An Introduction
to
Quantum Natural Language Processing (QNLP)

Part 3 :

Diagrammatic Quantum Theory

Outline of the lecture

❖ Process Theory – Boxes and Wires

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- ❖ States, Effects & Numbers – Kets, Bras & Scalars

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- ❖ States, Effects & Numbers – Kets, Bras & Scalars
- ❖ Circuit Diagrams – Parallel & Sequential Composition
- ❖ String Diagrams – Cups & Caps

Process Theory – Boxes & Wires

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A Process Theory consists of :

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i. A collection T of system-types represented by wires

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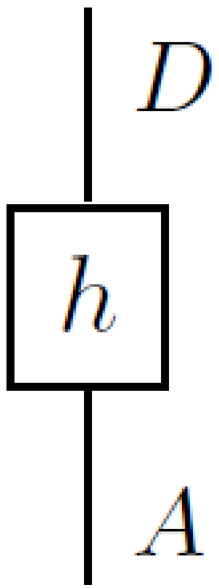
i. A collection T of system-types represented by wires

ii. A collection P of processes represented by boxes, where for each process in P, the input types and output types are taken from T

iii. A means of ‘wiring processes together’. An operation that interprets a diagram of processes in P as a process in P

Process Theory – Boxes & Wires



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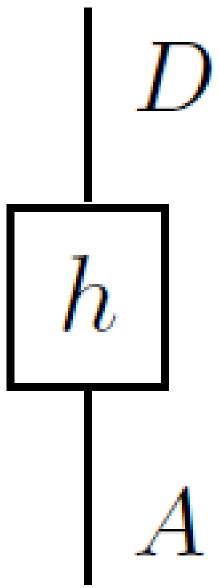
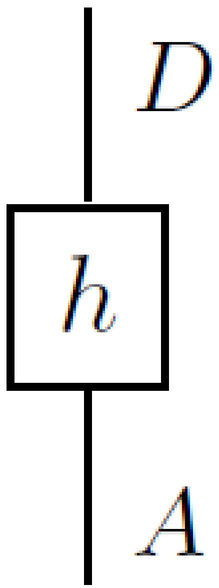


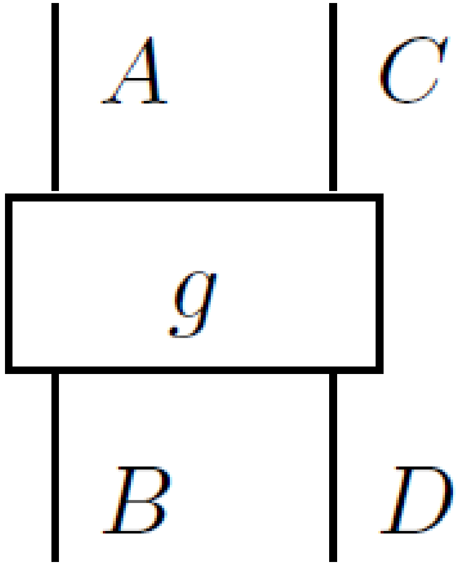
- Box named ‘h’ is called a process

Process Theory – Boxes & Wires



- Box named ‘h’ is called a process
- The wires ‘A’ and ‘D’ represent the inputs and outputs respectively. These wires are the system-types or types

Process Theory – Boxes & Wires



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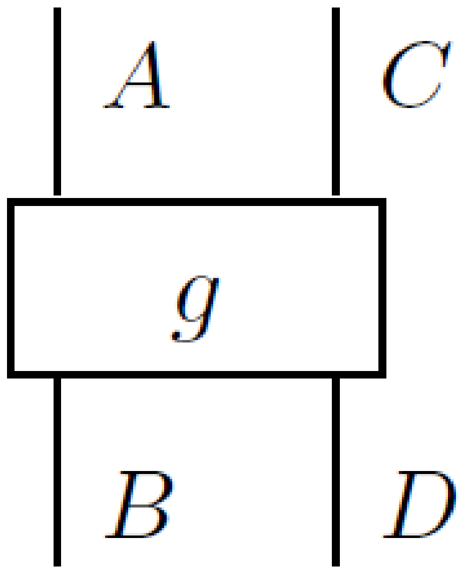
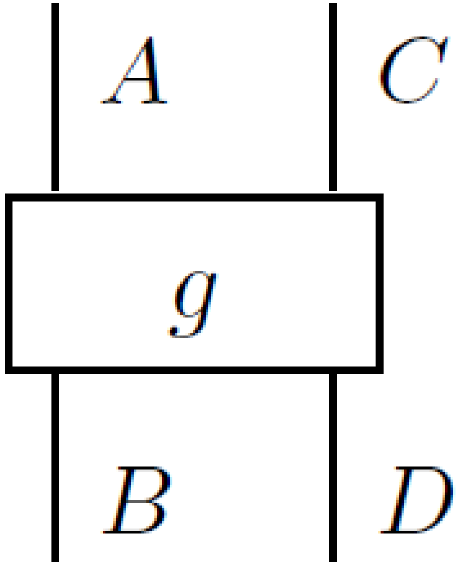


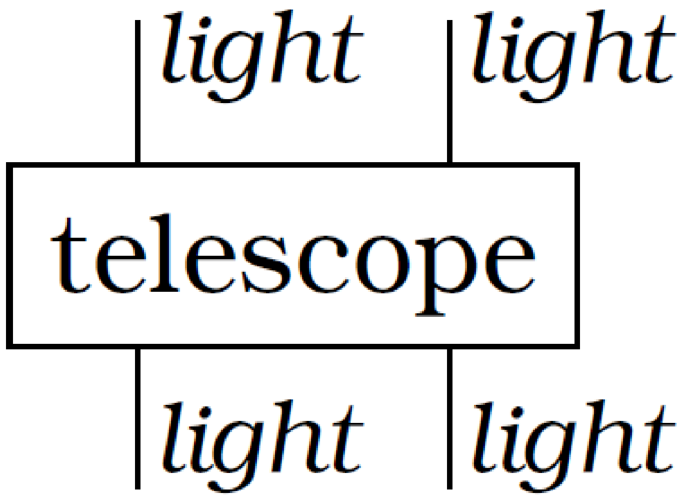
- Box named ‘g’ is called a process

Process Theory – Boxes & Wires



- Box named ‘g’ is called a process
- The wires ‘B’ and ‘D’ represent the inputs, ‘A’ and ‘C’ represent the outputs. These wires are the system-types or types

Process Theory – Boxes & Wires



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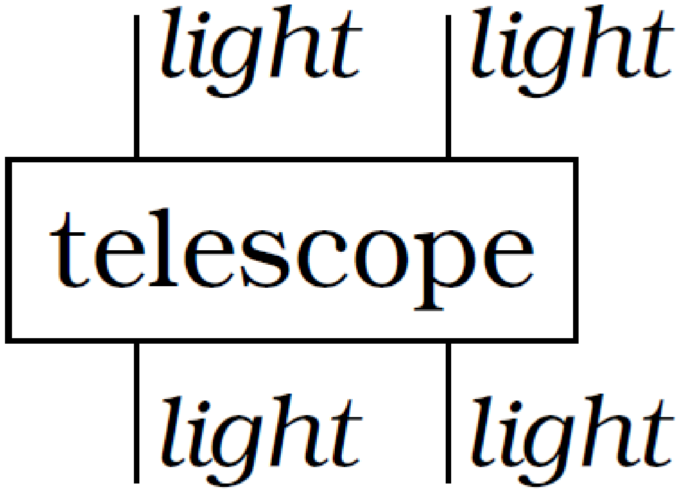

