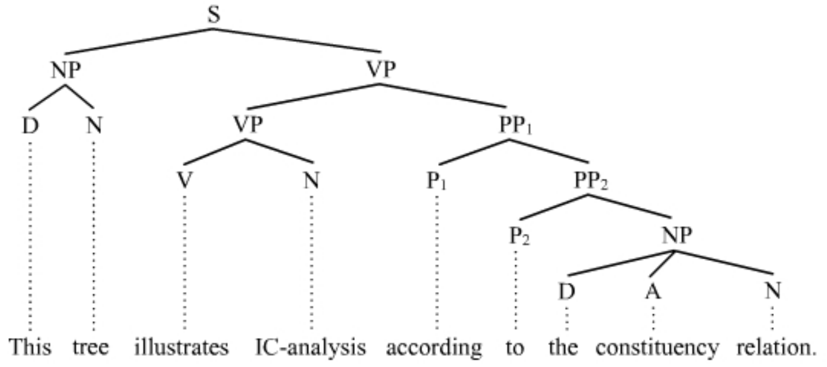
Context-Free Grammar

COMP90042

Natural Language Processing

Lecture 14


S
NP
VP
D
N
VP
PP₁
V
N
P₁
PP₂
P₂
NP
D
A
N
This tree illustrates IC-analysis according to the constituency relation.

Recap

- Center embedding
  - The cat loves Mozart
  - The cat the dog chased loves Mozart
  - The cat the dog the rat bit chased loves Mozart
  - The cat the dog the rat the elephant admired bit chased loves Mozart
- Cannot be captured by regular expressions ($\underline{\underline{S^nV^n}}$)
- **Context-free grammar!**

Basics of Context-Free Grammars

- **Symbols**
  - **Terminal**: word such as *book*
  - **Non-terminal**: syntactic label such as NP or VP
  - Convention to use upper and lower-case to distinguish, or else “quotes” for terminals
- **Productions** (rules) RHS. Replace W with XYZ.

  W → X Y Z

  LHS

  - Exactly one non-terminal on left-hand side (LHS)
  - An ordered list of symbols on right-hand side (RHS)
    — can be **Terminals** or **Non-terminals**
- **Start symbol:** S

Why "Context Free"

not matter W is where .

W → X Y Z

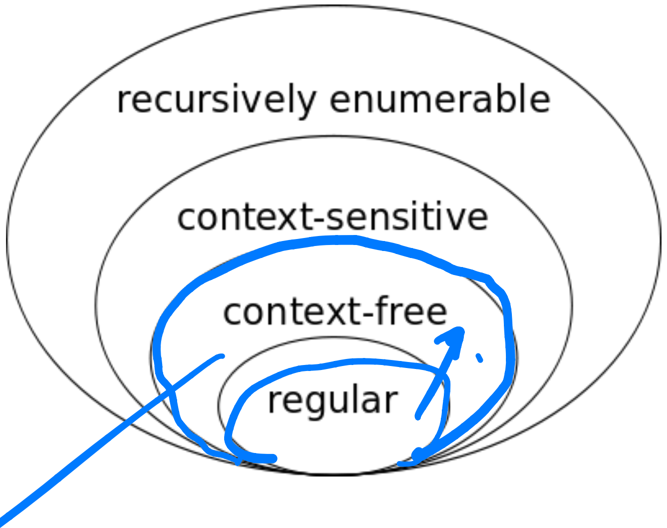
- Production rule depends only on the LHS (and not on ancestors, neighbours)
  - Analogous to Markov chain
  - Behaviour at each step depends only on current state

Context-Free vs. Regular

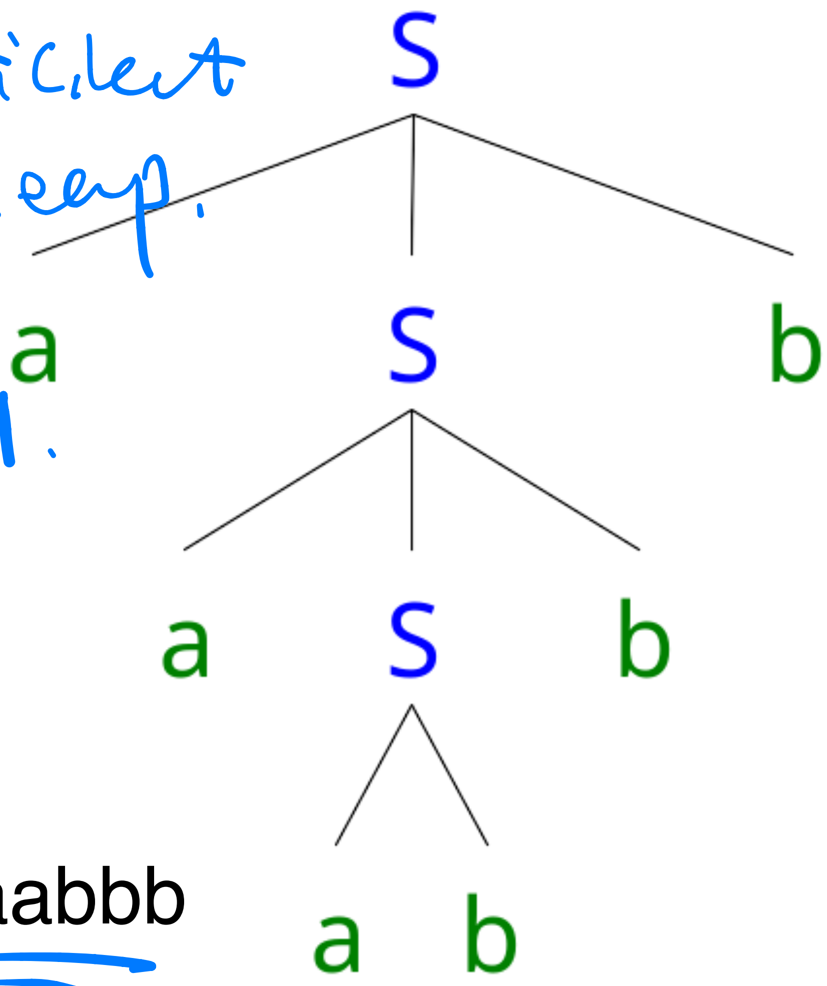
- Context-free languages more general than Regular languages
  - Allows recursive nesting
- CFG for $a^n b^n$:
  - S → a S b
  - S → a b

Term

NT – Start Symbol.



efficient
cheap.



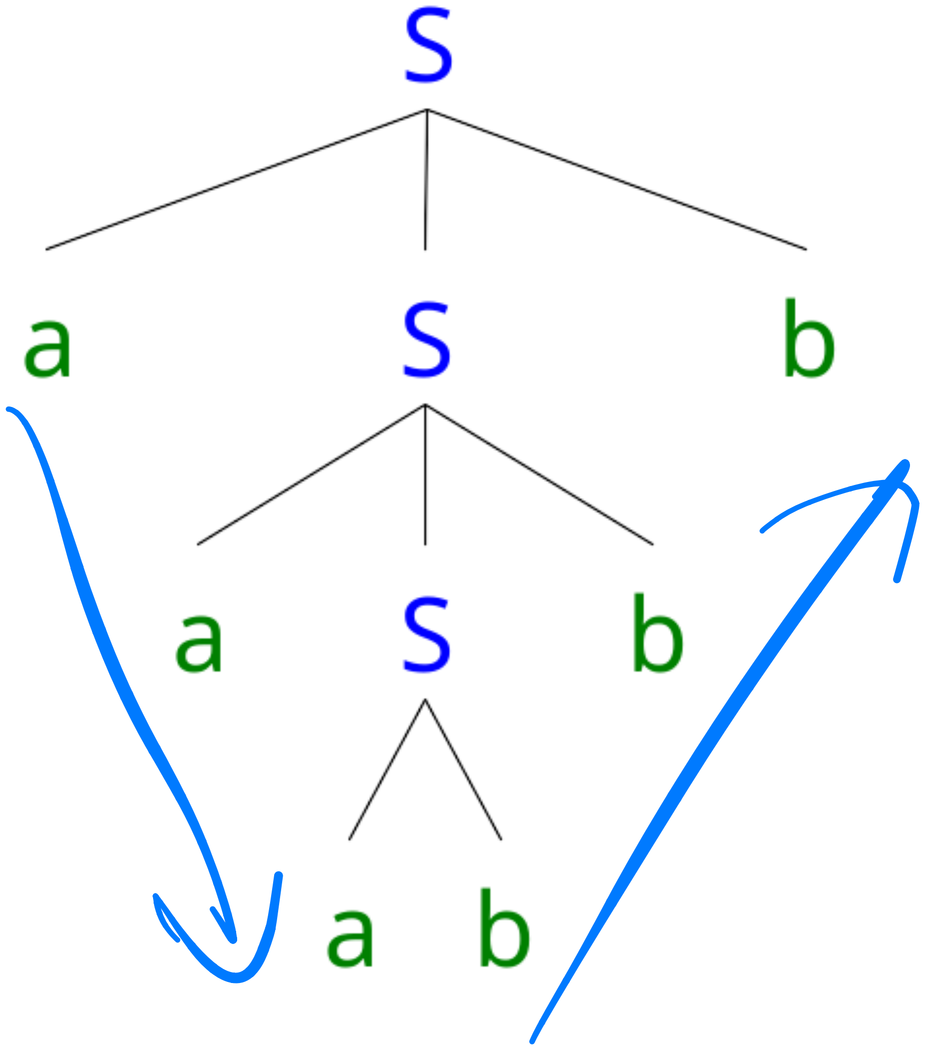
Parse tree for aaabbb

Parsing

- Given a string and production rules
- Produce a valid parse tree

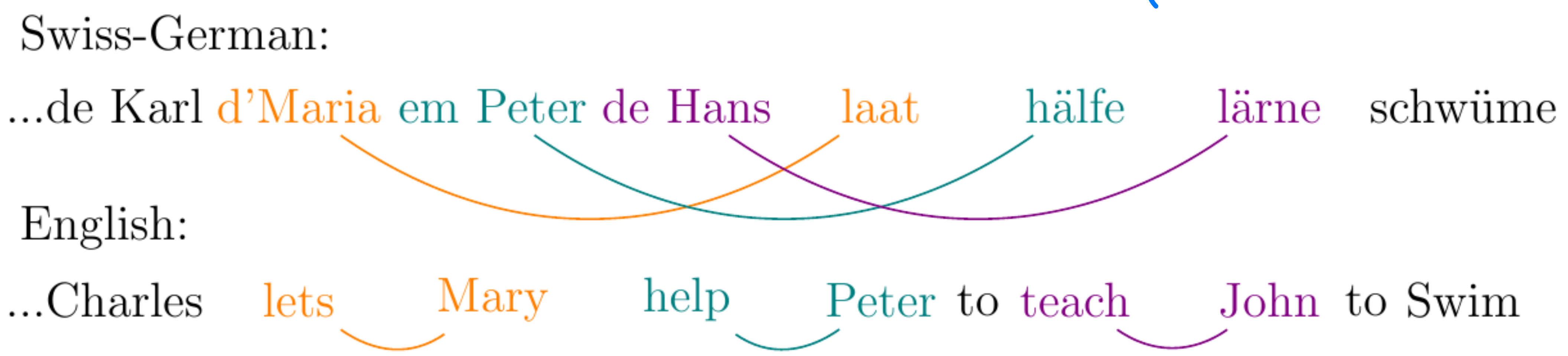
  - S → a S b
  - S → a b

aaabbb →



What This Means?

- If English can be represented with CFG:
  - we can build a "parser" to automatically judge whether a sentence is grammatical!
- But is natural language context-free?
- Not quite: cross-serial dependencies ($a^mb^nc^md^n$)



Swiss-German:

...de Karl d'Maria em Peter de Hans laat hälfe lärne schwüme

English:

...Charles lets Mary help Peter to teach John to Swim

But…

- Context-free representations strike a good balance:
  - CFG cover most syntactic patterns
  - CFG parsing is computational efficient
- We use CFG to describe a core fragment of English syntax

tell most sentences are good or not.

Syntactic Constituents

units.

