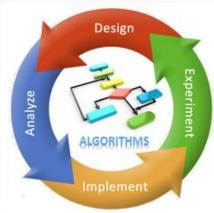
Analysis and Design of Algorithms
(ADA)
GTU # 3150703



Unit-5 Greedy Algorithms





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Outline

- General Characteristics of greedy algorithms
- Elements of Greedy Strategy
- Make change Problem
- Minimum Spanning trees (Kruskal's algorithm, Prim's algorithm)
- The Knapsack Problem
- Job Scheduling Problem
- Huffman code







Introduction



Characteristics of Greedy Algorithms

- ☐ Greedy algorithms are characterized by the following features.
 - 1. Greedy approach forms a set or list of candidates *C*.
 - 2. Once a candidate is selected in the solution, it is there forever: once a candidate is excluded from the solution, it is never reconsidered.
 - 3. To construct the solution in an optimal way, Greedy Algorithm maintains two sets.
 - 4. One set contains candidates that have already been considered and chosen, while the other set contains candidates that have been considered but rejected.
- ☐ The greedy algorithm consists of four functions.
 - **i. Solution Function:** A function that checks whether chosen set of items provides a solution.
 - ii. Feasible Function: A function that checks the feasibility of a set.
 - iii. Selection Function:- The selection function tells which of the candidates is the most promising.
 - iv. Objective Function:- An objective function, which does not appear explicitly, but gives the value of a solution.







Make a Change Problem



Problem Definition

- Suppose following coins are available with unlimited quantity:
 - 1. ₹10
 - 2. ₹5
 - 3. ₹2
 - 4. ₹1
 - 5. 50 paisa
- Our problem is to devise an algorithm for paying a given amount to a customer using the smallest possible number of coins.



Make Change - Greedy Solution

- If suppose, we need to pay an amount of ₹ 28/- using the available coins.
- Here we have a candidate (coins) set $C = \{10, 5, 2, 1, 0.5\}$
- The greedy solution is,



Amount 28

Total required coins = 5
Selected coins = {10, 5, 2,
1}



Make Change - Algorithm

```
# Input: C = \{10, 5, 2, 1, 0.5\} //C is a candidate set
# Output: S: set of selected coins
Function make-change(n): set of coins
S \leftarrow \emptyset {S is a set that will hold the solution}
sum \leftarrow 0 {sum of the items in solution set S}
while sum ≠ n do
   x \leftarrow the largest item in C such that sum + x \leq n
   if there is no such item then
          return "no solution found"
   S \leftarrow S \cup \{a \text{ coin of value } x\}
   sum \leftarrow sum + x
return S
```

Make Change – The Greedy Property

- The algorithm is greedy because,
 - → At every step it chooses **the largest available coin**, without worrying whether this will prove to be a **correct** decision later.
 - → It never changes the decision, i.e., once a coin has been included in the solution, it is there forever.
- **Examples**:
 - 1. Some coins with denominations 50, 20, 10, 5, 1 are available.
 - How many minimum coins required to make change for 37 cents?
 - How many minimum coins required to make change for 91 cents?
 - 2. Denominations: $d_1=6$, $d_2=4$, $d_3=1$. Make a change of ξ 8.



The minimum coins required are

2







Minimum Spanning Tree

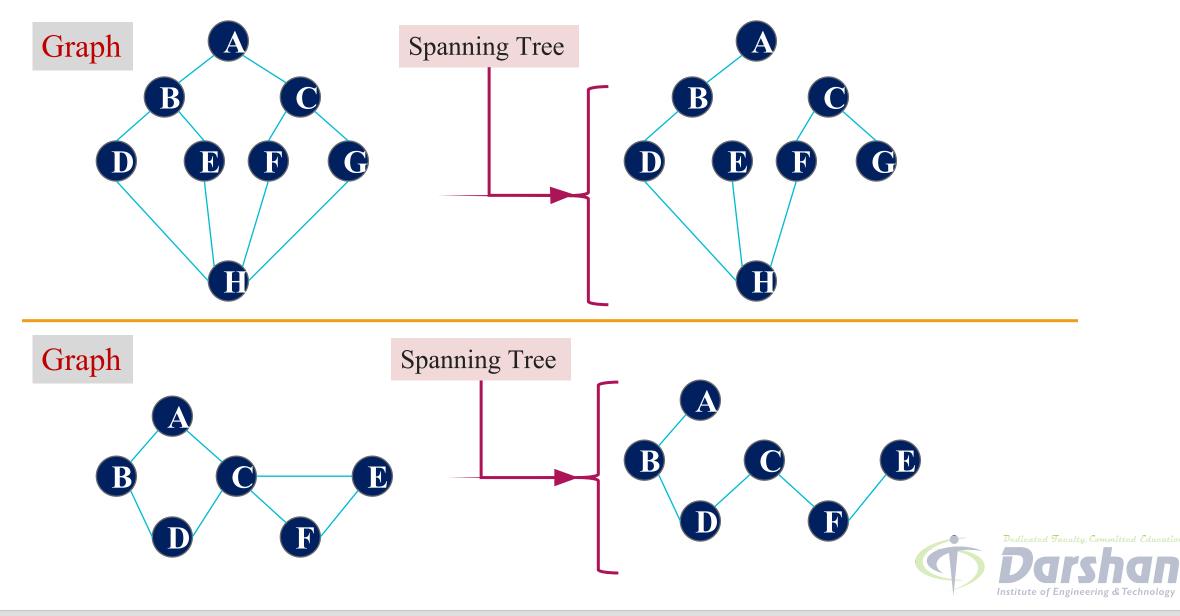


Introduction to Minimum Spanning Tree (MST)

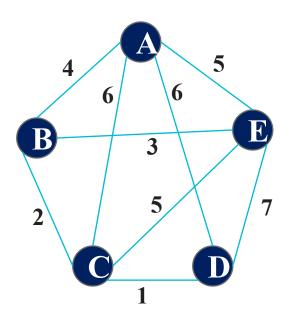
- Let $G = \langle N, A \rangle$ be a connected, undirected graph where,
 - 1. N is the set of nodes and
 - 2. A is the set of edges.
- Each edge has a given positive length or weight.
- ▶ A spanning tree of a graph *G* is a sub-graph which is basically a tree and it contains all the vertices of *G* but does not contain cycle.
- ▶ A minimum spanning tree (MST) of a **weighted connected graph** *G* is a spanning tree with minimum or smallest weight of edges.
- Two Algorithms for constructing minimum spanning tree are,
 - 1. Kruskal's Algorithm
 - 2. Prim's Algorithm



