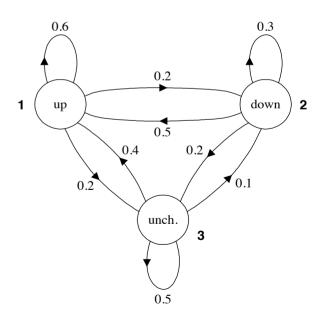
Sequence Tagging: Hidden Markov Models

COMP90042

Natural Language Processing

Lecture 6





POS Tagging Recap

- Janet will back the bill
- Janet/NNP will/MB back/VP the/DT bill/NN
- Local classifier: prone to error propagation.
 What about treating the full sequence as a "class"?
- - Output: "NNP MB VP DT NN"
- · Problems: Global classifier
 - ► Exponentially many combinations: ITagsIM, for length M
 - How to tag sequences of different lengths?

A Better Approach

- Tagging is a sentence-level task but as humans we decompose it into small word-level tasks.
 - Janet/NNP will/MB back/VP the/DT bill/NN
- Solution:
 - Define a model that decomposes process into individual word level steps
 - But that takes into account the whole sequence when learning and predicting (no error propagation)
- This is the idea of sequence labelling, and more general, structured prediction.

ops over the whole 52 grande

A Probabilistic Model

Goal: obtain best tag sequence t from sentence w

$$\hat{t} = argmax_t P(t \mid w)$$

$$\hat{t} = argmax_t \frac{P(w \mid t)P(t)}{P(w)} = argmax_t P(w \mid t) P(t)$$
[Bayes]

Let's decompose:

$$P(\boldsymbol{w}\,|\,\boldsymbol{t}) = \prod_{i=1}^n P(w_i\,|\,t_i)$$
 [Prob. of a word depends only on the tag]
$$P(\boldsymbol{t}) = \prod_{i=1}^n P(t_i\,|\,t_{i-1})$$
 [Prob. of a tag depends only on the previous tag]

- These are independence assumptions (bigram language models?)
- This is a Hidden Markov Model (HMM)

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Hidden Markov Model

$$\hat{t} = argmax_t P(w | t) P(t)$$

$$P(w | t) = \prod_{i=1}^{n} P(w_i | t_i)$$

$$P(t) = \prod_{i=1}^{n} P(t_i | t_{i-1})$$

- Why "Markov"?
 - Because it assumes the sequence follows a Markov chain: probability of an event (tag) depends only on the previous event (last tag)
- Why "Hidden"?
 - Because the events (tags) are not seen: goal is to find the best sequence

HMMs - Training

- Toy emits words. Parameters are the individual probabilities $\mathbf{P}(w_i | t_i)$ and $P(t_i \mid t_{i-1})$
 - ▶ Respective y, emission (O) and transition (A) probabilities
- Training uses Maximum Likelihood Estimation (MLE)
 - In Naïve Bayes & n-gram LMs, this is done by simply counting e) every VB that
 e) eviss like word frequencies according to the class.
- We do **exactly the same** in HMMs!

$$P(like | VB) = \frac{count(VB, like)}{count(VB)}$$

$$P(NN | DT) = \frac{count(DT, NN)}{count(DT)}$$

$$\frac{d}{dt}$$

$$P(NN \mid DT) = \frac{count(DT, NN)}{(DT)}$$

HMMs - Training

- What about the first tag?
 - Assume we have a symbol "<s>" that represents the start of your sentence.

$$P(NN \mid \langle s \rangle) = \frac{count(\langle s \rangle, NN)}{count(\langle s \rangle)}$$

- What about the last tag?
 - Assume we have a symbol "</s>" that represents the end of sentence.
- What about unseen (word, tag) and (tag, previous) combinations?

Smoothing techniques, like NB/n-gram LMs

Jive some mos mass to them.

Transition Matrix China Australia.



	NNP	MD	VB	JJ	NN	RB	DT
< <i>s</i> >	0.2767	0.0006	0.0031	0.0453	0.0449	0.0510	0.2026
NNP	0.3777	0.0110	0.0009	0.0084	0.0584	0.0090	0.0025
MD	0.0008	0.0002	0.7968	0.0005	0.0008	0.1698	0.0041
VB	0.0322	0.0005	0.0050	0.0837	0.0615	0.0514	0.2231
JJ	0.0366	0.0004	0.0001	0.0733	0.4509	0.0036	0.0036
NN	0.0096	0.0176	0.0014	0.0086	0.1216	0.0177	0.0068
RB	0.0068	0.0102	0.1011	0.1012	0.0120	0.0728	0.0479
DT	0.1147	0.0021	0.0002	0.2157	0.4744	0.0102	0.0017

The *A* transition probabilities $P(t_i|t_{i-1})$ computed from the WSJ corpus without Figure 8.7 smoothing. Rows are labeled with the conditioning event; thus P(VB|MD) is 0.7968.