- Sentences are broken into **constituents**
  - word sequence that function as a **coherent unit** for linguistic analysis
- Constituents have certain key properties:
  - movement
  - substitution
  - coordination

any sequene of words satisfy

Movement

- Constituents can be moved around sentences
  - Abigail gave [her brother] [a fish]
  - Abigail gave [a fish] to [her brother]
- Contrast: [gave her], [brother a]

✓

not constituents

Substitution

- Constituents can be substituted by other phrases of the same type
  - Max thanked [his older sister]
  - Max thanked [her] subs.
- Contrast: [Max thanked], [thanked his]

X X

Coordination

- Constituents can be conjoined with coordinators like *and* and *or*
  - [Abigail] and [her young brother] brought a fish
  - Abigail [bought a fish] and [gave it to Max]
  - Abigail [bought] and [greedily ate] a fish
- Contrast: [brother brought], [bought a]

✗ ✗

Constituents and Phrases

break units

- Once we identify constituents, we use **phrases** to describe them

label them

- Phrases are determined by their **head word:**
  - noun phrase: her younger **brother**
  - verb phrase: greedily **ate** it
- We can use CFG to formalise these intuitions

A Simple CFG for English

Terminal symbols: *rat, the, ate, cheese*

Non-terminal symbols: S, NP, VP, DT, VBD, NN

T → T

Productions:

S → NP VP

NP → DT NN

VP → VBD NP

DT → *the*

NN → *rat*

NN → *cheese*

VBD → *ate*

NT → T

Generating Sentences with CFGs

Always start with S (the sentence/start symbol)

S

Apply a rule with S on LHS (*S* → *NP VP*), i.e substitute RHS

NP VP

Apply a rule with NP on LHS (*NP* → *DT NN*)

DT NN VP

Apply rule with DT on LHS (DT → *the*)

the NN VP

Apply rule with NN on LHS (NN → *rat*)

the rat VP

In each step we rewrite the left-most non-terminal

Generating Sentences with CFGs

Apply rule with VP on LHS (VP → VBD NP)

the rat VBD NP

Apply rule with VBD on LHS (VBD → *ate*)

the rat ate NP

Apply rule with NP on LHS (NP → DT NN)

the rat ate DT NN

Apply rule with DT on LHS (DT → *the*)

the rat ate the NN

Apply rule with NN on LHS (NN → *cheese*)

the rat ate the cheese

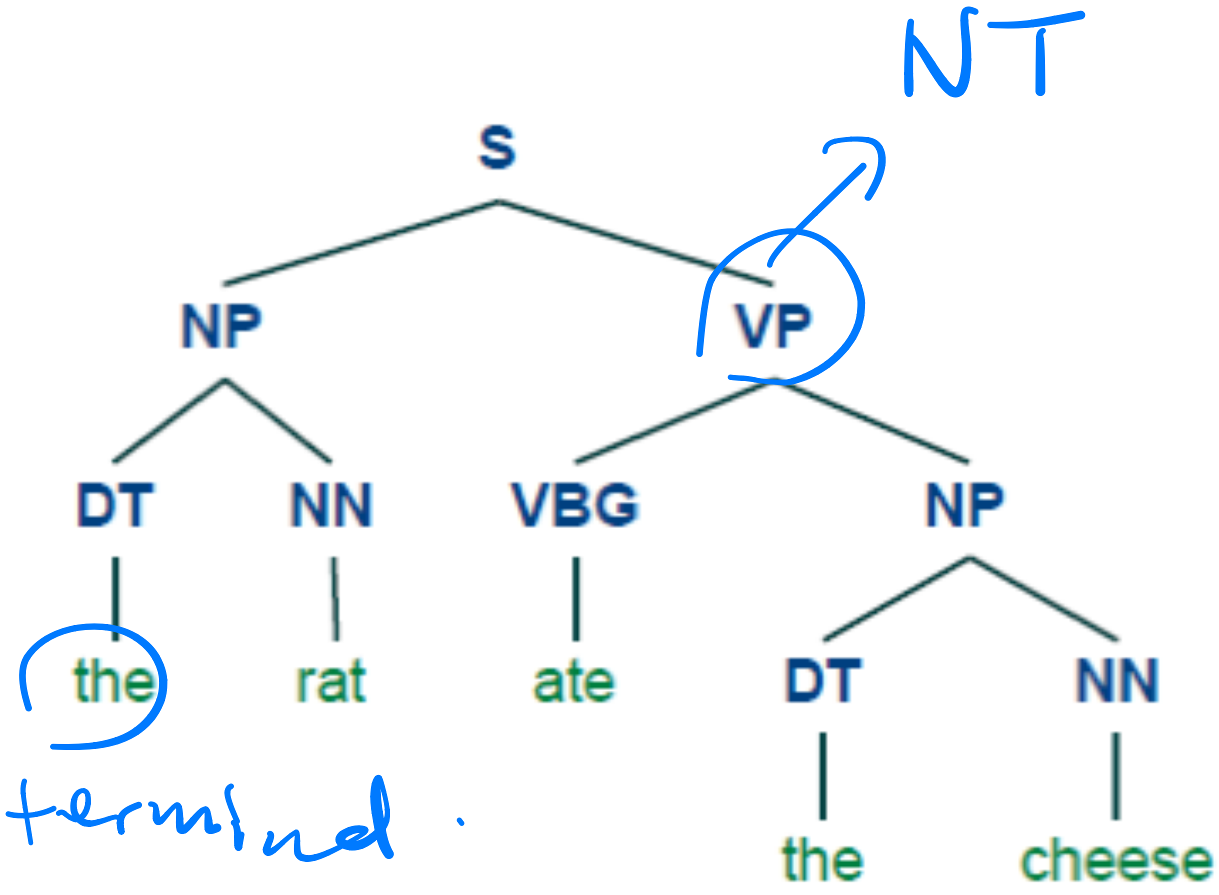
Terminals.

No non-terminals left, we're done!

CFG Trees

- Generation corresponds to a syntactic tree
- Non-terminals are internal nodes
- Terminals are leaves

```
(S (NP (DT the)
       (NN rat))
   (VP (VBG ate)
       (NP (DT the)
           (NN cheese))))
```



- Parsing is the **reverse** process

Reverse Geration

A CFG for Arithmetic Expressions

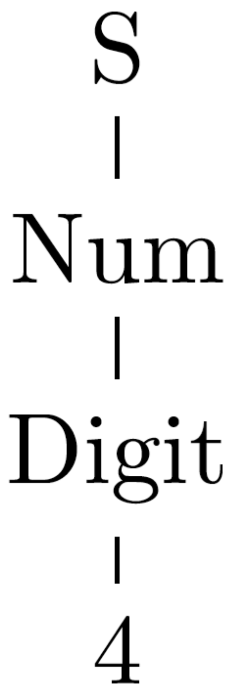
$$
\begin{aligned}
\textsc{S} &\rightarrow \textsc{S Op S} \mid \textsc{Num} \\
\textsc{Op} &\rightarrow + \mid - \mid \times \mid \div \\
\textsc{Num} &\rightarrow \textsc{Num Digit} \mid \textsc{Digit} \\
\textsc{Digit} &\rightarrow 0 \mid 1 \mid 2 \mid \ldots \mid 9
\end{aligned}
$$

- S = starting symbol
- ‘|’ = operator OR
- Recursive, NUM and S can produce themselves

Parsing

$$\begin{aligned} \textsc{S} &\rightarrow \textsc{S Op S} \mid \textsc{Num} \\ \textsc{Op} &\rightarrow + \mid - \mid \times \mid \div \\ \textsc{Num} &\rightarrow \textsc{Num Digit} \mid \textsc{Digit} \\ \textsc{Digit} &\rightarrow 0 \mid 1 \mid 2 \mid \ldots \mid 9 \end{aligned}$$

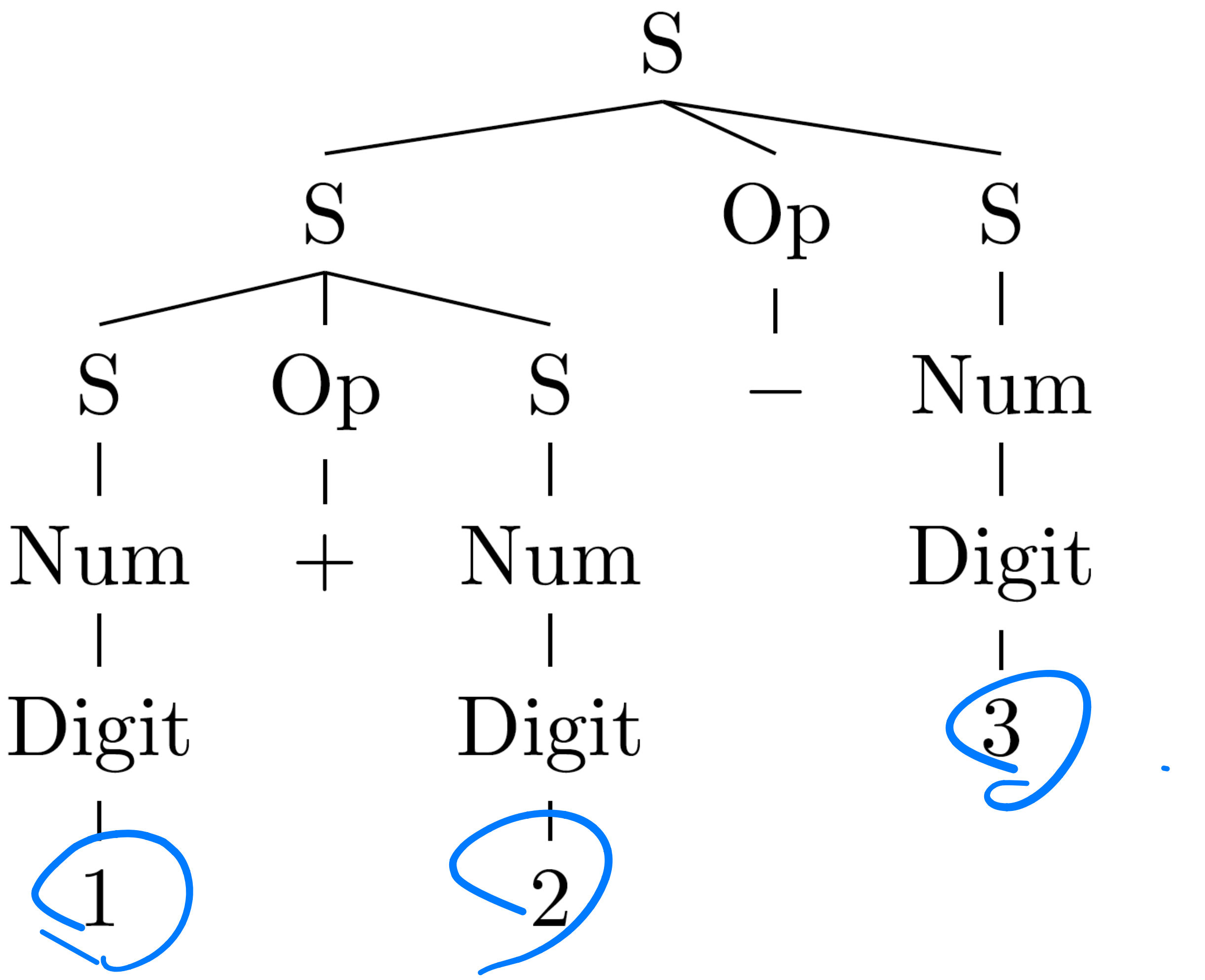
- Is ‘4’ a valid string?