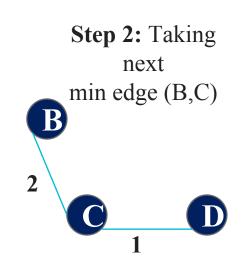
Spanning Tree Examples



Kruskal's Algorithm for MST – Example 1



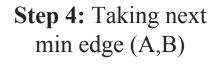
Step 1: Taking min edge (C,D)

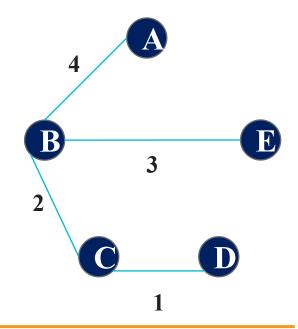


min edge (B,E)

B
3
2
C
1

Step 3: Taking next



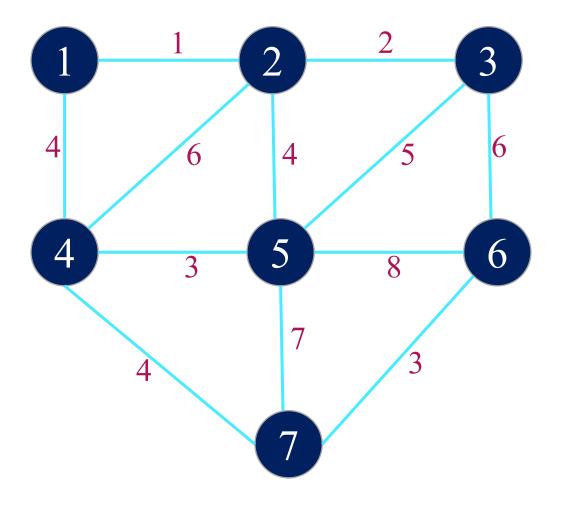


So, we obtained a minimum spanning tree of cost: 4 + 2 + 1 + 3 = 10



Kruskal's Algorithm for MST –

Example 2



Step:

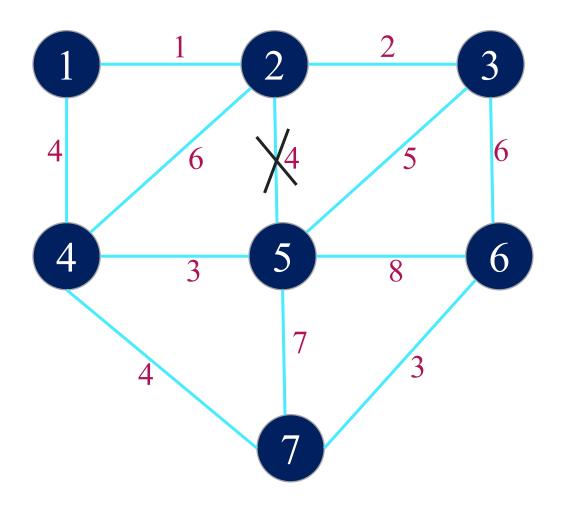
Sort the edges in increasing order of their

<u>weight</u>

Edges	Weight	
{1, 2}	1	
{2, 3}	2	
{4, 5}	3	
<i>{</i> 6 <i>,</i> 7 <i>}</i>	3	
{1, 4}	4	
{2, 5}	4	
<i>{</i> 4 <i>,</i> 7 <i>}</i>	4	
{3, 5}	5	
{2, 4}	6	
{3, 6}	6	
{5, 7}	7	
{5, 6}	8	

Kruskal's Algorithm for MST –

Example 2



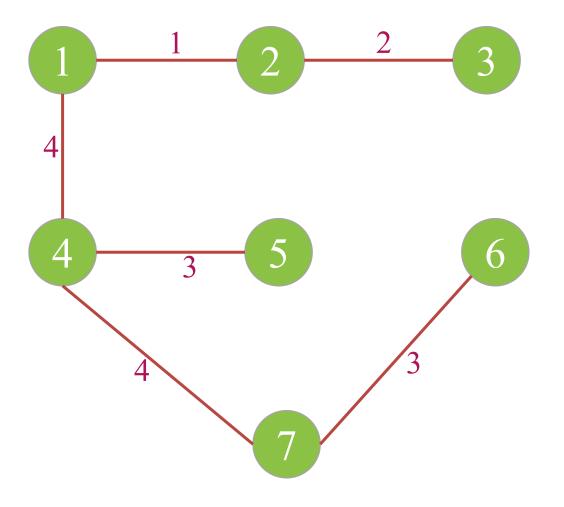
Step:

²Select the minimum weight edge but no cycle.

Edges	Weight	
{1, 2}	1	
{2, 3}	2	
{4, 5}	3	
<i>{</i> 6 <i>,</i> 7 <i>}</i>	3	$\sqrt{}$
{1, 4}	4	
{2,5}	4	
<i>{</i> 4 <i>,</i> 7 <i>}</i>	4	V
{3, 5}	5	
{2, 4}	6	
{3, 6}	6	
{5, 7}	7	
{5, 6}	8	

Kruskal's Algorithm for MST –

Example 2



Step:

The minimum spanning tree for the given

oranh N. 1.					
Edges	Weight				
{1, 2}	1				
{2, 3}	2				
{4, 5}	3	$\sqrt{}$			
{6, 7}	3				
{1, 4}	4	V			
<i>{</i> 4 <i>,</i> 7 <i>}</i>	4	V			