Emission (Observation) Matrix

	Janet	will	back	the	bill
NNP	0.000032	0	0	0.000048	0
MD	0	0.308431	0	0	0
VB	0	0.000028	0.000672	0	0.000028
JJ	0	0	0.000340	0	0
NN	0	0.000200	0.000223	0	0.002337
RB	0	0	0.010446	0	0
DT	0	0	0	0.506099	0

Figure 8.8 Observation likelihoods B computed from the WSJ corpus without smoothing, simplified slightly.

HMMs – Prediction (Decoding)

$$\hat{t} = argmax_{t} P(w \mid t) P(t)$$

$$= argmax_{t} \prod_{i=1}^{n} P(w_{i} \mid t_{i}) P(t_{i} \mid t_{i-1})$$
• Simple idea: for each word, take the tag that maximises $P(w_{i} \mid t_{i}) P(t_{i} \mid t_{t-1})$. Do it left-to-right, in *greedy* fashion.

- This is wrong! We are looking for argmax_t, not individual $argmax_{t_i}$ terms.
 - ▶ This is a local classifier: error propagation
- Correct way: take all possible tag combinations, evaluate them, take the max (like Naïve Bayes)
 - Problem: exponential number of sequences.

- Dynamic Programming to the rescue!
 - We can still proceed sequentially, as long as we are careful.
- "can play" -> can/MD play/VB
- Best tag for "can" is easy: $argmax_t P(can | t)P(t | < s >)$
 - We can do that because first "tag" is always "<s>"
- Suppose best tag for "can" is NN. To get the tag for "play", we can take $argmax_t P(\operatorname{play} \mid t) P(t \mid \operatorname{NN})$ but this is wrong.
- Instead, we keep track of scores for each tag for "can" and check what would happen if "can" had a different tag.

	Janet	will	back	the	bill
NNP	Janet 2(Janet D				
MD					
VB					
JJ					
NN					
RB					
DT					

	Janet	will	back	the	bill
NNP	P(JanetINNP) * P(NNPI <s>)</s>				
MD	P(JanetIMD) * P(MDI <s>)</s>				
VB					
JJ					
NN					
RB					
DT					

Transition and Emission Matrix

	NNP	MD	VB	JJ	NN	RB	DT
< <i>s</i> >	0.2767	0.0006	0.0031	0.0453	0.0449	0.0510	0.2026
NNP	0.3777	0.0110	0.0009	0.0084	0.0584	0.0090	0.0025
MD	0.0008	0.0002	0.7968	0.0005	0.0008	0.1698	0.0041
VB	0.0322	0.0005	0.0050	0.0837	0.0615	0.0514	0.2231
JJ	0.0366	0.0004	0.0001	0.0733	0.4509	0.0036	0.0036
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RB	0.0068	0.0102	0.1011	0.1012	0.0120	0.0728	0.0479
DT	0.1147	0.0021	0.0002	0.2157	0.4744	0.0102	0.0017

Figure 8.7 The A transition probabilities $P(t_i|t_{i-1})$ computed from the WSJ corpus without smoothing. Rows are labeled with the conditioning event; thus P(VB|MD) is 0.7968.

	Janet	will	back	the	bill
NNP	0.000032	0	0	0.000048	0
MD	0	0.308431	0	0	0
VB	0	0.000028	0.000672	0	0.000028
JJ	0	0	0.000340	0	0
NN	0	0.000200	0.000223	0	0.002337
RB	0	0	0.010446	0	0
DT	0	0	0	0.506099	0

Figure 8.8 Observation likelihoods *B* computed from the WSJ corpus without smoothing, simplified slightly.

	Janet	will	back	the	bill
NNP	0.000032 * 0.2767				
MD	0 * 0.0006				
VB	0				
JJ	0				
NN	0				
RB)				
DT					

	Janet	will	back	the	bill
NNP	8.8544e-06				
MD	0				
VB	0				
JJ	0				
NN	0				
RB	0				
DT	0				

	Janet	will	back	the	bill
NNP	8.8544e-06	P(willINNP) * P(NNPIt _{Janet}) * s(t _{Janet} IJanet)			
MD	0				
VB	0 /7				
JJ	0				
NN	0				
RB	0				
DT	0				