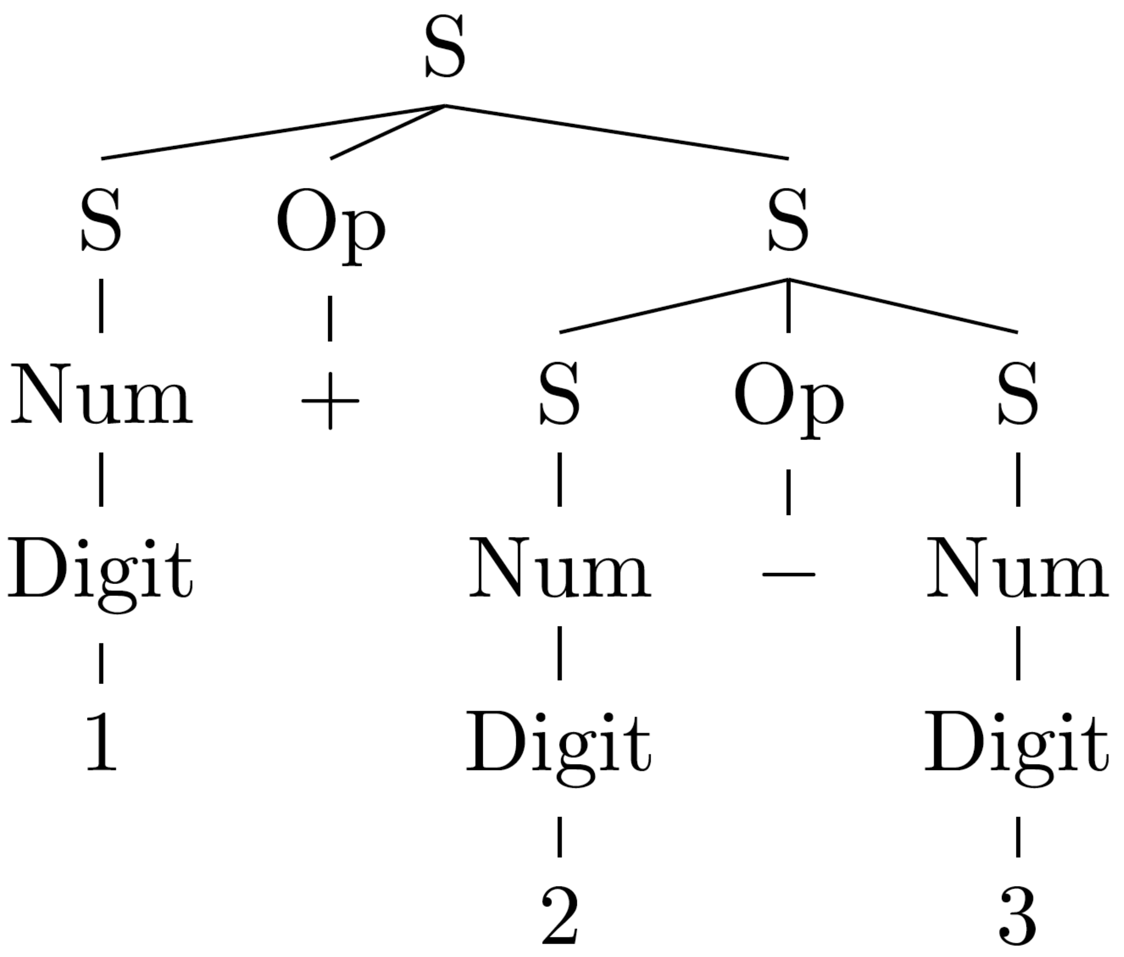


Parsing

$$
\begin{aligned}
\text{S} &\rightarrow \text{S Op S} \mid \text{Num}\\
\text{Op} &\rightarrow + \mid - \mid \times \mid \div\\
\text{Num} &\rightarrow \text{Num Digit} \mid \text{Digit}\\
\text{Digit} &\rightarrow 0 \mid 1 \mid 2 \mid \ldots \mid 9
\end{aligned}
$$

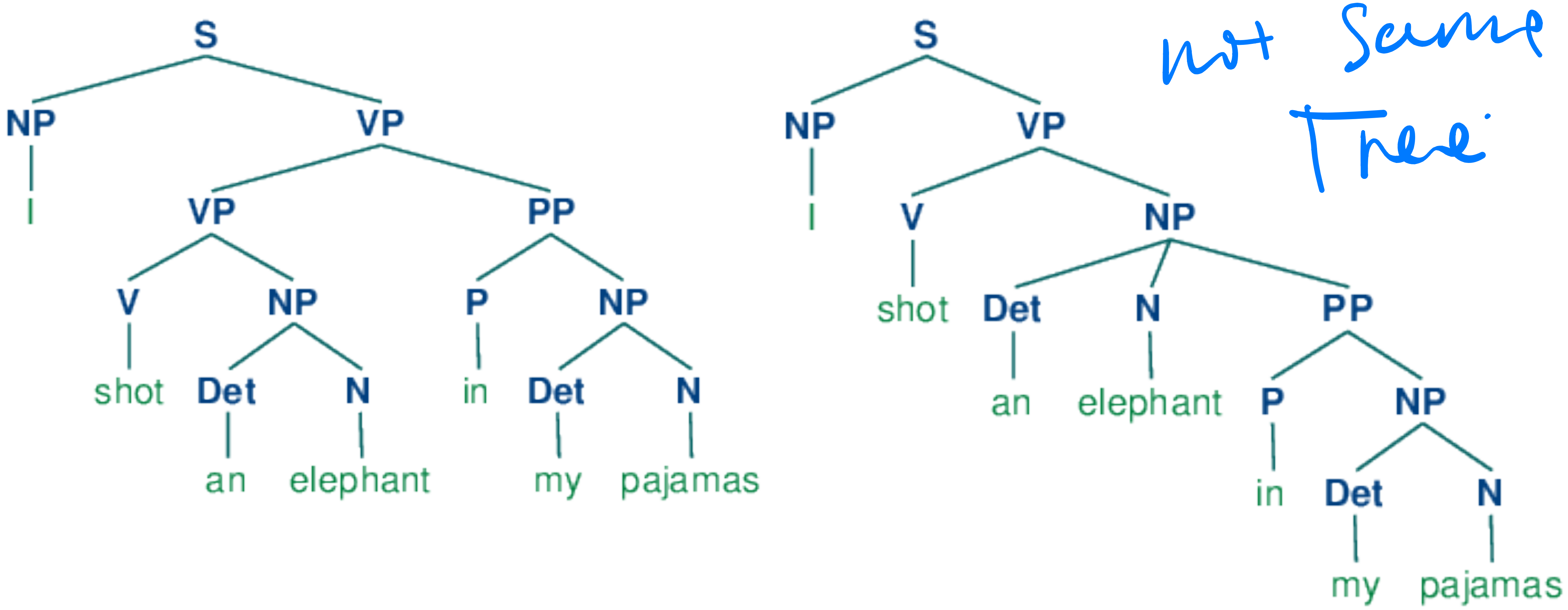
- Is ‘1+2-3’ a valid string?





Parse Ambiguity

- Often more than one tree can describe a string
- “*While hunting in Africa, I shot an elephant in my pajamas. How he got into my pajamas, I don't know.*”
  Animal Crackers (1930)



not same Tree

http://www.nltk.org/book/ch08.html

Parsing CFG

CYK Algorithm

chart

- Bottom-up approach to parsing in CFG
- Tests whether a string is valid given a CFG, without enumerating all possible parses
- Core idea: form small constituents first, and merge them into larger constituents

merge.

- Requirement: CFGs must be in **Chomsky Normal Forms**

Convert to Chomsky Normal Form

- Change grammar so all rules of form:
  - $\overset{NT}{A} \rightarrow \overset{NT}{B}\ \overset{NT}{C}$
  - $\overset{NT}{A} \rightarrow \overset{T}{a}$

Requirement

- Convert rules of form A → B c into:
  - A → B X
  - X → c

chomsky Normal Form.

Convert to Chomsky Normal Form

- Convert rules A → B C D into:
  - A → B Y
  - Y → C D
  - E.g., VP → VP NP NP
    for ditransitive cases, "*sold [her] [the book]*"
- X, Y are new symbols we have introduced

CNF (cont)

reason :

- CNF disallows unary rules, A → B. X .
- Imagine NP → S; and S → NP … leads to infinitely many trees with same yield. infinitely many trees.

one sentenc

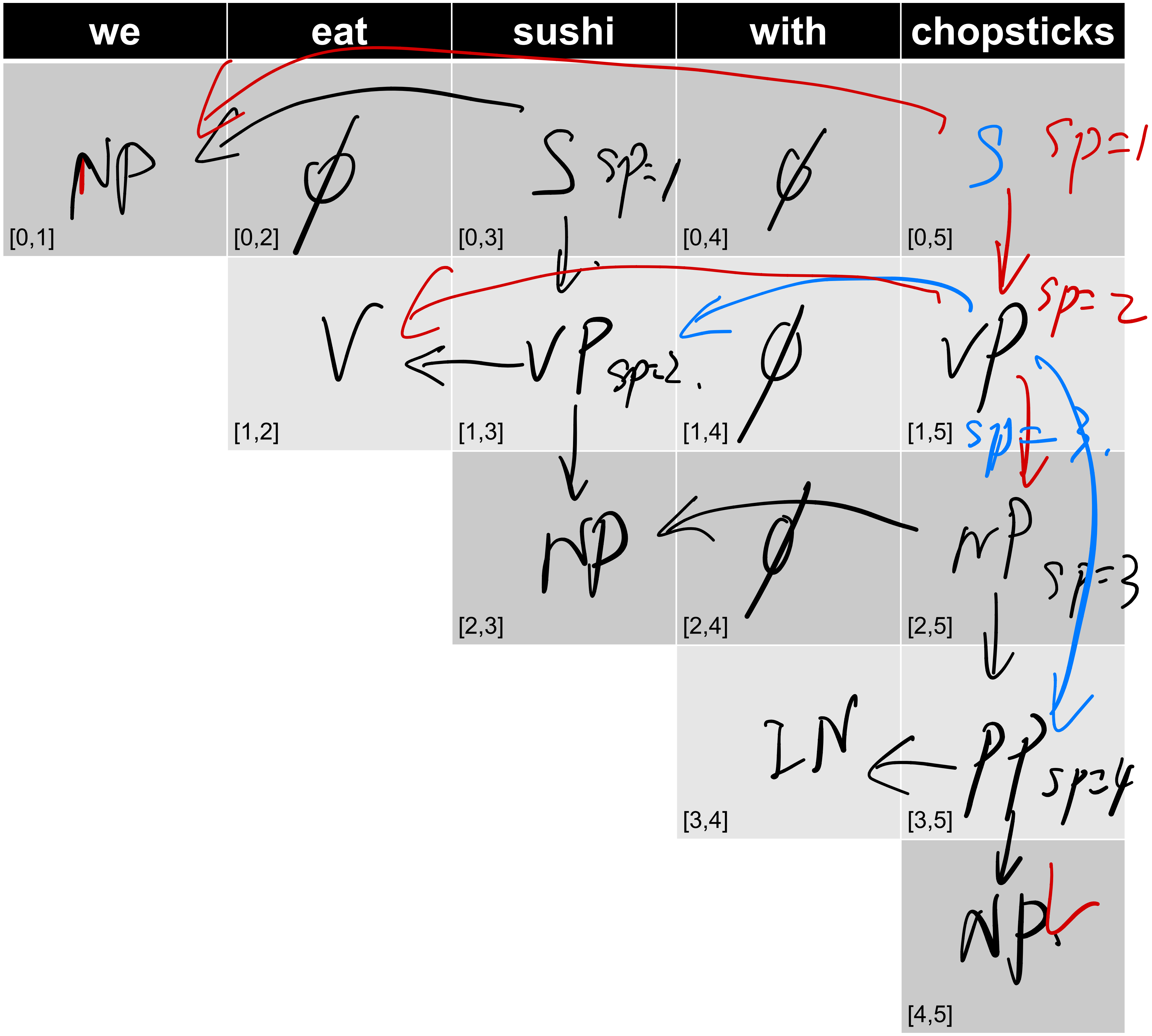
↓

- Replace RHS non-terminal with its productions
  - E.g convert A → B, B → 1, B → 2 into:

    drop
  - A → 1, A →2

The CYK Parsing Algorithm

- Convert grammar to Chomsky Normal Form (CNF)
- Fill in a parse table
- Use table to derive parse
- S in top right corner of table = success!
- Convert result back to original grammar



we	eat	sushi	with	chopsticks
NP [0,1]	∅ [0,2]	S sp=1 [0,3]	∅ [0,4]	S sp=1 [0,5]
	V [1,2]	VP sp=2 [1,3]	∅ [1,4]	VP sp=2 sp=3. [1,5]
		NP [2,3]	∅ [2,4]	NP sp=3 [2,5]
			IN [3,4]	PP sp=4 [3,5]
				NP [4,5]

