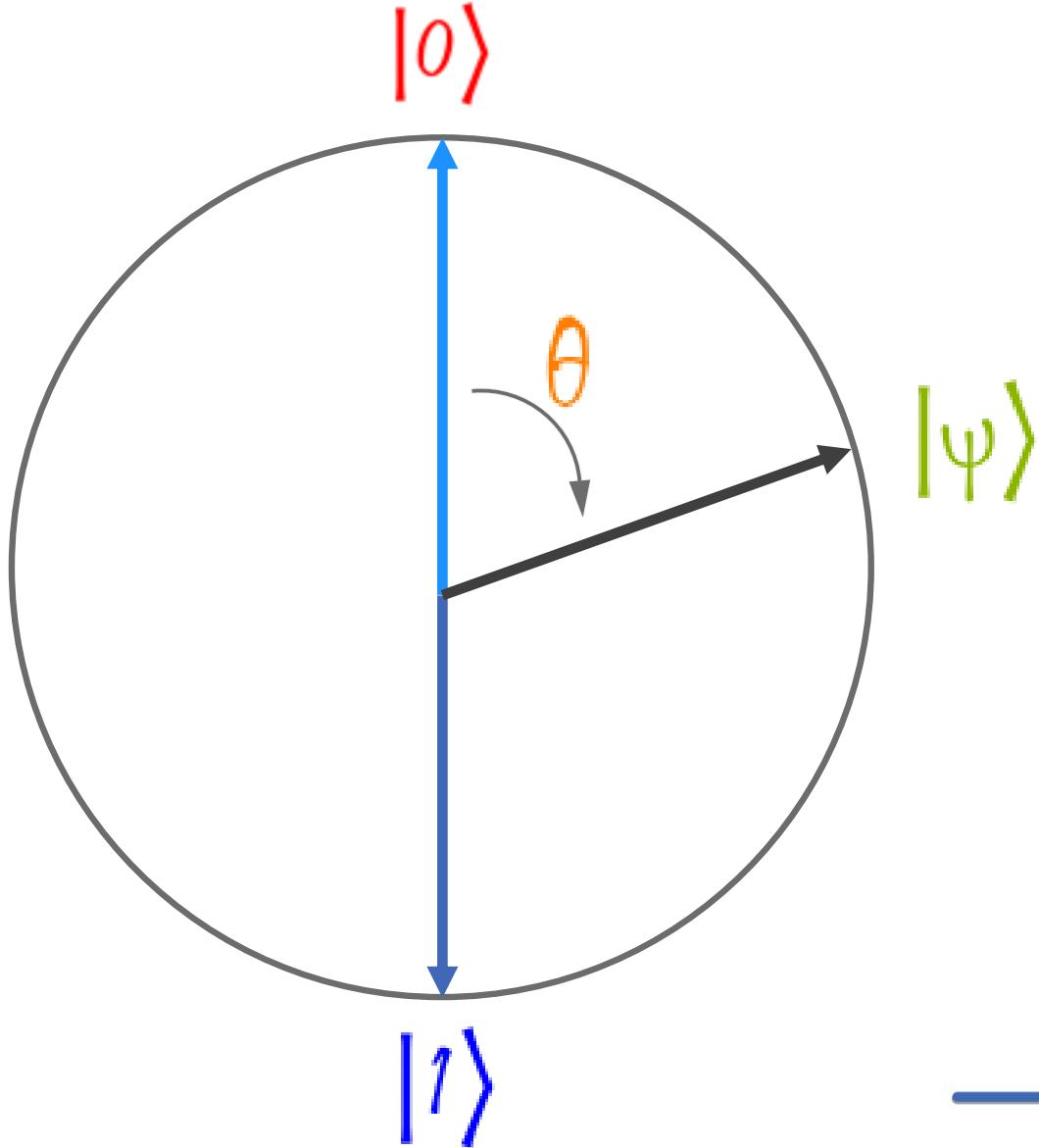
$$|0\rangle = \left( \cos\left(\frac{0}{2}\right), \sin\left(\frac{0}{2}\right) \right) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$|1\rangle = \left( \cos\left(\frac{\pi}{2}\right), \sin\left(\frac{\pi}{2}\right) \right) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

With this choice of coordinates, the classical states  $|0\rangle$  and  $|1\rangle$  represent a base.  
Any other state of the system can therefore be represented as a linear combination of the classical states

# Qubits and Bloch Sphere

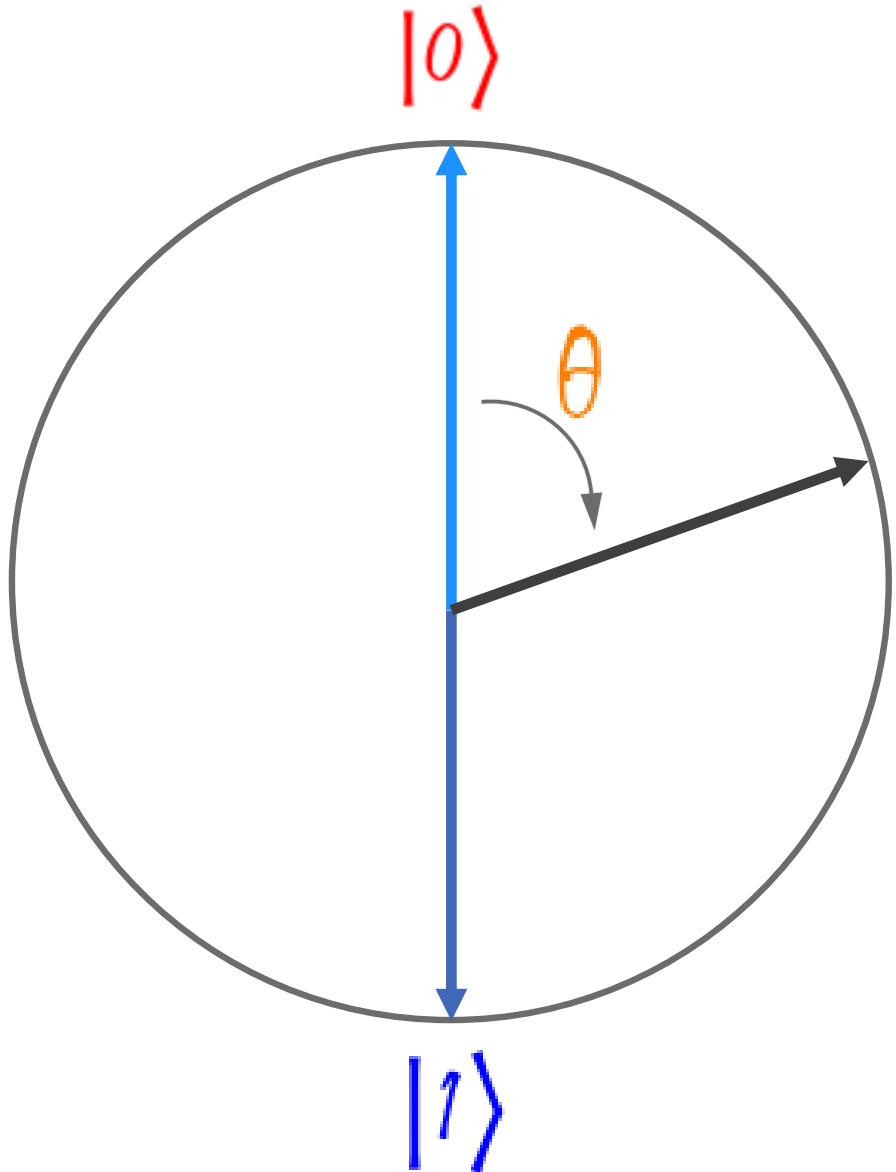
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$$|\psi\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \left( \cos\left(\frac{\theta}{2}\right), \sin\left(\frac{\theta}{2}\right) \right)$$

# Qubits and Bloch Sphere

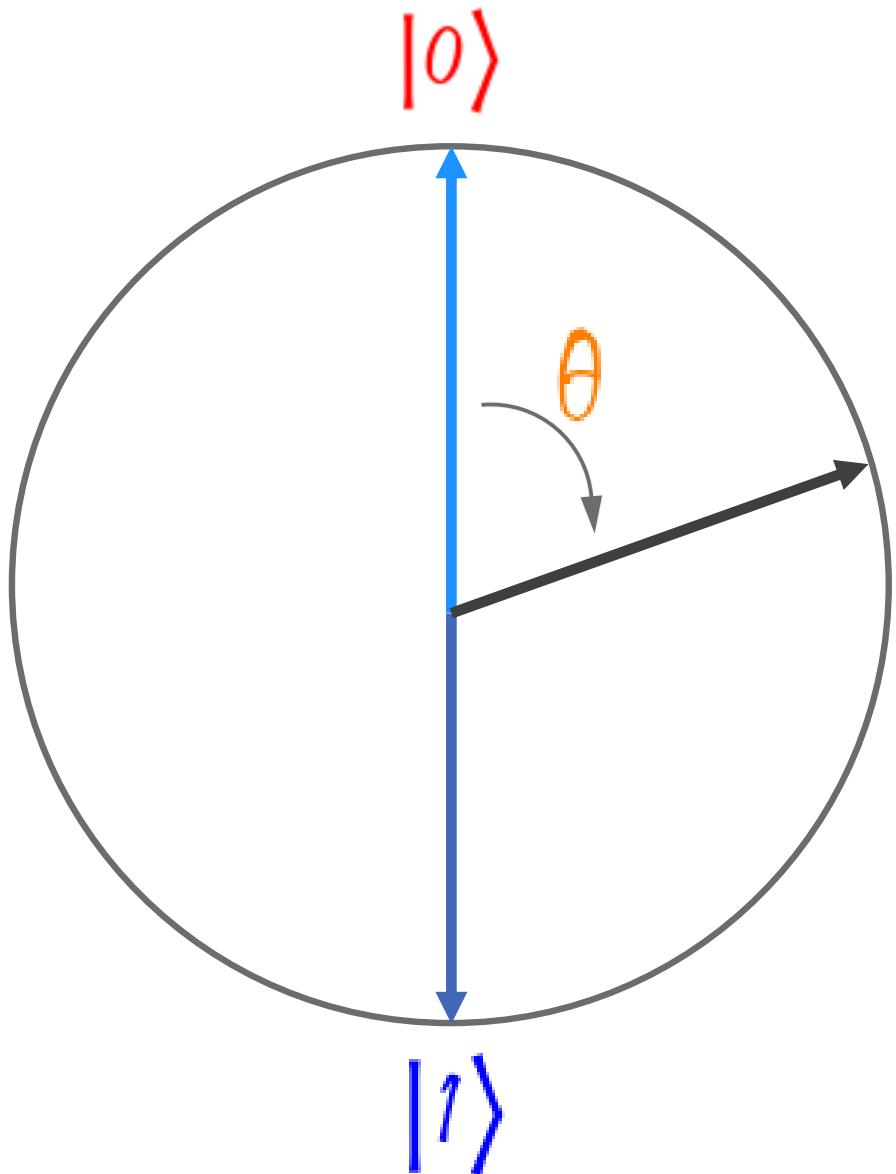
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$$|\psi\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \left( \cos\left(\frac{\theta}{2}\right), \sin\left(\frac{\theta}{2}\right) \right)$$

$$|\psi\rangle = \alpha \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \beta \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \alpha \cdot |0\rangle + \beta \cdot |1\rangle$$

# Qubits and Bloch Sphere



$$|\Psi\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \left( \cos\left(\frac{\theta}{2}\right), \sin\left(\frac{\theta}{2}\right) \right)$$

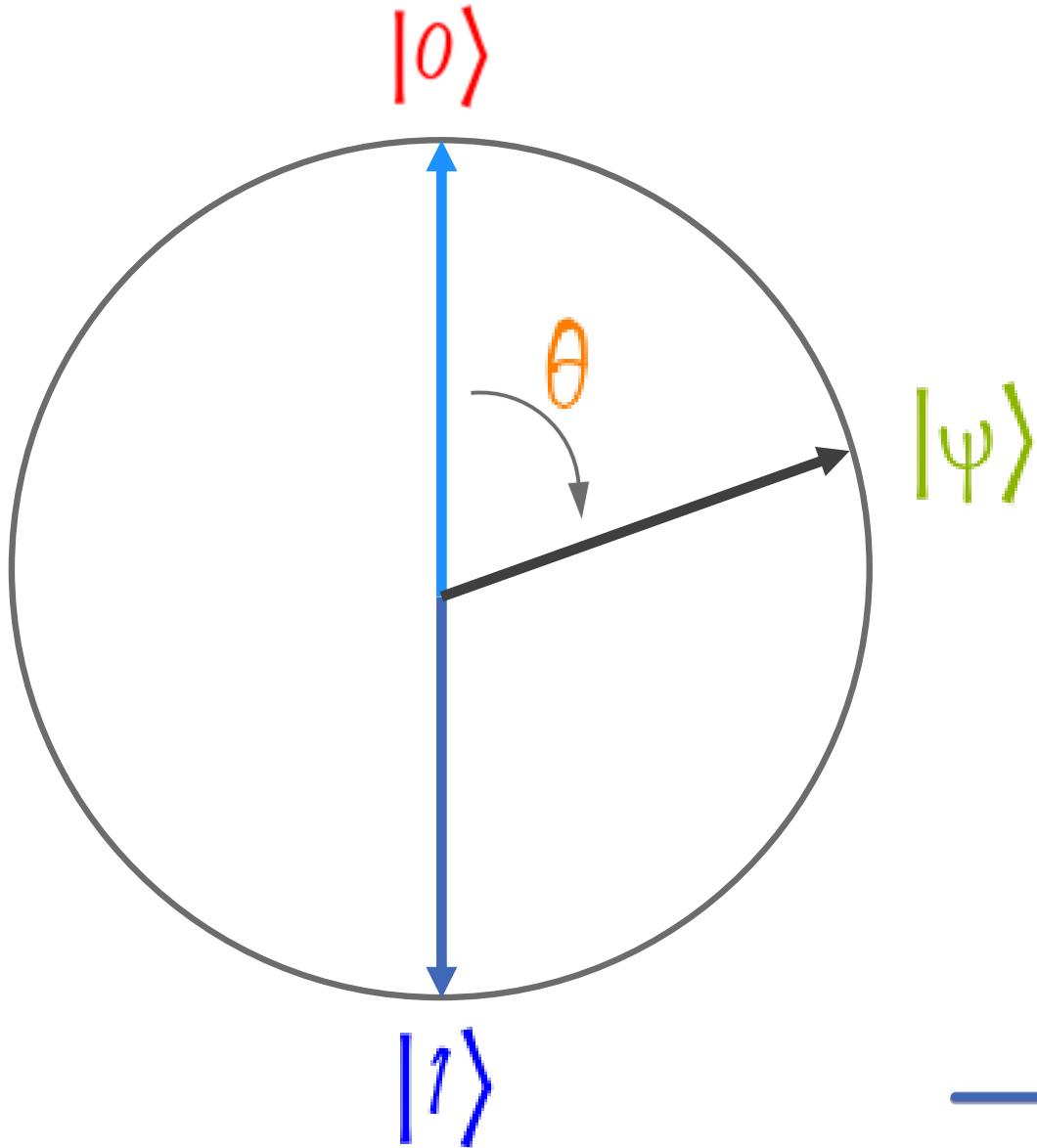
$$|\Psi\rangle = \alpha \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \beta \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \alpha \cdot |0\rangle + \beta \cdot |1\rangle$$

## Superposition

Written in this way, the relationship between the coordinates of a qubit and the superposition principle begins to appear clear.

In mathematical terms, the superposition state manifested by a qubit is expressed as a linear combination of the classical states 0 and 1

# Qubits and Bloch Sphere



$$|\psi\rangle = \alpha \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \beta \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \alpha \cdot |0\rangle + \beta \cdot |1\rangle$$

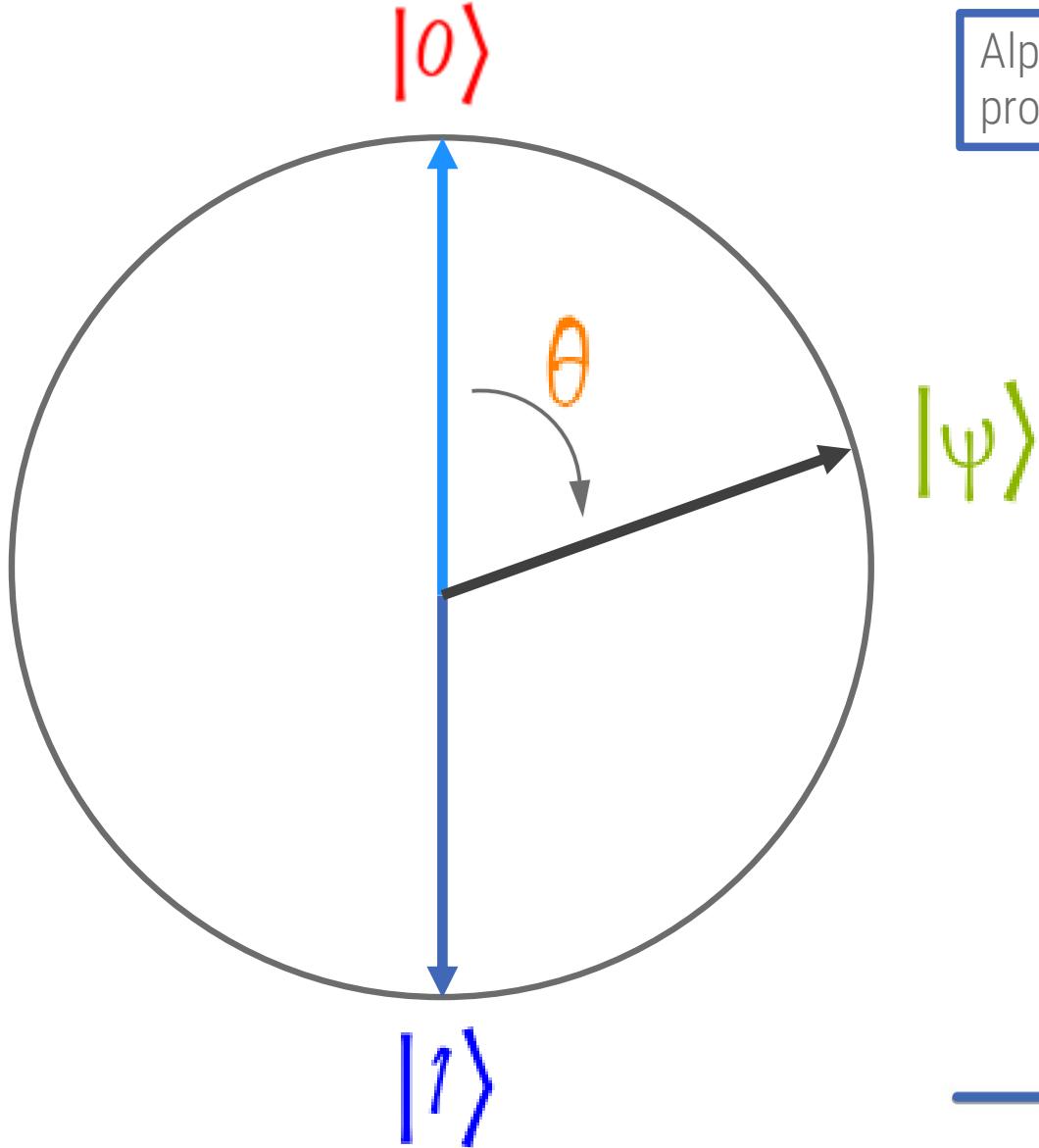
## Measuring a qubit: the wave function

Even if it can be represented mathematically, it will never be possible to observe a state of superposition.

Quantum computing, like classical computing, involves the measurement operation, which aims to make known the value of a certain qubit / bit

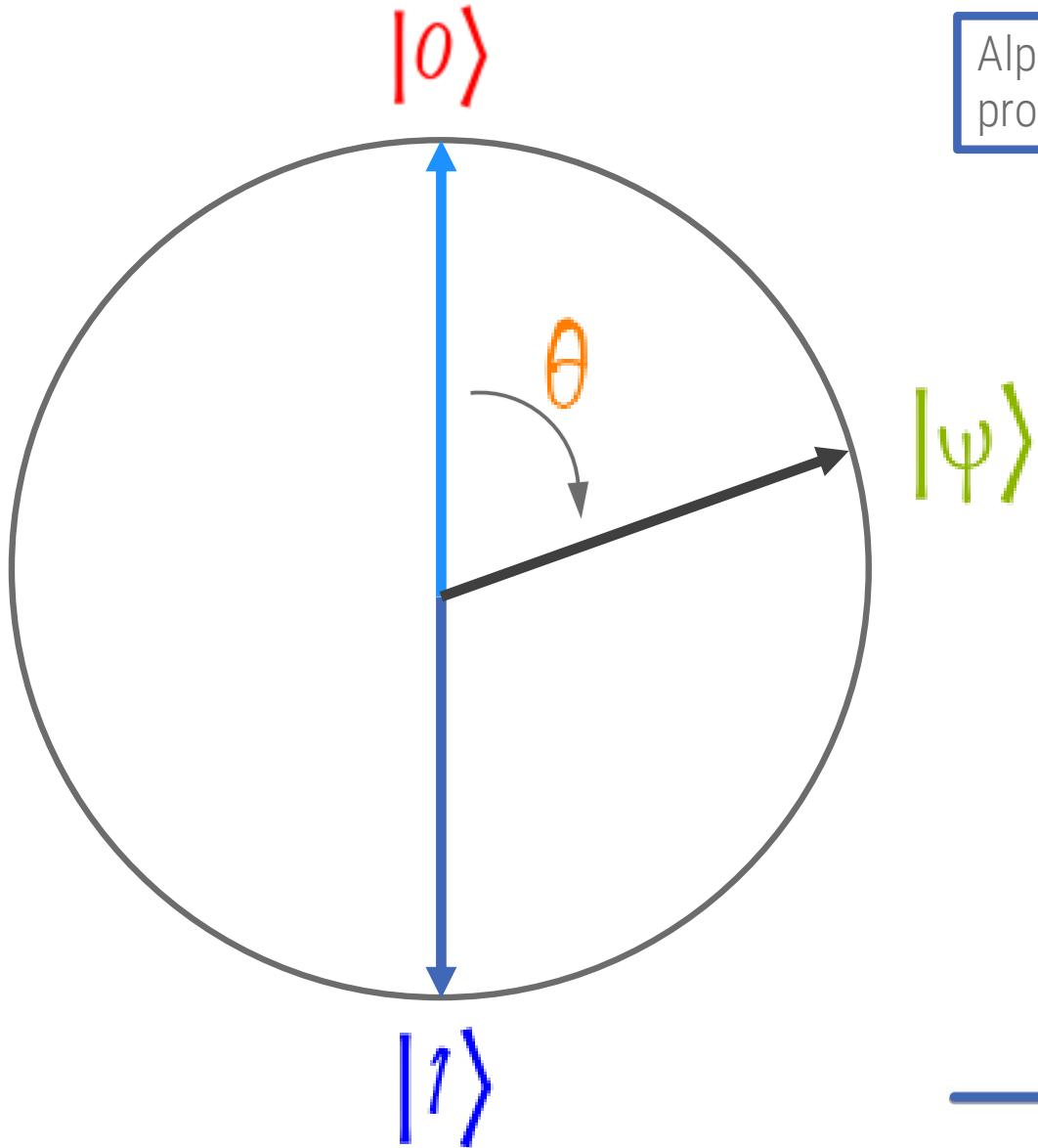
The fundamental difference is that the measurement of a qubit destroys the superposition state, forcing the qubit to assume one of the two classical values (collapse of the wave function)

# Qubits and Bloch Sphere



Alpha and Beta, commonly called amplitudes, are directly related to the probability of observing a qubit in one of the corresponding classical states

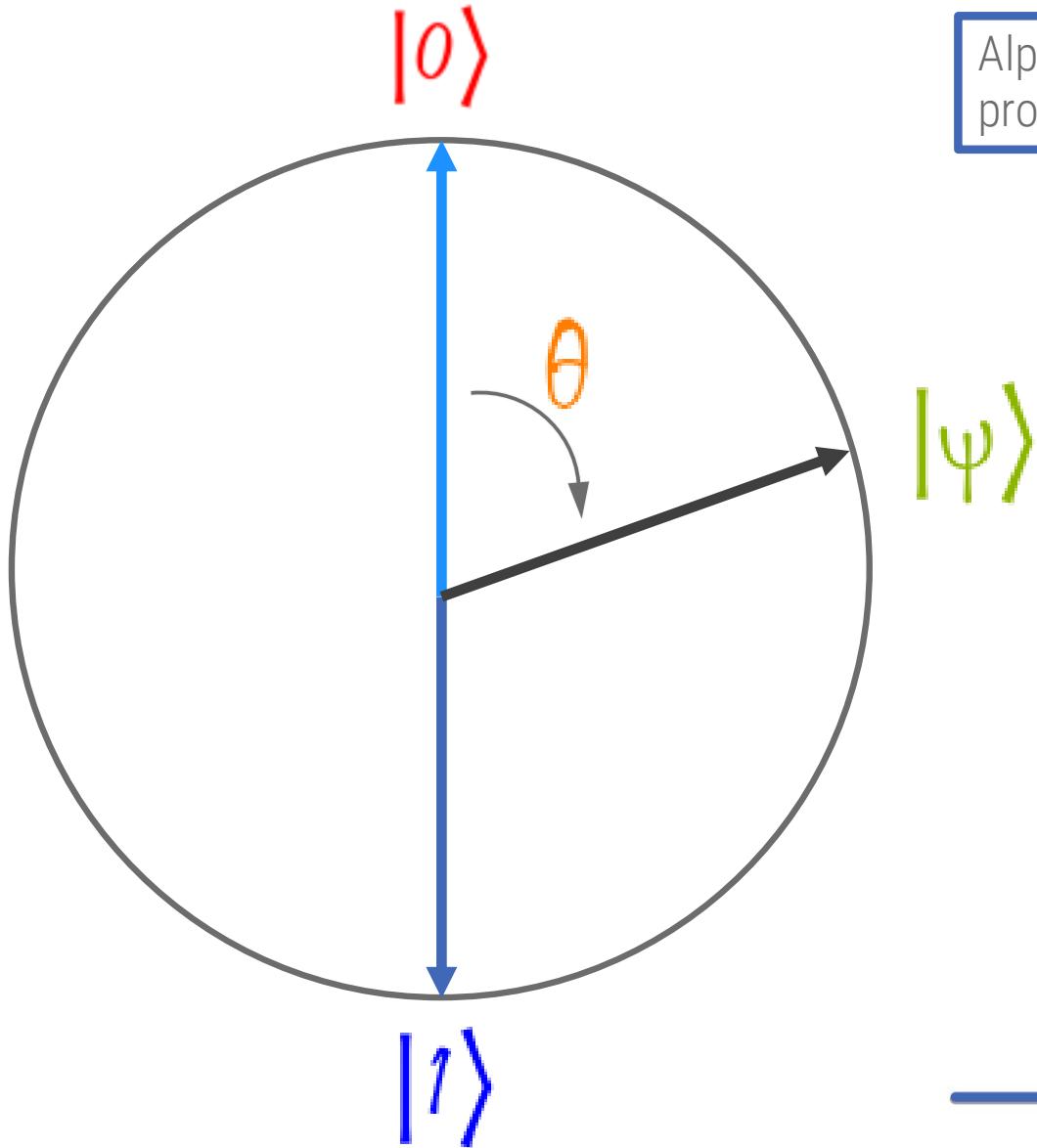
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$$|\psi\rangle = \alpha \cdot |0\rangle + \beta \cdot |1\rangle$$

# Qubits and Bloch Sphere



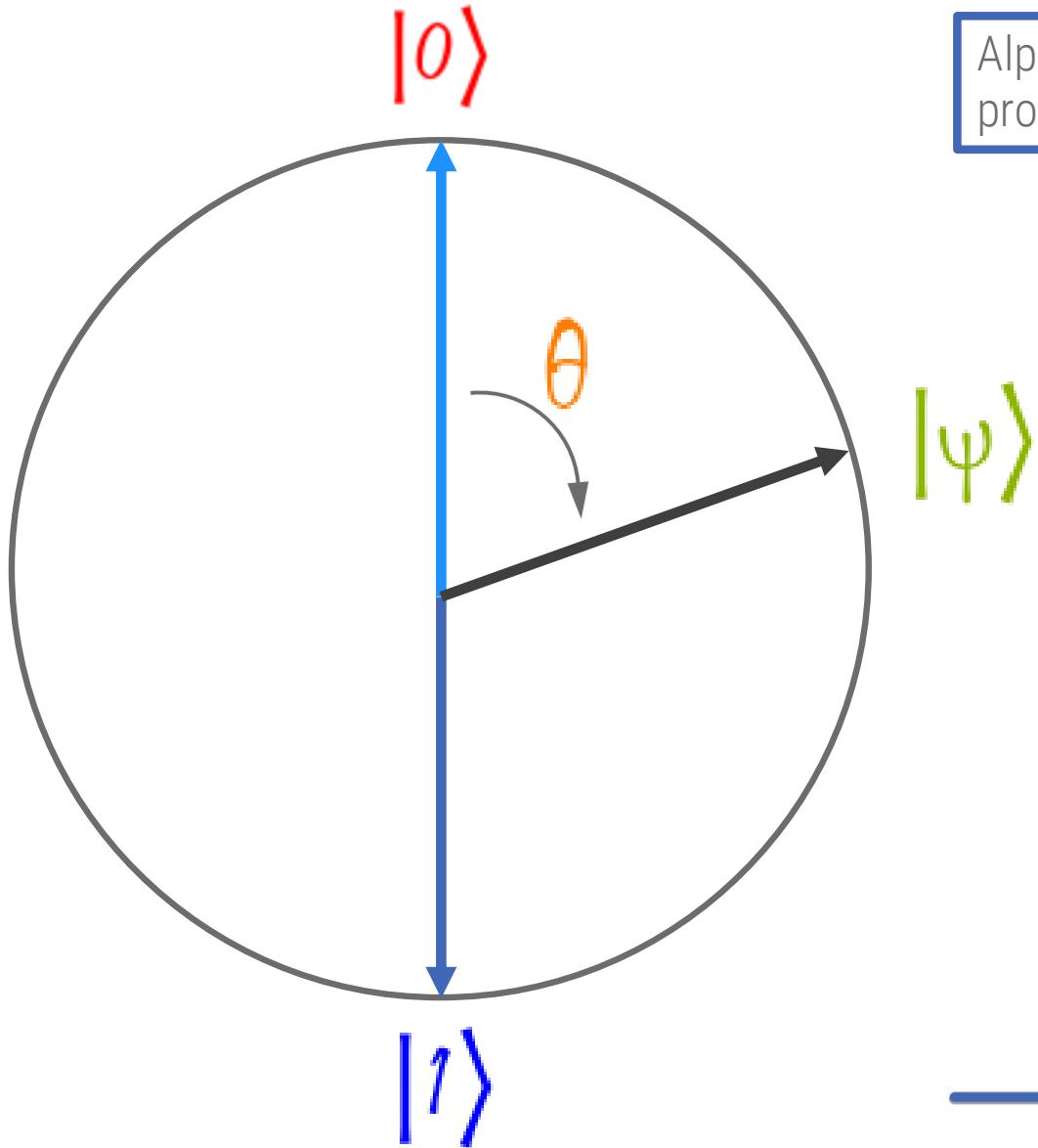
Alpha and Beta, commonly called amplitudes, are directly related to the probability of observing a qubit in one of the corresponding classical states

$$|\psi\rangle = \alpha \cdot |0\rangle + \beta \cdot |1\rangle$$

$$P(|\psi\rangle = |0\rangle) = \alpha^2$$

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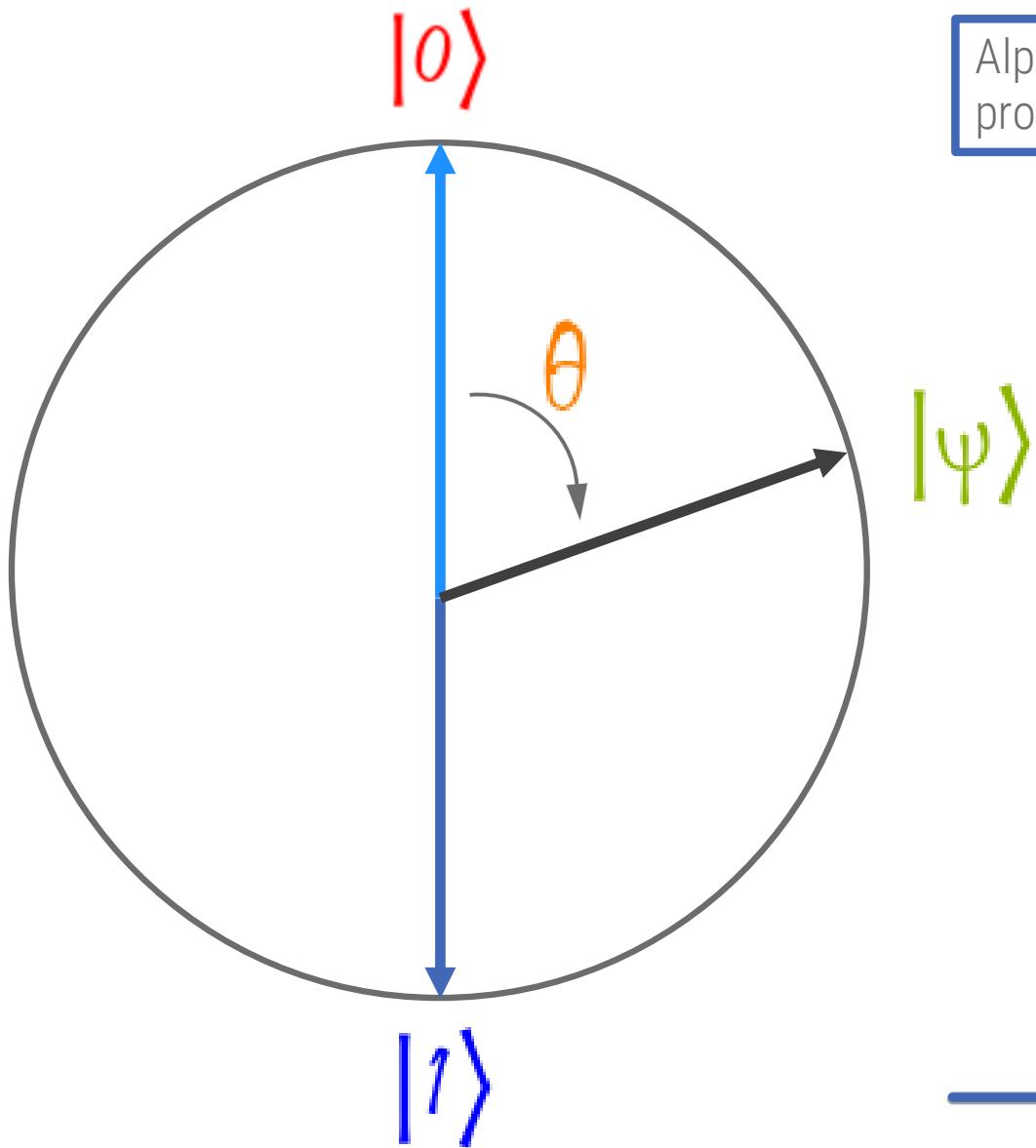
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$$\alpha^2 + \beta^2 = 1$$

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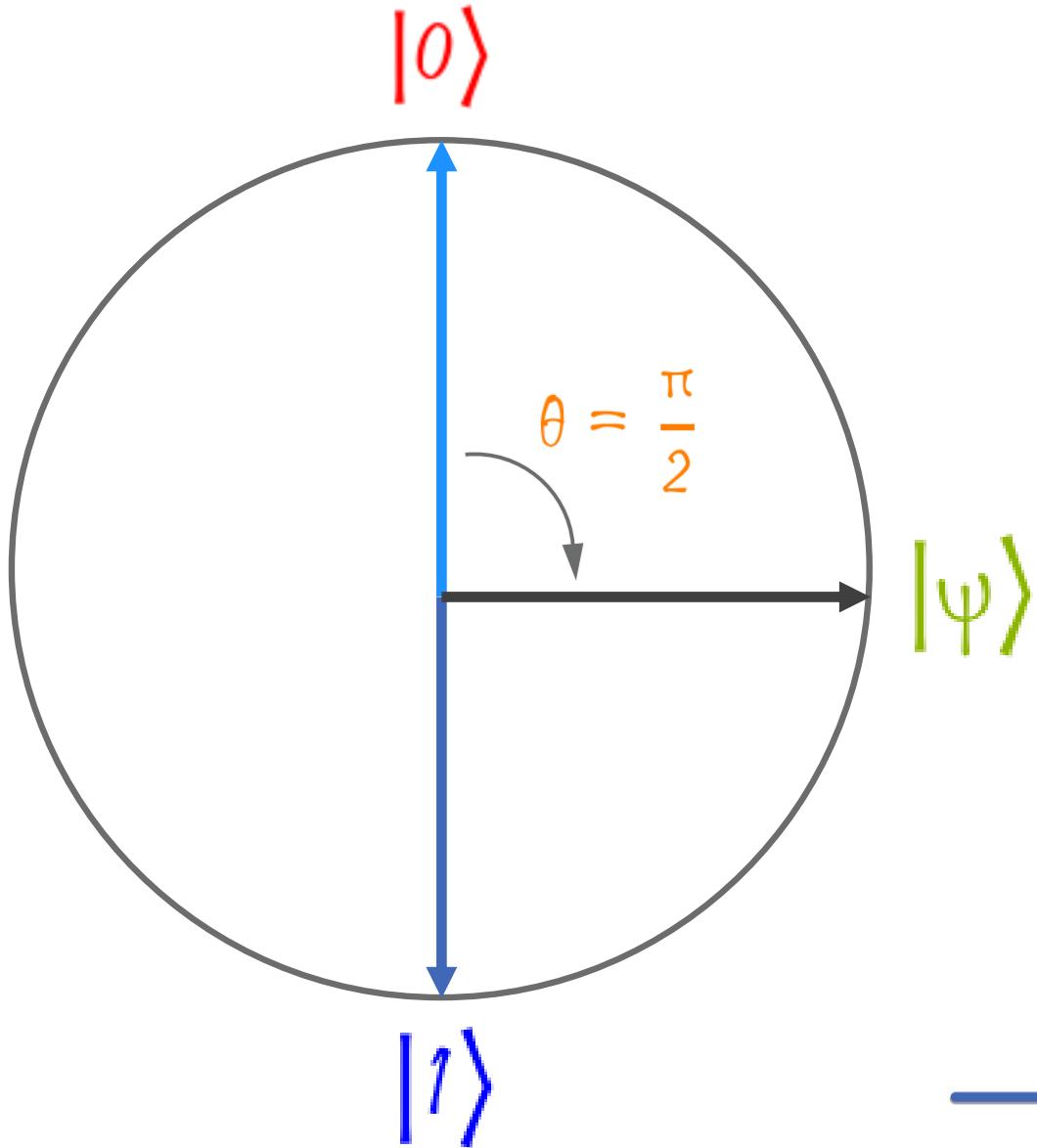
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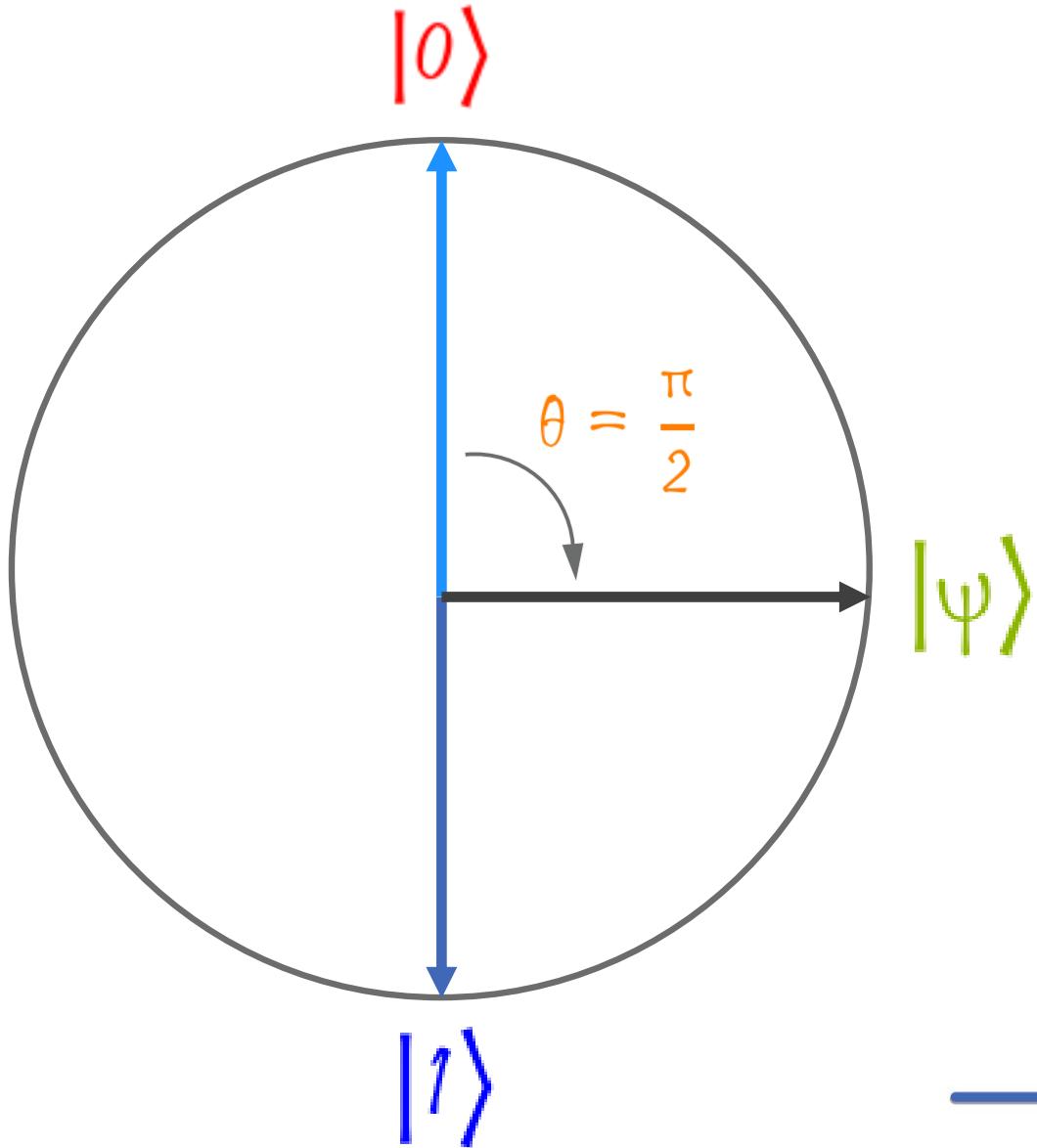
$$\left( \cos^2\left(\frac{\theta}{2}\right) + \sin^2\left(\frac{\theta}{2}\right) \right) = 1 \quad \forall \theta$$

# Qubits and Bloch Sphere



$$|\psi\rangle = \left( \cos\left(\frac{\theta}{2}\right), \sin\left(\frac{\theta}{2}\right) \right)$$

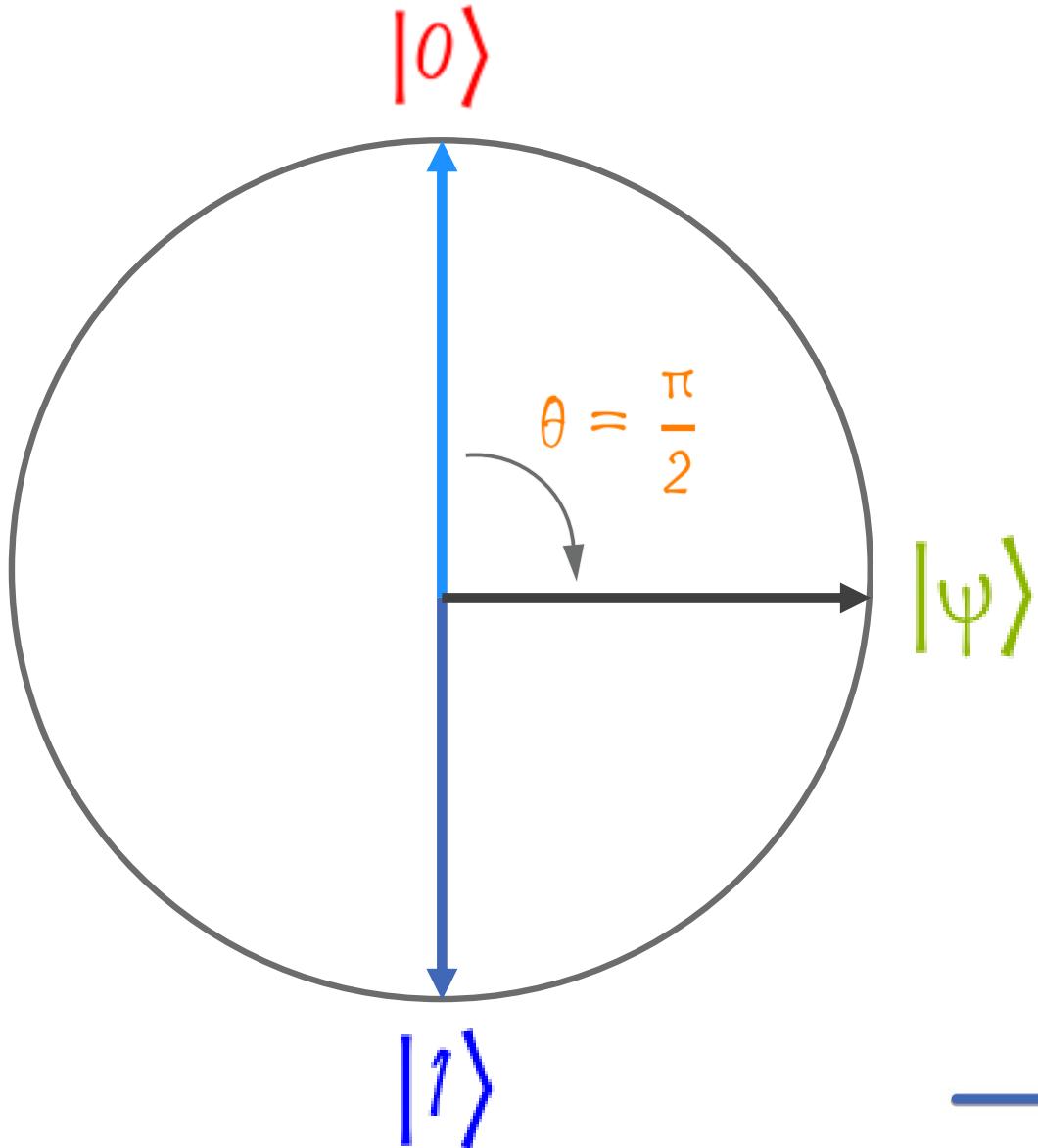
# Qubits and Bloch Sphere



$$\begin{aligned} |\psi\rangle &= \left( \cos\left(\frac{\theta}{2}\right), \sin\left(\frac{\theta}{2}\right) \right) \\ &= \left( \cos\left(\frac{\pi}{4}\right), \sin\left(\frac{\pi}{4}\right) \right) \end{aligned}$$

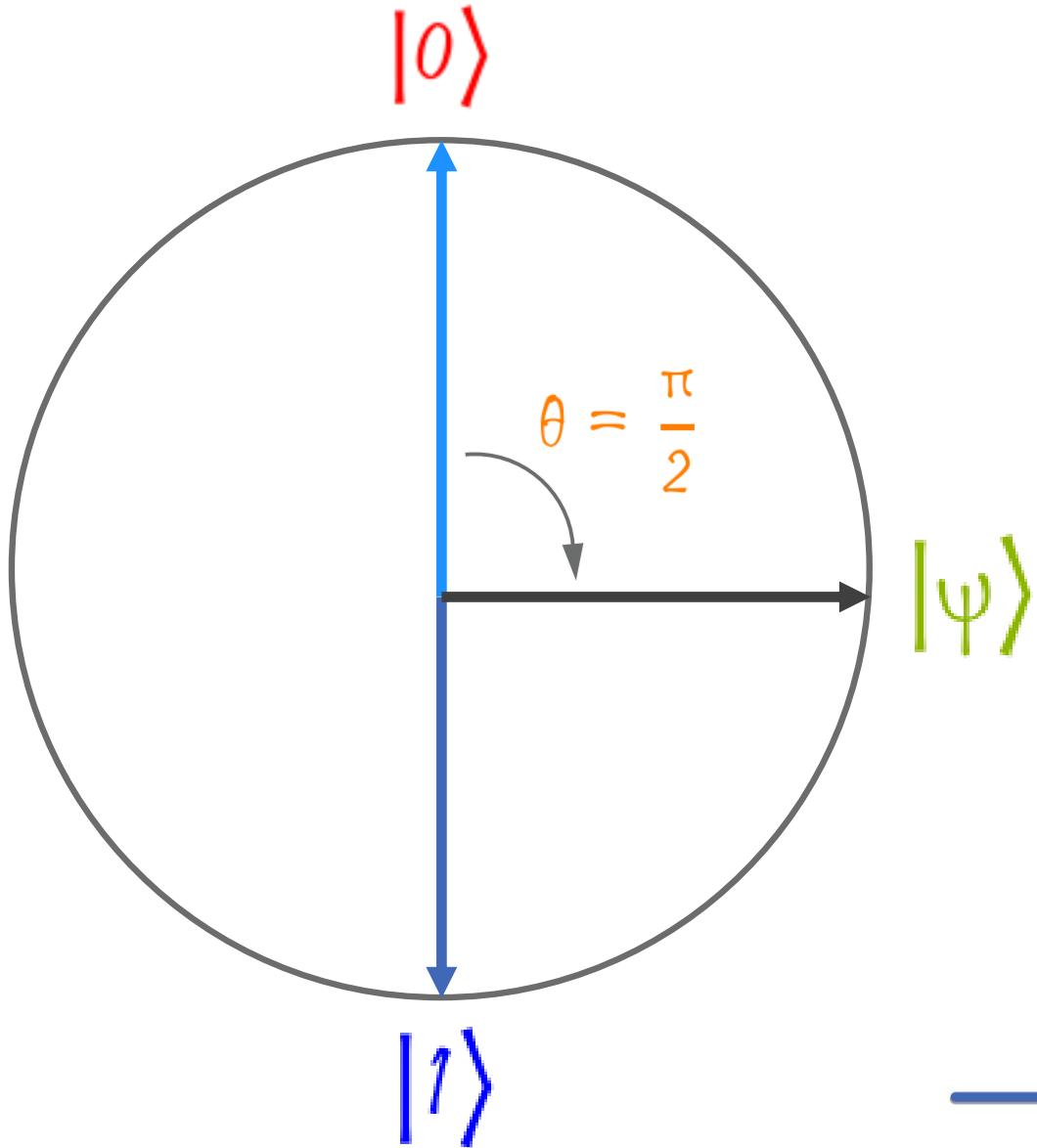
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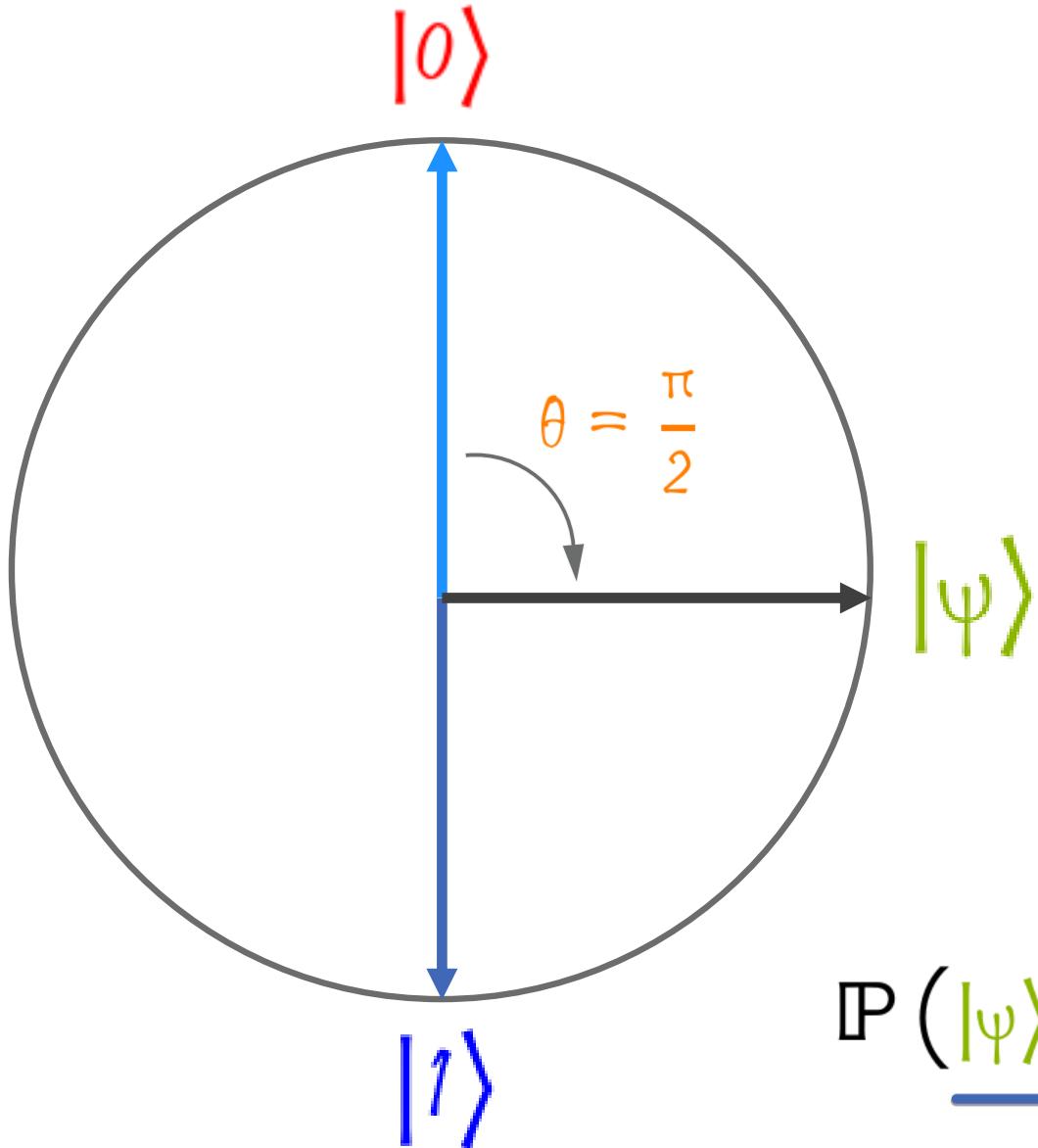
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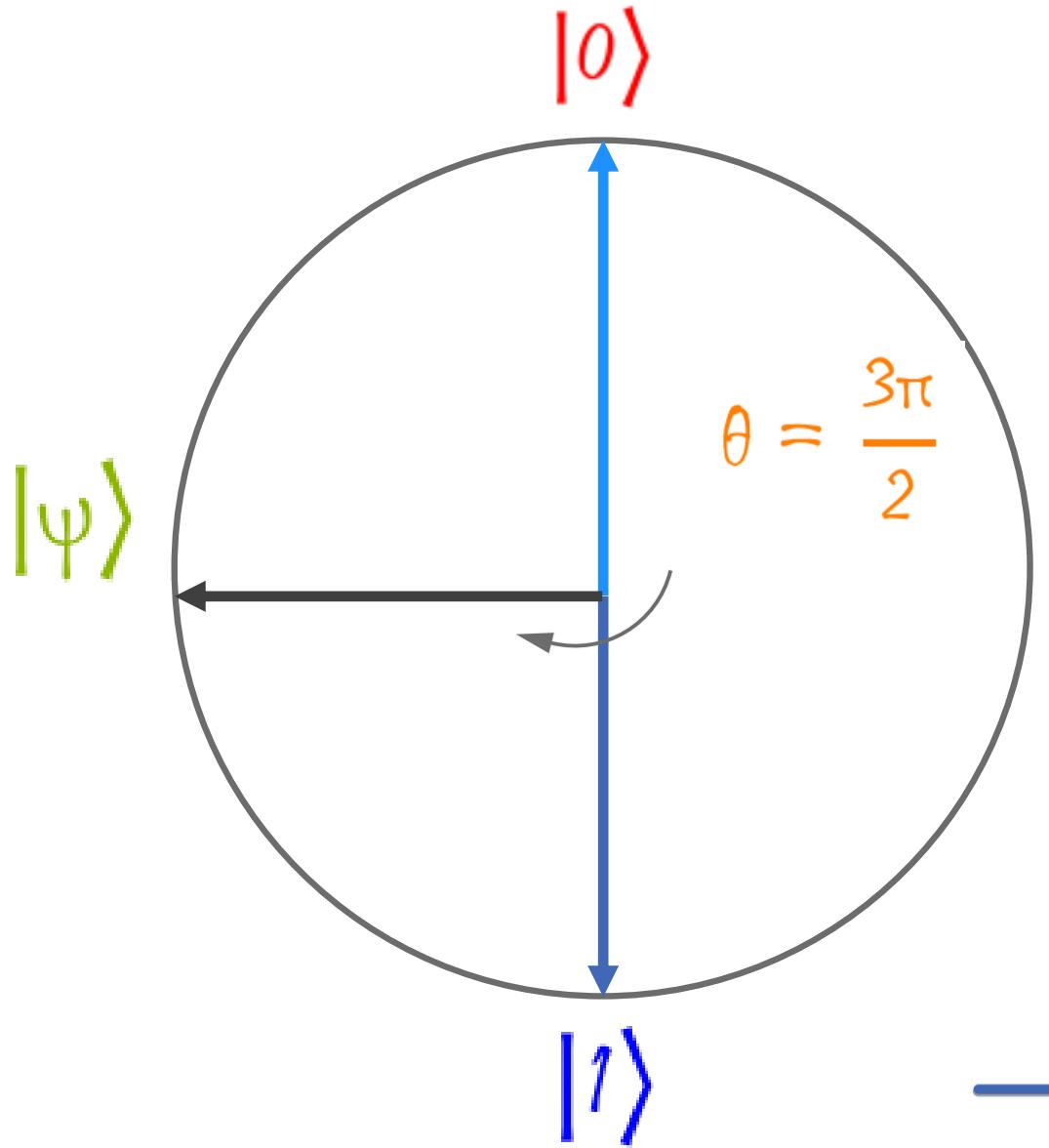
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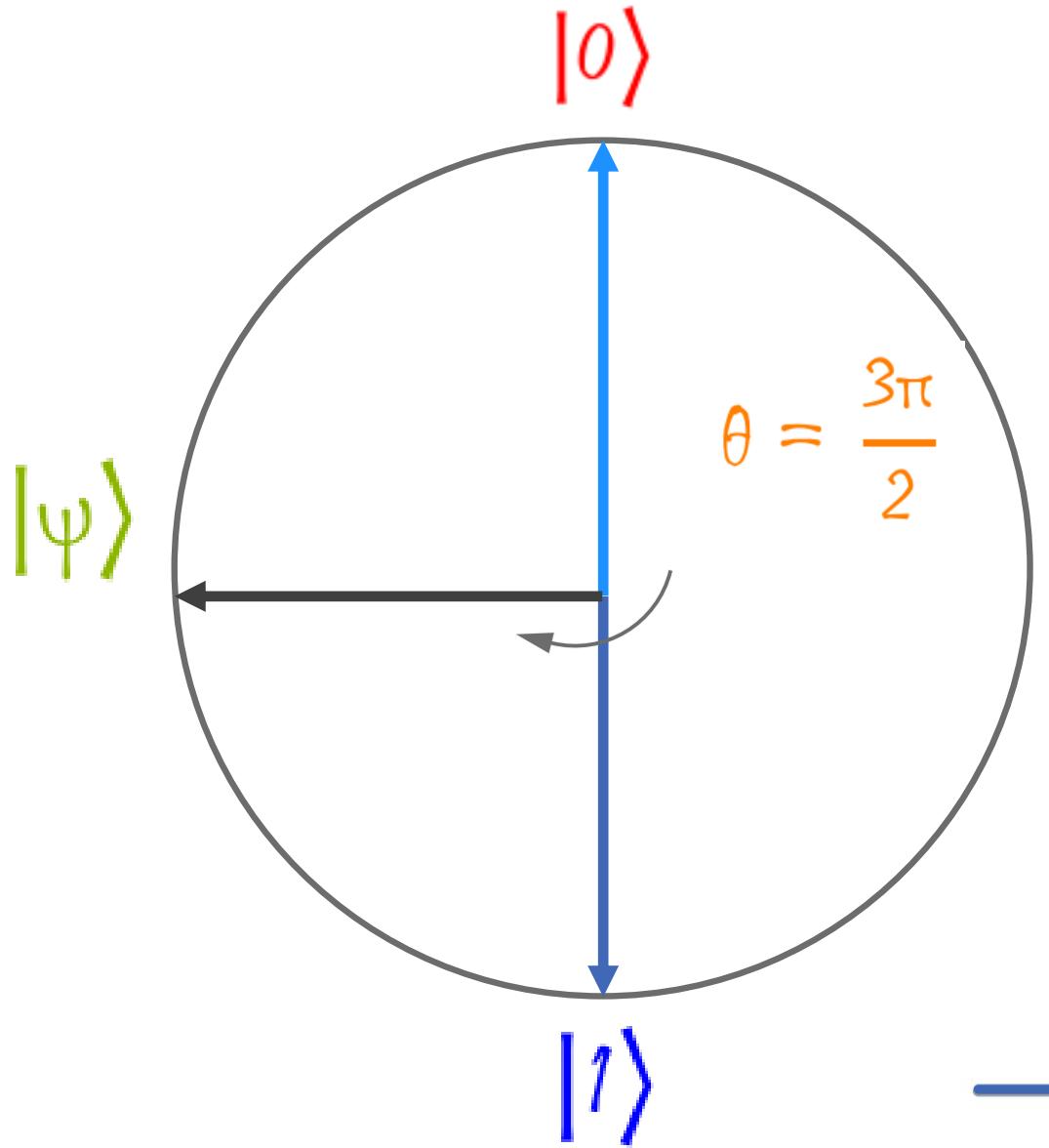
$$P(|\psi\rangle = |0\rangle) = P(|\psi\rangle = |1\rangle) = \frac{1}{2} \quad (50\%)$$

# Qubits and Bloch Sphere



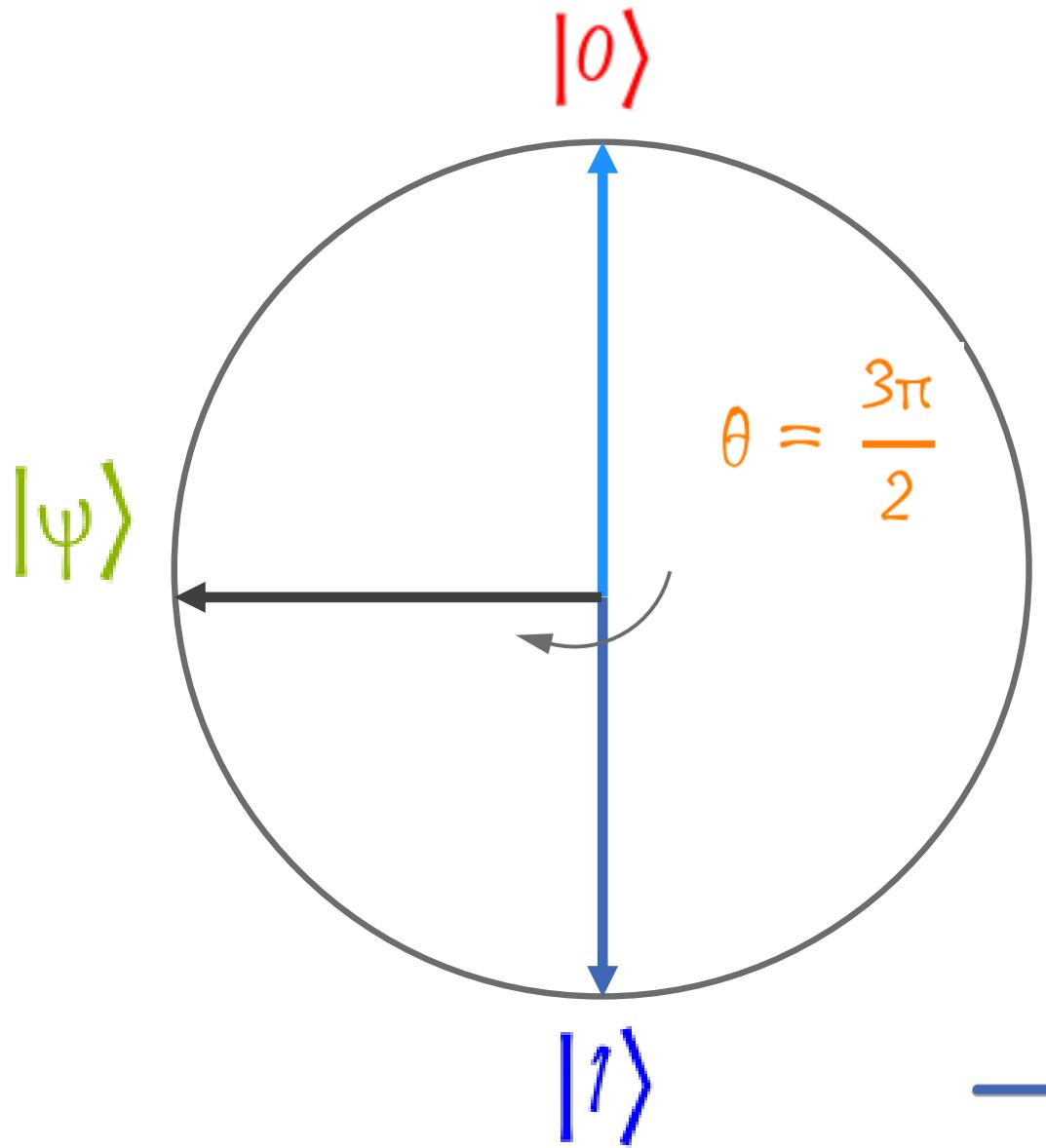
$$|\psi\rangle = \left( \cos\left(\frac{\theta}{2}\right), \sin\left(\frac{\theta}{2}\right) \right)$$

# Qubits and Bloch Sphere



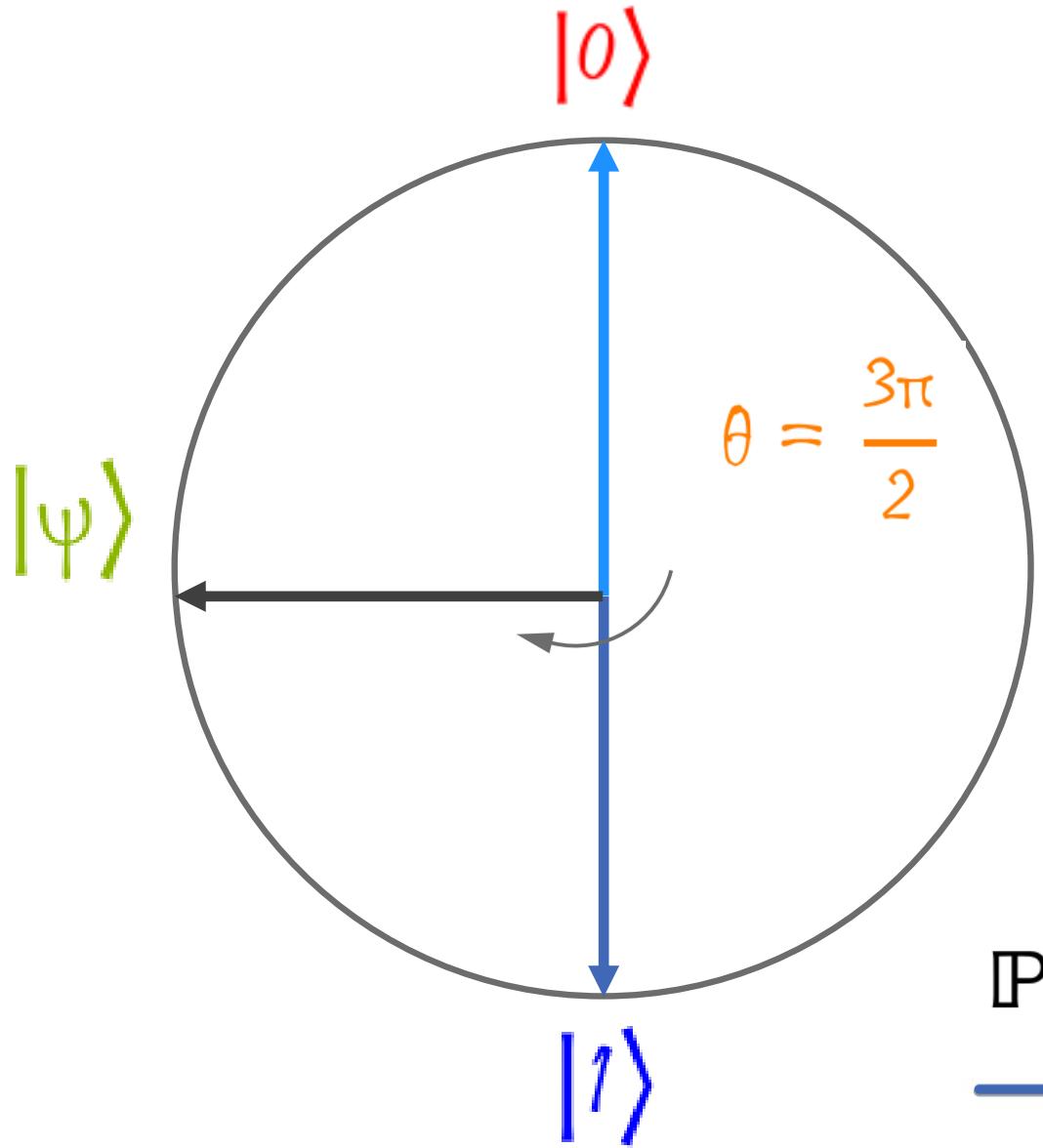
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# Qubits and Bloch Sphere



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# Qubits and Bloch Sphere

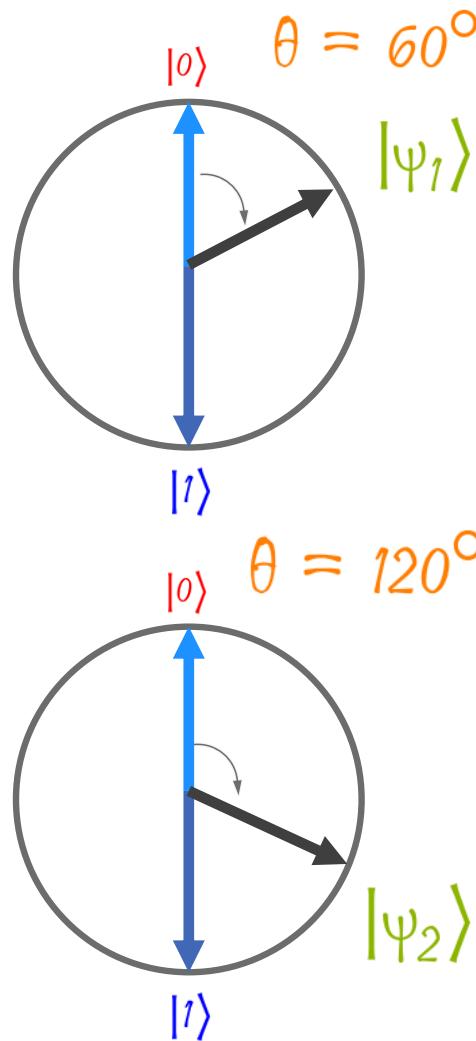


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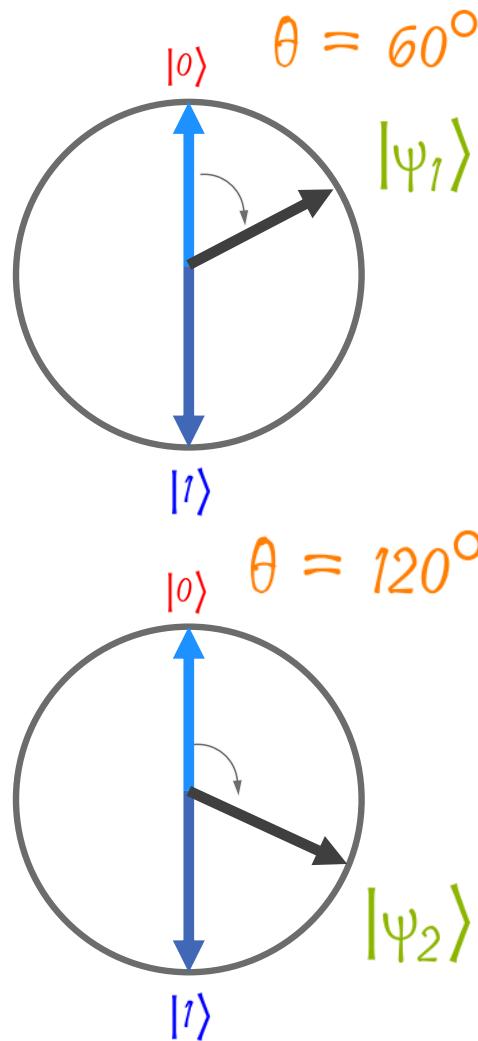
$$\mathbb{P}(|\psi\rangle = |0\rangle) = \mathbb{P}(|\psi\rangle = |1\rangle) = \frac{1}{2} \quad (50\%)$$

# Two Qubits System

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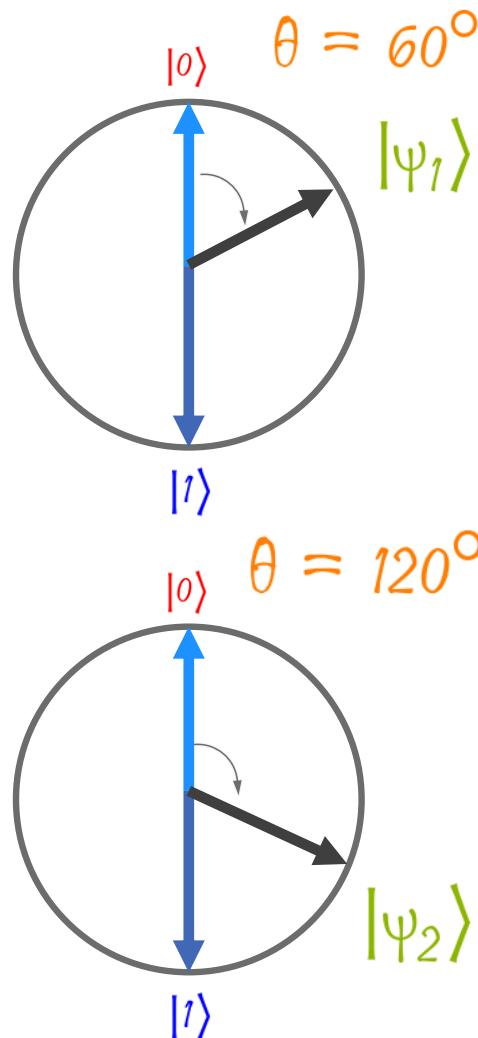


# Two Qubits System



- What are the mathematical representations of these two qubits as a function of the classical states?