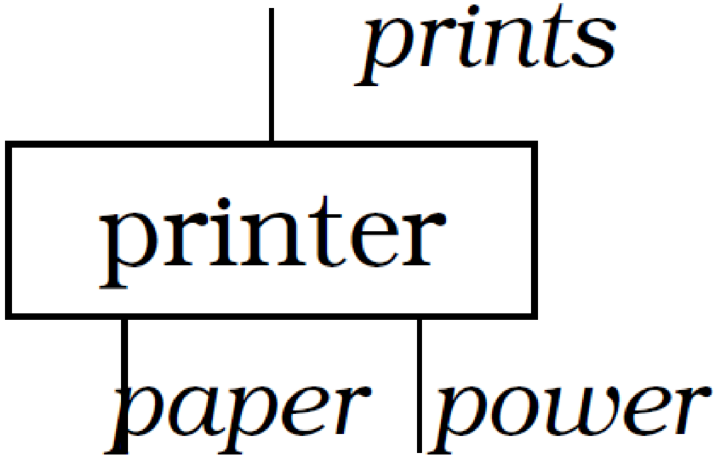

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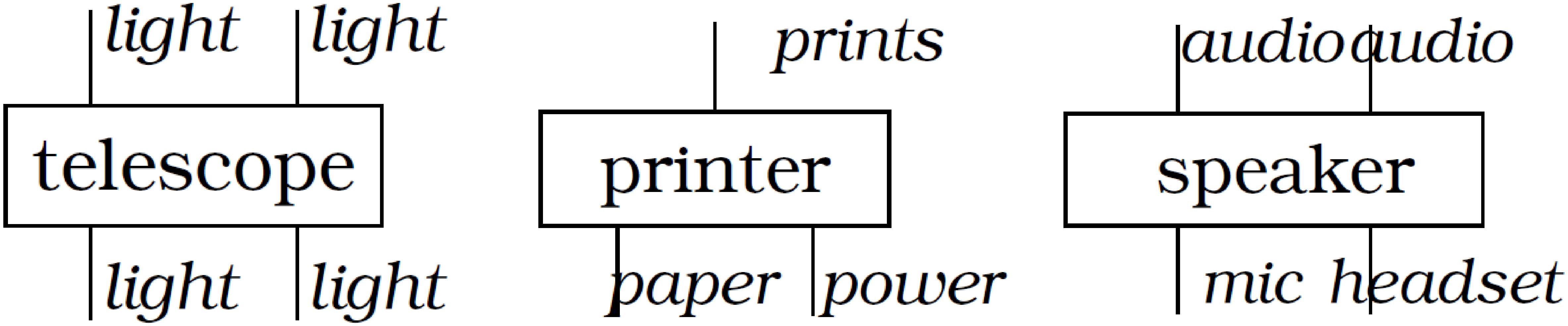

Process Theory – Boxes & Wires

Examples

- Functions (types = sets)

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- Relations (types = sets)

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Examples

- Functions (types = sets)
- Relations (types = sets)
- Linear maps (types = vector or Hilbert spaces)
- Classical Processes (types = classical systems)
- Quantum Processes (types = classical & quantum systems)

Process Theory – Boxes & Wires

Process Theory Concludes

States, Effects & Numbers – Kets, Bras & Scalars

States – Dirac Kets

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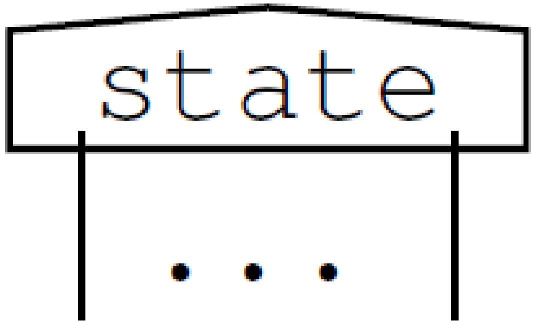
States – Dirac Kets



States, Effects & Numbers – Kets, Bras & Scalars

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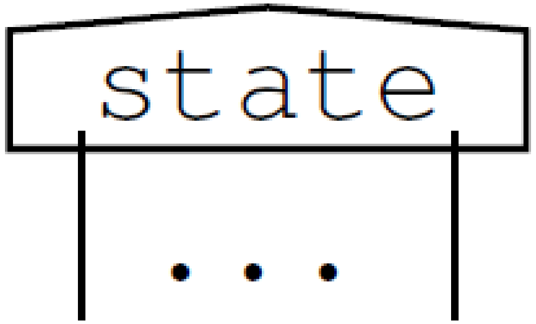

- Denoted by |state>

States, Effects & Numbers – Kets, Bras & Scalars

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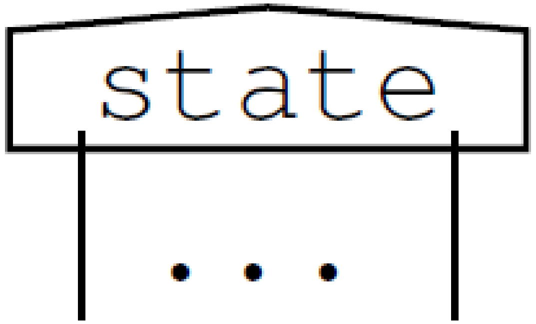

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- These processes do not have any inputs

States, Effects & Numbers – Kets, Bras & Scalars

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- Used to model tests

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Effects – Dirac Bras

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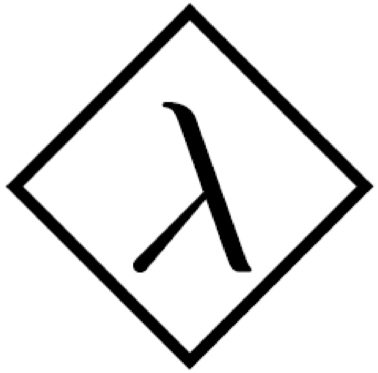
- These processes do not have any outputs and dual to states
- Used to model tests
- Example is discarding of a system

States, Effects & Numbers – Kets, Bras & Scalars

Numbers – Scalars

States, Effects & Numbers – Kets, Bras & Scalars

Numbers – Scalars



States, Effects & Numbers – Kets, Bras & Scalars

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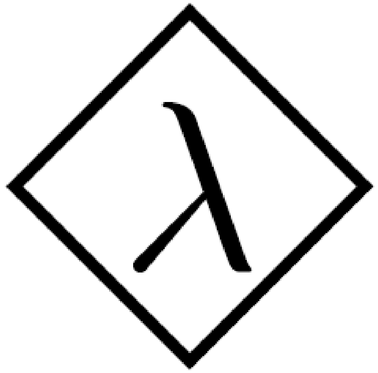

or

States, Effects & Numbers – Kets, Bras & Scalars

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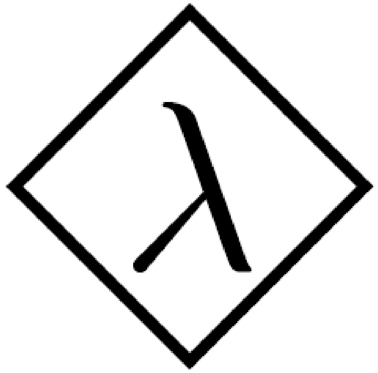

or

$$\lambda$$

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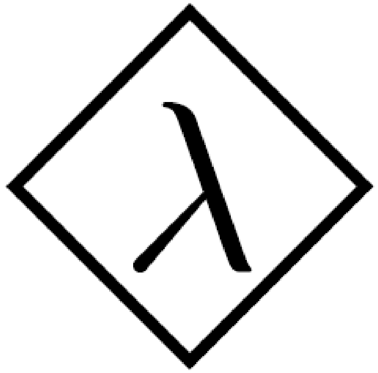

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States, Effects & Numbers – Kets, Bras & Scalars

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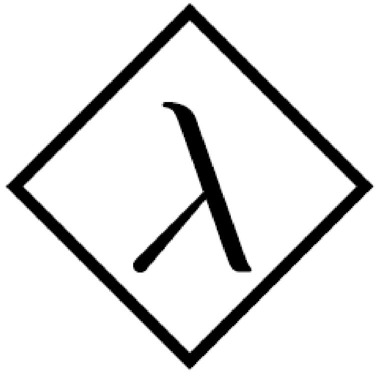

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- Processes without any inputs and outputs
- Represent scalar values