S → NP VP
NP → NP PP
PP → IN NP
VP → V NP
VP → VP PP
NP → we
NP → sushi
NP → chopsticks
IN → with
V → eat

we	eat	sushi	with	chopsticks
NP [0,1]	[0,2]	[0,3]	[0,4]	[0,5]
	V [1,2]	[1,3]	[1,4]	[1,5]
		NP [2,3]	[2,4]	[2,5]
			IN [3,4]	[3,5]
				NP [4,5]

S → NP VP
NP → NP PP
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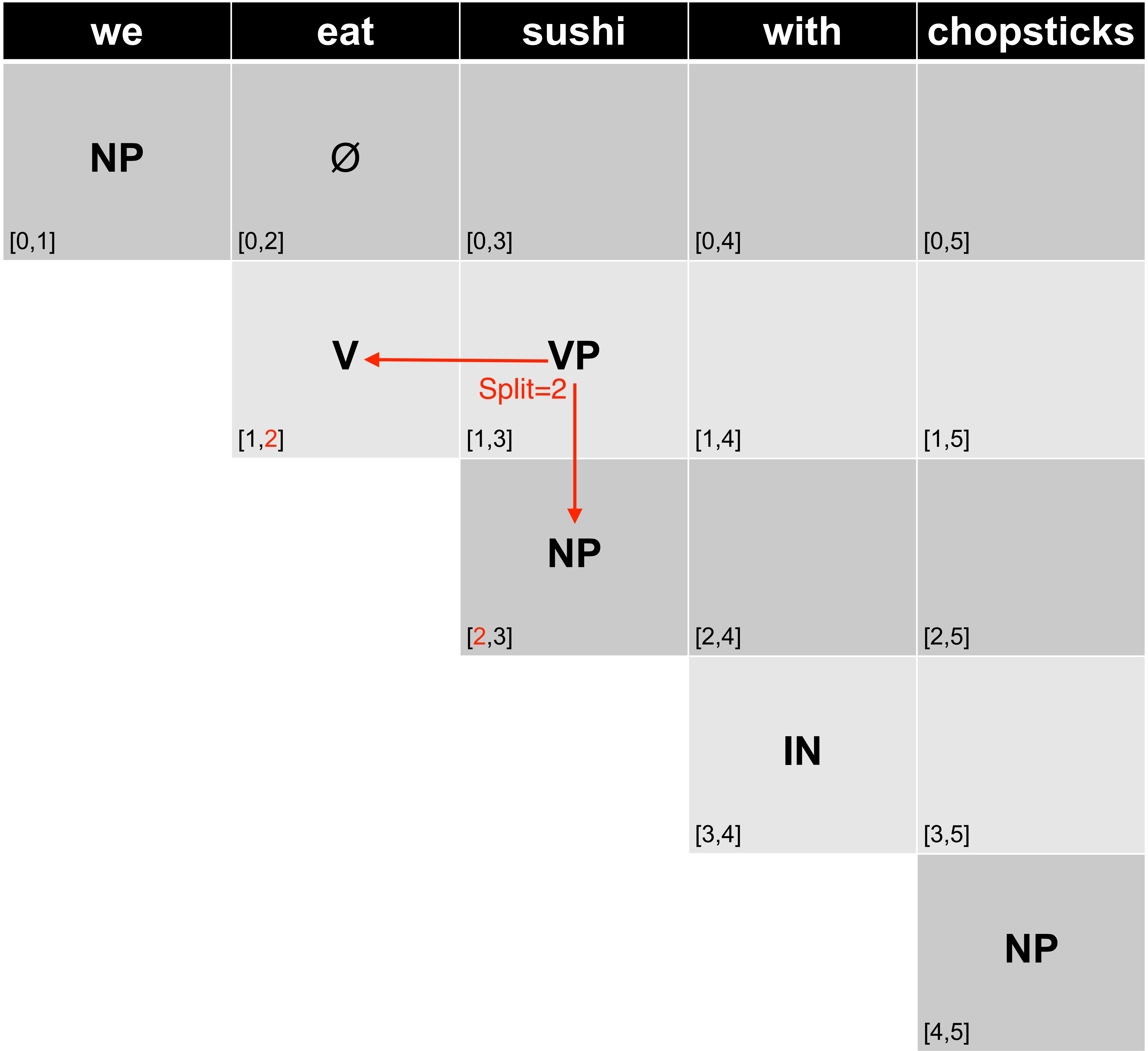
VP → V NP
VP → VP PP
NP → we
NP → sushi
NP → chopsticks
IN → with
V → eat

check production rule.

we	eat	sushi	with	chopsticks
NP [0,1]	Ø [0,2]	S sp=1 [0,3]	∅ [0,4]	S sp=1 [0,5]
	V [1,2]	split=2. VP [1,3]	∅ [1,4]	sp=2 VP sp=3. [1,5]
		NP [2,3]	∅ [2,4]	sp=3 NP [2,5]
			IN [3,4]	sp=4 PP [3,5]
				NP [4,5]

S → NP VP
NP → NP PP
PP → IN NP
VP → V NP
VP → VP PP
NP → we
NP → sushi
NP → chopsticks
IN → with
V → eat

nothing produce NP & V.



we	eat	sushi	with	chopsticks
NP [0,1]	Ø [0,2]	[0,3]	[0,4]	[0,5]
	V [1,2]	VP Split=2 [1,3]	[1,4]	[1,5]
		NP [2,3]	[2,4]	[2,5]
			IN [3,4]	[3,5]
				NP [4,5]

S → NP VP
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VP → VP PP
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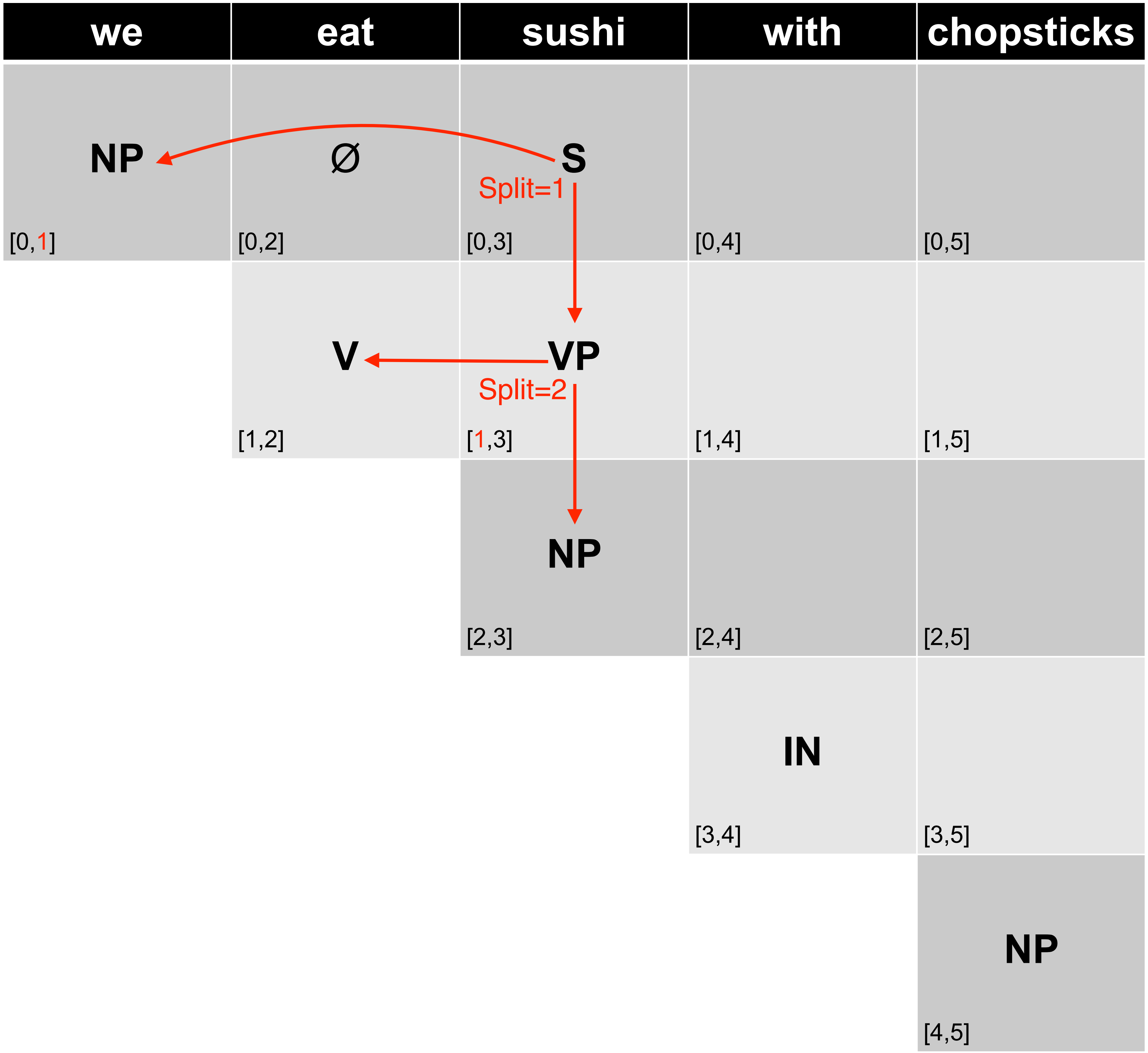
V → eat



we	eat	sushi	with	chopsticks
NP [0,1]	Ø [0,2]	S Split=1 [0,3]	[0,4]	[0,5]
	V [1,2]	VP Split=2 [1,3]	[1,4]	[1,5]
		NP [2,3]	[2,4]	[2,5]
			IN [3,4]	[3,5]
				NP [4,5]

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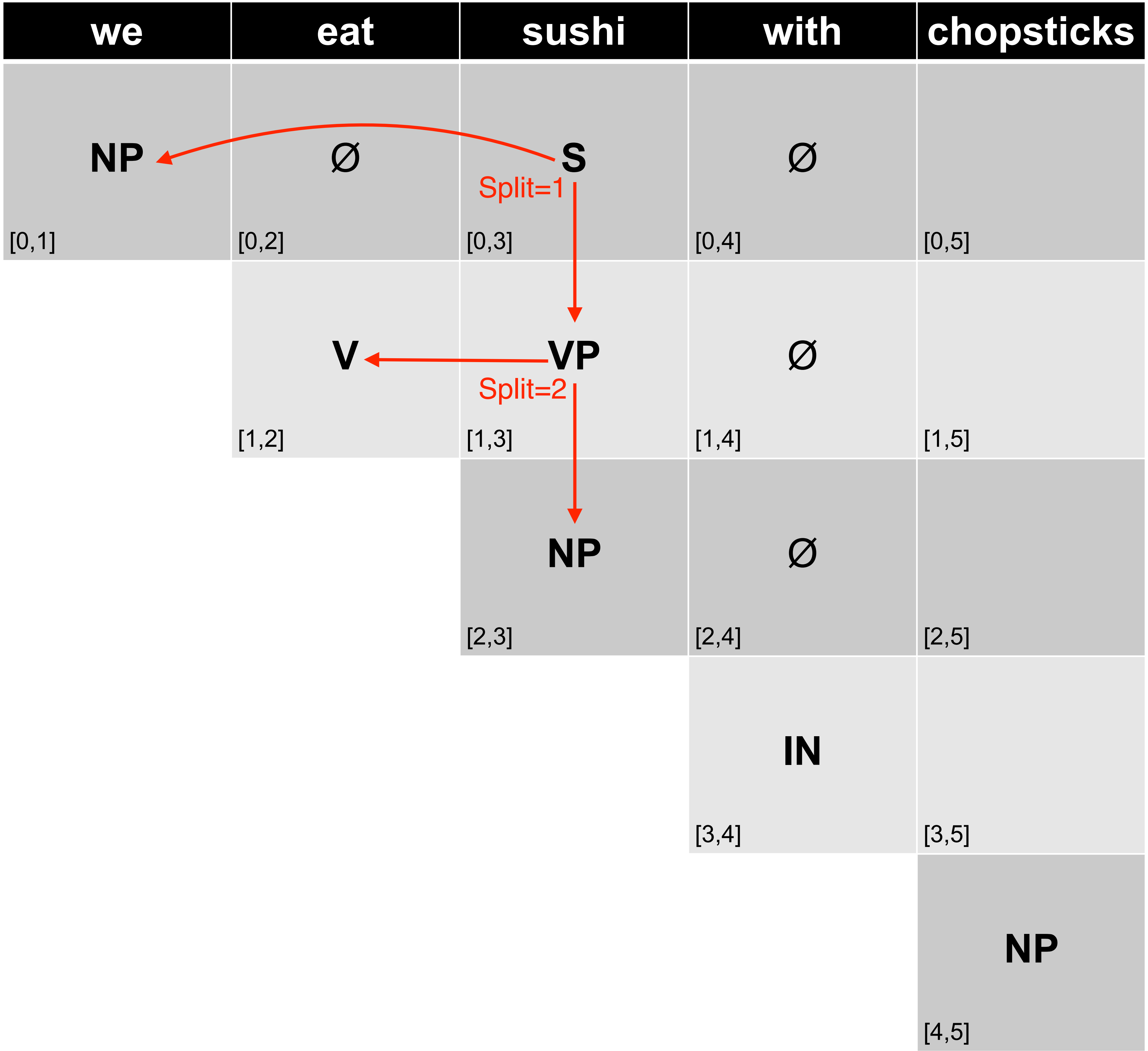