Kruskal's Algorithm – Example 2

Step	Edges considered -	Connected Components
	{ u , v }	

Edges	Weight
>	

Total Cost = 17



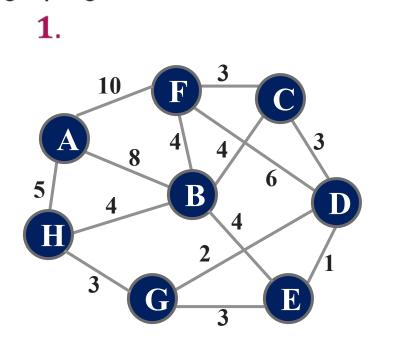
Kruskal's Algorithm for MST

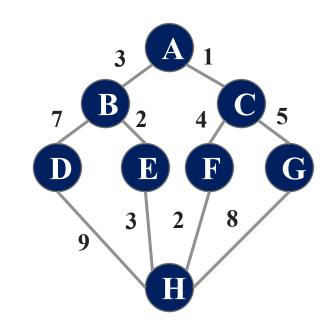
```
Function Kruskal(G = (N, A))
Sort A by increasing length
n ← the number of nodes in N
T \leftarrow \emptyset {edges of the minimum spanning tree}
Define n sets, containing a different element of set N
repeat
   e \leftarrow \{u, v\} //e is the shortest edge not yet considered
   ucomp \leftarrow find(u)
   vcomp \leftarrow find(v) find(u) tells in which connected component a node u is found
   if ucomp ≠ vcomp then merge(ucomp, vcomp)
   T \leftarrow T \cup \{e\}
                                  merge(ucomp, vcomp) is used to merge two connected components.
until T contains n - 1 edges
return T
```

Exercises – Home Work

The complexity for the Kruskal's algorithm is in $\theta(a \log n)$ where a is total number of edges and n is the total number of nodes in the graph G.

Write the kruskal's Algorithm to find out Minimum Spanning Tree. Apply the same and find MST for the graph given below.





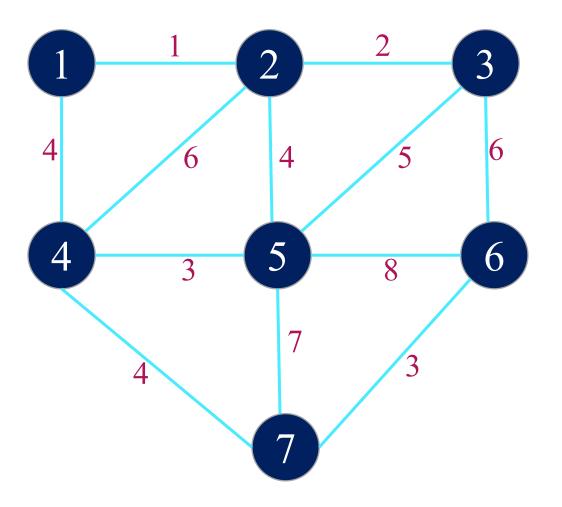


Prim's Algorithm

- ▶ In Prim's algorithm, the minimum spanning tree grows in a natural way, starting from an arbitrary root.
- ▶ At each stage we add a new branch to the tree already constructed; the algorithm stops when all the nodes have been reached.
- ▶ The complexity for the Prim's algorithm is $\theta(n^2)$ where n is the total number of nodes in the graph G.



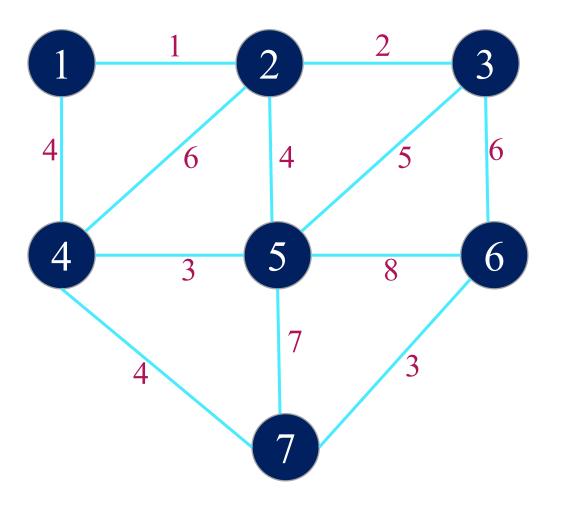
Prim's Algorithm for MST – Example 1





Node - Set B	Edges
1	

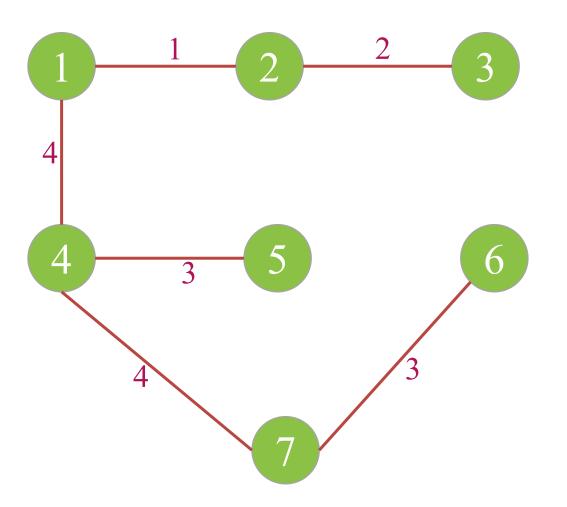
Prim's Algorithm for MST – Example 1



Step: Find an edge with minimum weight.

Node - Set B	Edges
1	
1, 2	
1, 2, 3	{1,4}, {2,4}, {2,5}, {3,5}, {3,6}
1, 2, 3, 4	{2,4} {2,5} {3,5} {3,6} {4,5} {4,7}
1, 2, 3, 4, 5	{2,4} {2,5} {3,5} {3,6} {4,7} {5,6} {5,7}
1, 2, 3, 4, 5, 7	{2,4} {2,5} {3,5} {3,6} {5,0} {5,7} {6,7}
1, 2, 3, 4, 5, 6, 7	

Prim's Algorithm for MST – Example 1

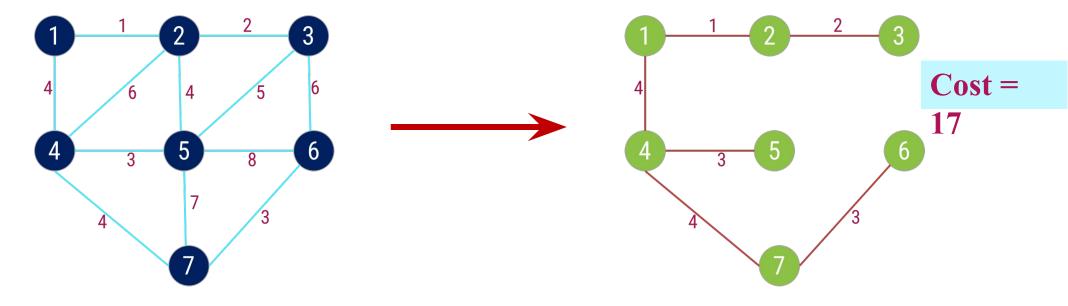




The minimum spanning tree for the given

aranh					
Node	Edges				
1					
1, 2	{1, 2}				
1, 2, 3	{2, 3}				
1, 2, 3, 4	{1, 4}				
1, 2, 3, 4, 5	{4, 5}				
1, 2, 3, 4, 5, 7	{4, 7}				
1, 2, 3, 4, 5, 6, 7	{6, 7}				

