Q1

W21

(a) Define algorithm. Discuss key characteristics of algorithms. **03**

- A step-by-step procedure, to solve the different kinds of problems.
- An algorithm is any well-defined computational procedure that takes some value, or a set of values as input and produces some value, or a set of values as output.
- Characteristics: finiteness, input, output, optimization

(b) Explain why analysis of algorithms is important? Explain: Worst Case, Best Case and Average Case Complexity with suitable example. **04**

- By analyzing some of the candidate algorithms for a problem, the most efficient one can be easily identified.
- Eg. linear search:
  - Worst Case (O(n)) ,
  - Best Case(O(1)) ,
  - Average Case (O(n))

(c) Write and analyze an insertion sort algorithm to arrange n items into ascending order. **07**

- Greedy approach

```
insertion_sort(arr,num)

for i ← 0 ,i < length(arr) than i++
    minindex ← i
    for j ← i ,j< length(arr) than j++
        if arr[minindex] < arr[j] than
            minindex ← j
   (now, swap elements indexed at minindex and ... than repeat the swapping for i=i+1 till it
   gets to minindex) or
   (
   for k ← i, k<minindex than k++
            swap(k,minindex,arr)
   )
```

Best Case: O(N^2) , Average Case: O(N^2) , Worst Case: O(N^2)

W22

(a) Sort the best case running times of all these algorithms in a non-decreasing order. LCS, Quick-Sort, Merge-Sort, Counting-Sort, Heap-Sort, Selection-Sort, Insertion-Sort, Bucket-Sort, Strassen's Algorithm. 03

- **Counting Sort**: Θ(n+k) , **Bucket Sort**: Θ(n) , **Insertion Sort**: Θ(n) , **Selection Sort**: Θ(n2) , **Quick Sort**: Θ(nlogn) , **Merge Sort**: Θ(nlogn) , **Heap Sort**: Θ(nlogn) , **Strassen's Algorithm**: Θ(nlog27)≈Θ(n2.81) , **Longest Common Subsequence (LCS)**: Θ(mn)

(b) State whether the statements are correct or incorrect with reasons.

1. O(f(n)) + O(f(n)) = O (2f(n))

2. If 3n + 5 = O(n2) , then 3n + 5 = o(n2) 04

- 1. False: To make notation we remove any constant
- 2. False: Since f(n) = 3n+5 is close to n2 hence condition o(n2) fails.

(c) Explain asymptotic analysis with all the notations and its mathematical inequalities. 07

- Asymptotic notation is a way to describe how the running time (or complexity) of an algorithm changes as the size of the input grows
  - O-Notation (Big O notation) (Upper Bound) (f(n) <= g(n))
  - o-Notation (Small o notation) (Strict-Upper Bound) (f(n) < g(n))
  - Ω-Notation (Omega notation) (Lower Bound) (f(n) >= g(n))
  - Ω-Notation (Small Omega notation) (Strict-Lower Bound) (f(n) > g(n))
  - θ-Notation (Theta notation) (Average order)

W23

(a) What is an algorithm? Explain various properties of an algorithm.RE 03

(b) Solve the following using Master's theorem:

a. T(n) = 2T(n/4) + 1

b. T(n)=3T(n/4) + nlgn 04

- **a. T(n) = 2T(n/4) + 1**

  Step 1 : a = 2 , b = 4 , f(n)=1

  Step 2 : now, $n^{\log_b a} = n^{\log_4 2} = n^{1/2}$

  Step 3 : f(n) = $n^{\log_b a - E}$

  Step 4 : T(n) = $\theta(n^{\log_b a}) = \theta(n^{1/2})$

- **b. T(n)=3T(n/4) + nlgn**

  Step 1 : a = 3 , b = 4 , f(n)=n(logn)

  Step 2 : now, $n^{\log_b a} = n^{\log_4 3} = n^{0.80}$

  Step 3 : f(n) = $n^{\log_b a+E}$

  Step 4 : T(n) = θ(f(n)) = θ(n(logn))