we	eat	sushi	with	chopsticks
NP [0,1]	Ø [0,2]	S Split=1 [0,3]	Ø [0,4]	[0,5]
	V [1,2]	VP Split=2 [1,3]	Ø [1,4]	[1,5]
		NP [2,3]	Ø [2,4]	[2,5]
			IN [3,4]	[3,5]
				NP [4,5]

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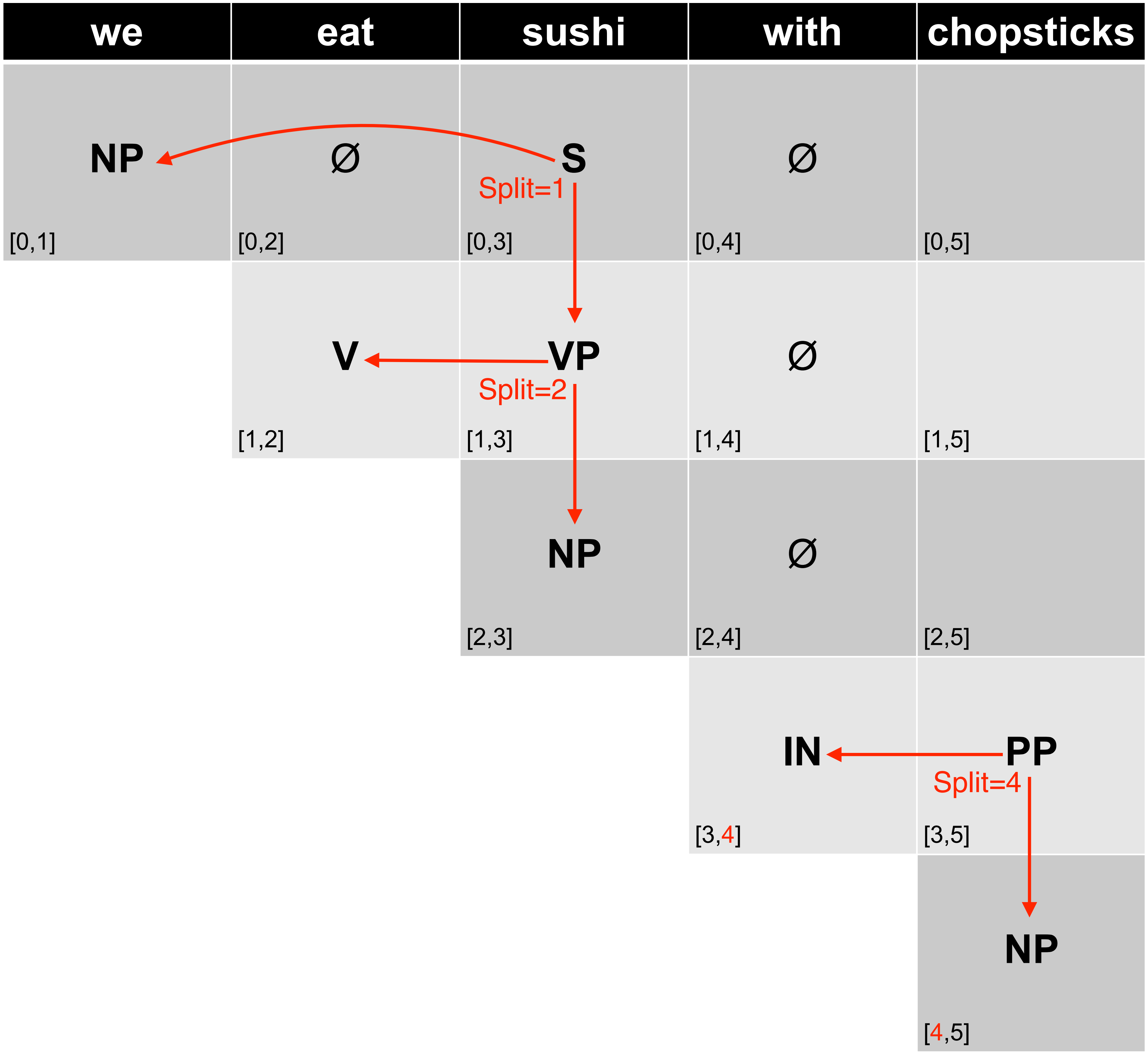

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NP [0,1]	Ø [0,2]	S Split=1 [0,3]	Ø [0,4]	[0,5]
	V [1,2]	VP Split=2 [1,3]	Ø [1,4]	[1,5]
		NP [2,3]	Ø [2,4]	[2,5]
			IN [3,4]	PP Split=4 [3,5]
				NP [4,5]

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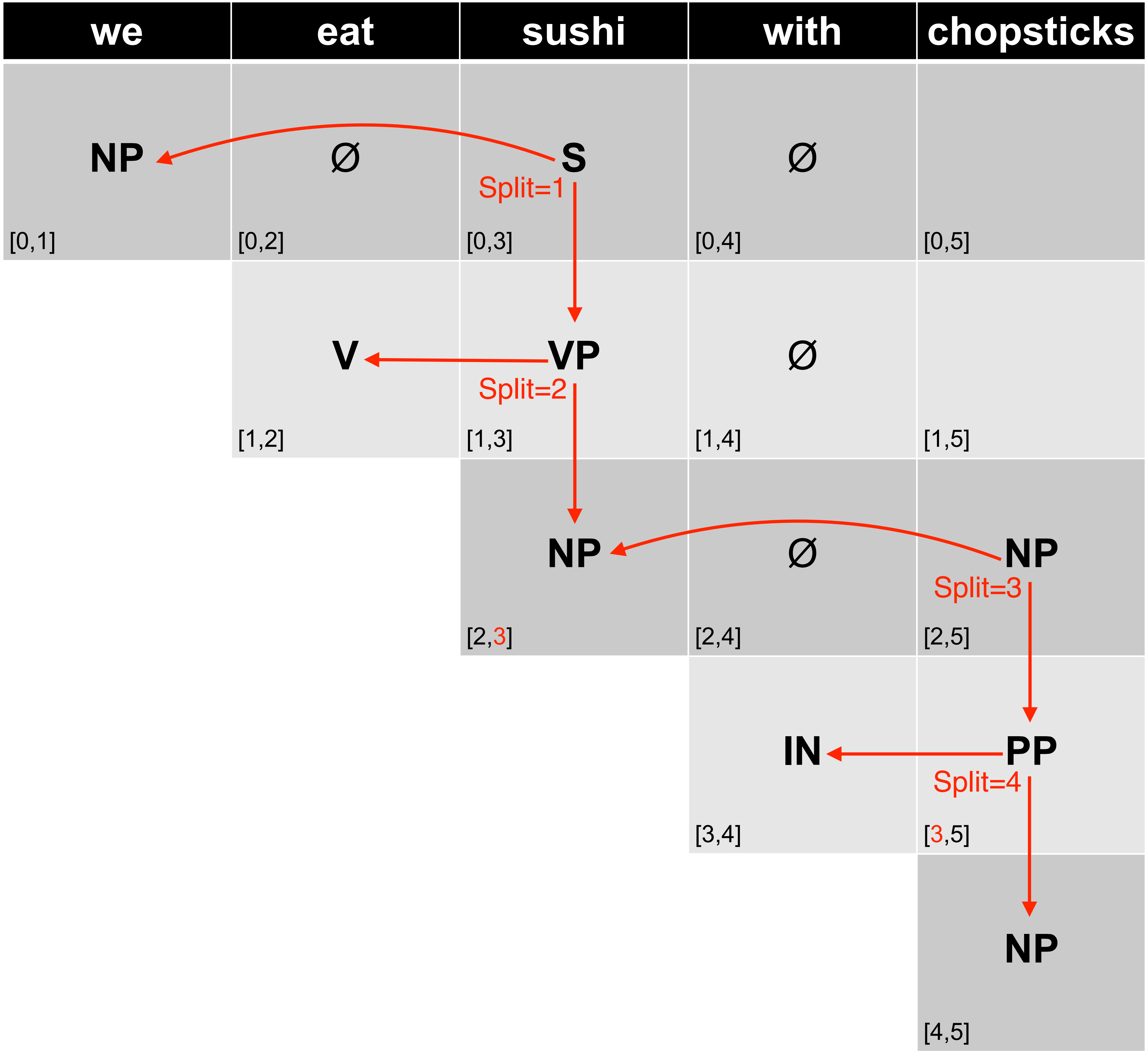

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	V [1,2]	VP Split=2 [1,3]	Ø [1,4]	[1,5]
		NP [2,3]	Ø [2,4]	NP Split=3 [2,5]
			IN [3,4]	PP Split=4 [3,5]
				NP [4,5]

