1 du

1.1

1.1.1 $(a_n) = (1, -1, 2, -1, 3, -3, ...)$

$(a_n) = (1, 0, 2, 0, 3, 0...) - (0, 1, 0, 2, 0, 3) + (0, 0, 0, 1, 0, 0...)$
$f(x) = \frac{1-x}{(1-x^2)^2} + x^3$

1.1.2 $(b_n) = (1, -3, 5, -7, 9, -11, ...)$

prohodíme $-x$ za x, přičteme $(1, 1, 1, ...)$ a vydělíme 2

$\sum_{n\geq 0}(2n+1)x^n = \frac{1-x}{(1+x)^2}$

1.1.3 $(c_n) = (1, 4, 9, 16, 25, 36, ...)$

$\sum_{n\geq 0} n^2x^n = \frac{2}{(1-x)^3} - \frac{3}{(1-x)^2} + \frac{1}{1-x} = \frac{x(x+1)}{(1-x)^3}$

1.2

1.2.1 $[x^5] : (2x-1)^-2$

$\frac{1}{(1-2x)^2} = \sum_{n=0}^{\infty} x^n 2^n (1+n)$

$\left[x^5\right] : 192$

1.2.2 $[x^5] : (1+x)^{-1/3}$

$\sum_{n=0}^{\infty} \binom{-\frac{1}{3}}{n} x^n$

$\left[x^5\right] : -\frac{91}{729}$

1.3

1.3.1 $a_0 = 0, a_1 = 1, a_n = a_{n-1} + a_{n-2} + 2$

$a_0 \to (0, 1, f_1 + f_2 + 2, ...)$
$= (0, 1, 0, 0...) = x$
$+ (0, f_0, f_1, f_2, ...) = xf(x)$
$+ (0, 0, f_0, f_1, ...) = x^2f(x)$
$+ (0, 0, 2, 2, ...) = \frac{2x^2}{1-x}$

$f(x) = x + xf(x) + x^2f(x) + \frac{2x^2}{1-x}$
$1 = \frac{x}{f(x)} + x + x^2 + \frac{\frac{2x^2}{1-x}}{f(x)}$
$f(x) = \frac{x+\frac{2x^2}{1-x}}{1-x-x^2}$
$\to x \neq \frac{-1-\sqrt{5}}{2}, x \neq \frac{\sqrt{5}-1}{2}, x \neq 1$

$a_n = \left(-2 + \frac{2^{-n}\left((1-\sqrt{5})^n(-2+\sqrt{5})+(1+\sqrt{5})^n(2+\sqrt{5})\right)}{\sqrt{5}}\right)$

1.3.2 $b_0 = 2, b_1 = 3, b_n = 3b_{n-2} - 2b_{n-1}$

$a_0 \to (2, 3, 3f_0 - 2f_1)$
$= (2, 3, 0, 0) = 2 + 3x$
$+ (0, 0, 3f_0, 3f_1) = 3x^2 f(x)$
$+ (0, 0, -2f_1, -2f_2) = -2xf(x) + 4x$

$f(x) = 2 + 3x + 3x^2 f(x) - 2f(x) + 4x$
$f(x) - 3x^2 f(x) + 2xf(x) = 2 + 7x$
$1 - 3x^2 + 2x = \frac{2+7x}{f(x)}$
$f(x) = \frac{2+7x}{1-3x^2+2x}$
$\to x \neq -\frac{1}{3}, x \neq 1$

$a_n = \frac{9-(-3)^n}{4}$