DEPARTMENT OF MATHEMATICS

UIET, CSJMU KANPUR

End Sem. (Even Sem.) Examination 2022

B.Tech. I Sem. (ECE)

Subject- Mathematics I **Subject Code – MTH-S-101**

TIME- 3 Hours **Maximum Marks -50**

SECTION-A

All questions are compulsory. Each carries 01 mark.

Question 1.

(i) Find the degree of $u = \tan^{-1}\left(\frac{x^3+y^3}{\sqrt{x}+\sqrt{y}}\right)$.

(ii) Find the value of $\Gamma\left(-\frac{5}{2}\right)$.

(iii) Find the unit normal vector of to the surface $xy^3z^2 = 4$ at (-1, -1, 2).

(iv) If $\vec{V} = xy^2i + 2yx^2zj - 3yz^2k$ then find $curl\,V$ at the point (1, -1, 1).

(v) Write the mean and variance of Poisson distribution.

(vi) If $u = x(1-y), v = xy$, find the value of the Jacobian $\frac{\partial(u,v)}{\partial(x,y)}$.

(vii) Find the POINT for maximum value of the function $x^3y^2(1-x-y)$.

(viii) Find the value of integral $\int_1^0 \int_0^1 (x+y)\,dxdy$.

(ix) Evaluate the integral $\int_0^a \int_0^{\sqrt{a^2-y^2}} (x^2+y^2)dxdy$ by changing into polar coordinates.

(x) Find $\int_0^a \int_0^{\frac{bx}{a}} \int_0^{c+xy} dzdydx$.

Section B

All questions are compulsory. Each carries 04 mark.

Question 2. Apply Green's theorem to evaluate $\int[(2x^2-y^2)dx + (x^2+y^2)dy]$, where C is the boundary of the area enclosed by the x-axis and upper half of circle $x^2+y^2=a^2$.

Question 3. State and Prove Eulers homogenous theorem.

Question 4. Find the extreme values of $u = x^2y^2 - 5x^2 - 5y^2 - 8xy$.

Question 5. Suppose $F(x,y,z) = x^3i + yj + zk$ is the force field. Find the work done F along the line from (1, 2, 3) to (3, 5, 7).

Question 6. Test the series $x + \frac{2}{5}x^2 + \frac{8}{10}x^3 + \cdots + \frac{n^2-1}{n^2+1}x^n + \cdots$

(0, -1)

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(0, -1)

Section C (2*10)

All questions are compulsory.

Question 7. **(a)** Show that $\int\int\int \frac{dxdydz}{(x+y+z+1)^3} = \frac{1}{2}log2 - \frac{5}{16}$,the integral being taken throughout the volume bounded by planes $x = 0, y = 0, z = 0, x + y + z + 1 = 1$.

(b) Trace the curve $y^2(2a - x) = x^3$. .

Question 8. (a) **If** $F = 2xzi - xj + y^2k$, evaluate $\int\int\int F\, dV$ over the region bounded by the surface $x = 0, y = 0, y = 6$ and $z = x^2, z = 4$.

$x = 2$

(b) **Two** players A and B play tennis games. Their chances of wining a game are in the ratio 3:2 respectively . Find A's chance of wining at least two games out of four games played

DEPARTMENT OF MATHEMATICS
C.S.J.M. UNIVERSITY, KANPUR.

Mathematics-I (MTH-S101) (ECE)
Electronic Communication and Engineering

Semester: 2022-23 (Odd Sem) **Year: Ist Year**

SECOND MID SEMESTER EXAMINATION

Time: 1.5 h **Maximum marks: 30**

All questions are compulsory

Section A

1. Attempt all the questions and each question contains 1 Marks.
   a) Evaluate $\int_1^{\ln 8}\int_0^{lny} e^{x+y}\,dx\,dy$.
   b) Find the value of $\Gamma\left(-\frac{7}{2}\right)$.
   c) Write the definition of sequence with suitable example.
   d) Find the degree of the homogenous function $logy - logx$.
   e) Find the first order partial derivative of $u = x^{xy}$.
   f) Test the convergence of the series $\sum_{n=1}^{\infty} cos\frac{1}{n}$.
   g) If $f(x,y) = \begin{cases} \frac{(x^2-y^2)}{(x^2+y^2)}, & when\, x \neq 0, y \neq 0 \\ 0 & otherwise \end{cases}$
   Show that f is discontinuous at origin.
   h) Evaluate $\lim_{(x,y)\to(0,0)} \frac{x^2y}{x^4+y^2}$.
   i) Determine, following functions are functionally dependent or independent
   $$u = e^x siny, v = e^x cosy.$$