Total Cost = 17



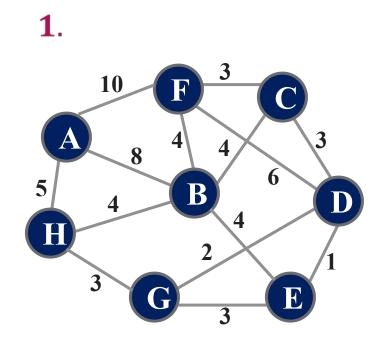
Step	Edge	Set B	Edges Considered
	Selected		
	{ u , v }		
Init.	-	{1}	
1	{1, 2}	{1,2}	{1,2 } {1,4}
2	{2, 3}	{1,2,3}	{1,4} {2,3 } {2,4} {2,5}
3	{1, 4}	{1,2,3,4}	{1,4 } {2,4} {2,5} {3,5} {3,6}
4	{4, 5}	{1,2,3,4,5}	{2,4} {2,5} {3,5} {3,6} {4,5 } {4,7}
5	{4, 7}	{1,2,3,4,5,7}	{2,4} {2,5} {3,5} {3,6} {4,7 } {5,6}
			{5,7}
6	{6,7}	{1,2,3,4,5,6,7}	{2,4} {2,5} {3,5} {3,6} {5,6} {5,7}

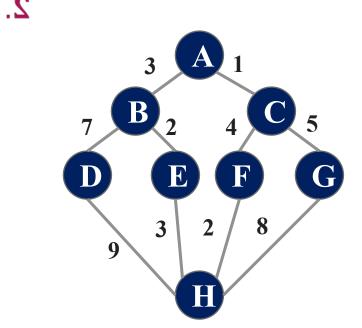
Prim's Algorithm

```
Function Prim(G = (N, A): graph; length: A - R+): set of edges
T \leftarrow \emptyset
B ← {an arbitrary member of N}
while B \neq N do
       find e = \{u, v\} of minimum length such that
              u \in B and v \in N \setminus B
       T \leftarrow T \cup \{e\}
       B \leftarrow B \cup \{v\}
return T
```

Exercises – Home Work

☐ Write the Prim's Algorithm to find out Minimum Spanning Tree. Apply the same and find MST for the graph given below.











Single Source Shortest Path — Dijkstra's Algorithm

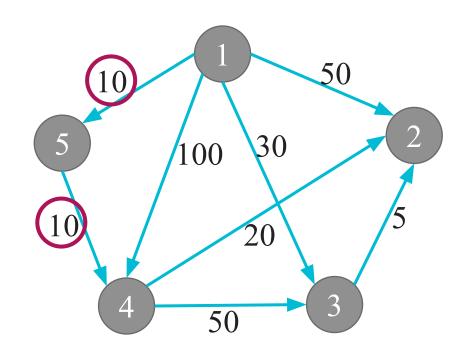


Introduction

- Consider now a directed graph G = (N, A) where N is the set of nodes and A is the set of directed edges of graph G.
- Each edge has a positive length.
- ▶ One of the nodes is designated as the source node.
- ▶ The problem is to determine the length of the shortest path from the source to each of the other nodes of the graph.
- Dijkstra's Algorithm is for finding the shortest paths between the nodes in a graph.
- ▶ For a given source node, the algorithm finds the shortest path between the source node and every other node.
- ▶ The **algorithm maintains a matrix** *L* which gives the length of each directed edge:



Dijkstra's Algorithm - Example



Single source shortest path algorithm

\sim	1	4
Source	node =	
Source	Houc —	1

Step	V	C	2	3	4	5
Init.	-	{2, 3, 4, 5}	50	30	<u>100</u>	10
1	5	{2, 3, 4}	50	30	20	10

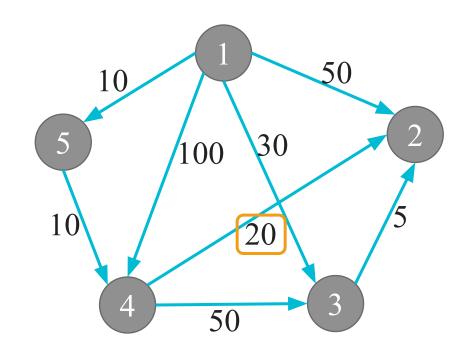
Is there path from 1 - 5 - 4

Yes

Compare cost of 1–5–4 (20) and 1–4 (100)



Dijkstra's Algorithm - Example



Single source shortest path algorithm

		Sou	rce 1	node =	= 1	
4				4	_	

Step	V	C	2	3	4	5
Init.	-	{2, 3, 4, 5}	50	30	100	10
1	5	{2, 3, 4}	50	30	20	10
2	4	{2, 3}	40	30	20	10

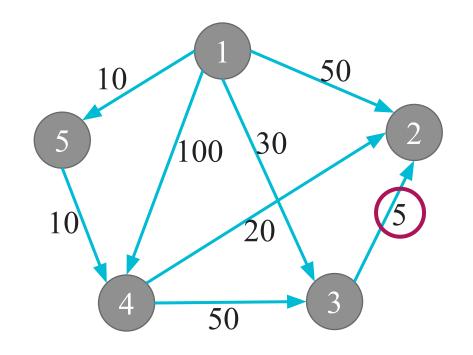
Is there path from 1 - 4 - 3

No

Compare cost of 1–4–3 (70) and 1–3 (30)