(c) Write selection sort algorithm and compute running time of algorithm. 07

- Greedy approach

```
Selection_sort(arr,num)

for i ← 0 ,i < length(arr) than i++
    minindex ← i
    for j ← i ,j< length(arr) than j++
        if arr[minindex] < arr[j] than
            minindex ← j
    (now, swap elements indexed at minindex and ... ) or
    (
            swap(i,minindex,arr)
    )
```

Best Case: O(N^2) , Average Case: O(N^2) , Worst Case: O(N^2)

S22

(a) Define Algorithm, Time Complexity and Space Complexity RE 03

(b) Explain: Worst Case, Best Case and Average Case Complexity with suitable example. RE 04

(c) Sort the following list using quick sort algorithm:< 5, 3 ,8 ,1 ,4 ,6 ,2 ,7 > Also write Worst and Best case and Average case of quick sort algorithm. 07

- Initial p = 7, i = 5 , j = 2.

5	3	8	1	4	6	2	7
i						j	p

- For i = 8 the condition fails for element at i<element at p hence for an element that doesn’t meet the condition element at j>element at p will be replaced.

5	3	2	1	4	6	8	7
		i				j	p

- Now i will keep getting incrimented till the element at i < element at p hence for j it will keep getting decrimented till element at j > element at p. but since i>j p will be swapped with i and its elements too.

5	3	2	1	4	6	8	7
					j	i	p

- Now element at the pivot p will be swapped with i+1 th element
- doing this we will get all element of left side which are smaller than p and on the other hand all elements which are on right side will be greater than p

5	3	2	1	4	6	7	8
					j	ip	

- Since for the right side of the pivot p there is only an element therefor it doent meet the condition i < j hence its complete

5	3	2	1	4	6	7
i					j	p

- Since i+1 = p therefor p = i.

5	3	2	1	4	6	7
					ijp	

- Since for the right side of the pivot p there is only an element therefor it doent meet the condition i < j hence its complete

5	3	2	1	4	6
i				j	p

- Since i+1 = p therefor p = i.

5	3	2	1	4	6
				ijp	

- Since for the right side of the pivot p there is only an element therefor it doent meet the condition i < j hence its complete

5	3	2	1	4
i			j	p

- For i = 5 the condition fails for element at i<element at p hence for an element that doesn’t meet the condition element at j>element at p will be replaced

1	3	2	5	4
i		j		p

- Now i will keep getting incrimented till the element at i < element at p hence for j it will keep getting decrimented till element at j > element at p. but since i>j p will be swapped with i and its elements too.

1	3	2	4	5
		j	ip	

- Since for the right side of the pivot p there is only an element therefor it doent meet the condition i < j hence its complete

1	3	2	4
i		j	p

- Since i+1 = p therefor p = i.

1	3	2	4

ijp

- Since for the right side of the pivot p there is only an element therefor it doent meet the condition i < j hence its complete

1	3	2

i j p

- The first condition which is about
- Now since i>j hence element at p index and I index will get swapped and p will become i

1	3	2

j ip

Now for both sides of p, left and right i<j doesn't meet the condition hence its sorted.

S23

(a) Define following terms:
(i) Big O Notation
(ii) Big Theta Notation
(iii) Big Omega Notation. RE **03**

(b) Perform Bucket sort for following sequence: 30, 12, 22, 66, 48, 27, 35, 43, 47, 41. **04**

- make array buckets of size 10.
- than store elemenets based on 0<x<10,10<x<20,20<x<30,30<x<40,40<x<5.
- after than sort elements in buckets than merge them orderly.

(c) Explain the bubble sort algorithm and derive its best case, worst case, and average case time complexity. **07**

- Greedy approach

```
Bubble_sort(arr,num)

for i ← 0 ,i < length(arr) than i++
    for j ← 0 ,j< length(arr)-1 than j++
        if arr[j+1] < arr[j] than
            swap(j,j+1,arr)
```

- Best Case: $O(N^2)$ (Already sorted)
- Average Case: $O(N^2)$
- Worst Case: $O(N^2)$ (Sorted in reverse order)