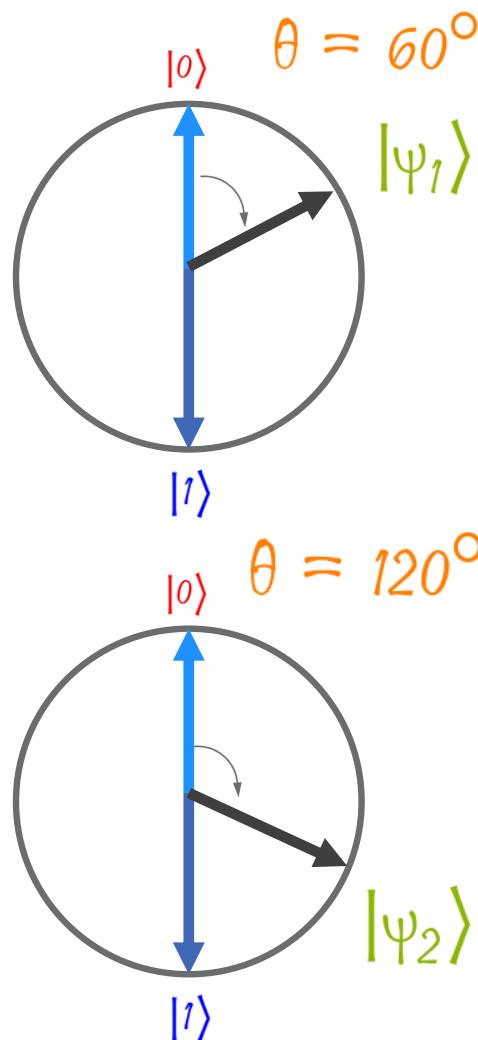
# Two Qubits System



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$$|\Psi_1\rangle = \frac{\sqrt{3}}{2} |0\rangle + \frac{1}{2} |1\rangle, \quad |\Psi_2\rangle = \frac{1}{2} |0\rangle + \frac{\sqrt{3}}{2} |1\rangle$$

# Two Qubits System

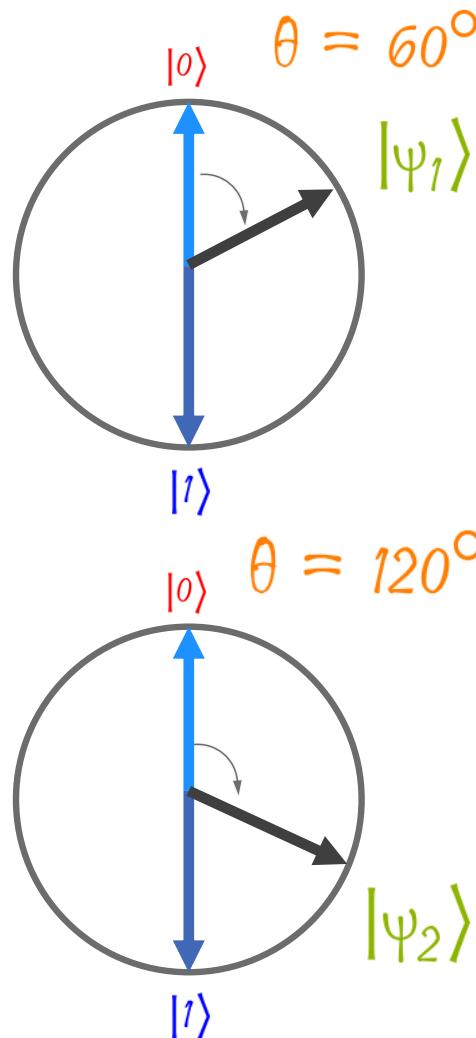


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# Two Qubits System



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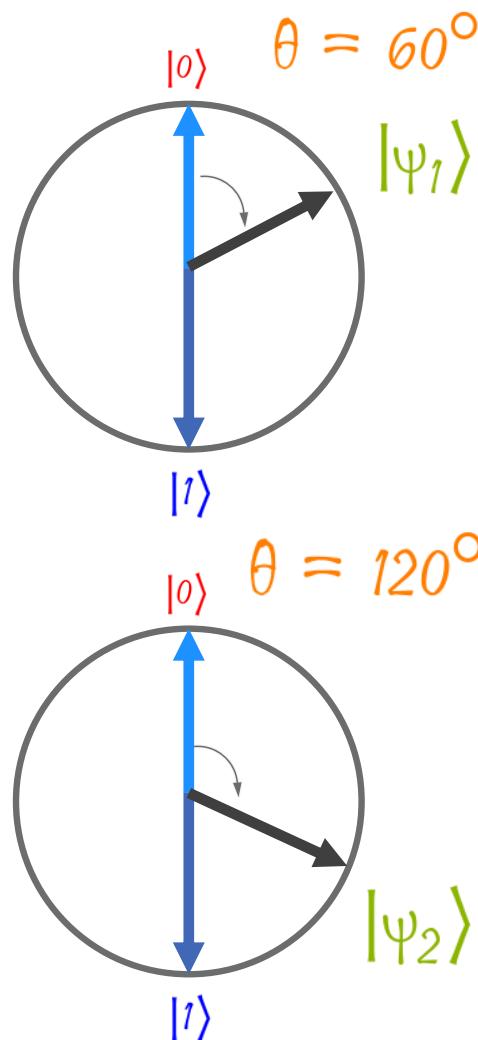
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- What are the probabilities of obtaining the classical states 0 and 1 after taking a measurement?

$ \Psi_1\rangle$	P
$ 0\rangle$	75%
$ 1\rangle$	25%

$ \Psi_2\rangle$	P
$ 0\rangle$	25%
$ 1\rangle$	75%

# Two Qubits System



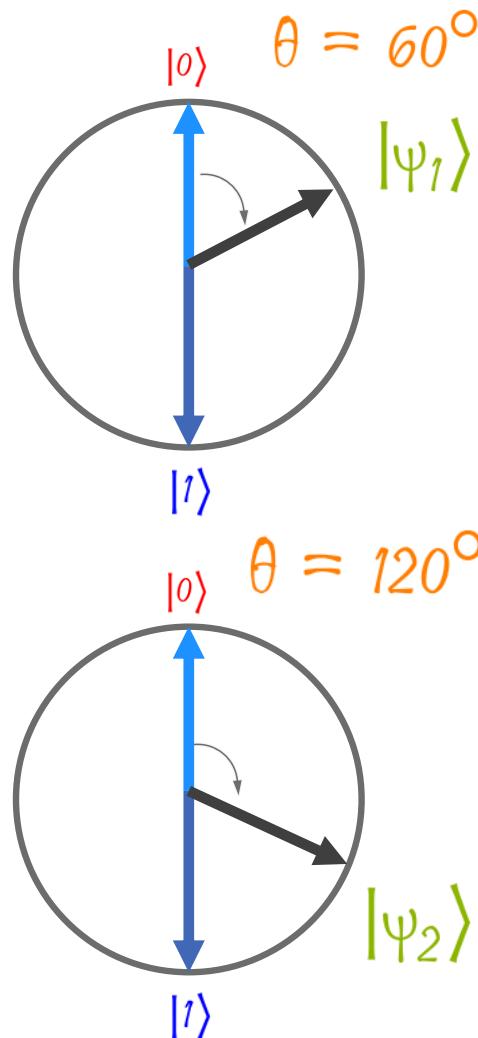
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- Considering that the two measurements are independent event, what could be the probability of observing one of the four possible combinations of classical states?

# Two Qubits System



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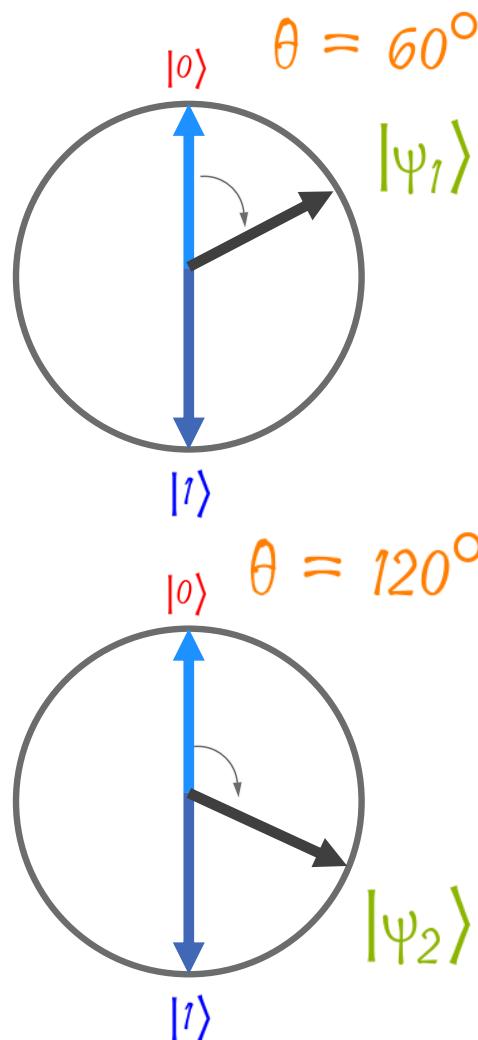
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$ \psi_2\rangle$	P
$ 0\rangle$	25%
$ 1\rangle$	75%

- Considering that the two measurements are independent event, what could be the probability of observing one of the four possible combinations of classical states?

$ \psi_1\psi_2\rangle$	$ 0\rangle$	$ 1\rangle$
$ 0\rangle$	18.75%	56.25%
$ 1\rangle$	6.25%	18.75%

# Two Qubits System

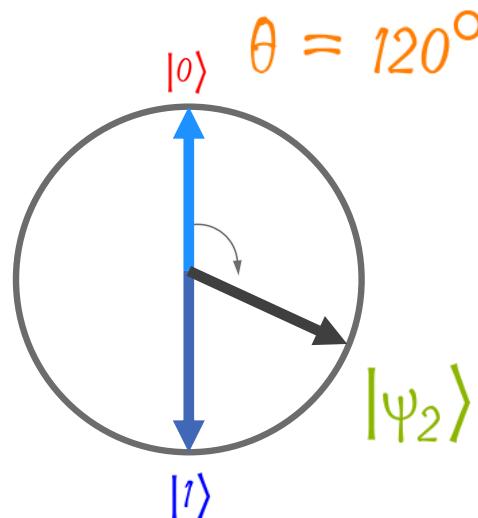
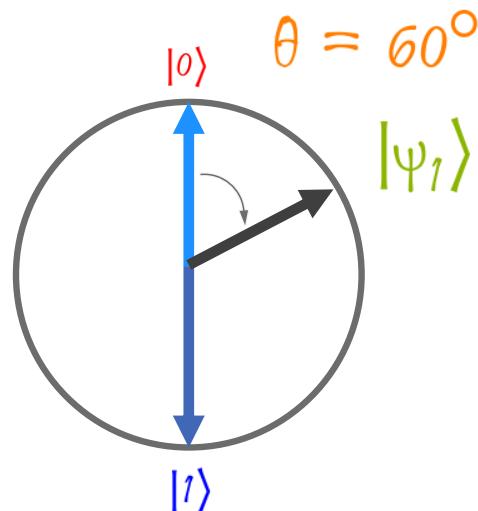


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- The wave function that represents the possible states of a system composed of two qubits is a function with 4 different terms ( $2^2$ )

# Two Qubits System

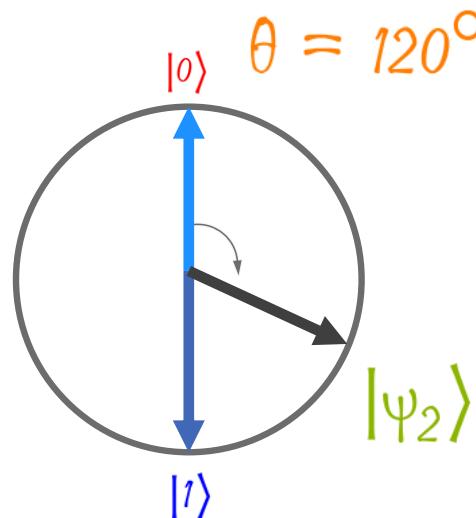
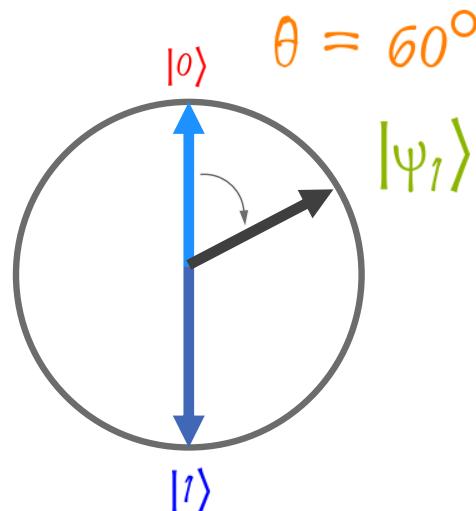


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$ \Psi_1\Psi_2\rangle$	$ 0\rangle$	$ 1\rangle$
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- By extracting the square root of the values obtained as the probability of obtaining a combination of classical states, I can therefore obtain a description of the wave function relative to the system composed of the two qubits

# Two Qubits System



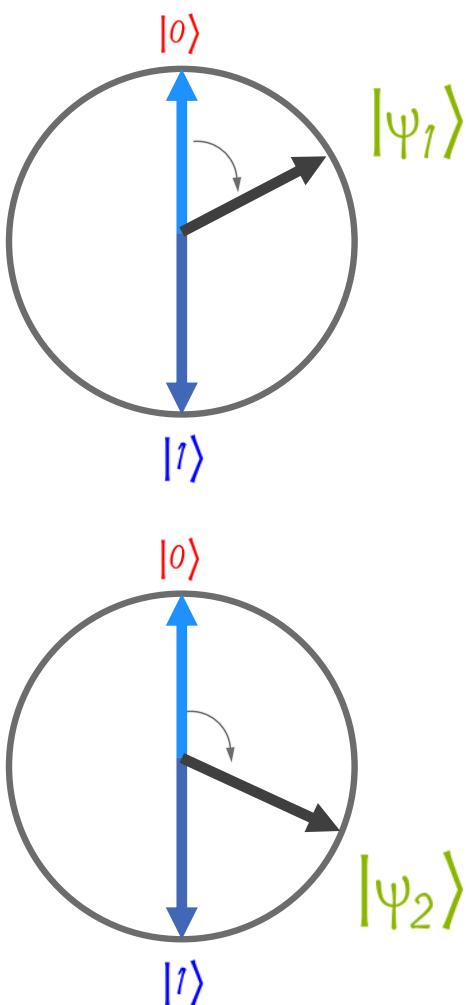
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$$|\Psi_1\Psi_2\rangle = 0.43|00\rangle + 0.75|01\rangle + 0.25|10\rangle + 0.43|11\rangle$$

# Two Qubits System



$$|\psi_1\rangle = \alpha_1|0\rangle + \beta_1|1\rangle, \quad |\psi_2\rangle = \alpha_2|0\rangle + \beta_2|1\rangle$$

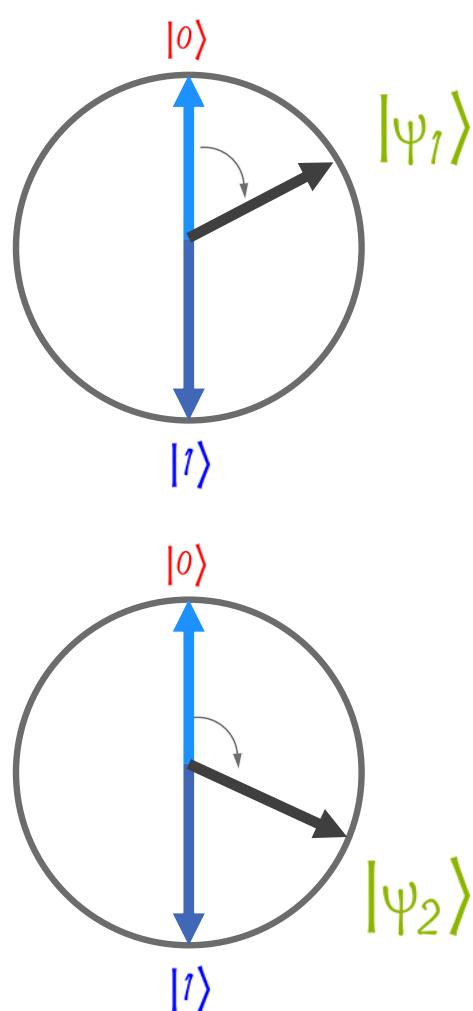
$ \psi_1\rangle$	$P$
$ 0\rangle$	$\alpha_1^2$
$ 1\rangle$	$\beta_1^2$

$ \psi_2\rangle$	$P$
$ 0\rangle$	$\alpha_2^2$
$ 1\rangle$	$\beta_2^2$

$ \psi_1\psi_2\rangle$	$ 0\rangle$	$ 1\rangle$
$ 0\rangle$	$\alpha_1^2\alpha_2^2$	$\alpha_1^2\beta_2^2$
$ 1\rangle$	$\beta_1^2\alpha_2^2$	$\beta_1^2\beta_2^2$

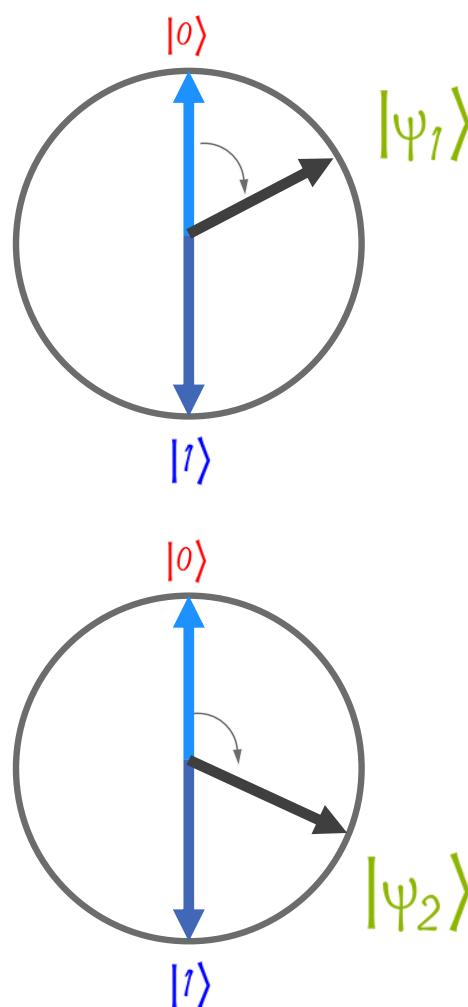
$$|\psi_1\psi_2\rangle = \alpha_1\alpha_2|00\rangle + \alpha_1\beta_2|01\rangle + \beta_1\alpha_2|10\rangle + \beta_1\beta_2|11\rangle$$

# Multi Qubits System



$$|\Psi_1\Psi_2\rangle = \alpha_1\alpha_2|00\rangle + \alpha_1\beta_2|01\rangle + \beta_1\alpha_2|10\rangle + \beta_1\beta_2|11\rangle$$

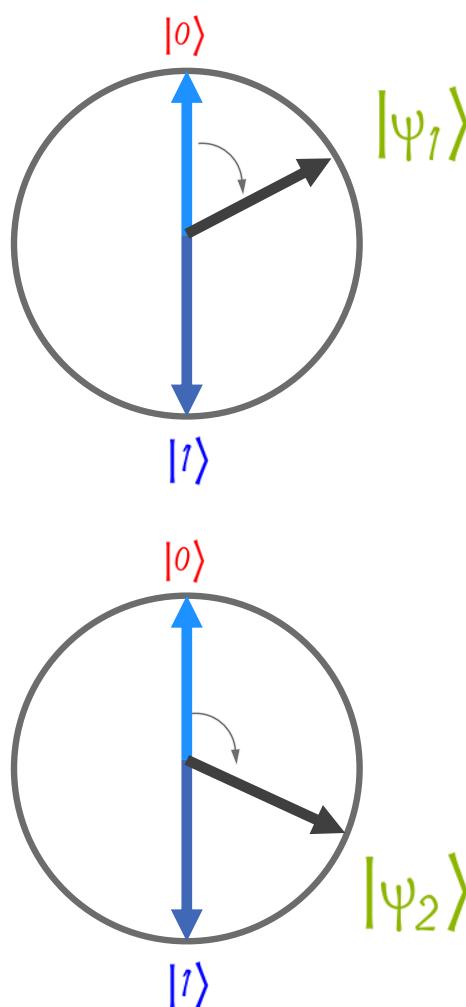
# Multi Qubits System



$$|\Psi_1\Psi_2\rangle = \alpha_1\alpha_2|00\rangle + \alpha_1\beta_2|01\rangle + \beta_1\alpha_2|10\rangle + \beta_1\beta_2|11\rangle$$

$$\begin{aligned} |\Psi_1\Psi_2\Psi_3\rangle &= \alpha_1\alpha_2\alpha_3|000\rangle + \alpha_1\alpha_2\beta_3|001\rangle + \dots \\ &\quad + \beta_1\beta_2\alpha_3|110\rangle + \beta_1\beta_2\beta_3|111\rangle \end{aligned}$$

# Multi Qubits System



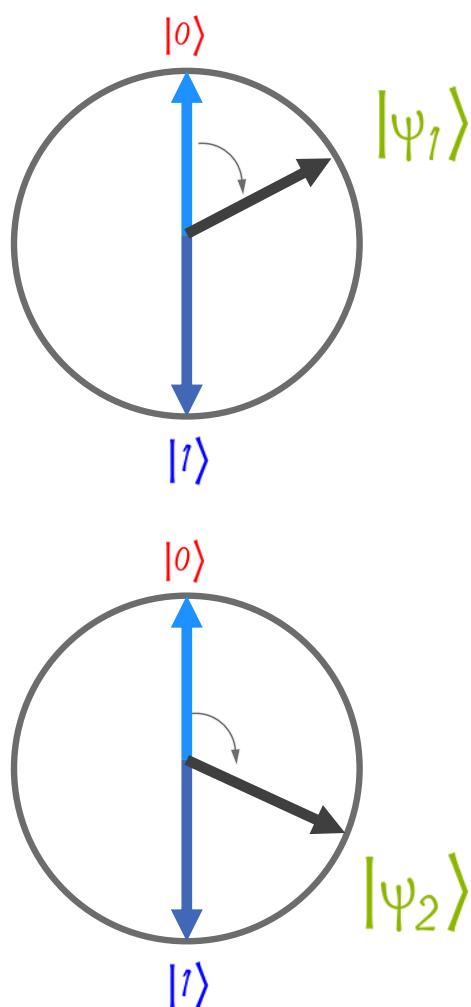
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...

$$\begin{aligned} |\Psi_1\Psi_2\dots\Psi_N\rangle &= \alpha_1\alpha_2\dots\alpha_N|00\dots0\rangle + \alpha_1\alpha_2\dots\alpha_{N-1}\beta_N|00\dots01\rangle + \dots \\ &\quad + \beta_1\beta_2\dots\beta_{N-1}\alpha_N|11\dots10\rangle + \beta_1\beta_2\dots\beta_N|11\dots11\rangle \end{aligned}$$

# Multi Qubits System



$$|\psi_1\psi_2\rangle = \alpha_1\alpha_2|00\rangle + \alpha_1\beta_2|01\rangle + \beta_1\alpha_2|10\rangle + \beta_1\beta_2|11\rangle$$

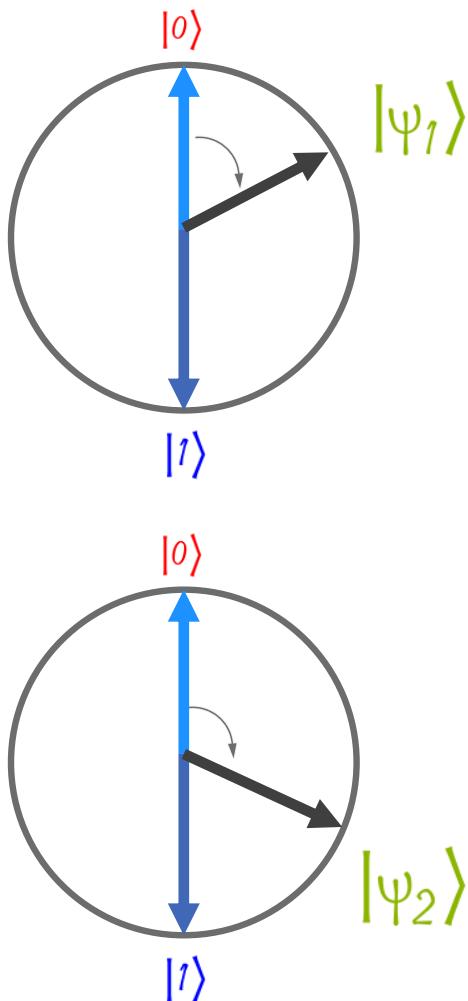
$$\begin{aligned} |\psi_1\psi_2\psi_3\rangle &= \alpha_1\alpha_2\alpha_3|000\rangle + \alpha_1\alpha_2\beta_3|001\rangle + \dots \\ &\quad + \beta_1\beta_2\alpha_3|110\rangle + \beta_1\beta_2\beta_3|111\rangle \end{aligned}$$

...

$$\begin{aligned} |\psi_1\psi_2\dots\psi_N\rangle &= \alpha_1\alpha_2\dots\alpha_N|00\dots0\rangle + \alpha_1\alpha_2\dots\alpha_{N-1}\beta_N|00\dots01\rangle + \dots \\ &\quad + \beta_1\beta_2\dots\beta_{N-1}\alpha_N|11\dots10\rangle + \beta_1\beta_2\dots\beta_N|11\dots11\rangle \end{aligned}$$

In general, a system with  $N$  qubits is completely described by a vector with  $2^N$  elements

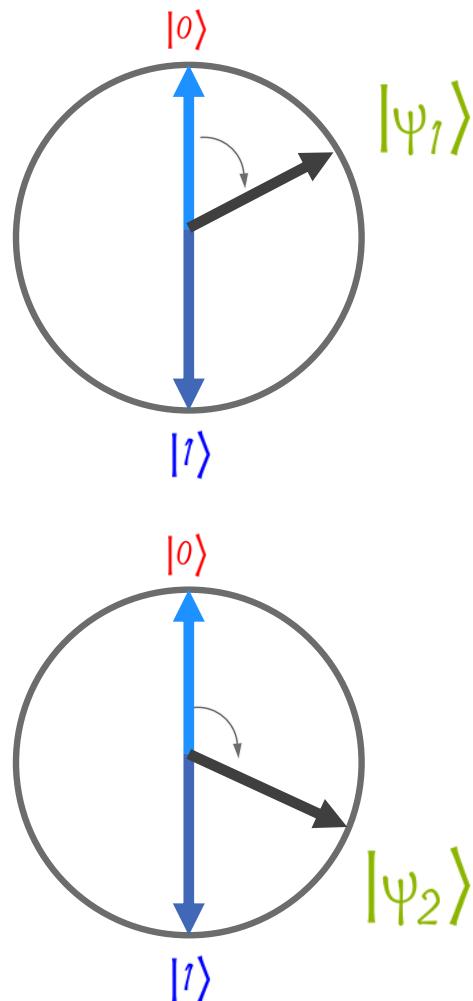
# Multi Qubits System



Another way to describe the representation of a system of qubits using the wave functions of the qubits of the system is through the concept of Kronecker product.  
In formulas we have:

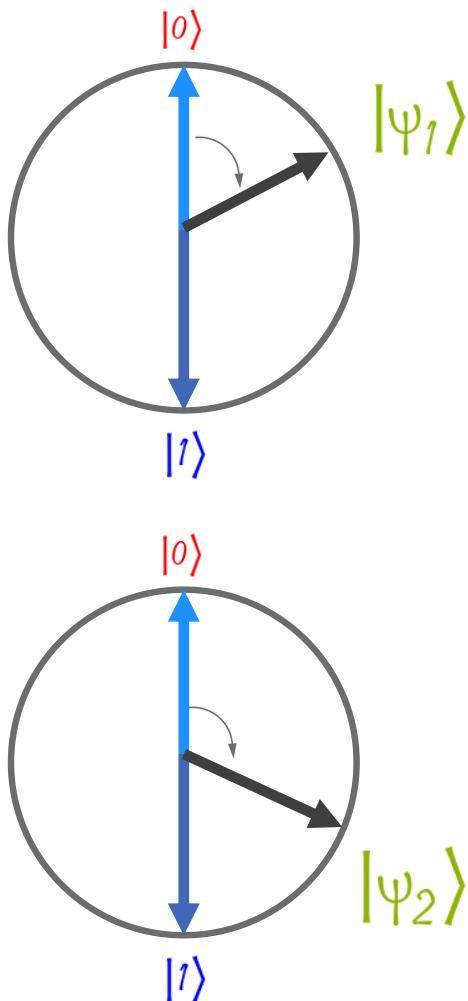
$$|\psi_1\psi_2 \dots \psi_N\rangle = |\psi_1\rangle \otimes |\psi_2\rangle \otimes |\psi_3\rangle \otimes \dots \otimes |\psi_N\rangle$$

# Multi Qubits System



We have seen, therefore, how it is possible to describe the wave function of a system of qubits starting from the wave functions of each single qubit

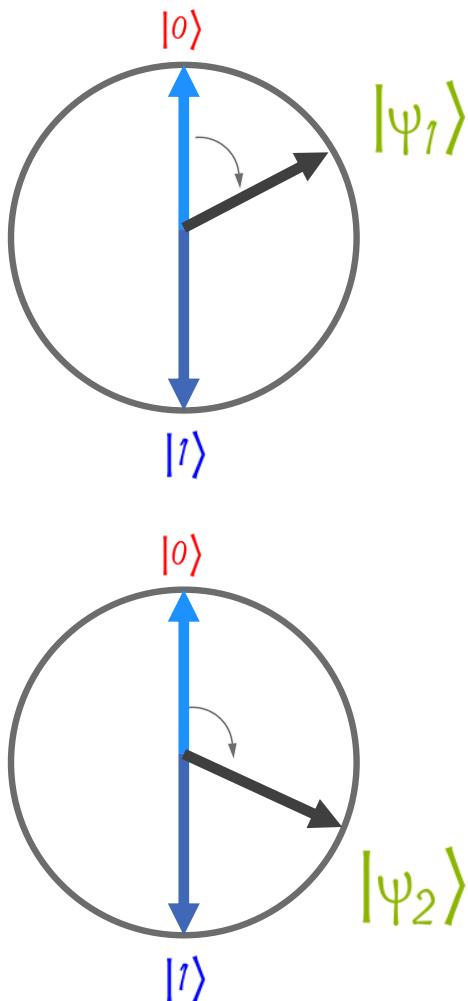
# Multi Qubits System



We have seen, therefore, how it is possible to describe the wave function of a system of qubits starting from the wave functions of each single qubit

$$|\psi_1\psi_2 \dots \psi_N\rangle = \alpha_1\alpha_2 \dots \alpha_N|00 \dots 0\rangle + \alpha_1\alpha_2 \dots \alpha_{N-1}\beta_N|00 \dots 01\rangle + \dots \\ + \beta_1\beta_2 \dots \beta_{N-1}\alpha_N|11 \dots 10\rangle + \beta_1\beta_2 \dots \beta_N|11 \dots 1\rangle$$

# Multi Qubits System

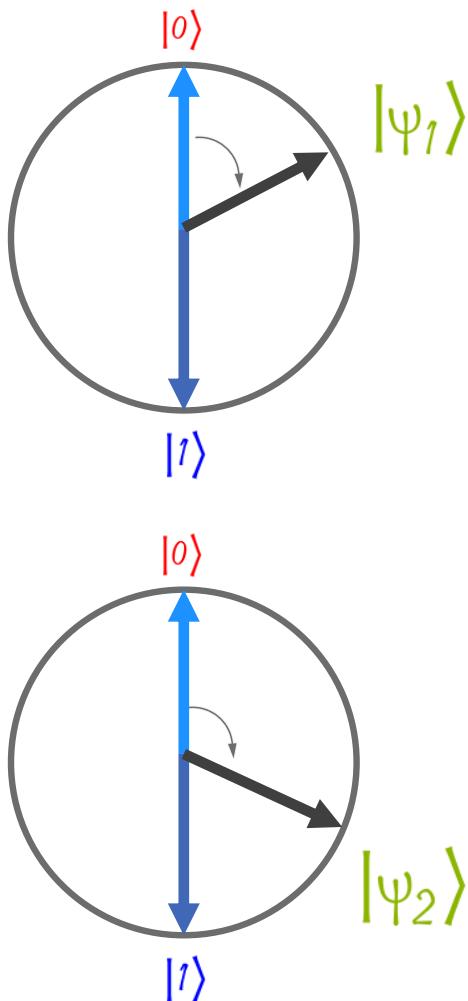


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$$|\psi_1\psi_2 \dots \psi_N\rangle = \alpha_1\alpha_2 \dots \alpha_N |00 \dots 0\rangle + \alpha_1\alpha_2 \dots \alpha_{N-1}\beta_N |00 \dots 01\rangle + \dots + \beta_1\beta_2 \dots \beta_{N-1}\alpha_N |11 \dots 10\rangle + \beta_1\beta_2 \dots \beta_N |11 \dots 1\rangle$$



# Multi Qubits System



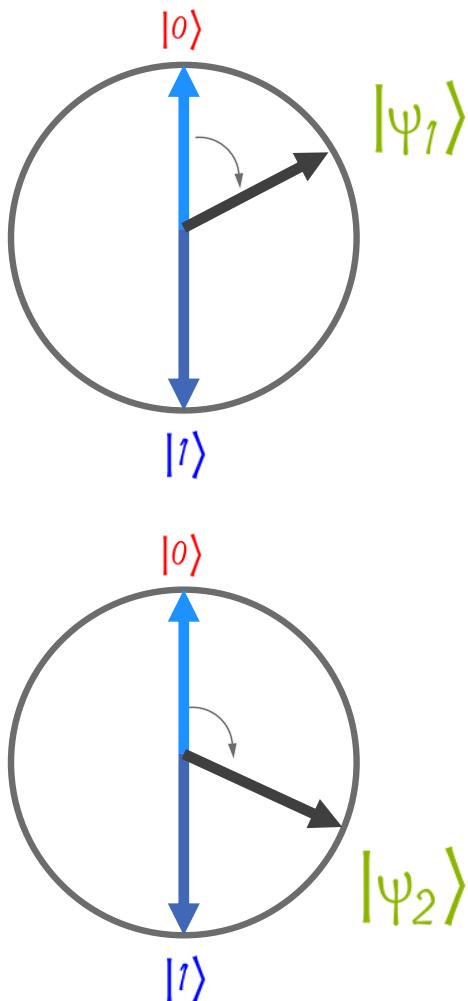
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$$|\psi_1\psi_2 \dots \psi_N\rangle = \alpha_1\alpha_2 \dots \alpha_N |00 \dots 0\rangle + \alpha_1\alpha_2 \dots \alpha_{N-1}\beta_N |00 \dots 01\rangle + \dots + \beta_1\beta_2 \dots \beta_{N-1}\alpha_N |11 \dots 10\rangle + \beta_1\beta_2 \dots \beta_N |11 \dots 1\rangle$$



$$|\psi_1\psi_2 \dots \psi_N\rangle = \gamma_1 |00 \dots 0\rangle + \gamma_2 |00 \dots 01\rangle + \dots + \gamma_{N-1} |11 \dots 10\rangle + \gamma_N |11 \dots 1\rangle$$

# Multi Qubits System



We have seen, therefore, how it is possible to describe the wave function of a system of qubits starting from the wave functions of each single qubit

However, the reverse statement is not so true. We will see in the course of the lesson an example of a system of qubits that cannot be decomposed into the tensor product of several qubits

$$|\psi_1\psi_2 \dots \psi_N\rangle = \alpha_1\alpha_2 \dots \alpha_N |00 \dots 0\rangle + \alpha_1\alpha_2 \dots \alpha_{N-1}\beta_N |00 \dots 01\rangle + \dots + \beta_1\beta_2 \dots \beta_{N-1}\alpha_N |11 \dots 10\rangle + \beta_1\beta_2 \dots \beta_N |11 \dots 1\rangle$$



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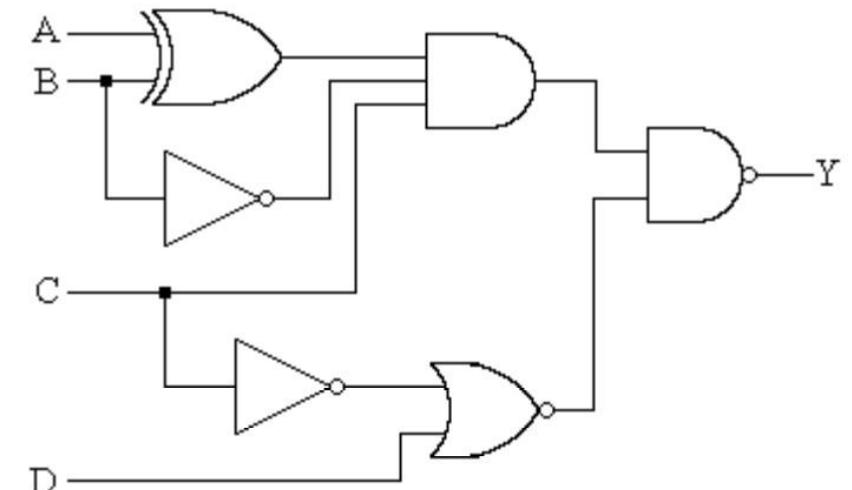
# Classical bit manipulation

---

```
#include <stdio.h>
#include <string.h>
#include <math.h>
int main() {
    int input = 0;
    scanf("%d", &input);
    int y = 0;
    int x = 2;
    for (; y = 1; x++) {
        int newInput = input - (x * x);
        newInput = sqrt(newInput);
        if ((input - (x * x)) == ((newInput * newInput) && (newInput != (x * x)))) {
            printf(x, " + ", newInput * newInput);
            y = 1;
        }
    }
    return 0;
}
```

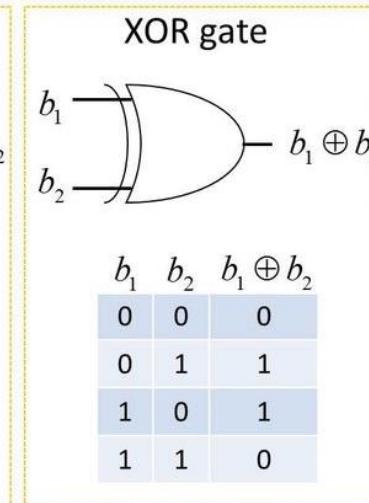
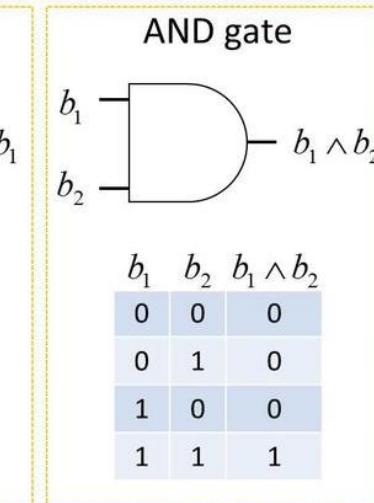
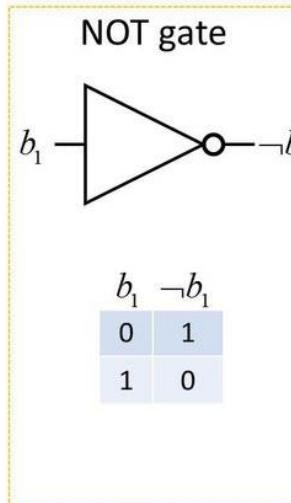
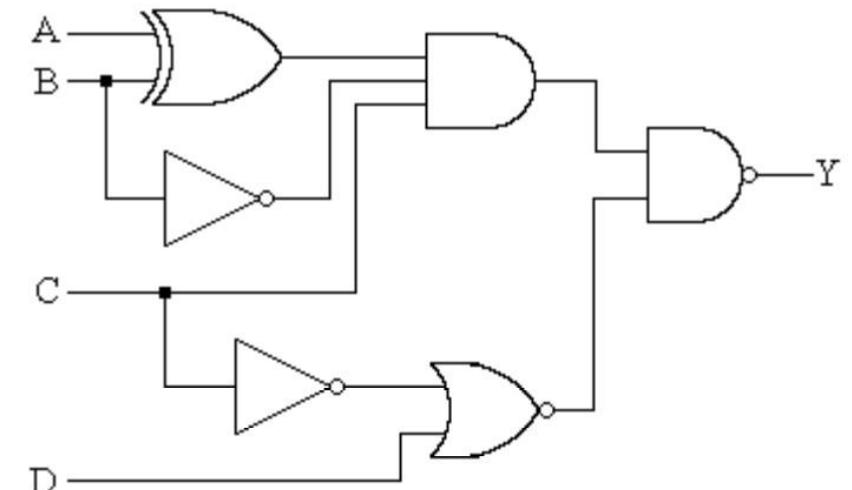
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# Classical bit manipulation