Dijkstra's Algorithm - Example



Single source shortest path algorithm

	1
Source node =	:

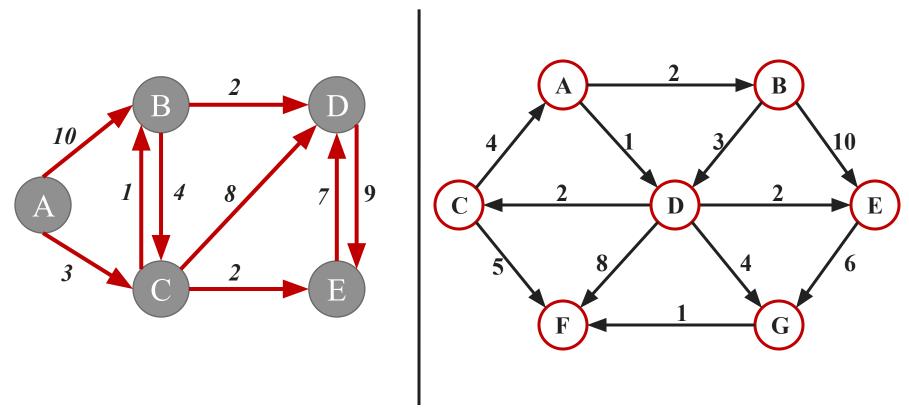
Step	V	C	2	3	4	5
Init.	-	{2, 3, 4, 5}	50	30	100	10
1	5	{2, 3, 4}	50	30	20	10
2	4	{2, 3}	40	30	20	10
3	3	{2}	35	30	20	10

Compare cost of 1-3-2 and 1-2



Exercises – Home Work

□ Write Dijkstra's Algorithm for shortest path. Use the algorithm to find the shortest path from the following graph.





Dijkstra's Algorithm

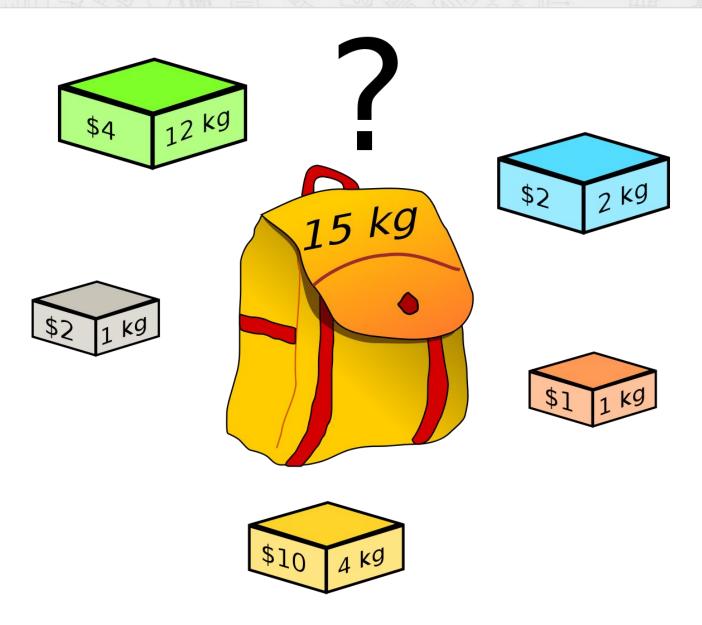




Knapsack Problem



Knapsack Problem









Fractional Knapsack Problem

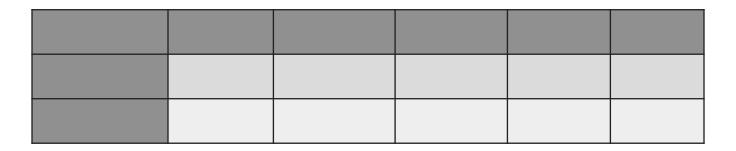


Introduction



Fractional Knapsack Problem - Example

- We are given 5 objects and the weight carrying capacity of knapsack is W = 100.
- For each object, weight w_i and value v_i are given in the following table.



▶ Fill the knapsack with given objects such that the total value of knapsack is **maximized**.

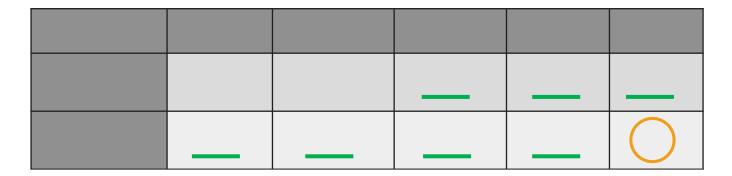


Fractional Knapsack Problem - Greedy Solution

- ☐ Three Selection Functions can be defined as,
 - 1. Sort the items in **descending order of their values** and select the items till weight criteria is satisfied.
 - 2. Sort the items in **ascending order of their weight** and select the items till weight criteria is satisfied.
 - 3. To calculate the **ratio value/weight** for each item and sort the item on basis of this ratio. Then take the item with the highest ratio and add it.



Fractional Knapsack Problem - Greedy Solution



Selection		Objects				
	1	2	3	4	5	

Weight Capacity 100

30	50	20	
10	20	30	40
30	10	20	40

Profit = 66 + 20 + 30 + 48 = 164



Fractional Knapsack Problem - Algorithm

W = 100 and Current weight in knapsack= 60Object weight = 50
The fraction of object to be included will be (100 - 60) / 50 = 0.8



Exercises – Home Work







Activity Selection Problem



Introduction

- ▶ The Activity Selection Problem is an optimization problem which deals with the selection of non-overlapping activities that needs to be executed by a single person or a machine in a given time duration.
- ▶ An activity-selection can also be applicable for scheduling a resource among several competing activities.
- We are given a set S of n activities with start time s_i and finish time f_i , of an i^{th} activity. Find the maximum size set of mutually compatible activities.
- Activities i and j are compatible if the half-open internal $[s_i, f_i)$ and $[s_j, f_j)$ do not overlap, that is, i and j are compatible if $s_i \ge f_j$ or $s_j \ge f_i$.