States, Effects & Numbers – Kets, Bras & Scalars

States, Effects & Numbers Concludes

Circuit Diagrams – Parallel & Sequential Composition

Parallel Composition

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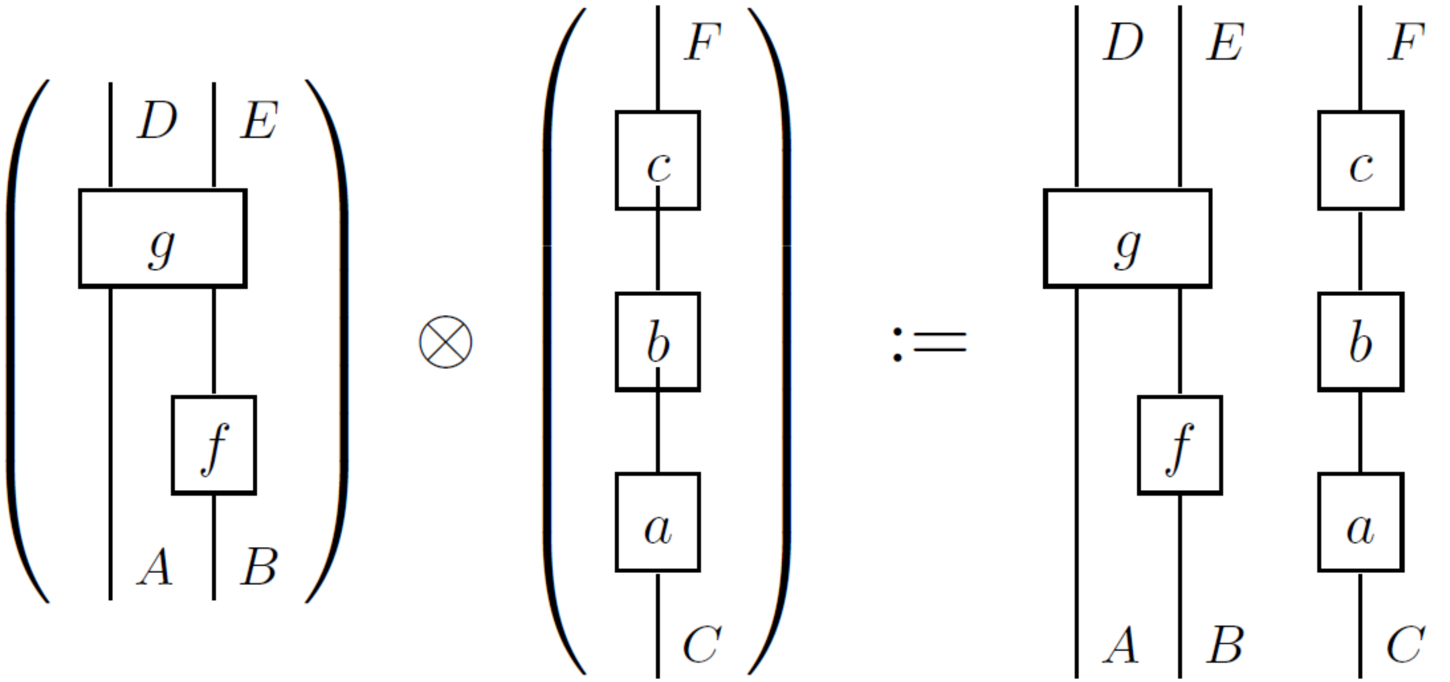
Parallel Composition



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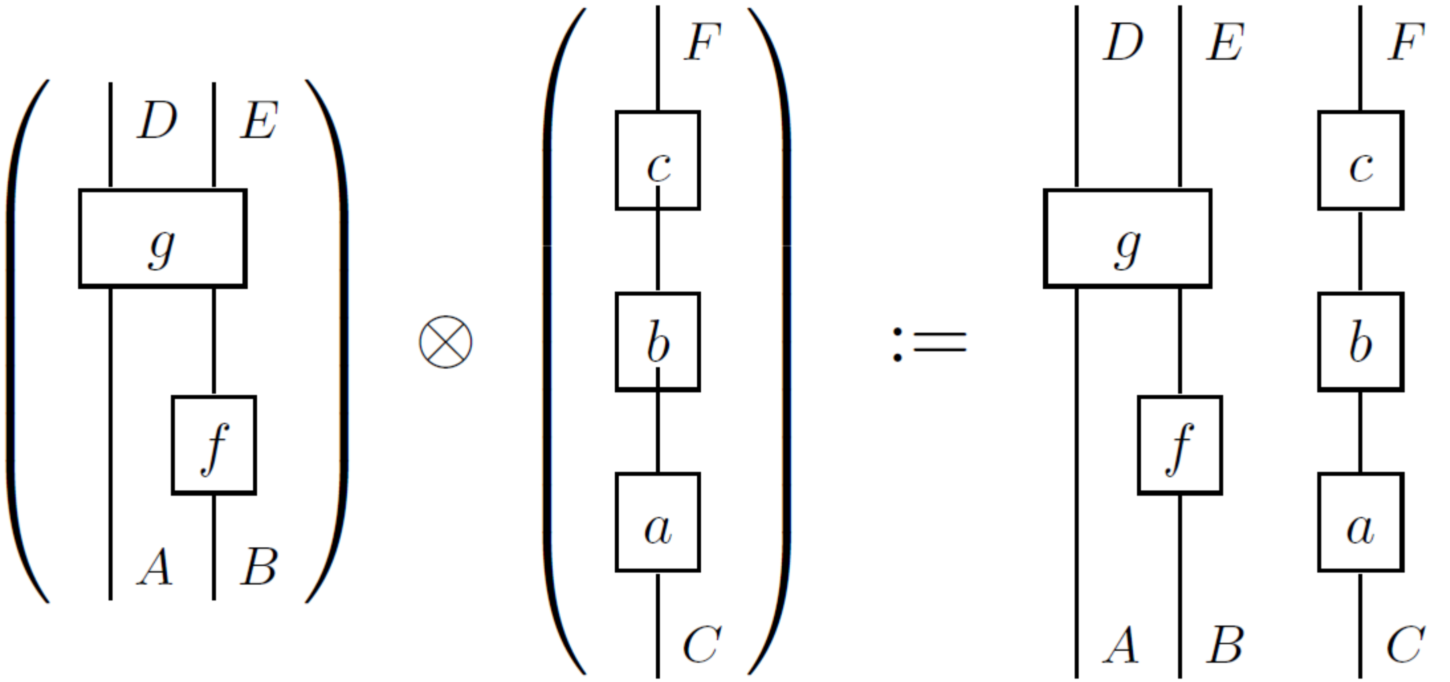