Q2

W21

(a) Write an algorithm of Selection Sort Method. RE 03
(b) Sort the following numbers using heap sort.20, 10, 50, 40, 30 RE 04
(c) Sort the following list using quick sort algorithm: <50, 40, 20, 60, 80, 100,45, 70, 105, 30, 90, 75> Also discuss worst and best case of quick sort algorithm. RE 07
OR (c) Apply merge sort algorithm on array A = {2,7,3,5,1,9,4,8}. What is time complexity of merge sort in worst case? 07

2	**7**	**3**	**5**	**1**	**9**	**4**	**8**
i			**mid**	**mid+1**			**j**

- Than it will get devided

2	**7**	**3**	**5**	**1**	**9**	**4**	**8**
i	**mid**	**mid+1**	**j**	**i**	**mid**	**mid+1**	**j**

- Than they will get devide too

2	**7**	**3**	**5**	**1**	**9**	**4**	**8**
i	**j**	**i**	**j**	**i**	**j**	**i**	**j**

- Since it has been devide now we can merge by sorting them

2	**7**	**3**	**5**	**1**	**9**	**4**	**8**
i	**mid**	**mid+1**	**j**	**i**	**mid**	**mid+1**	**j**

- Now we will merge two array by sorting them while merging them eg.

  itteration1: 2,7 3,5 2
  itteration2: 7 5 2,3
  itteration3: 7 2,3,5
  itteration4: 2,3,5,7

2	**3**	**5**	**7**	**1**	**4**	**8**	**9**
i			**mid**	**mid+1**			**j**

Again we will merge two array by sorting them while merging them.

1	**2**	**3**	**4**	**5**	**7**	**8**	**9**
i							**j**

- Time complexity=nlogn

W22

(a) What is the use of Loop Invariant? What should be shown to prove that an algorithm is correct? 03

- A loop invariant is a property of a program that remains unchanged throughout the execution of a loop. In other words, it is a condition that holds true before and after each iteration of the loop.
- To prove that algorithm is correct one can use Flags , one can trace the algo for few itterations , one can check whether outcome us equvalent to prediction or not etc...

(b) Apply LCS on sequence <A,B,A,C,B,C> for pattern <A,B,C> 04
(c) Write and explain the recurrence relation of Merge Sort. 07
OR (c) Perform the analysis of a recurrence relation T(n)= $2T(n2) + (\theta n2)$ **by drawing its recurrence tree. 07**

W23

(a) Explain general characteristics of greedy algorithms. 03
(b) What is asymptotic notation? Find out big-oh notation of the f(n) = 3n2+5n+10 04
(c) Illustrate the working of the quick sort on input instance: 25, 29, 30, 35, 42, 47, 50, 52, 60. Comment on the nature of input i.e. best case, average case or worst case. Also discuss worst and best case of quick sort algorithm. RE 07
OR (c) Give the properties of Heap Tree. Sort the following data using Heap Sort Method: 20, 50, 30, 75, 90, 60, 80, 25, 10, 40. 07

S22

(a) Write an algorithm of Selection Sort Method. RE 03
(b) Demonstrate Binary Search method to search Key = 14, form the array A=<2,4,7,8,10,13,14,60> 04
(c) Write the Master theorem. Solve following recurrence using it.
(i)T(n)= T(n/2) + 1
(ii) T(n)=2T(n/2) + n log n 07
OR (c) Solve following recurrence relation using iterative method T(n) = T(n - 1) + 1 with T(0) = 0 as initial condition. Also find big oh notation 07

S23

(a) Define Algorithms and characteristics of algorithms. RE 03
(b) What is a recurrence? Solve recurrence equation for T (n) =T (n-1) + 1 using substitution method. RE 04
(c) Discuss Binary search algorithm, also write and solve its recurrence relation. RE 07
OR (c) Explain Merge Sort algorithm with suitable example. RE 07