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            printf(x, " + ", newInput * newInput);
            y = 1;
        }
    }
    return 0;
}
```



# Quantum Gates

- Mathematically speaking, the Quantum Gates (operators capable of acting on a system composed of N qubits) can be represented as square matrices of size equal to  $2^N$

$$\begin{pmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{pmatrix} \begin{pmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} & \alpha_{14} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} & \alpha_{24} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} & \alpha_{34} \\ \alpha_{41} & \alpha_{42} & \alpha_{43} & \alpha_{44} \end{pmatrix}$$
$$\left[ \begin{array}{cccccc} \alpha_{11} & \alpha_{12} & \alpha_{13} & \cdots & \alpha_{1M} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} & \cdots & \alpha_{2M} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} & \cdots & \alpha_{3M} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \alpha_{M1} & \alpha_{M2} & \alpha_{M3} & \cdots & \alpha_{MM} \end{array} \right] \quad M=2^N$$

# Quantum Gates

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$$\left[ \begin{array}{cccc|c} \alpha_{11} & \alpha_{12} & \alpha_{13} & \cdots & \alpha_{1M} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} & \cdots & \alpha_{2M} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} & \cdots & \alpha_{3M} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \alpha_{M1} & \alpha_{M2} & \alpha_{M3} & \cdots & \alpha_{MM} \end{array} \right] \quad M=2^N$$

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- In order to act as quantum gates, matrices must meet a couple of important prerequisites

# Quantum Gates

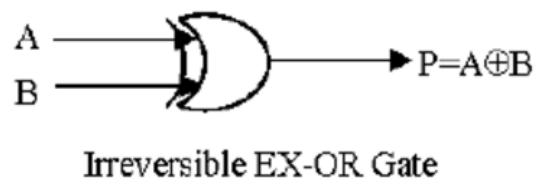
$$\left( \begin{array}{cc} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{array} \right) \left( \begin{array}{cccc} \alpha_{11} & \alpha_{12} & \alpha_{13} & \alpha_{14} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} & \alpha_{24} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} & \alpha_{34} \\ \alpha_{41} & \alpha_{42} & \alpha_{43} & \alpha_{44} \end{array} \right)$$
$$\left[ \begin{array}{ccccc} \alpha_{11} & \alpha_{12} & \alpha_{13} & \cdots & \alpha_{1M} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} & \cdots & \alpha_{2M} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} & \cdots & \alpha_{3M} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \alpha_{M1} & \alpha_{M2} & \alpha_{M3} & \cdots & \alpha_{MM} \end{array} \right] \quad M=2^N$$

- Mathematically speaking, the Quantum Gates (operators capable of acting on a system composed of N qubits) can be represented as square matrices of size equal to  $2^N$
- In order to act as quantum gates, matrices must meet a couple of important prerequisites

1: They must be unitary matrices: one of the reasons for this choice is that quantum gates are defined as operators that take qubits in input and return qubits in output. Consequently, it is important that the application of quantum gates does not modify the length of the input vector

# Quantum Gates

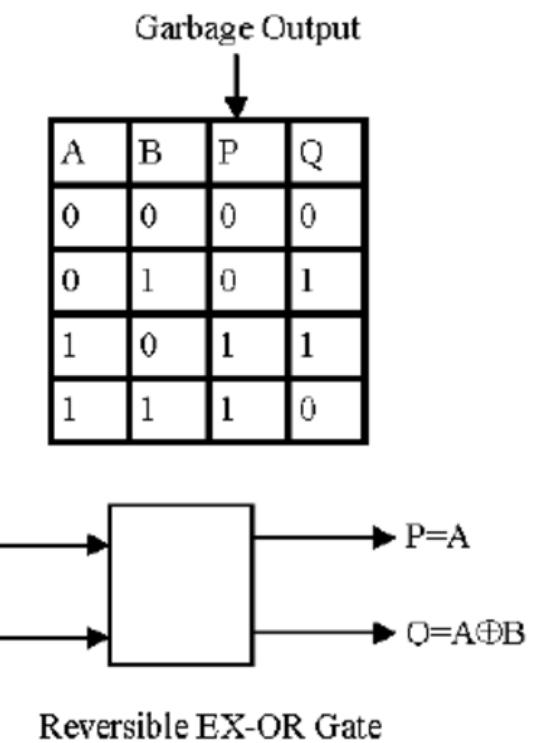
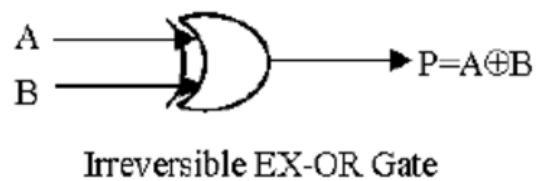
A	B	P
0	0	0
0	1	1
1	0	1
1	1	0



- Mathematically speaking, the Quantum Gates (operators capable of acting on a system composed of N qubits) can be represented as square matrices of size equal to  $2^N$
- In order to act as quantum gates, matrices must meet a couple of important prerequisites
  - 2: The number of input qubits must equal the number of output qubits. This implies that it is not possible to construct quantum gates for non-reversible operations, ie where it is not possible to trace the input states by observing only the output states (such as the XOR gate). However, it is possible to make these operations reversible

# Quantum Gates

A	B	P
0	0	0
0	1	1
1	0	1
1	1	0



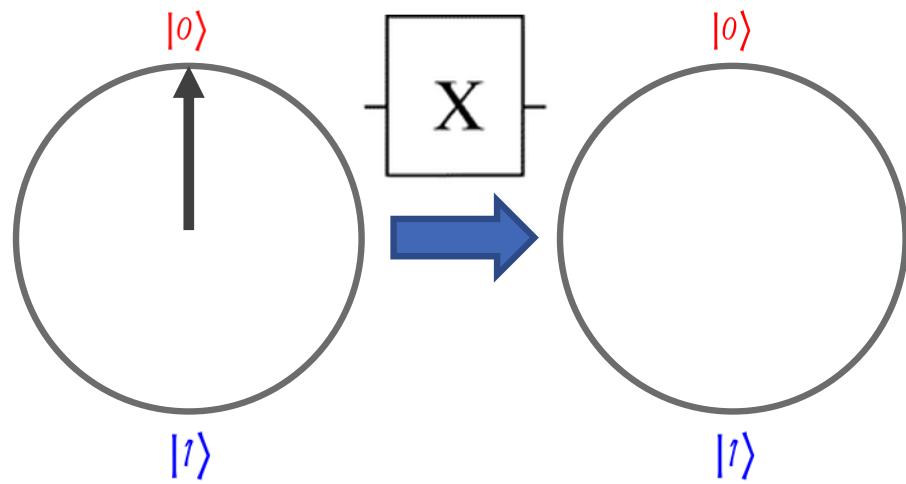
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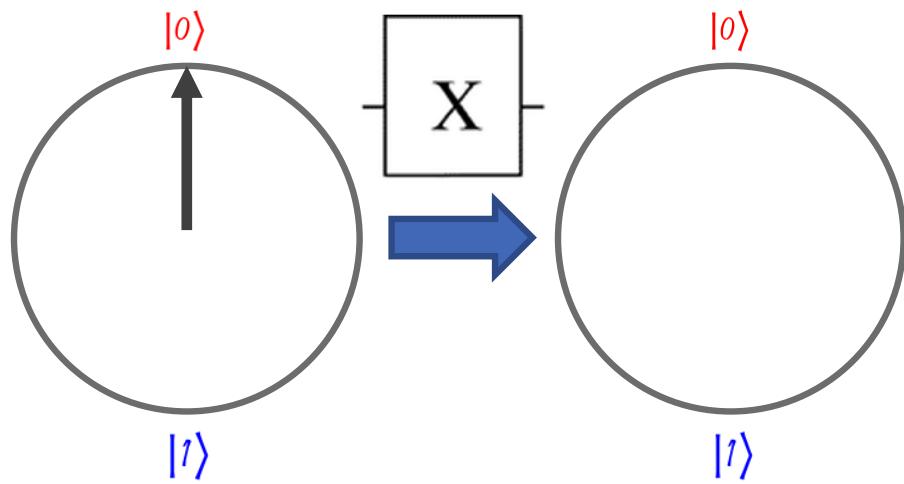
# X (NOT) Gate

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# X (NOT) Gate

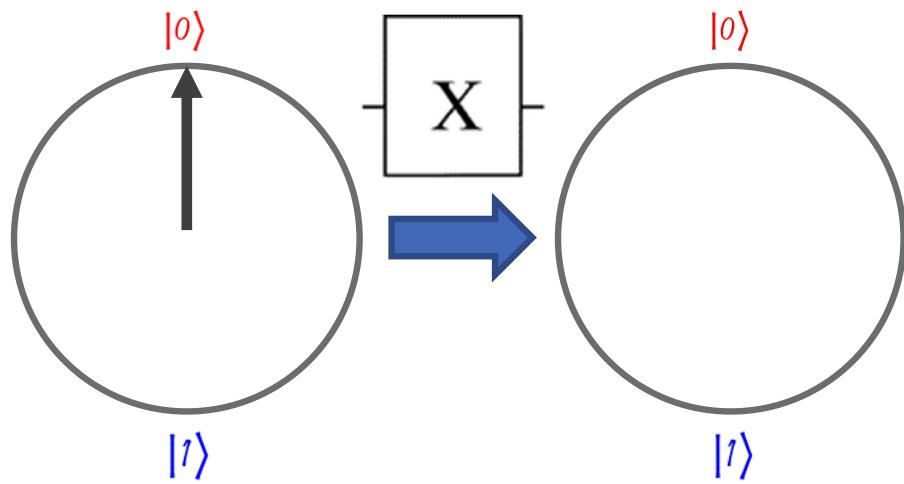
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$$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

# X (NOT) Gate

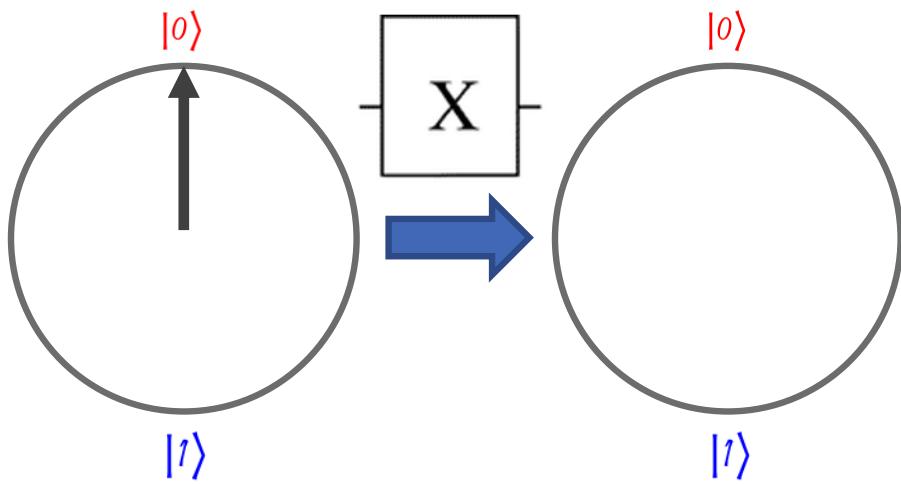
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$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

# X (NOT) Gate

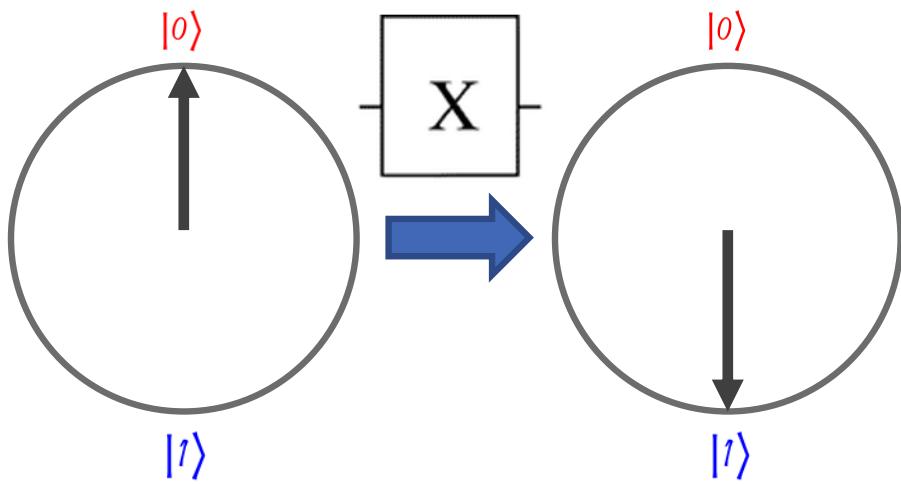
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$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

# X (NOT) Gate

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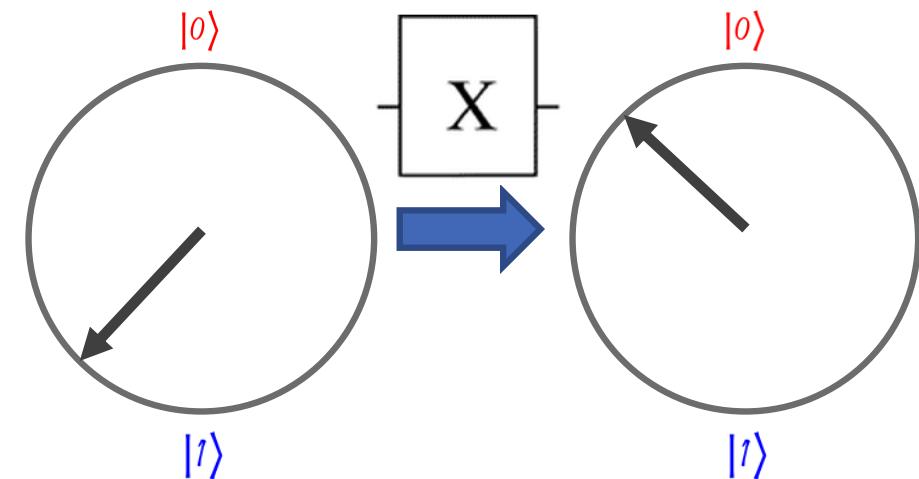
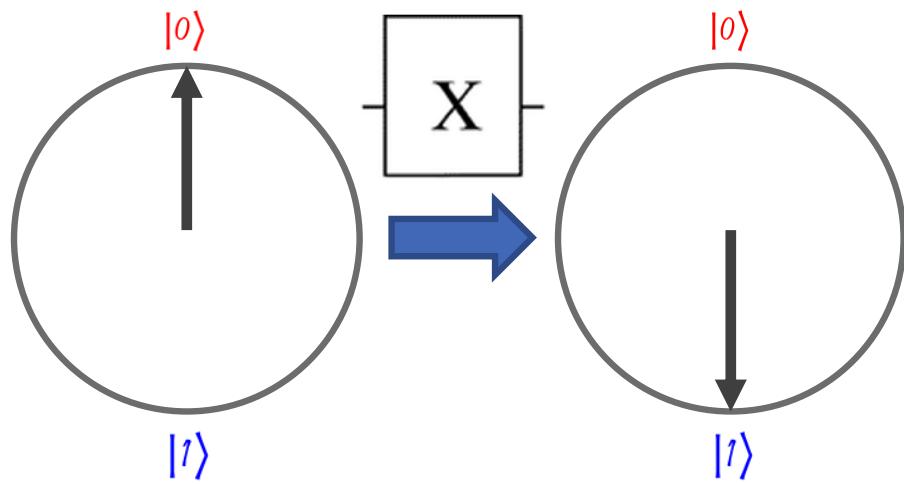


$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$|0\rangle \rightarrow |1\rangle$$

# X (NOT) Gate

---



$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$|0\rangle \rightarrow |1\rangle$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} \beta \\ \alpha \end{pmatrix}$$

$$\alpha \cdot |0\rangle + \beta \cdot |1\rangle \rightarrow \beta \cdot |0\rangle + \alpha \cdot |1\rangle$$

# Single Qubit Quantum Gates

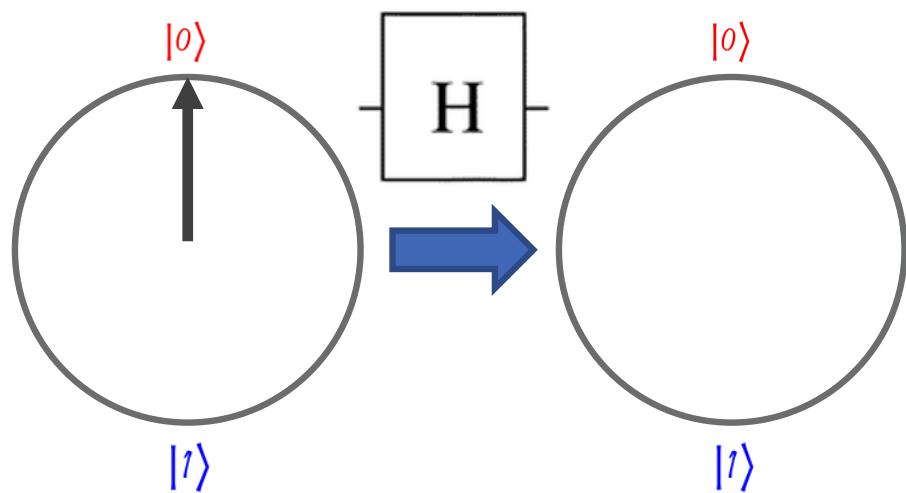
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X Gate  
Bit-flip, Not

$$\begin{array}{c} \text{X Gate} \\ \text{Bit-flip, Not} \end{array} \quad \boxed{\text{X}} \quad \equiv \quad \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \quad \beta |0\rangle + \alpha |1\rangle$$

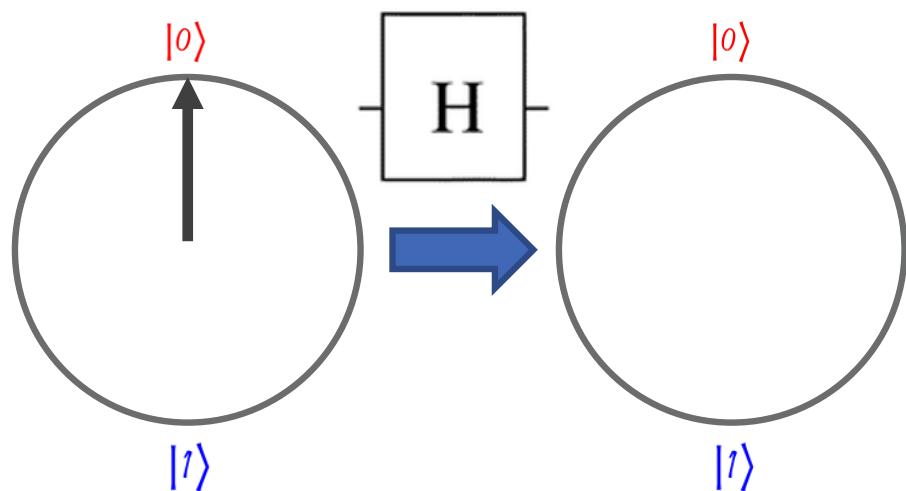
# Hadamard Gate

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# Hadamard Gate

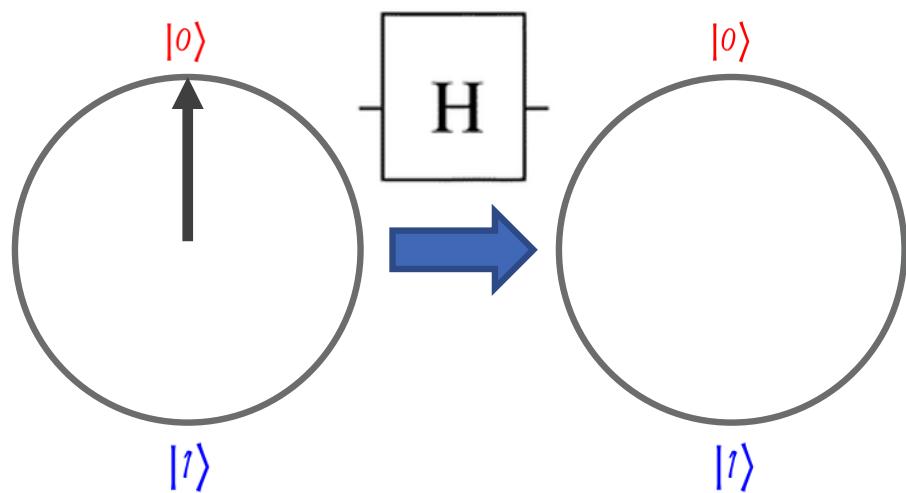
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$$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

# Hadamard Gate

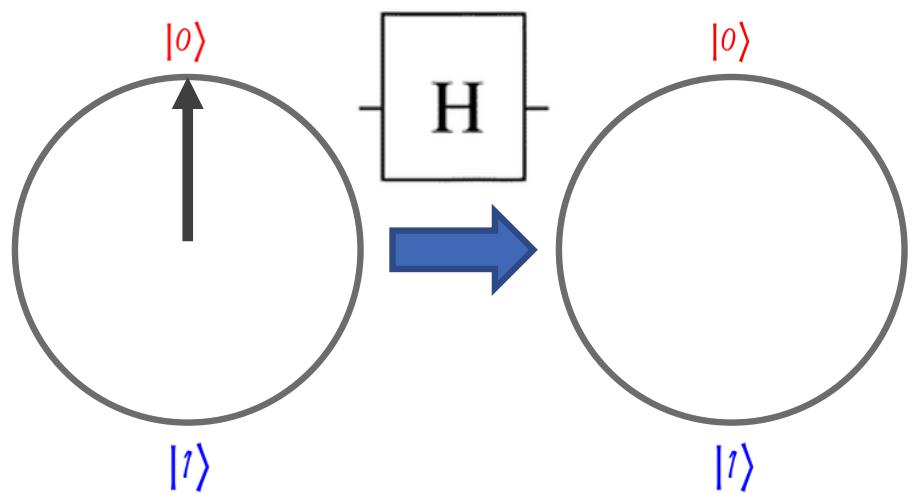
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$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

# Hadamard Gate

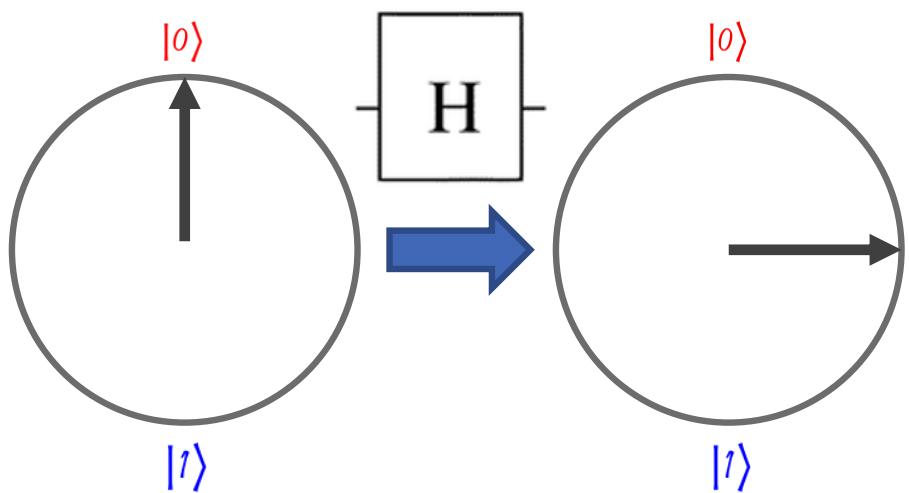
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$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

# Hadamard Gate

---

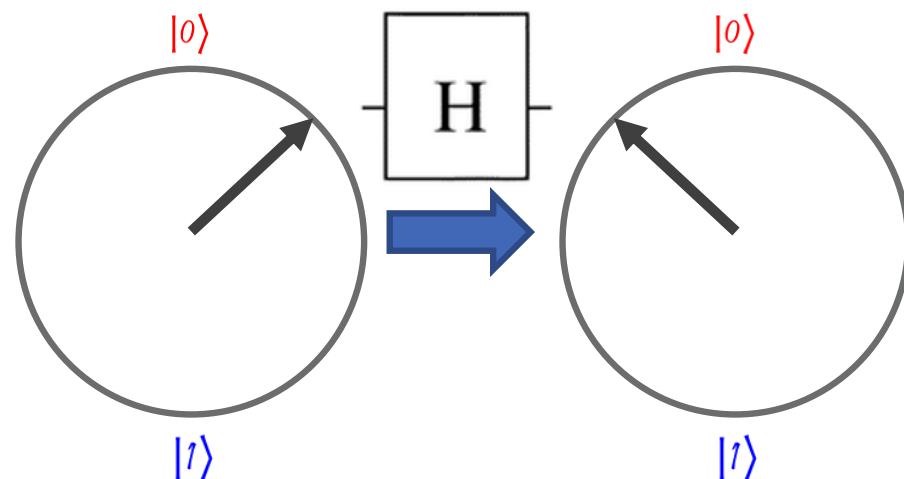
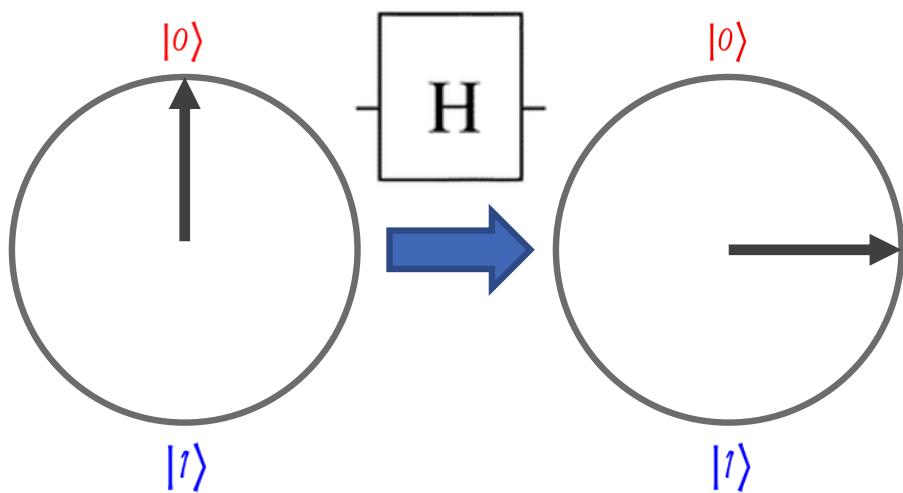


$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$|0\rangle \rightarrow \frac{1}{\sqrt{2}} \cdot |0\rangle + \frac{1}{\sqrt{2}} \cdot |1\rangle$$

# Hadamard Gate

---



$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \cdot \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \alpha + \beta \\ \alpha - \beta \end{pmatrix}$$

$$|0\rangle \rightarrow \frac{1}{\sqrt{2}} \cdot |0\rangle + \frac{1}{\sqrt{2}} \cdot |1\rangle$$

$$\alpha|0\rangle + \beta|1\rangle \rightarrow \frac{\alpha + \beta}{\sqrt{2}} \cdot |0\rangle + \frac{\alpha - \beta}{\sqrt{2}} \cdot |1\rangle$$

# Single Qubit Quantum Gates

X Gate  
Bit-flip, Not

$$\begin{array}{c|c} X & \equiv \\ \hline \end{array} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \beta |0\rangle + \alpha |1\rangle$$

Z Gate  
Phase-flip

$$\begin{array}{c|c} Z & \equiv \\ \hline \end{array} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \alpha |0\rangle - \beta |1\rangle$$

H Gate  
Hadamard

$$\begin{array}{c|c} H & \equiv \frac{1}{\sqrt{2}} \\ \hline \end{array} = \frac{\alpha+\beta|0\rangle + \alpha-\beta|1\rangle}{\sqrt{2}}$$

T Gate

$$\begin{array}{c|c} T & \equiv \\ \hline \end{array} = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \alpha |0\rangle + e^{i\pi/4} \beta |1\rangle$$

# Two qubits Gates: Swap gate

---

$$|\psi_1\rangle = \alpha_1|0\rangle + \beta_1|1\rangle, \quad |\psi_2\rangle = \alpha_2|0\rangle + \beta_2|1\rangle$$

# Two qubits Gates: Swap gate

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The two qubits involved in the transformation are identified with the names of control qubit (the black dot) and target qubit (the white circle with the cross)



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invert the amplitudes of the target qubit if and only  
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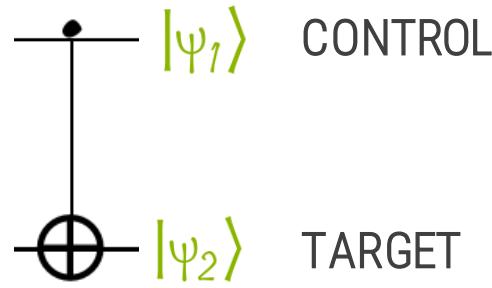
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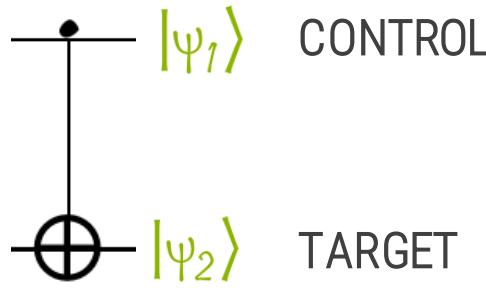
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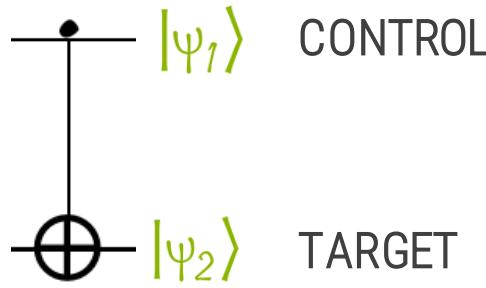
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# Two qubits Gates

---

Controlled Not  
Controlled X  
CNot

$$\begin{array}{c} \text{Controlled Not} \\ \text{Controlled X} \\ \text{CNot} \end{array} \quad \begin{array}{c} \text{Quantum Circuit Diagram} \\ \text{Matrix Form} \\ \text{ket Expansion} \end{array}$$

Quantum Circuit Diagram:

Matrix Form:

$$\equiv \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

ket Expansion:

$$= a|00\rangle + b|01\rangle + d|10\rangle + c|11\rangle$$

Swap

$$\begin{array}{c} \text{Swap} \end{array} \quad \begin{array}{c} \text{Quantum Circuit Diagram} \\ \text{Matrix Form} \\ \text{ket Expansion} \end{array}$$

Quantum Circuit Diagram:

Matrix Form:

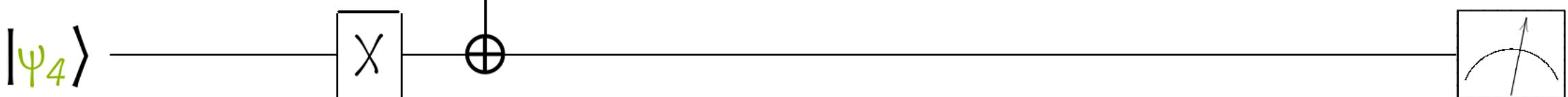
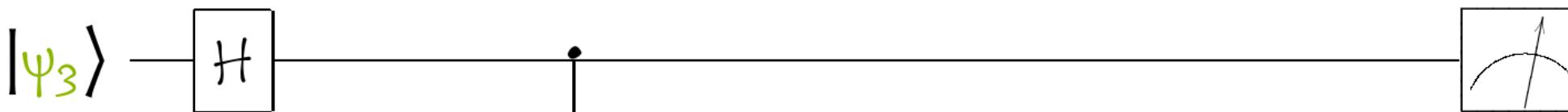
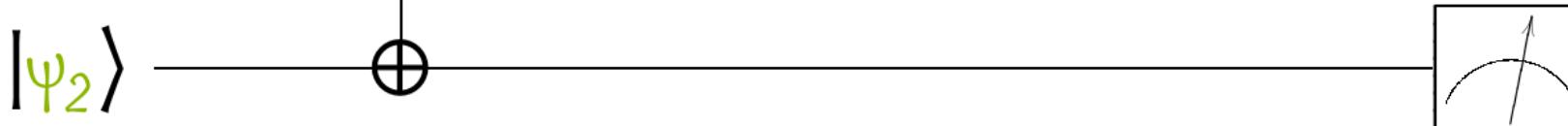
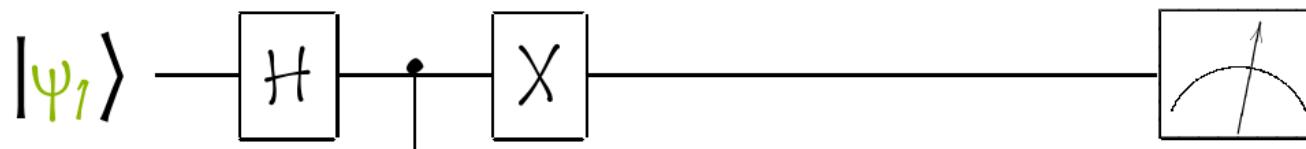
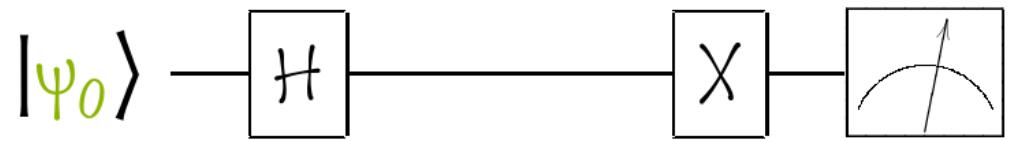
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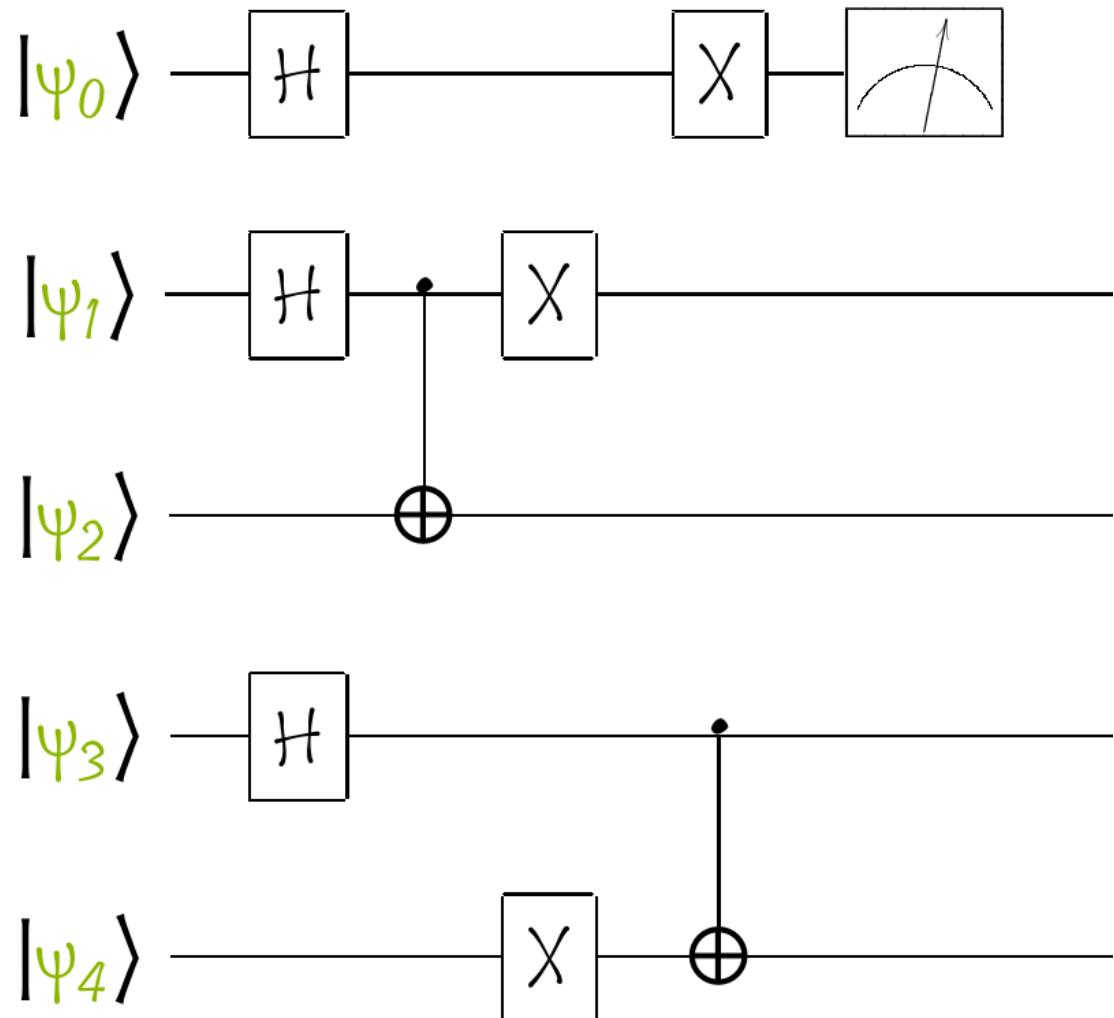
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# Quantum Circuits

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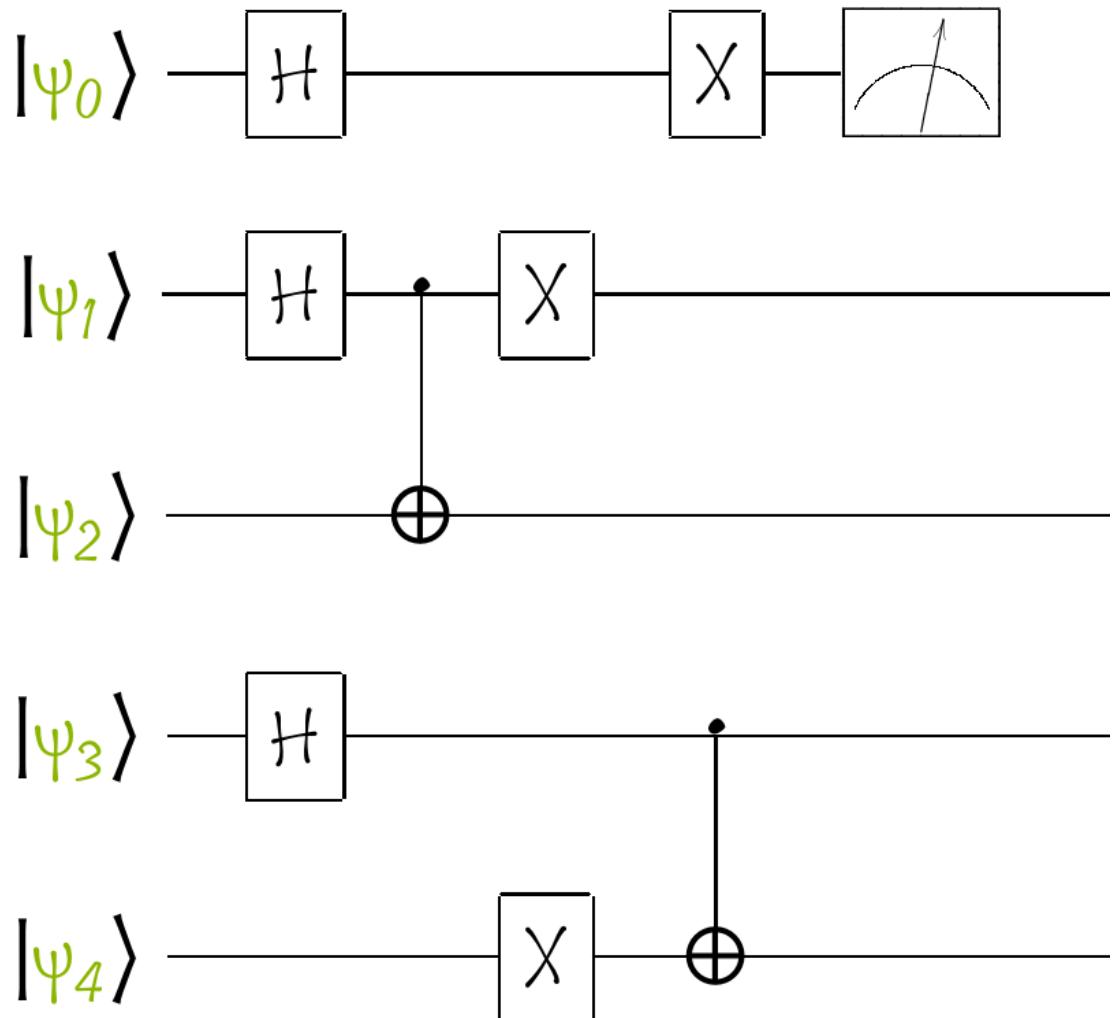


# Quantum Circuits



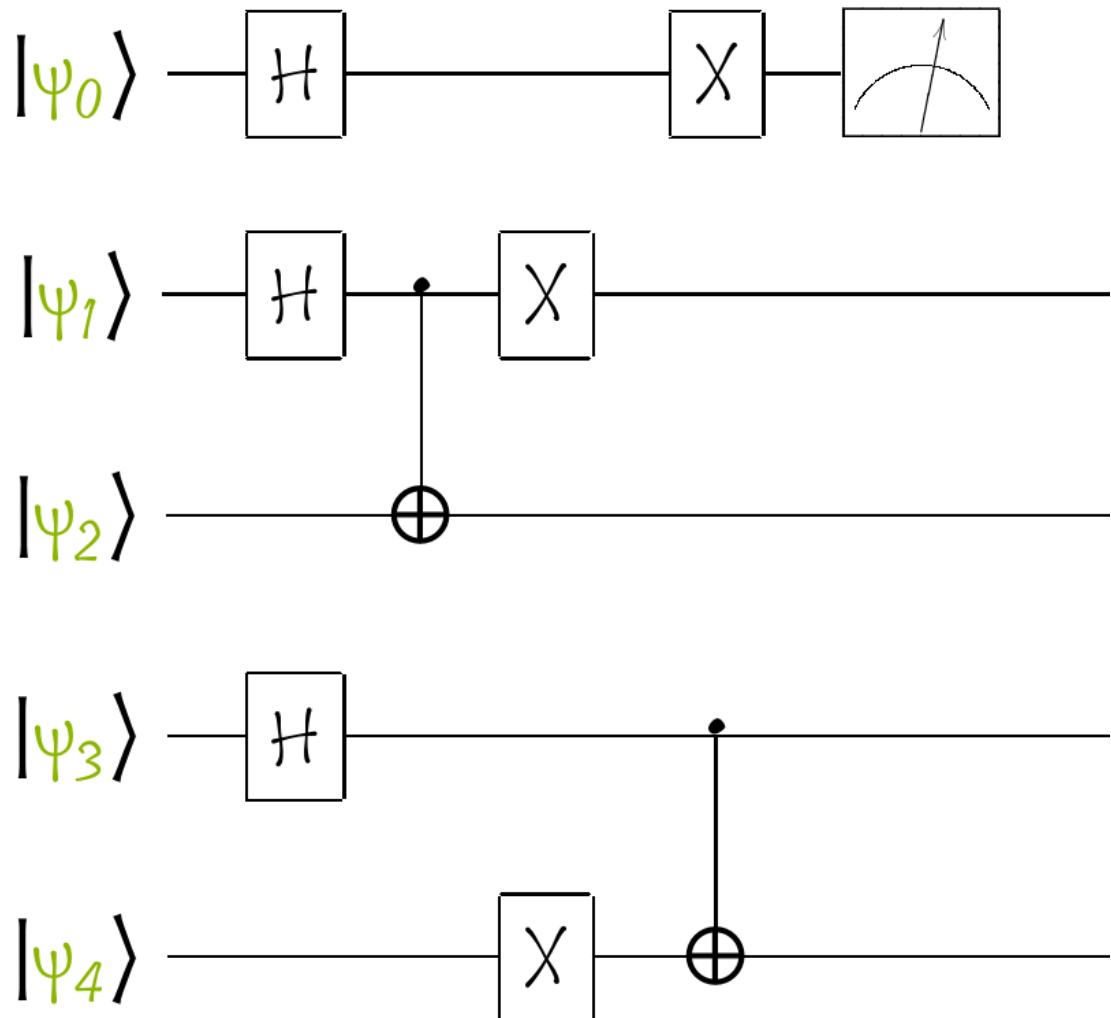
- A quantum circuit is a set of quantum gates acting on a system of qubits

# Quantum Circuits



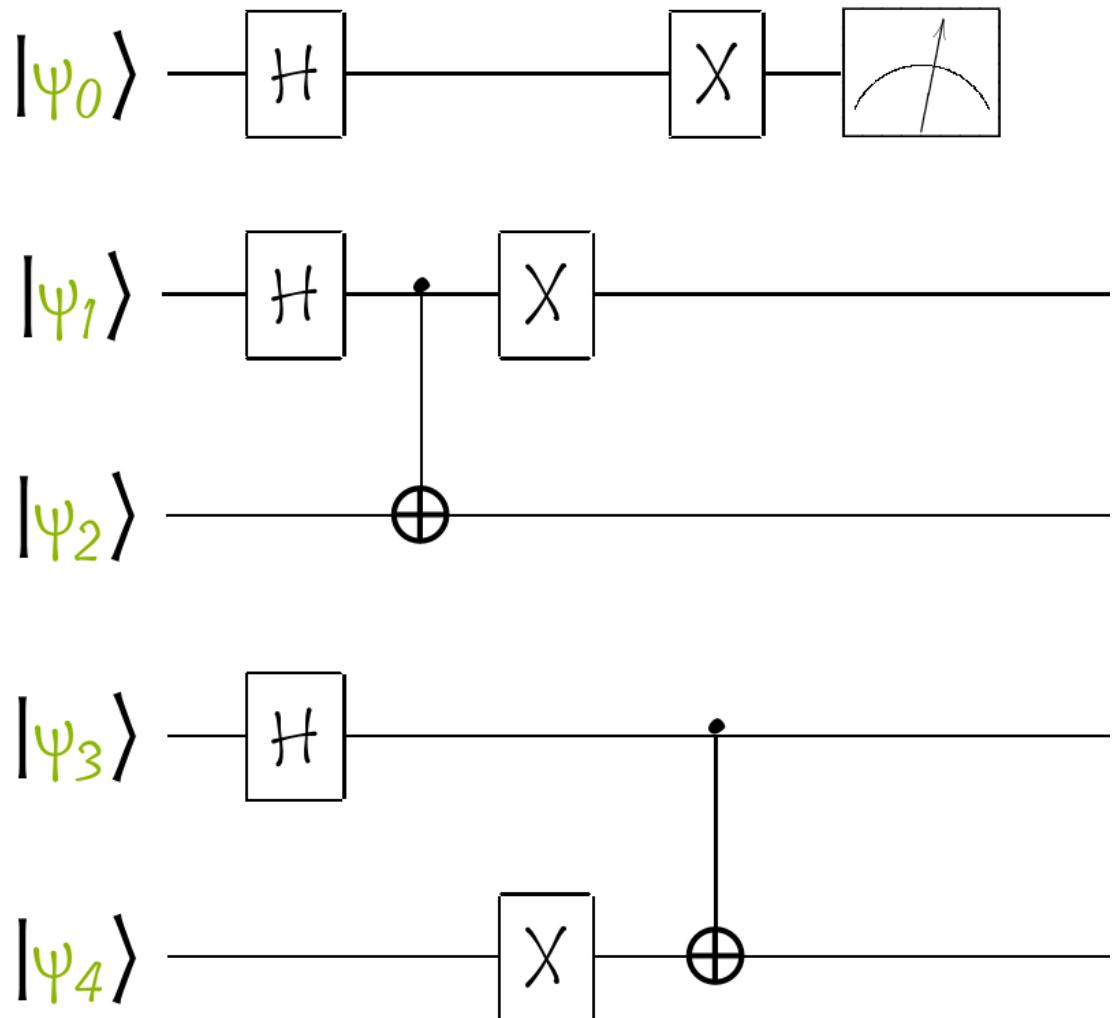
- A quantum circuit is a set of quantum gates acting on a system of qubits
- Each quantum circuit is characterized by a depth, i.e. the longest path from the input (or from a preparation) to the output (or a measurement gate), moving forward in time along qubit wires.

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- Gates on the same level act simultaneously on the entire system of qubits. The order is from left to right

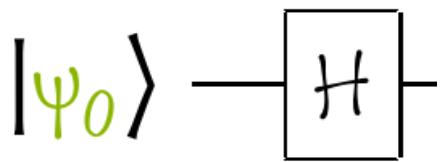
# Quantum Circuits



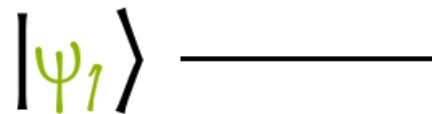
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- By convention, the system always starts from a state of all at rest (i.e. all qubits in the classic state 0)

# Combine Quantum Gates

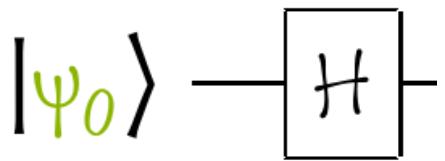
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- Let's suppose we have a circuit like the one in the figure



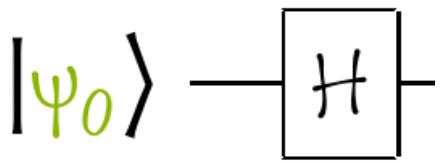
# Combine Quantum Gates



- Let's suppose we have a circuit like the one in the figure
- Furthermore, suppose we are in the middle of a larger quantum circuit, which however only involves two qubits. In this case, the wave function of the system can be represented by the product of the wave functions of the individual qubits

$$|\Psi_1\Psi_2\rangle = \alpha_1\alpha_2|00\rangle + \alpha_1\beta_2|01\rangle + \beta_1\alpha_2|10\rangle + \beta_1\beta_2|11\rangle$$

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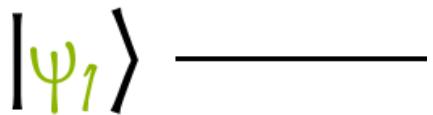
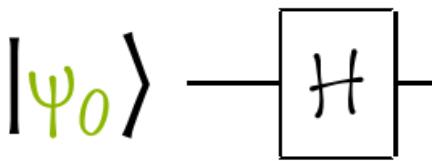