- Denoted by ⊗ symbol and works by placing diagrams side-by-side

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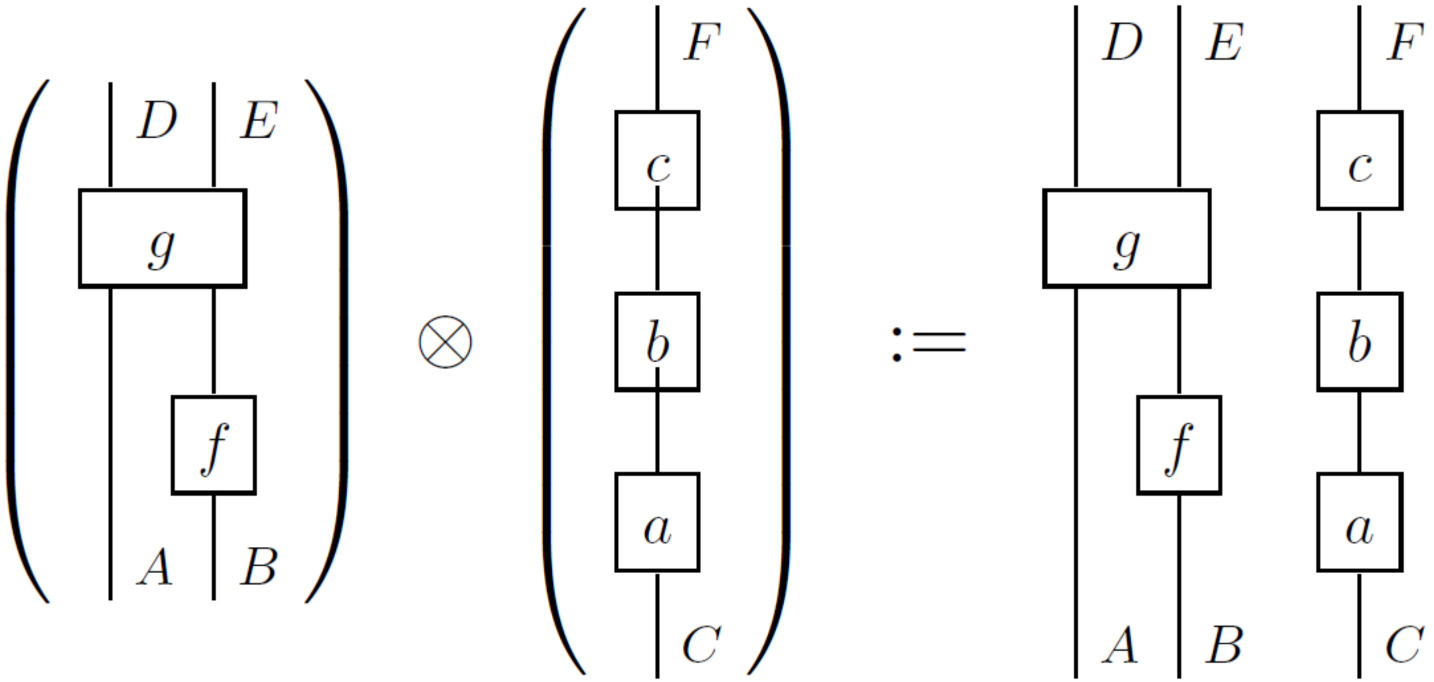

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- Follows associativity property and has unit too

Circuit Diagrams – Parallel & Sequential Composition

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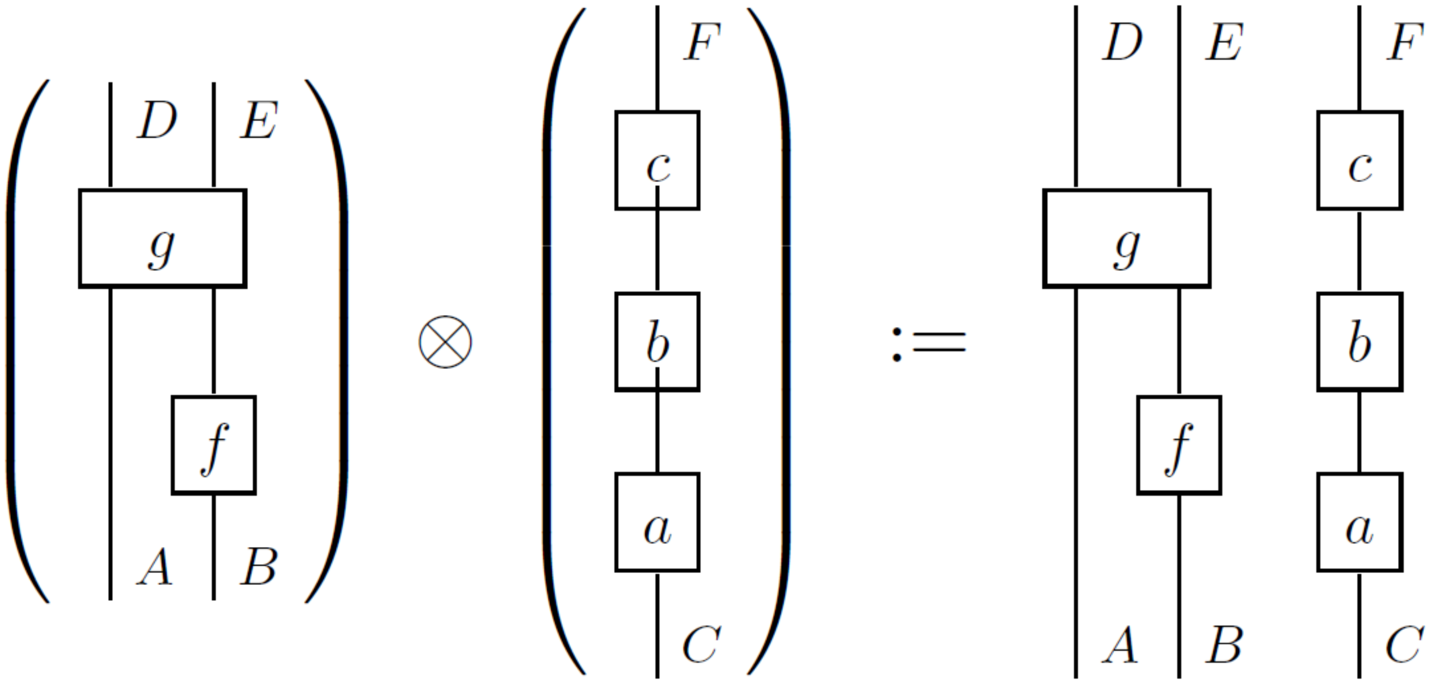

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- Follows associativity property and has unit too
- Also valid for system types

Circuit Diagrams – Parallel & Sequential Composition

Sequential Composition

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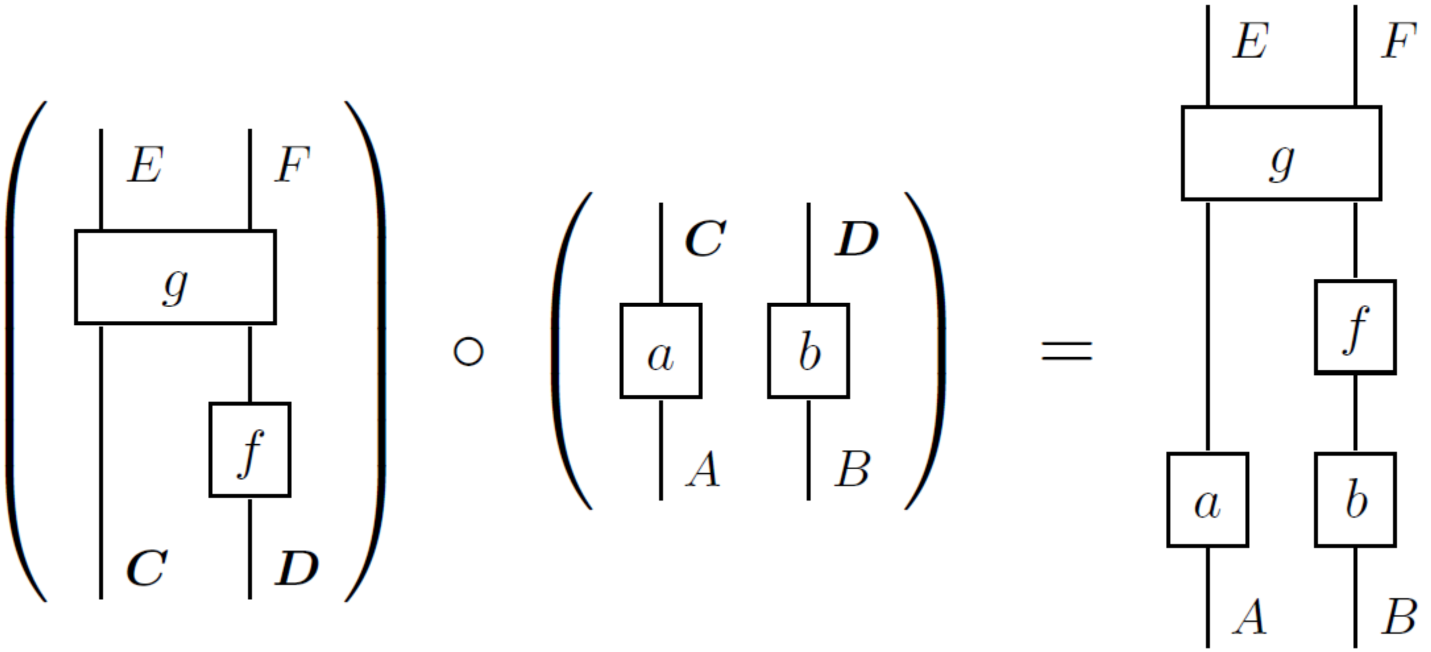
Sequential Composition