Activity Selection Problem - Example

Sr	Activity	
•		
1	P	(1, 4)
2	Q	(3, 5)
3	R	(0, 6)
4	S	(5, 7)
5	T	(3, 8)
6	U	(5, 9)
7	V	(6, 10)
8	W	(8, 11)
9	X	(8, 12)
10	Y	(2, 13)
11	Z	(12, 14)

Example: 11 activities are given as,

Solution:

Step 1:

Sort the activities of set S as per increasing finish time to directly identify mutually compatible activities by comparing finish time of first activity and start time of next activity

Activity Selection Problem - Example

Sr	Activity	
•		
1	Р	(1, 4)
2	Q	(3, 5)
3	R	(0, 6)
4	S	(5,7)
5	T	(3, 8)
6	U	(5, 9)
7	V	(6, 10)
8	W	(8, 11)
9	X	(8/12)
10	Y	(2, 13)
11	Z	(12, 14)

Example: 11 activities are given as,

Solution:

Step 2:

- 1. $A = \{P\}$
- 2. $A = \{P, S\}$
- 3. $A = \{P, S, W\}$
- 4. $A = \{P, S, W, Z\}$

Answer: $A = \{P, S, W, Z\}$

Activity Selection - Algorithm

```
Algorithm: Activity Selection
Step I: Sort the input activities by increasing finishing time. f₁ ≤
f_2 \leq \ldots \leq f_n
Step II: Call GREEDY-ACTIVITY-SELECTOR (s, f)
  n = length [s]
  A = \{i\}
  j = 1
  for i = 2 to n
        do if s_i \ge f_i
             then A = A U \{i\}
                       j = i
  return set A
```

Exercise - HW

Given arrival and departure times of all trains that reach a railway station, find the minimum number of platforms required for the railway station so that no train waits. We are given two arrays which represent arrival and departure times of trains that stop. Arr[] = {9:00, 9:40, 9:50, 11:00, 15:00, 18:00} dep[] = {9:10, 12:00, 11:20, 11:30, 19:00, 20:00}

Time	Event Type	Total Platforms Needed
		at this Time
9:00	Arrival	1
9:10	Departure	0
9:40	Arrival	1
9:50	Arrival	2
11:00	Arrival	3
11:20	Departure	2
11:30	Departure	1
12:00	Departure	0
15:00	Arrival	1
18:00	Arrival	2
19:00	Departure	1
20:00	Departure	0
Minimum	Platforms needed	on railway station

⁼ Maximum platforms needed at any time



^{= 3}





Job Scheduling with Deadlines



Introduction

- ightharpoonup We have set of n jobs to execute, each of which takes unit time.
- ▶ At any point of time we can **execute only one job**.
- \blacktriangleright Job i earns profit $g_i > 0$ if and only if it is executed **no later than** its deadline d_i .
- ▶ We have to find an optimal sequence of jobs such that our total **profit is maximized**.
- ▶ Feasible jobs: A set of job is feasible if there exits at least one sequence that allows all the jobs in the set to be executed no later than their respective deadlines.



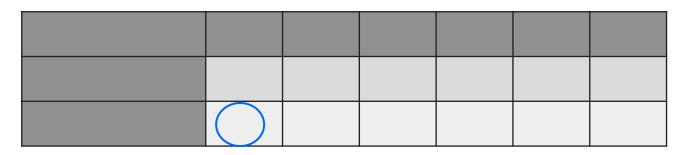
- brack Using greedy algorithm find an optimal schedule for following jobs with n=6.
- ▶ Profits: $(P_1, P_2, P_3, P_4, P_5, P_6) = (15,20,10,7,5,3)$ &
- ▶ Deadline: $(d_1, d_2, d_3, d_4, d_5, d_6) = (1)(3)(1, 3, 1, 3)$

Solution:

Step 1:

Sort the jobs in **decreasing order** of their profit.





Step 2:

Find total position $P = \min(n, \max(di))$

Here, $P = \min(6,3) = 3$

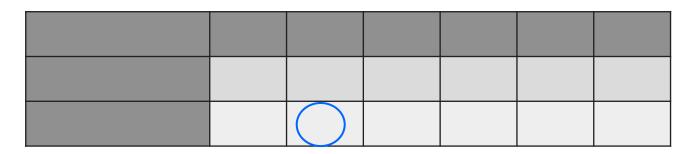
P	1	2	3
Job selected	0	0	0

Step 3:

 $d_1 = 3$: assign job 1 to position 3

P	1	2	3
Job selected	0	0	J1





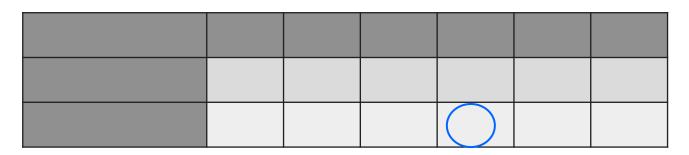
Step 4: $d_2 = 1$: assign job 2 to position 1

P	1	2	3
Job selected	J2	0	J1

Step 5:
$$d_3 = 1$$
: assign job 3 to position 1

But position 1 is already occupied and two jobs can not be executed in parallel, so reject job 3





Step 6:

 $d_4=3$: assign job 4 to position 2 as, position 3 is not free but position 2 is free.

P	1	2	3
Job selected	J2	J4	J1

Now **no more free position** is left so no more jobs can be scheduled. The final optimal sequence:

Execute the job in order 2, 4, 1 with total profit value 42.



Exercises – Home Work

- 1. Using greedy algorithm find an optimal schedule for following jobs with n=4.
 - Profits: (a, b, c, d) = (20,10,40,30) &
 - Deadline: $(d_1, d_2, d_3, d_4) = (4, 1, 1, 1)$
- 2. Using greedy algorithm find an optimal schedule for following jobs with n=5.
 - Profits: (a, b, c, d, e) = (100, 19, 27, 25, 15) &
 - Deadline: $(d_1, d_2, d_3, d_4, d_5) = (2, 1, 2, 1, 3)$



Job Scheduling with Deadlines - Algorithm

Algorithm: Job-Scheduling (P[1..n], D[1..n])

- 1. Sort all the n jobs in decreasing order of their profit.
- 2. Let total position P = min(n, max(d_i))
- 3. Each position 0, 1, 2..., P is in different set and T({i}) = i, for 0 ≤ i ≤
 P.
- 4. Find the set that contains d, let this set be K. if T(K) = 0 reject the job; otherwise:
 - 1. Assign the new job to position T(K).
 - 2. Find the set that contains T(K) 1. Call this set L.
 - 3. Merge K and L. the value for this new set is the old value of T(L).



Huffman Codes



Prefix Code

- ☐ Prefix code is used for encoding(compression) and Decoding(Decompression).
- ☐ Prefix Code: Any code that is not prefix of another code is called prefix code.