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- Or, more generally, it can be represented by non-decomposable coefficients (we will see an example shortly)

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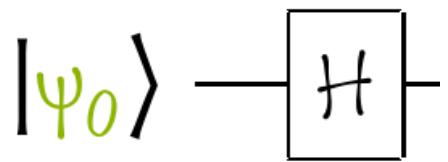


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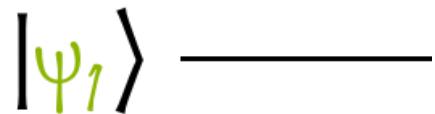
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- Furthermore, suppose we are in the middle of a larger quantum circuit, which however only involves two qubits. In this case, the wave function of the system can be represented by the product of the wave functions of the individual qubits
- Or, more generally, it can be represented by non-decomposable coefficients (we will see an example shortly)
- The question therefore is: how should I act on the total wave function of the system to reflect the application of the Hadamard gate only on the first qubit?

# Combine Quantum Gates

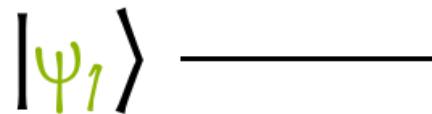
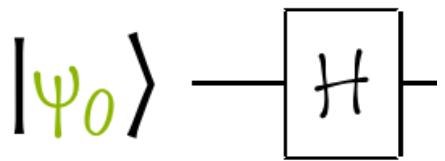


- The answer is: Kronecker Product



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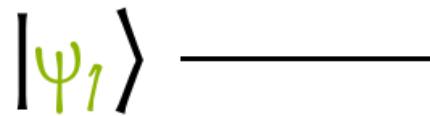
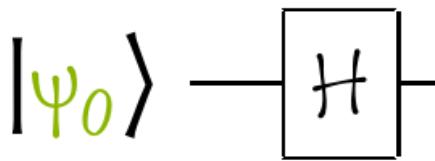
# Combine Quantum Gates



- The answer is: Kronecker Product
- To build the matrix that acts on a system composed of several qubits, it is sufficient to identify all the gates operating on the system at the same level (identifying the empty spaces with identity matrices)

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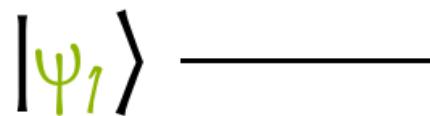
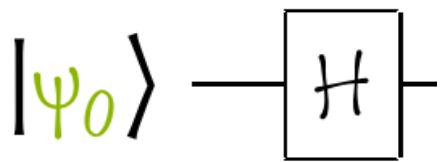
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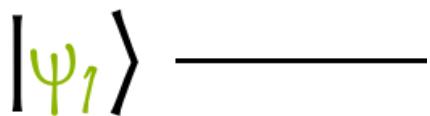
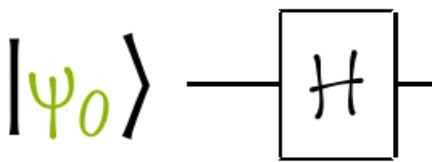


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$$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

# Combine Quantum Gates

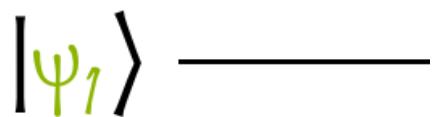
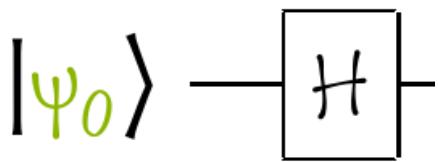


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$$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

# Combine Quantum Gates



$$|\Psi_{12}\rangle = a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle$$

- The answer is: Kronecker Product
- To build the matrix that acts on a system composed of several qubits, it is sufficient to identify all the gates operating on the system at the same level (identifying the empty spaces with identity matrices)
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# Combine Quantum Gates

$|\Psi_0\rangle$  —

$|\Psi_1\rangle$  —  $H$

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$|\Psi_1\rangle$  —  $H$

$|\Psi_0\rangle$  —

$|\Psi_1\rangle$  —  $X$

$|\Psi_0\rangle$  —  $H$

$|\Psi_1\rangle$  —  $X$

# Single Qubit Quantum Gates

X Gate  
Bit-flip, Not

$$\begin{array}{c|c} X & \equiv \\ \hline \end{array} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \beta |0\rangle + \alpha |1\rangle$$

Z Gate  
Phase-flip

$$\begin{array}{c|c} Z & \equiv \\ \hline \end{array} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \alpha |0\rangle - \beta |1\rangle$$

H Gate  
Hadamard

$$\begin{array}{c|c} H & \equiv \frac{1}{\sqrt{2}} \\ \hline \end{array} = \frac{\alpha+\beta|0\rangle + \alpha-\beta|1\rangle}{\sqrt{2}}$$

T Gate

$$\begin{array}{c|c} T & \equiv \\ \hline \end{array} = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \alpha |0\rangle + e^{i\pi/4} \beta |1\rangle$$

# Single Qubit Quantum Gates

$$\begin{array}{c|c} \boxed{X} & \equiv \\ \hline \end{array} \quad \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$
$$\begin{array}{c|c} \boxed{Z} & \equiv \\ \hline \end{array} \quad \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$
$$\begin{array}{c|c} \boxed{H} & \equiv \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \\ \hline \end{array}$$
$$\begin{array}{c|c} \boxed{T} & \equiv \\ \hline \end{array} \quad \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}$$

$|\Psi_0\rangle$  —

$|\Psi_1\rangle$  —  $\boxed{\mathcal{H}}$

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# Combine Quantum Gates

$$\begin{array}{c} |\Psi_0\rangle \xrightarrow{\quad} \\ |\Psi_1\rangle \xrightarrow{\boxed{H}} \end{array} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \end{pmatrix} \begin{array}{c} |\Psi_0\rangle \xrightarrow{\quad} \\ |\Psi_1\rangle \xrightarrow{\boxed{X}} \end{array} \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

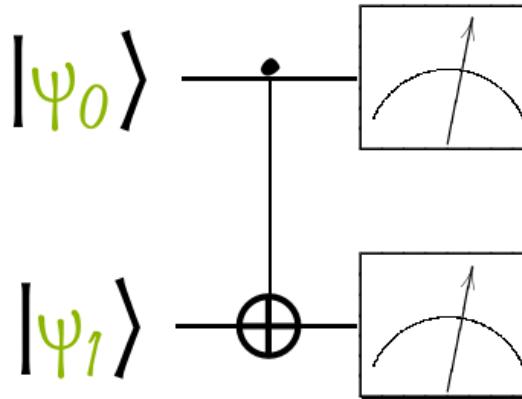
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$$\begin{array}{c} |\Psi_0\rangle \xrightarrow{\boxed{H}} \\ |\Psi_1\rangle \xrightarrow{\boxed{H}} \end{array} \frac{1}{2} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix} \begin{array}{c} |\Psi_0\rangle \xrightarrow{\boxed{H}} \\ |\Psi_1\rangle \xrightarrow{\boxed{X}} \end{array} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & -1 & 0 \end{pmatrix}$$

# Alternative Controlled Gates

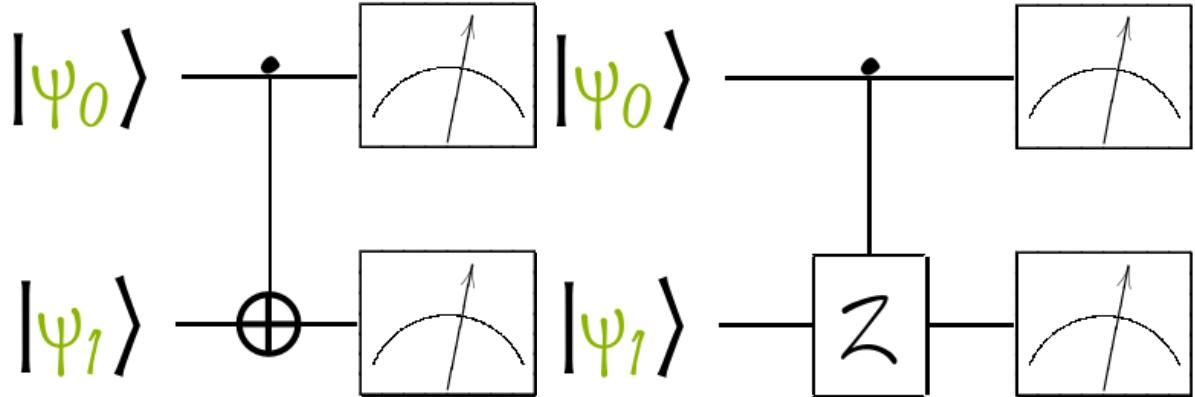
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- In this section, without going into too much detail, I would like to show you an alternative method for building the CNOT gate



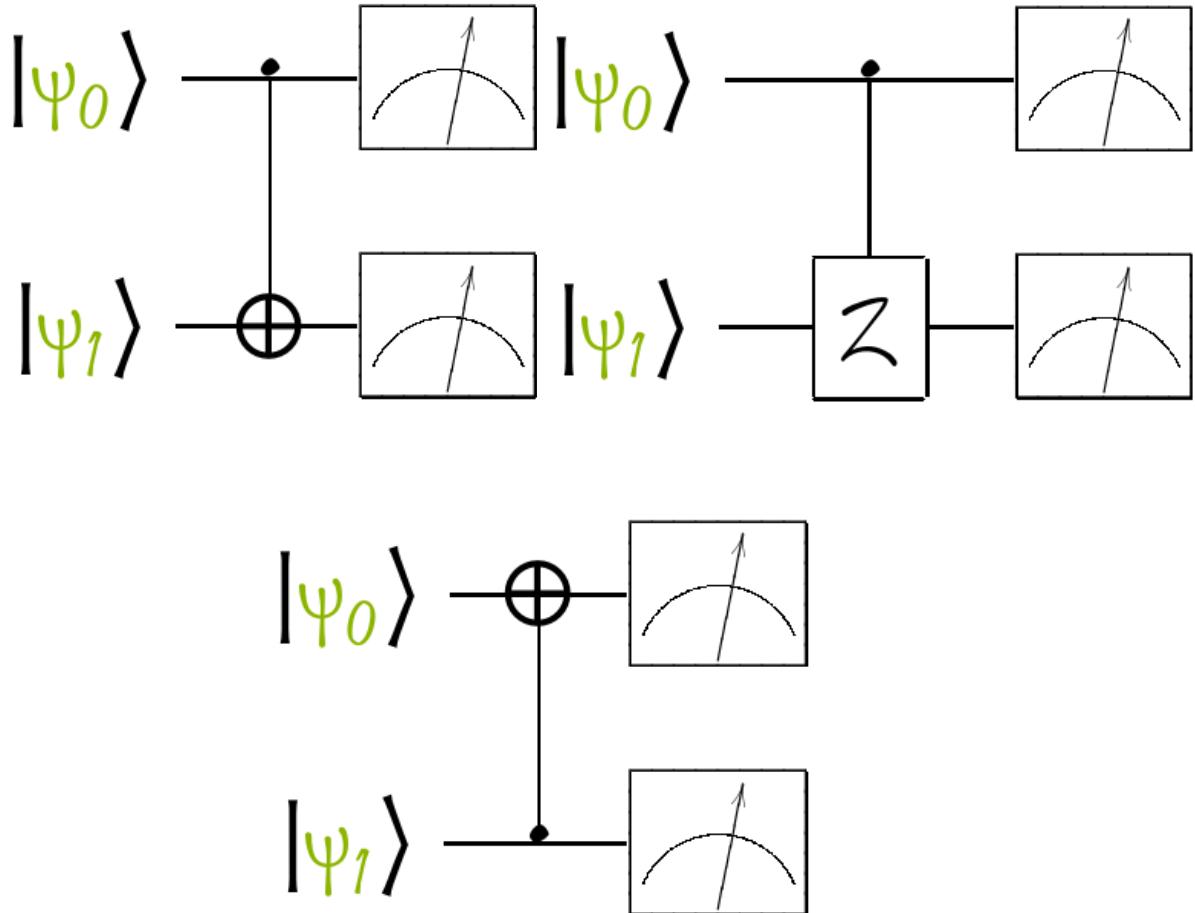
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- In general, this method can be used to build generic controlled gates



# Alternative Controlled Gates

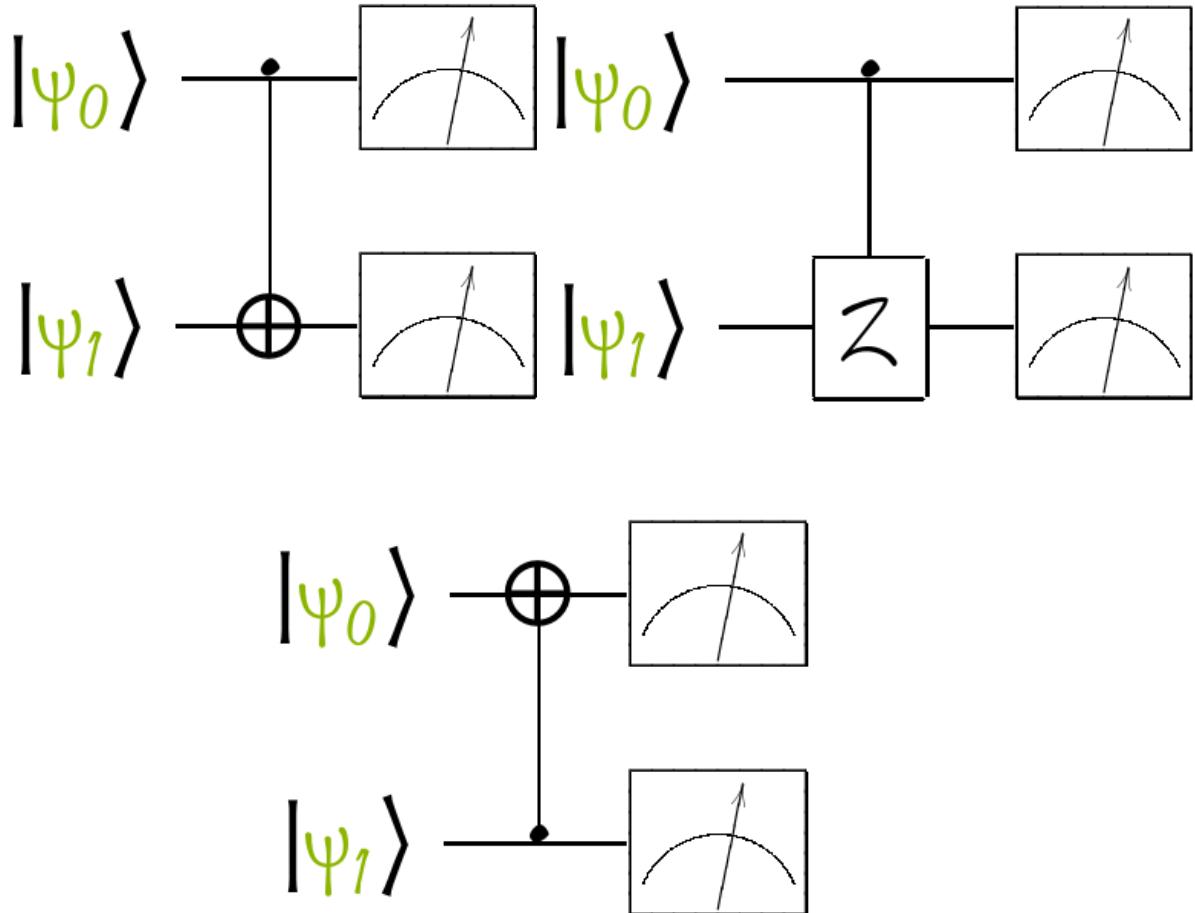
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# Alternative Controlled Gates

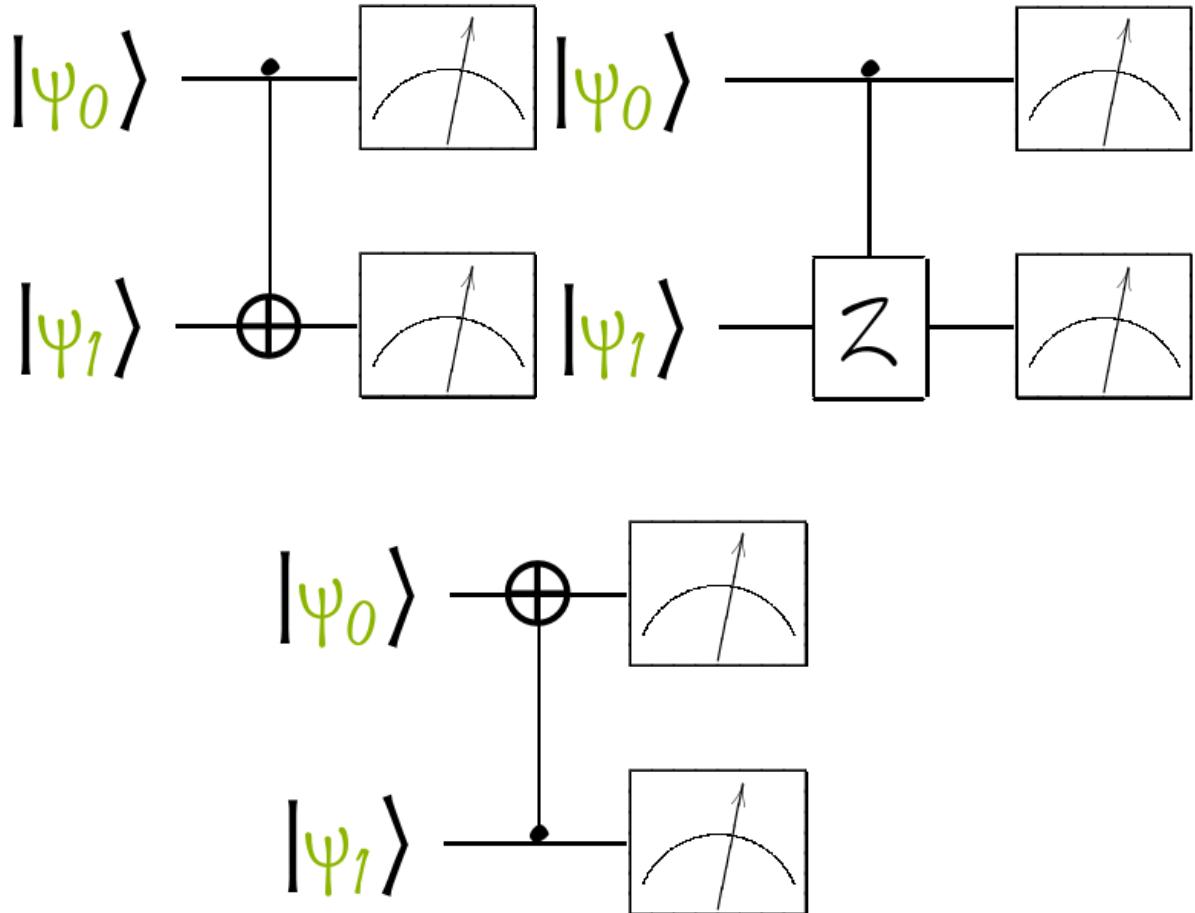
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$$|0\rangle\langle 0| \otimes I + |1\rangle\langle 1| \otimes X$$



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- That can be read as: "if the first qubit is 0, do nothing. If the first qubit is 1, apply X"



# Alternative Controlled Gates

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$$|0\rangle\langle 0| = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} (1, 0) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad |1\rangle\langle 1| = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} (0, 1) = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

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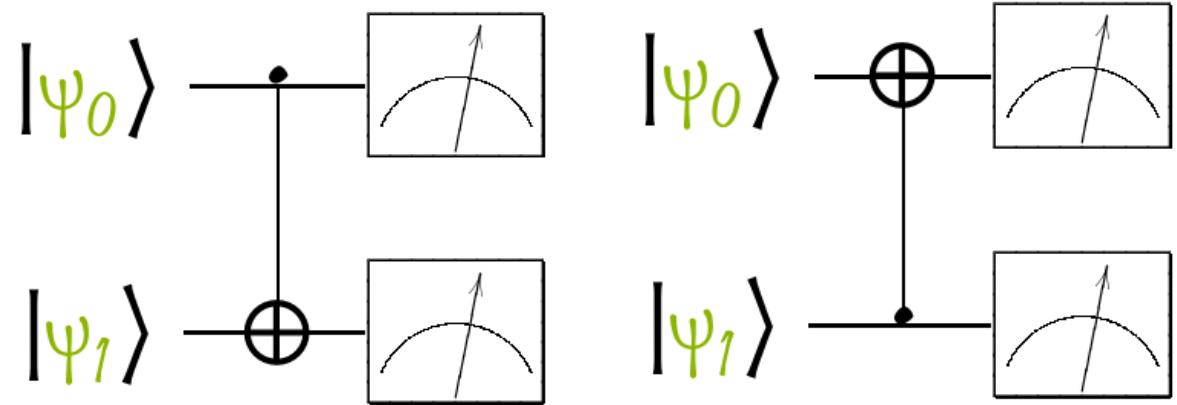
$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

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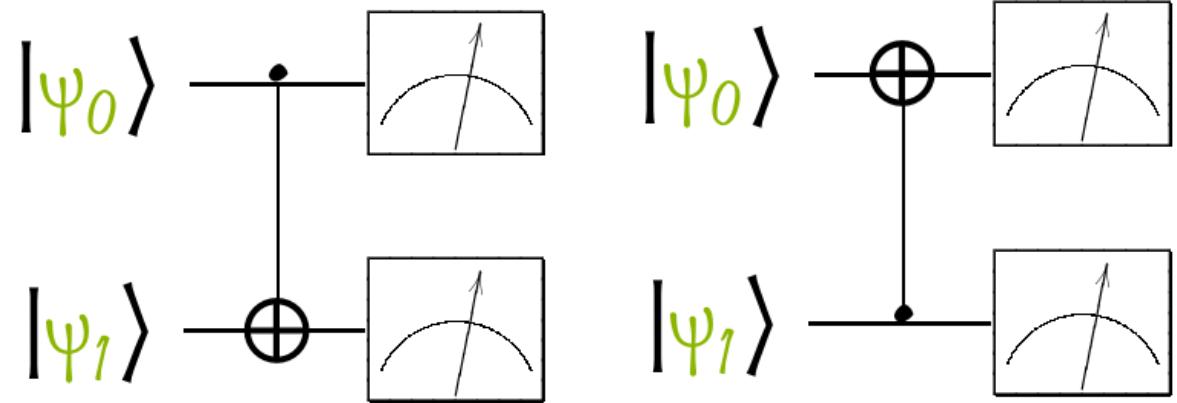
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- Reverse CNOT gate



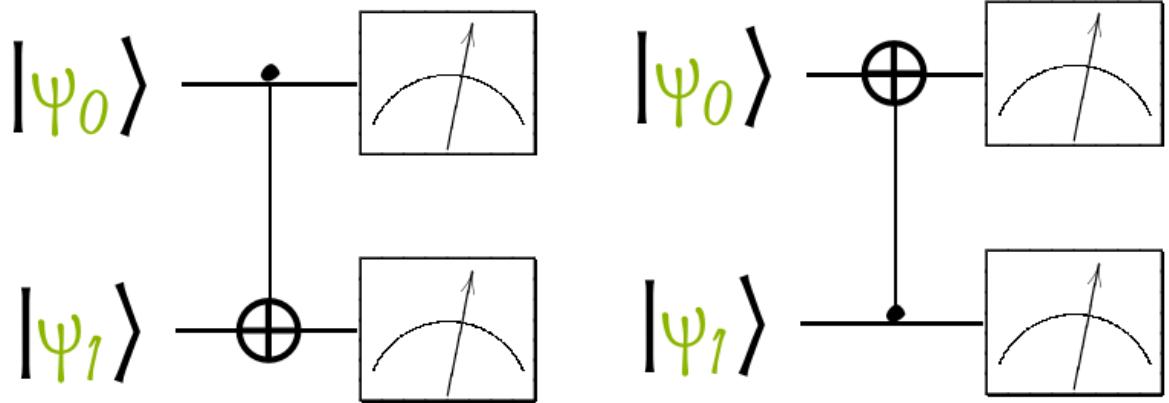
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- This time, if we want to use the method described above, we need to understand how to modify the summation expressed in the previous slide



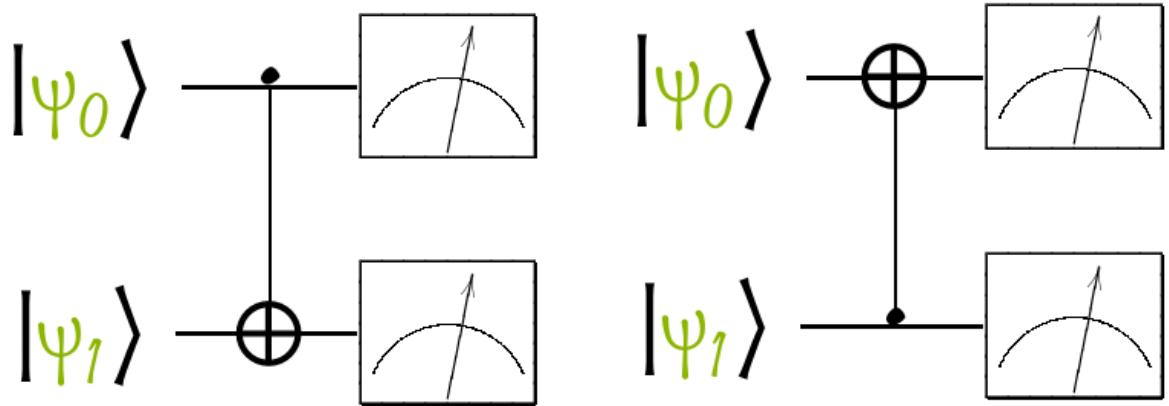
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- The reverse CNOT reverses the roles of target qubit and control qubit from the original CNOT. Its description, therefore, is:



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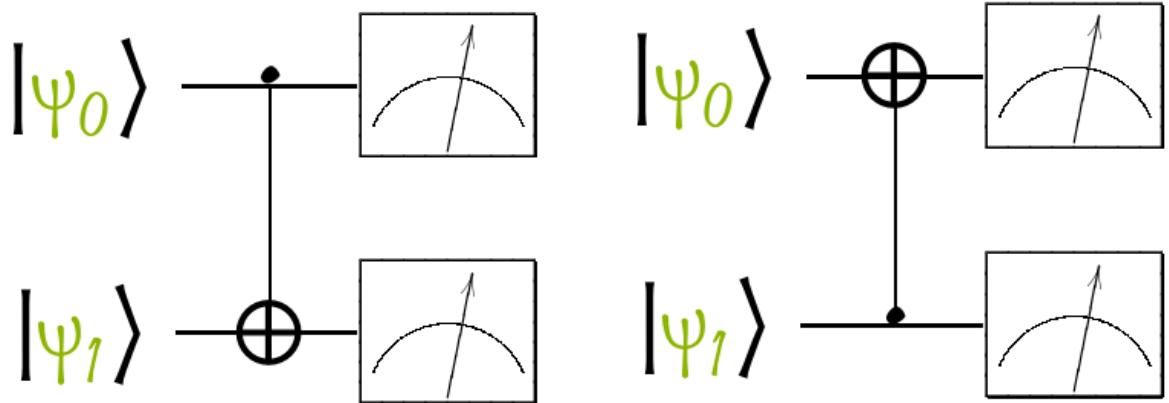
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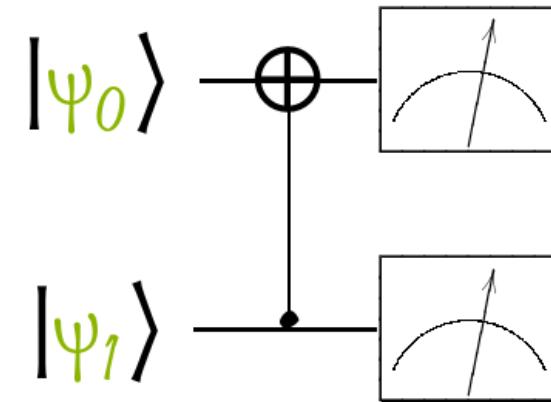
$$I \otimes |0\rangle\langle 0| + X \otimes |1\rangle\langle 1|$$



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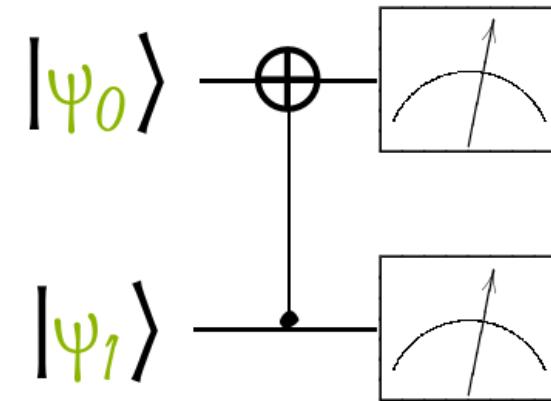


$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \otimes \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

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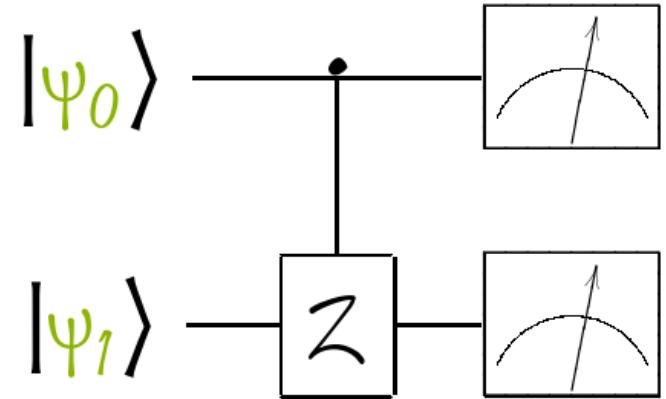
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# Alternative Controlled Gates

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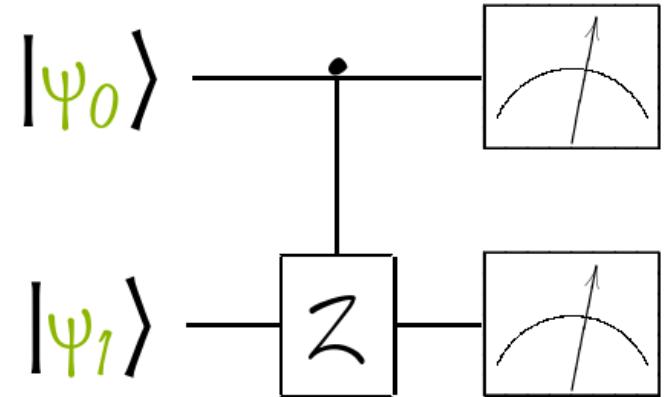
- Construction of generic controlled gates



# Alternative Controlled Gates

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- Construction of generic controlled gates
- The idea is always the same: you think about the gate you want to use and it applies only when the conditions we want to impose are satisfied

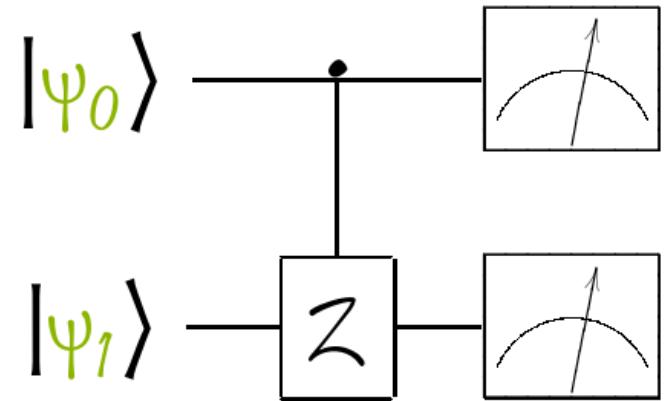


# Alternative Controlled Gates

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- Some examples are:

$$|0\rangle\langle 0| \otimes I + |1\rangle\langle 1| \otimes Z$$



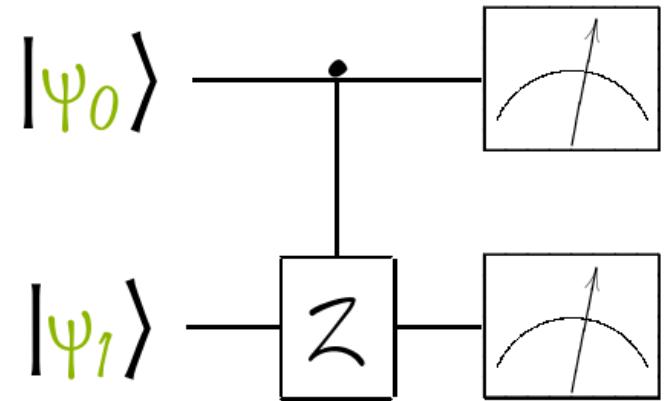
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$$I \otimes I \otimes |0\rangle\langle 0| \otimes I \otimes I \otimes I \otimes I \otimes I \otimes I + \\ I \otimes I \otimes |1\rangle\langle 1| \otimes I \otimes I \otimes I \otimes I \otimes X \otimes I$$



# Alternative Controlled Gates

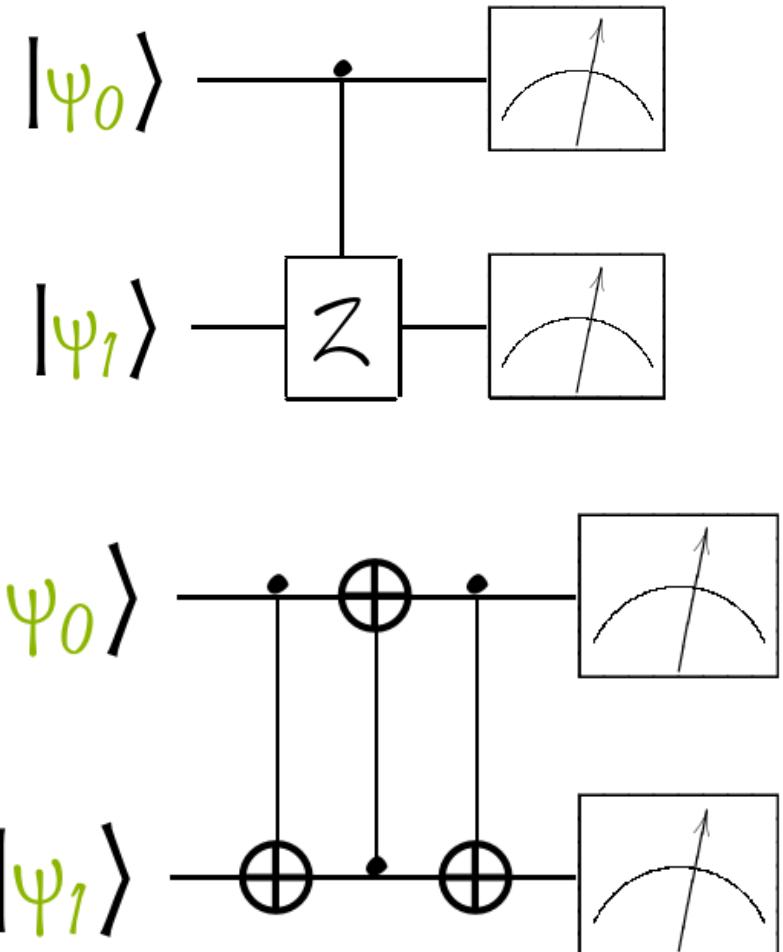
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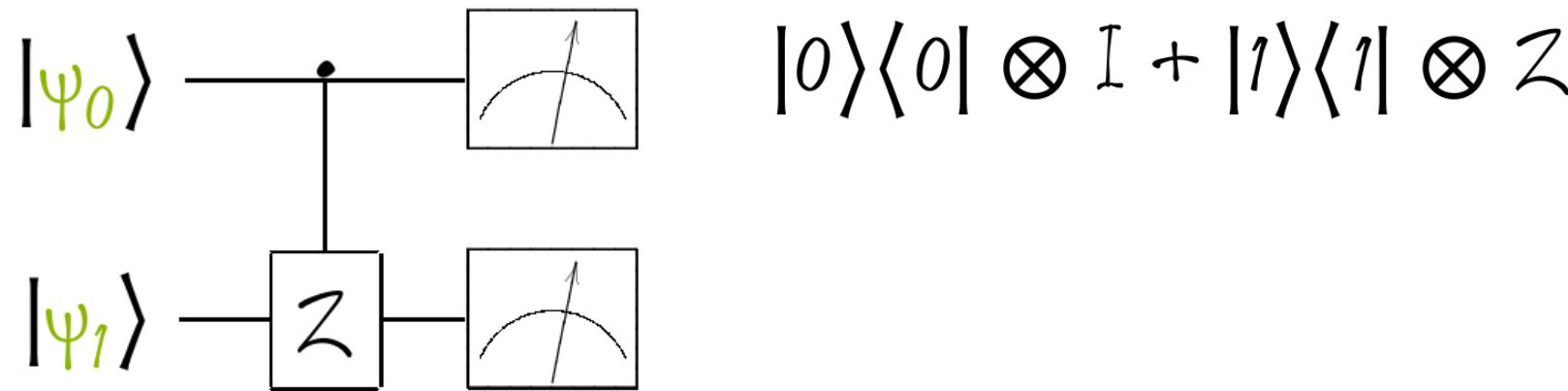
$$I \otimes I \otimes |1\rangle\langle 1| \otimes I \otimes I \otimes I \otimes I \otimes X \otimes I$$

- Exercise: find the gates



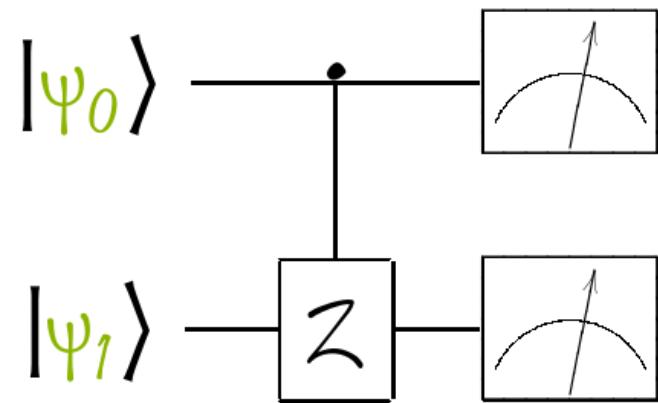
# Alternative Controlled Gates

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# Alternative Controlled Gates

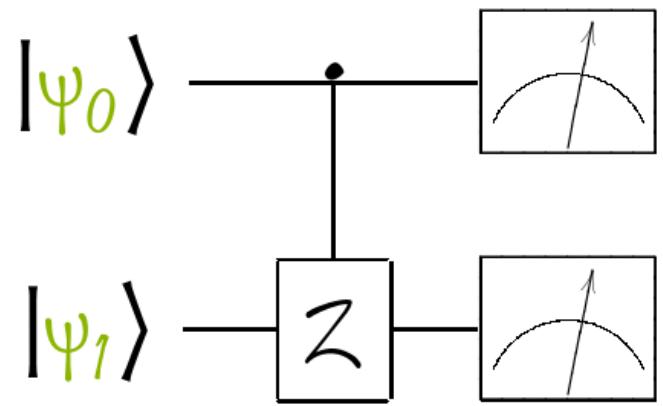
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$$|0\rangle\langle 0| \otimes I + |1\rangle\langle 1| \otimes Z$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

# Alternative Controlled Gates

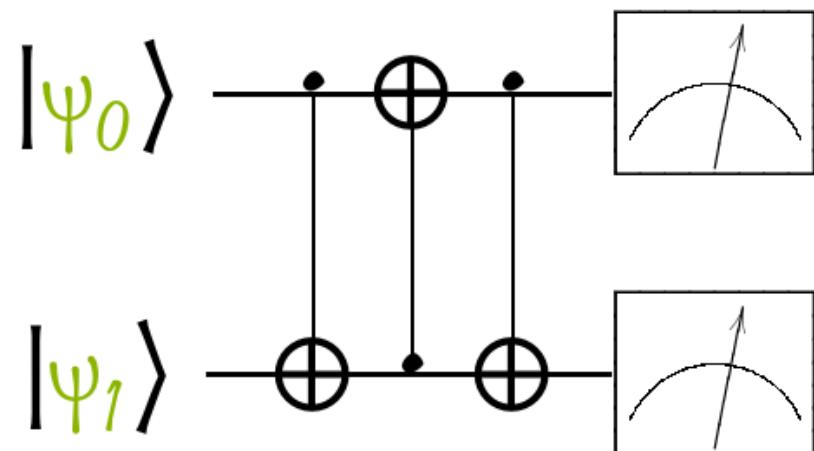
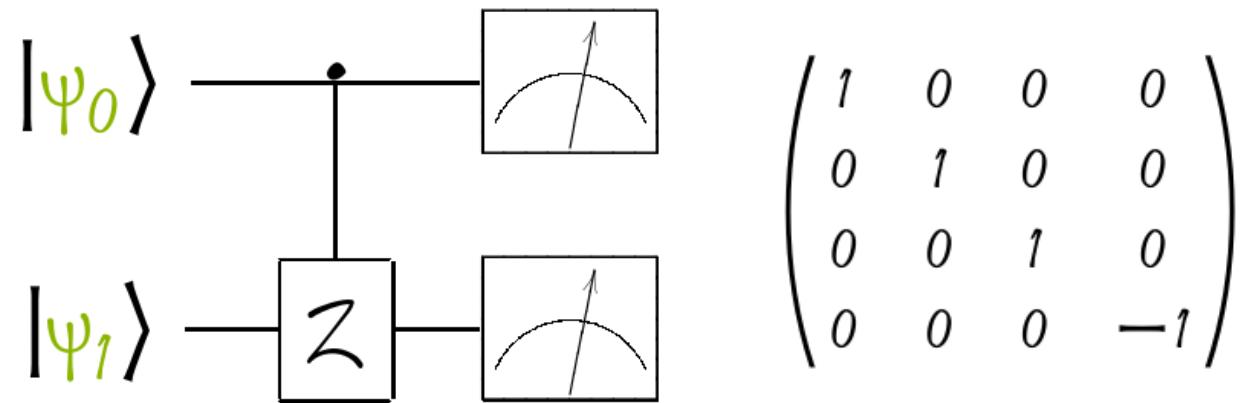


$$|0\rangle\langle 0| \otimes I + |1\rangle\langle 1| \otimes Z$$

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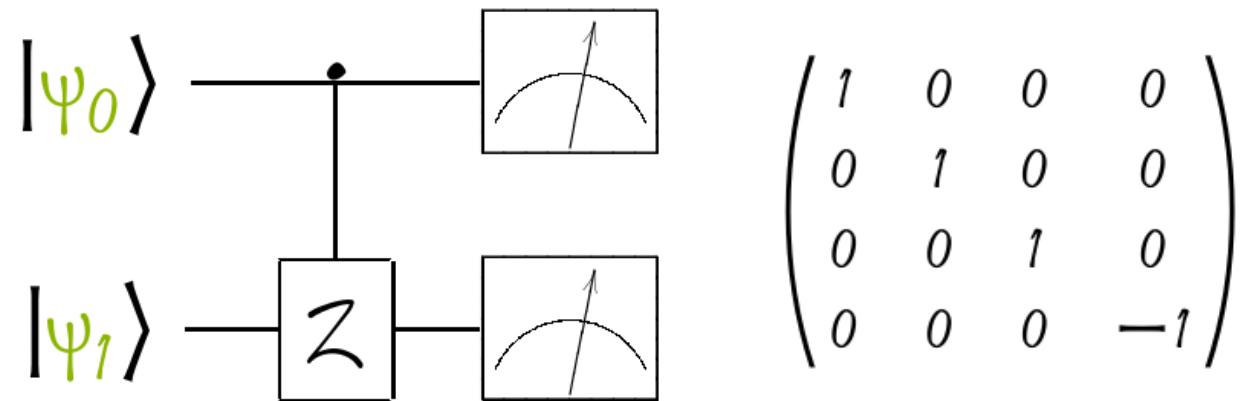
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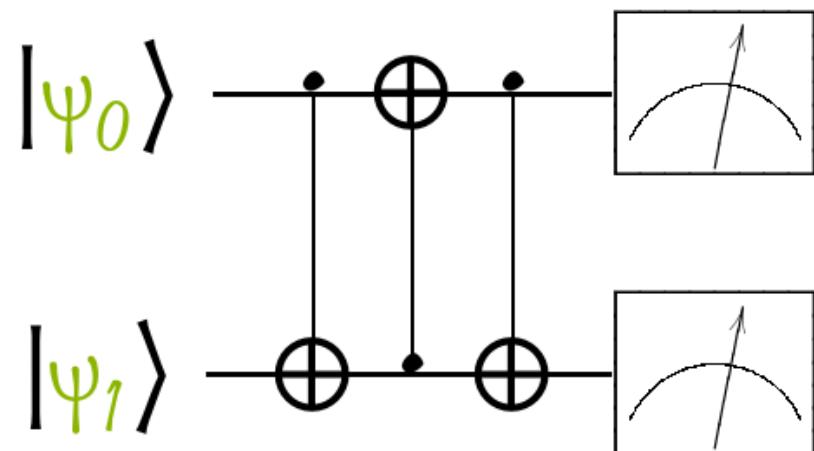
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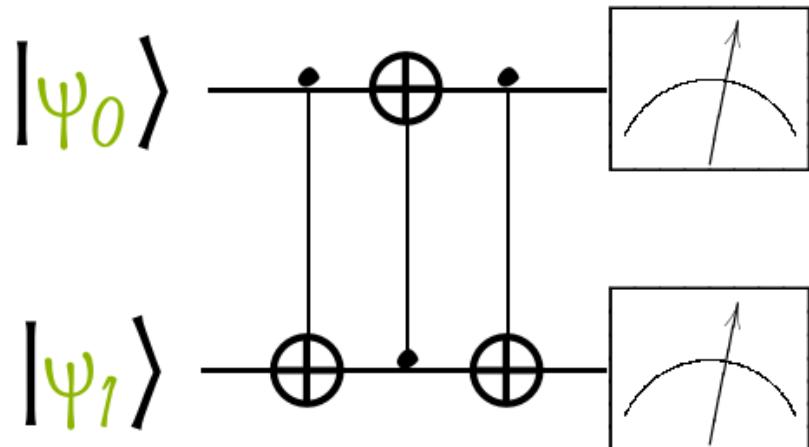
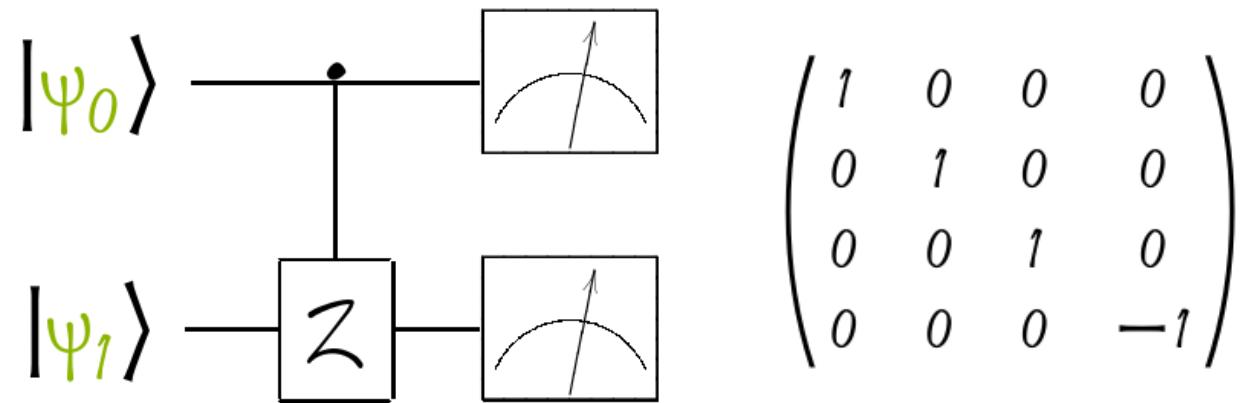


$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$CNOT_1 \rightarrow RCNOT_1 \rightarrow CNOT_2$

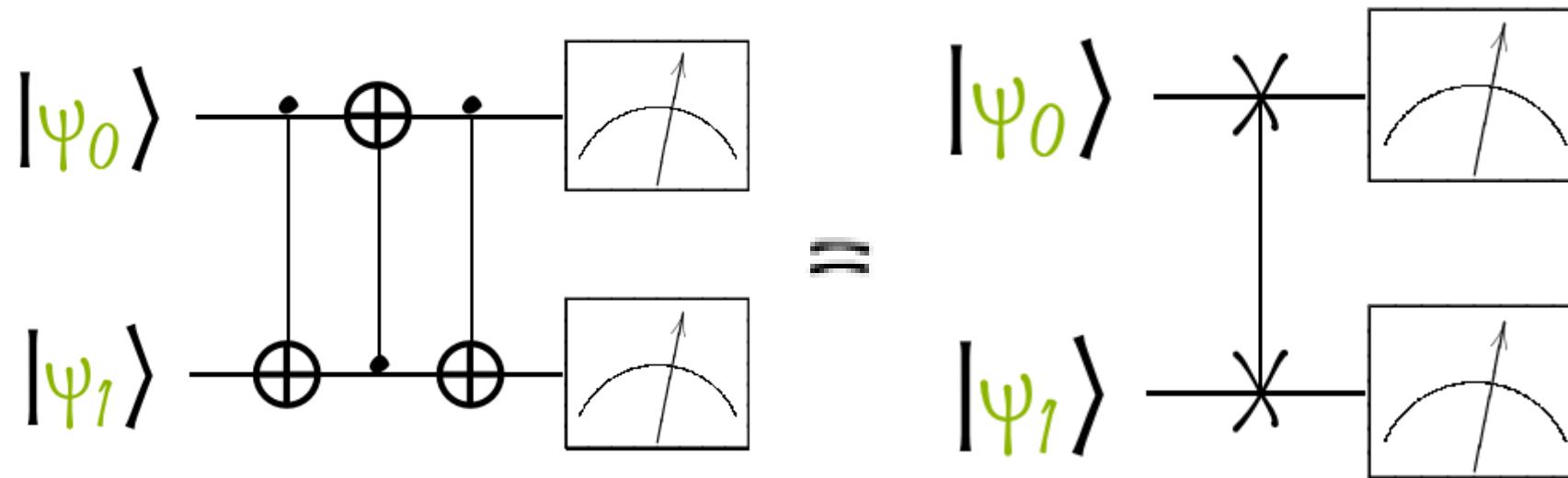
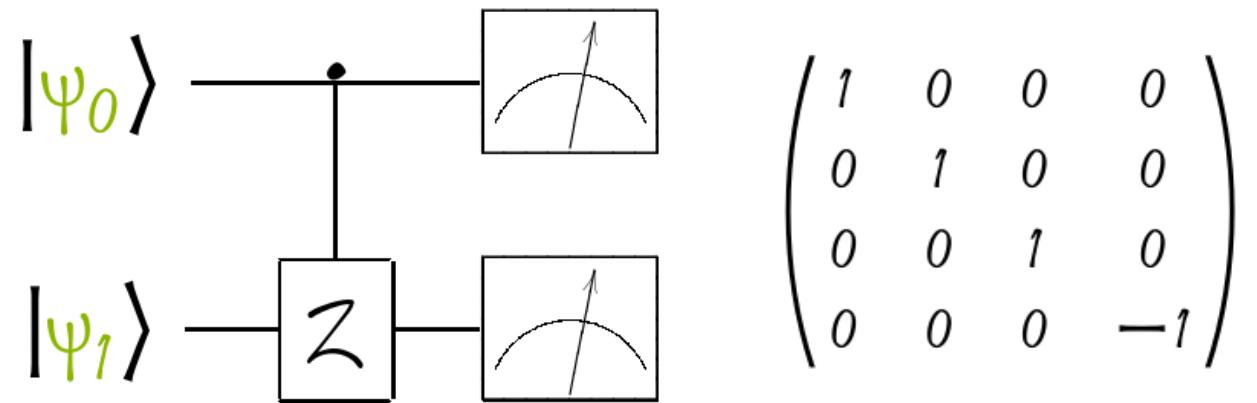


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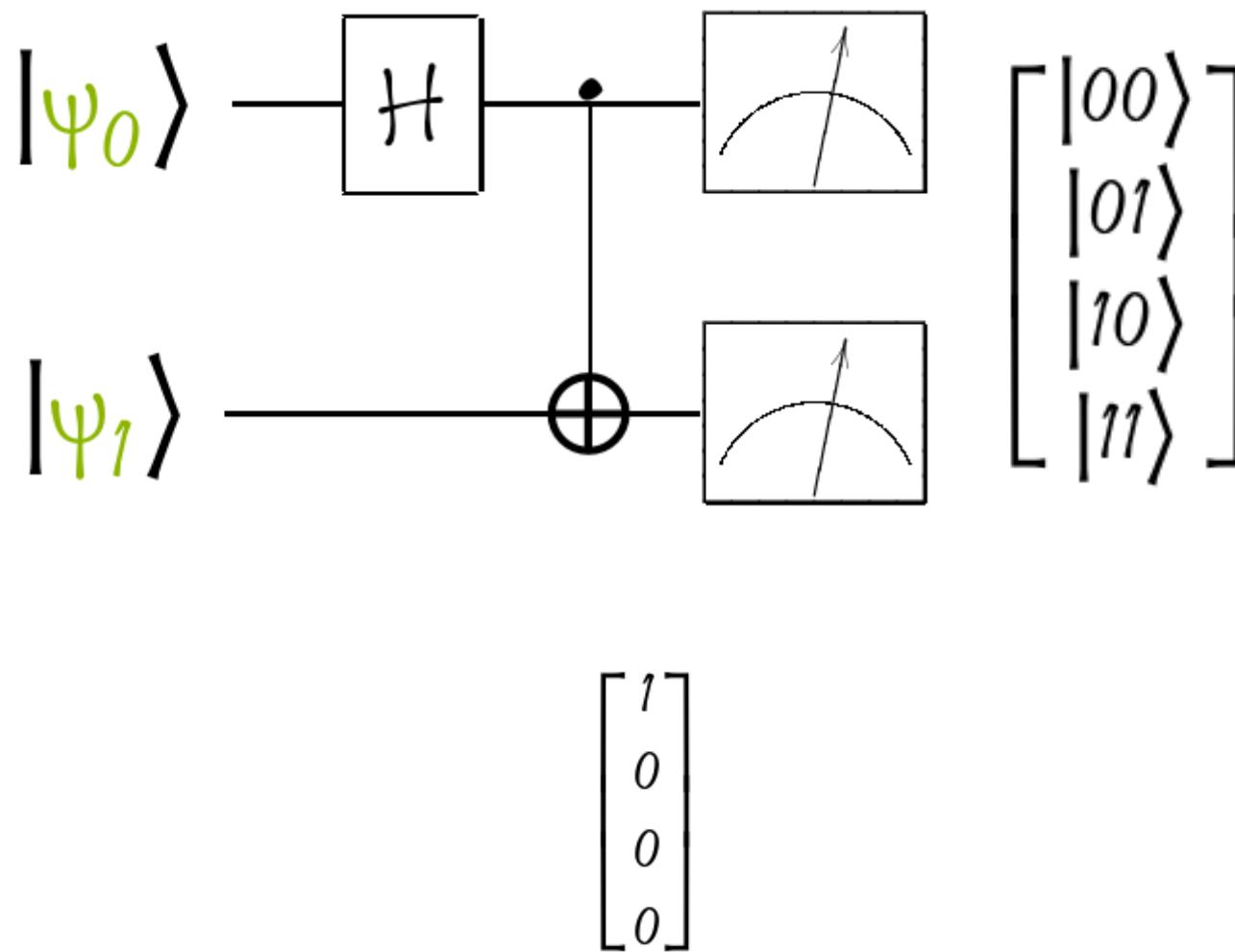
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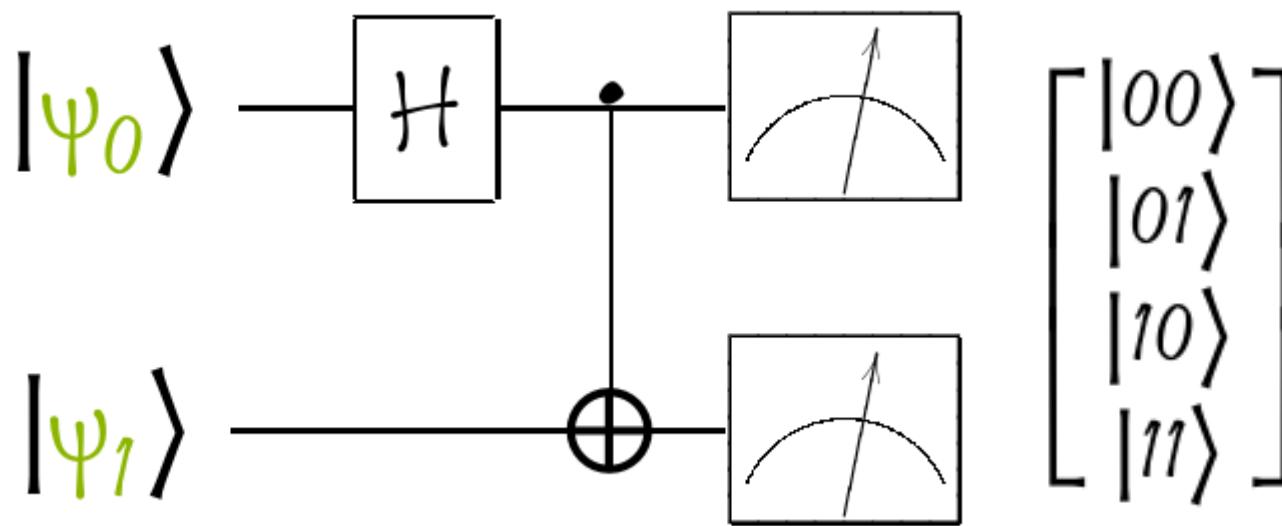
# First Circuit example

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# First Circuit example

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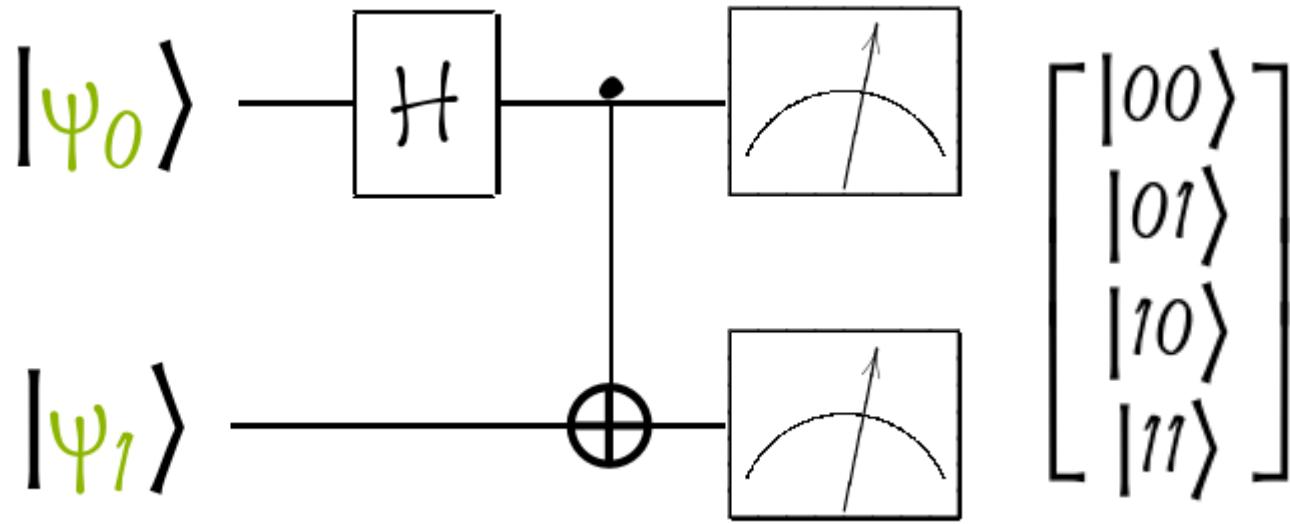


$$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

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# First Circuit example

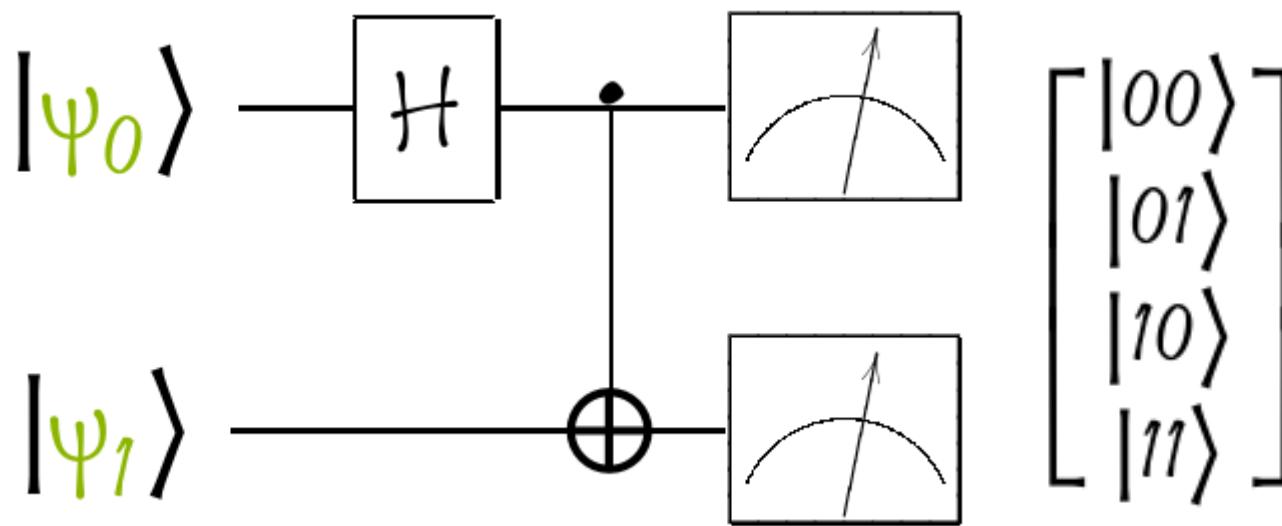
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# First Circuit example

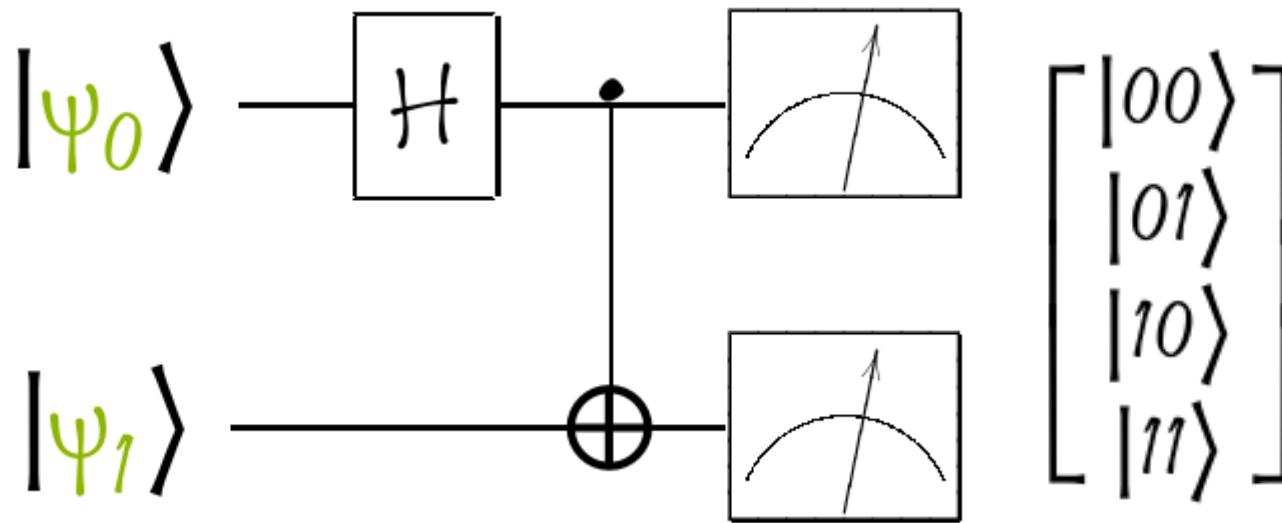
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$$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

# First Circuit example

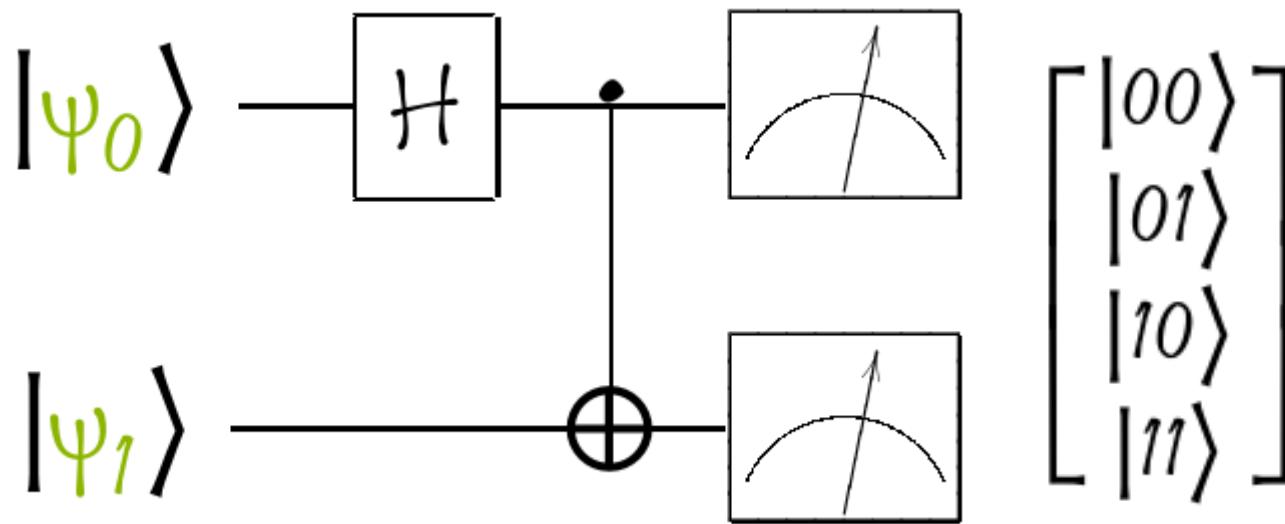
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$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \cdot \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

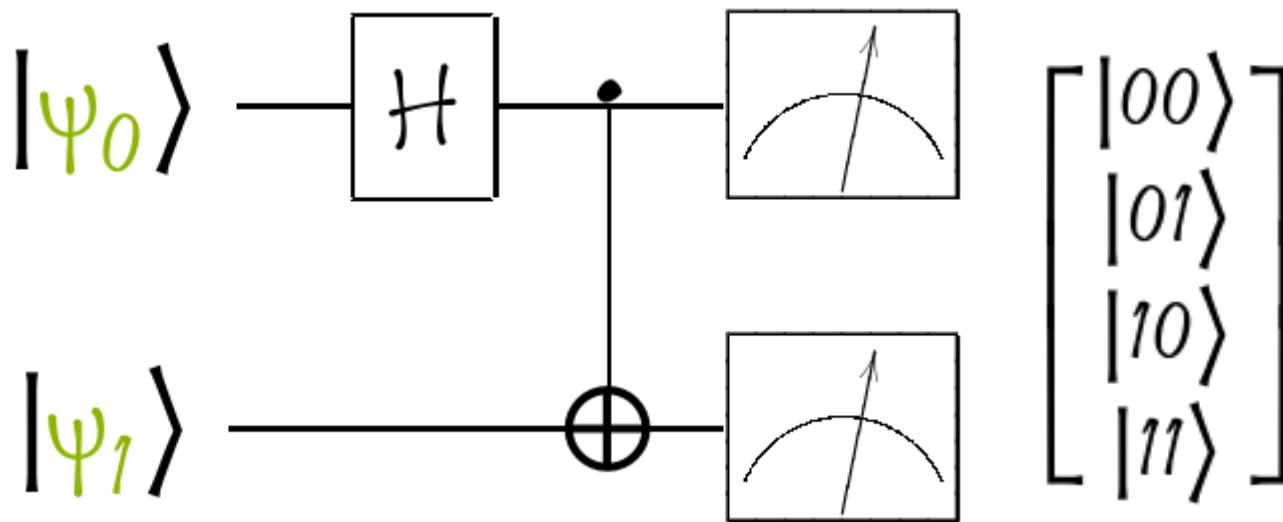
# First Circuit example

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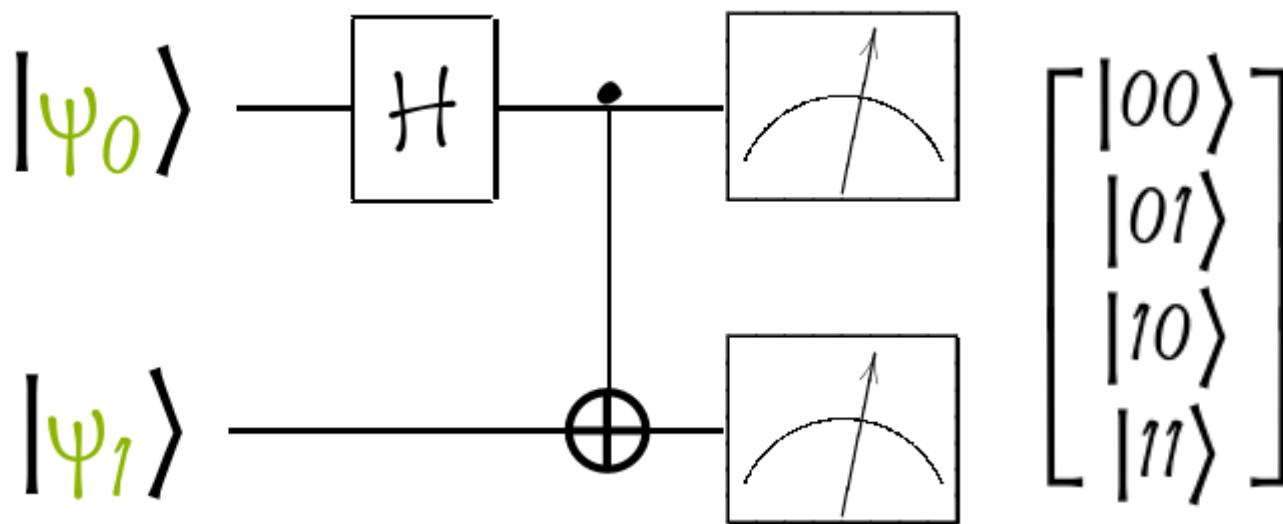
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \cdot \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

# First Circuit example



$$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \cdot \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