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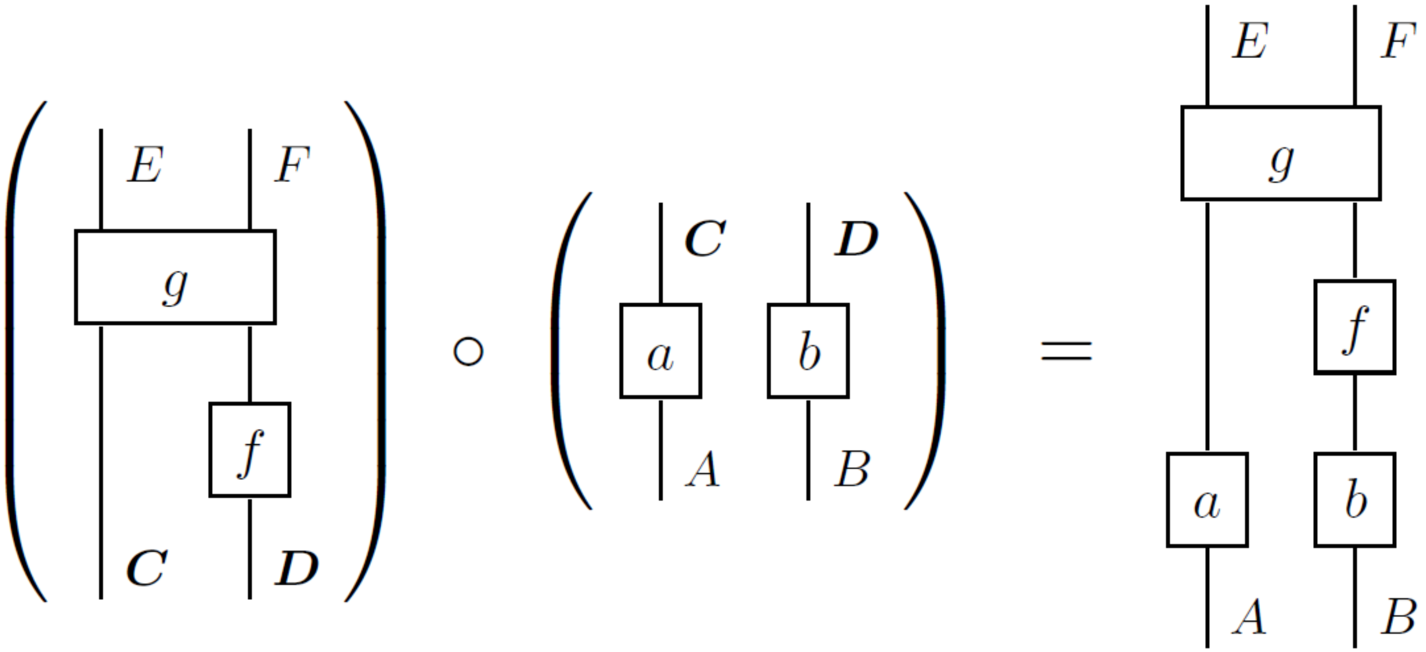
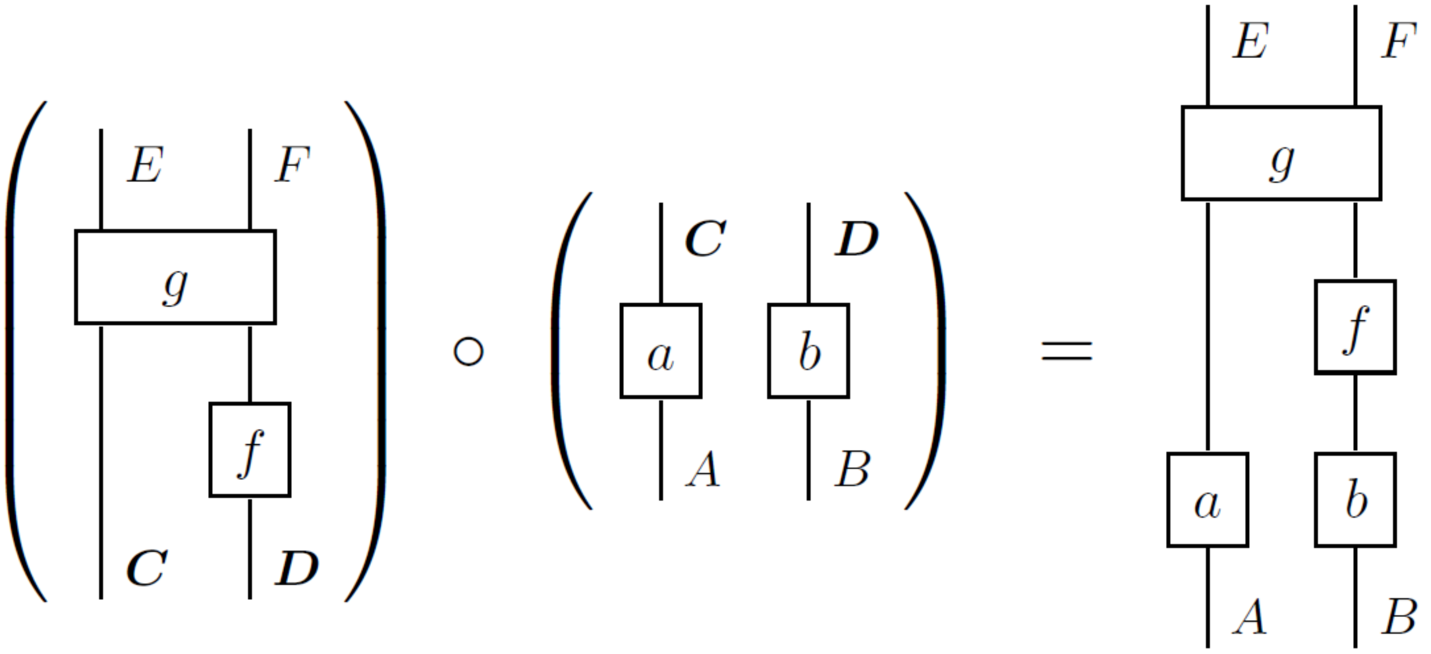