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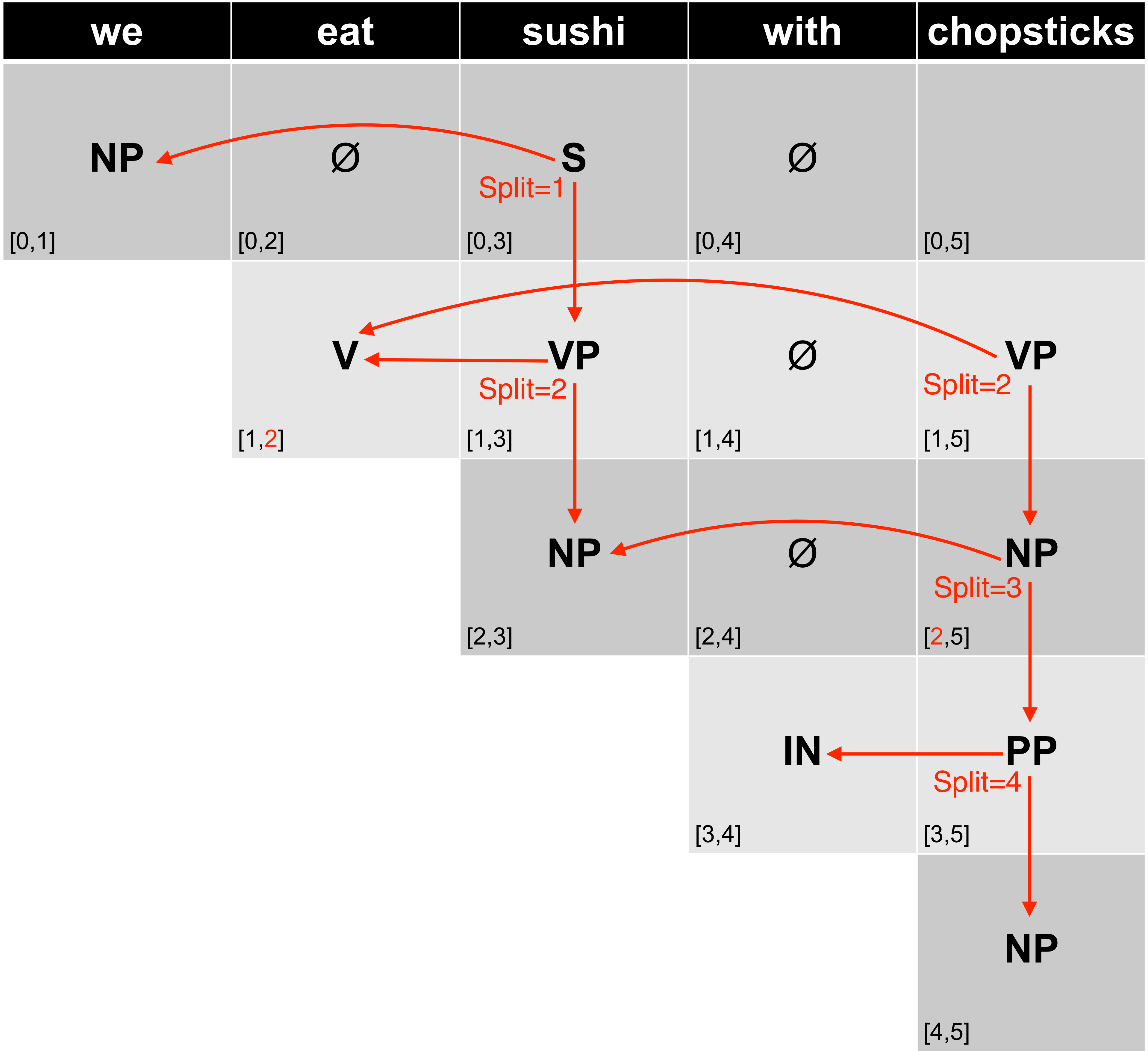

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NP [0,1]	Ø [0,2]	S Split=1 [0,3]	Ø [0,4]	[0,5]
	V [1,2]	VP Split=2 [1,3]	Ø [1,4]	VP Split=2 [1,5]
		NP [2,3]	Ø [2,4]	NP Split=3 [2,5]
			IN [3,4]	PP Split=4 [3,5]
				NP [4,5]

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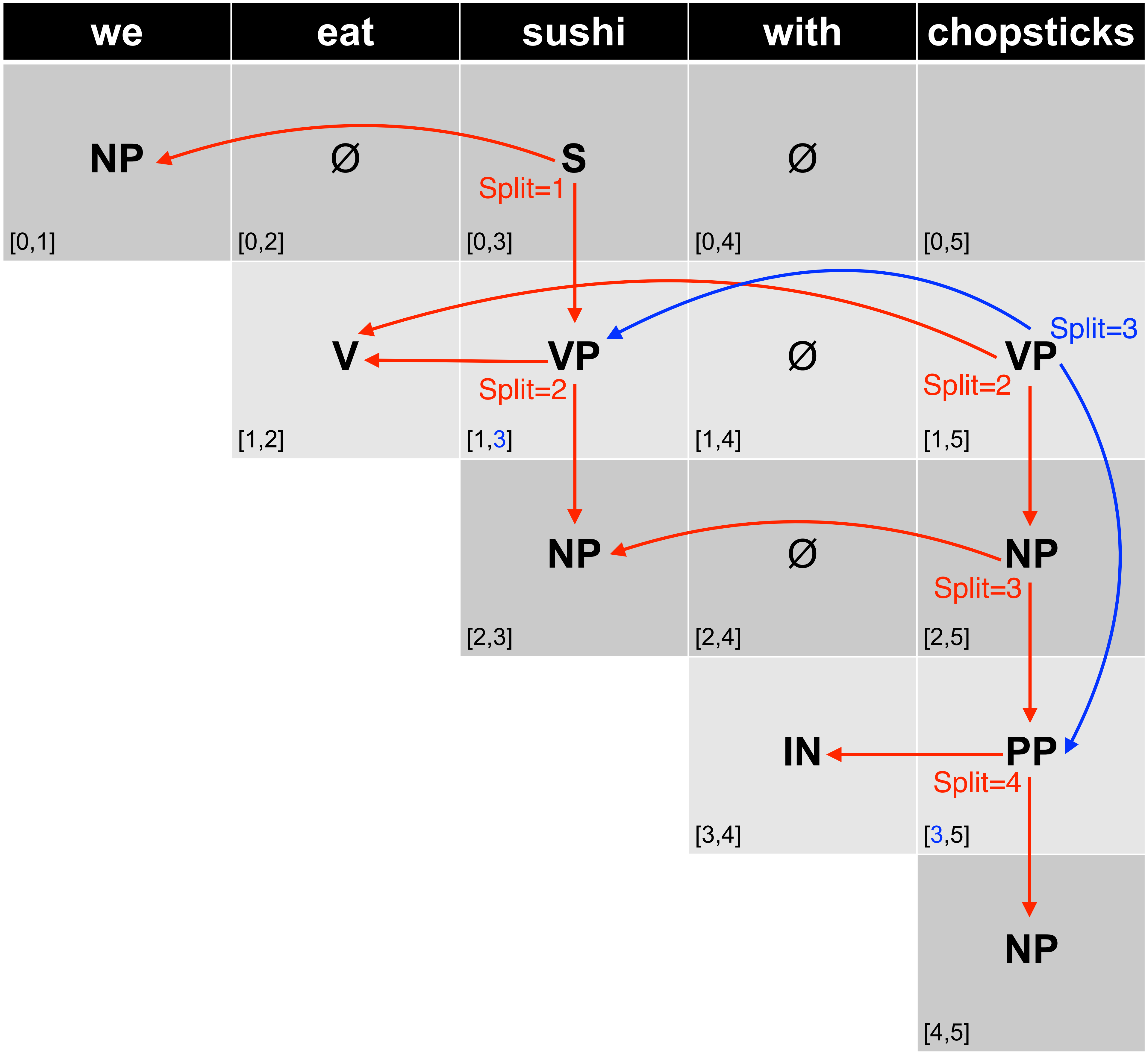

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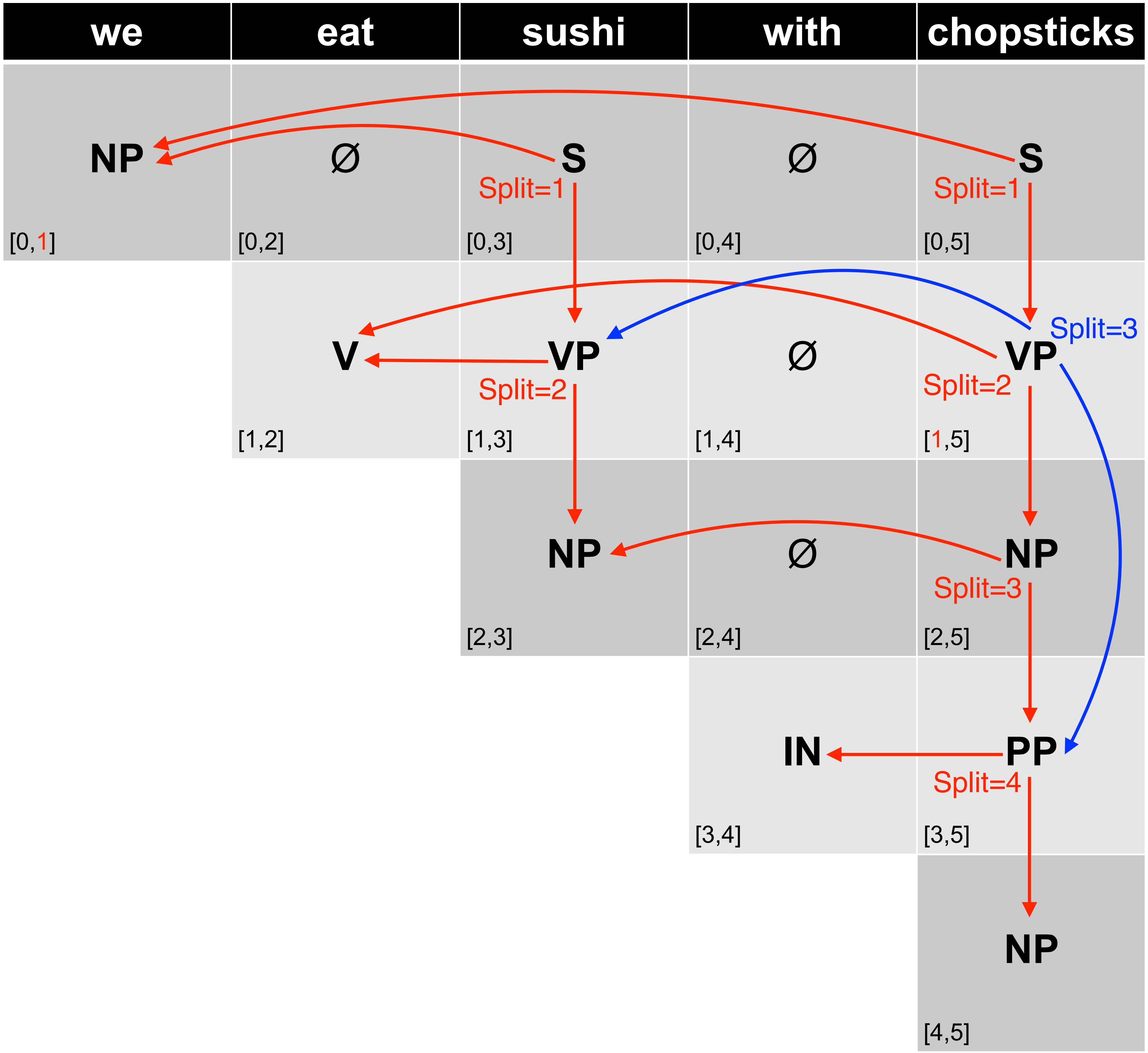

we	eat	sushi	with	chopsticks
NP [0,1]	Ø [0,2]	S Split=1 [0,3]	Ø [0,4]	S Split=1 [0,5]

	V
	[1,2]
		VP
		Split=2
		[1,3]
			Ø
			[1,4]
				Split=3
				VP
				Split=2
				[1,5]

		NP [2,3]	Ø [2,4]	NP Split=3 [2,5]
			IN [3,4]	PP Split=4 [3,5]
				NP [4,5]