# First Circuit example

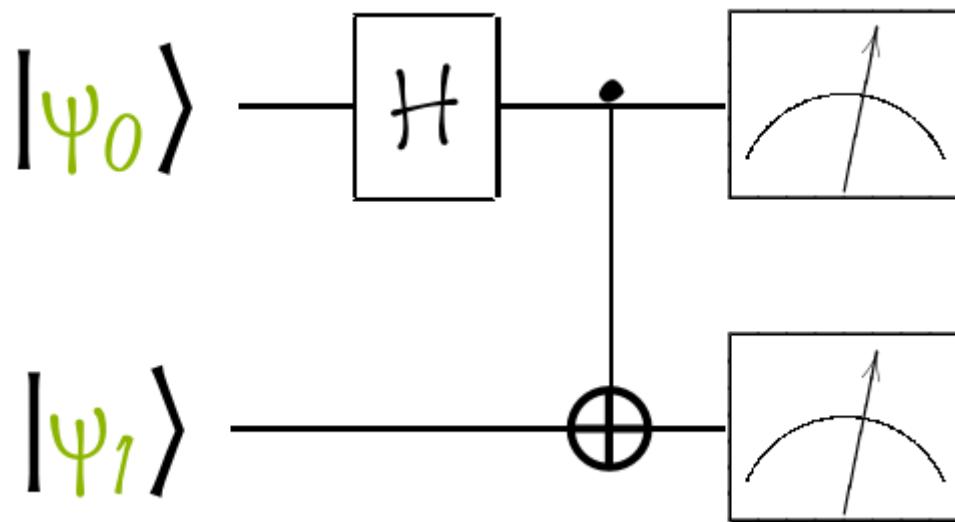


$ \Psi_1\Psi_2\rangle$	$ 0\rangle$	$ 1\rangle$
$ 0\rangle$	50%	0%
$ 1\rangle$	0%	50%

$$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \cdot \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

# First Circuit example

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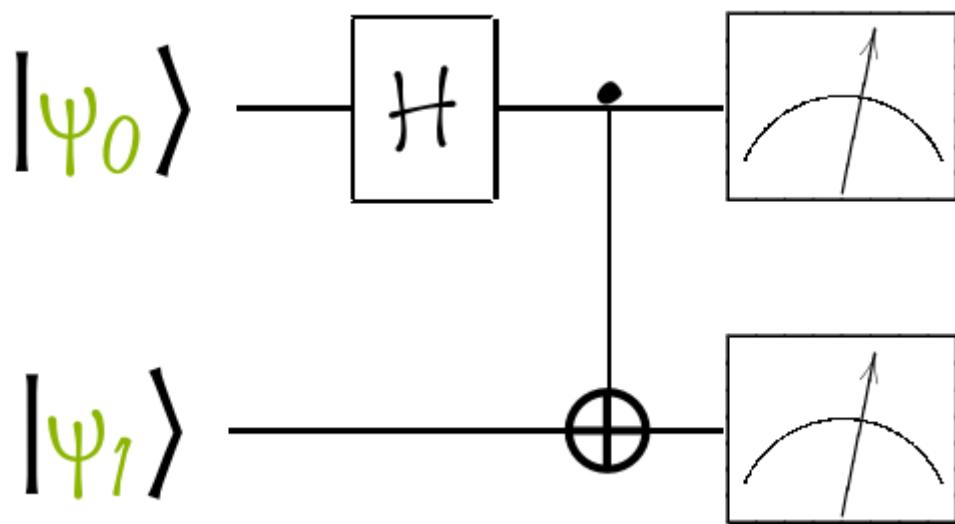
$$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$ \Psi_1\Psi_2\rangle$	$ 0\rangle$	$ 1\rangle$
$ 0\rangle$	50%	0%
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$$|\Psi_1\Psi_2\rangle = \alpha_1\alpha_2|00\rangle + \alpha_1\beta_2|01\rangle + \beta_1\alpha_2|10\rangle + \beta_1\beta_2|11\rangle$$

# First Circuit example

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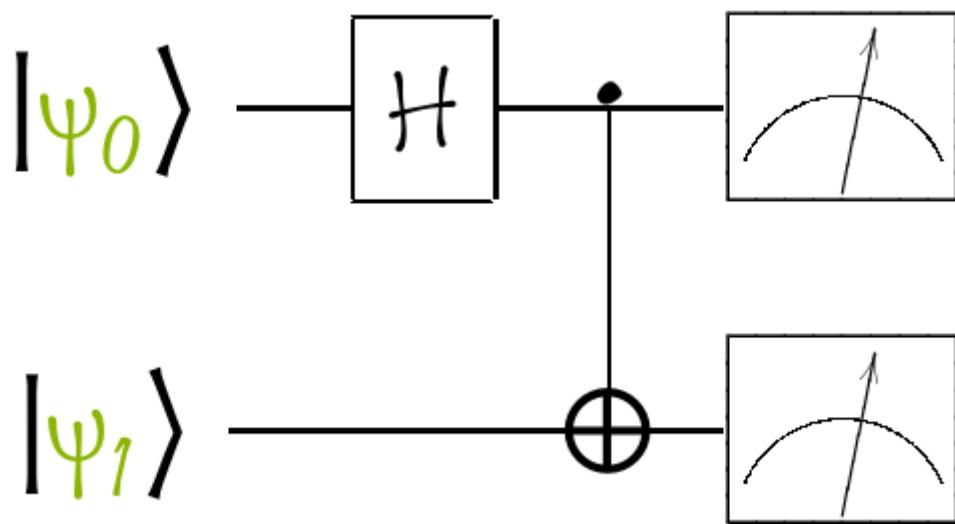
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# First Circuit example



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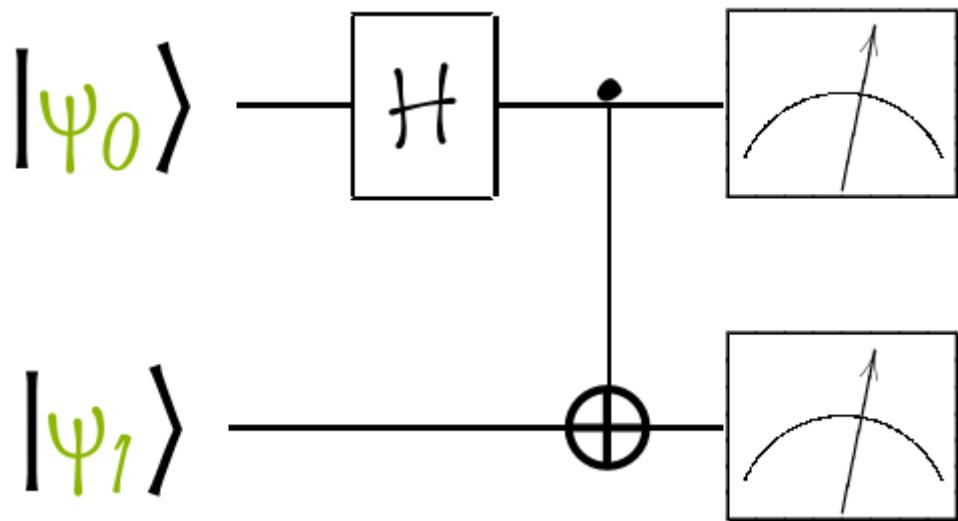
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No Way!

# First Circuit example

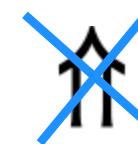


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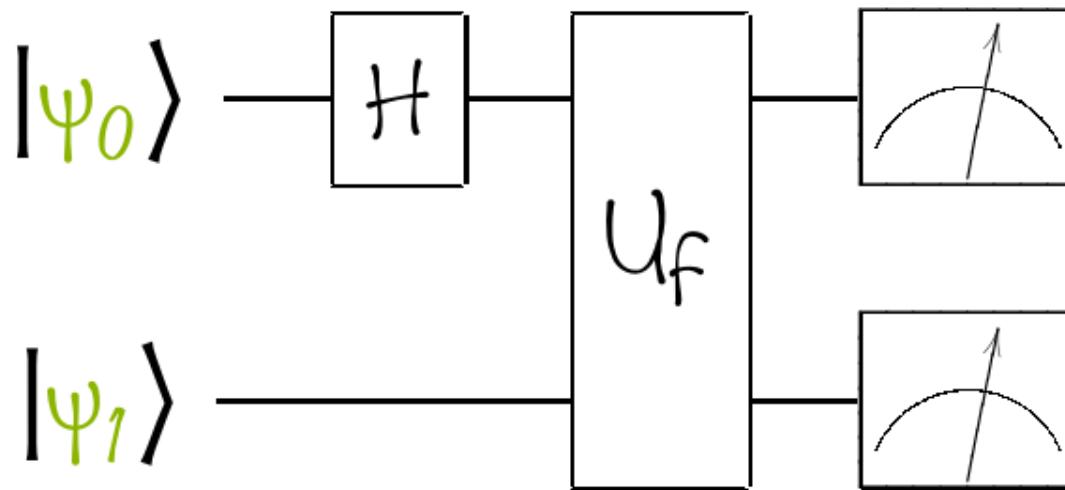
$$|\Psi_1\Psi_2 \dots \Psi_N\rangle = \alpha_1 \alpha_2 \dots \alpha_N |00 \dots 0\rangle + \alpha_1 \alpha_2 \dots \alpha_{N-1} \beta_N |00 \dots 01\rangle + \dots$$

$$+ \beta_1 \beta_2 \dots \beta_{N-1} \alpha_N |11 \dots 10\rangle + \beta_1 \beta_2 \dots \beta_N |11 \dots 1\rangle$$



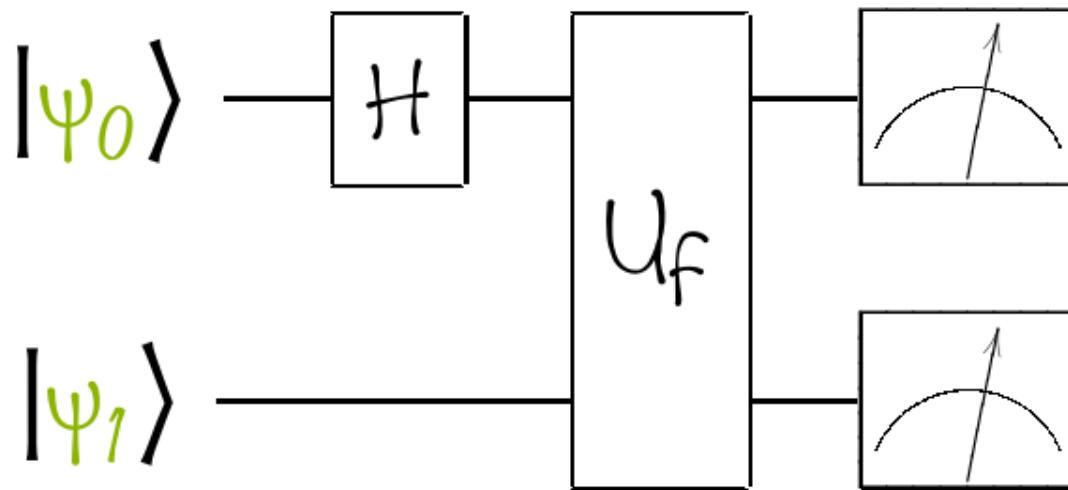
$$|\Psi_1\Psi_2 \dots \Psi_N\rangle = \gamma_1 |00 \dots 0\rangle + \gamma_2 |00 \dots 01\rangle + \dots + \gamma_{N-1} |11 \dots 10\rangle + \gamma_N |11 \dots 1\rangle$$

# Quantum Parallelism



- Let's now consider a circuit like the one in the figure

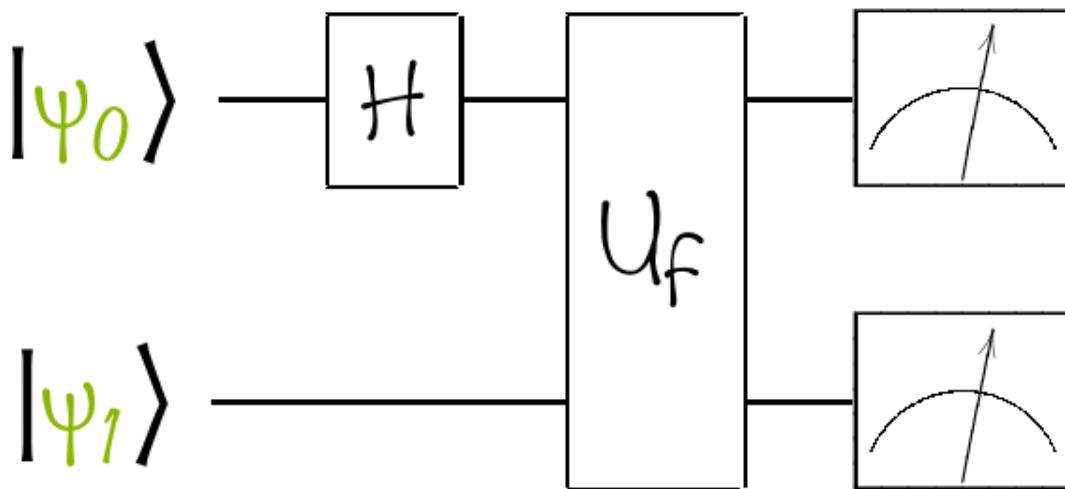
# Quantum Parallelism



- Let's now consider a circuit like the one in the figure
- A gate appears in this circuit that we have never seen before. This particular gate, called  $U_f$ , is a gate associated with a certain function  $f$

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# Quantum Parallelism

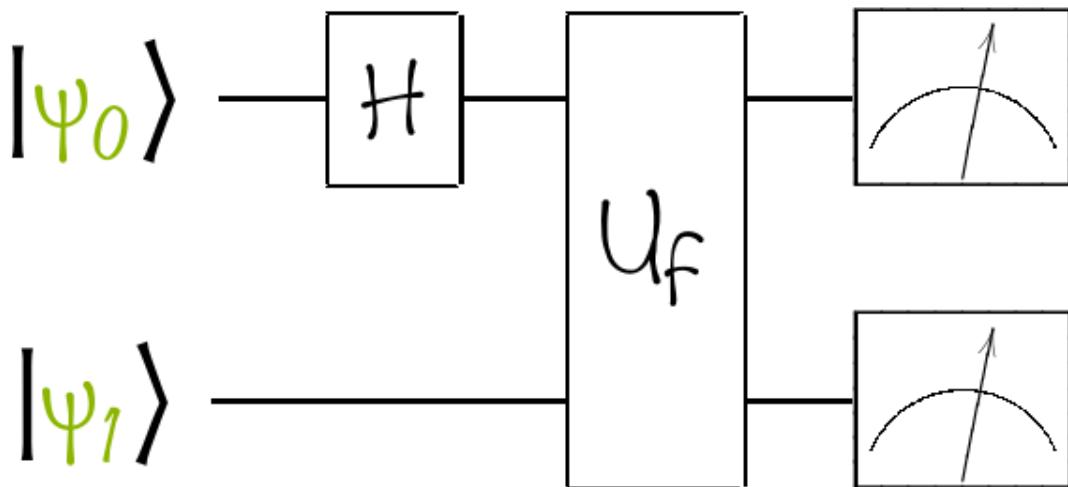


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$$|\Psi_1\rangle \xrightarrow{U_f} |\Psi_1\rangle$$

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# Quantum Parallelism

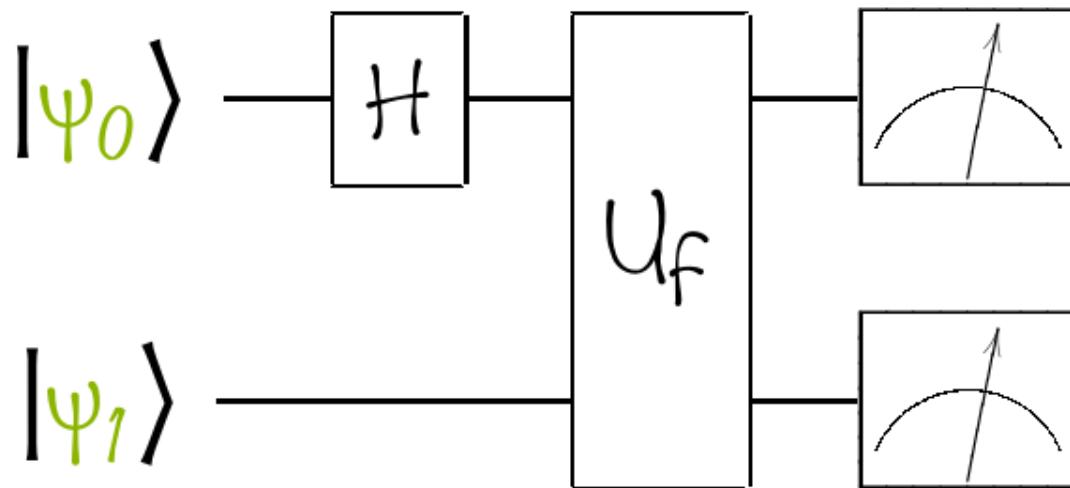


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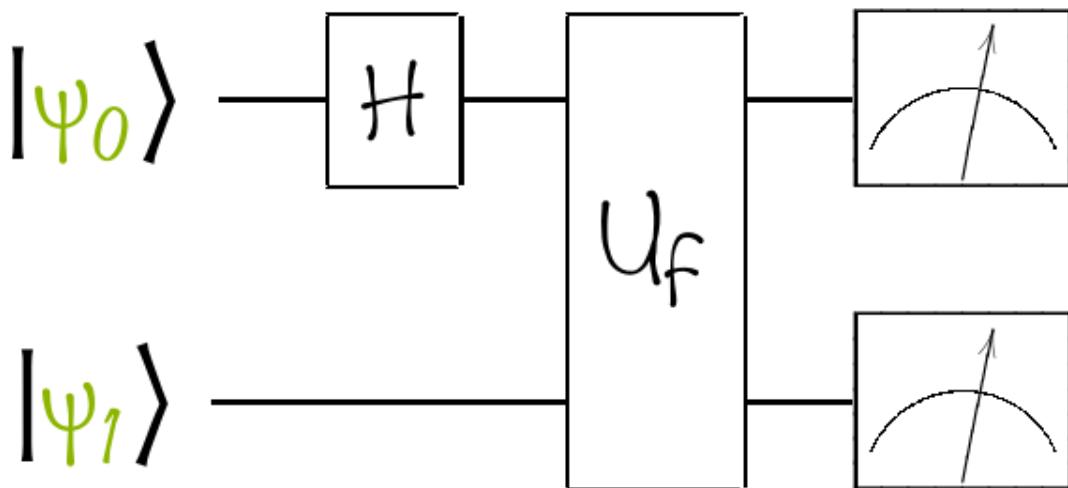


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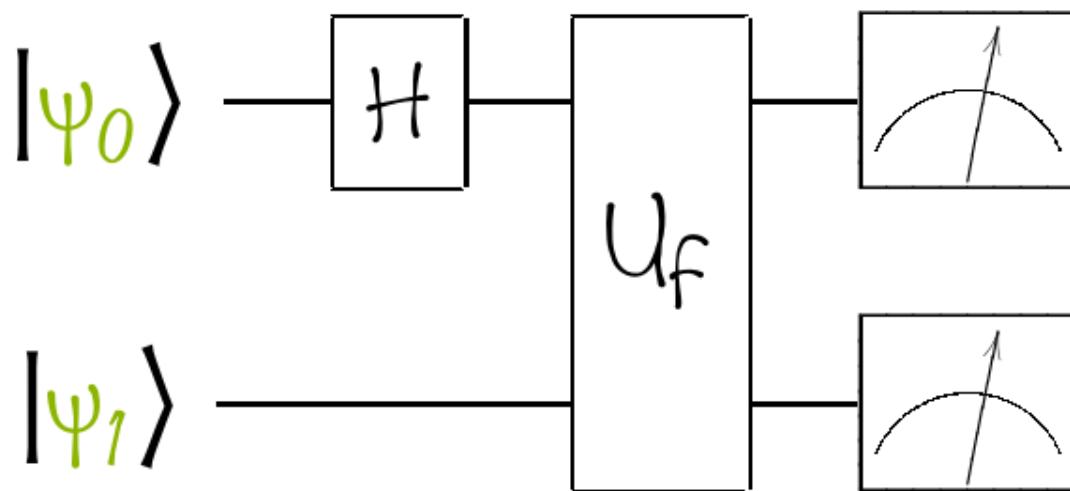


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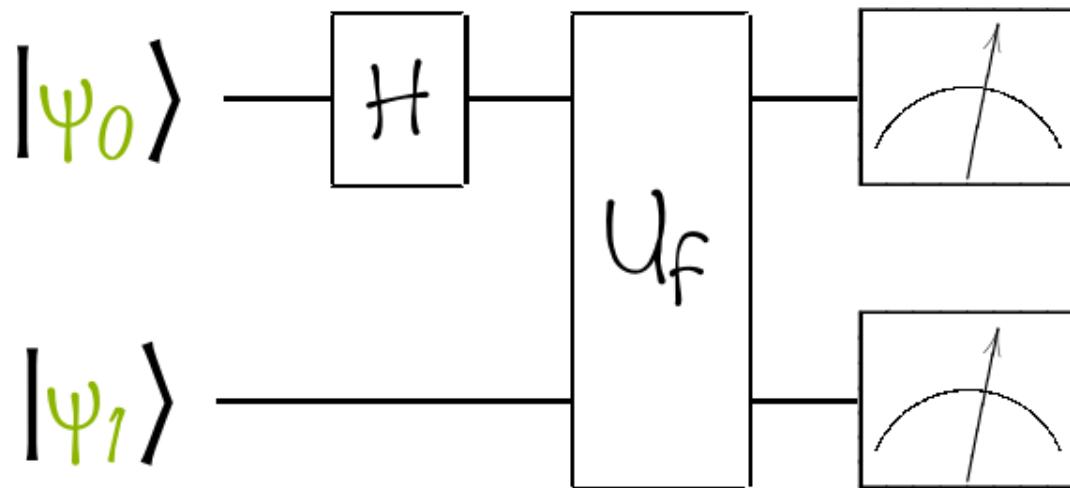
$$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|10\rangle \rightarrow$$

$$\frac{1}{\sqrt{2}}|0, 0 \oplus f(0)\rangle + \frac{1}{\sqrt{2}}|1, 0 \oplus f(1)\rangle$$

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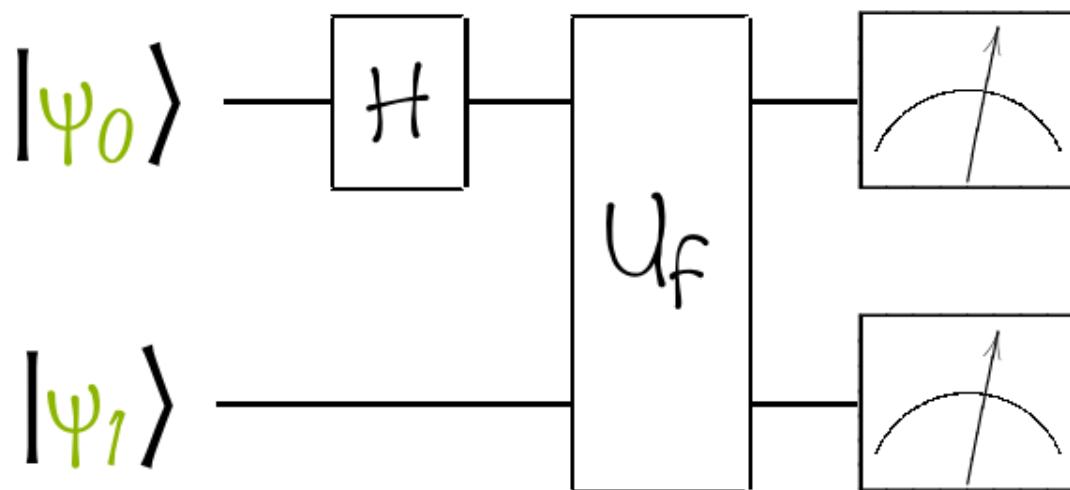
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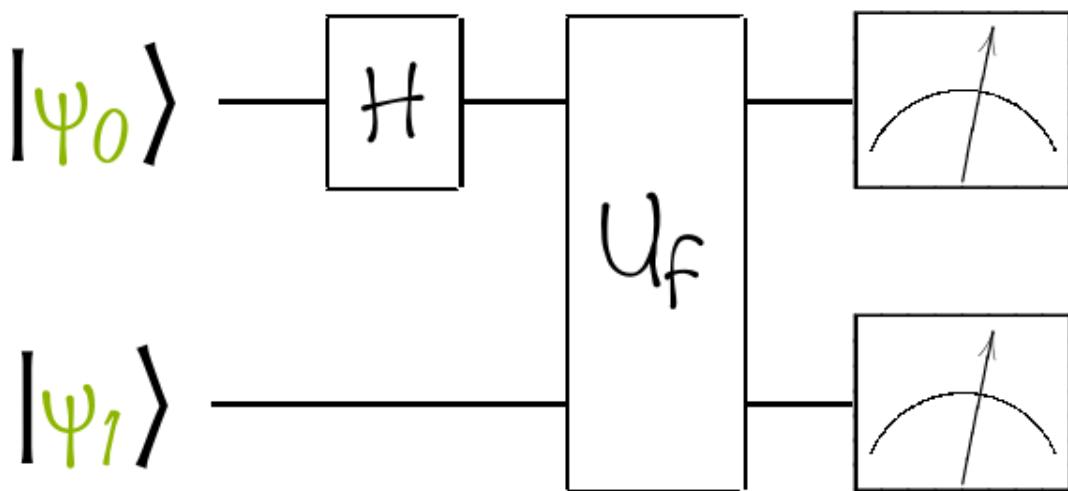


$$|\Psi_{01}\rangle = \frac{|0, f(0)\rangle + |1, f(1)\rangle}{\sqrt{2}}$$

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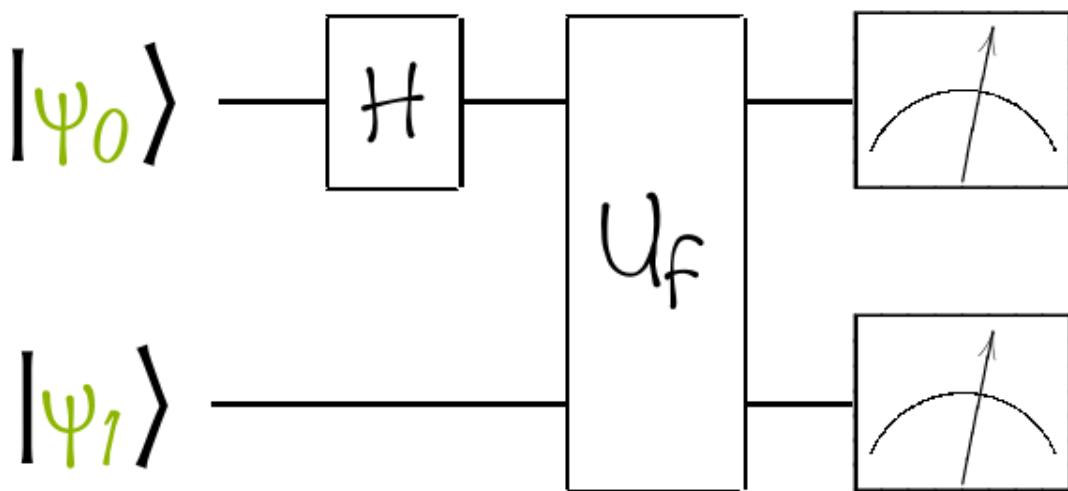
# Quantum Parallelism



- Let us pause for a moment on the quantum state we have obtained

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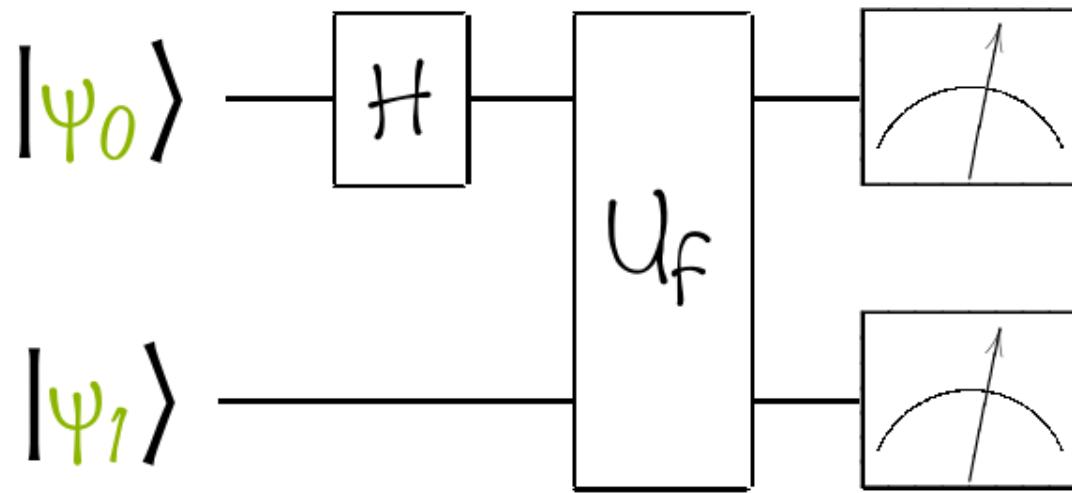
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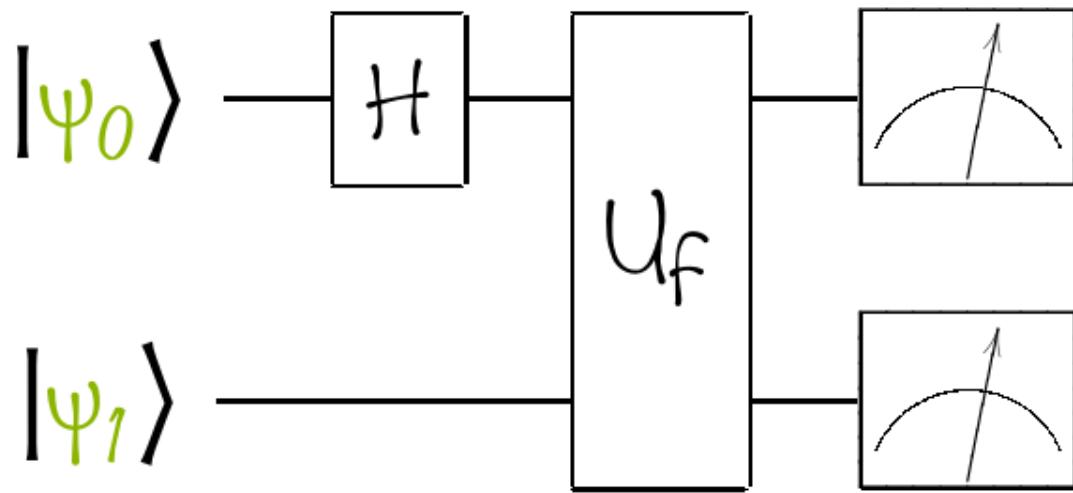
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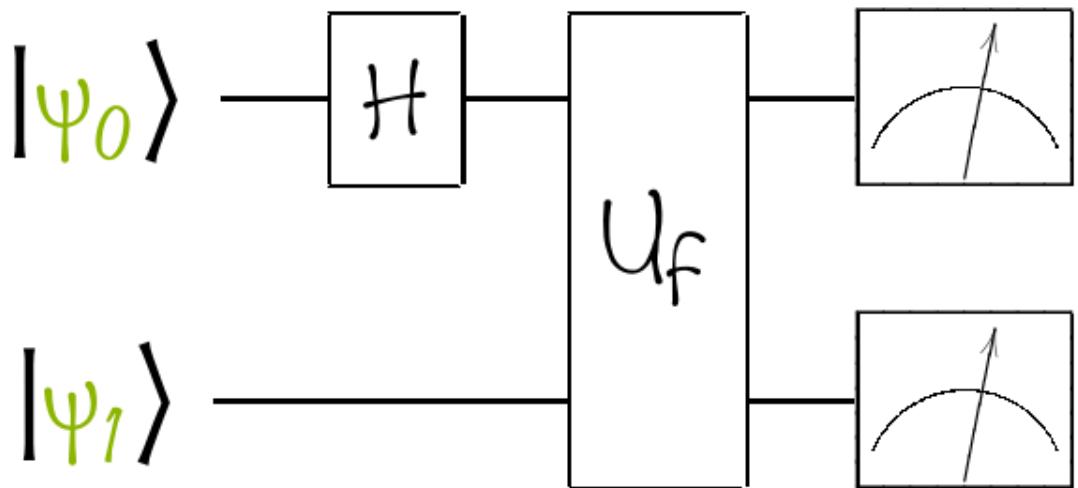
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- In our case, for example, the state we have obtained is completely useless. Why?

# Quantum Parallelism: Deutsch Algorithm

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- Problem is: try to evaluate the nature of the function  $f$  using as few operations as possible
  - Classically speaking, the most efficient algorithm to accomplish the task involves a minimum of two measurements: one for the value of  $f$  in point 0, the other for the value of  $f$  in point 1

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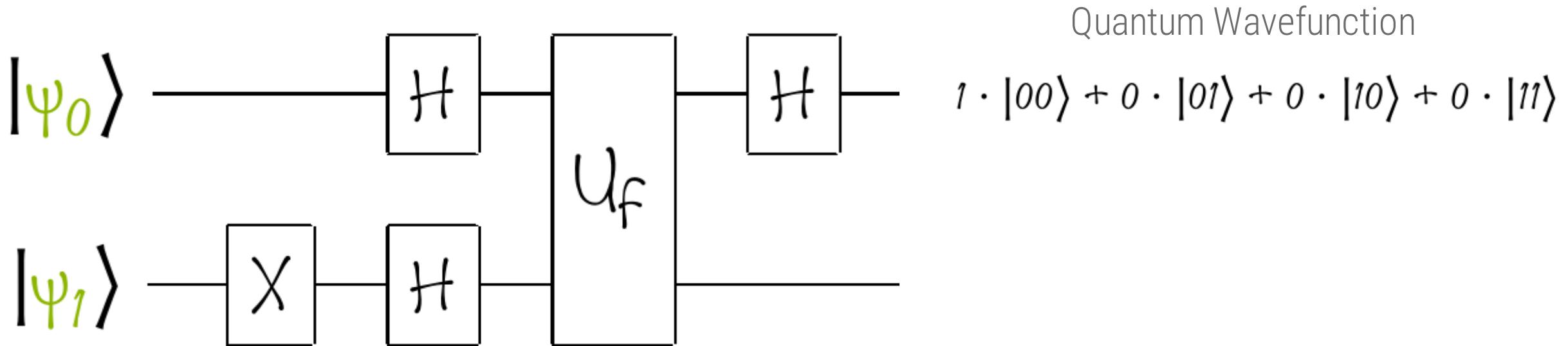
IF (  $f(0) == 0$  ):  
  IF (  $f(1) == 0$  ):  
    f is constant

ELSE:  
  f is balanced

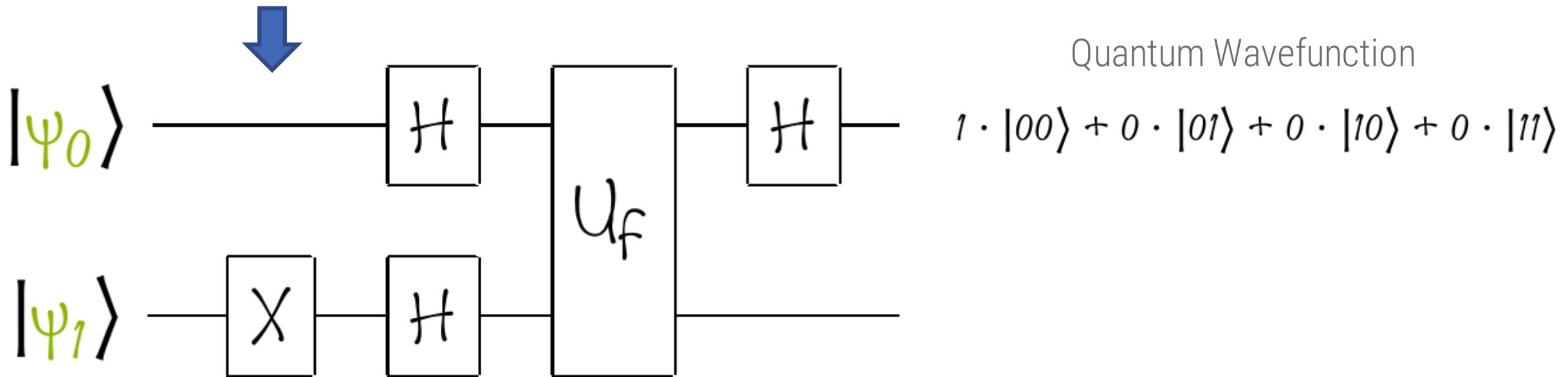
ELSE:  
  IF (  $f(1) == 0$  ):  
    f is balanced

ELSE:  
  f is constant

# Quantum Parallelism: Deutsch Algorithm

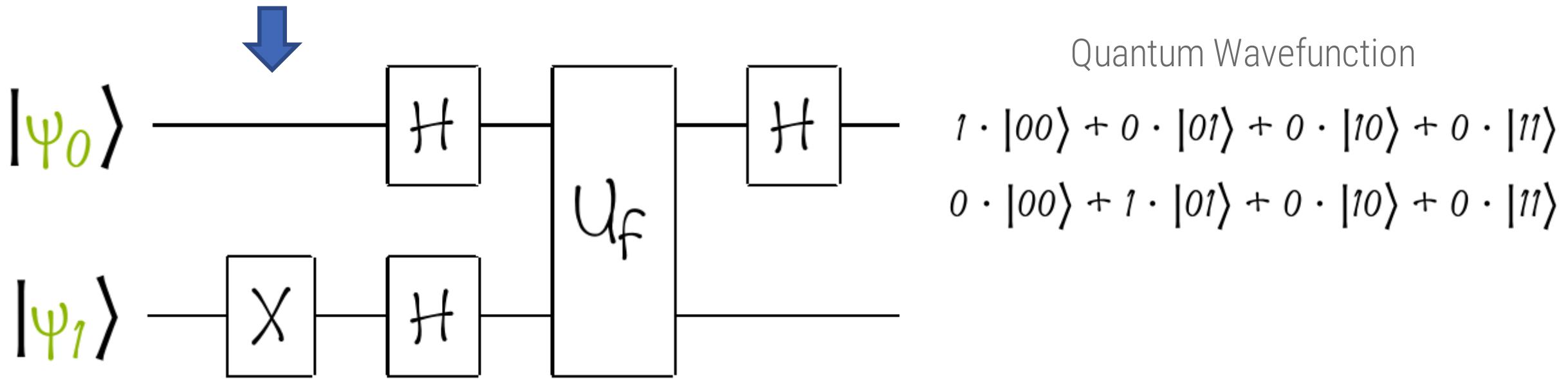


# Quantum Parallelism: Deutsch Algorithm



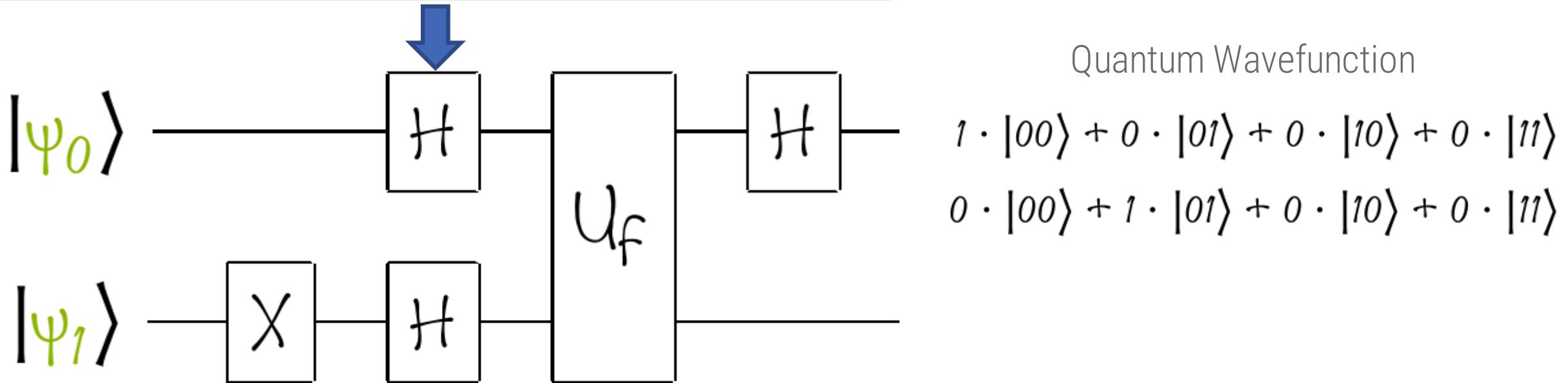
$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

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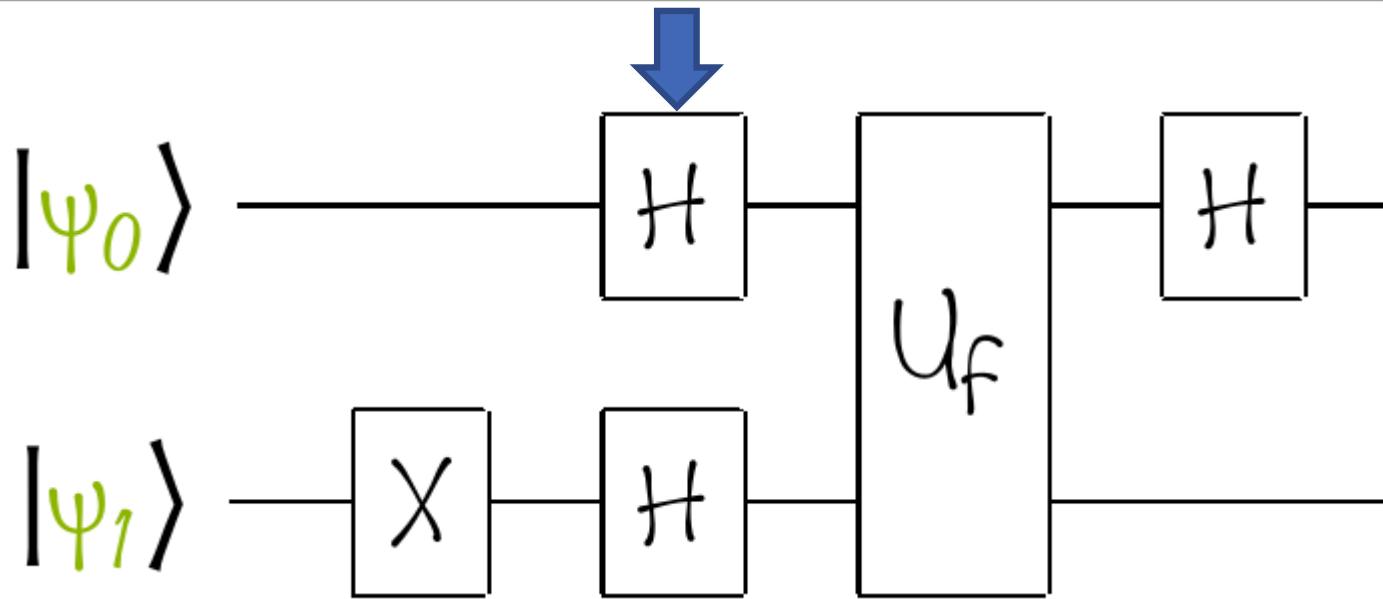
Quantum Wavefunction

$$1 \cdot |00\rangle + 0 \cdot |01\rangle + 0 \cdot |10\rangle + 0 \cdot |11\rangle$$

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Quantum Wavefunction

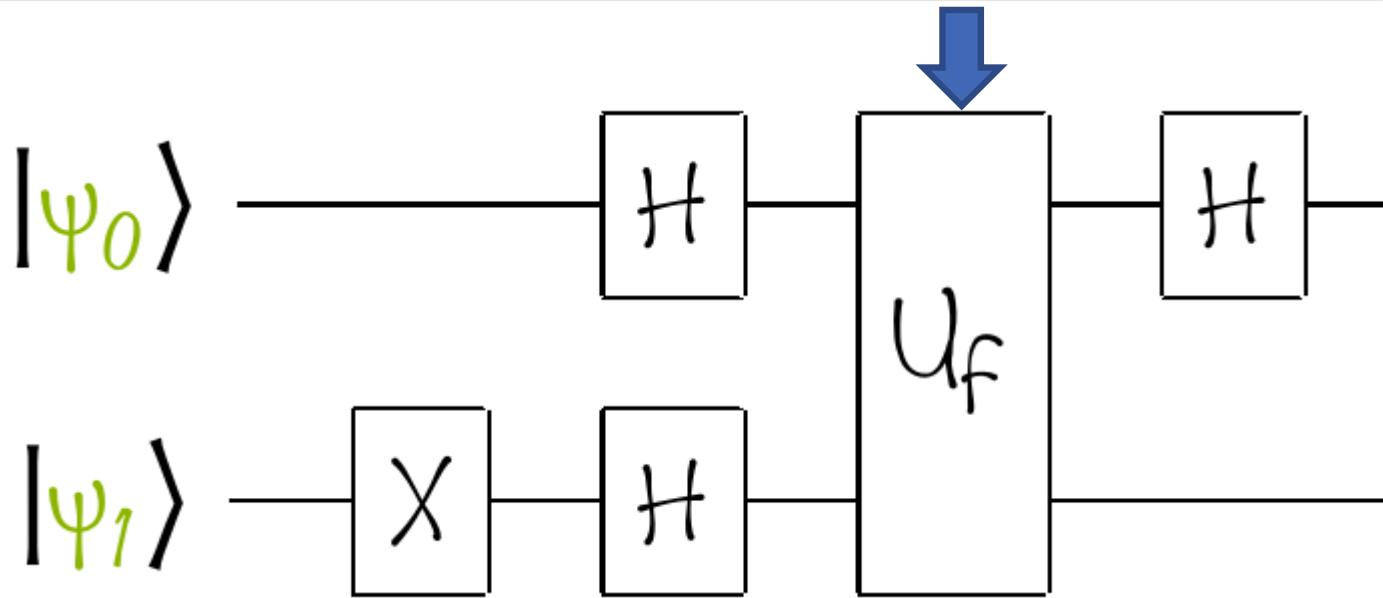
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$$\frac{1}{2} \cdot |00\rangle - \frac{1}{2} \cdot |01\rangle + \frac{1}{2} \cdot |10\rangle - \frac{1}{2} \cdot |11\rangle$$

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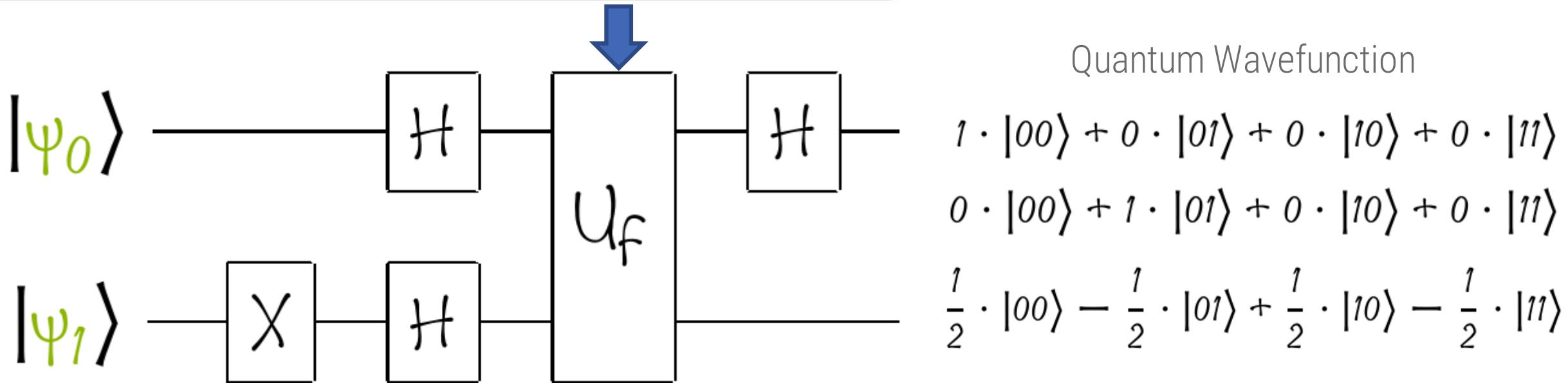
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$$|\Psi_1\rangle \xrightarrow{U_f} |\Psi_1'\rangle$$

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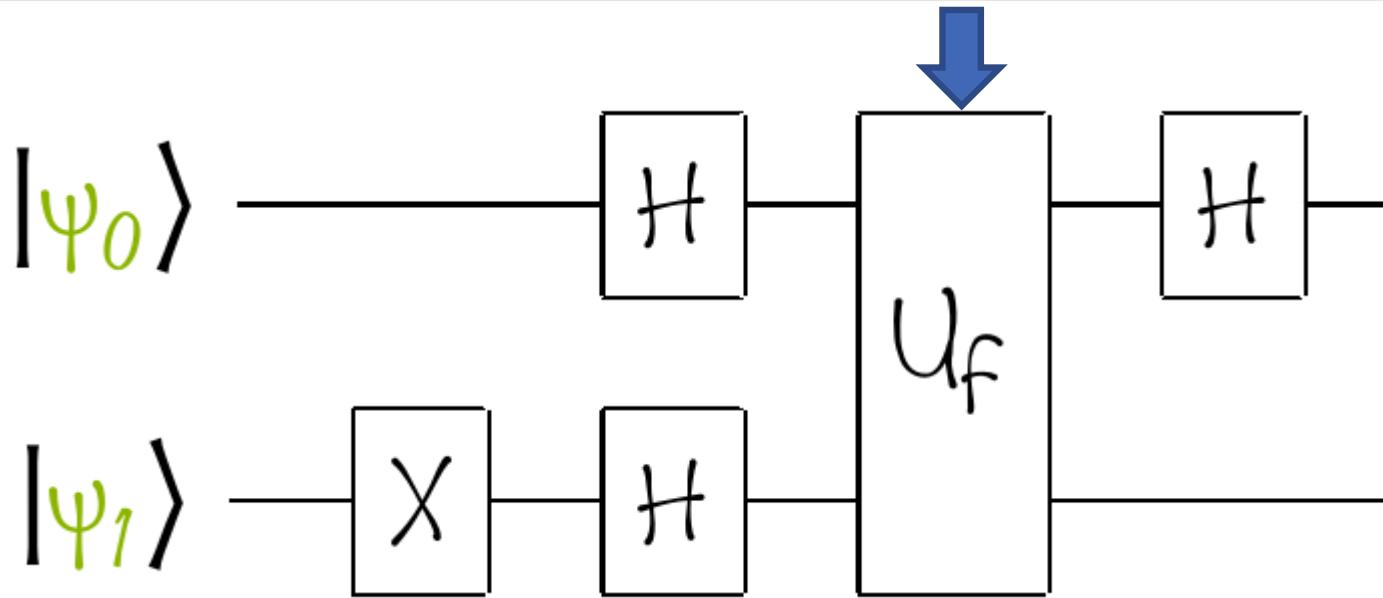
# Quantum Parallelism: Deutsch Algorithm



$$|00\rangle \rightarrow |0, f(0)\rangle \quad |01\rangle \rightarrow |0, 1 + f(0)\rangle$$

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# Quantum Parallelism: Deutsch Algorithm



Quantum Wavefunction

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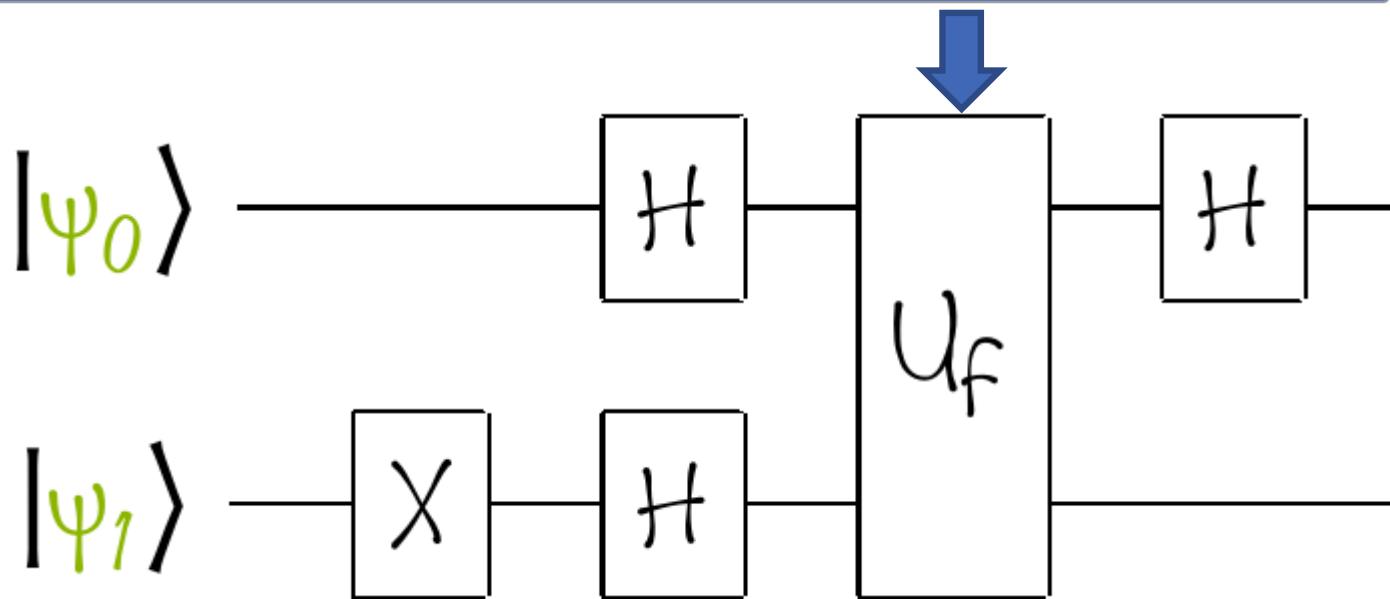
$$\frac{1}{2} \cdot |00\rangle - \frac{1}{2} \cdot |01\rangle + \frac{1}{2} \cdot |10\rangle - \frac{1}{2} \cdot |11\rangle$$

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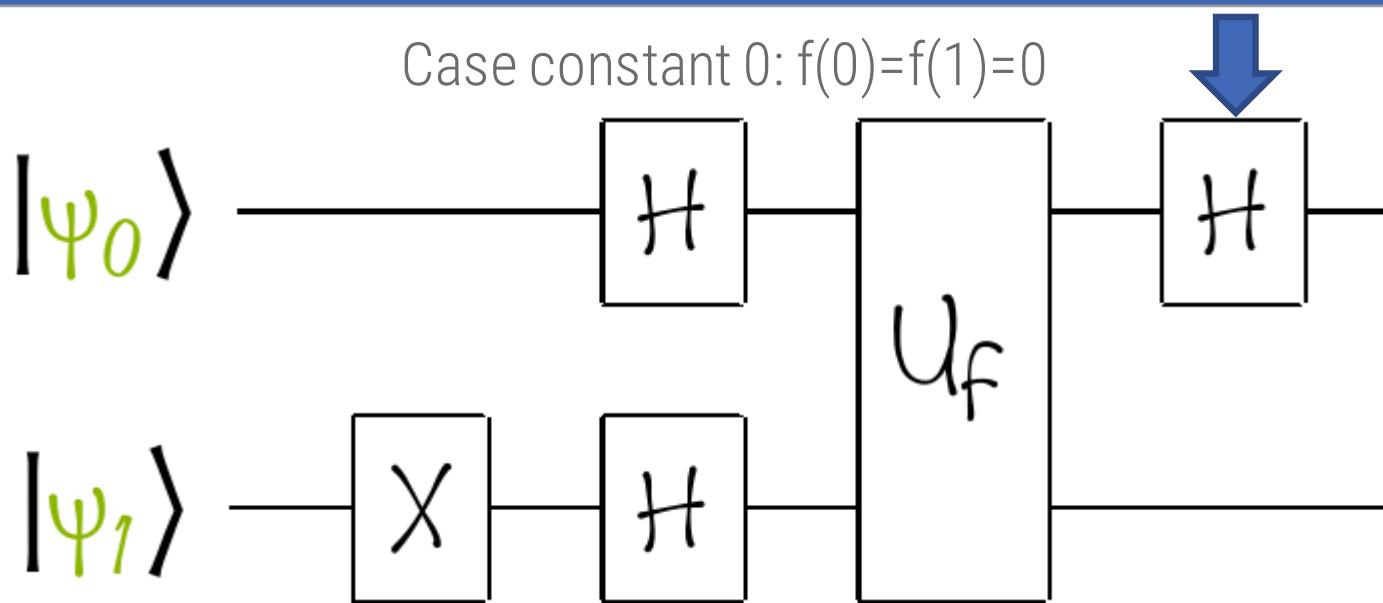
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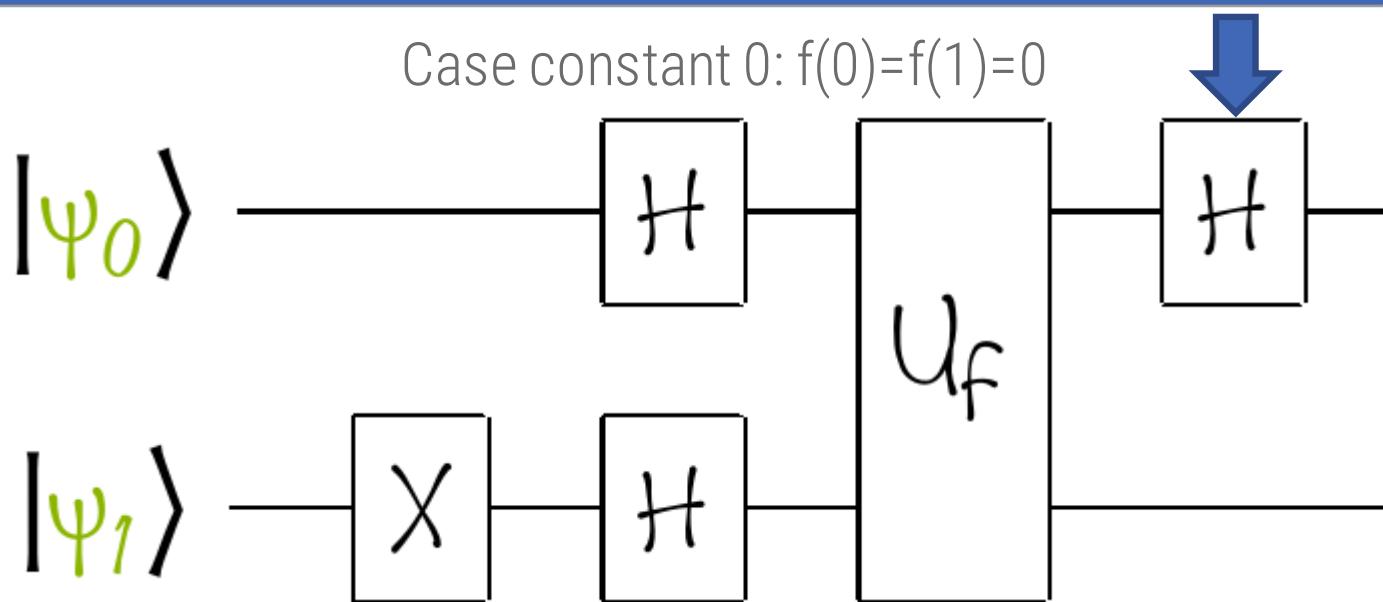
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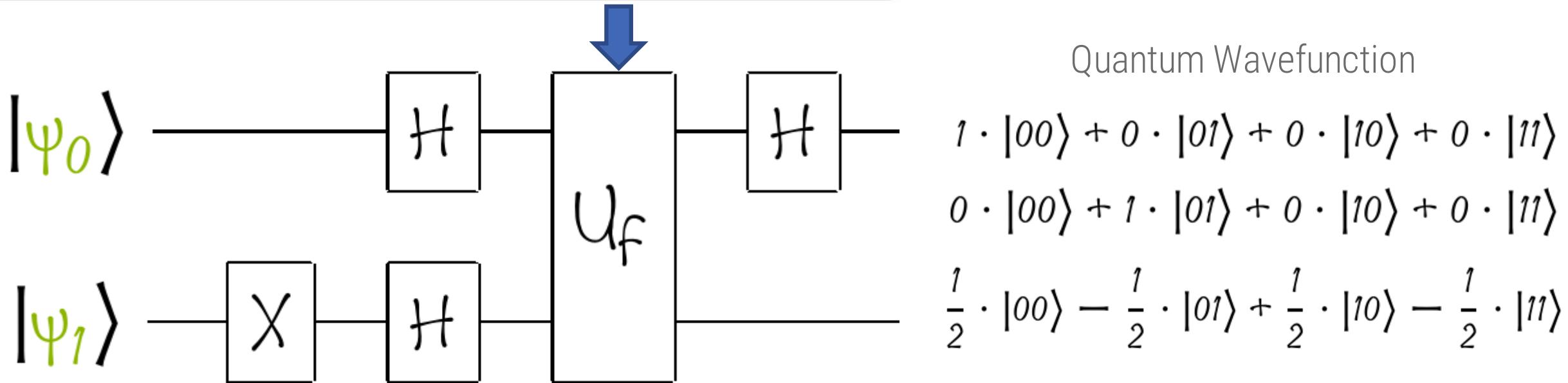
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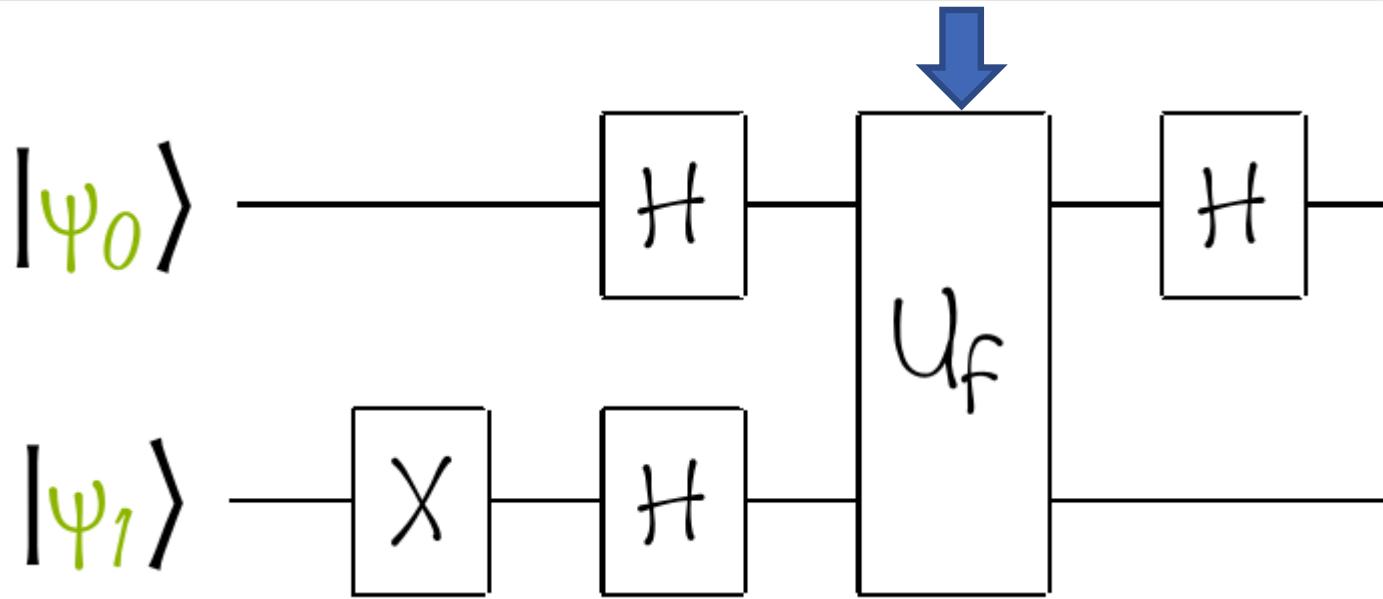
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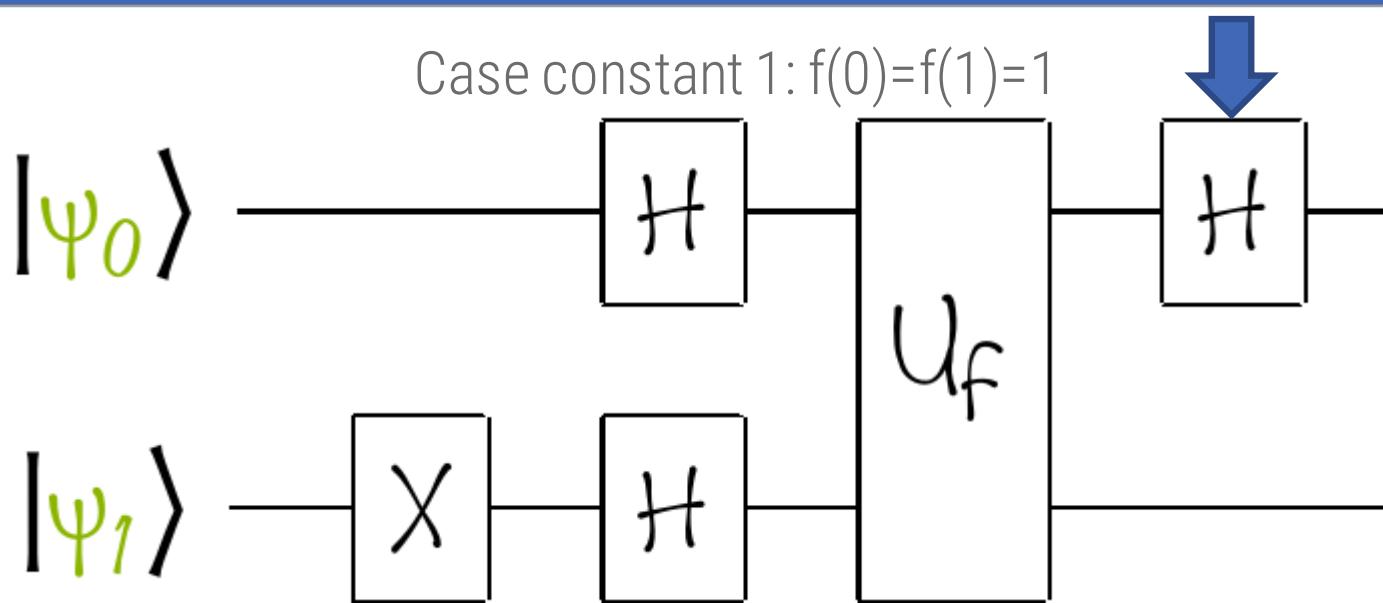
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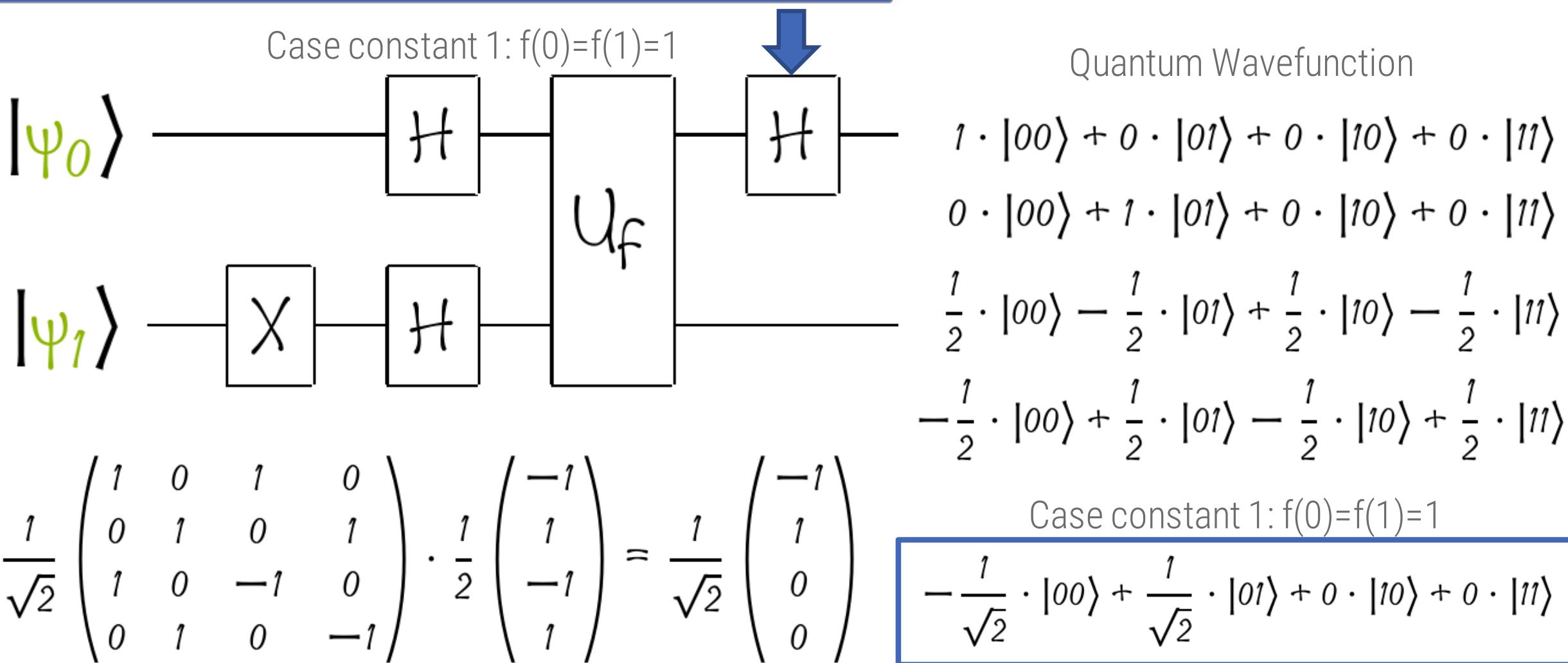
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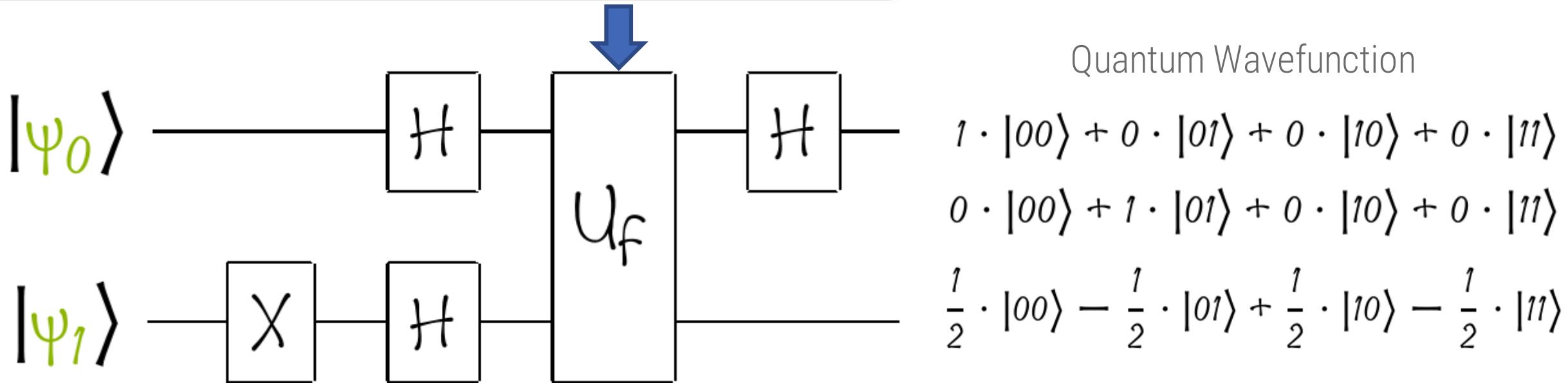
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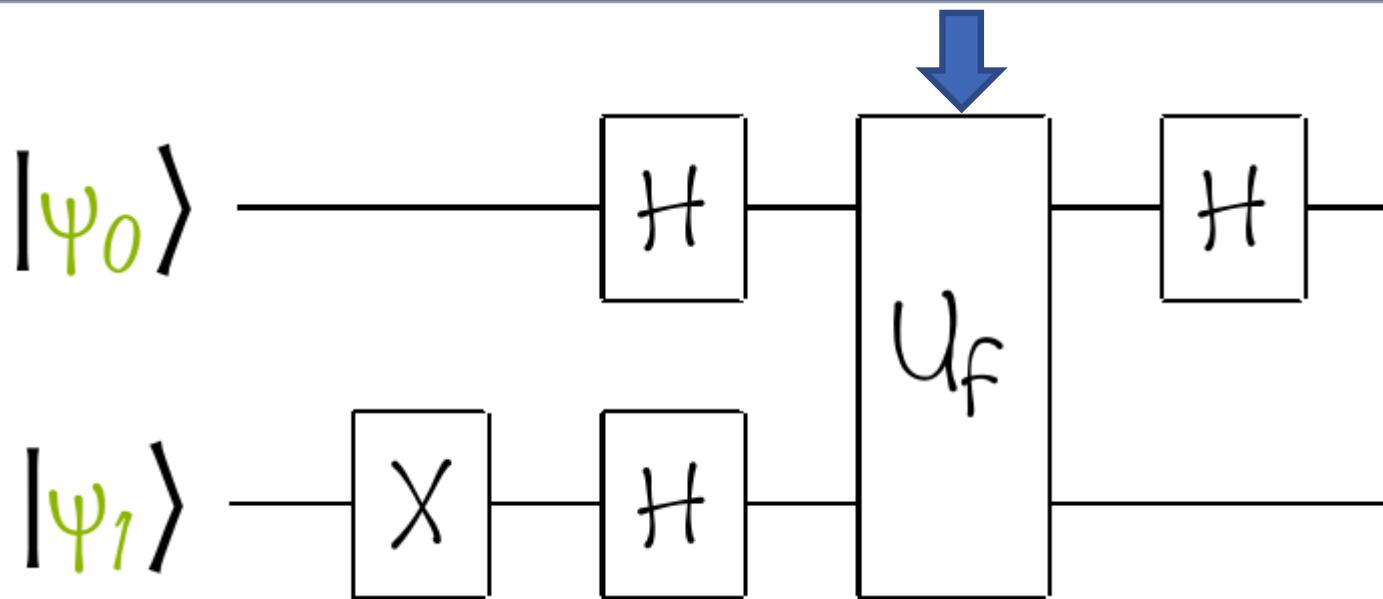
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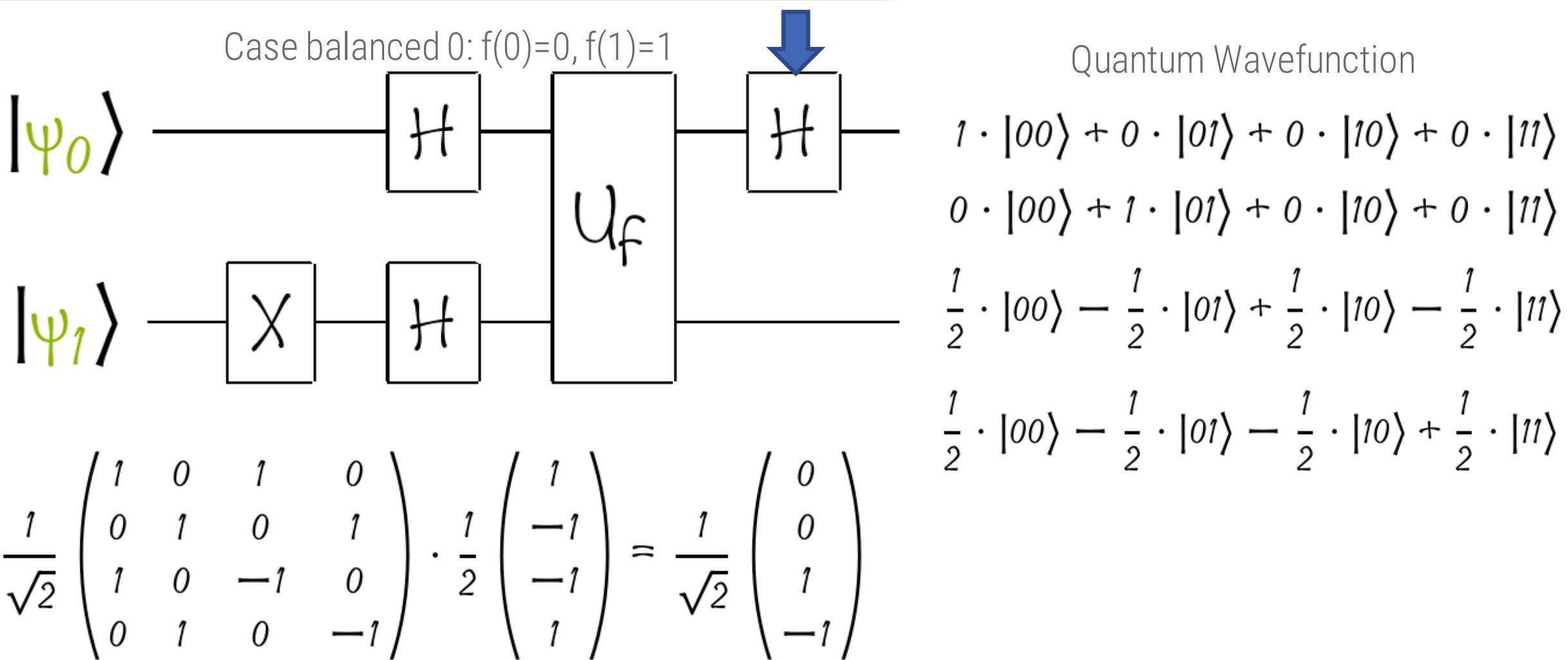
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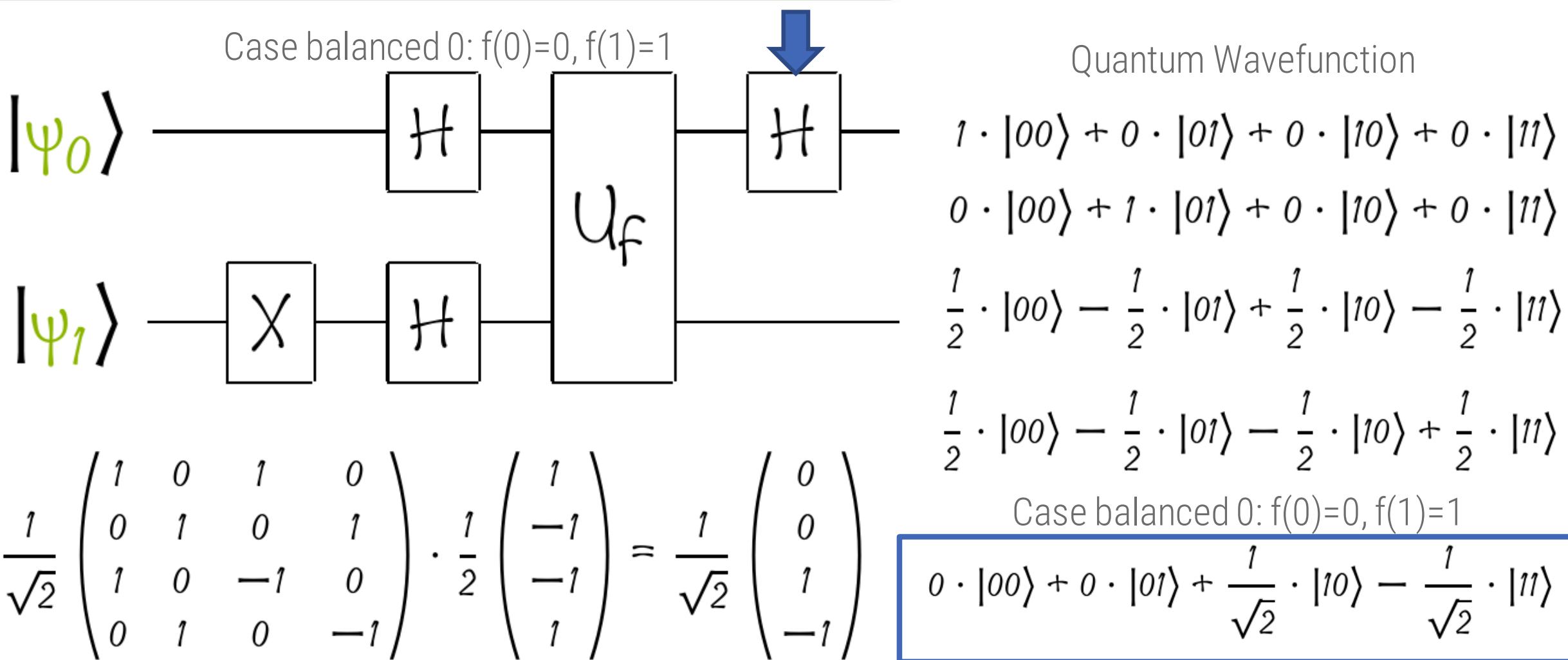
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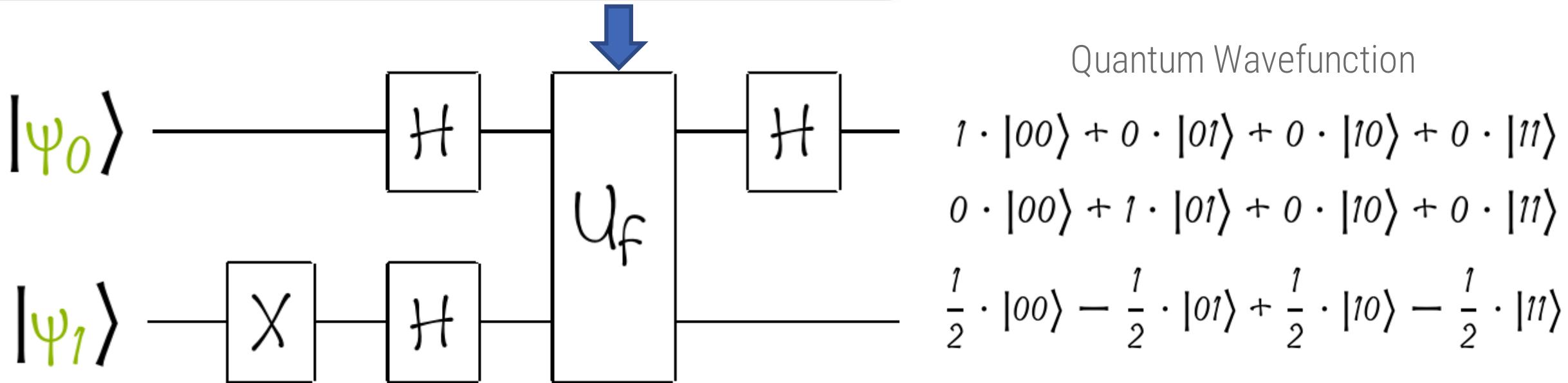
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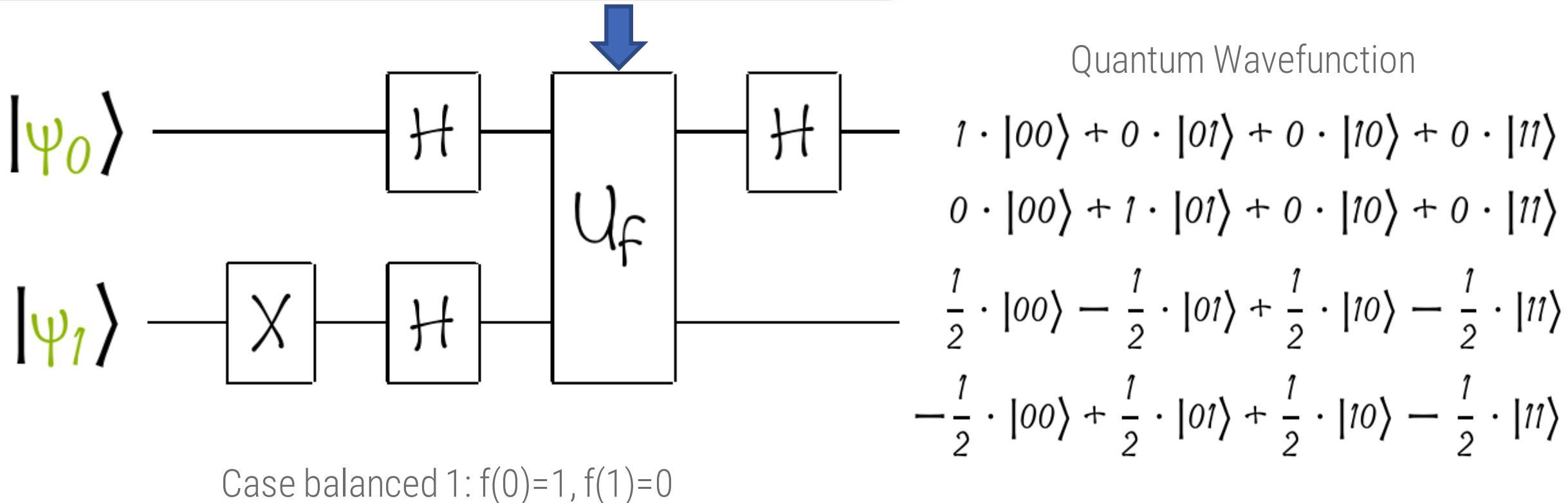
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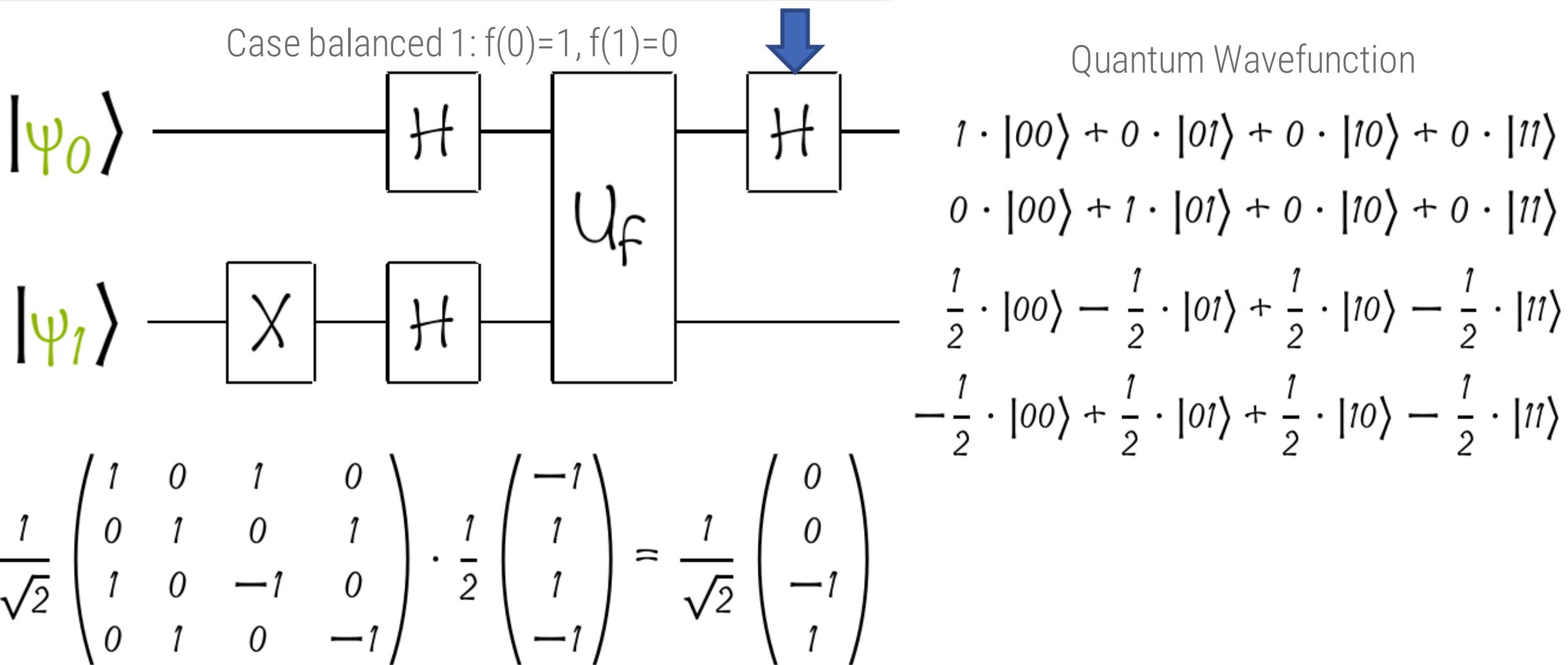


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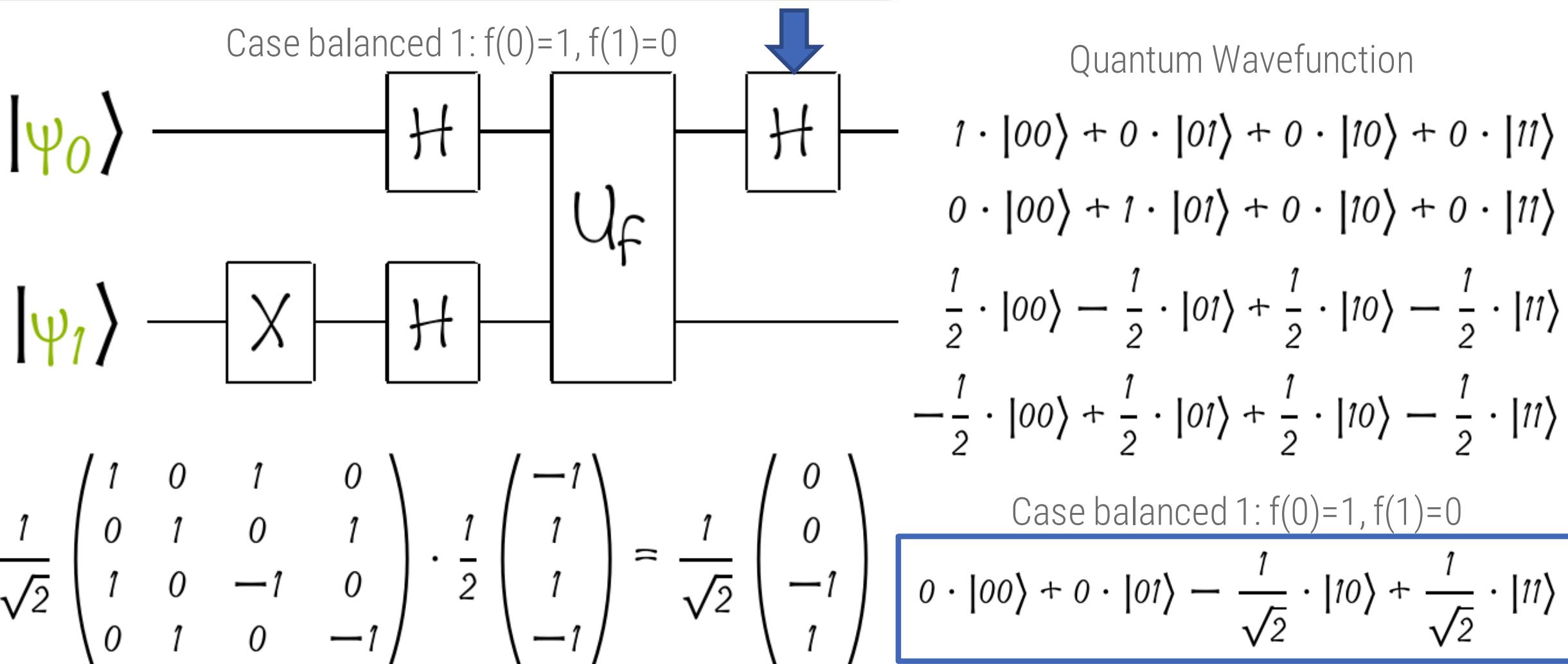
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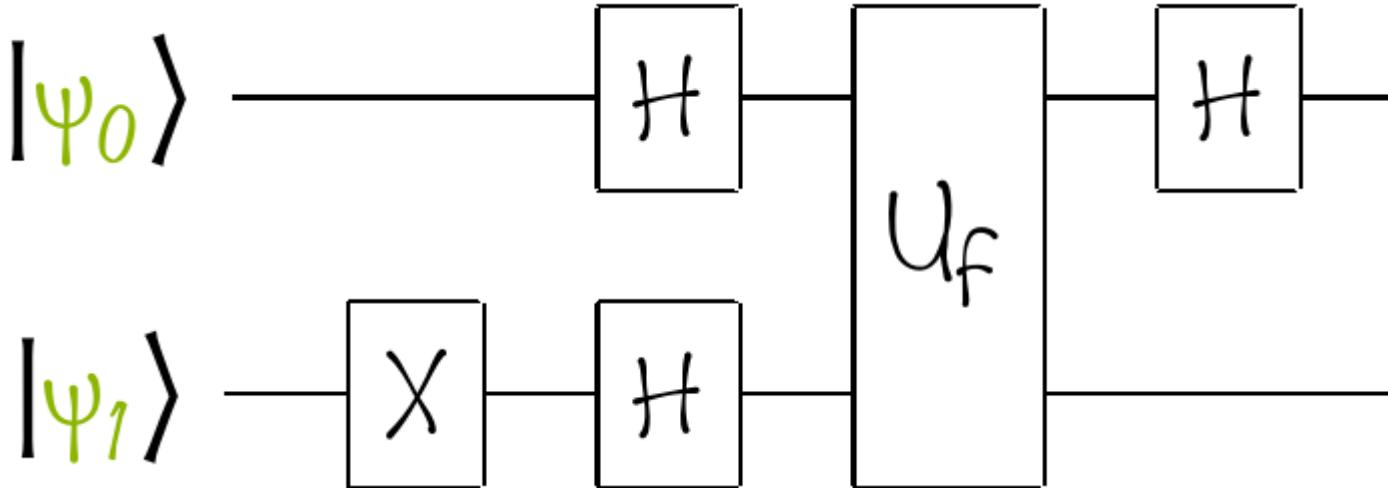
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Case constant 0:  $f(0)=f(1)=0$

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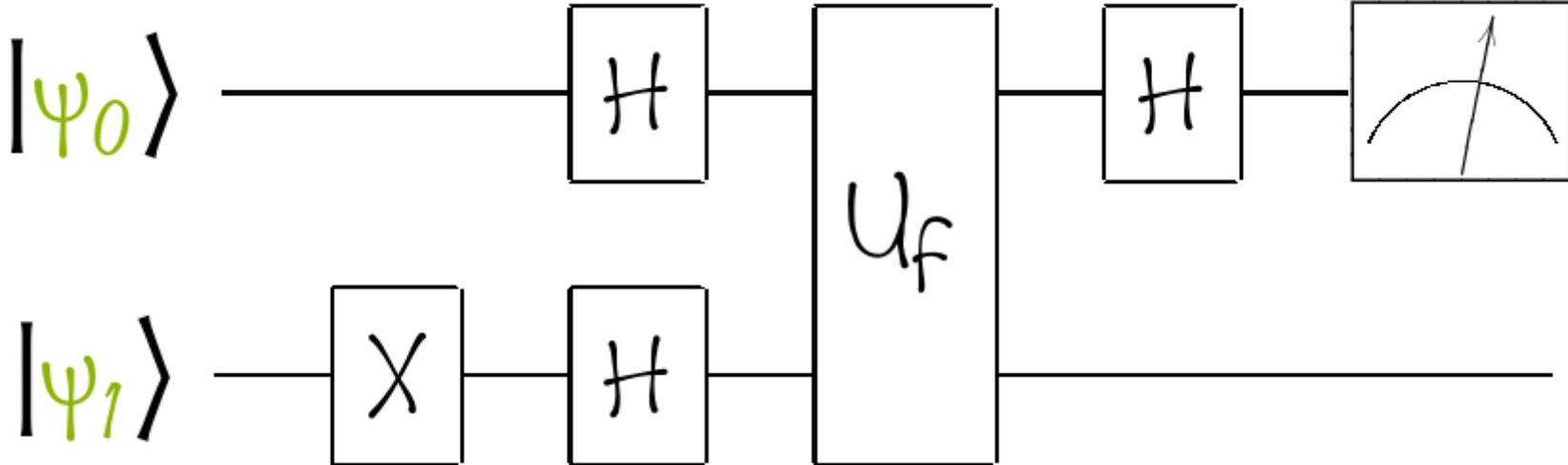
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Computational order of an algorithm: measures the number of operations required to solve a given problem. It is usually expressed as a function of the size of the problem to be solved (parameter N)

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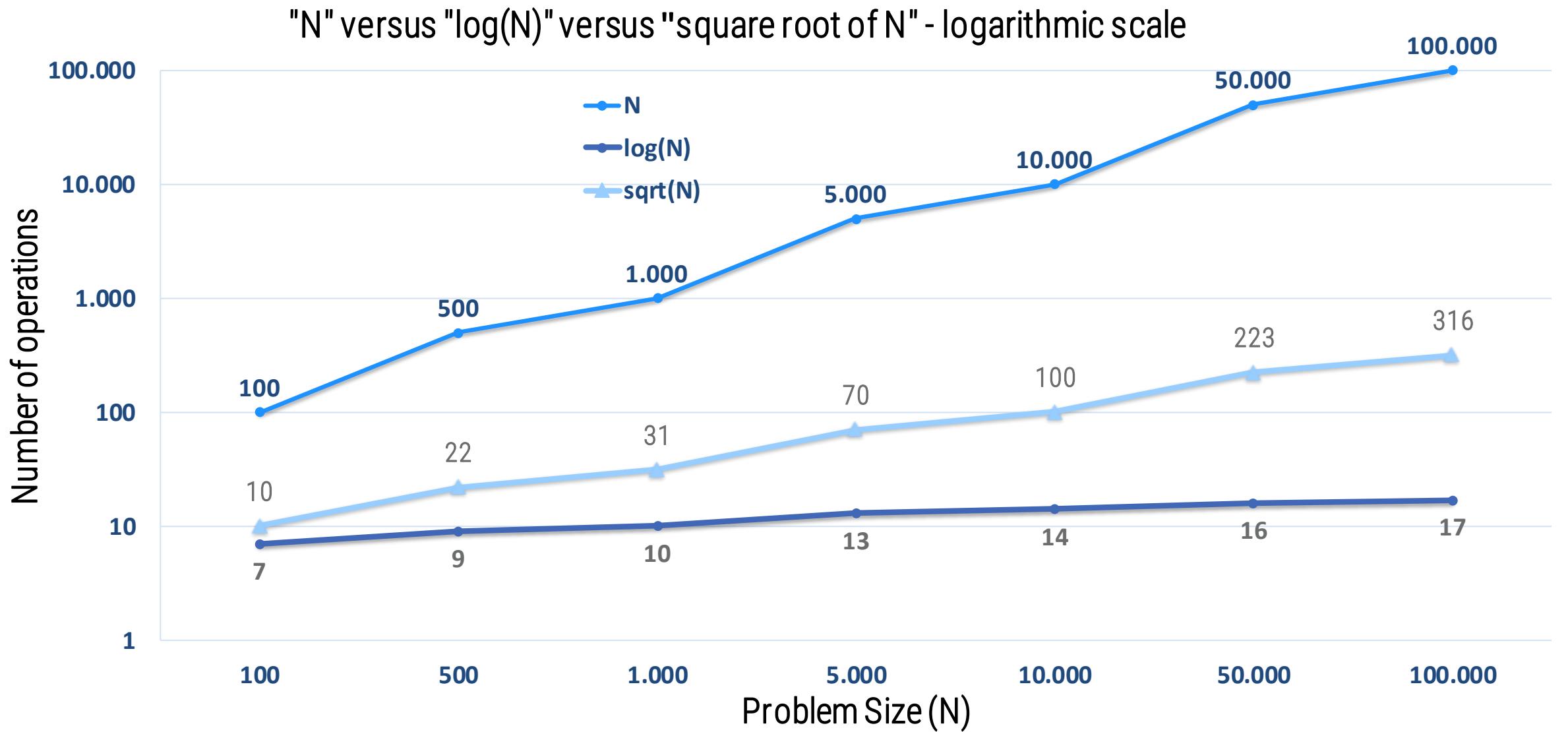
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Example: N to  $\sqrt{N}$

We move from one computational order to another exponentially smaller

Example: N to  $\log(N)$

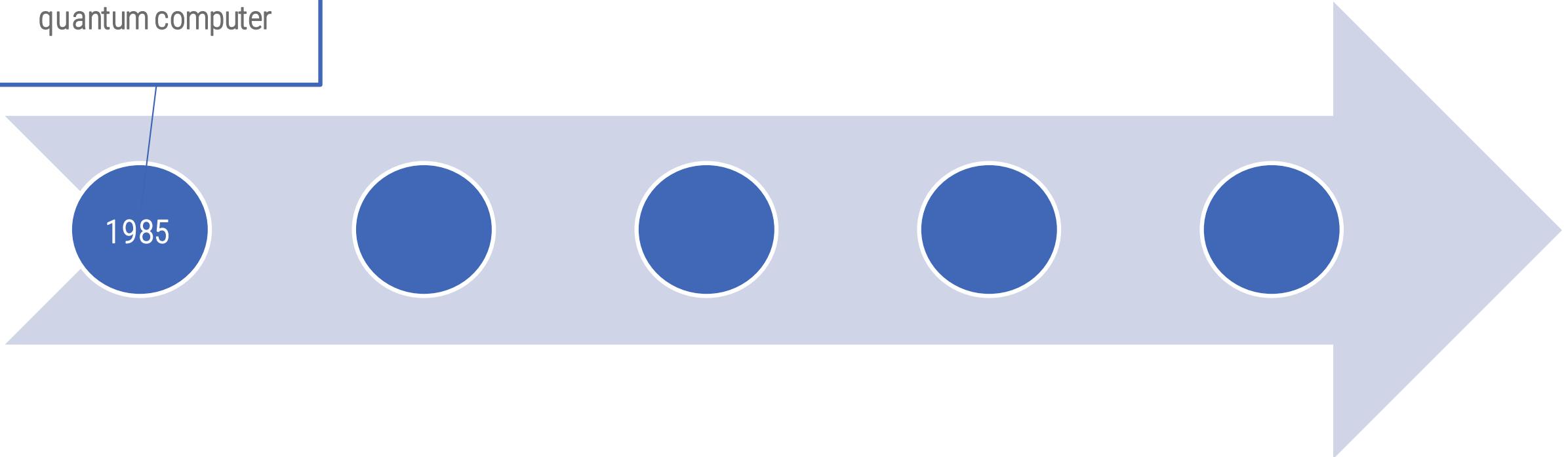
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# Brief Timeline of Quantum Algorithms

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HHL algorithm, for solving a system of linear equations

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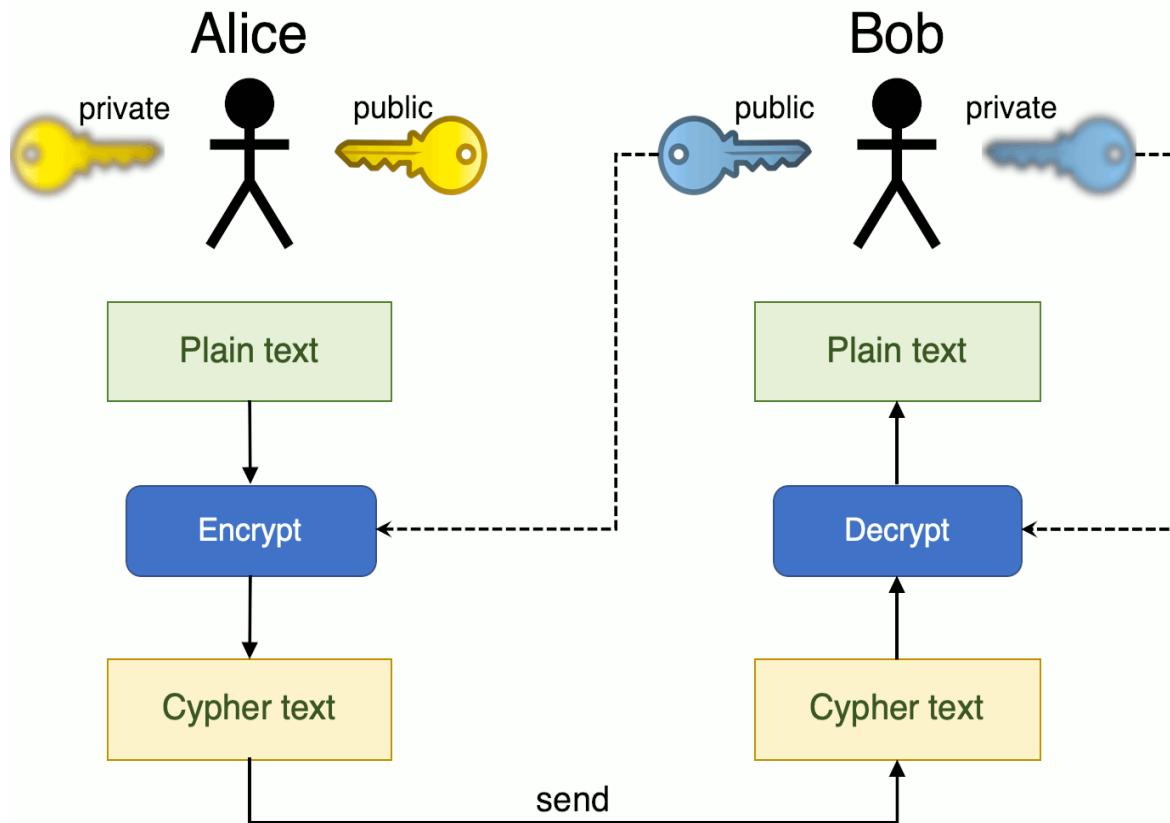
# The impact of QC in the world - Shor's algorithm

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Asymmetric encryption: how we protect our data

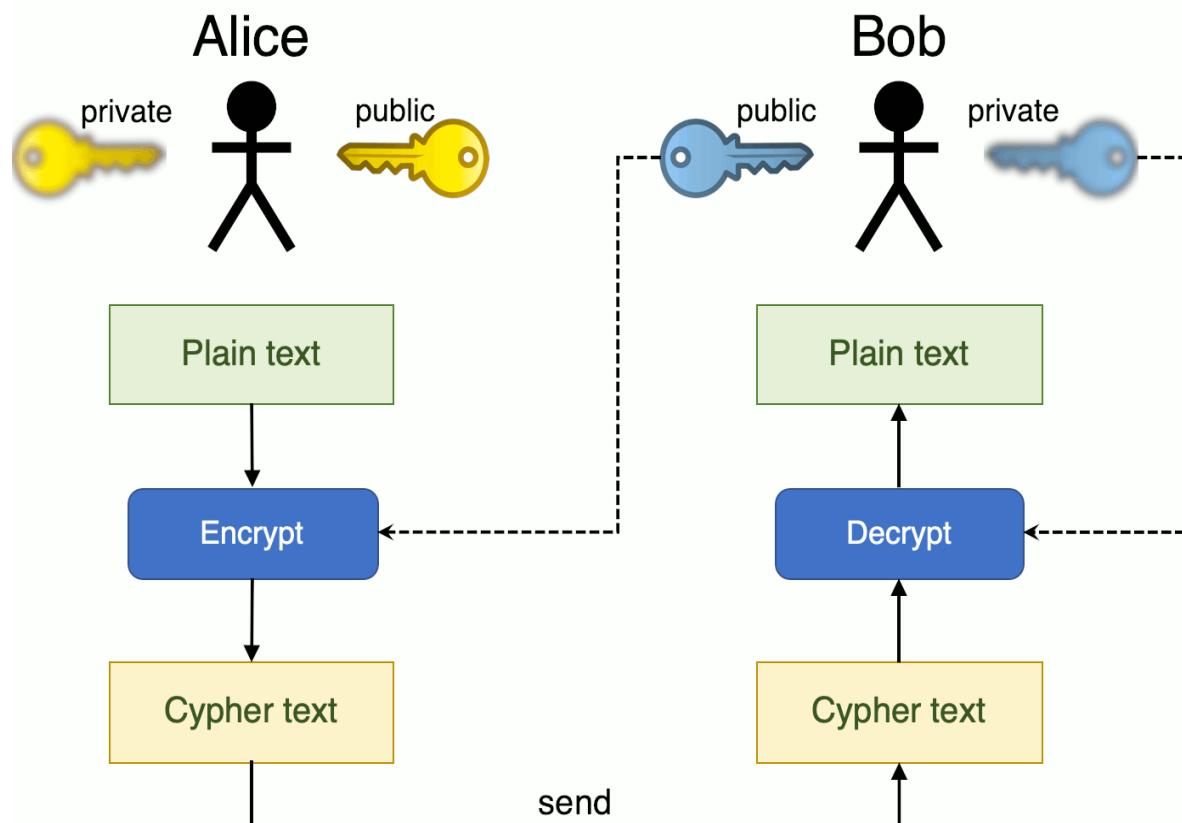
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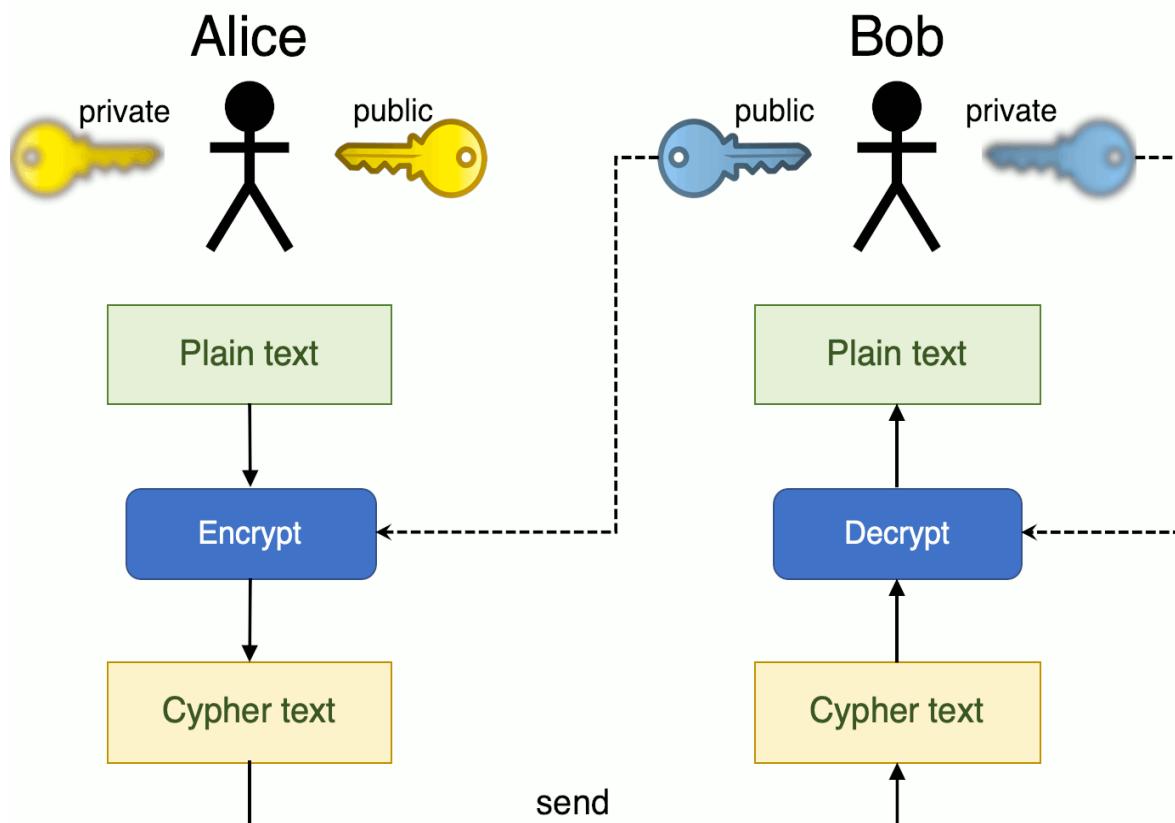
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# The impact of QC in the world - Shor's algorithm

## Asymmetric encryption: how we protect our data



### Public Key

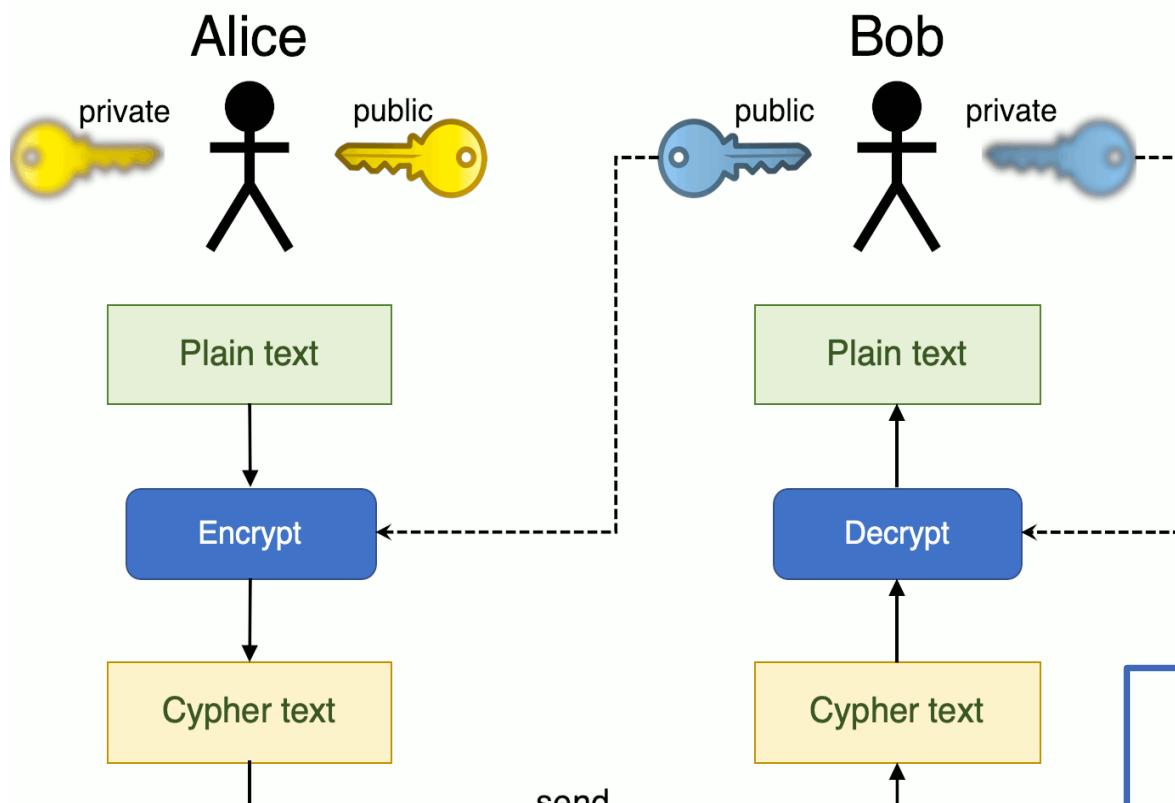
Known by everybody  
Used by the sender to encrypt a secret message

### Private Key

Known by the owner only  
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### Public Key

Known by everybody  
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Theoretically, it is possible to extrapolate the private key from the public key

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Asymmetric encryption: how we protect our data

In practice, it's not that simple

# The impact of QC in the world - Shor's algorithm

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Asymmetric encryption: how we protect our data

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The protection of the private key, in fact, is entrusted to a class of mathematical problems that are very difficult to solve

An example is given by the factoring of semi-prime numbers: given a number  $N$ , find two prime numbers  $p$  and  $q$  such that

$$N = p \times q$$

They are all problems of an exponential computational order

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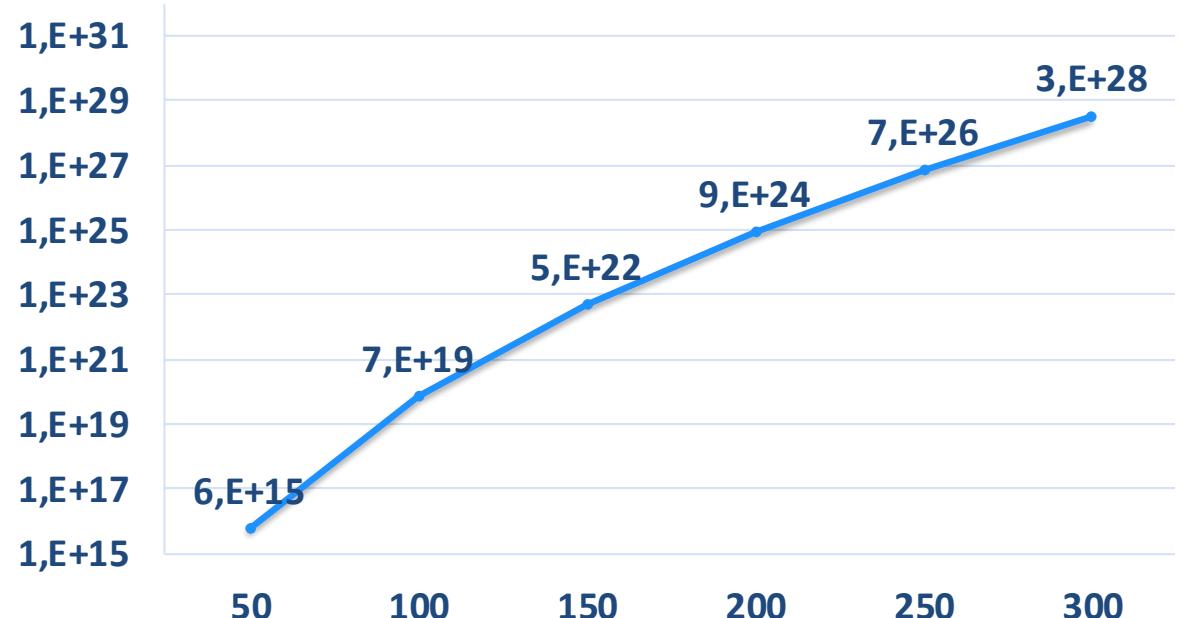
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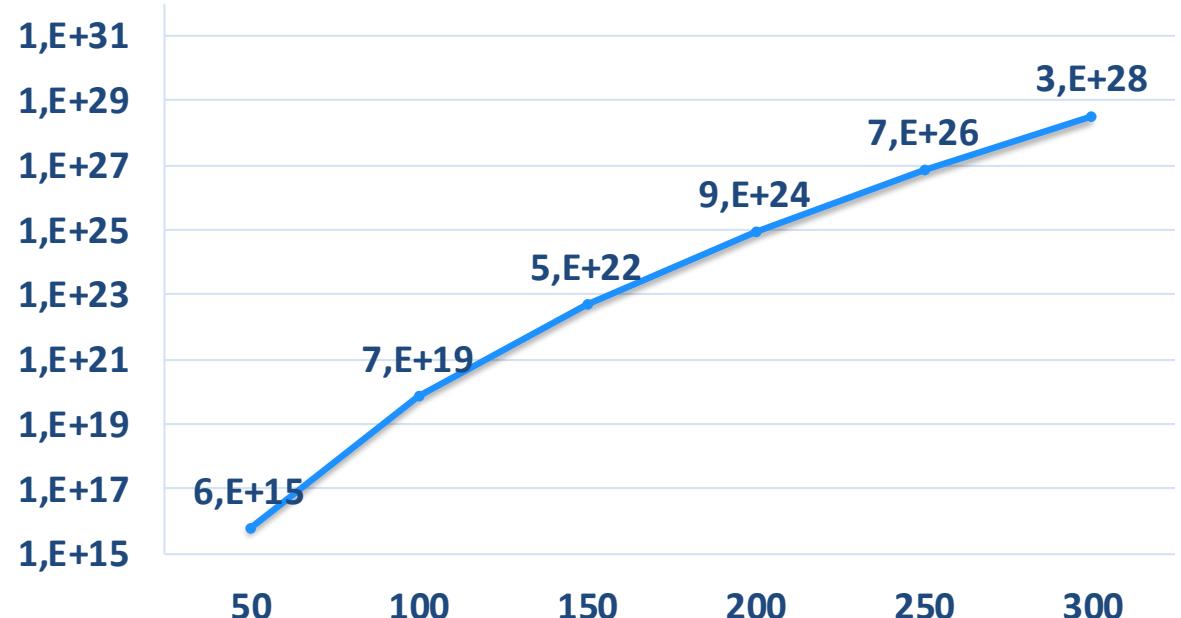
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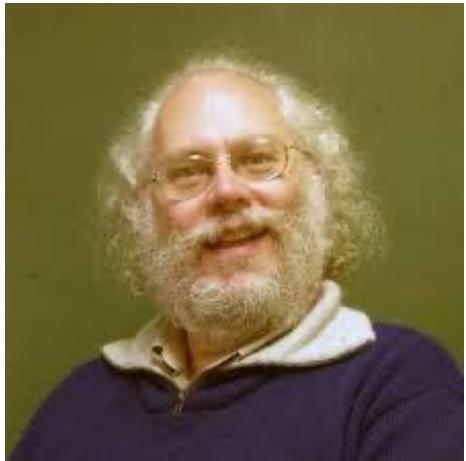
Summit, the most powerful supercomputer in the world, performs  $2E+17$  operations per second. Making  $3E+28$  would cost about 4,700 years

# The impact of QC in the world - Shor's algorithm

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## Asymmetric encryption: how we protect our data

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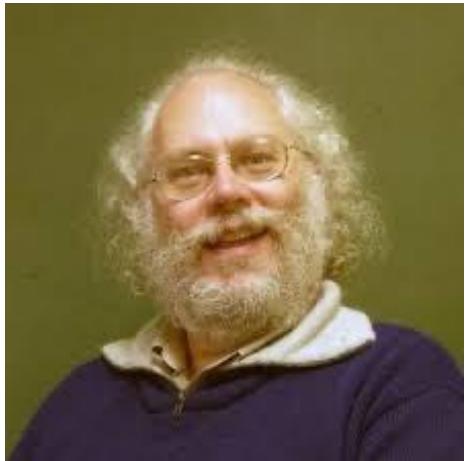
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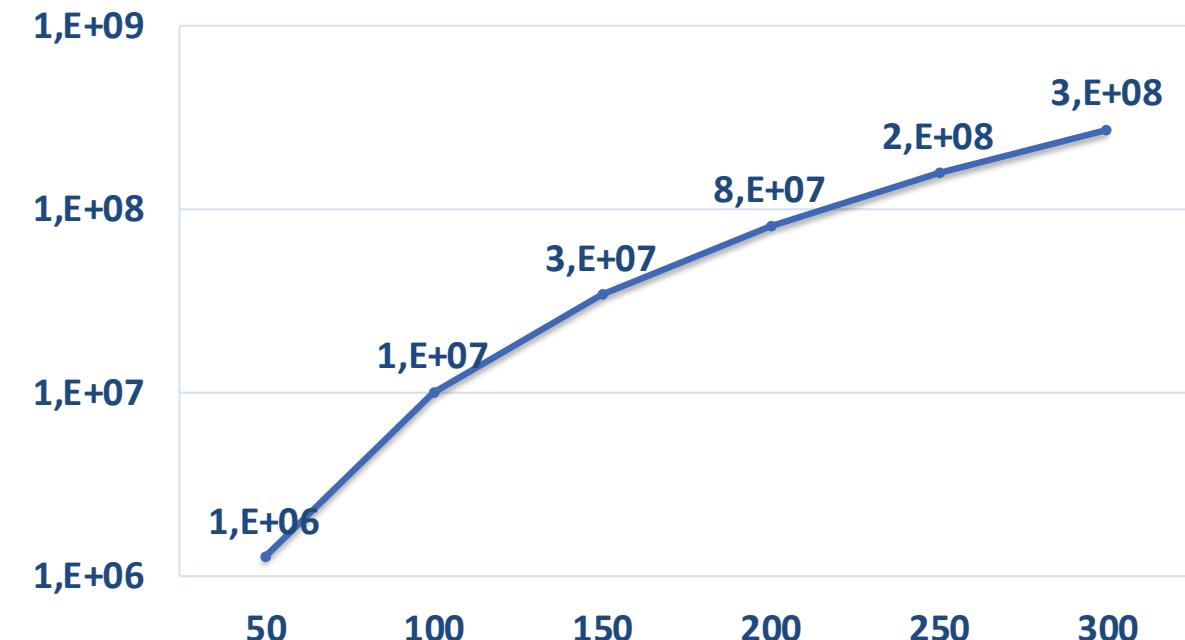
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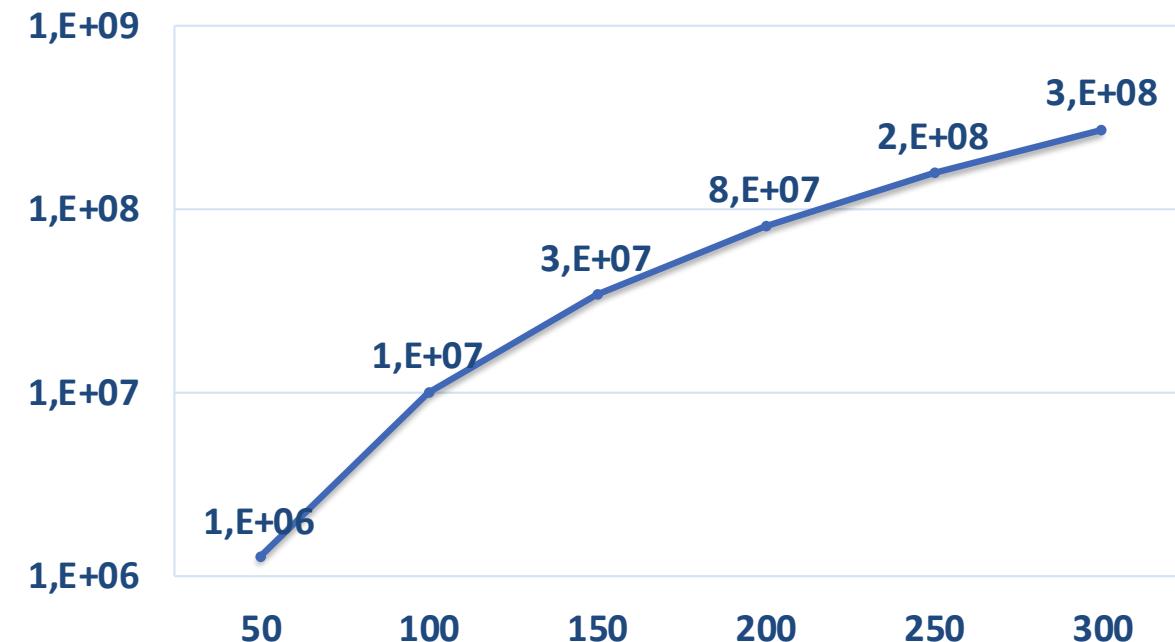
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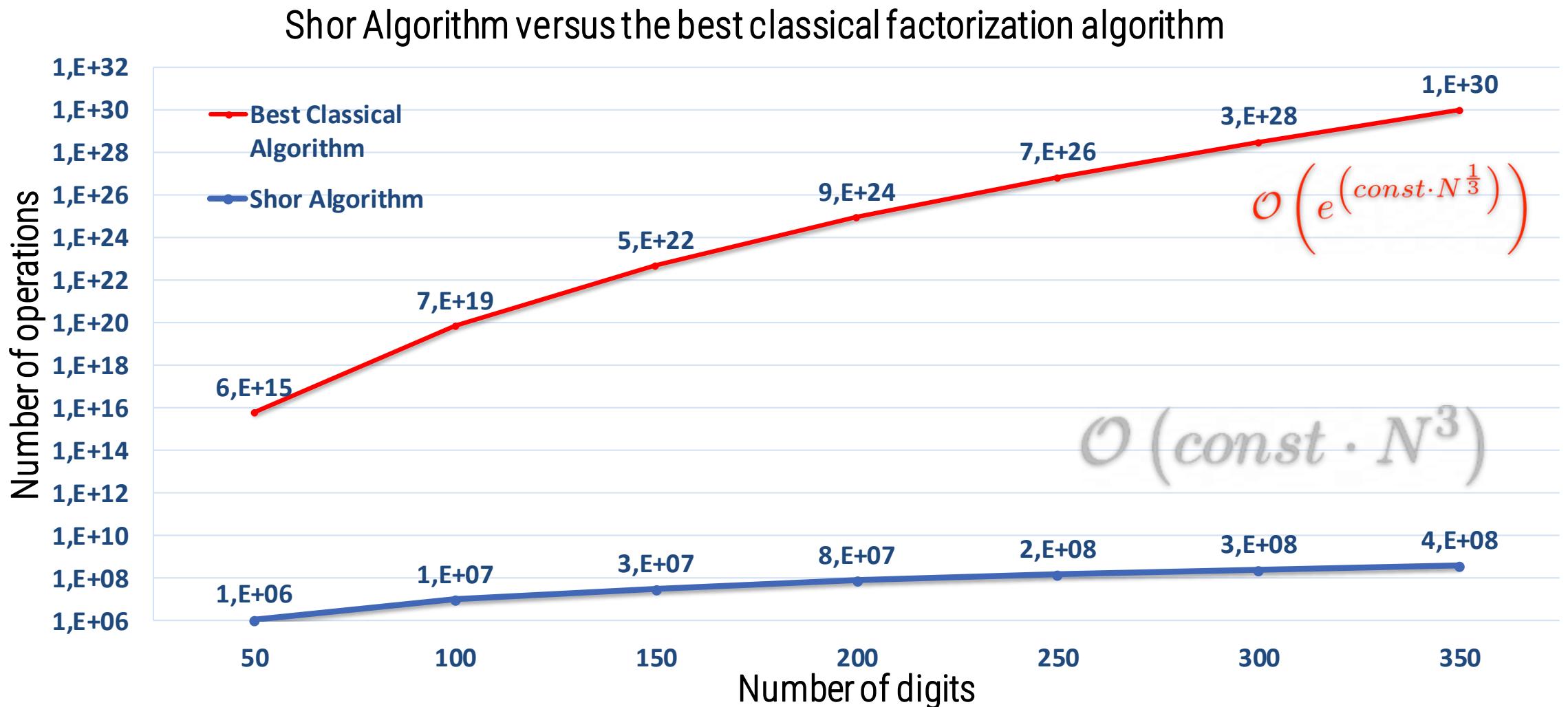
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If Summit could run Shor's algorithm, it would take just 1.5 nanoseconds to factor a number consisting of 300 digits

# The impact of QC in the world - Shor's algorithm

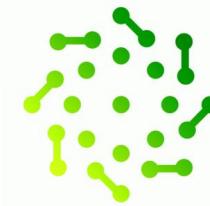


# Quantum computers: State of the Art

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- So far we have seen how quantum computing works mathematically.