Characters	Frequency	Code	Bits
a	45	000	135
b	13	111	39
С	12	101	36
d	16	110	48
e	9	011	27
f	5	001	5
	290		



Huffman code Introduction

- ☐ Huffman invented a greedy algorithm that constructs an optimal prefix code called a Huffman code.
- ☐ Huffman coding is a lossless data compression algorithm.
- ☐ It assigns variable-length codes to input characters.
- ☐ Lengths of the assigned codes are based on the frequencies of corresponding characters.
- ☐ The most frequent character gets the smallest code and the least frequent character gets the largest code.
- ☐ The variable-length codes assigned to input characters are Prefix Codes.



Huffman Codes

- ☐ In Prefix codes, the codes are assigned in such a way that the code assigned to one character is not a prefix of code assigned to any other character.
- ☐ For example,

$$a = 01$$
, $b = 010$ and $c = 11$ Not a prefix code

- ☐ This is how Huffman Coding makes sure that there is **no ambiguity** when decoding the generated bit stream.
- ☐ There are mainly two major parts in Huffman Coding
 - 1. Build a Huffman Tree from input characters.
 - 2. Traverse the Huffman Tree and assign codes to characters.



☐ Find the Huffman codes for the following characters.

Characters	a	b	c	d	e	f
Frequency (in thousand)	45	13	12	16	9	5

Step 1: Arrange the characters in the Ascending order of their frequency.

f:5

e:9

c:12

b:13

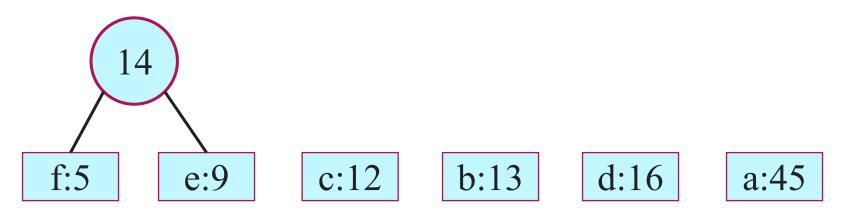
d:16

a:45



Step 2:

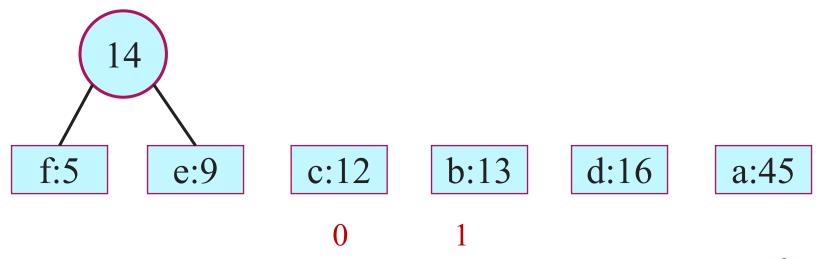
- Extract two nodes with the minimum frequency.
- ✓ Create a new internal node with frequency equal to the sum of the two nodes frequencies.
- ✓ Make the first extracted node as its left child and the other extracted node as its right child.



