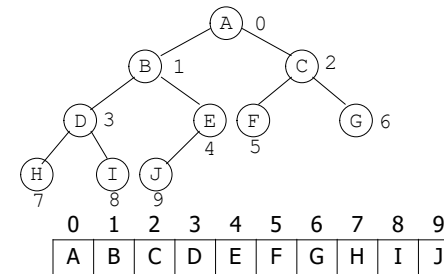


Threaded Binary Trees

5-1

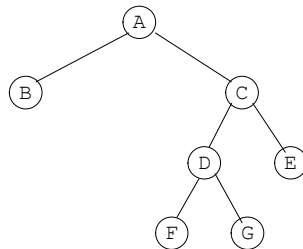
1

Implicit array representation for Binary Tree



5-2

2



5-3

3

Linked Representation of the Tree

5-4

4

Drawback with BT

- Too many null pointers representation of binary trees
 - n: number of nodes
 - number of non-null links: $n-1$
 - total links: $2n$
 - null links: $2n-(n-1)=n+1$
- Solution????
 - Replace these null pointers with some useful pointers known as “threads”.

5-5

5

Drawbacks with Normal BT continued ..

- Traversing operation is the most frequently used operation on tree.
- Temporary data structure(stack) is required to implement non recursive traversal algorithm.

5-6

6

Need for TBT

- > linked representation , there are more 0/NULL - links than actual pointers.
- There are
 - $n+1$ 0/NULL-links and $2n$ total links
 - Wastage of memory
- Thread : a pointer to other nodes in the tree for replacing the 0/NULL-link.**
 - By doing this we can reutilize the Null Pointers. Will result in :
 1. No Wastage of memory for null pointers.
 2. Non Recursive traversal without stack.

5-7

7

Traversal Without A Stack

Two methods:

1. Use of parent field to each node.
- 2. Use of two bits per node to represents binary trees as threaded binary trees**

5-8

8

Threaded Binary Trees (Continued)

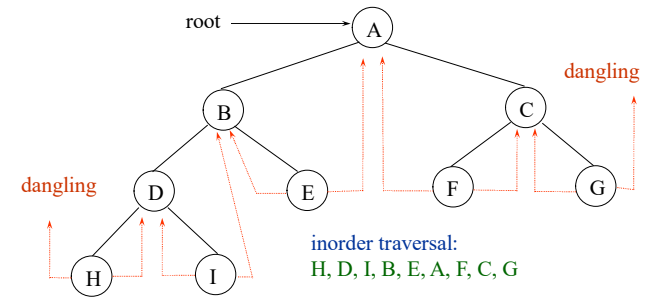
To construct threads we use following rules (assume that ptr represents a node):

- If ptr->left_child is null, replace it with a pointer to the node that would be visited *before* ptr in an *inorder traversal* (i.e. **Inorder predecessor**)
- If ptr->right_child is null, replace it with a pointer to the node that would be visited *after* ptr in an *inorder traversal* (i.e. **Inorder successor**)

5-9

9

A Threaded Binary Tree



5-10

10

Data Structures for Threaded BT

left_thread	left_child	data	right_child	right_thread
TRUE	•	—	•	FALSE

TRUE: thread

FALSE: child

```
typedef struct threaded_tree *threaded_pointer;
typedef struct threaded_tree
{
    threaded_pointer left_child;
    threaded_pointer right_child;
    char data;
    int left_thread;
    int right_thread;
};
```