	Janet	will	back	the	bill
NNP	8.854 4c 06	P(willINNP) * P(NNPIt s(t _{Janet} IJanet)			
MD	0		late this for al he max.	l tags,	
VB	0	·	•	•	s(NNP Janet),
JJ	0		will I NNP) * P(I	, ,	, .
NN	0	F(will I NNP) * P(I	NINPIDI) S(L	Ji i Janet))
RB	0				
DT	0				

Transition and Emission Matrix

	NNP	MD	VB	JJ	NN	RB	DT
< <i>s</i> >	0.2767	0.0006	0.0031	0.0453	0.0449	0.0510	0.2026
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MD	0.0008	0.0002	0.7968	0.0005	0.0008	0.1698	0.0041
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NN	0.0096	0.0176	0.0014	0.0086	0.1216	0.0177	0.0068
RB	0.0068	0.0102	0.1011	0.1012	0.0120	0.0728	0.0479
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MD	0	0.308431	0	0	0
VB	0	0.000028	0.000672	0	0.000028
JJ	0	0	0.000340	0	0
NN	0	0.000200	0.000223	0	0.002337
RB	0	0	0.010446	0	0
DT	0	0	0	0.506099	0

Figure 8.8 Observation likelihoods *B* computed from the WSJ corpus without smoothing, simplified slightly.

	Janet	will	back	the	bill
NNP	8.8544e-06	0 * P(NNPIt _{Janet}) * s(t _{Janet} lJanet)			
MD	0				
VB	0				
JJ	0				
NN	0				
RB	0				
DT	0				

	Janet	will		back	the	bill
NNP	8.8544e-06	0 max				
MD	0	P(willIM P(MDIt _J s(t _{Janet} IJ	anet) *			
VB	0			(will I MD) * P(I		
JJ	0			will I MD) * P(M	, ,	ŕ
NN	0		Г	will I MD) * P(M	S(DT)	r Janet))
RB	0					
DT	0					

	Janet	will	back	the	bill
NNP	8.8544e-06	0			
MD	0	3.004e-8			
VB	0				
JJ	0				
NN	0				
RB	0				
DT	0				

	Janet	will	back	the	bill
NNP	8.8544e-06	0			
MD	0	3.004e-8			
VB	0	2.231e-13			
JJ	0	0			
NN	0	1.034e-10			
RB	0	0			
DT	0	0			

Transition and Emission Matrix

	NNP	MD	VB	JJ	NN	RB	DT
< <i>s</i> >	0.2767	0.0006	0.0031	0.0453	0.0449	0.0510	0.2026
NNP	0.3777	0.0110	0.0009	0.0084	0.0584	0.0090	0.0025
MD	0.0008	0.0002	0.7968	0.0005	0.0008	0.1698	0.0041
VB	0.0322	0.0005	0.0050	0.0837	0.0615	0.0514	0.2231
JJ	0.0366	0.0004	0.0001	0.0733	0.4509	0.0036	0.0036
NN	0.0096	0.0176	0.0014	0.0086	0.1216	0.0177	0.0068
RB	0.0068	0.0102	0.1011	0.1012	0.0120	0.0728	0.0479
DT	0.1147	0.0021	0.0002	0.2157	0.4744	0.0102	0.0017

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	Janet	will	back	the	bill
NNP	0.000032	0	0	0.000048	0
MD	0	0.308431	0	0	0
VB	0	0.000028	0.000672	0	0.000028
JJ	0	0	0.000340	0	0
NN	0	0.000200	0.000223	0	0.002337
RB	0	0	0.010446	0	0
DT	0	0	0	0.506099	0

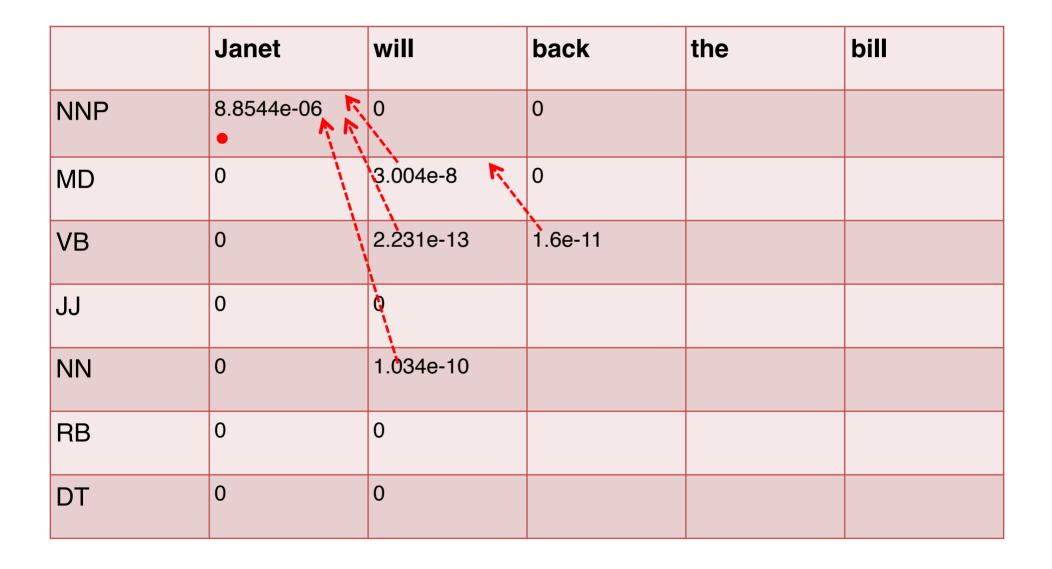
Figure 8.8 Observation likelihoods *B* computed from the WSJ corpus without smoothing, simplified slightly.