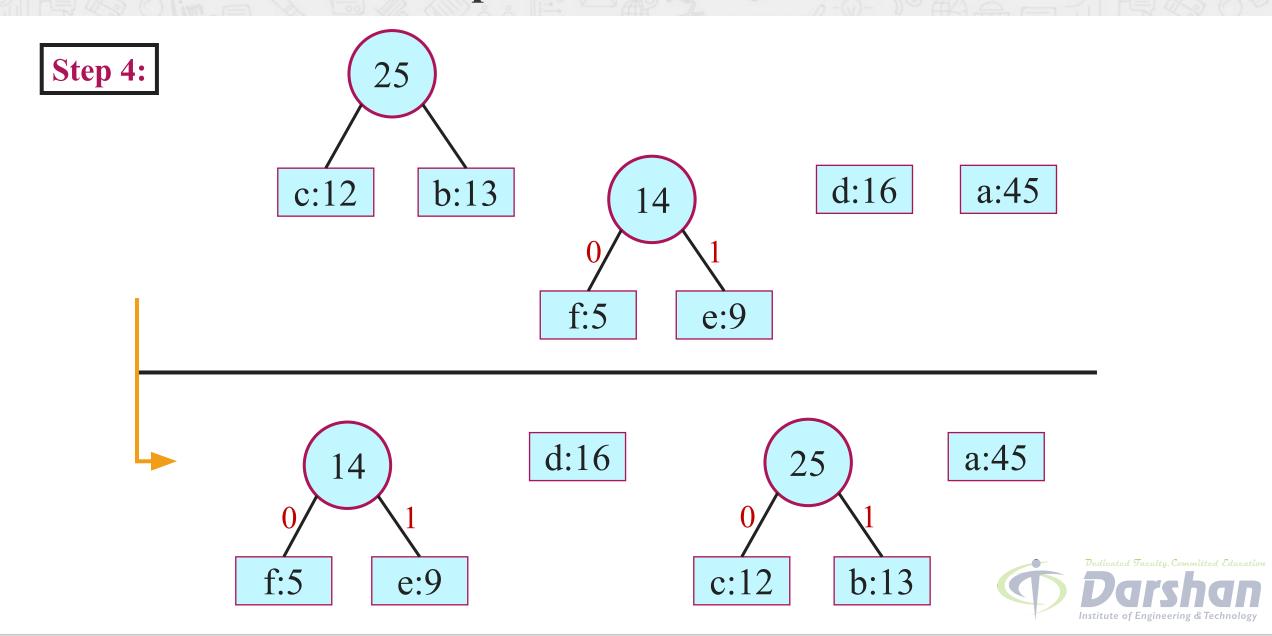


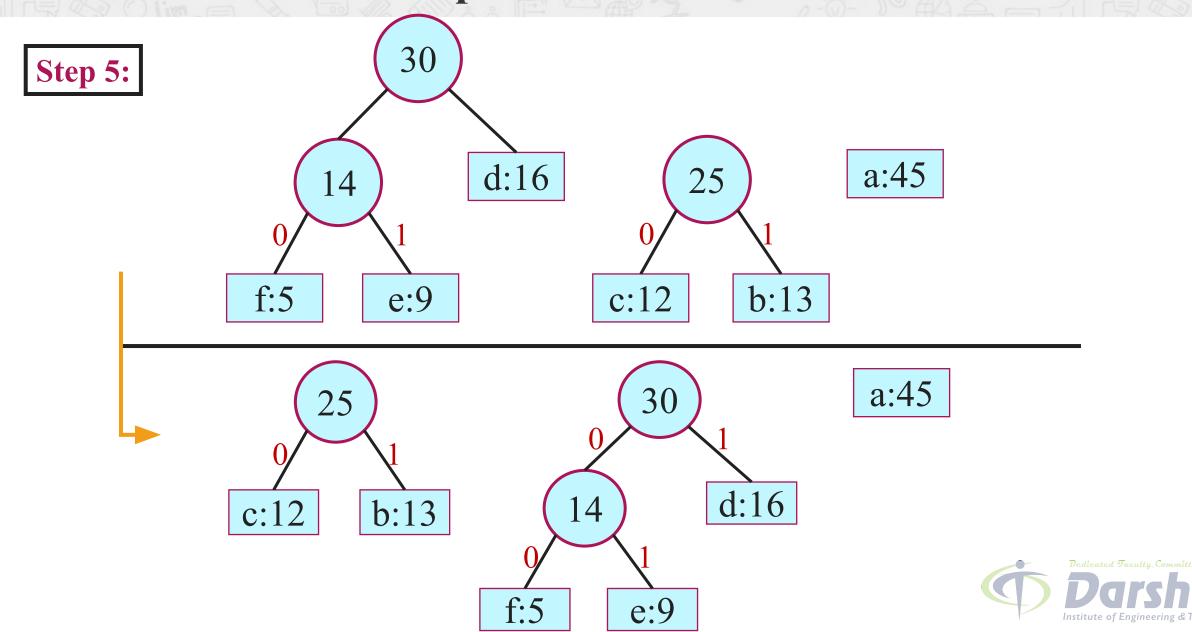
Step 3:

- Rearrange the tree in ascending order.
- ✓ Assign 0 to the left branch and 1 to the right branch.
- Repeat the process to complete the tree.

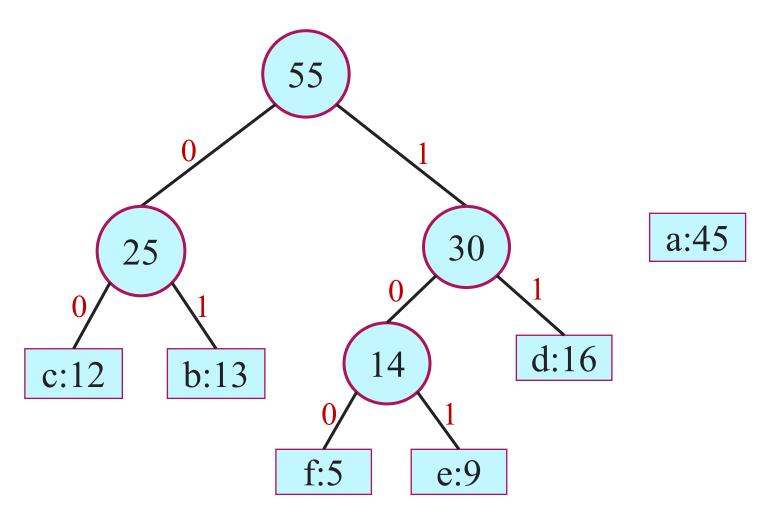






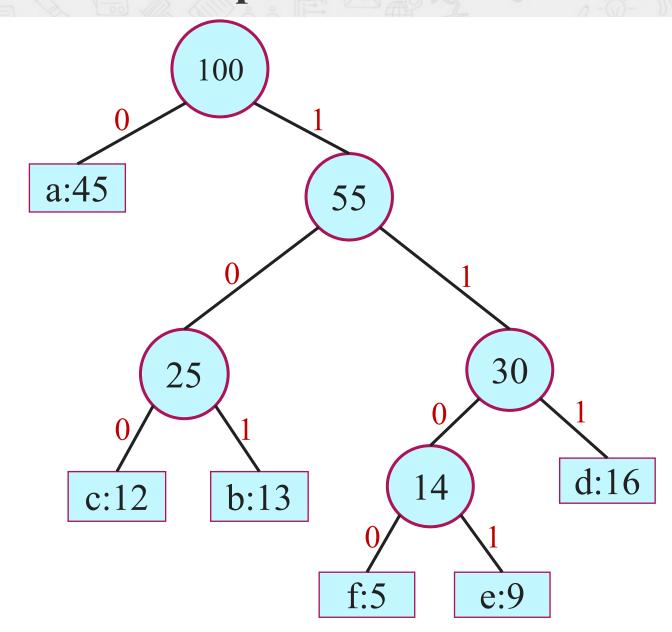


Step 6:





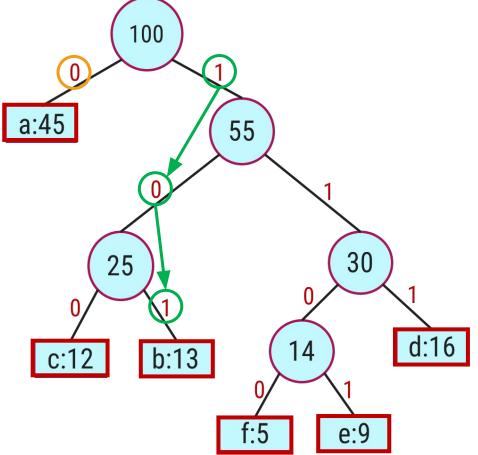
Step 7:





Step 8

Characters	a	b	c	d	e	f
Frequency (in thousand)	45	13	12	16	9	5
	0	101	100	111	1101	1100



Total bits: 224



Huffman Codes - Algorithm

```
Algorithm: HUFFMAN (C)
n = |C|
Q = C
for i = 1 to n-1
        allocate a new node z
        z.left = x = EXTRACT-MIN(Q)
        z.right = y = EXTRACT-MIN(Q)
        z.freq = x.freq + y.freq
        INSERT(Q,z)
return EXTRACT-MIN(Q) // return the root of the tree
```

Exercises – Home Work

- ☐ Find an optimal Huffman code for the following set of frequency.
- 1. a:50, b:20, c:15, d:30.
- 2. Frequency

Characters	A	В	C	D	E	F
Frequency (in thousand)	24	12	10	8	8	5

3. Frequency

Characters	a	b	c	d	e	f	g
Frequency (in thousand)	37	28	29	13	30	17	6





Thank You!