- Denoted by ∘ symbol

Circuit Diagrams – Parallel & Sequential Composition

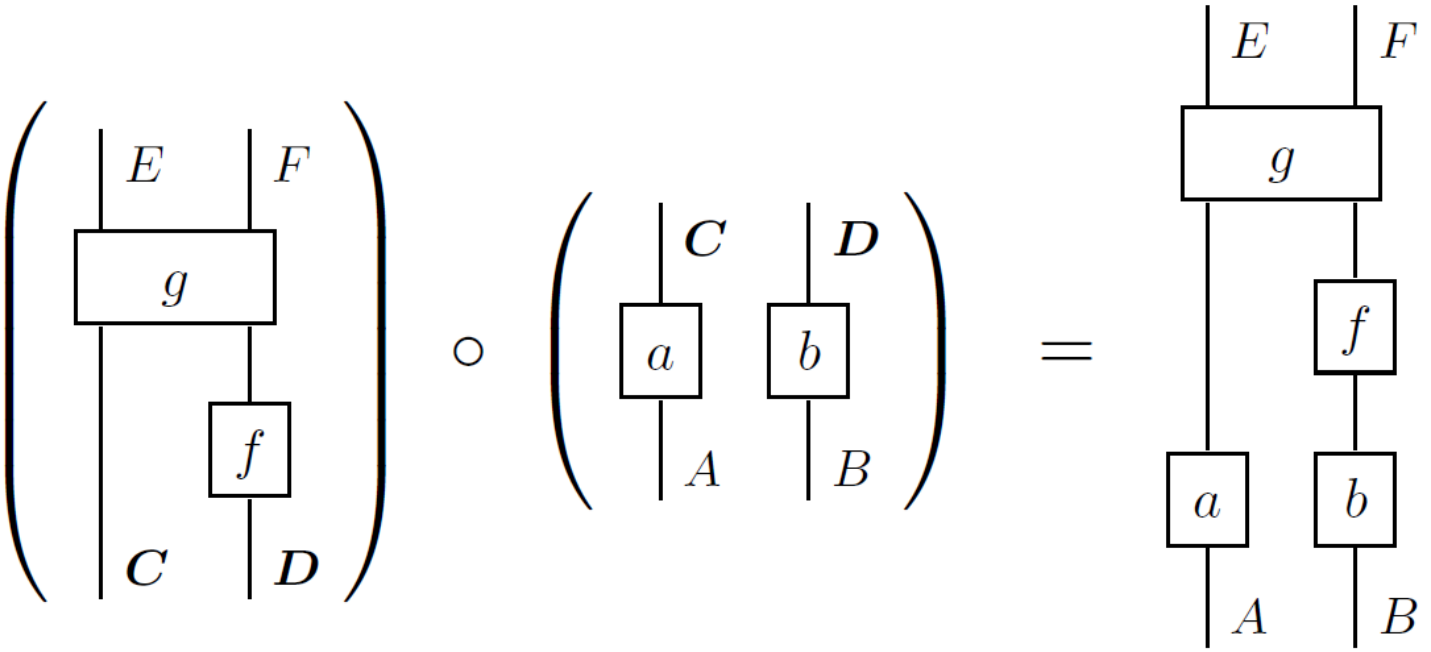
Sequential Composition



- Denoted by ∘ symbol
- Process on right happens first and then process on the left

Circuit Diagrams – Parallel & Sequential Composition

Sequential Composition



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Circuit Diagrams – Parallel & Sequential Composition

No Directed Cycles

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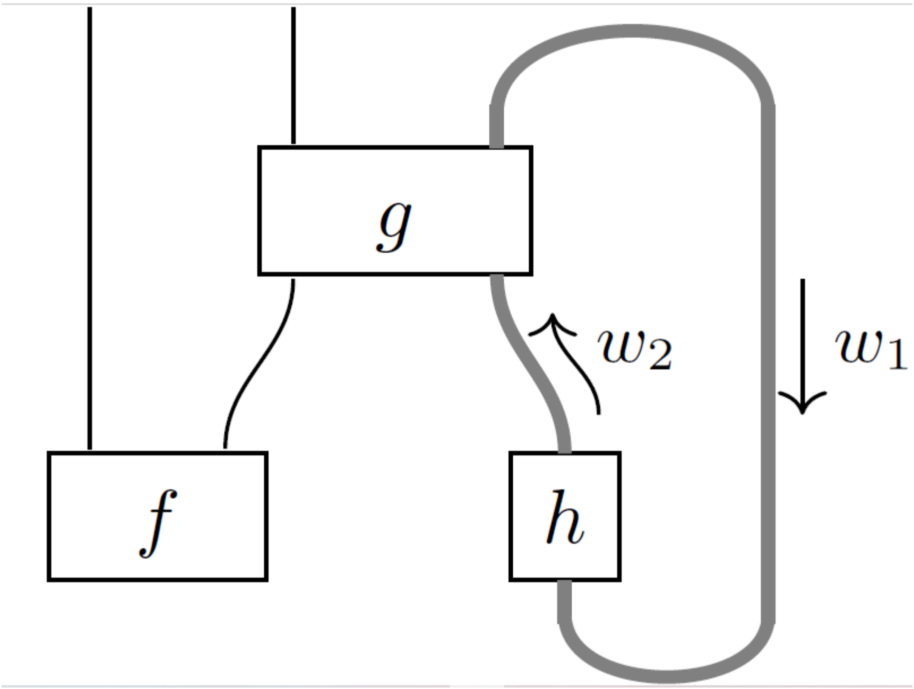
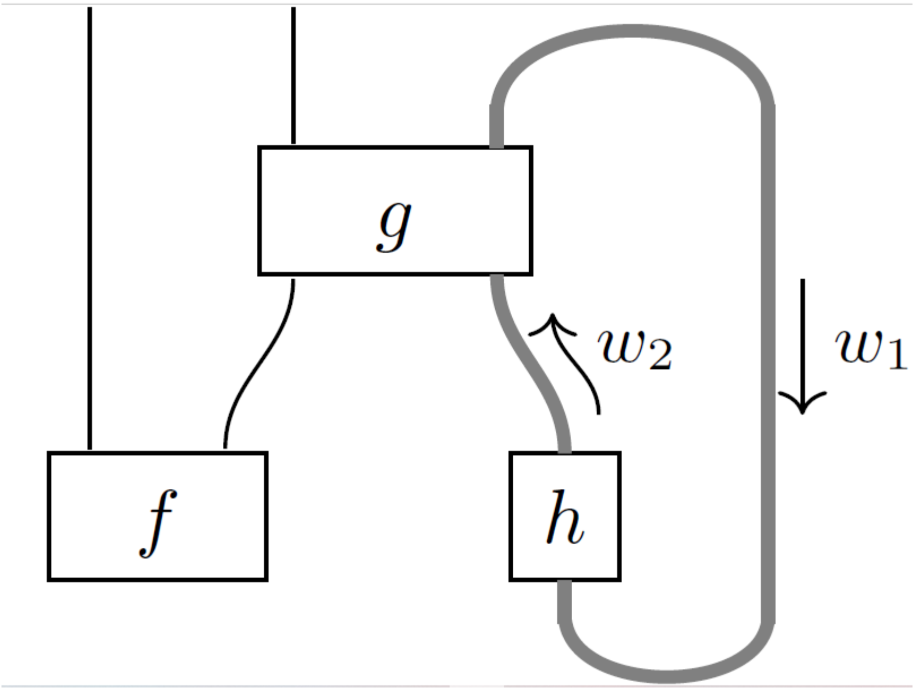


Circuit Diagrams – Parallel & Sequential Composition

No Directed Cycles



- No directed cycles should be present in a circuit diagram

Circuit Diagrams – Parallel & Sequential Composition

Circuit Diagrams Conclude

String Diagrams – Cups & Caps

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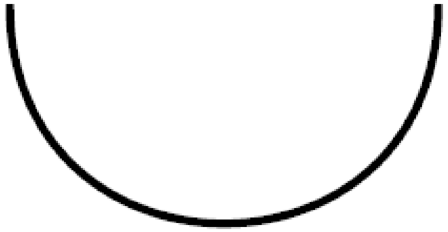
- ❖ String diagrams depicts the phenomena of entanglement in a pictorial way
- ❖ String diagrams provides a way to express non-separable and separable processes in a diagrammatic fashion
- ❖ Cups and Caps can be utilized to compose different states together
- ❖ String diagrams are very useful for the study of QNLP algorithms such as DisCoCat