	Janet	will	back	the	bill
NNP	8.8544e-06	0			
MD	0	3.004e-8			
VB	0	2.231e-13			
JJ	0	O.			
NN	0	1.034e-10			
RB	0	0			
DT	0	0			

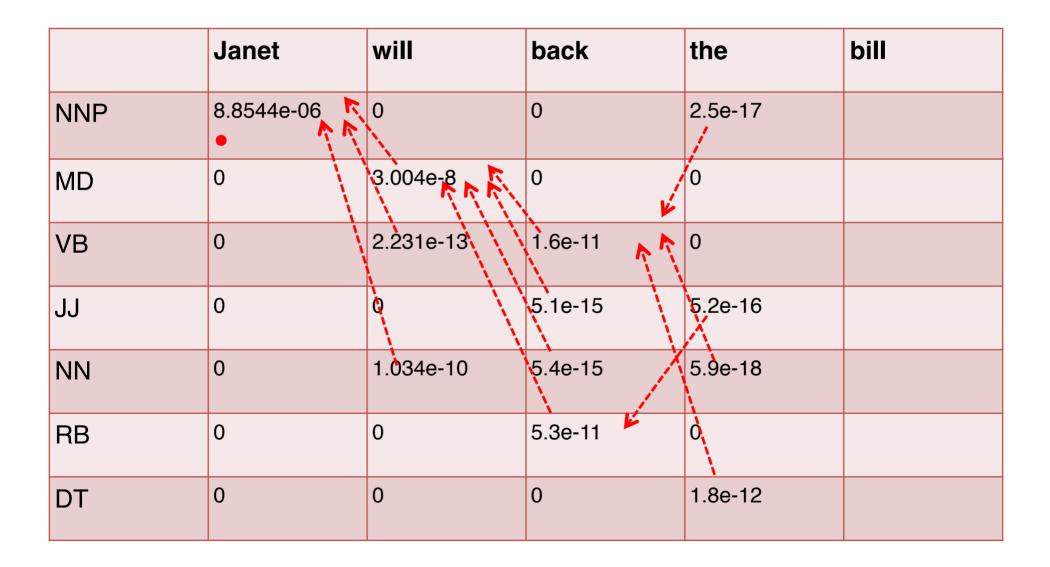
	Janet	will	back	the	bill
NNP	8.8544e-06	0	0		
MD	0	3.004e-8	0		
VB	0	2.231e-13	P(backIVB) * P(VBIt _{will}) * s(t _{will} Iwill)		
JJ	0	O			
NN	0	1.034e-10			
RB	0	0			
DT	0	0			