5-11

11

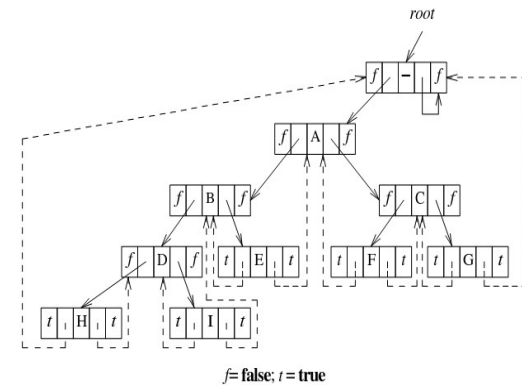


Figure 5.23: Memory representation of threaded tree

5-12

12

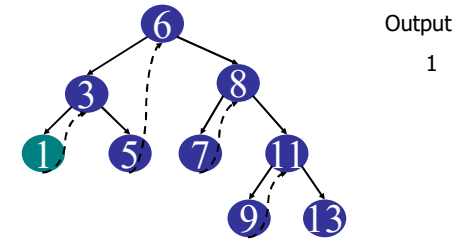
Advantage of TBT in Traversals

- By using threads, we can perform an inorder, preorder and postorder traversal without making use of a stack
- For any node, ptr, in a threaded binary tree, if $\text{ptr} \rightarrow \text{rightThread} = \text{TRUE}$, the inorder successor of ptr is ptr's parent. Otherwise, we obtain the inorder successor of ptr by following a path of left-child links from the right-child of ptr until we reach a node with $\text{left Thread} = \text{TRUE}$

5 -13

13

Threaded Tree Traversal



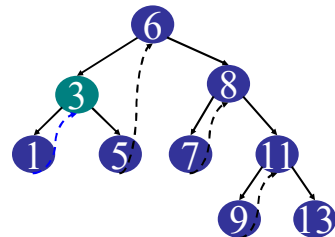
Start at leftmost node, print it

Output
1

5 -14

14

Threaded Tree Traversal



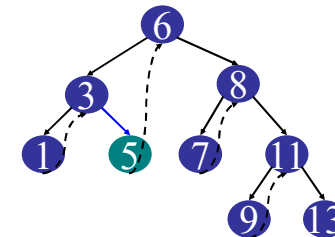
Follow thread to right, print node

Output
1
3

5 -15

15

Threaded Tree Traversal



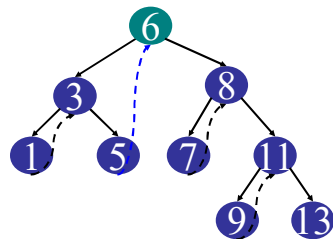
Follow link to right, go to leftmost node and print

Output
1
3
5

5 -16

16

Threaded Tree Traversal



Output

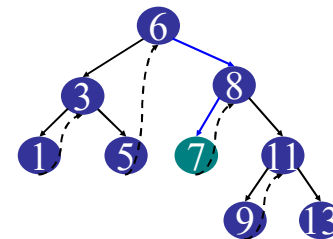
1
3
5
6

Follow thread to right, print node

5 -17

17

Threaded Tree Traversal (Inorder)



Output

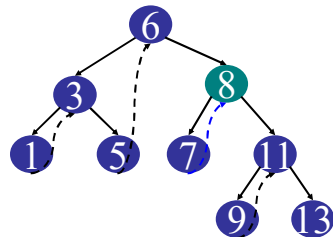
1
3
5
6
7

Follow link to right, go to
leftmost node and print

5 -18

18

Threaded Tree Traversal



Output

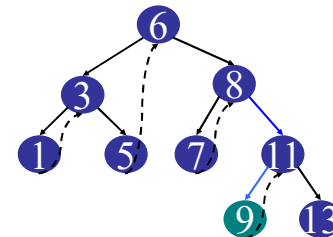
1
3
5
6
7
8

Follow thread to right, print node

5 -19

19

Threaded Tree Traversal



Output

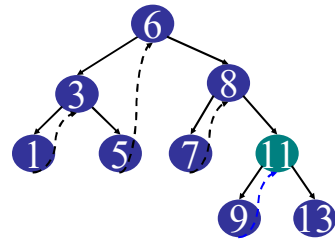
1
3
5
6
7
8
9

Follow link to right, go to
leftmost node and print

5 -20

20

Threaded Tree Traversal



Follow thread to right, print node

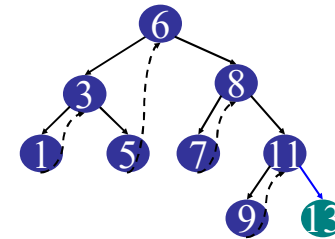
Output

1
3
5
6
7
8
9
11

5 -21

21

Threaded Tree Traversal



Follow link to right, go to
leftmost node and print

Output

1
3
5
6
7
8
9
11
13

5 -22

22

TBT root creation

FUNCTION CREATE_ROOT(ROOT):