IBM Q



PASQAL

rigetti

A 3D-style hexagon with a central white circle containing a blue 'Q' shape, with orange and grey gradients on the sides.

IONQ

D-WAVE  
The Quantum Computing Company™



XANADU

# Quantum computers: State of the Art

---

- So far we have seen how quantum computing works mathematically.
- We have discovered that it is possible to exploit certain phenomena, in particular quantum parallelism, to build quantum algorithms capable of solving some types of problems exponentially faster.

IBM Q



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- We then briefly mentioned the existence of other very powerful quantum algorithms, such as the Shor algorithm

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  - The example of Deutsch's algorithm made us begin to understand the potential of quantum computers
  - We then briefly mentioned the existence of other very powerful quantum algorithms, such as the Shor algorithm
  - A question now arises: why today's quantum computers are still not used to implement the most powerful algorithms?
- 

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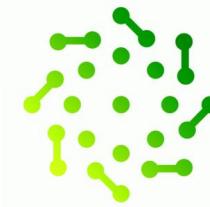


# Quantum computers: State of the Art

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- Short answer: today's quantum computers are not advanced enough. They are much more like advanced prototypes than production machines

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The IONQ logo features a stylized orange and grey hexagonal symbol followed by the word "IONQ" in a bold, sans-serif font.

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  - As you can imagine, building a quantum computer is not an easy task. There are many problems to overcome.
  - Let's see how to get an idea of the actual power of a quantum computer
- 

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# Building a Quantum Computer

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  - Few for a universal quantum computer, close in number for other applications
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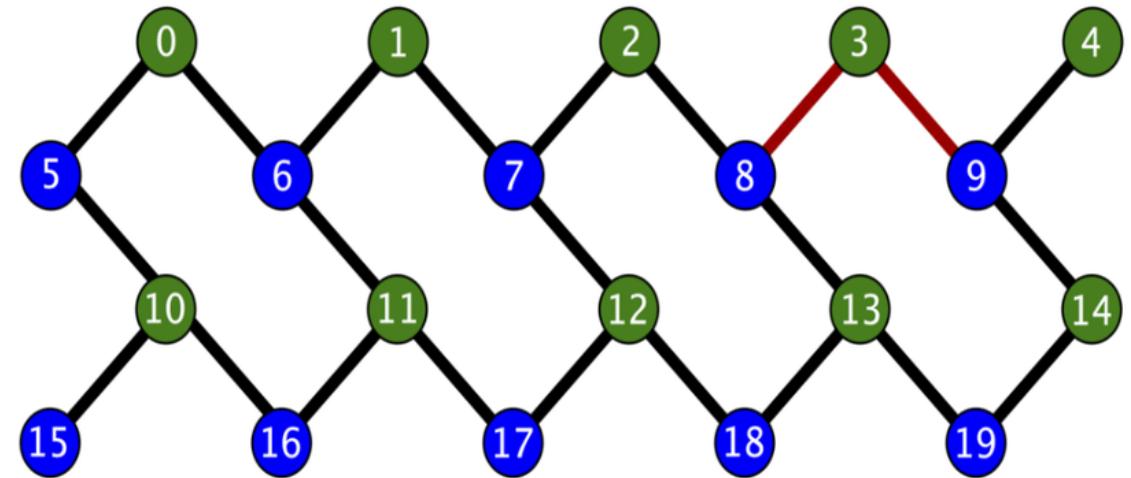
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- As we will see shortly, so far the most powerful General Purpose quantum computer models do not exceed one hundred qubits
- Few for a universal quantum computer, close in number for other applications
- Very important thing: bigger and bigger models keep coming out. The next milestone, which seems to apply to everyone, is to reach a thousand qubits by 2023
- Number of Qubits

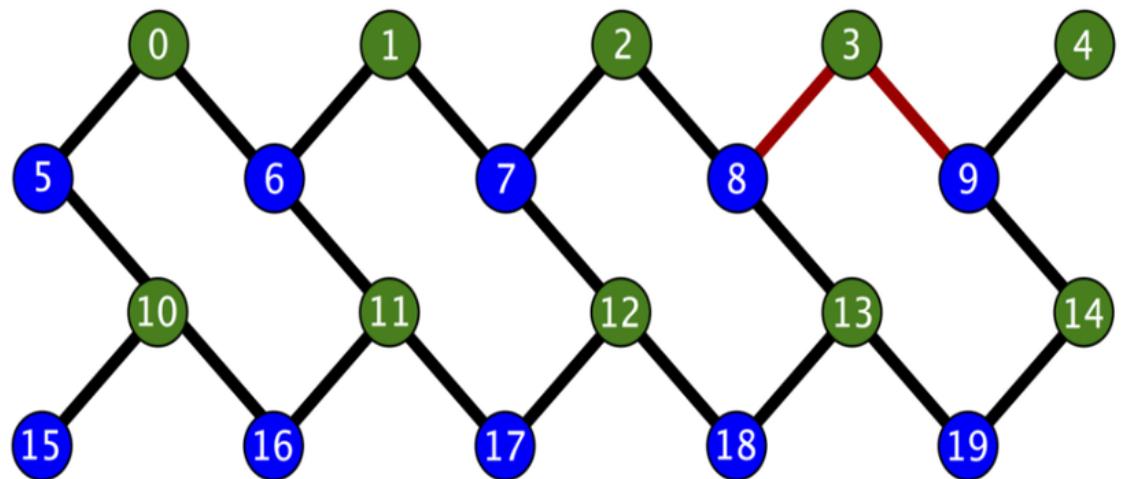
# Building a Quantum Computer

- The second thing we need to observe to evaluate a quantum computer is the connectivity between its qubits.
- Number of Qubits
- Connectivity of Qubits



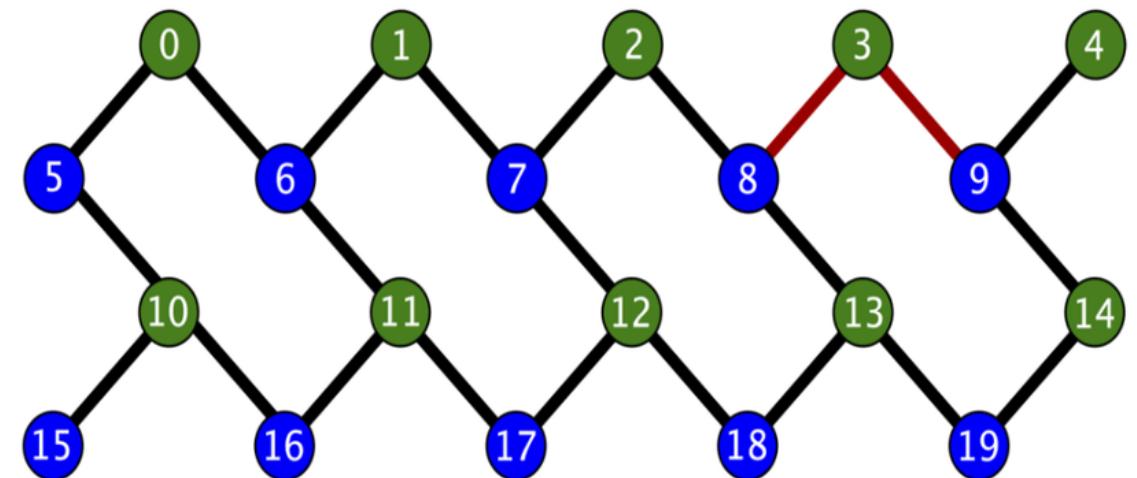
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- Number of Qubits
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# Building a Quantum Computer

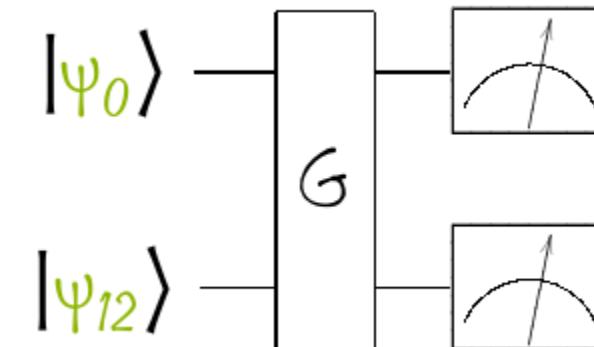
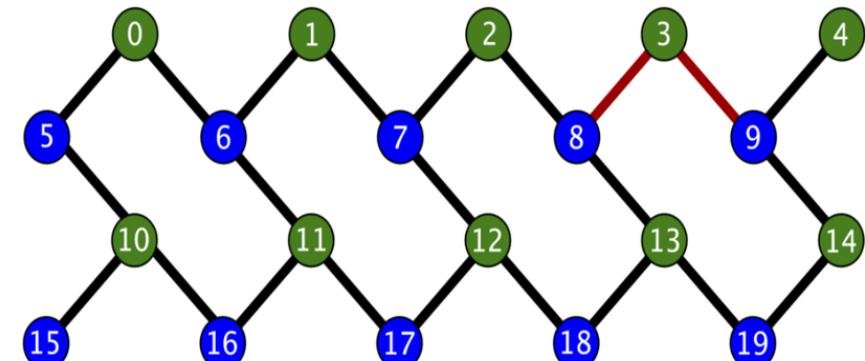
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- Number of Qubits
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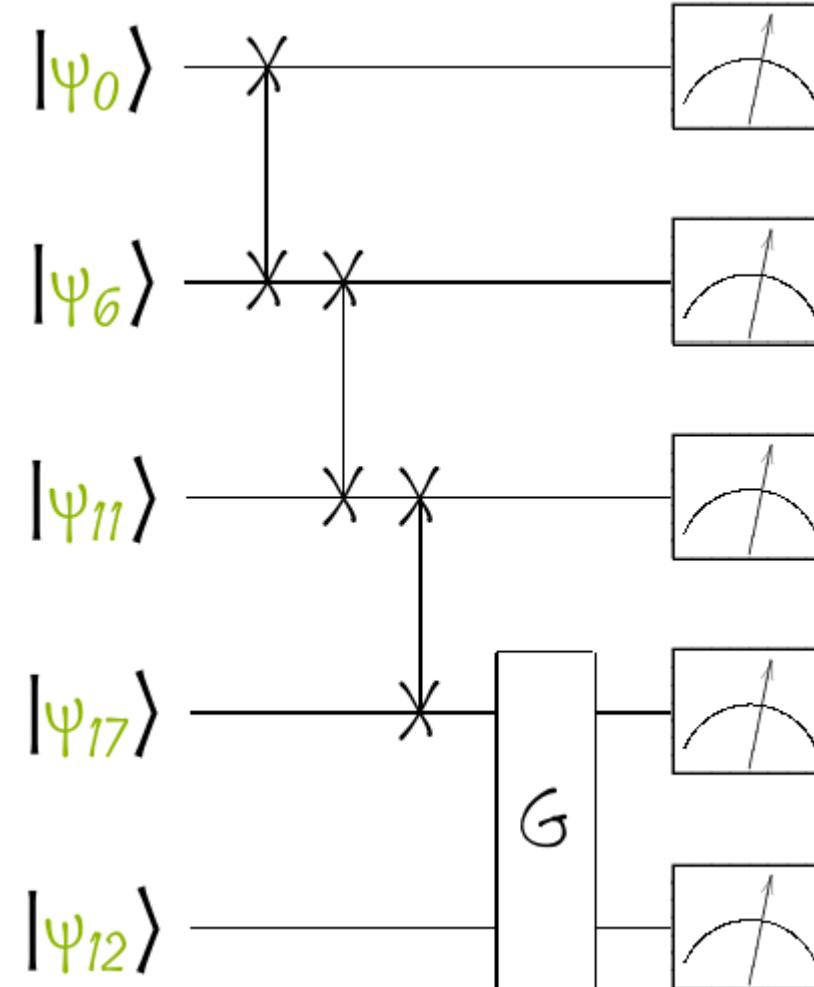
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  - Number of Qubits
  - Connectivity of Qubits
  - Coherence Time

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- Quantum superposition states are extremely fragile states.
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  - In order to be maintained for a sufficiently long time it is necessary that the quantum computer is protected from practically any form of noise
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  - Even the qubits themselves contribute to producing noise capable of destroying quantum states: this is one of the reasons why it is difficult to make large chipsets.
- Number of Qubits
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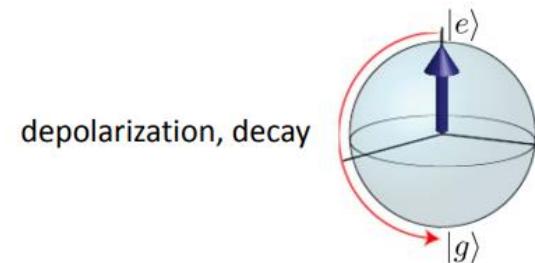
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- How can the problem be addressed? In addition to isolating the QPU from the outside world as much as possible, it is necessary to improve the technology of the qubits themselves
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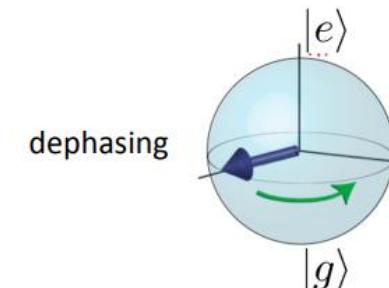
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- Number of Qubits
- Connectivity of Qubits
- Coherence Time
  - $T_1$ : energy relaxation time



perturbation orthogonal to quantization axis ( $\propto \sigma_{x,y}$ ); e.g. fast charge fluctuations causing transitions

- $T_2$ : dephasing time



slow perturbation along quantization axis ( $\propto \sigma_z$ ); e.g. magnetic flux noise causing phase randomization

# Building a Quantum Computer

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- The fourth thing to consider is the application of quantum gates to qubits circuits.
- Number of Qubits
- Connectivity of Qubits
- Coherence Time
- Operations on qubits

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- Several techniques are used to physically build a quantum gate
  - Number of Qubits
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- The fourth thing to consider is the application of quantum gates to qubits circuits.
- Several techniques are used to physically build a quantum gate
- What must be taken into account when thinking about a real quantum computer is that not all quantum gates may be available for a certain hardware.
  - Number of Qubits
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  - Coherence Time
  - Operations on qubits
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- Number of Qubits
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- Several techniques are used to physically build a quantum gate
- What must be taken into account when thinking about a real quantum computer is that not all quantum gates may be available for a certain hardware.
- In addition, they have an application time that affects the maximum time available to complete a circuit
- And remember: the time available to complete a quantum circuit is not infinite, quite the opposite. The quantum computer works as long as it can maintain its superposition state
- Number of Qubits
- Connectivity of Qubits
- Coherence Time
- Operations on qubits
  - Not every gate is available
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# Building a Quantum Computer

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- Last but not least, the applications of quantum gates are not free from errors
  - Number of Qubits
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    - Errors on applications (gate fidelity)

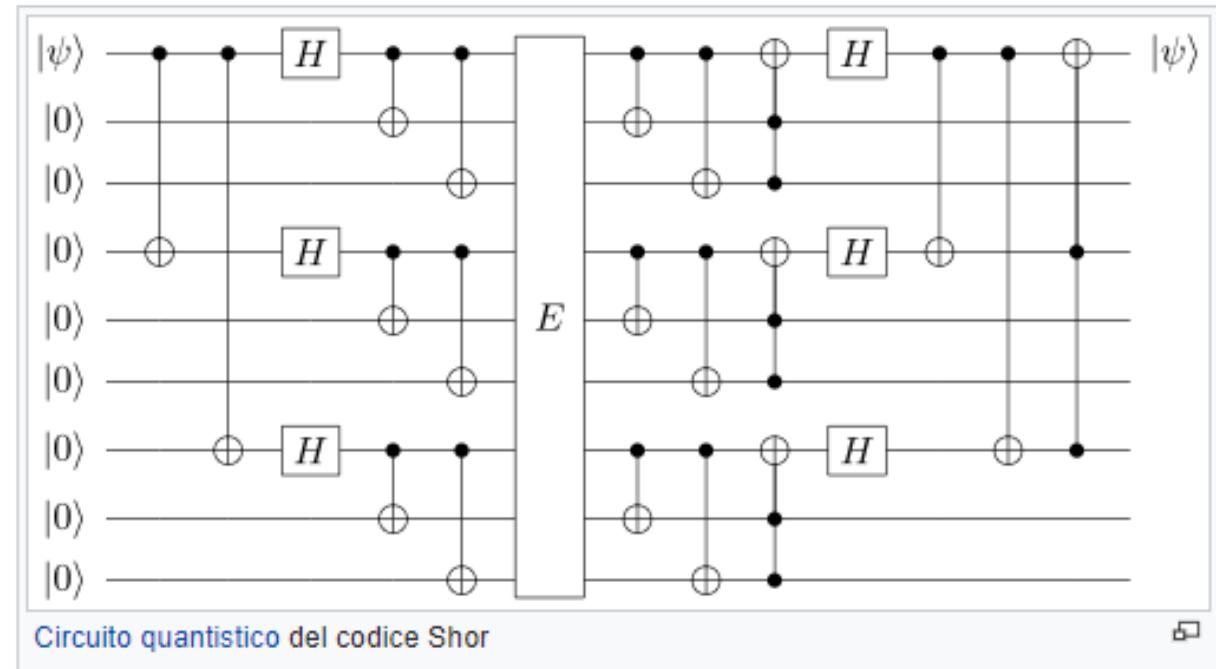
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  - This is a very important problem: to have a reliable quantum computer, the error percentage on the application of the gates must be zero
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- This is a very important problem: to have a reliable quantum computer, the error percentage on the application of the gates must be zero
- This is not true in reality, unfortunately. There are error mitigation techniques but they require an exaggerated number of additional qubits per hour (example of Shor's error correction code)



# Building a Quantum Computer

- On May 14, 2021, Google made public the technical characteristics of its latest quantum computer, Sycamore

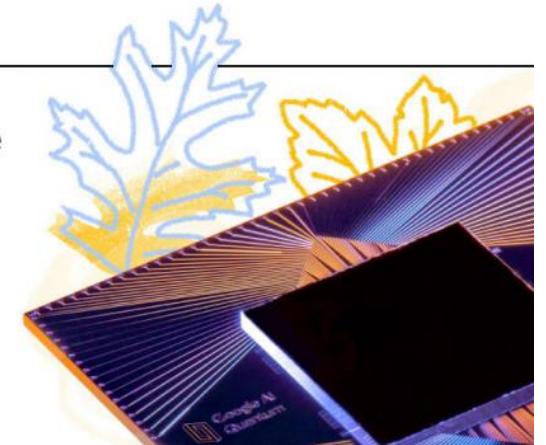


## Quantum Computer Datasheet

Published May 14, 2021

### Description

At the core of Google's Quantum Computing Service is the Sycamore processor. It has up to 54 superconducting qubits in a square grid lattice suitable for general Noisy Intermediate Scale Quantum (NISQ) algorithms like Hartree-Fock (chemistry), QAOA (optimization), and machine learning. Standard single- and two-qubit gates are



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Quantum AI



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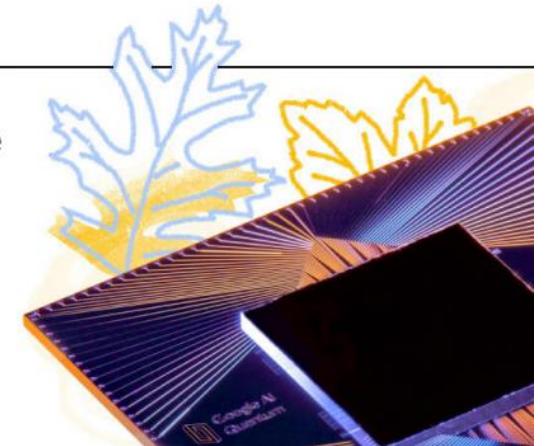


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- Let's take a look at the power of Sycamore

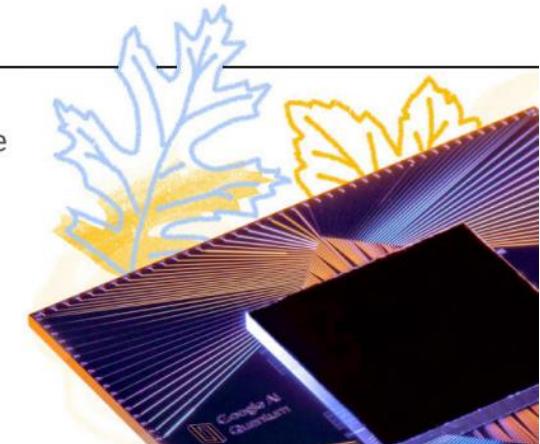


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# Building a Quantum Computer

## Weber Quantum Computer

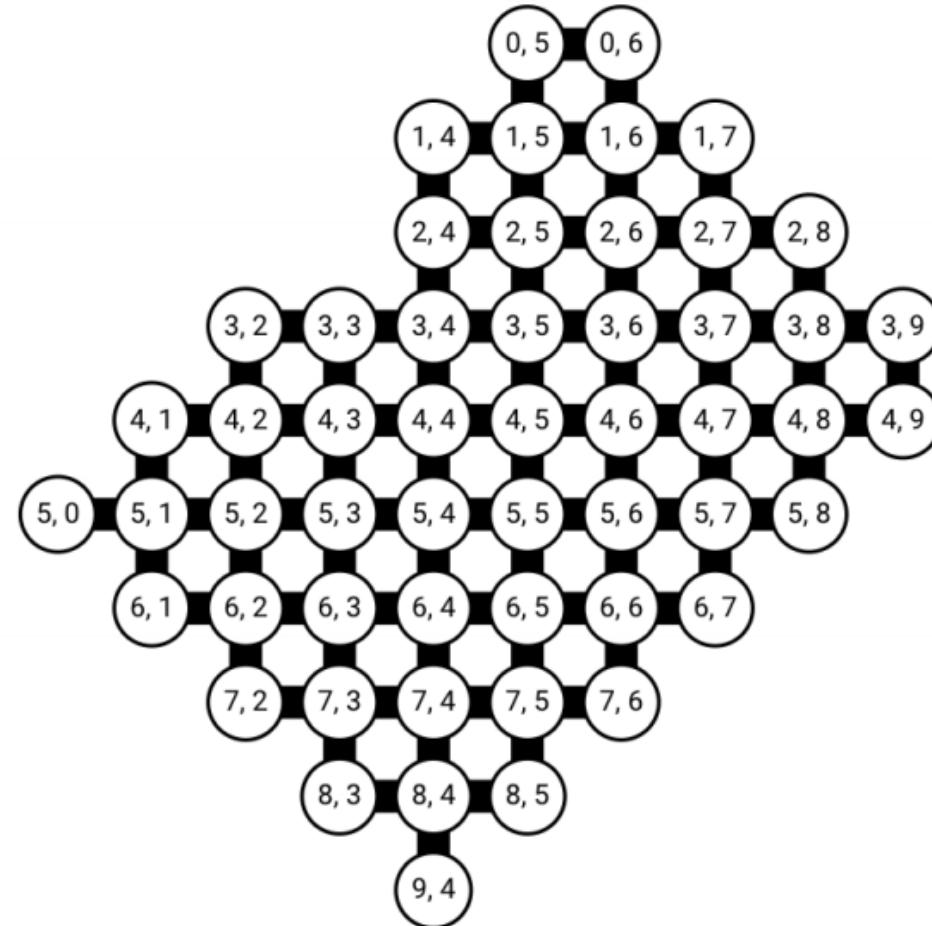


### Qubit Grid

processor_id	Weber
Family	Sycamore
Supported two-qubit gates	$\sqrt{\text{SWAP}}$ , Sycamore
Number of qubits in the grid	53

### Qubit Layout

Example grid for Weber. [Use QCS Console](#) for up-to-date layout.



# Building a Quantum Computer

## Qubit Operations

Type	Gate	Duration <sup>1</sup>	Matrix
Single Qubit Gates	Phased XZ	25 ns	$\begin{bmatrix} \cos(\pi x/2) & -i\sin(\pi x/2)e^{i\pi a} \\ -i\sin(\pi x/2)e^{i\pi(a+z)} & \cos(\pi x/2)e^{i\pi z} \end{bmatrix}$
	Virtual Z	0 ns	$\begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi t} \end{bmatrix}$
	Physical Z	20 ns	$\begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi t} \end{bmatrix}$
Two Qubit Gates	Sycamore	12 ns	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & -i & 0 & 0 \\ 0 & 0 & 0 & e^{-i\pi/6} \end{bmatrix}$
	$\sqrt{i}\text{SWAP}$	32 ns	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \pm 1/\sqrt{2} & \pm i/\sqrt{2} & 0 \\ 0 & \pm i/\sqrt{2} & \pm 1/\sqrt{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
	CZ	In development	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$

# Building a Quantum Computer

## Performance

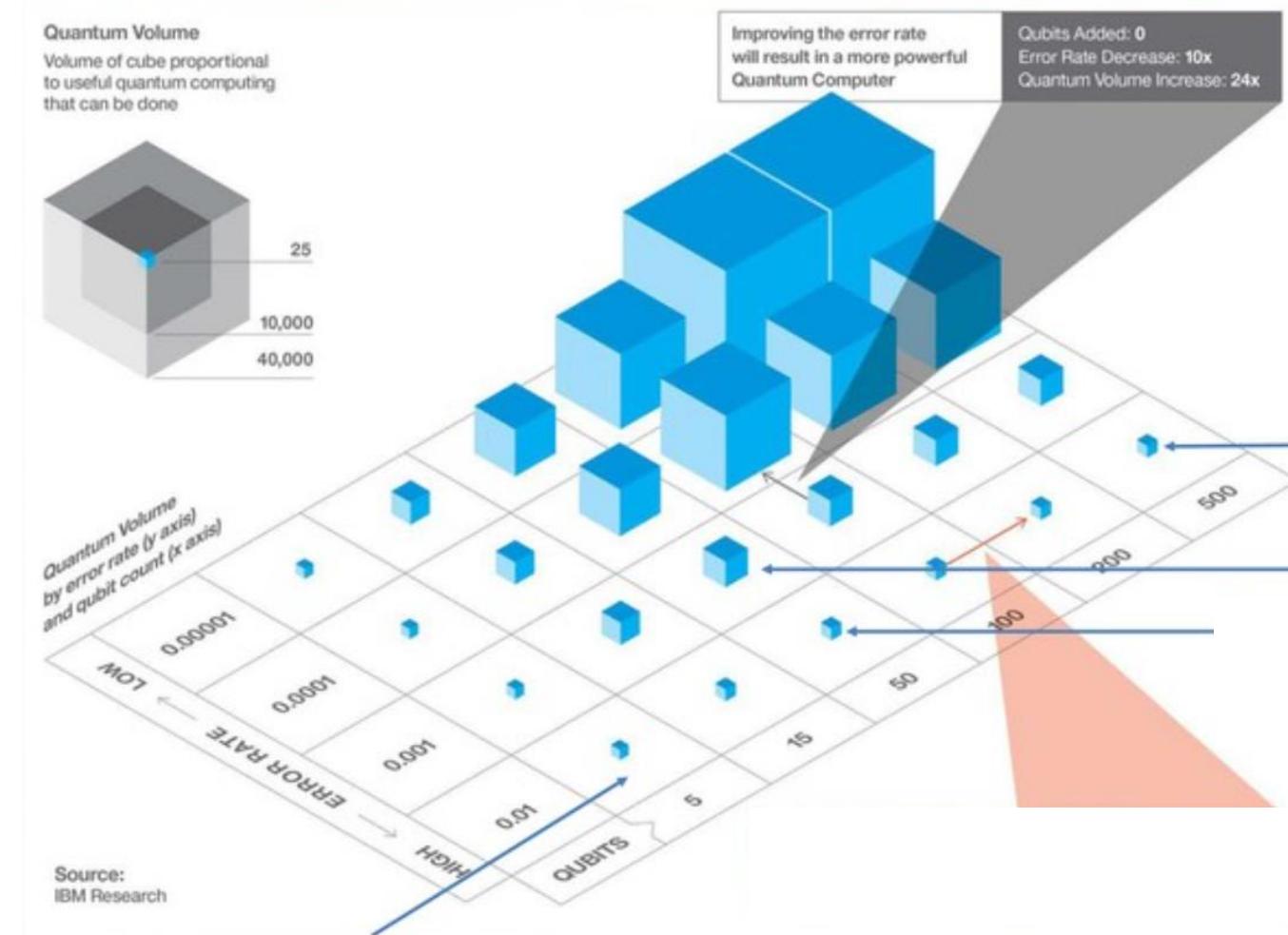
Metric	Symbol	Condition	Low <sup>1</sup>	Typ <sup>2</sup>	High <sup>3</sup>	Units	Description
Single-qubit gate error rate	e1	Isolated	0.1	0.1	0.2	% error per gate	Randomized benchmarking
Two-qubit gate error rate ( $\sqrt{iSWAP}$ )	e2 ( $\sqrt{iSWAP}$ )	Isolated	0.7	0.9	1.9	% error per gate	Cross-entropy benchmarking (XEB)
		Parallel	0.8	1.4	3.3	% error per gate	Cross-entropy benchmarking

# Building a Quantum Computer

Readout error $ 0\rangle$	$e_{r0}$	Isolated	0.5	1.1	2.6	% error	Confusion matrix: prepare $ 0\rangle$ and observe $ 1\rangle$ ; includes state prep error
		Simultaneous	1	2	3	% error	Confusion matrix: prepare $ 0\rangle$ and observe $ 1\rangle$ ; includes state prep error
Readout error $ 1\rangle$	$e_{r1}$	Isolated	3	5	9	% error	Confusion matrix: prepare $ 1\rangle$ and observe $ 0\rangle$ ; includes state prep error
		Simultaneous	3	7	9	% error	Confusion matrix: prepare $ 1\rangle$ and observe $ 0\rangle$ ; includes state prep error
Relaxation	$T_1$	Isolated	11	15	21	$\mu s$	Direct measurement of $ 1\rangle$ population relaxation

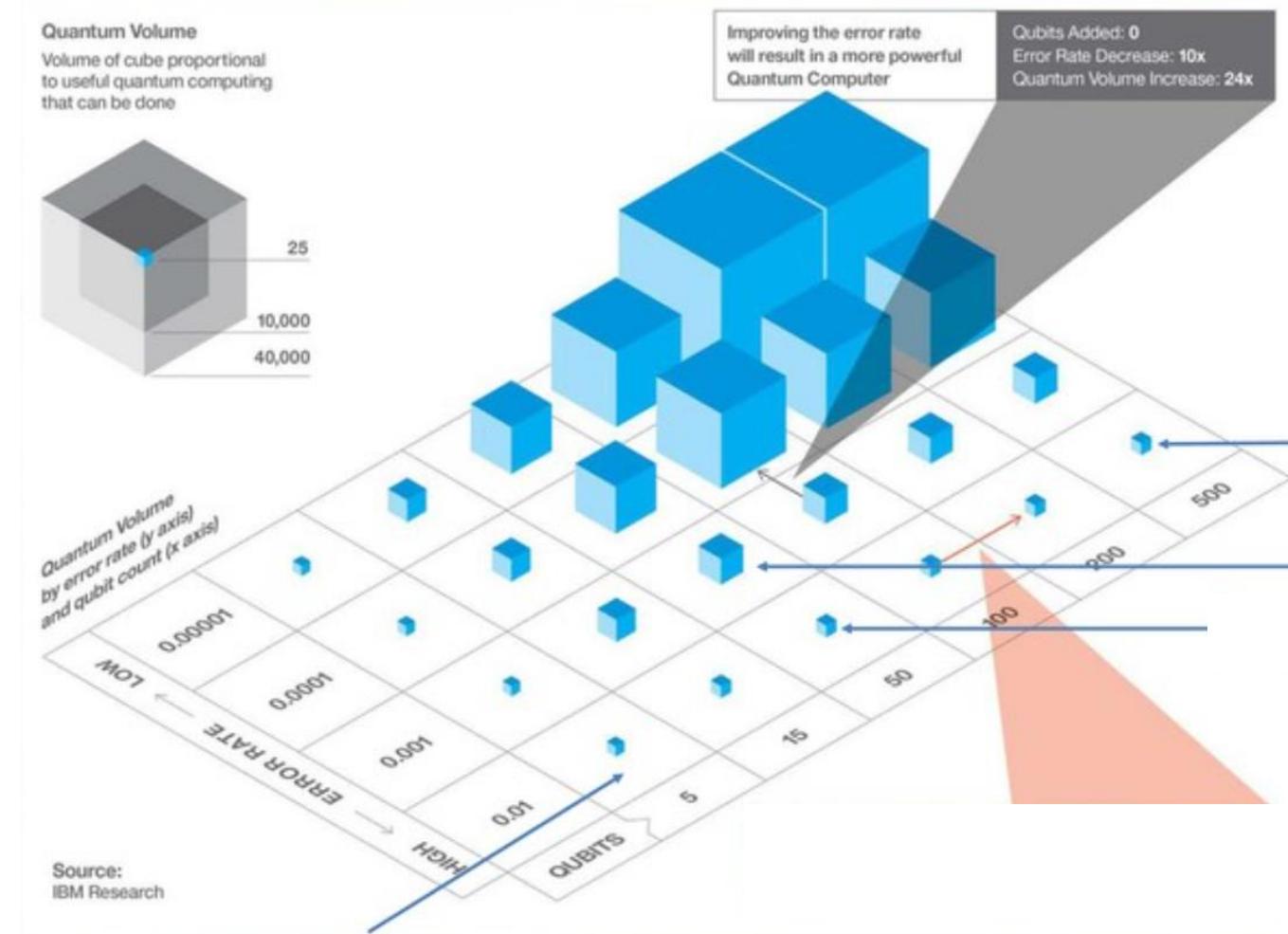
# Quantum Volume

- Quantum Volume is a metric that can be used to express the power of a quantum computer.



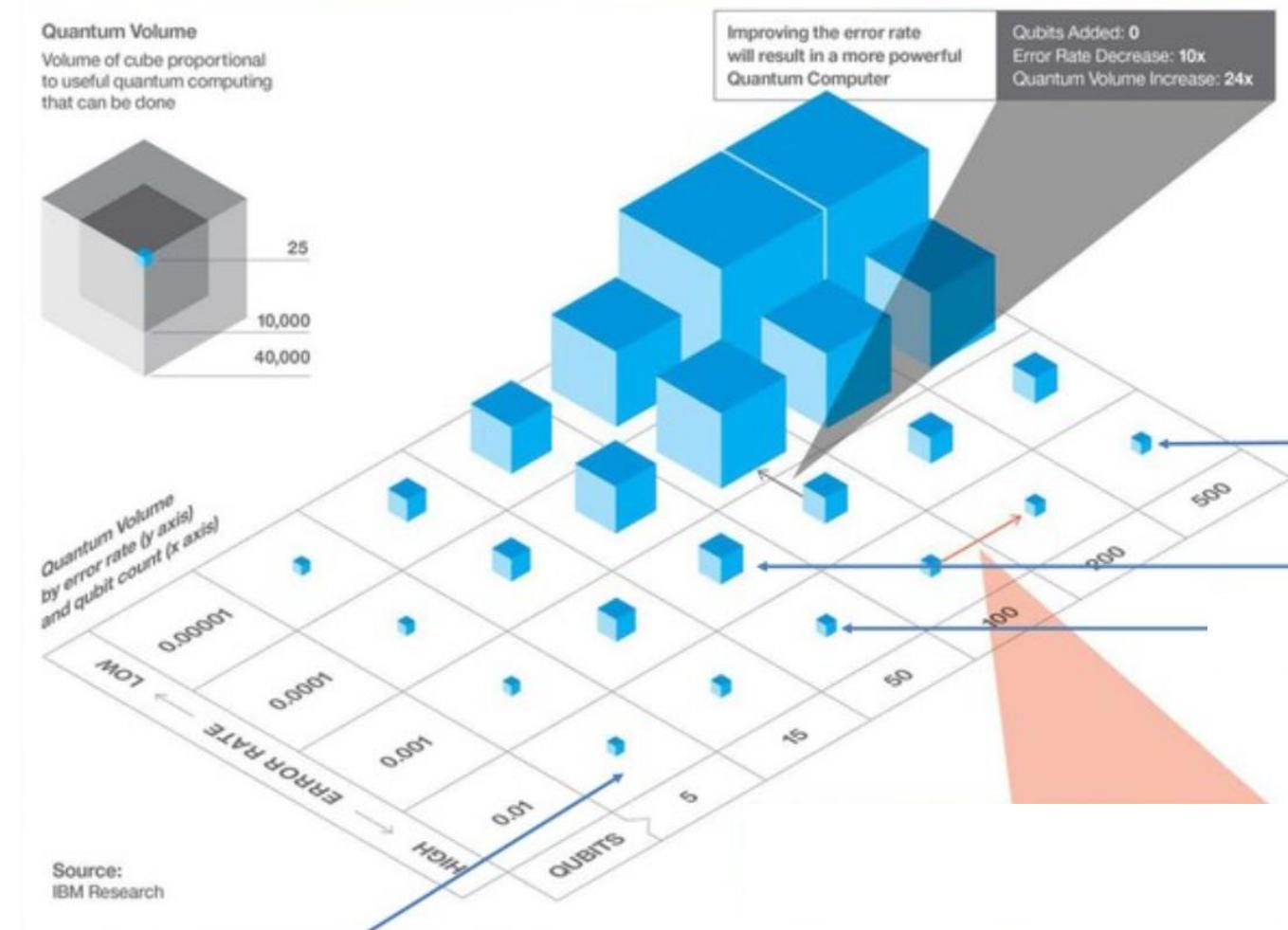
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# Quantum Volume

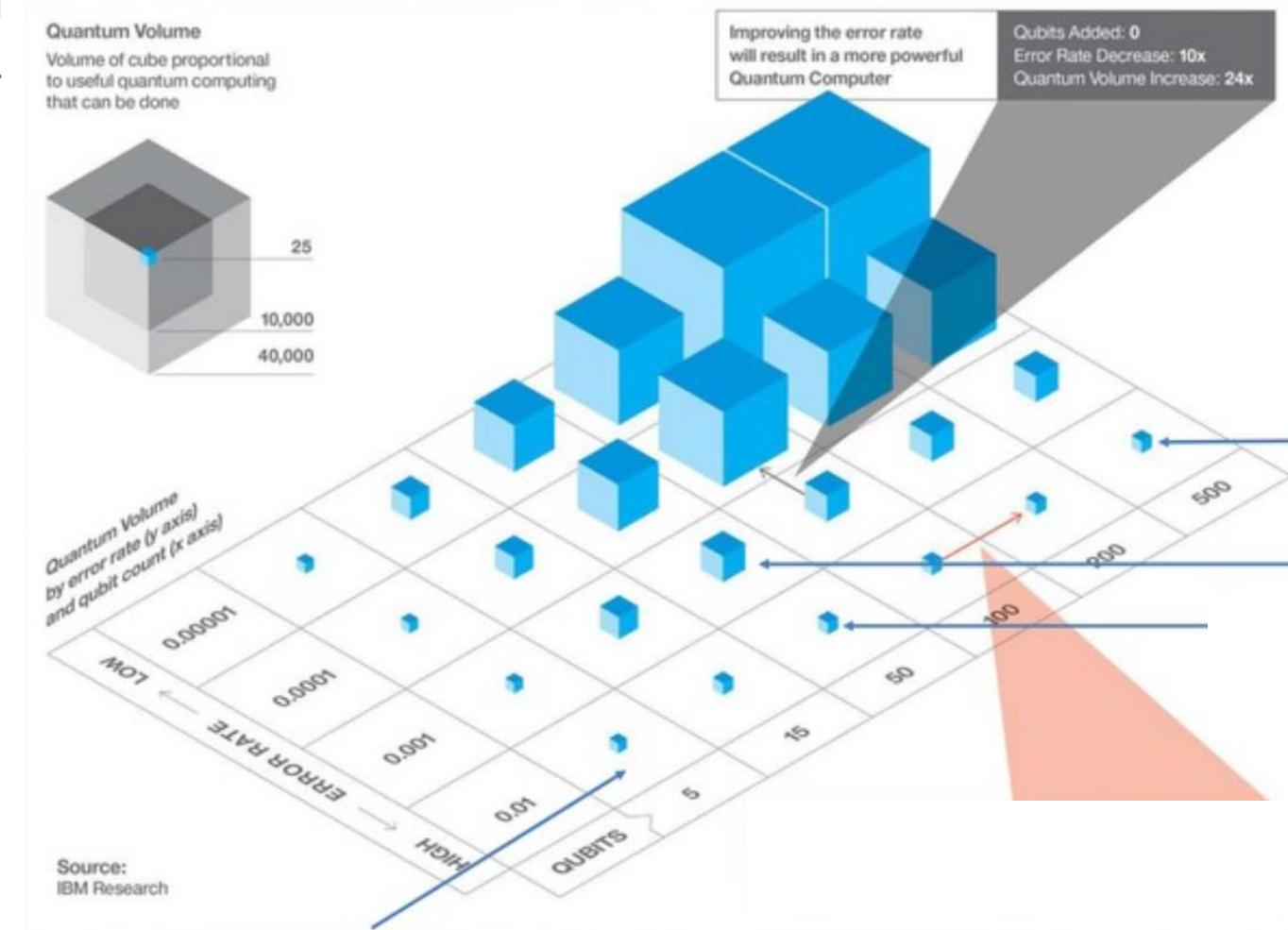
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- In particular, it takes into account connectivity, number of qubits and error rates.



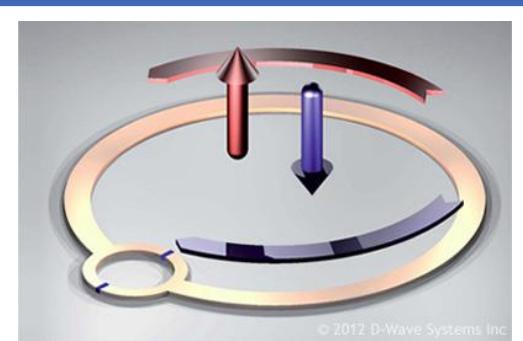
# Quantum Volume

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- In particular, it takes into account connectivity, number of qubits and error rates.