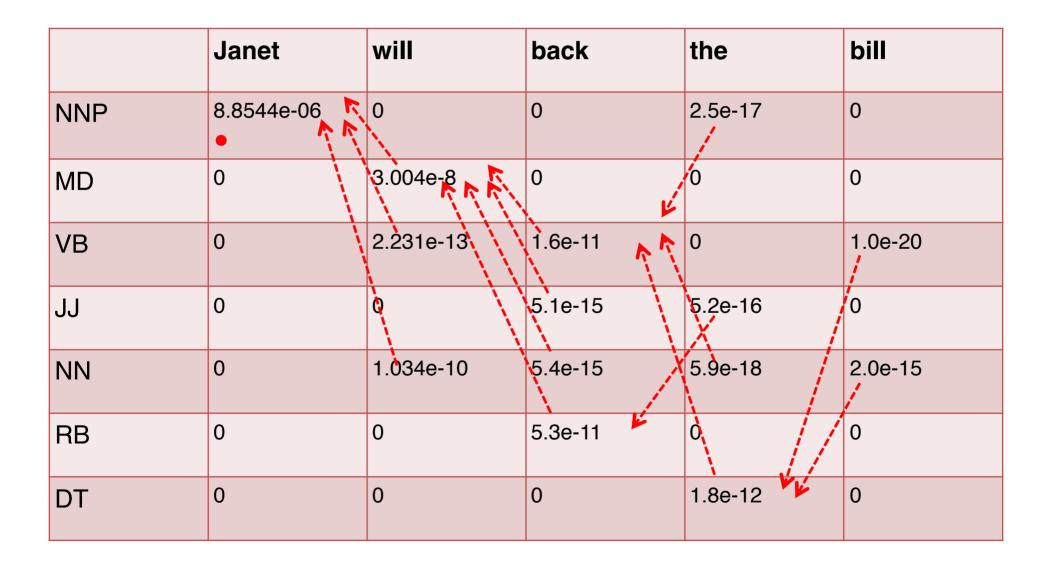
	Janet	will	back	the	bill
NNP	8.8544e-06	0	0		
MD	0	3.004e-8	0		
VB	0	2.231e-13	MD: 1.6e-11 VB: 7.5e-19 NN: 9.7e-17		
JJ	0	d			
NN	0	1.034e-10			
RB	0	0			
DT	0	0			

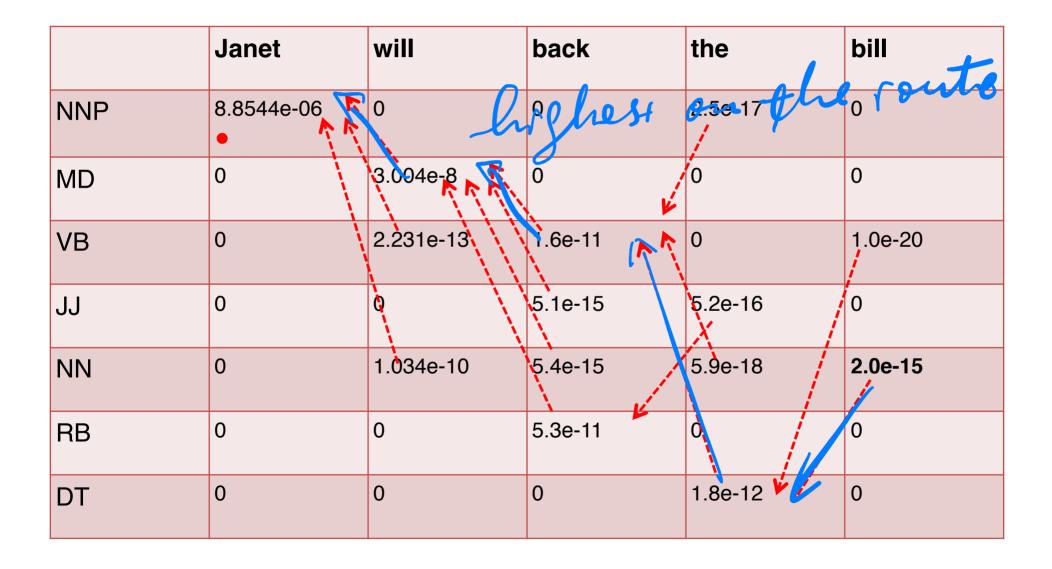
	Janet	will	back	the	bill
NNP	8.8544e-06	0	0		
MD	0	3.004e-8	0		
VB	0	2.231e-13	MD: 1.6e-11 VB: 7.5e-19 NN: 9.7e-17		
JJ	0	d			
NN	0	1.034e-10			
RB	0	0			
DT	0	0			