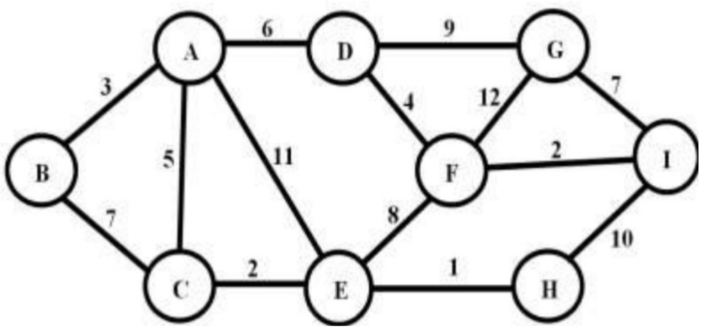
Q3

W21

(a) What is Principle of Optimality? Explain its use in Dynamic Programming Method 03
(b) Explain Binomial Coefficient algorithm using dynamic programming. 04
(c) Solve the following 0/1 Knapsack Problem using Dynamic Programming. There are five items whose weights and values are given in following arrays. Weight w [] = {1,2,5,6,7} Value v [] = {1, 6, 18, 22, 28} Show your equation and find out the optimal knapsack items for weight capacity of 11 units. 07
OR (a) Compare Dynamic Programming Technique with Greedy Algorithms 03
OR (b) Give the characteristics of Greedy Algorithms. 04
OR (c) Obtain longest common subsequence using dynamic programming. Given A = "acabaca" and B = "bacac". 07

W22

(a) Consider the array 2,4,6,7,8,9,10,12,14,15,17,19,20. Show (without actually sorting), how the quick sort performance will be affected with such input. 03
(b) "A greedy strategy will work for fractional Knapsack problem but not for 0/1", is this true or false? Explain. 04
(c) Apply Kruskal's algorithm on the given graph and step by step generate the MST. 07



OR (a) Consider an array of size 2048 elements sorted in non-decreasing order. Show how the Binary Search will perform on this size by analysis of its recurrence relation. Derive the running time. 03
OR (b) Explain the steps of greedy strategy for solving a problem. 04
OR (c) Apply Prim's algorithm on the given graph in Q.3 (C) FIG:1 Graph G(V,E) and step by step generate the MST. 07

W23

(a) Sort the List "G,U,J,A,R,A,T,S,A,R,K,A,R" in alphabetical order using merge sort. 03
(b) Following are the details of various jobs to be scheduled on multiple processors such that no two processes execute at the same on the same processor. Show schedule of these jobs on minimum number of processors using greedy approach.

Jobs	J1	J2	J3	J4	J5	J6	J7	
Start time	0	3	4	9	7	1	6	
Finish time	2	7	7	11	10	5	8	04

(c) Using algorithm find an optimal parenthesization of a matrix chain product whose sequence of dimension is (5,10,3,12,5,50,6) (use dynamic programming). 07
OR (a) Apply counting sort for the following numbers to sort in ascending order.3, 1, 2, 3, 3, 1 03
(b) Find the Optimal Huffman code for each symbol in following text ABCCDEBABFFBACBEBDFAAAABCDEEDCCBFEBFCAE 04
(c) Solve following knapsack problem using dynamic programming algorithm with given capacity W=5, Weight and Value are as follows (2,12),(1,10),(3,20),(2,15) 07

S22

(a) What is Principle of Optimality? Explain its use in Dynamic Programming Method 03
(b) Find out LCS of A={K,A,N,D,L,A,P} and B = {A,N,D,L} 04
(c) Discuss Assembly Line Scheduling problem using dynamic programming with example.07
OR (a) Give the characteristics of Greedy Algorithms 03
OR (b) Give difference between greedy approach and dynamic programming. 04
(c) Consider Knapsack capacity W=15, w = (4, 5, 6, 3) and v=(10, 15, 12, 8) find the maximum profit using greedy method. 07

S23

(a) Explain principle of optimality with suitable example. 03
(b) Explain advantages and disadvantages of dynamic programming. 04
(c) Given the denominations: d1=1, d2=4, d3=6. Calculate for making change of Rs. 8 using dynamic programming. 07
OR (a) Explain Weighted Graph, Undirected Graph, Directed Graph. 03
OR (b) Discuss advantages and disadvantages of greedy algorithm. 04

OR (c) Consider weights w=(3,4,6,5) and profit v=(2,3,1,4) and Knapsack capacity W=8. Find the maximum profit using dynamic approach. **07**