- It is not intuitive to calculate

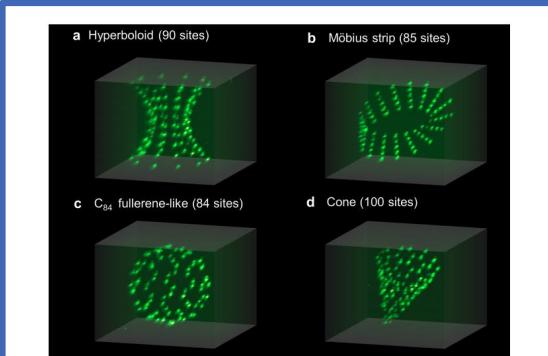
$$\log_2 V_Q = \arg \max_{n \leq N} \{ \min [n, d(n)] \}$$



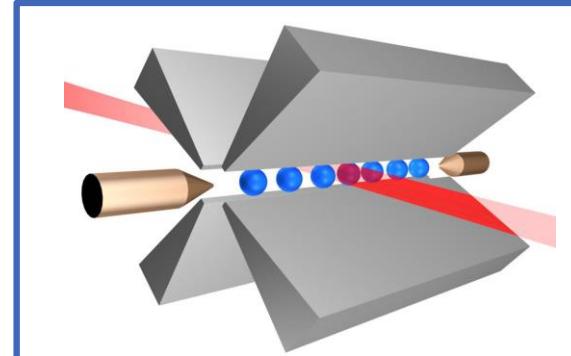
# Making a Qubit



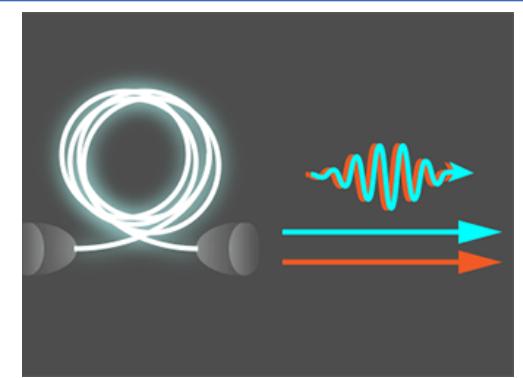
Superconducting



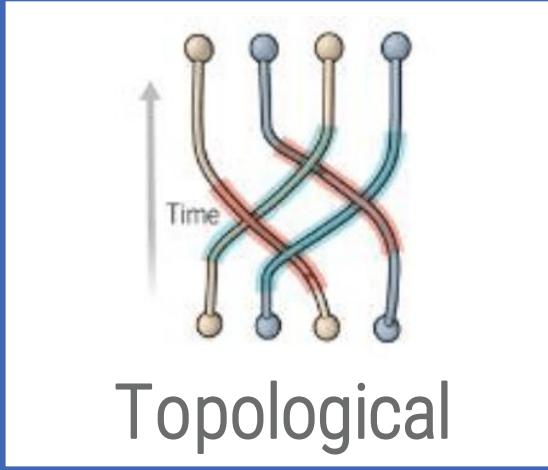
Neutral Atoms



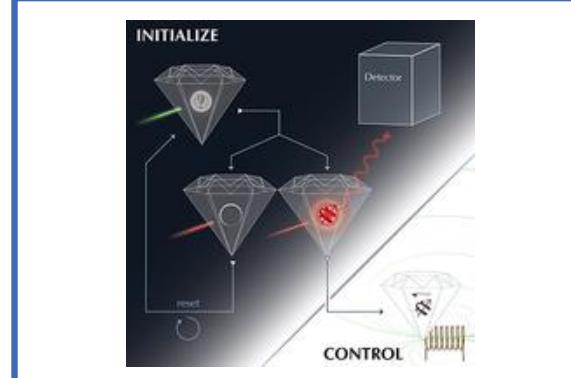
Trapped-ions



Photonic



Topological



Diamond Center

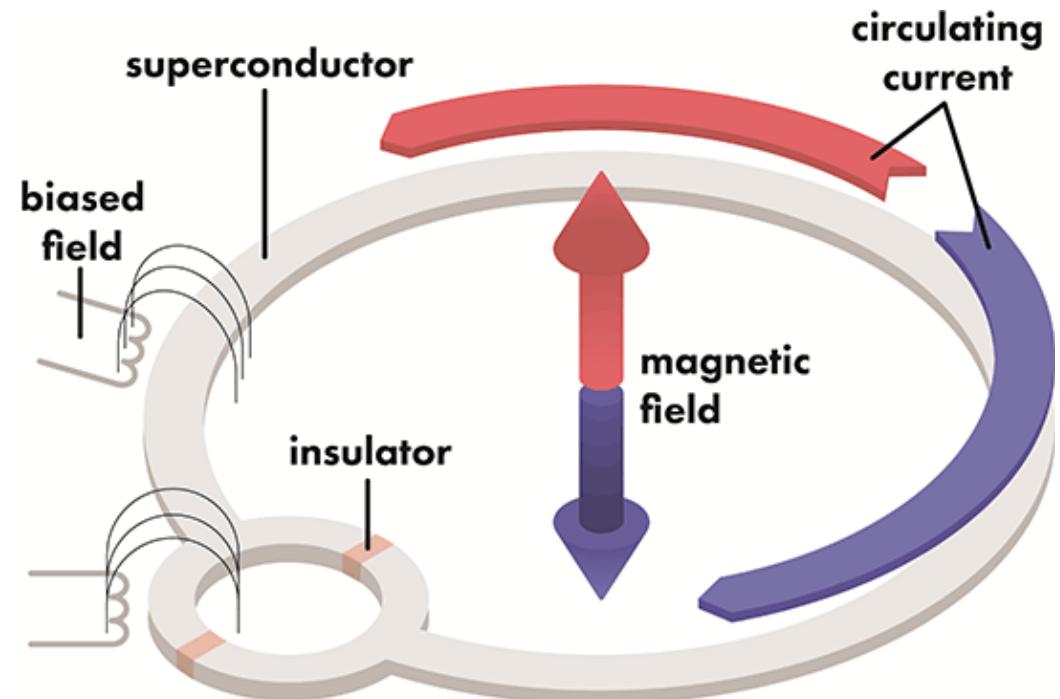
# Superconducting Qubits Technology

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- The first technology we are going to analyze is also the most widespread

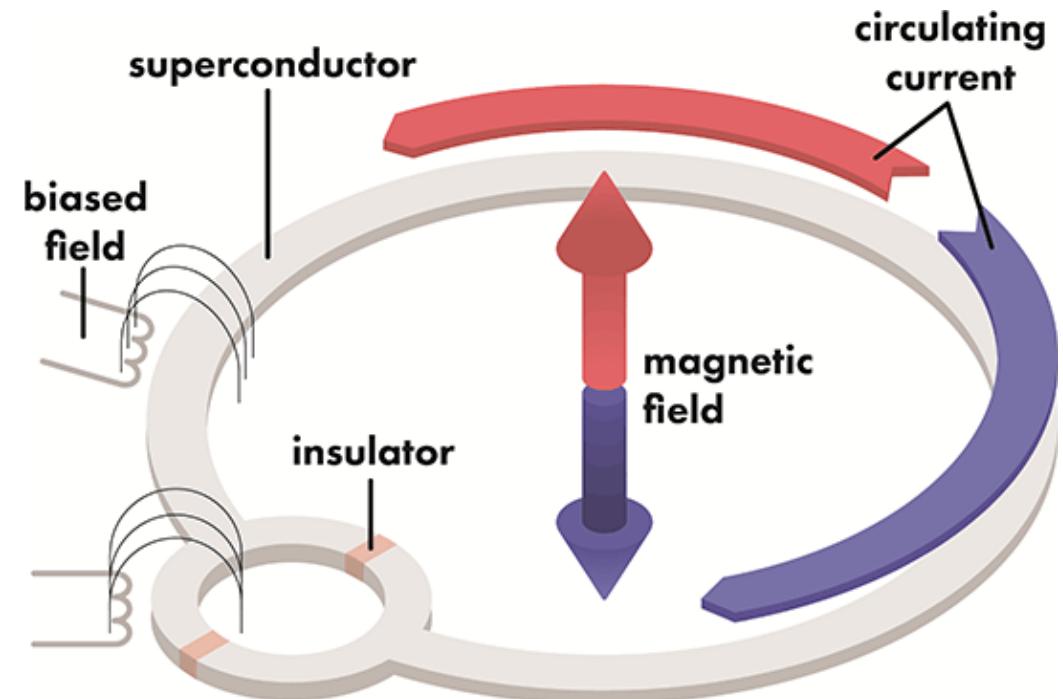
# Superconducting Qubits Technology

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- Superconducting qubits are rings of superconducting material crossed by electric current



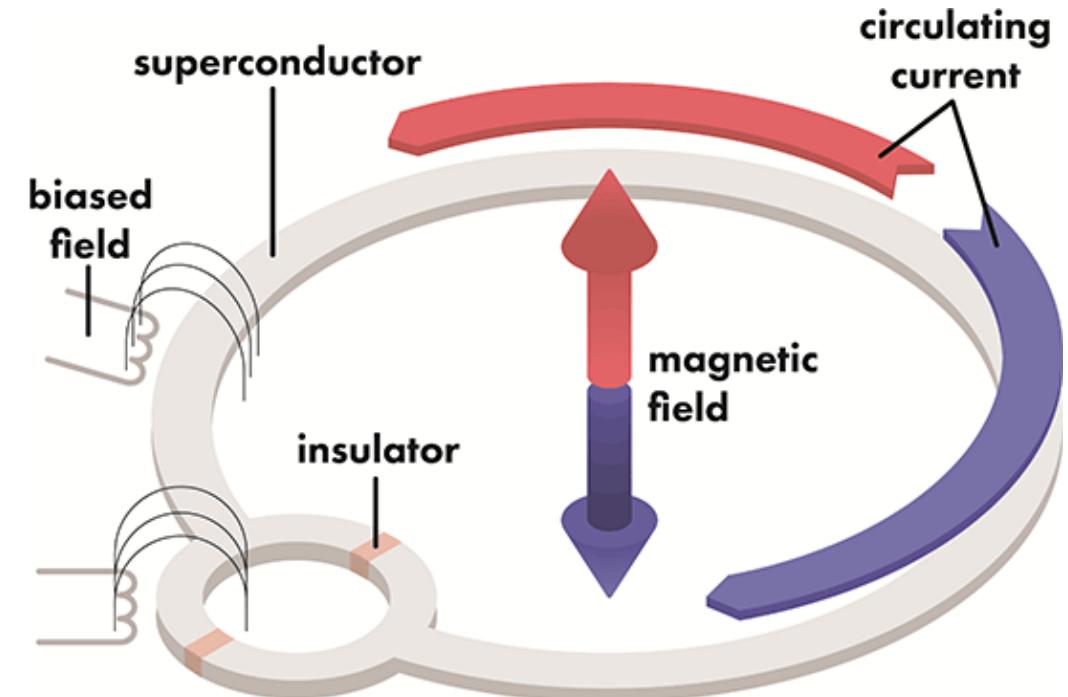
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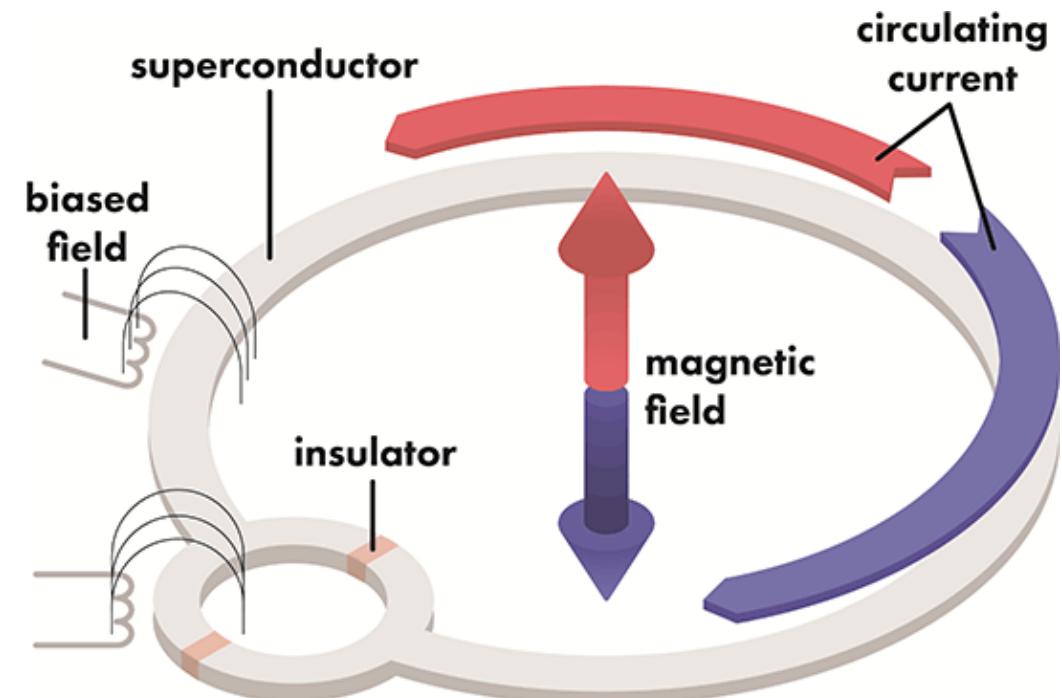
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# Superconducting Qubits Technology

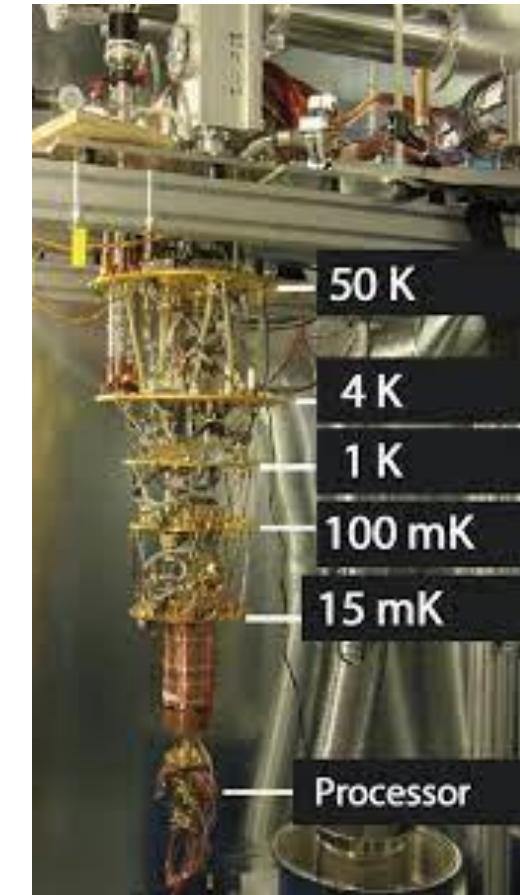
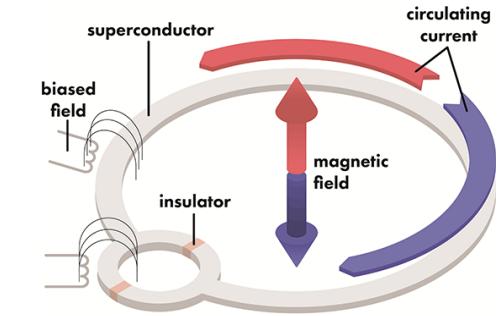
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- In this way, the superconducting qubit is able to reproduce superposition effects



# Superconducting Qubits Technology

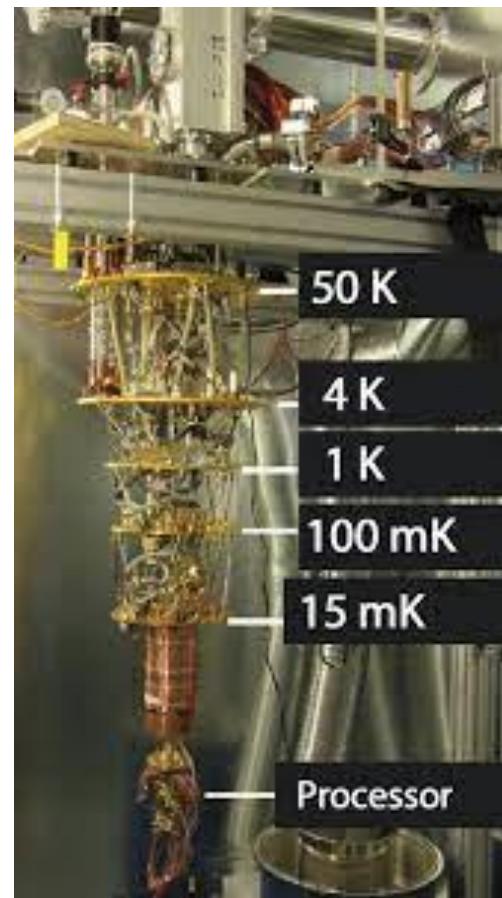
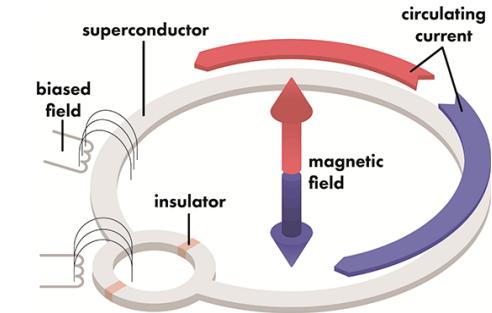
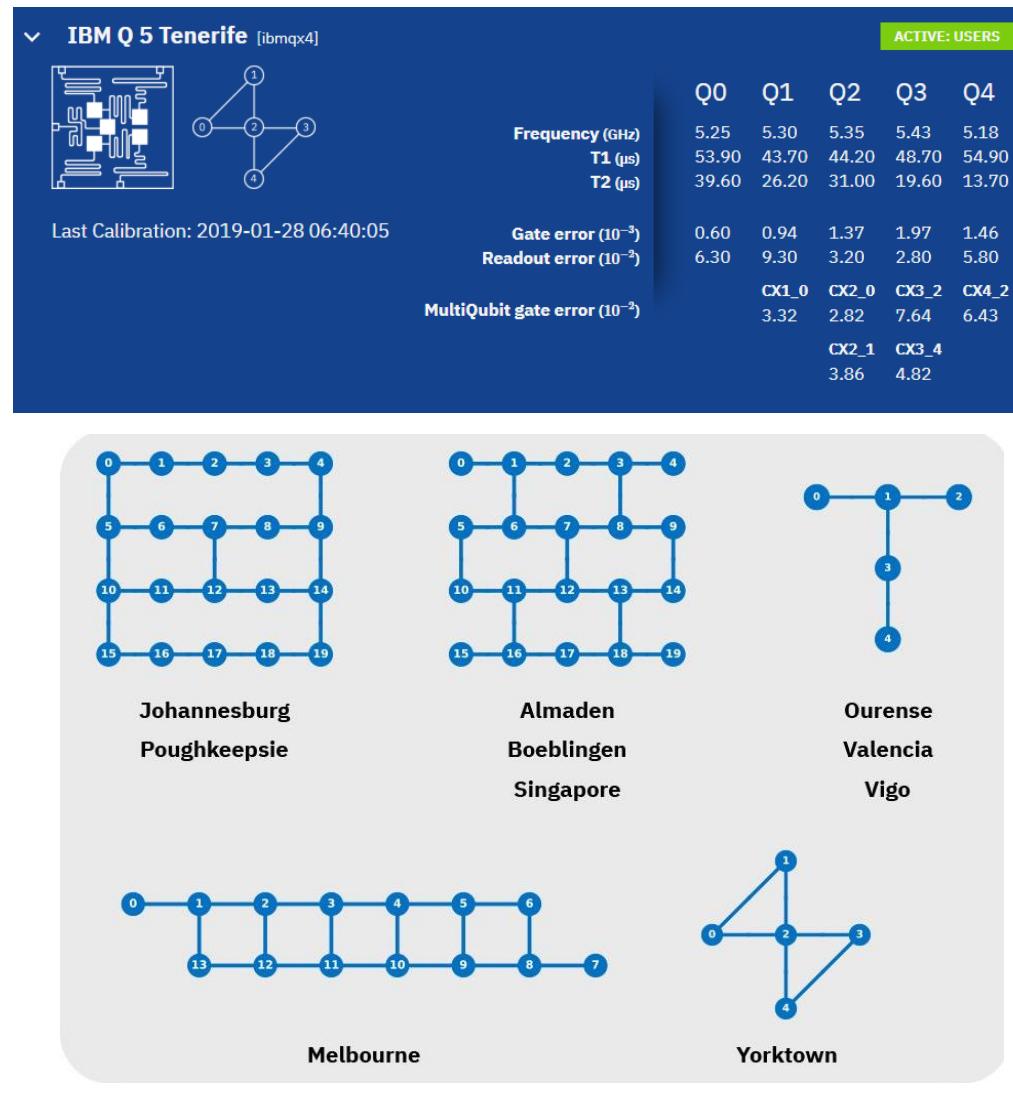
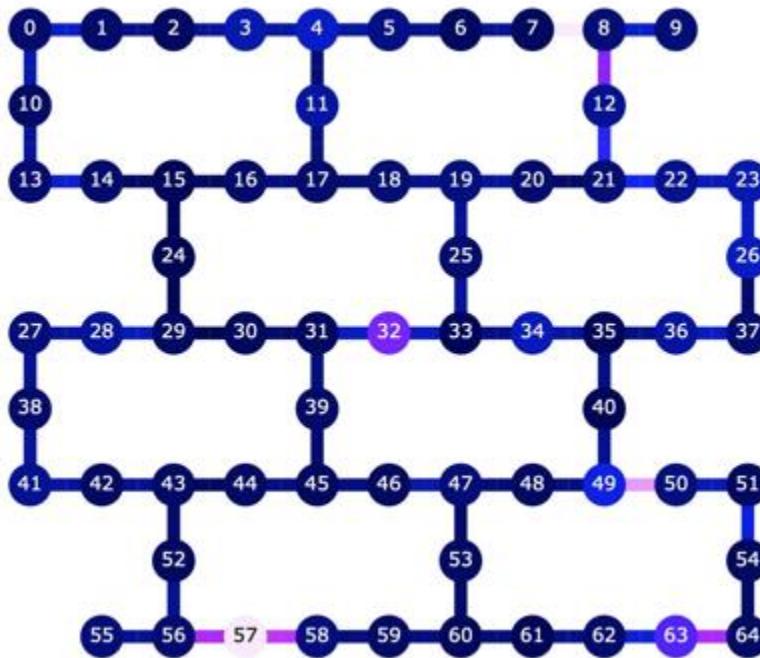
IBM Q

- IBM Quantum Division: American Big Tech, already owner of the most powerful supercomputer in the world (Summit), IBM was one of the first large companies to seriously embark on the development of quantum computers
- In 2016 they created *ibmquantumexperience*, a website where anybody can use their smallest quantum computers, made available for free to the public.
- To date, all models with 5 and 16 qubits can be used for free
- Their largest computer is 65 qubits
- They have a very aggressive roadmap: they predict 1000 qubits in 2023



# Superconducting Qubits Technology

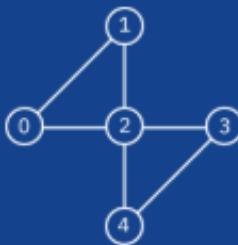
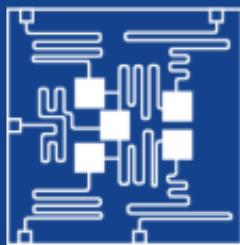
# IBM Q



# Superconducting Qubits Technology

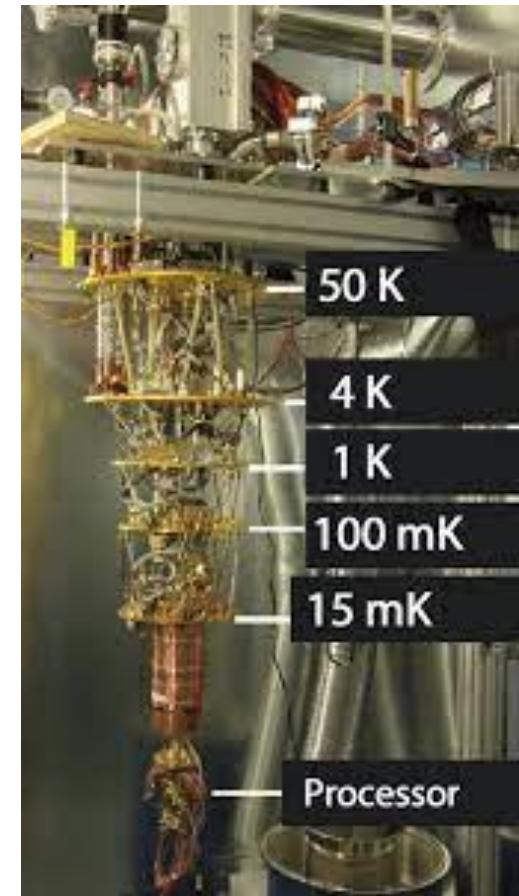
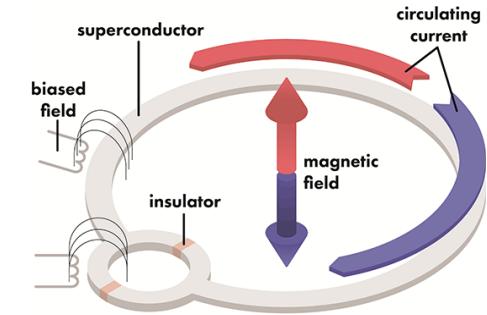
# IBM Q

## IBM Q 5 Tenerife [ibmqx4]

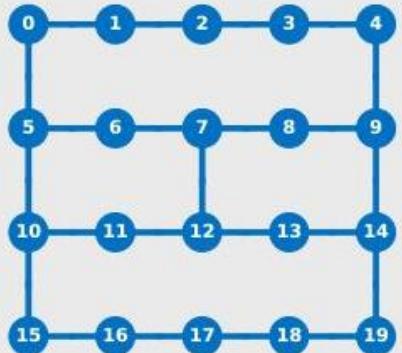


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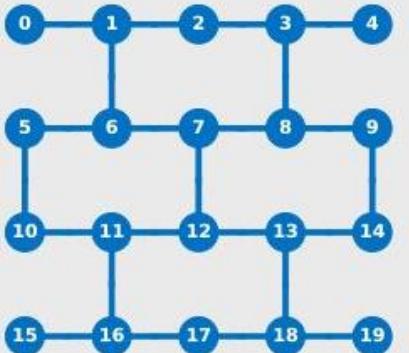
	ACTIVE: USERS				
	Q0	Q1	Q2	Q3	Q4
<b>Frequency (GHz)</b>	5.25	5.30	5.35	5.43	5.18
<b>T1 (μs)</b>	53.90	43.70	44.20	48.70	54.90
<b>T2 (μs)</b>	39.60	26.20	31.00	19.60	13.70
<b>Gate error (<math>10^{-3}</math>)</b>	0.60	0.94	1.37	1.97	1.46
<b>Readout error (<math>10^{-2}</math>)</b>	6.30	9.30	3.20	2.80	5.80
<b>MultiQubit gate error (<math>10^{-2}</math>)</b>	CX1_0 3.32	CX2_0 2.82	CX3_2 7.64	CX4_2 6.43	
	CX2_1 3.86	CX3_4 4.82			



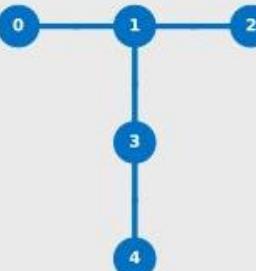
# Superconducting Qubits Technology



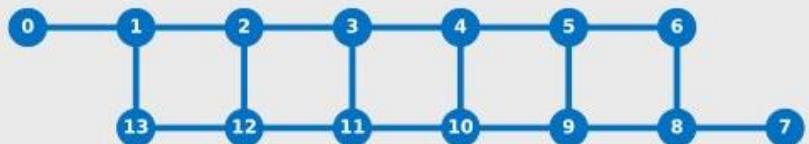
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Poughkeepsie



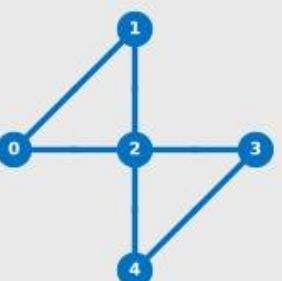
Almaden  
Boeblingen  
Singapore



Ourense  
Valencia  
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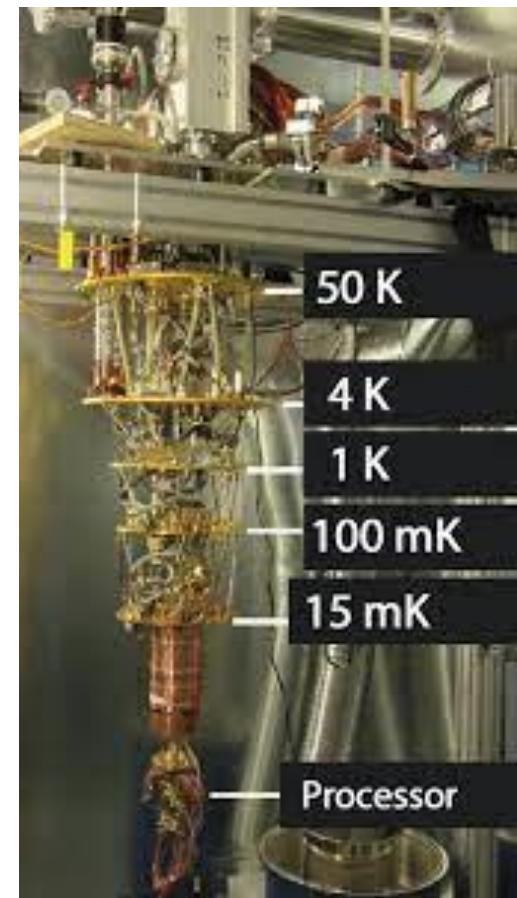
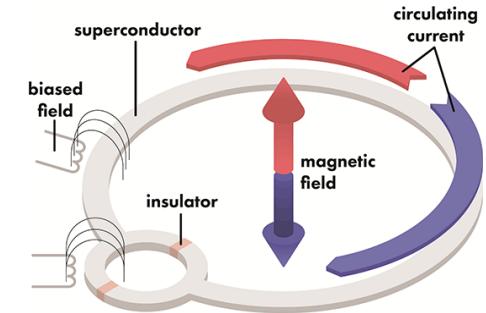


Melbourne

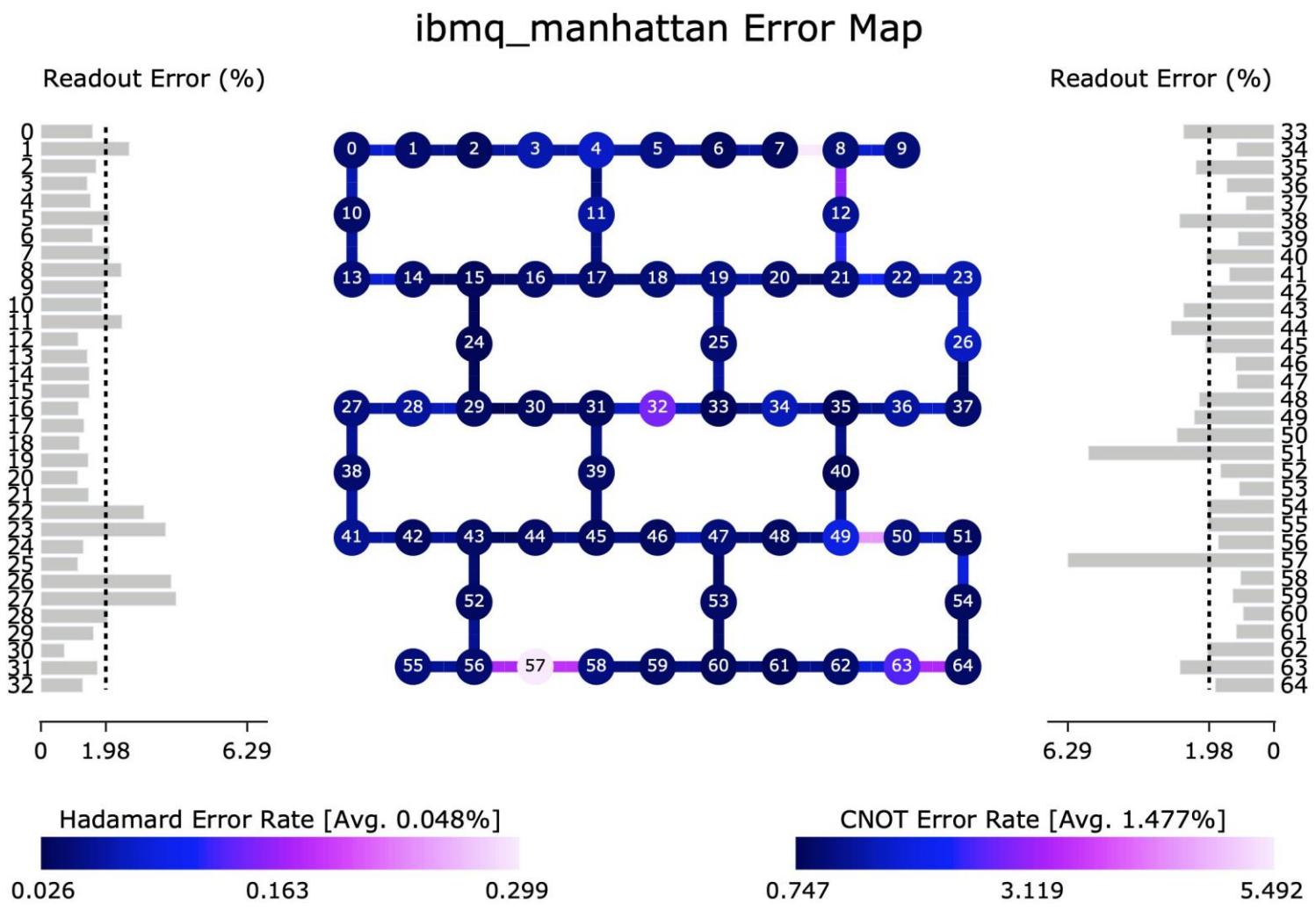


Yorktown

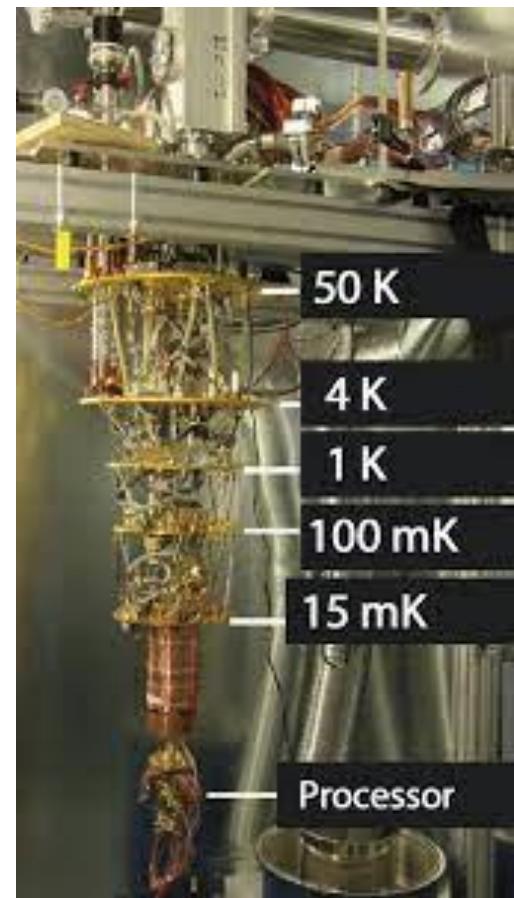
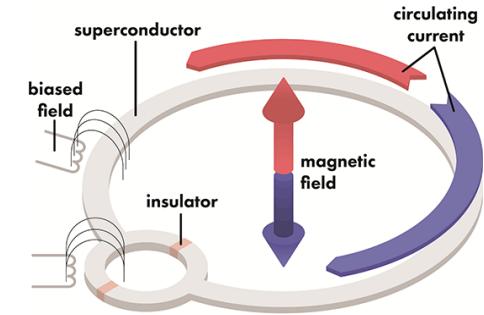
IBM Q



# Superconducting Qubits Technology



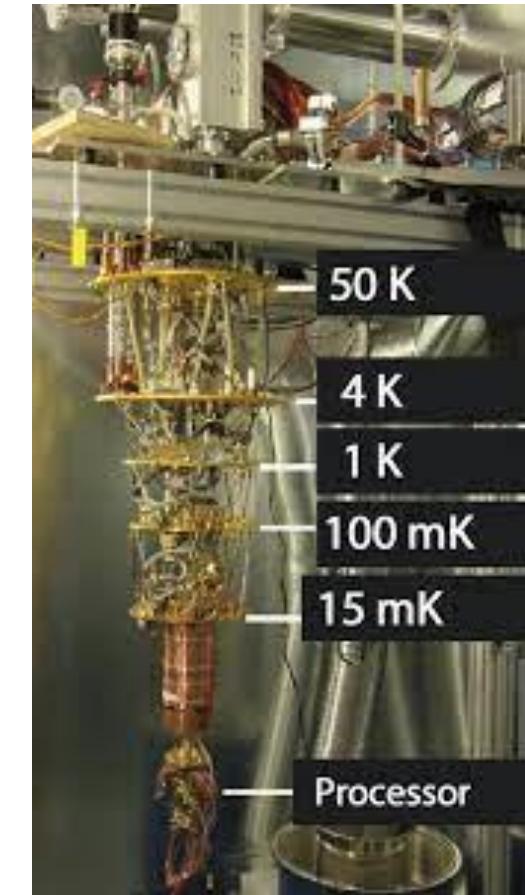
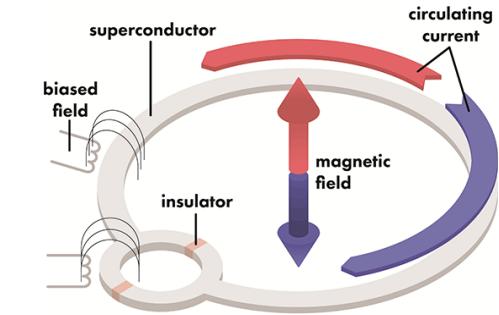
IBM Q



# Superconducting Qubits Technology



- American startup born in 2013 in California
- They also manufacture their own general purpose quantum computers using superconducting qubits
- Most powerful machine: Aspen-9 (32 qubits)
- They have announced that they are close to building a model with 128 qubits



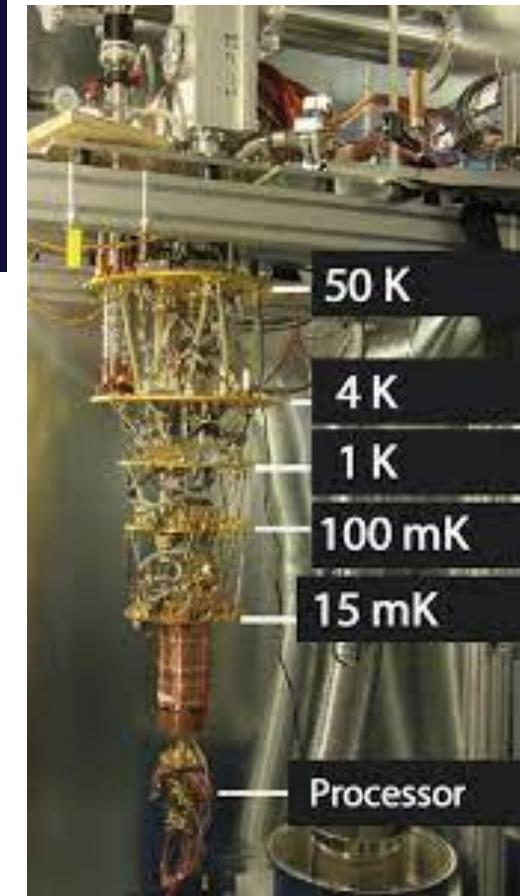
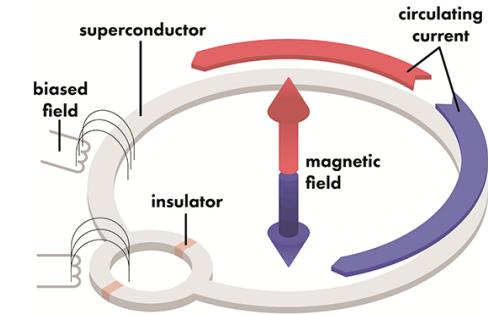
# Superconducting Qubits Technology

## Rigetti right now

### Aspen-9

Median Time Duration ( $\mu$ s)

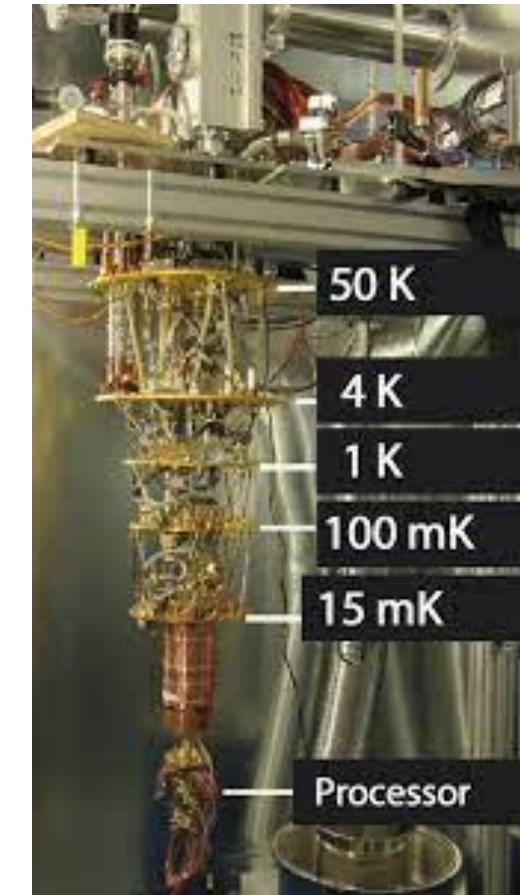
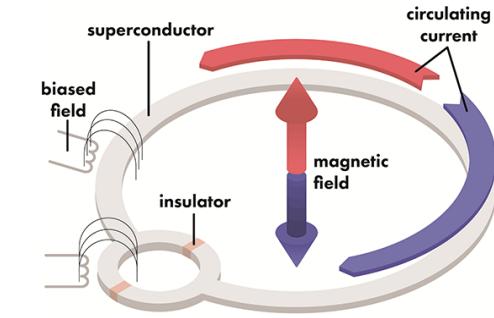
Deployed	07.02.21	T1 Lifetime	27	Single-qubit gates	99.8%
Qubits	31	T2 Lifetime	19	Two-qubit gates (CZ)	95.8%
				Two-qubit gates (XY)	95.4%



# Superconducting Qubits Technology



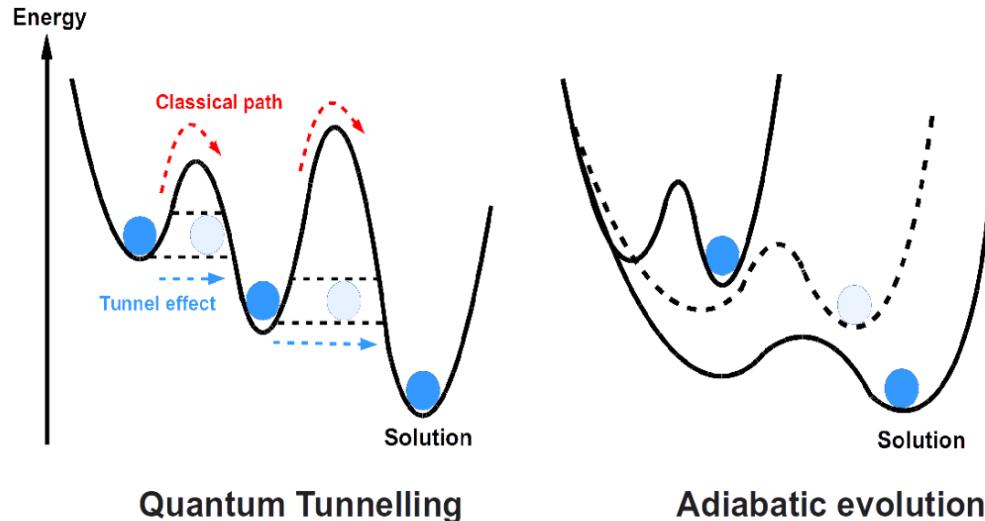
- D-Wave is our latest example of a quantum computer company with superconducting qubits
- Among all those we have seen so far, it is undoubtedly the most particular: it does not produce general purpose quantum computers, but special purpose
- What does it mean? In practice it means that it is not possible to use their computers to implement any quantum algorithm
- Their computers are made with the sole purpose of solving optimization problems by implementing a particular algorithm called Quantum Annealing
- With this choice, they have shown that it is possible to make great chipsets already: their latest model has more than 5600 qubits!



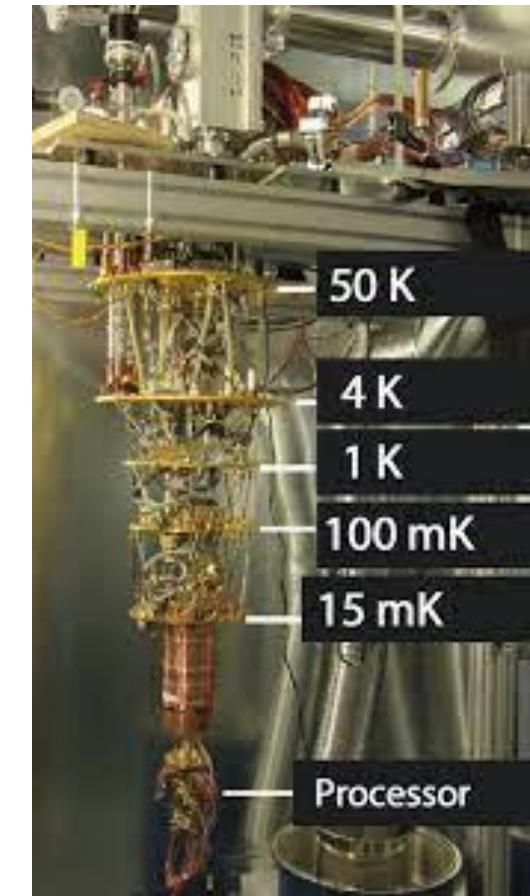
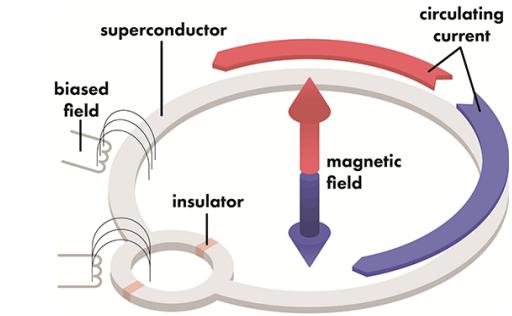
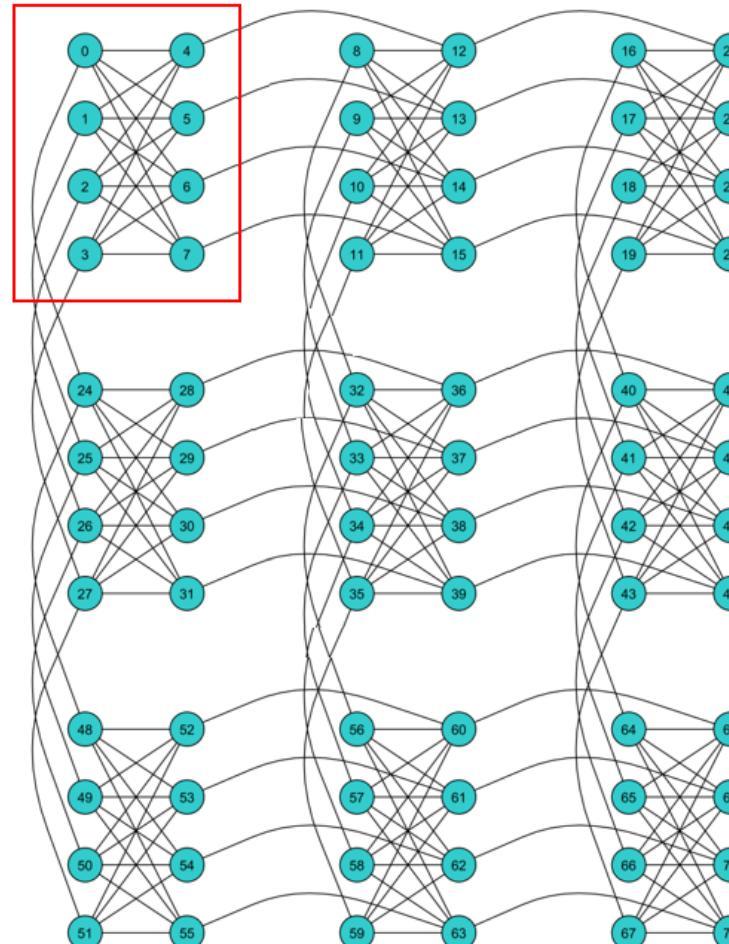
# Superconducting Qubits Technology



The Quantum Computing Company™

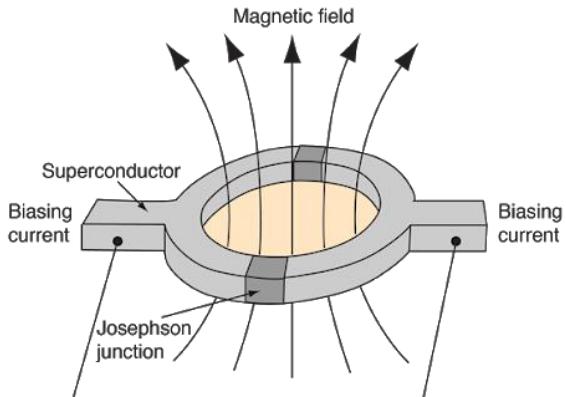


Unit Cell



# Superconducting Qubits Technology

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## SUPERCONDUCTING PRO

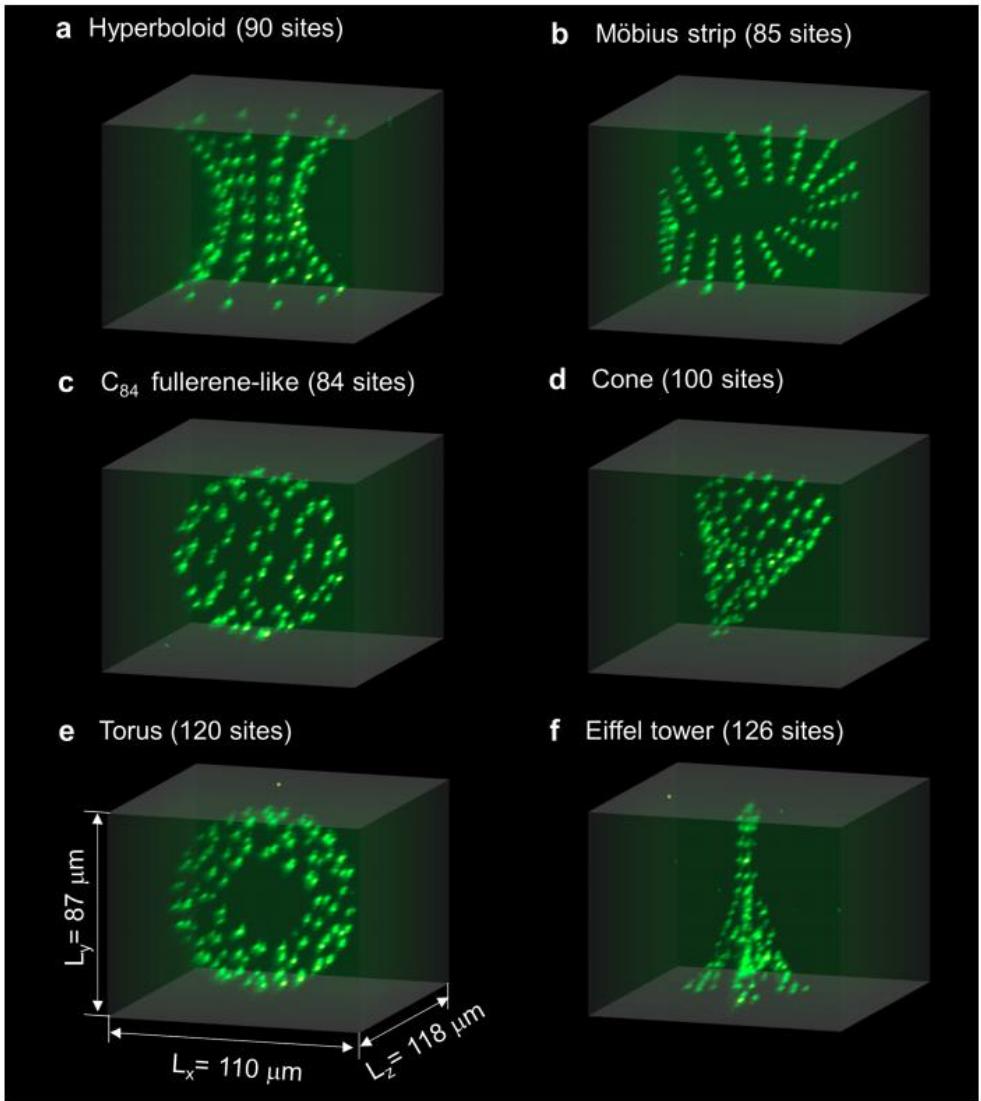
- Macroscopic
- Very large experience
- Fast operations

## CONS

- Very low coherence time
- Low fidelity
- Need to be cooled down

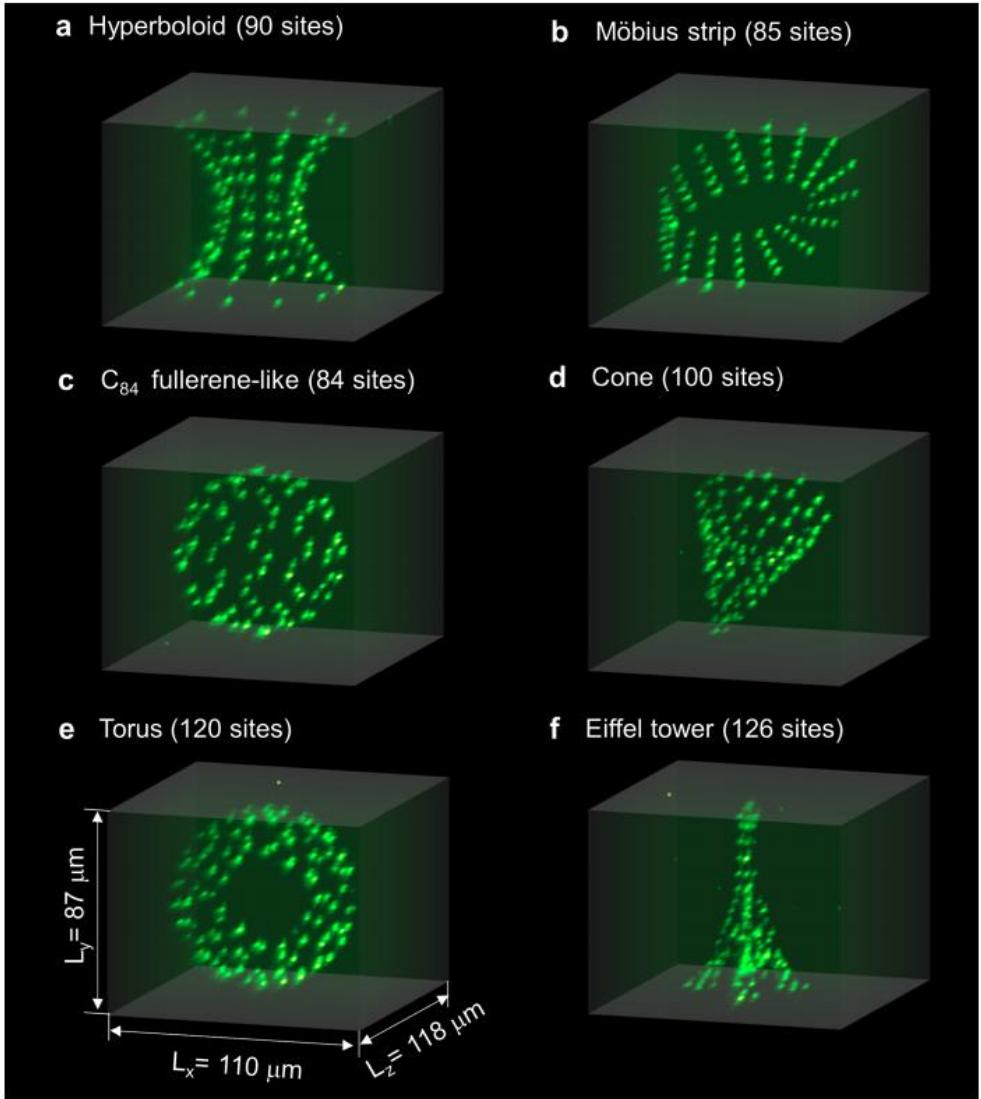
# Neutral Atoms Qubits Technology

- The second qubits technology that we are going to analyze is the Neutral Atoms technology



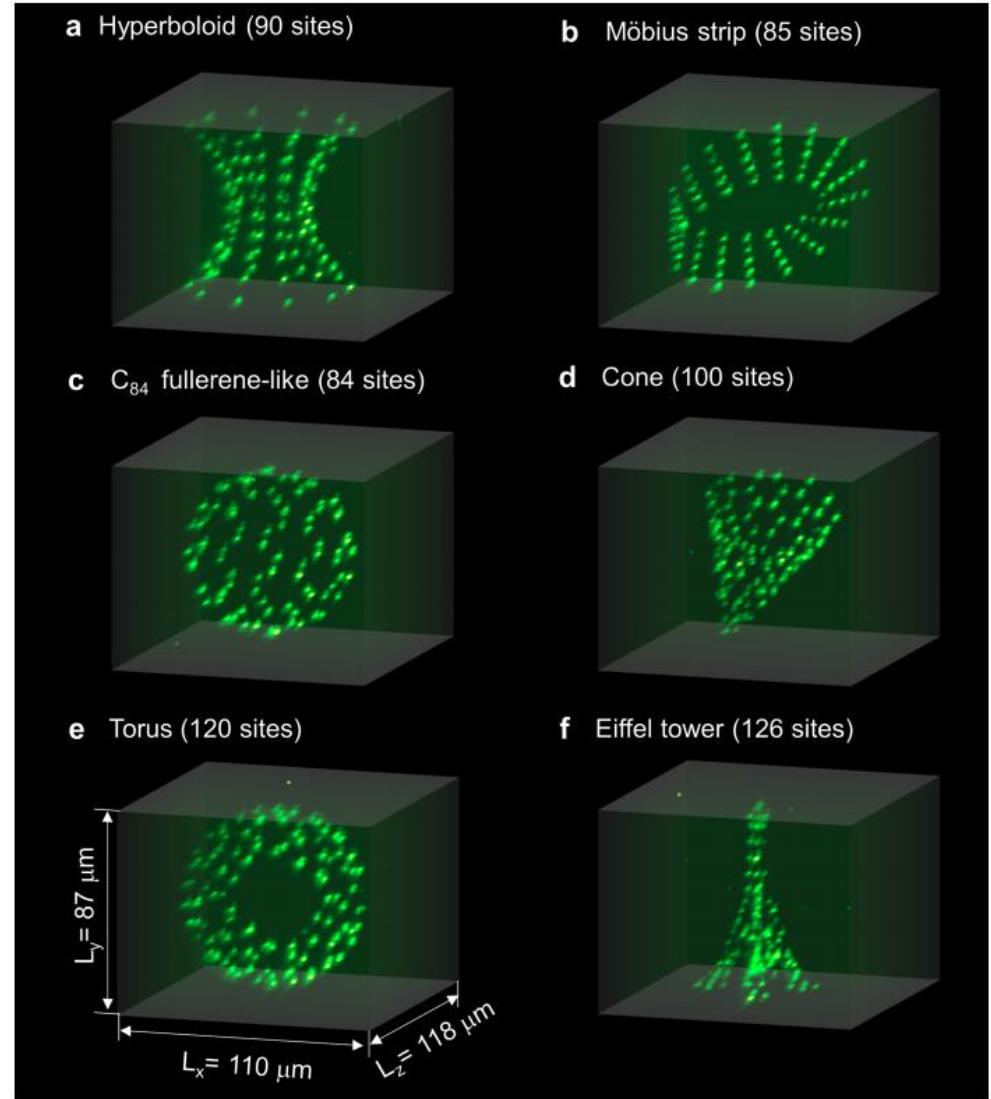
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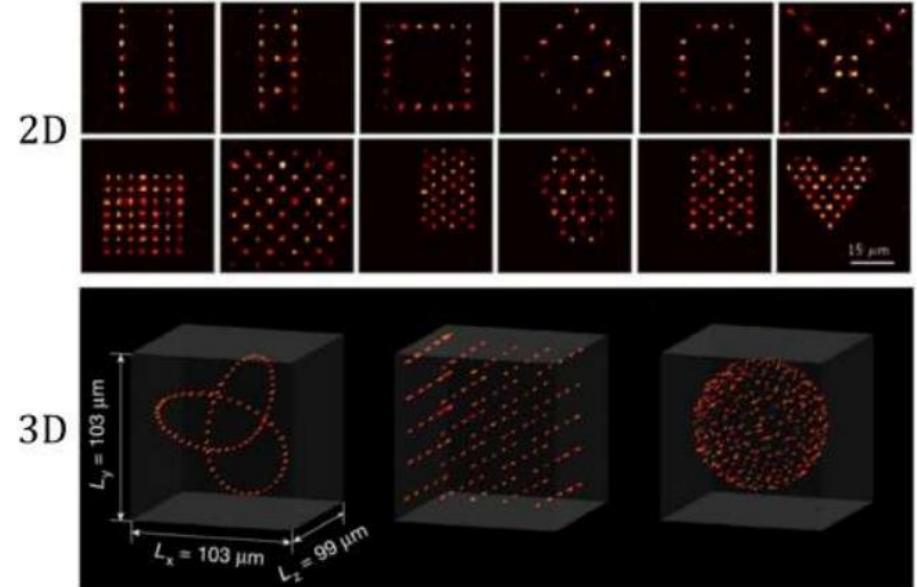
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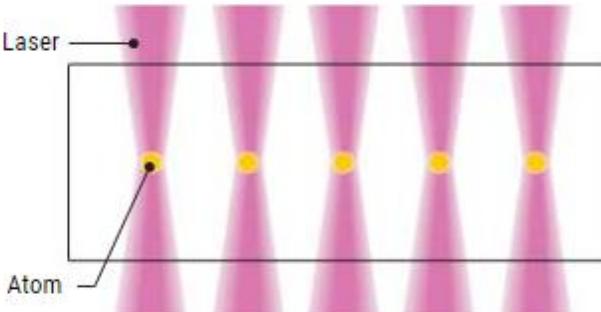


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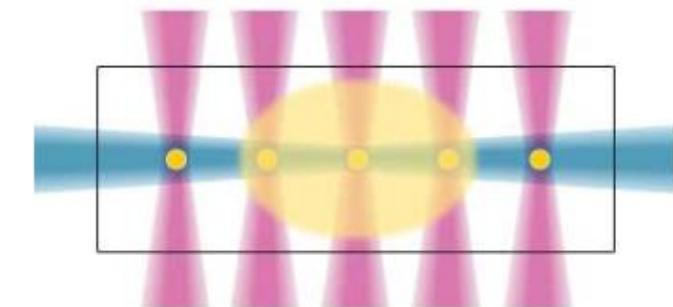
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- Qubits Neutral Atoms are Rubidium atoms, which can be manipulated through the use of special lasers



**1** Individual laser beams are used to trap arrays of atoms in vacuum chambers.

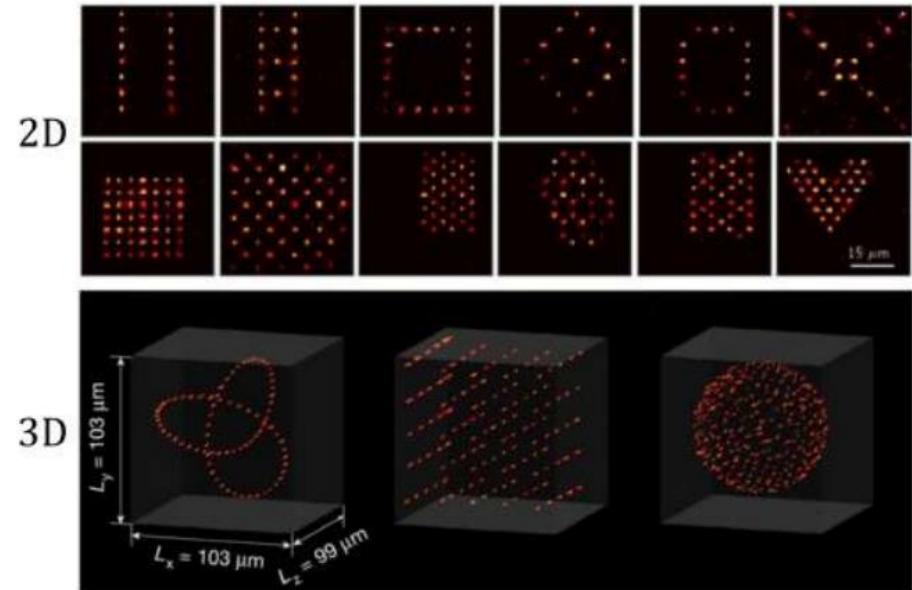


**2** Other lasers excite an atom's outermost electron. The massively enlarged atom can interact with its neighbors, enabling the formation of entangled quantum bits.

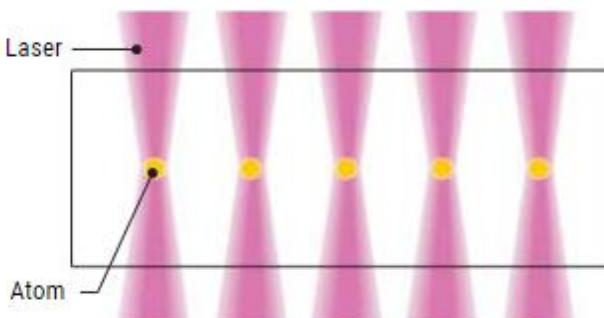


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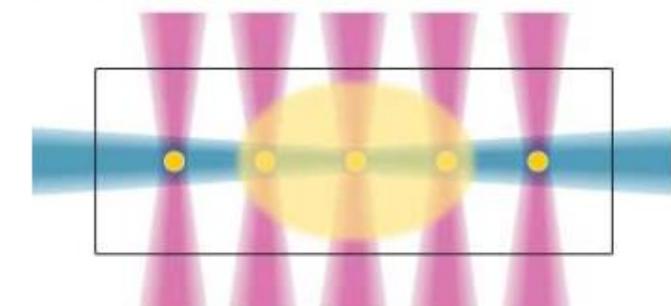
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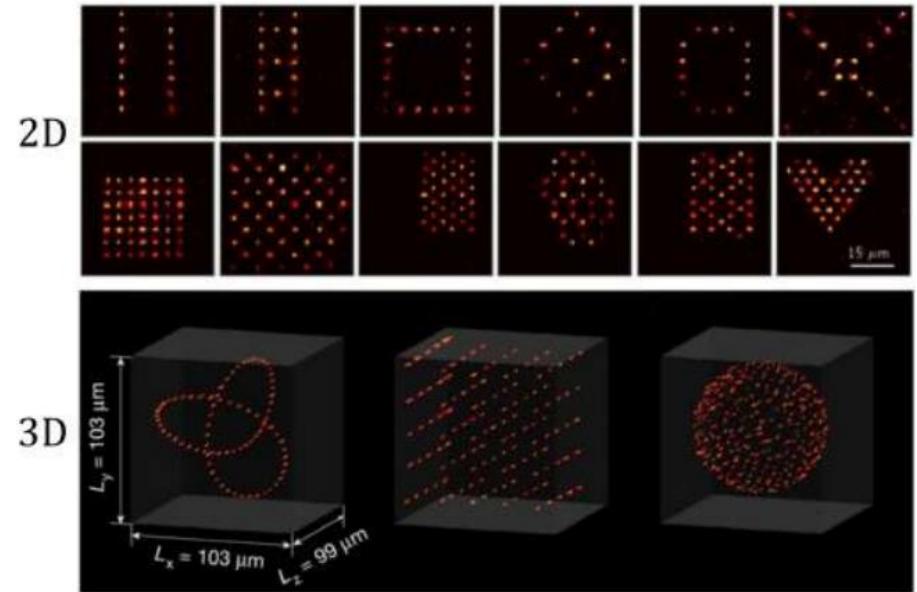


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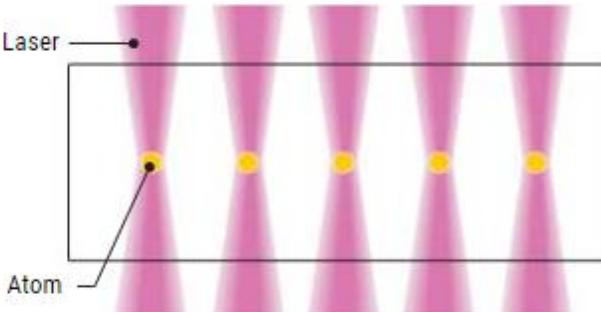


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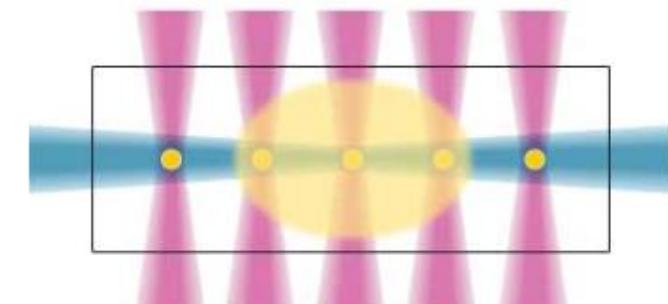
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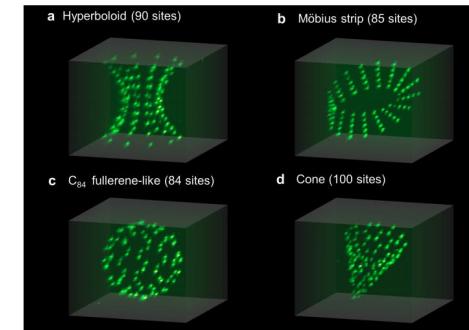


# Neutral Atoms Qubits Technology



PASQAL

- Pasqal is a French startup born in Paris in 2019
- Spin-off of the Institut d'Optique Graduate School, its team boasts among the greatest experts in quantum computing using neutral atoms
- The company was born after demonstrating the effectiveness of their qubits with a working laboratory prototype
- In this first phase, Pasqal's quantum machine is a machine capable of performing analog calculations
- In other words, it is not yet possible to program it as a real QC, but it can be used to simulate the evolution of particular Hamiltonians
- They plan to have a computer with 100 qubits by September this year and to increase the number of qubits to a thousand by 2023.

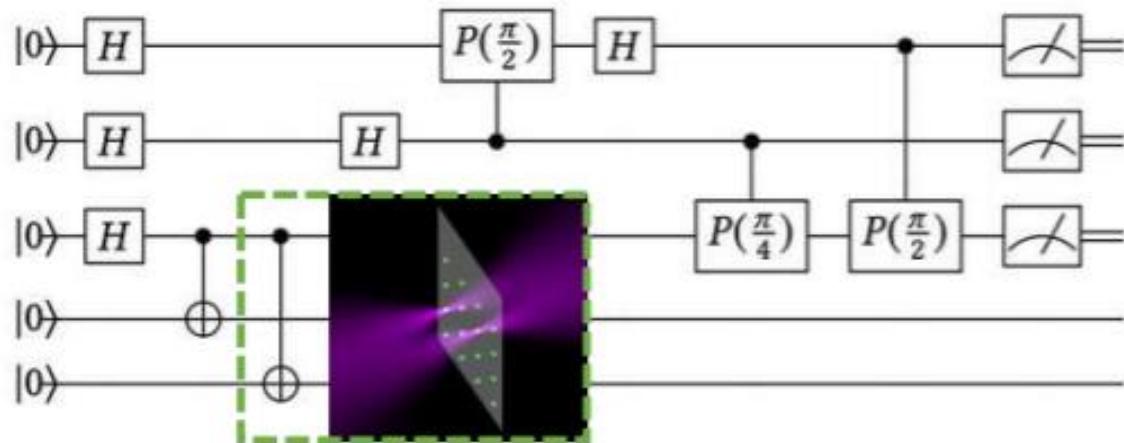


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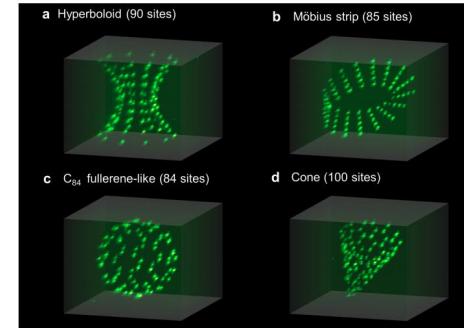
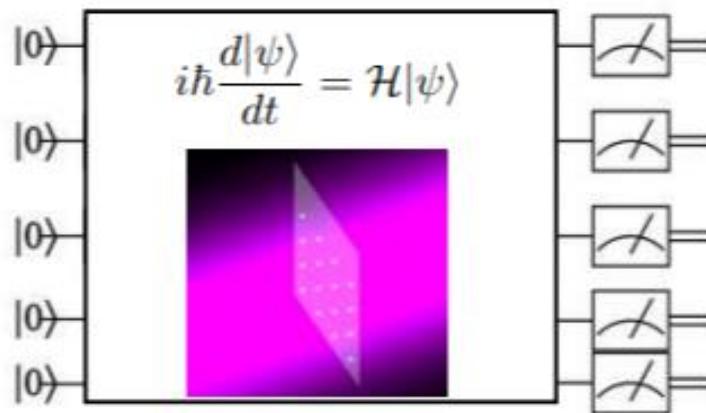


## PASQAL

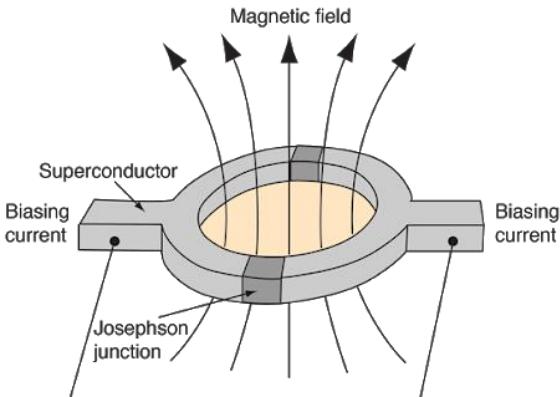
(a) Digital processing



(b) Analog processing



# Building a Quantum Computer

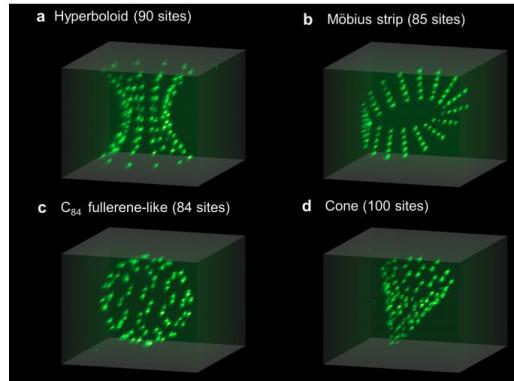


## SUPERCONDUCTING PRO

- Macroscopic
- Very large experience
- Fast operations

## CONS

- Very low coherence time
- Low fidelity
- Need to be cooled down



## NEUTRAL ATOMS PRO

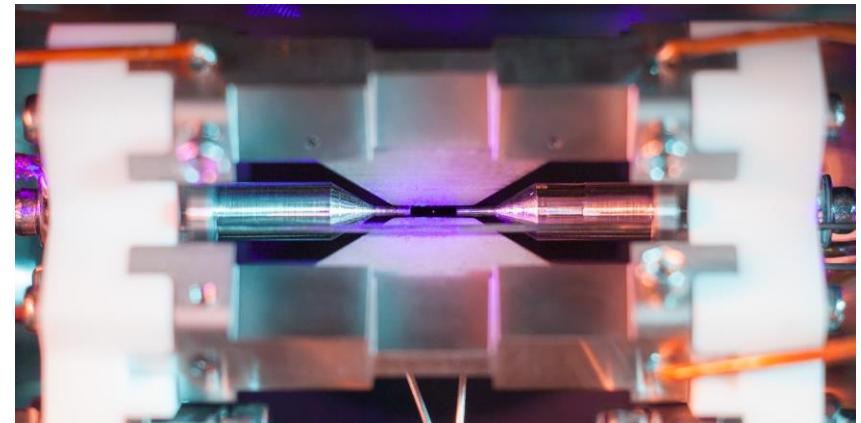
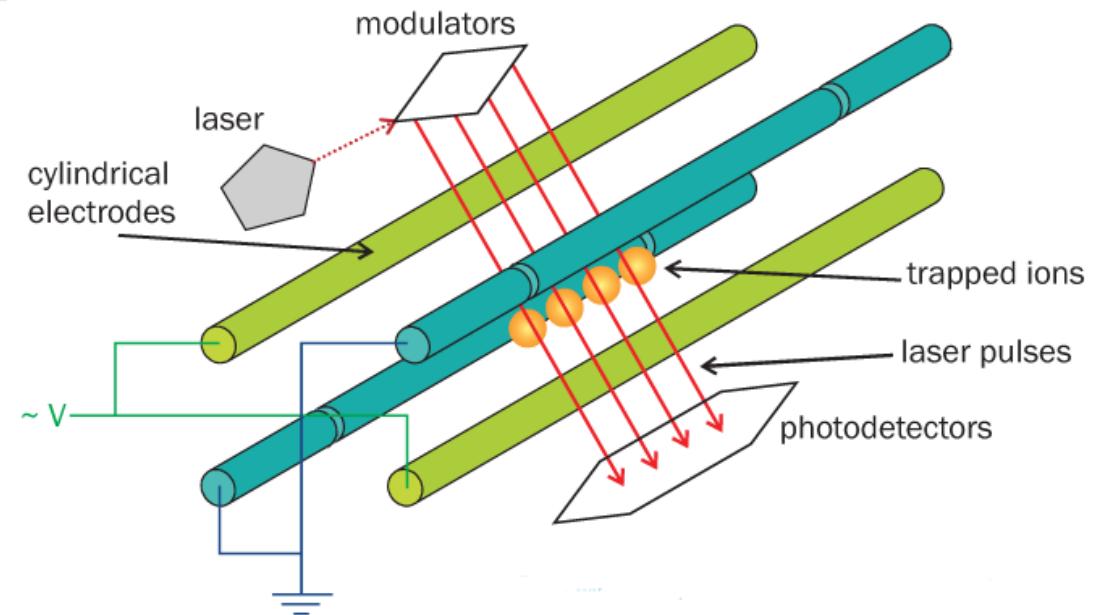
- Good coherence
- interchangeable topology
- Room temperature

## CONS

- Experimental technology

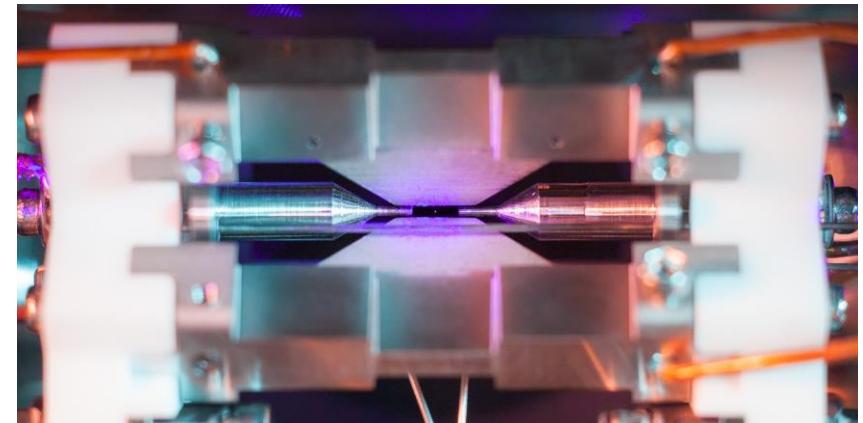
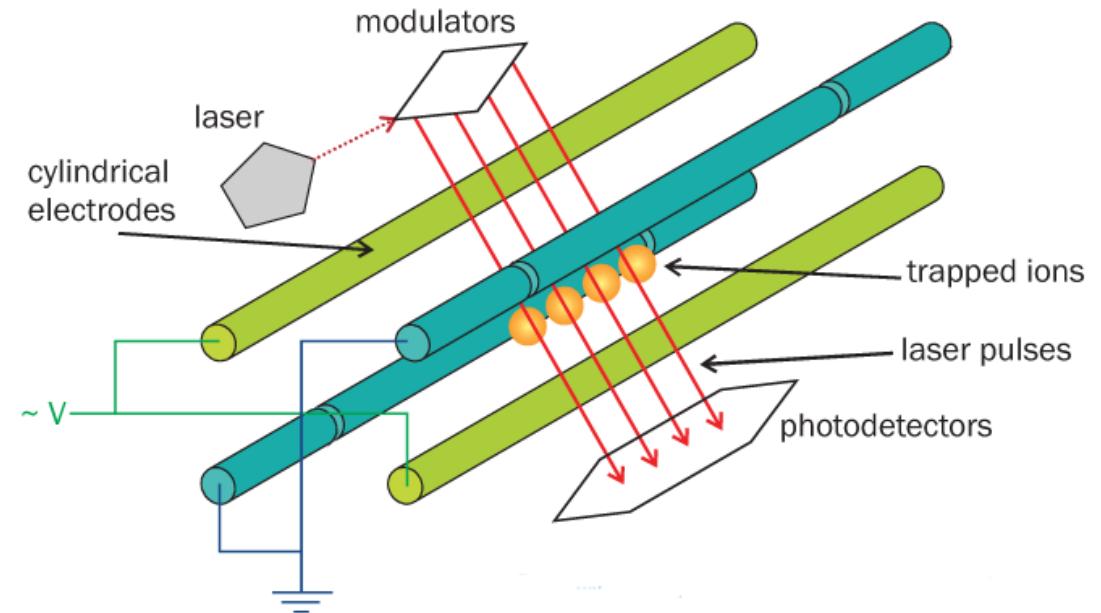
# Trapped Ions Qubits Technology

- Trapped-ions is the third and qubit technology that we see in detail today



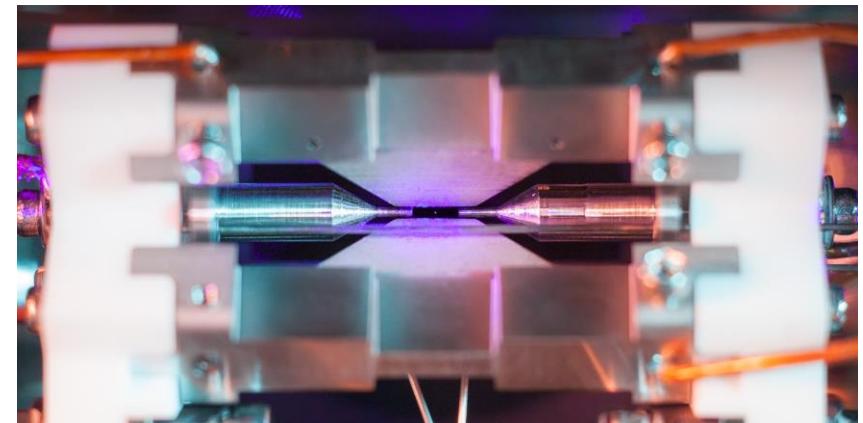
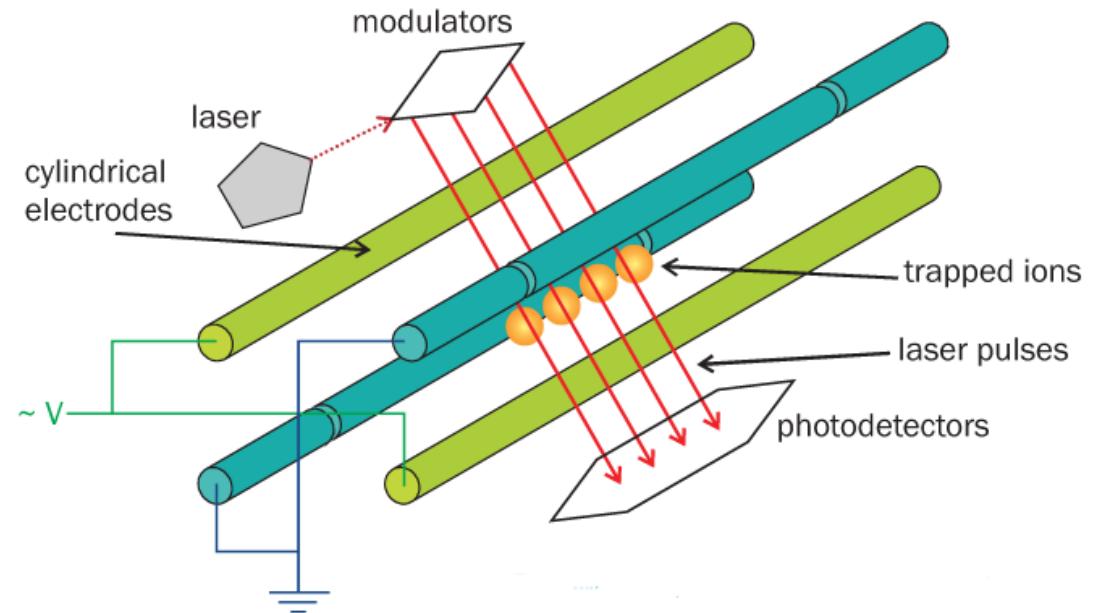
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- In this case, trapped ions are used as qubits



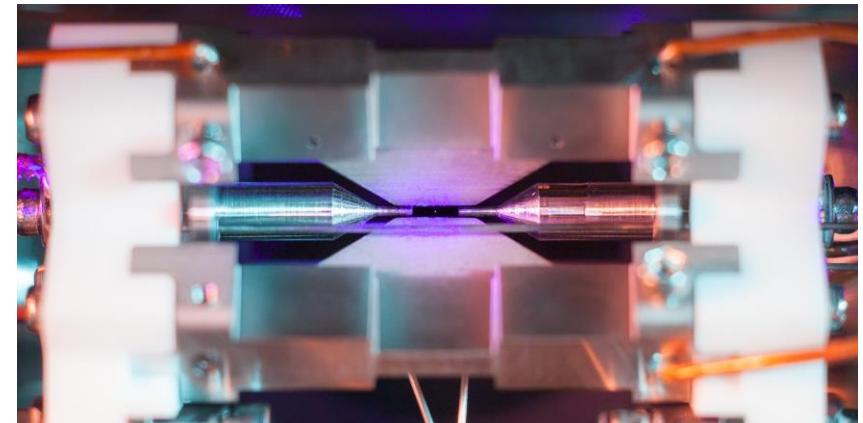
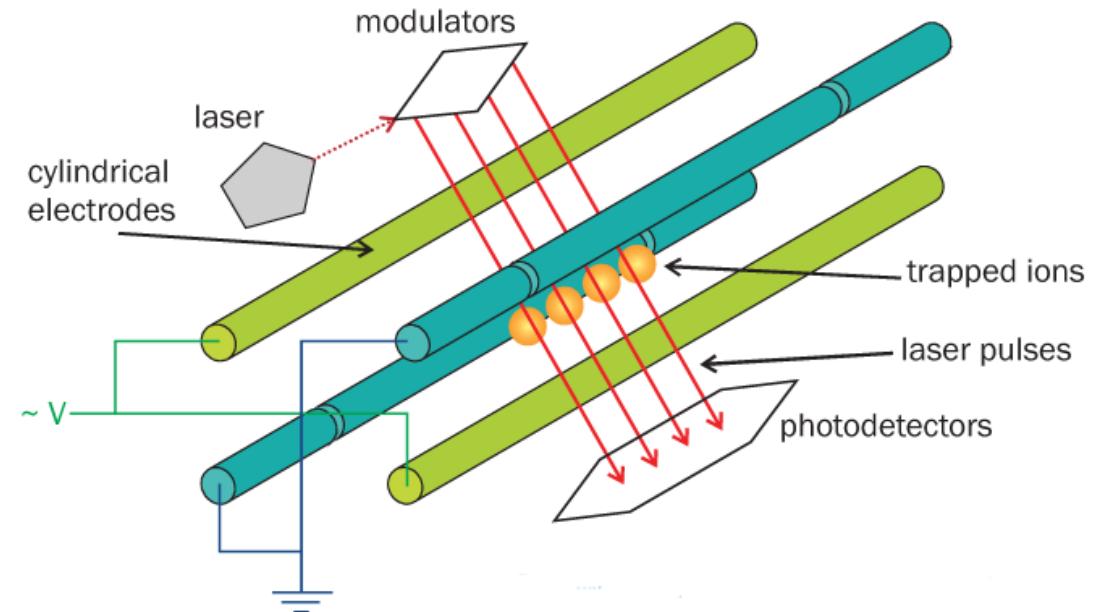
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- Through a mechanism like the one in the figure it is possible to trap an ion which can be manipulated using laser pulses



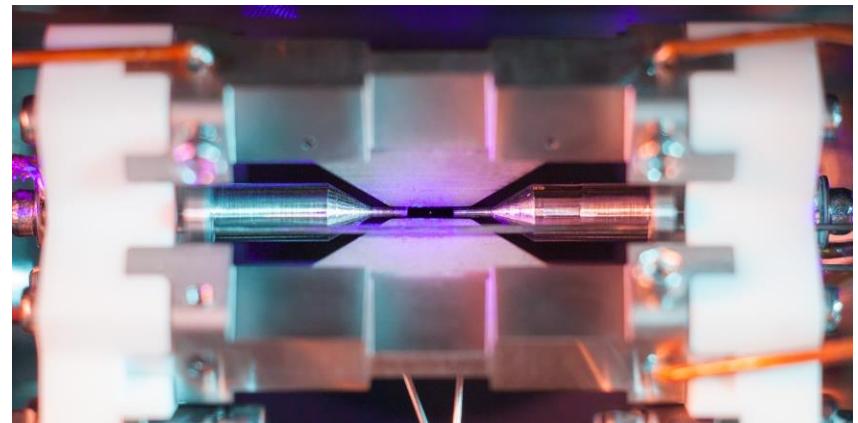
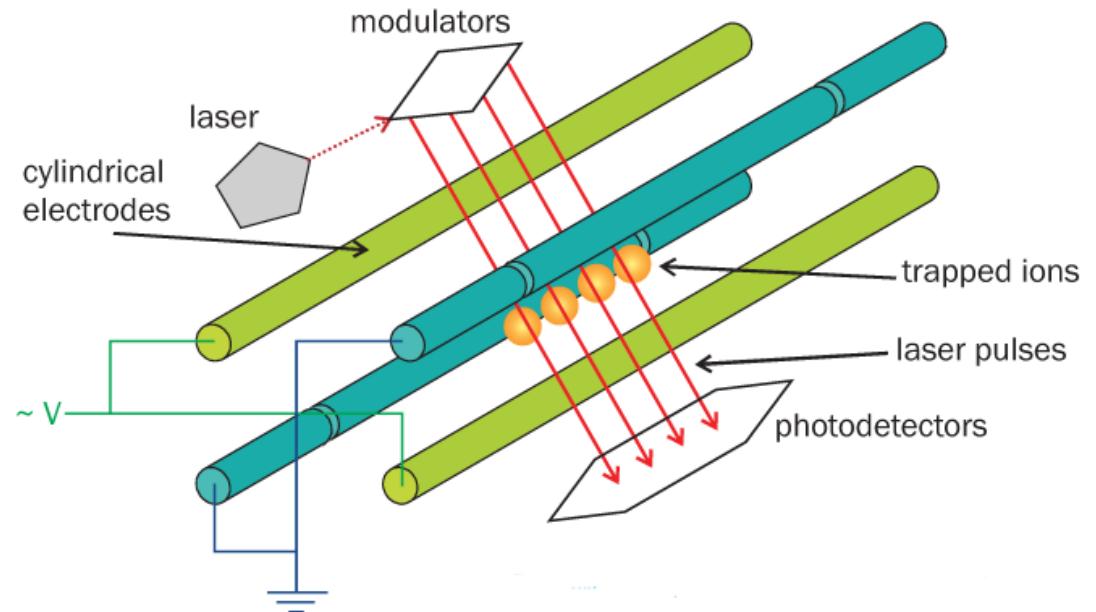
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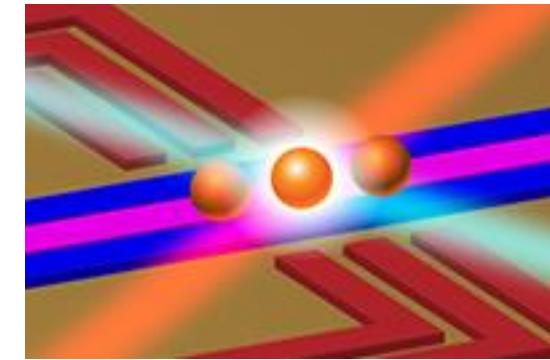
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- Through a mechanism like the one in the figure it is possible to trap an ion which can be manipulated using laser pulses
- This is also a very interesting technology, albeit not so mature
- A great strength of this technology is the connectivity of the qubits: thanks to the Coulomb forces, every chipset made with trapped-ion atoms will always naturally be completely connected.



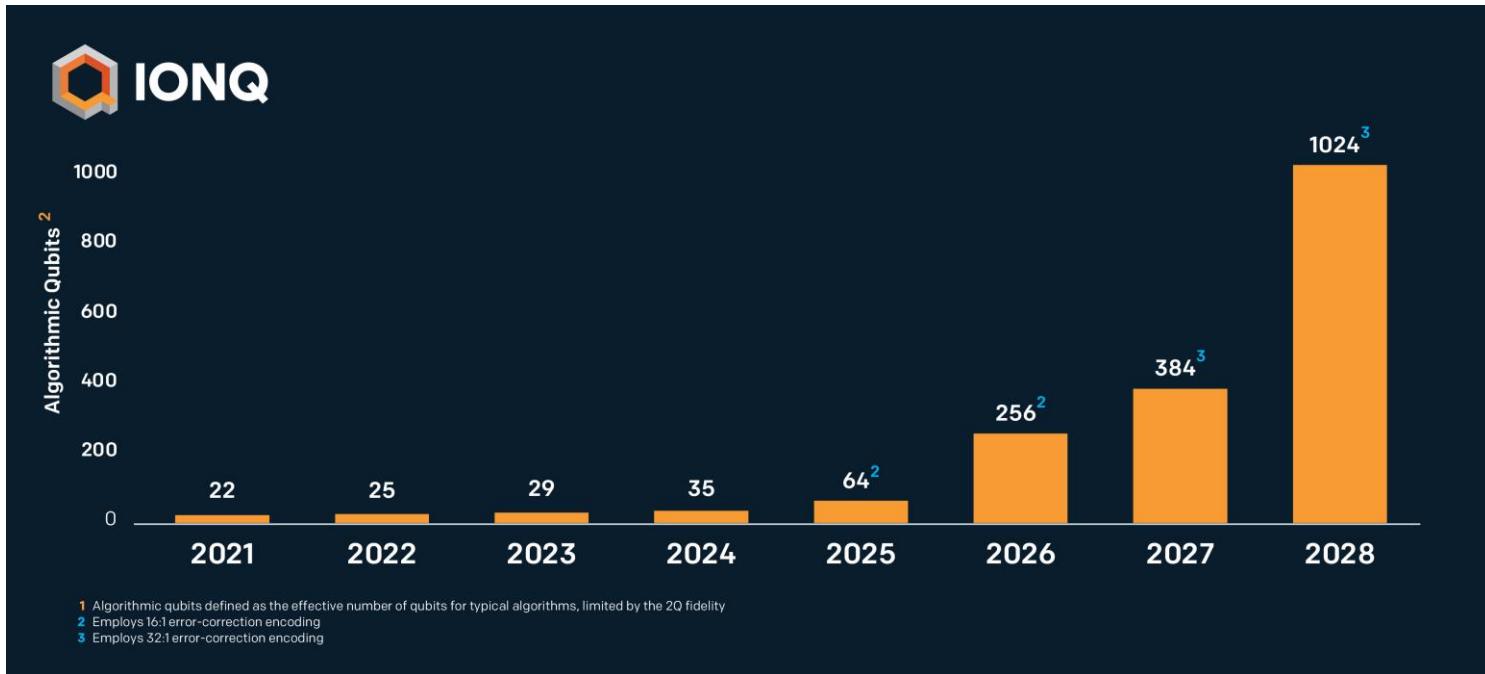
# Trapped Ions Qubits Technology



- IonQ is a quantum computing hardware and software company based in College Park, Maryland. They are developing a general-purpose trapped ion quantum computer and software to generate, optimize, and execute quantum circuits
- It was founded in 2015
- They are considered the most advanced startup in their field
- Their biggest machine is a fully connected quantum computer with 32 qubits
- They consider it the most powerful quantum computer in the world, but to date no work has yet come out confirming this.



# Trapped Ions Qubits Technology



## Algorithmic Qubit Calculator

System Parameters

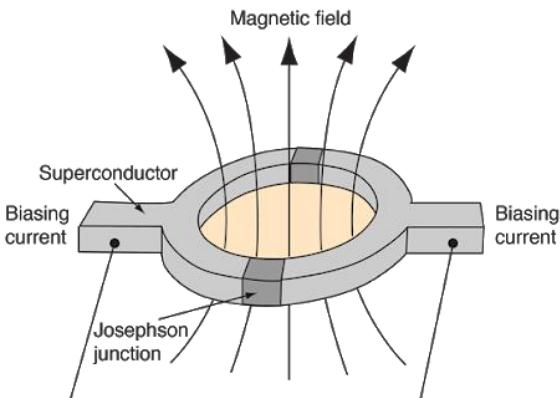
Physical Qubits <small>?</small>	Average 2Q Fidelity <small>?</small>	Error Correction Overhead <small>?</small>
32	99.90%	16:1
Connectivity <small>?</small>	Weight of Errors Corrected <small>?</small>	
fully-connected	1	

Quantum Compute Power

Algorithmic Qubits	Expected QV <small>?</small>	Methodology <small>?</small>
22	<b>4.19E+06</b>	Physical Qubits

Hence, we introduce Algorithmic Qubits (AQ), which is defined as the largest number of effectively perfect qubits you can deploy for a typical quantum program<sup>1</sup>. It's a similar idea to Quantum Volume, but takes error-correction into account and has a clear, direct relationship to qubit count. In the absence of error-correction encoding,  $AQ = \log_2(QV)$ , or inversely,  $QV = 2^{AQ}$ . AQ represents the number of "useful" encoded qubits in a particular quantum computer and is a simple proxy for the ability to execute real quantum algorithms for a given input size.