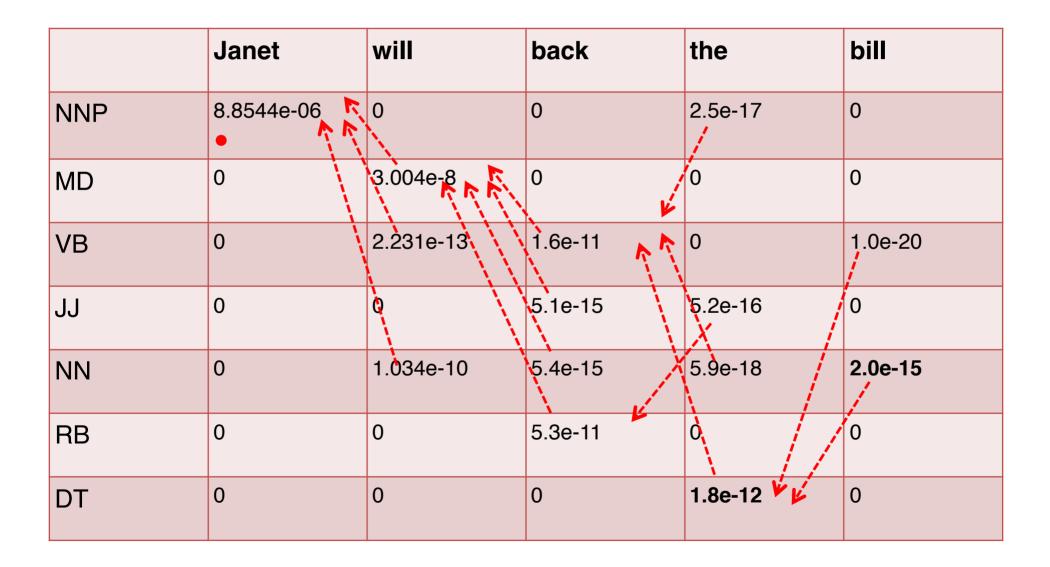


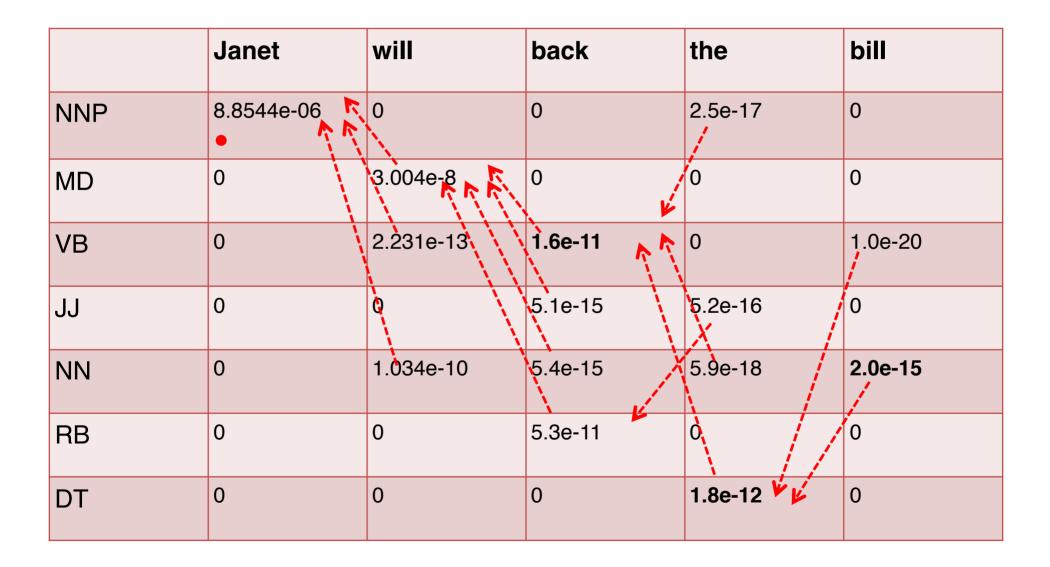
	Janet	will	back	the	bill
NNP	8.8544e-06	0	0		
MD	0	3.004e-8	0		
VB	0	2.231e-13	1.6e-11		
JJ	0	0	5.1e-15		
NN	0	1.034e-10	5.4e-15		
RB	0	0	5.3e-11		
DT	0	0	0		