1. IF ROOT=NULL
THEN TEMP ← GETNODE(X)
LTHREAD(TEMP) ← RTHREAD(TEMP) ← 1
LPTR(TEMP) ← RPTR(TEMP) ← HEAD
LPTR(HEAD) ← TEMP
ROOT ← TEMP
2. RETURN(ROOT)

5 -23

23

TBT Insertion

FUNCTION INSERT_NODE(HEAD, X):

1. IF LPTR(HEAD) = HEAD
THEN WRITE('CREATE ROOT FIRST')
LPTR(HEAD) ← CREATE_ROOT(HEAD)
RETURN HEAD
2. PARENT ← LPTR(HEAD)
FLAG ← TRUE
3. REPEAT THRU STEP 4 WHILE FLAG = TRUE
4. WRITE('ROOT IS', DATA(PARENT))
WRITE('1. INSERT AT LEFT, 2. INSERT AT RIGHT')
WRITE('ENTER CHOICE')
INPUT(CH)

5 -24

24

TBT Insertion contd..

```

SELECT CH
CASE 1: IF (LTHREAD (PARENT) = 1)
    THEN NEV ← GETNODE(X)
    LPTR(NEV) ← LPTR(PARENT)
    RPTR(NEV) ← PARENT
    LTHREAD(NEV) ← RTHREAD(NEV) ← 1
    LPTR(PARENT) ← NEV
    LTHREAD(PARENT) ← 0
    FLAG ← FALSE
ELSE
    PARENT ← LPTR(PARENT)

CASE 2: IF (RTHREAD (PARENT) = 1)
    THEN NEV ← GETNODE(X)
    RPTR(NEV) ← RPTR(PARENT)
    LPTR(NEV) ← PARENT
    LTHREAD(NEV) ← RTHREAD(NEV) ← 1
    RPTR(PARENT) ← NEV
    RTHREAD(PARENT) ← 0
    FLAG ← FALSE
ELSE
    PARENT ← RPTR(PARENT)

5. RETURN(PARENT)
    
```

5 -25

25

TBT-Preorder Traversal

PROCEDURE PRETHREAD(HEAD):

```

1. CURRENT ← LPTR(HEAD)

2. IF (CURRENT = HEAD)
    THEN WRITE('EMPTY TREE')
    RETURN

3. REPEAT THRU STEP 5 WHILE CURRENT != HEAD

4. WRITE(DATA(CURRENT))

5. IF (LTHREAD(CURRENT) = 0)
    CURRENT ← LPTR(CURRENT)
ELSE
    REPEAT WHILE (RTHREAD(CURRENT) = 1)
        CURRENT ← RPTR(CURRENT)
    CURRENT ← RPTR(CURRENT)

6. RETURN
    
```

5 -26

26

TBT-Inorder Traversal

PROCEDURE INTREAD(HEAD):

```

1. CURRENT ← LPTR(HEAD)

2. IF (CURRENT = HEAD)
    THEN WRITE('EMPTY TREE')
    RETURN

3. REPEAT WHILE LTHREAD(CURRENT) = 0
    CURRENT ← LPTR(CURRENT)

4. REPEAT THRU STEP 6 WHILE CURRENT != HEAD

5. WRITE(DATA(CURRENT))

6. IF RTHREAD(CURRENT) = 1
    CURRENT ← RPTR(CURRENT)
ELSE
    CURRENT ← RPTR(CURRENT)
    REPEAT WHILE (LTHREAD(CURRENT) = 0)
        CURRENT ← LPTR(CURRENT)

7. RETURN
    
```

5 -27

27

TBT-Postorder Traversal (Initialize)

PROCEDURE INITIALIZE(HEAD):

```

1. CURRENT ← LPTR(HEAD)

2. IF (CURRENT = HEAD)
    THEN WRITE('EMPTY TREE')
    RETURN

3. REPEAT THRU STEP 4 WHILE CURRENT != HEAD

5. IF (LTHREAD(CURRENT) = 0)
    CURRENT ← LPTR(CURRENT)
    FLAG(CURRENT) = 0
ELSE
    REPEAT WHILE (LTHREAD(CURRENT) = 1)
        CURRENT ← RPTR(CURRENT)
    FLAG(CURRENT) = 0

    CURRENT ← RPTR(CURRENT)
    FLAG(CURRENT) = 0

6. RETURN
    
```