S → NP VP
NP → NP PP
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VP → V NP
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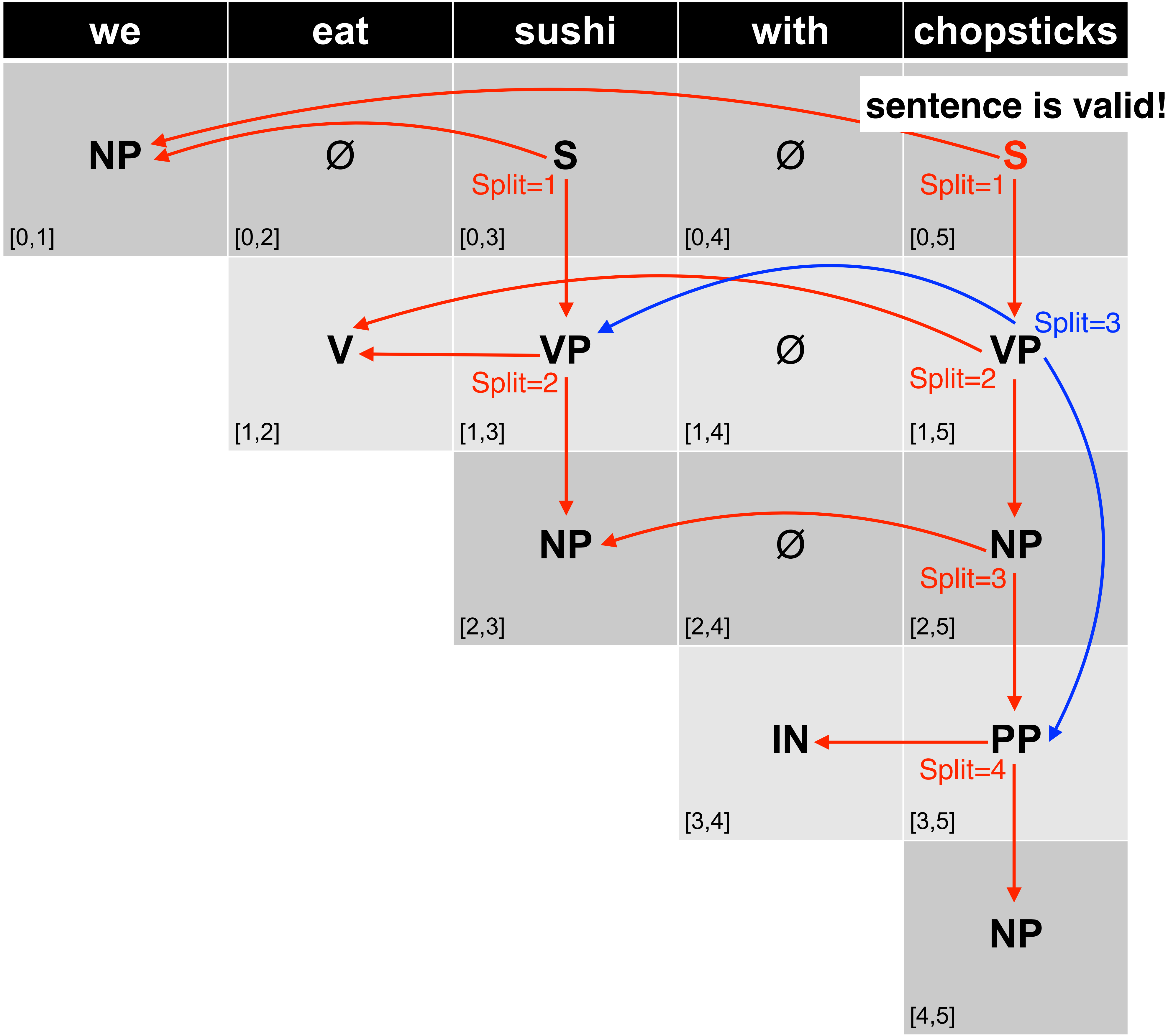

we	eat	sushi	with	chopsticks
NP [0,1]	Ø [0,2]	S Split=1 [0,3]	Ø [0,4]	sentence is valid! S Split=1 [0,5]

	V
	[1,2]
		VP
		Split=2
		[1,3]
			Ø
			[1,4]
				Split=3
				VP
				Split=2
				[1,5]

		NP [2,3]	Ø [2,4]	NP Split=3 [2,5]
			IN [3,4]	PP Split=4 [3,5]
				NP [4,5]

S → NP VP
NP → NP PP
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VP → VP PP
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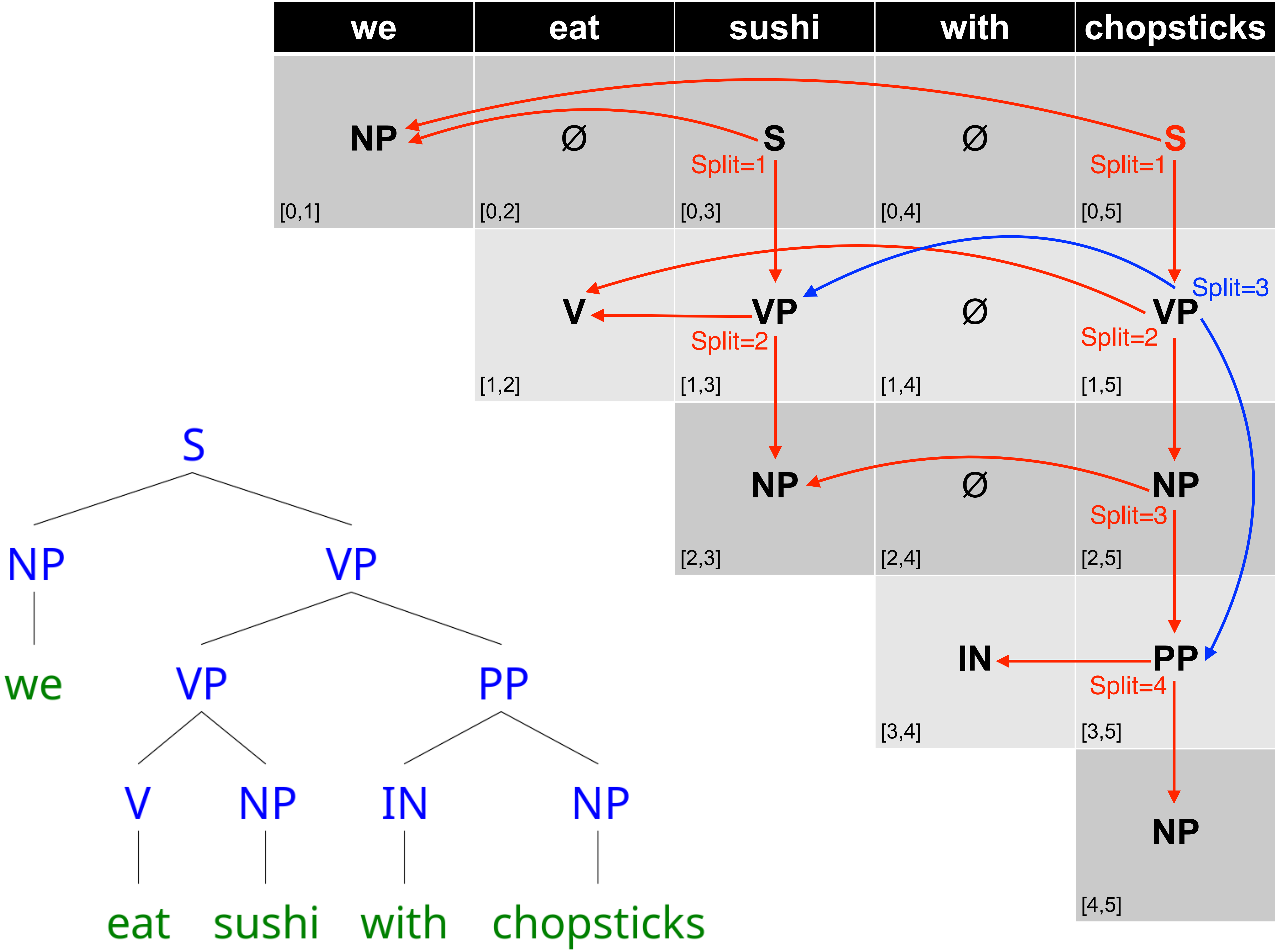
CYK: Retrieving the Parses

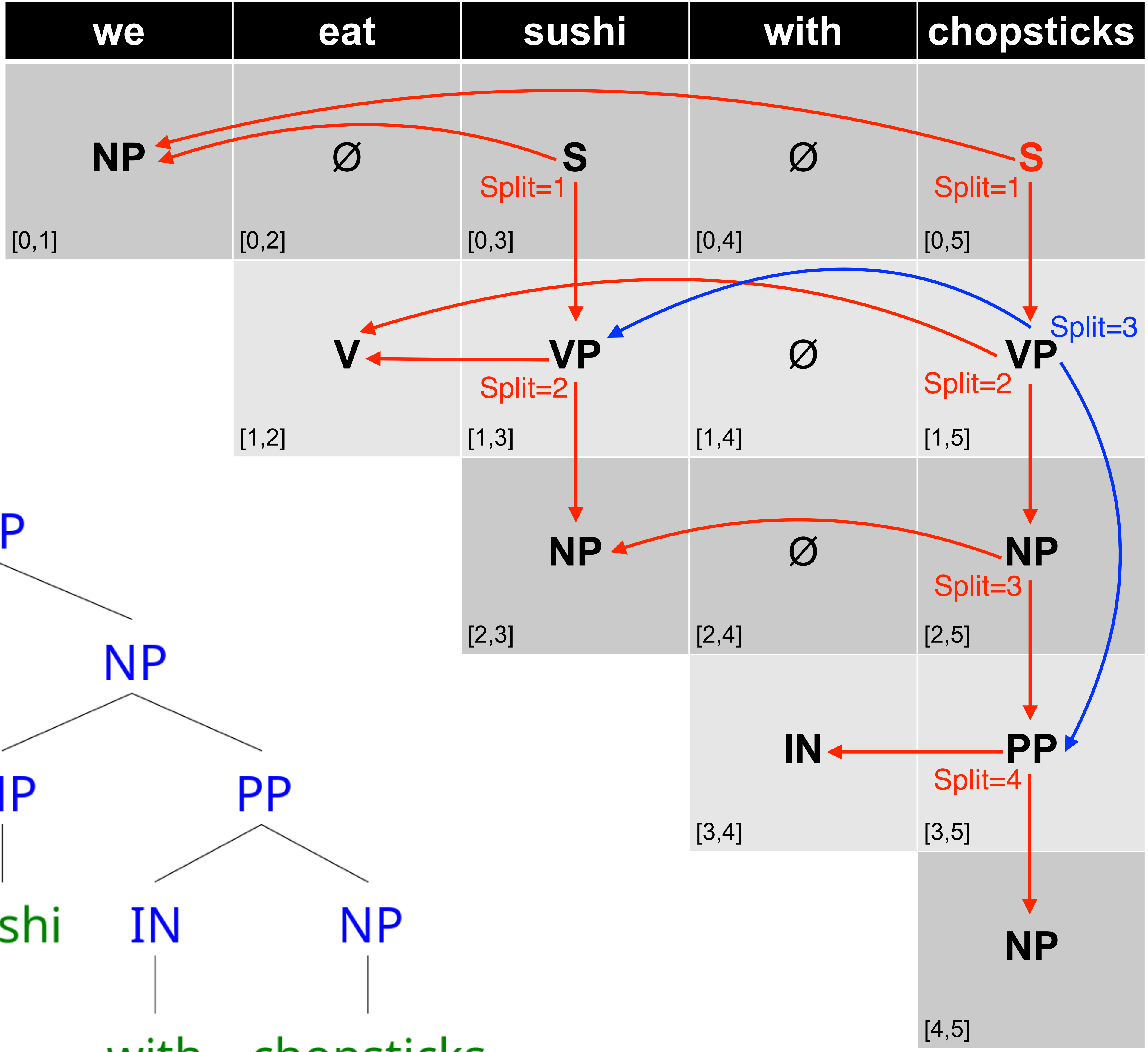
- S in the top-left corner of parse table indicates success
- To get parse(s), follow pointers back for each match



we	eat	sushi	with	chopsticks
NP [0,1]	∅ [0,2]	S Split=1 [0,3]	∅ [0,4]	S Split=1 [0,5]

	V
	[1,2]
		VP
		Split=2
		[1,3]
			∅
			[1,4]
				Split=3
				VP
				Split=2
				[1,5]

