1 du

1.1

$$(a_n) = (1, -1, 2, -1, 3, -3, ...)$$

$(a_n) = (1, 0, 2, 0, 3, 0...) - (0, 1, 0, 2, 0, 3) + (0, 0, 0, 1, 0, 0...)$
$f(x) = \frac{1-x}{(1-x^2)^2} + x^3$

$$(b_n) = (1, -3, 5, -7, 9, -11, ...)$$

prohodíme $-x$ za x, přičteme $(1, 1, 1, ...)$ a vydělíme 2

$\sum_{n\geq 0}(2n+1)x^n = \frac{1-x}{(1+x)^2}$

$$(c_n) = (1, 4, 9, 16, 25, 36, ...)$$

$\sum_{n\geq 0} n^2x^n = \frac{2}{(1-x)^3} - \frac{3}{(1-x)^2} + \frac{1}{1-x} = \frac{x(x+1)}{(1-x)^3}$

1.2

$$[x^5] : (2x-1)^-2$$

$\frac{1}{(1-2x)^2} = \sum_{n=0}^{\infty} x^n 2^n (1+n)$

$\left[x^5\right] : 192$

$$[x^5] : (1+x)^{-1/3}$$

$\sum_{n=0}^{\infty} \binom{-\frac{1}{3}}{n} x^n$

$\left[x^5\right] : -\frac{91}{729}$

1.3

$$a_0 = 0, a_1 = 1, a_n = a_{n-1} + a_{n-2} + 2$$

$a_0 \leftarrow (0, 1, f_1 + f_2 + 2, ...)$
$= (0, 1, 0, 0...) = x$
$+ (0, f_0, f_1, f_2, ...) = xf(a)$
$+ (0, 0, f_0, f_1, ...) = x^2 f(a)$
$+ (0, 0, 2, 2, ...) = \frac{2x^2}{1-x}$

$a_n = 2^n - 1$

$$b_0 = 2, b_1 = 3, b_n = 3_{n-2} - 2b_{n-1}$$

$a_n = \frac{9-(-3)^n}{4}$