5 -28

28

TBT-Postorder Traversal

PROCEDURE POSTTHREAD(HEAD):

```
1. TEMP ← left(HEAD)
   FLAG(TEMP) ← 0

2. REPEAT THRU STEP 5 WHILE TEMP != HEAD

3. IF Lbit(TEMP)=0 AND FLAG(left(TEMP))!=2)
   then TEMP ← left(TEMP)
      FLAG(TEMP) ← 0
   return

4. IF Rbit(TEMP)=0 AND FLAG(right(TEMP))!=2)
   then TEMP ← right(TEMP)
      FLAG(TEMP) ← 1
   return
```

```
5. WRITE (DATA(TEMP))
   IF(FLAG(TEMP) = 0)
   then FLAG(TEMP)=2
      WHILE(Rbit(TEMP)=0)
      TEMP ← right(TEMP)
      TEMP ← right(TEMP)
   else
      FLAG(TEMP)=2
      WHILE(Lbit(TEMP)=0)
      TEMP ← left(TEMP)
      TEMP ← left(TEMP)

6. return
```

TBT-Postorder Traversal

PROCEDURE POSTTHREAD(HEAD):

```
1. TEMP ← LPTR(HEAD)
   FLAG(TEMP) ← 0

2. REPEAT THRU STEP 5 WHILE TEMP != HEAD

3. IF LTHREAD(TEMP)=0 AND FLAG(LPTR(TEMP))!=2)
   THEN TEMP ← LPTR(TEMP)
      FLAG(TEMP) ← 0
   RETURN

4. IF RTHREAD(TEMP)=0 AND FLAG(RPTR(TEMP))!=2)
   THEN TEMP ← RPTR(TEMP)
      FLAG(TEMP) ← 1
   RETURN
```

5 -30

29

30

TBT-Postorder contd..

```
5. WRITE (DATA(TEMP))
   IF(FLAG(TEMP) = 0)
   THEN FLAG(TEMP)=2
      WHILE(RTHREAD(TEMP)=0)
      TEMP ← RPTR(TEMP)
      TEMP ← RPTR(TEMP)
   ELSE
      FLAG(TEMP)=2
      WHILE(LTHREAD(TEMP)=0)
      TEMP ← LPTR(TEMP)
      TEMP ← LPTR(TEMP)

6. RETURN
```

5 -31

TBT EXP creation

```
TBT_ExpTee()
{
   while(str1(i)
   !='\0') {
      if(isalpha(str1[i]))
      {parent = new node
       parent->data = str1[i]
       parent->lthd = false
       parent->rthd = false
       parent->rlink = pop()
       parent->llink = pop()
       t1= parent ->llink
       t2 =parent->rlink
       if(t2->llink==NULL)
       t2->llink = parent
       if(t1->rlink==NULL)
       t1->rlink = parent
       push( stack_data,parent)
      }
      else // when the token is
            oerator
      { parent = new node
       parent->data = str1[i]
       parent->lthd = false
       parent->rthd = false
       parent->rlink = pop()
       parent->llink = pop()
       t1= parent ->llink
       t2 =parent->rlink
       if(t2->llink==NULL)
       t2->llink = parent
       if(t1->rlink==NULL)
       t1->rlink = parent
       push( stack_data,parent)
      }
   }
}
```

31

32

TBT Expt Tree Creation

```
temp1 = t1 temp2 = t2      root = stack_data.pop_data()
while(!(temp1->rthd))      head->llink=root
    temp1 = temp1->rlink    temp1 = root->llink
temp1->rlink = parent      temp2 = root->rlink
while(!(temp2->lthd))      while(!(temp1->lthd))
    temp2 = temp2->llink    temp1 = temp1->llink
temp2->llink = parent;     temp1->llink = head
    push(parent);         while(!(temp2->rthd))
    }                     temp2 = temp2->rlink
    i++ // take next token temp2->rlink = head
    }
```