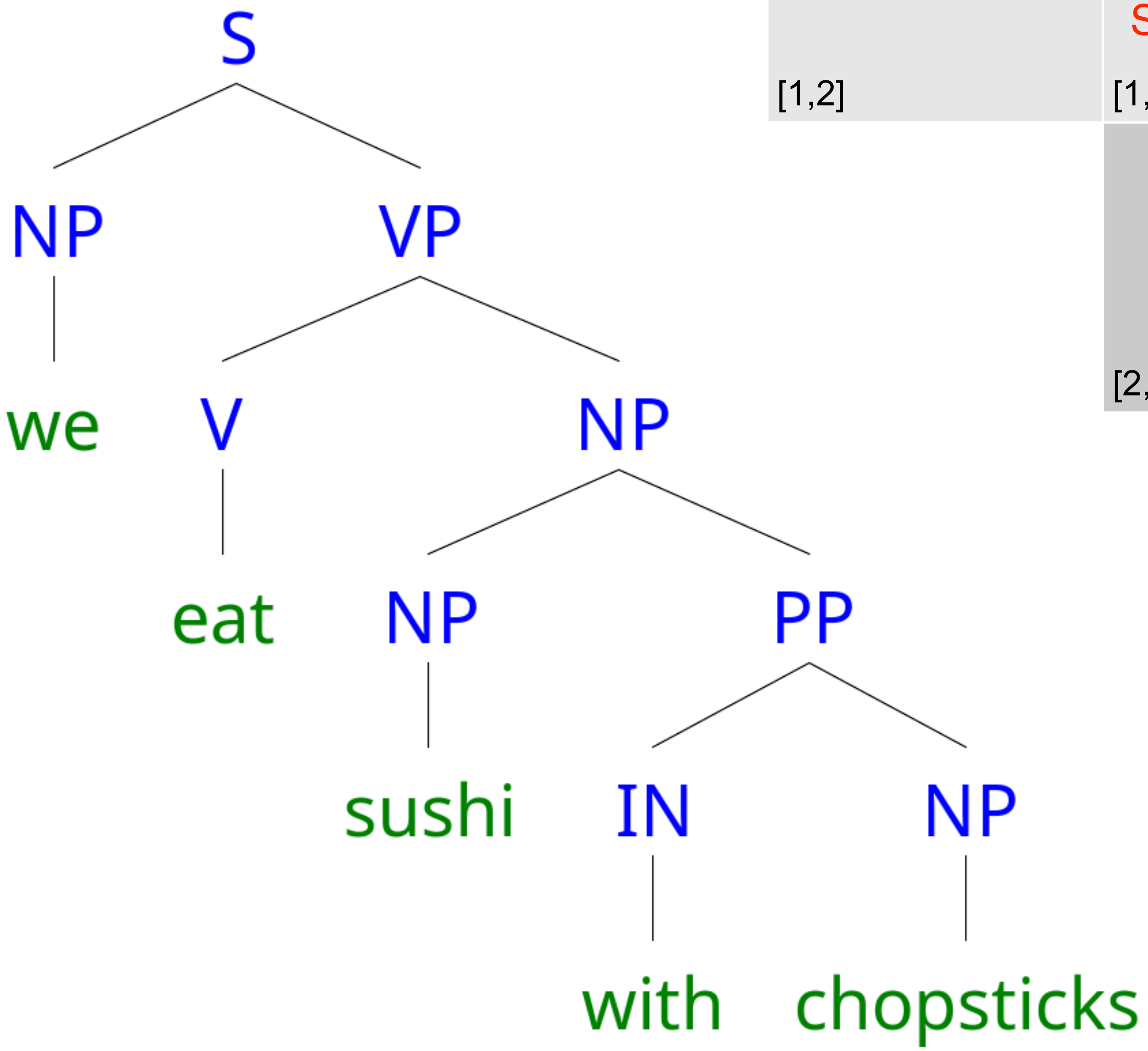
		NP [2,3]	∅ [2,4]	NP Split=3 [2,5]
			IN [3,4]	PP Split=4 [3,5]
				NP [4,5]



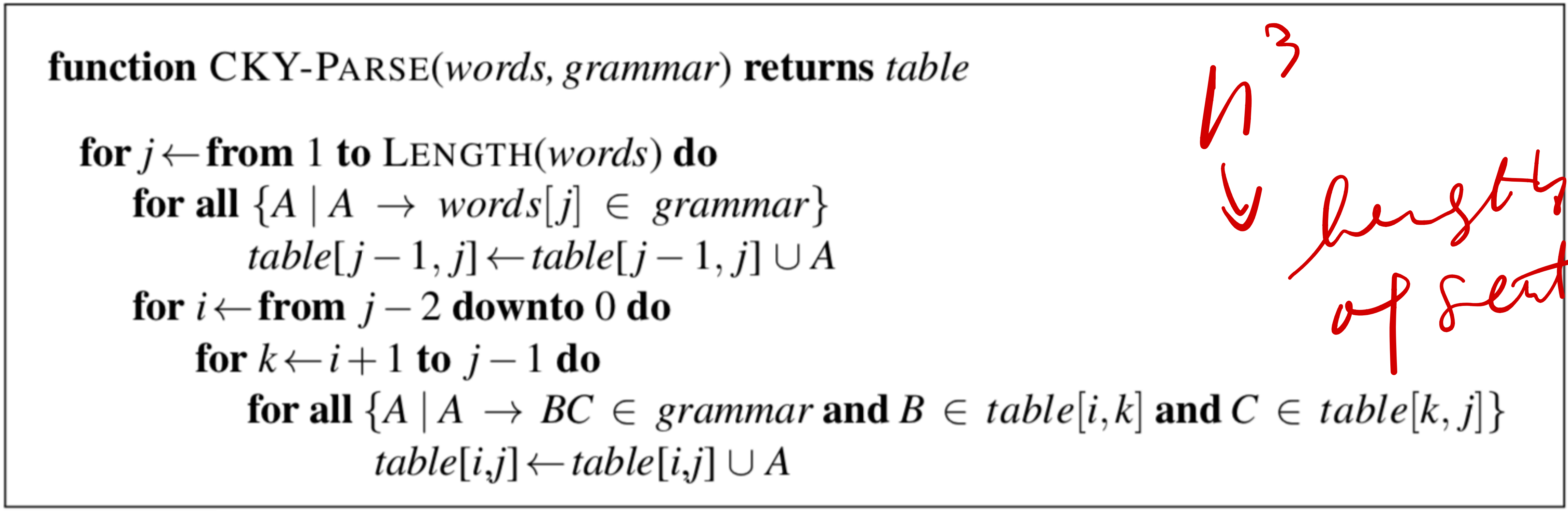
we	eat	sushi	with	chopsticks
NP [0,1]	Ø [0,2]	S Split=1 [0,3]	Ø [0,4]	S Split=1 [0,5]

	V
	[1,2]
		VP
		Split=2
		[1,3]
			Ø
			[1,4]
				VP
				Split=3
				Split=2
				[1,5]

		NP [2,3]	Ø [2,4]	NP Split=3 [2,5]
			IN [3,4]	PP Split=4 [3,5]
				NP [4,5]



CYK Algorithm



```
function CKY-PARSE(words, grammar) returns table

  for $j \leftarrow$ from 1 to LENGTH(words) do
      for all $\{A \mid A \rightarrow words[j] \in grammar\}$
          $table[j-1, j] \leftarrow table[j-1, j] \cup A$
      for $i \leftarrow$ from $j-2$ downto 0 do
          for $k \leftarrow i+1$ to $j-1$ do
              for all $\{A \mid A \rightarrow BC \in grammar$ and $B \in table[i,k]$ and $C \in table[k,j]\}$
                  $table[i,j] \leftarrow table[i,j] \cup A$
```

Figure 12.5 The CKY algorithm.

n^3

↓ length of sentence

Representing English with CFGs

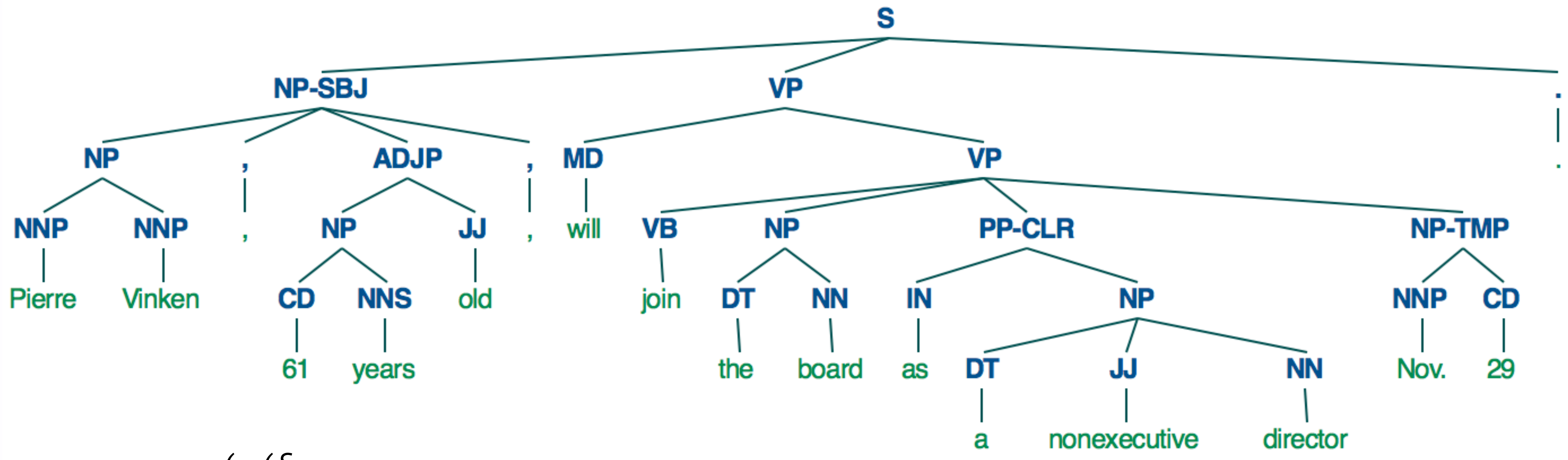
From Toy Grammars to Real Grammars

- Toy grammars with handful of productions good for demonstration or extremely limited domains
- For real texts, we need real grammars
- Many thousands of production rules

Key Constituents in Penn Treebank

- Sentence (S)
- Noun phrase (NP)
- Verb phrase (VP)
- Prepositional phrase (PP)
- Adjective phrase (AdjP)
- Adverbial phrase (AdvP)
- Subordinate clause (SBAR)

Example PTB/0001



```
( (S
    (NP-SBJ
      (NP (NNP Pierre) (NNP Vinken) )
      (, ,)
      (ADJP
        (NP (CD 61) (NNS years) )
        (JJ old) )
      (, ,) )
    (VP (MD will)
      (VP (VB join)
        (NP (DT the) (NN board) )
        (PP-CLR (IN as)
          (NP (DT a) (JJ nonexecutive) (NN director) ))
        (NP-TMP (NNP Nov.) (CD 29) )))
    (. .) ))
```

plaintex

Basic English Sentence Structures

- Declarative sentences (S → NP VP)
    - *The rat ate the cheese*
- Imperative sentences (S → VP)
    - *Eat the cheese!*
- Yes/no questions (S → VB NP VP)
    - *Did the rat eat the cheese?*
- *Wh*-subject-questions (S → WH VP)
    - *Who ate the cheese?*
- *Wh*-object-questions (S → WH VB NP VP)
    - *What did the rat eat?*

English Noun Phrases

- Pre-modifiers
  - DT, CD, ADJP, NNP, NN
  - E.g. *the two very best Philly cheese steaks*
- Post-modifiers
  - PP, VP, SBAR
  - A delivery from Bob coming today that I don’t want to miss

NP → DT? CD? ADJP? (NN|NNP)+ PP* VP? SBAR?

NP → PRP

Verb Phrases

- Auxiliaries in the front
  - MD, AdvP, VB, TO
  - E.g *should really have tried to wait*

VP → (MD|VB|TO) AdvP? VP

- Arguments and adjuncts post modifiers.
  - NP, PP, SBAR, VP, AdvP
  - E.g *told him yesterday that I was ready*
  - E.g. *gave John a gift for his birthday to make amends*

VP → VB NP? NP? PP* AdvP* VP? SBAR?

Other Constituents

- Prepositional phrase
  - PP → IN NP *in the house*
- Adjective phrase
  - AdjP → (AdvP) JJ *really nice*
- Adverb phrase
  - AdvP → (AdvP) RB *not too well*
- Subordinate clause
  - SBAR → (IN) S *since I came here*
- Coordination
  - NP → NP CC NP; VP → VP CC VP; etc. *Jack and Jill*
- Complex sentences
  - S → S SBAR; S → SBAR S; etc. *if he goes, I'll go*

A Final Word

- Context-free grammars can represent linguistic structure
- There are relatively fast dynamic programming algorithms to retrieve this structure
- But what about ambiguity?
  - Extreme ambiguity will slow down parsing
  - If multiple possible parses, which is best?

Readings

- E18 Ch. 9.2, 10.1