String Diagrams – Cups & Caps

Cups

String Diagrams – Cups & Caps

Cups



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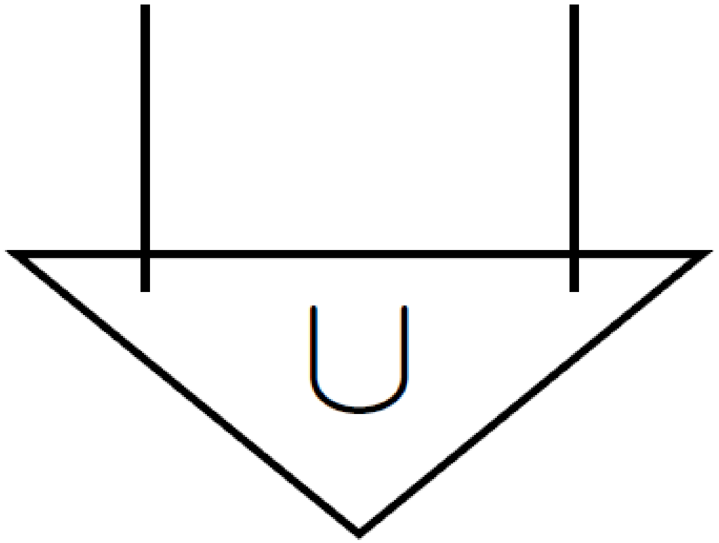
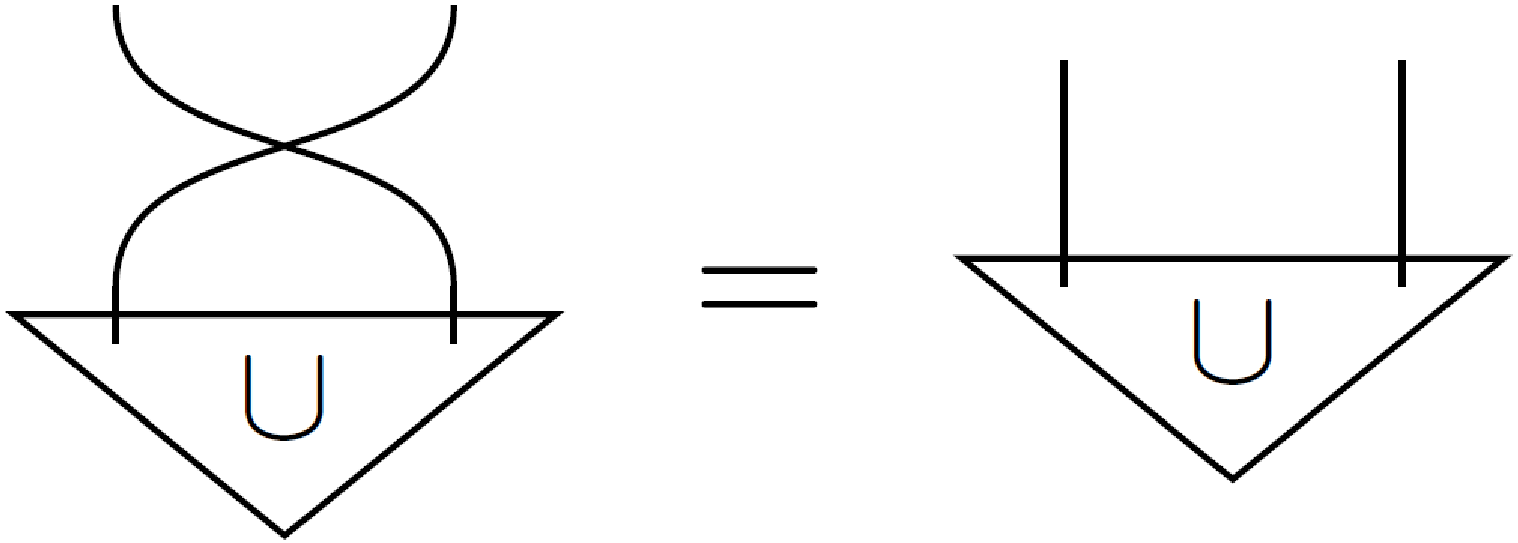


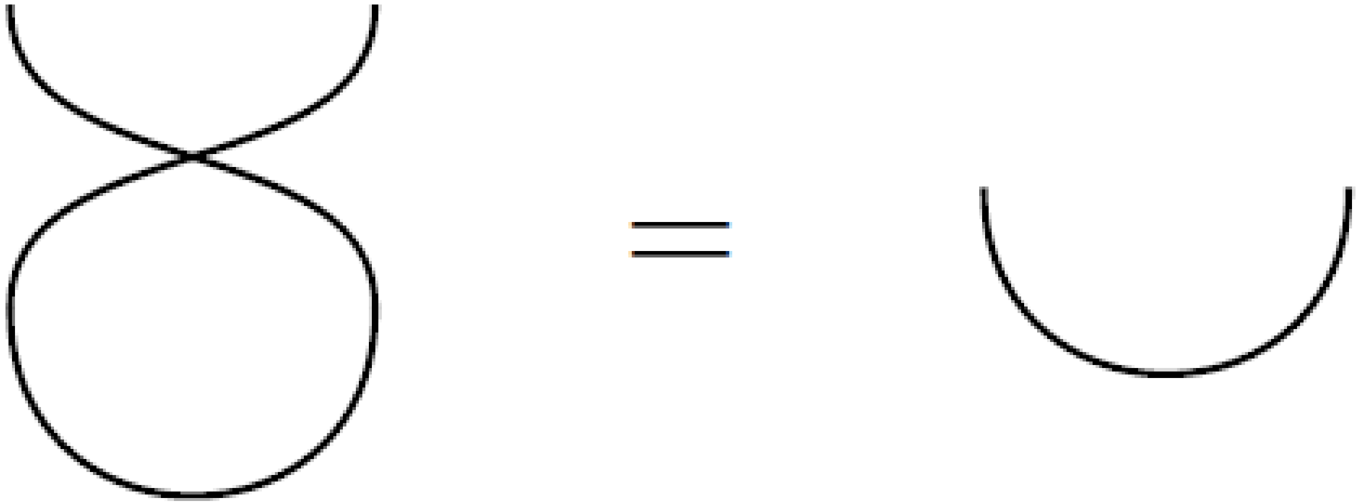
String Diagrams – Cups & Caps

Cups – Rule 1



String Diagrams – Cups & Caps

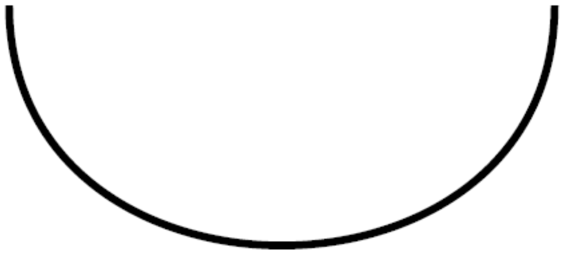
Cups – Rule 2



String Diagrams – Cups & Caps

Cups – Bell Effects

$$\langle Bell| =$$



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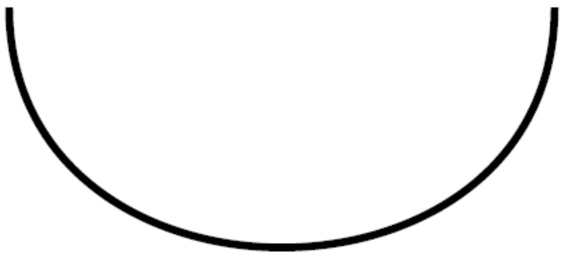
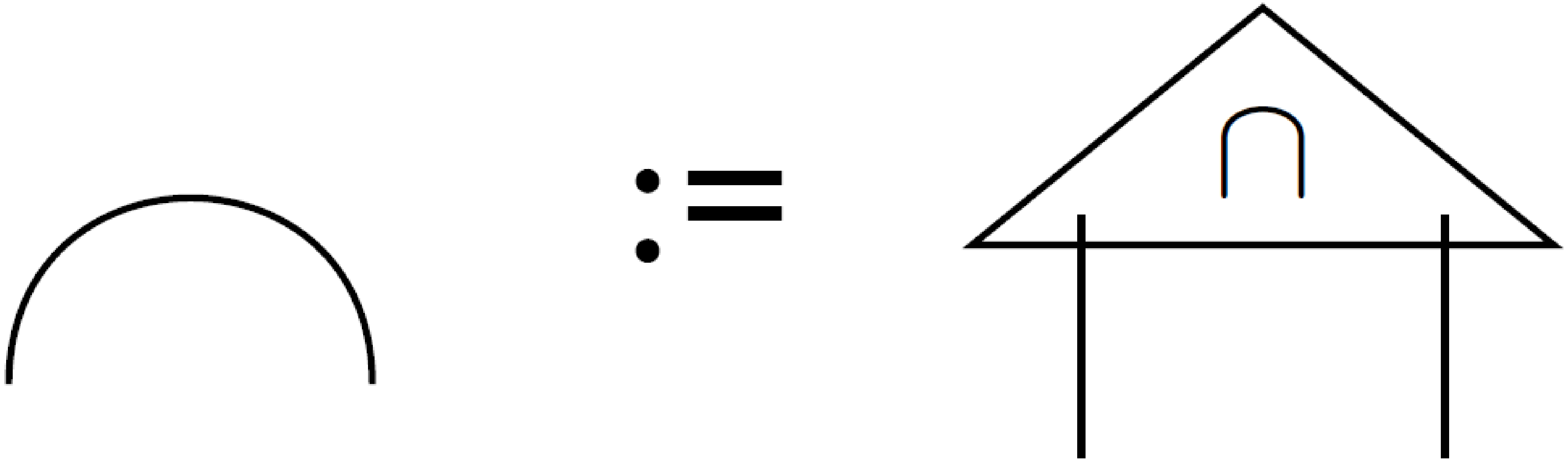