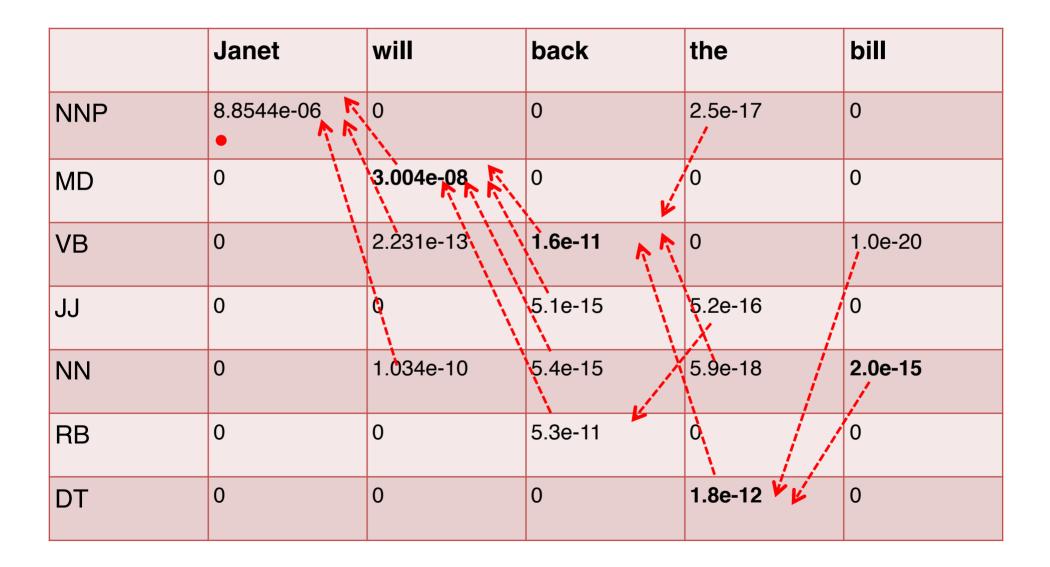


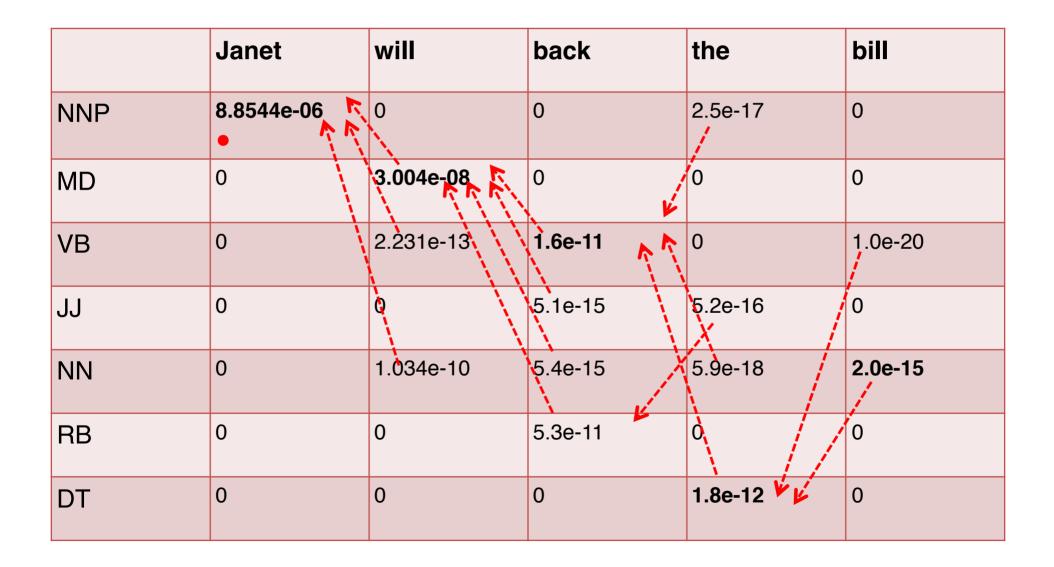










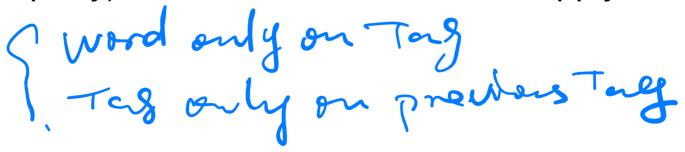


	Janet	will	back	the	bill
NNP	8.8544e-06	0	0	2.5e-17	0
MD	0	3.004e-08	0	0	0
VB	0	2.231e-13	1.6e-11	0	1.0e-20
JJ	0	0	5.1e-15	5.2e-16	0
NN	0	1.034e-10	5.4e-15	5.9e-18	2.0e-15
RB	0	0	5.3e-11	O ₁	0
DT	0	0	0	1.8e-12 💆	0

	Janet/NNP	will/MD	back/VB	the/DT	bill/NN
NNP	8.8544e-06	0	0	2.5e-17	0
MD	0	3.004e-08	0	0	0
VB	0	2.231e-13	1.6e-11	0	1.0e-20
JJ	0	0	5.1e-15	5.2e-16	0
NN	0	1.034e-10	5.4e-15	5.9e-18	2.0e-15
RB	0	0	5.3e-11	O ₁	0
DT	0	0	0	1.8e-12 📂	0

T*N. The Viterbi Algorithm every numbers compute Toperations

- Complexity: O(T²N), where T is the size of the tagset and N is the length of the sequence.
 - T * N matrix, each cell performs T operations.
- Why does it work?
 - ▶ Because of the independence assumptions that decompose the problem (specifically, the Markov property). Without these, we cannot apply DP.



