

Title : Assignment 8 : Shortest Path finding

Aim : To implement shortest path using Dijkstras algorithm

Problem Statement : Represent a graph of city using adjacency matrix / adjacency list. Nodes should represent the various landmarks and links should represent the distance between them. Find the shortest path using Dijkstra's algorithm from single source to all destination.

Theory :

What is shortest path?

In graph theory, the shortest path is the path between two vertices such that the sum of the weights of its edges is minimized.

The problem of finding the shortest path in a graph from one vertex to another. Shortest can be least number of edges, least total weight etc.

various algorithm to find shortest path

- 1) Dijkstra's algorithm
- 2) Bellman-Ford algorithm
- 3) Floyd-Warshall algorithm
- 4) Johnson's algorithm
- 5) Viterbi algorithm

Greedy approach:

An algorithm is designed to achieve optimum solution for a given problem. In greedy algorithm approach, decisions are made from given solution domain. As being greedy, the closest solution that seems to provide an optimum solution is chosen.

Greedy algorithm builds up solution piece by piece, always choosing the next piece that offers the most obvious & immediate benefits.

• Dijkstra's algorithm:

It is an algorithm for finding the shortest paths between nodes in a graph.

The algorithm creates a tree of shortest path from the starting vertex (source) to all other points in graph.

Dijkstra algorithm finds a shortest path tree from a single source node by building a set of nodes that have minimum distance from source.

Real time uses of Dijkstra's algorithm:

- 1) Social networking applications
- 2) Telephone network
- 3) Digital mapping services in Google map
- 4) IP routing to find open shortest path first
- 5) Flighting agenda

Algorithms for Dijkstra's single source to multiple destinations

Procedure Dijkstra

//src is the source vertex

for $i=0$ to V

// find initial distance

If $\text{weight}[\text{src}][i] = 0$

$\text{dist}[i] = \text{weight}[\text{src}][i]$

else

$\text{dist}[i] = 32767$

$\text{path}[i] = \text{src}$

$\text{visited}[i] = 0$

End for

// take source as current vertex & make it as visited

$\text{current} = \text{src}$

$\text{visited}[\text{src}] = 1$

// repeat for all vertices

for $j=0$ to $V-1$

$\text{mindist} = 32767$

// find minimum distance from current to all other

for $i=0$ to V

If $\text{visited}[i] \neq 0$ and $\text{dist}[i] < \text{mindist}$

$\text{mindist} = \text{dist}[i]$

$\text{current} = i$

End for

// make current as visited

$\text{visited}[\text{current}] = 1$

// find shortest path from current

for $i=0$ to v

If $visited[i] = 0$ and $(dist[current] + weight[current][i]) < dist[i]$

$dist[i] = dist[current] + weight[current][i]$

$path[i] = current$

End for

End for

// display shortest path

for $i=0$ to v

If $i \neq src$

print $i, dist[i]$

int $j=i$

do

$j = path[j]$

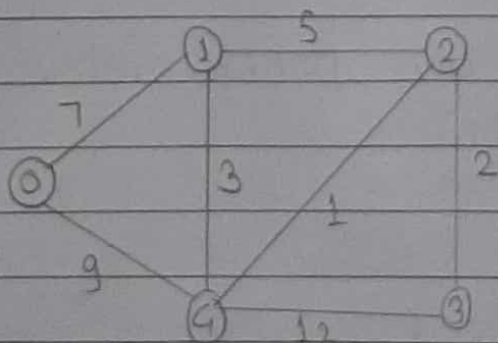
print $path[j]$

while $j \neq src$

End for

End

Example:



Select 0 as source vertex,

From 0, 7 is minimum distance, select it

Initial:

| vertex | Path | distance |
|--------|------|----------|
| 0 | 0 | 0 |
| 1 | 0 | 7 |
| 2 | 0 | ∞ |
| 3 | 0 | ∞ |
| 4 | 0 | 9 |

Selected vertex: 2

cost: 10

| vertex | Path | distance |
|--------|------|----------|
| 0 | 0 | 0 |
| 1 | 0 | 7 |
| 2 | 4 | 10 |
| 3 | 2 | 12 |
| 4 | 0 | 9 |

Selected vertex: 1 cost: 7

| vertex | Path | Distance |
|--------|------|----------|
| 0 | 0 | 0 |
| 1 | 0 | 7 |
| 2 | 1 | 12 |
| 3 | 0 | ∞ |
| 4 | 0 | 9 |

Shortest path:

Vertex 1: 0 \rightarrow 1 Distance = 7

Vertex 2: 0 \rightarrow 4 \rightarrow 2 Distance: 10

Vertex 3: 0 \rightarrow 4 \rightarrow 2 \rightarrow 3 Distance: 12

Vertex 4: 0 \rightarrow 4 Distance: 9

Selected vertex: 4 cost: 9

| vertex | Path | distance |
|--------|------|----------|
| 0 | 0 | 0 |
| 1 | 0 | 7 |
| 2 | 04 | 10 |
| 3 | 4 | 21 |
| 4 | 0 | 9 |

Test cases :

- 1) Directed graph with no loops & parallel edges
- 2) Undirected graph with no loop & parallel edges

Validations:

Number of vertices & edges are positive

Conclusion:

Time complexity of Dijkstra algorithm is $O(V^2)$. It can be reduced to $O(E \log V)$ if graph is represented using adjacency list.