Cups denote the entangling effect or entanglement in Dirac Bra form!

String Diagrams – Cups & Caps

Caps



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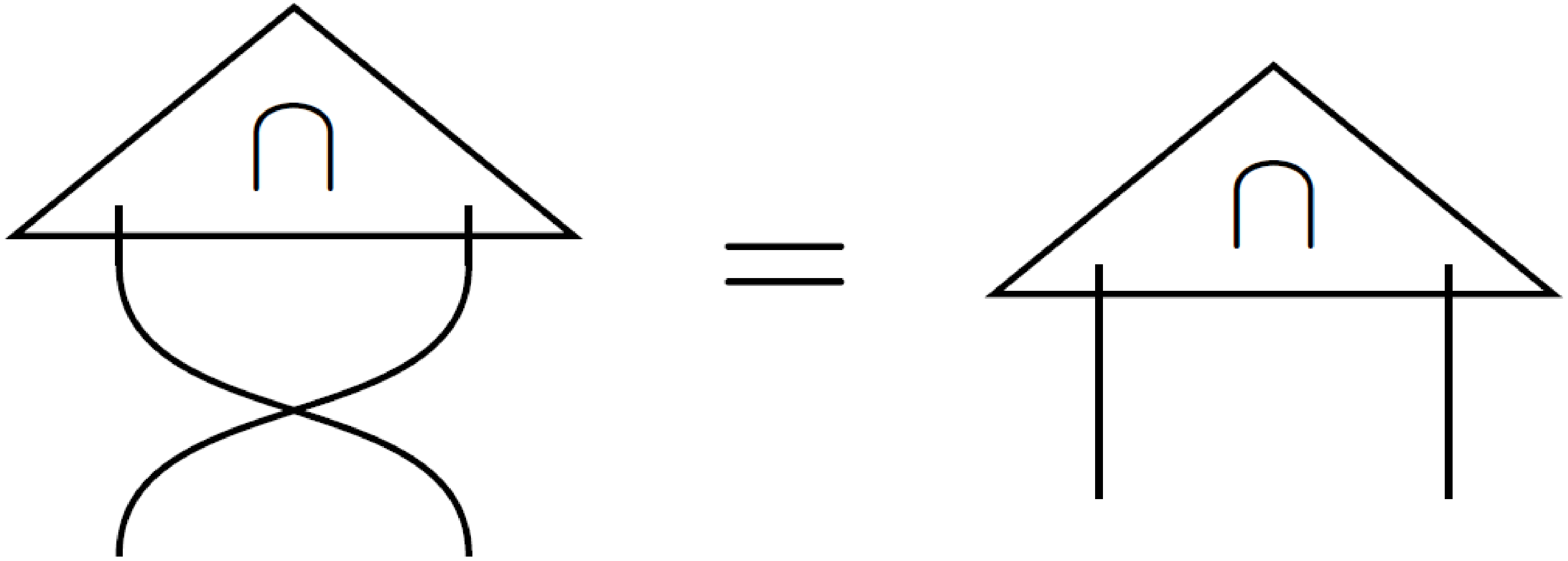
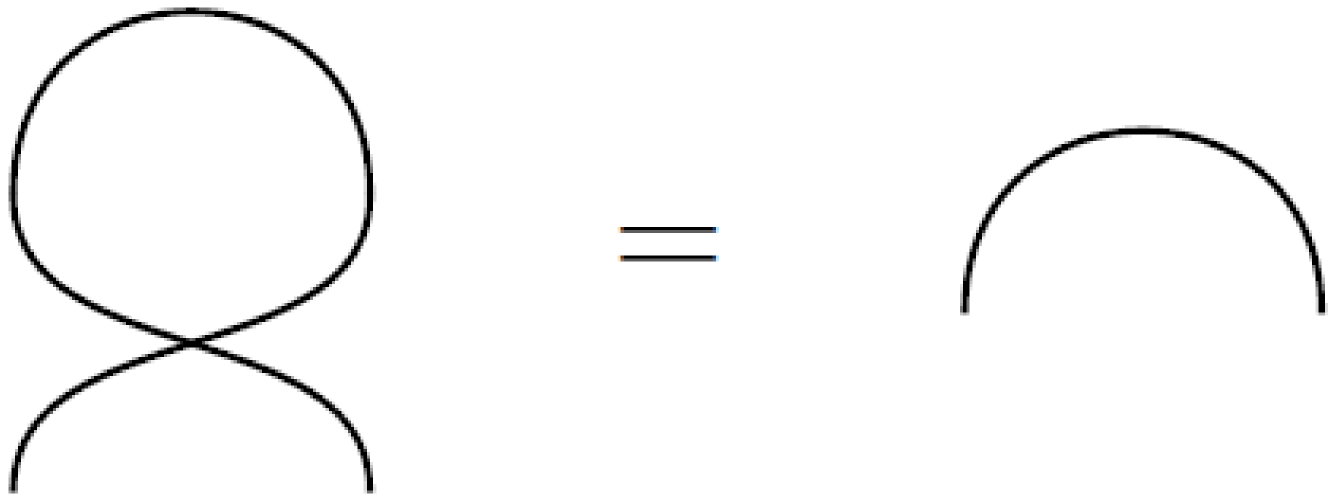


String Diagrams – Cups & Caps

Caps – Rule 2



String Diagrams – Cups & Caps

Caps – Bell States

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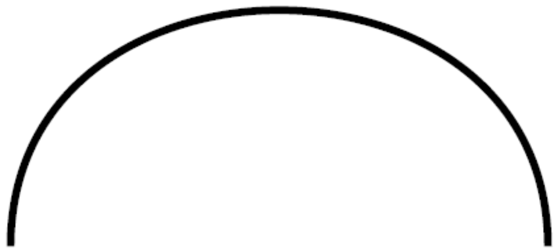

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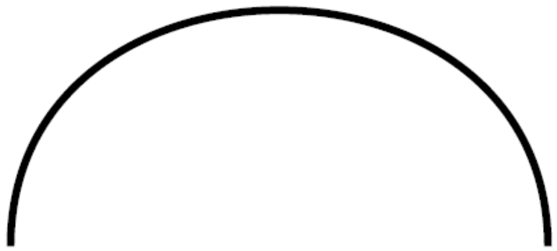

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String Diagrams Conclude

References

- ❖ Bob Coecke, “Foundations for Near Term Quantum Natural Language Processing”, https://arxiv.org/abs/2012.03755
- ❖ Bob Coecke, “Compositionality as we see it, everywhere around us”, https://arxiv.org/abs/2110.05327
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- ❖ Bob Coecke, Aleks Kissinger, “Picturing Quantum Processes”, Cambridge University Press, 2017
- ❖ Joachim Lambek, “From Word to Sentence: A Computational Algebraic Approach to Grammar”, Polimetrica s.a.s., 2008

Thank you so much!