Section B (3*3=9)

2. $I_D = \iint_D x^3y\,dxdy$ where D is the region enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ in the first quadrant.
3. Evaluate $\int_0^{\frac{\pi}{2}} \left(\frac{\sqrt[3]{sin^8x}}{\sqrt{cosx}}\right) dx$.
4. Evaluate $I = \int_0^1 x^{\frac{3}{2}}(1-x^2)^{\frac{5}{2}}dx$.

Section C (2*6=12)

5. (a) Find the radius of convergence of $\sum_{n=0}^{\infty}(-1)^n \frac{z^{2n}}{(2n)!}$
   (b) Prove that $xu_x + yu_y = \frac{5}{2} tanu$ if $u = \sin^{-1}\left(\frac{x^3+y^3}{\sqrt{x}+\sqrt{y}}\right)$.
6. (a) Trace the curve $y^2(a+x) = x^2(b-x)$.
   (b) Test the function $f(x) = x^4 + y^4 - x^2 - y^2 +$ for maxima, minima and saddle points.

University Institute of Engineering and Technology, CSJM University, Kanpur

Mathematics I MTH-S101 (ECE)

Semester: 22-23 (Odd Semester) | Year: I year (2K22)

Maximum marks: 30 | Time : 90 min.

First MID SEMSTER EXAMINATION

SECTION- A

Attempt all question

Question 1.

a. If $u = e^x(x\cos y - y\sin y)$, then $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \cdots$

b. If $x = uv, y = \frac{u}{v}$, then $\frac{\partial(x,y)}{\partial(u,v)} = \cdots$

c. $\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}$ is a homogeneous function of degree....

d. If $u = x^4 + y^4 + 3x^2y^2$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \cdots$

e. If u is the composite number of t, defined by the relation $u = f(x,y); x = \phi(t), y = \psi(t)$, then total derivative $\frac{du}{dt} = \cdots$.

f. If $A = f_{xx}(a,b), B = f_{xy}(a,b)$, $C = f_{yy}(a,b)$, then $f(x,y)$ will have a maximum at (a,b), if?

g. Show that the sequence $< s_n >$ where $s_n = \frac{3n}{n+5n^{\frac{1}{2}}}$ has limit 3.

h. Test the series $1 - \frac{1}{2}x + \frac{1}{3}x^2 - \frac{1}{4}x^3 + \cdots$

i. Test for convergence $\sum \frac{1}{(\log)^n}$.

Section B 3*3=9

2. Test the series $x + \frac{3}{5}x^2 + \frac{8}{10}x^3 + \cdots + \frac{n^2-1}{n^2+1}x^n + \cdots$.
3. Find the extreme value of $f(x,y,z) = 2x + 3y + z$ such that $x^4 + y^4 = 5$ and $x + z = 1$.
4. Expand $f(x,y) = 21 + x - 20y + 4x^2 + xy + 6y^2$ in Taylor series of maximum order about the point (-1, 2).

Section C 4*3=12

5. (a). Test the series $\frac{1}{a.1^2+b} + \frac{2}{a.2^2+b} + \frac{3}{a.3^2+b} + \cdots$

   (b) State and proof Euler Theorem for homogenous function.

6. (a) Find the relative maximum and minimum value of the function
$$f(x,y) = 2(x^2 - y^2) - x^4 + y^4.$$

   (b) If $u = x^2 - y^2, v = 2xy$ and $x = r\cos\theta, y = r\sin\theta$, find $\frac{\partial(u,v)}{\partial(r,\theta)}$.