# Building a Quantum Computer

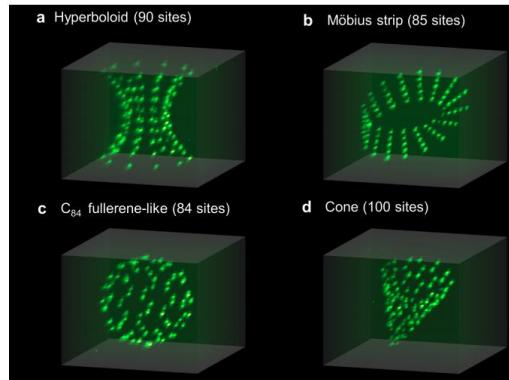


## SUPERCONDUCTING PRO

- Macroscopic
- Very large experience
- Fast operations

## CONS

- Very low coherence time
- Low fidelity
- Need to be cooled down

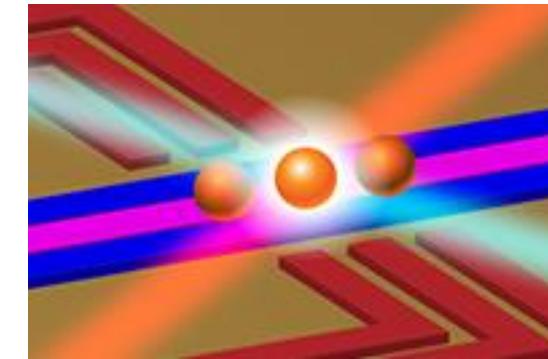


## NEUTRAL ATOMS PRO

- Good coherence
- interchangeable topology
- Room temperature

## CONS

- Experimental technology



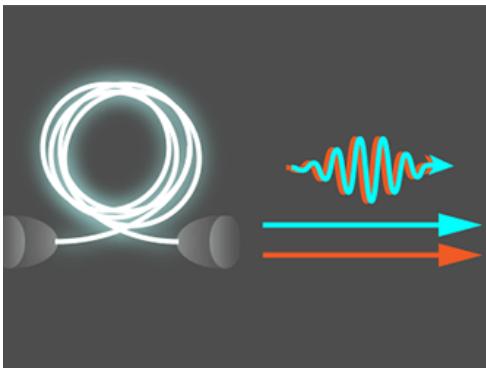
## TRAPPED IONS PRO

- Very long coherence
- Promising fidelity
- Room temperature

## CONS

- Slow operations
- Many lasers needed

# Building a Quantum Computer

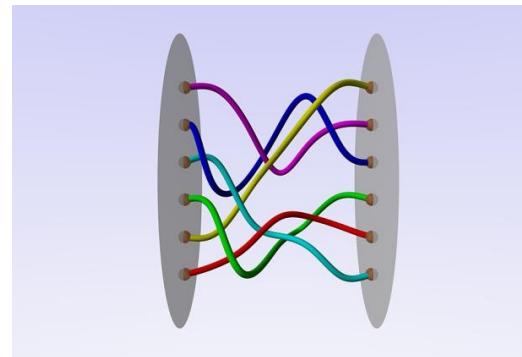


## PHOTONIC PRO

- Good scalability
- Room temperature

## CONS

- Very experimental technology
- Connectivity not clear

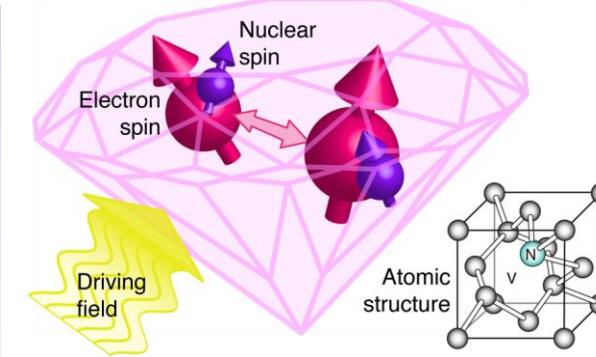


## TOPOLOGICAL PRO

- Almost perfect on paper

## CONS

- Existence not confirmed



## DIAMOND VACANCIES PRO

- Good coherence time
- Room temperature

## CONS

- Not so precise
- Difficult to entangle

# Emulate a Quantum Computer

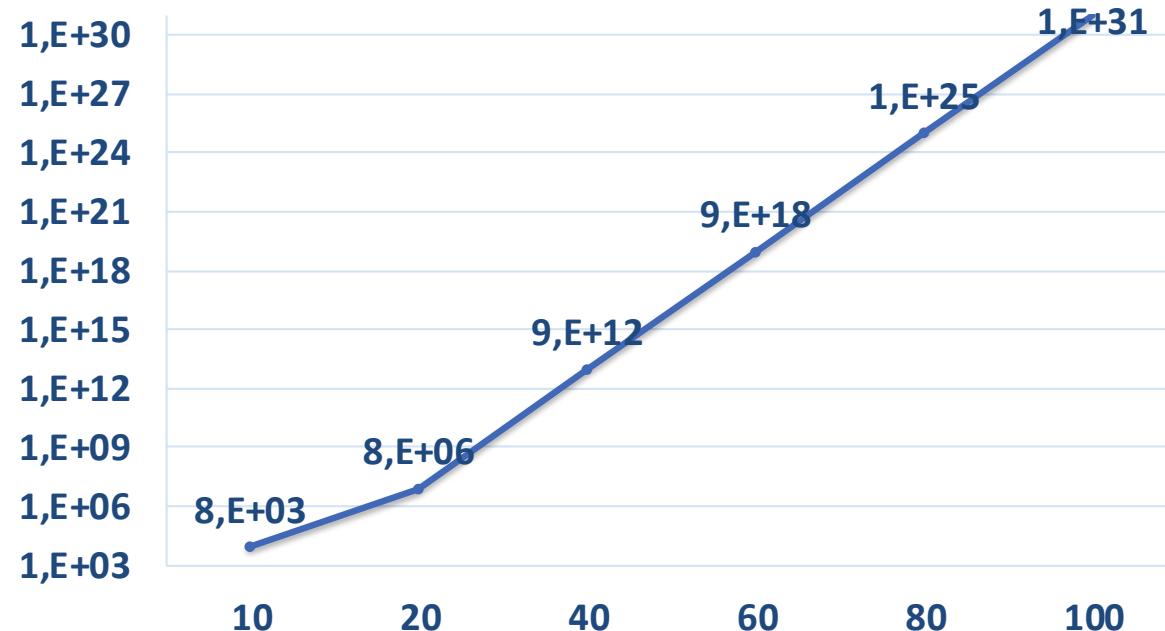
- Emulating a quantum computer on a classic computer is possible, but it requires a considerable amount of resources, especially if you want to simulate a fair amount of qubits.
- Emulating a qubit requires a lot of RAM:

8 (4 + 4) bytes (for the coefficient) X 2 bits (the states)

In general, emulating N qubits requires

$8 \times 2^N$  bytes

- Furthermore, the emulation of a quantum circuit requires computational time for the application of quantum gates. Natural phenomena in real quantum computers, in classical computers they are calculations to be made, often very expensive



kilo-	k or K **	$10^3$
mega-	M	$10^6$
giga-	G	$10^9$
tera-	T	$10^{12}$

peta-	P	$10^{15}$
exa-	E	$10^{18} *$
zetta-	Z	$10^{21} *$
yotta-	Y	$10^{24} *$

# Emulate a Quantum Computer

MARCONI - 100

**Nodes: 980**

Processors: 2x16 cores IBM POWER9

AC922 at 3.1 GHz

Accelerators: 4 x NVIDIA Volta V100

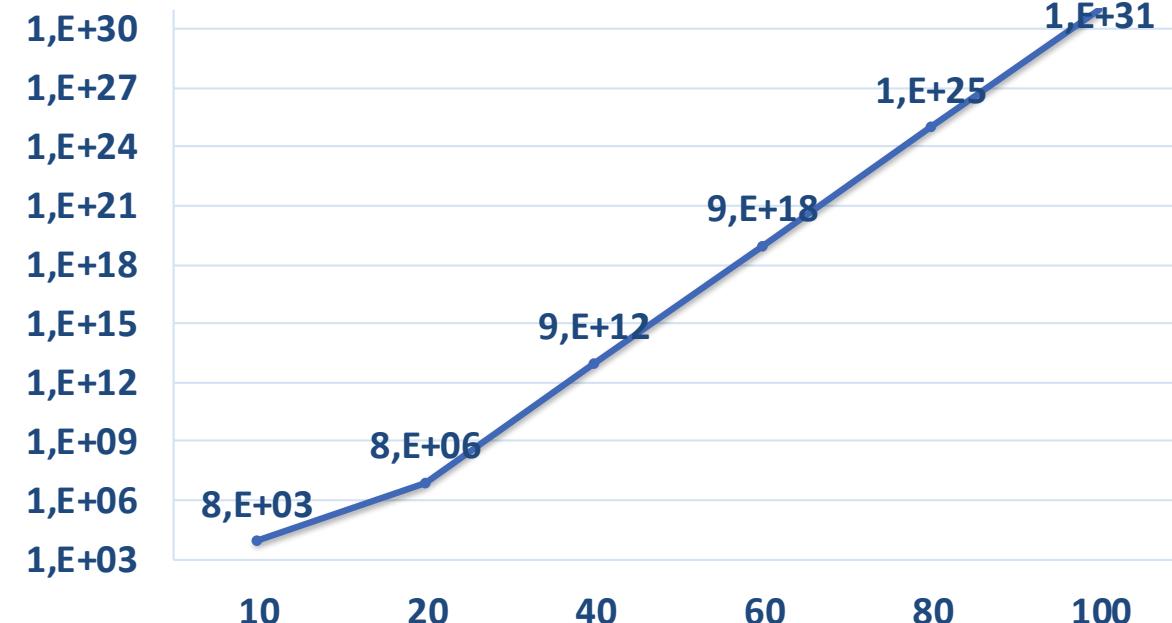
GPUs, Nvlink 2.0, 16GB

Cores: 32 cores/node

**RAM: 256 GB/node**

Peak Performance: ~32 PFlop/s

$$256 \times 980 = 250880 \text{ GB} = 2,5E+14$$



kilo-	k or K **	$10^3$
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# Quantum Computing In The World

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# Quantum Computing In The World

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- 2018: National Quantum Initiative Act
- 1.2 Billion \$ over 10 years + 625 million \$ over 5 years for five research centres + 340 million \$ from the private sector
- Big Tech Companies: Google, IBM, Intel, Microsoft (<100 qubits)
  - Google: *Quantum Supremacy* with superconducting qubits (October 2019).
  - IBM: very aggressive roadmap, more than 1000 qubits in 2023
  - Microsoft: no QC yet, developed Azure Quantum Network. Searching for the Holy Grail of the Qubits: The topological realization
- Very Promising Start-up:
  - Rigetti Computing: QC producer, superconducting qubits technology. Announced 128 qubits model
  - IonQ: Trapped-Ion quantum computers
  - Honeywell: Trapped-Ion quantum computers
  - Xanadu: Photonic quantum computers
  - D-WAVE (Canadian start-up, to be precise): manufacturers of *Quantum Annealers*, special purpose quantum computers able to solve optimization problems. More than 5000 qubits

# Quantum Computing In The World

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- China had made quantum technology a key priority in its thirteenth five-year plan (2016-2020)
- *Micius: Quantum Experiments at Space Scale*: carrying out a range of ground-to-space experiments in quantum communication
- 2017: \$10 billion for national laboratory for quantum information sciences
- December 2020: China claims to have achieved **quantum supremacy** with *Jiuzhang*, a photonic quantum computer capable of detecting up to 76 photons (USTC: University of Science and Technology of China)
- Big Tech Companies: Alibaba, Baidu
  - In 2015, Alibaba set up its own quantum computing laboratory in order to produce a **prototype 50 to 100 qubit quantum computer** for general use by 2030
  - Baidu, the company behind the eponymous search engine, announced the creation of a quantum computing institute in March 2018



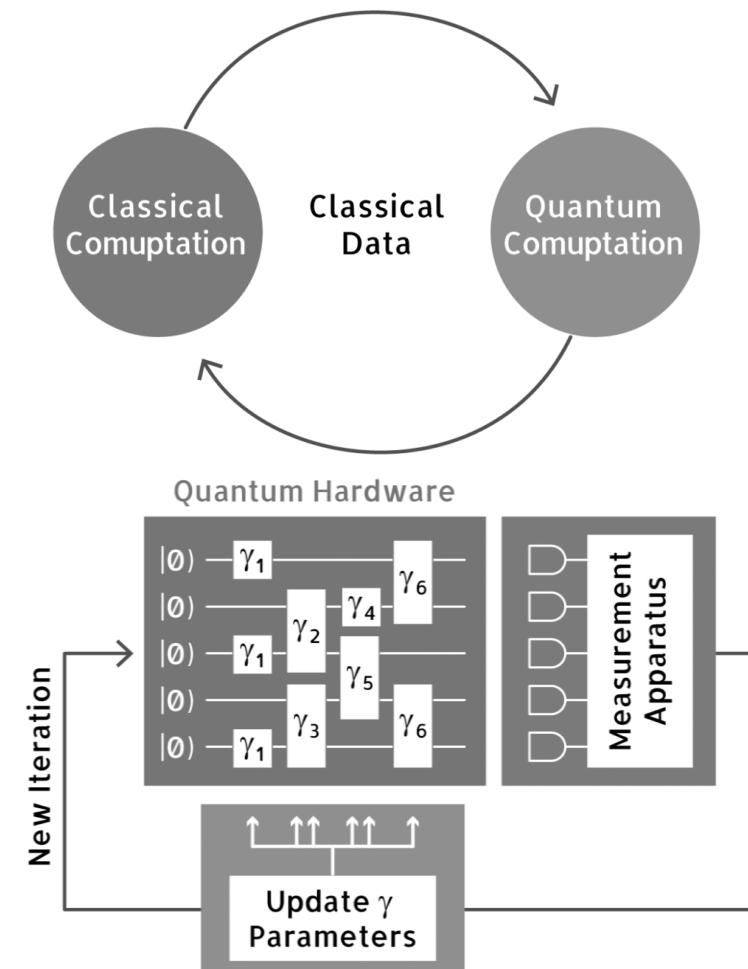
# Quantum Computing In The World

Company	Type	Technology	Now	Next Goal
Intel	Gate	Superconducting	49	TBD
Google	Gate	Superconducting	53 / 72	TBD
IBM	Gate	Superconducting	64	127
Rigetti	Gate	Superconducting	32	128
USTC (China)	Gate	Superconducting	10	20
USTC (China)	Gate	Photonic	76	TBD
IonQ	Gate	Ion Trap	32	79
IQOQI/Univ. Ulm/Univ. Innsbruck	Gate	Ion Trap	20	TBD
NSF STAQ Project	Gate	Ion Trap	N/A	≥64
Intel	Gate	Spin	26	TBD
Silicon Quantum Computing	Gate	Spin	N/A	10
CEA-Leti/INAC/Institut Néel	Gate	Spin	N/A	100
Pasqal	Gate	Neutral Atoms	100	1000
D-Wave	Annealing	Superconducting	5000+	TBD

# Far from Universal QC, entering the NISQ era

- Although the results obtained so far do not allow us to observe significant applications of the most famous quantum algorithms, this does not mean that the computers produced so far are completely useless.
- Some examples of quantum supremacy have been achieved
- Before reaching universal quantum supremacy, we will arrive at local quantum supremacy
- By combining the power of quantum computers and current supercomputers, we can achieve great benefits
- Quantum computers that exist today are about to reach the power needed to run hybrid algorithms (NISQ Computers)

## HYBRID ALGORITHM



# And what about Europe?



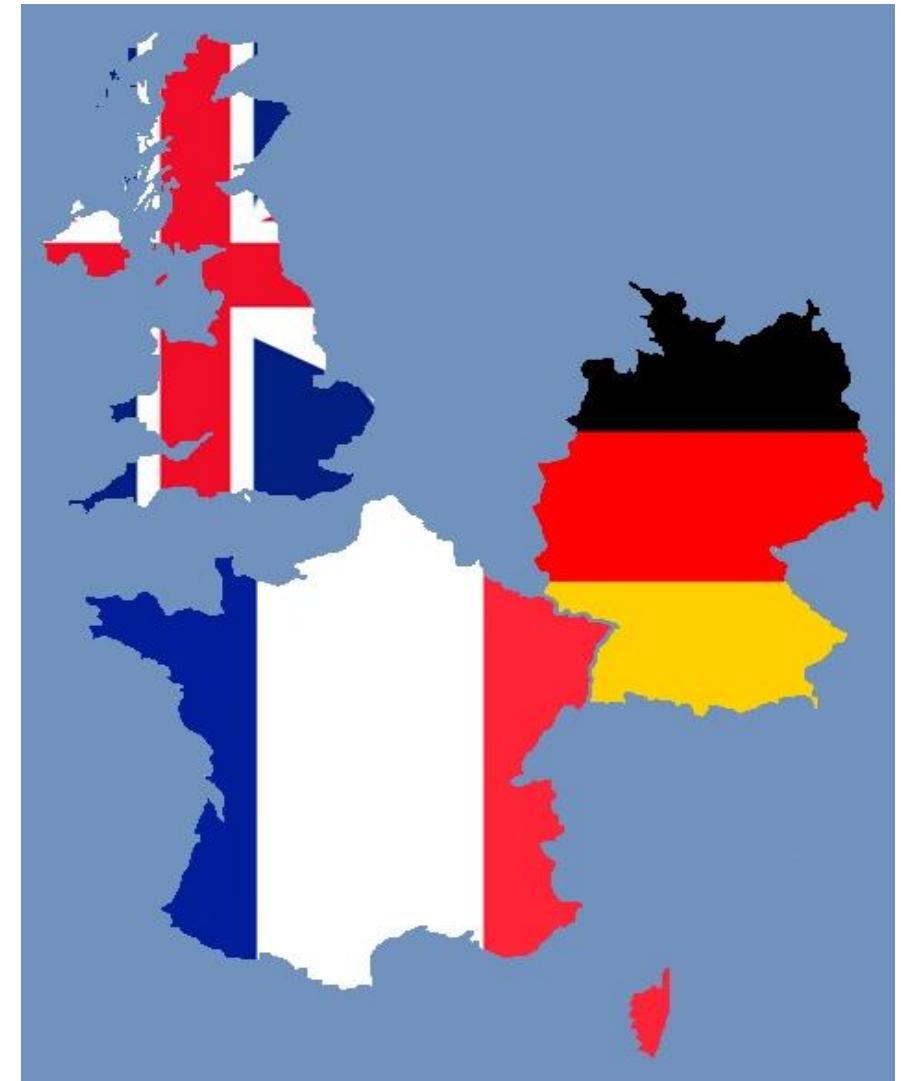
**EuroHPC**  
Joint Undertaking



- May 2016: over 3400 figures from research and corporate world signed the *Quantum Manifesto*
  - The Manifesto, addressed to the European Commission, said in essence: *we have the opportunity to compete for a new kind of technological independence, let's take it.*
- October 2018: The European Commission launched the **Quantum Flagship programme**: 1.3 billions of Euro to support 10 year of quantum technologies research and development.
- The European High Performance Computing Joint Undertaking (EuroHPC JU) is a joint initiative between the EU, European countries and private partners to develop a World Class Supercomputing Ecosystem in Europe.
- The European Processor Initiative (EPI) is a project whose aim is to design and implement a roadmap for a new family of low-power European processors for extreme scale computing, high-performance Big-Data and a range of emerging applications.

# European Union countries quantum initiatives

- 2013: UK (Still in Europe!) became the **first country in Europe to announce its own quantum strategy**, investing €370 million over five years
  - 2018: realization of a National Quantum Computing centre with the aim to **build a Quantum Computer**
  - 2019: additional investment of £153 million in quantum computing
- 2018: Germany announced a framework programme with the aim to **develop quantum technologies**
  - The framework programme was funded with over €650 million
  - May 2021: Germany will spend about **2 billion euros** to support the development of its first quantum computer and related technologies in the next four years
- 2019: the French government instructs a task force to implement a **national strategy for quantum technologies**
  - January 2021: Emmanuel Macron announced the **National Quantum Plan**
  - The Quantum Plan provides for actions in support of research (especially for quantum computers, sensors, and communications), industry, and academic and professional training. It is financed with €1.8 billion

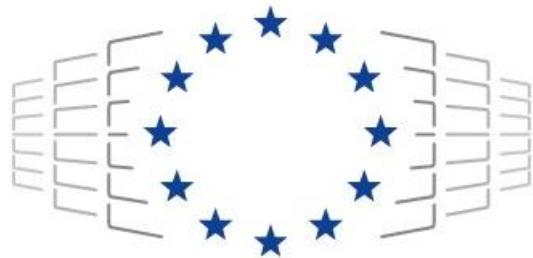


# Quantum Computing in Italy

- Quantum Computing is also becoming a reality in Italy.
- With the Italian Recovery Fund (*PNRR*) the Italian government has provided a large amount of money to be donated to **research on strategic issues**. Quantum Computing has been defined as such.
- For technology transfer, the government aims to use €1.3 billion to create 20 regional innovation hubs for research and development, jointly funded by the public and private sectors and modelled after the Fraunhofer institutes for applied research in Germany.
- Another €1.6 billion will be used to launch seven new centres on key technologies: artificial intelligence, **quantum computing**, agricultural technology, energy, hydrogen, technologies for finance and pharmaceutical research



# HPC and Quantum Computing: HPCQS Consortium

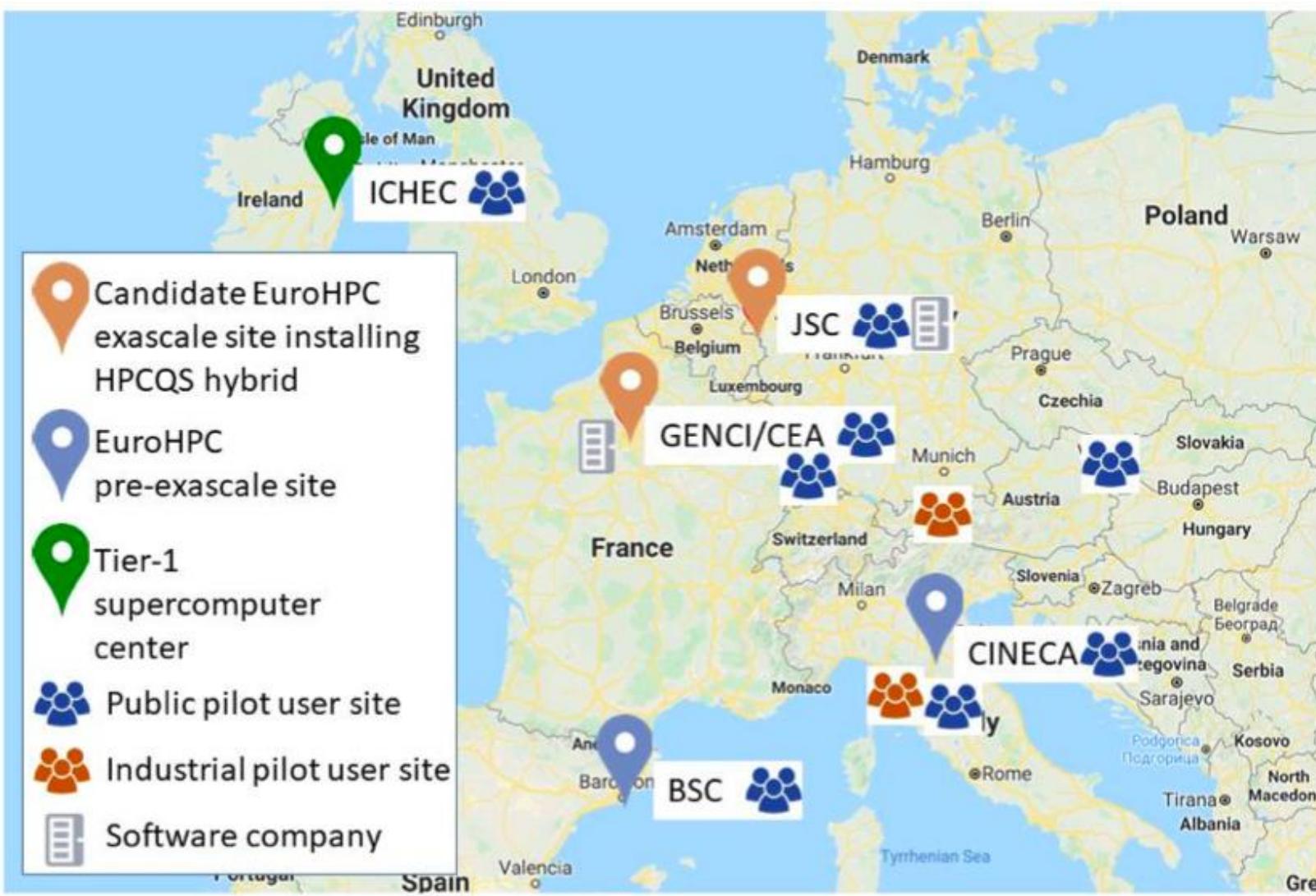


**EuroHPC**  
Joint Undertaking

FZJ (Coordinator)	CNRS
ParTec (LTP* <sup>1</sup> )	Sorbonne (LTP*)
CEA	SUPELEC (LTP*)
GENCI	INRIA
ATOS	Pasqal
CNR	CINECA
NUIG-ICHEC	BSC
UIBK	FLS
EURICE	Parity QC
	Fraunhofer IAF

- The HPCQS consortium was born with the idea of combining HPC and QC hardware and software.
- For the realization of Quantum Computers, the French company PASQAL was chosen, which produces quantum computers based on Neutral Atoms technology
- During the 4 years of the project, the most efficient way to connect Pasqal computers to EuroHPC supercomputers will be studied.
- In fact, latency between computing systems is a significant problem
- The ultimate goal of the project is the creation of an interconnected network of quantum computers throughout Europe, able to communicate with each other and through the support of EUROHPC supercomputers.

# HPC and Quantum Computing: HPCQS Consortium



EuroHPC  
Joint Undertaking



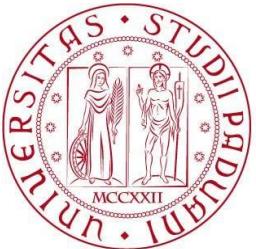
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# HPC and QC in Italy: role of CINECA

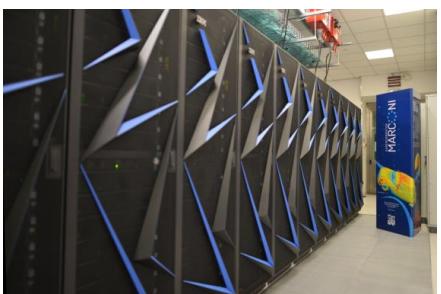


- CINECA, the Italian supercomputing center and inter-university consortium, started working in the field of quantum computing 3 years ago, in 2018.
- Today CINECA, together with the most important European computing centers, **is part of the HPCQS consortium**
- From March 2021 it distributes D-WAVE quantum computing hours and General Purpose Quantum Computing emulators free of charge to Italian universities through the ISCRA project.
- At the beginning of 2021 the CINECA Quantum Computing Lab was inaugurated ([www.quantumcomputinglab.cineca.it](http://www.quantumcomputinglab.cineca.it))

# HPC and QC in Italy: role of CINECA



UNIVERSITÀ  
DEGLI STUDI  
DI PADOVA



Also in March 2021 a collaboration with Pasqal began. This collaboration, which runs parallel to the HPCQS project, was born with the aim of testing the hardware status of the Pasqal machine.

- In addition to scientific research, CINECA is also developing a series of "HPC-ready" emulators based on different systems (including *tensor network* emulators of not perfect quantum systems, able to make the most of the available computational resources)
- On this front collaborations are proceeding with Pasqal and UniPD
- Not only software and algorithm development: organization of annual conferences (HPCQC, this year in its fourth edition) and schools (Introduction to Quantum Computing, first edition at the end of June 2021)