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Viterbi Pseudocode

```
alpha = np.zeros(M, T)
for t in range(T):
  alpha[1, t] = pi[t] * O[w[1], t]
for i in range(2, M):
  for t i in range(T):
    for t last in range(T): # t last means t \{i-1\}
      s = alpha[i-1, t_last] * A[t_last, t_i] * O[w[i], t_i]
      if s > alpha[i,t i]:
        alpha[i,t i] = s
       back[i,t i] = t last
best = np.max(alpha[M-1,:])
return backtrace(best, back)
```

- Good practice: work with log probabilities to prevent underflow (multiplications become sums)
- Vectorisation (use matrix-vector operations)

HMMs In Practice

 We saw HMM taggers based on bigrams. State-of-theart use tag trigrams.

$$P(t) = \prod_{i=1}^{n} P(t_i | t_{i-1}, t_{i-2}) \text{ Viterbi now O(T³N)}$$

- Need to deal with sparsity: some tag trigram sequences might not be present in training data
 - ▶ Backoff: $P(t_i | t_{i-1}, t_{i-2}) = \lambda_3 \hat{P}(t_i | t_{i-1}, t_{i-2}) + \lambda_2 \hat{P}(t_i | t_{i-1}) + \lambda_1 \hat{P}(t_i)$ ▶ $\lambda_1 + \lambda_2 + \lambda_3 = 1$
- With additional features, reach 96.5% accuracy on Penn Treebank (Brants, 2000)

Other Variant Taggers

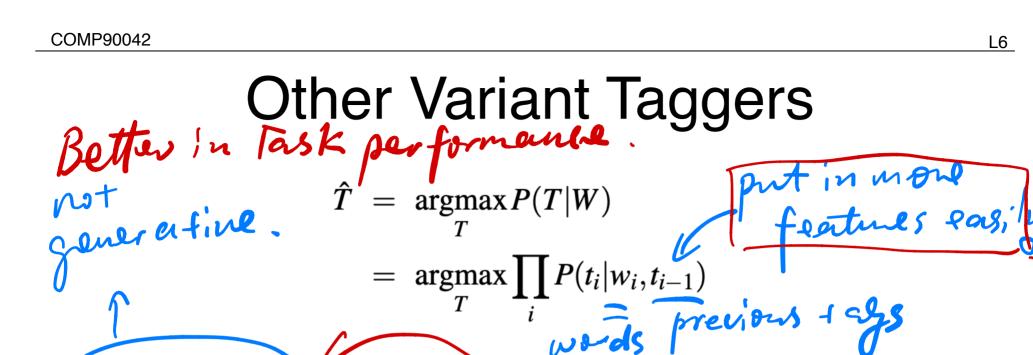
- HMM is generative
 - allows for unsupervised HMMs: learn model without any tagged data!

$$\hat{T} = \underset{T}{\operatorname{argmax}} P(T|W)$$

$$= \underset{T}{\operatorname{argmax}} P(W|T)P(T)$$

$$= \underset{T}{\operatorname{argmax}} \prod_{i} P(word_{i}|tag_{i}) \prod_{i} P(tag_{i}|tag_{i-1})$$

(an do text governion, Not with Bert)



Discriminative models describe P(t I w) directly

- supports richer feature set, generally better accuracy when trained over large supervised datasets.
- E.g., Maximum Entropy Markov Model (MEMM), Conditional random field (CRF), Connectionist Temporal Classification (CTC)
- Most deep learning models of sequences are discriminative (e.g., encoder-decoders for translation), similar to an MEMM

HMMs in NLP

- HMMs are highly effective for part-of-speech tagging
 - trigram HMM gets 96.5% accuracy (TnT)
 - related models are state of the art
 - ▶ MEMMs 97%
 - ▶ CRFs 97.6%
 - ▶ Deep CRF 97.9%
 - ► English Penn Treebank tagging accuracy https://aclweb.org/aclwidex.php?title=POS Tagging (State of the art)
- Apply out-of-the box to other sequence labelling tasks
 - named entity recognition, shallow parsing, alignment ...
 - ▶ In other fields: DNA, protein sequences, image lattices...

A Final Word

- HMMs are a simple, yet effective way to perform sequence labelling.
- Can still be competitive, and fast. Natural baseline / for other sequence labelling tasks.
- Main drawback: not very flexible in terms of feature representation, compared to MEMMs and CRFs.

Readings

- JM3 Appendix A A.1-A.2, A.4
- See also E18 Chapter 7.3
- References:
 - Rabiner's HMM tutorial http://tinyurl.com/2hqaf8
 - Lafferty et al, Conditional random fields: Probabilistic models for segmenting and labeling sequence data (2001)