Multivariate Control Charts

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Statistical Quality Control

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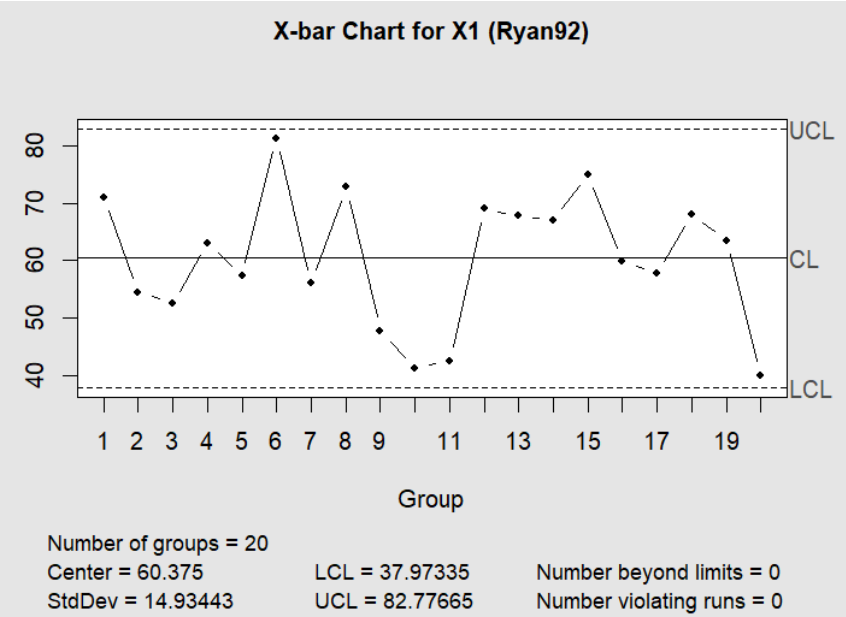
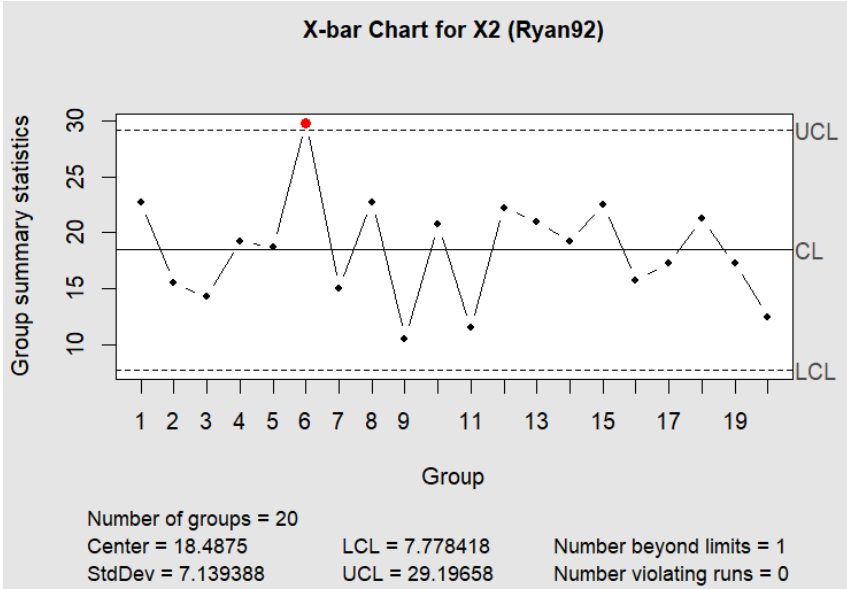
Why Multivariate?



X-bar Chart for X1



X-bar Chart for X2

- Univariate charts monitor each variable separately.
- X2 flags one out-of-control point, but overall, the process seems stable.

Real-World Motivation: Baking Cookies

Scenario: In a bakery, quality control checks:

- **Size** of cookies (diameter in cm)
- **Color** (brownness index)
- **Moisture content**

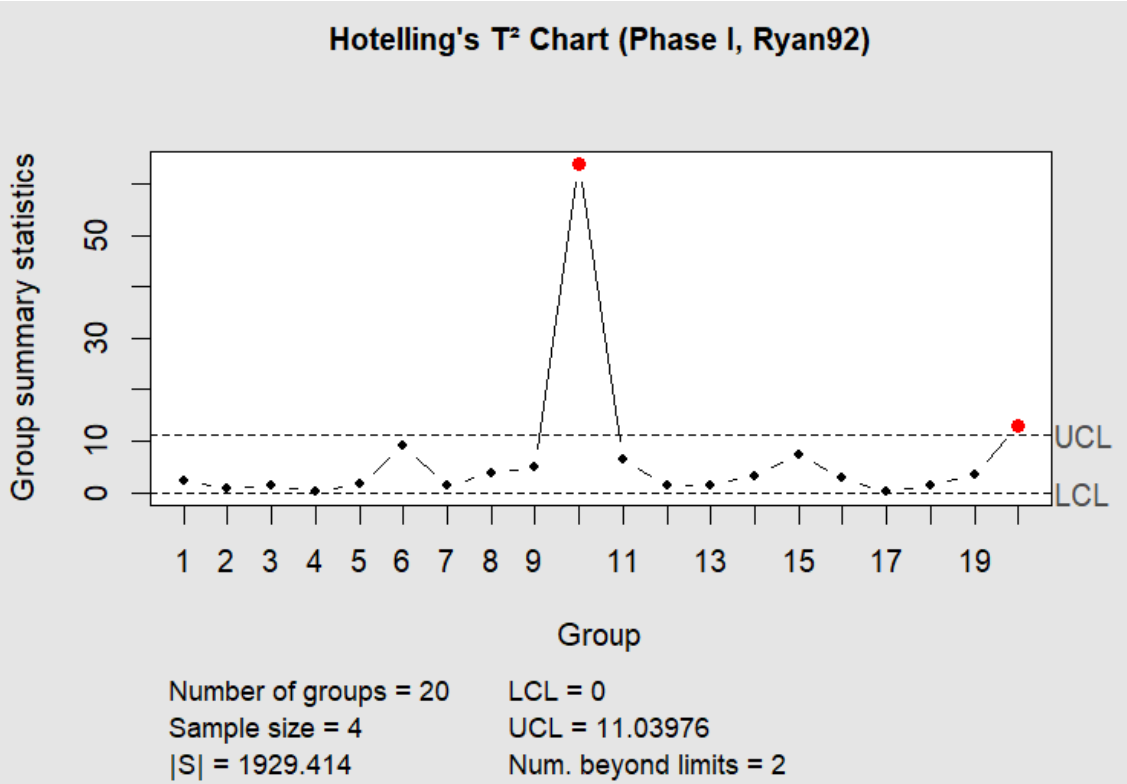
The Problem:

- These traits are often *correlated*.
- For example, overbaking reduces size and moisture, but increases color.
- Univariate charts might miss these subtle joint effects.

Solution: A **Multivariate Control Chart**:

- Looks at *all three traits together*.
- Catches when all variables are slightly off in the same batch.
- Helps bakers fix oven settings early — before batches go to waste.

What Is a Multivariate Control Chart?



The Hotelling T^2 chart jointly monitors correlated quality variables.
Result: Subgroups 10 and 20 are out-of-control — missed by univariate charts!

Hotelling's T^2 Chart

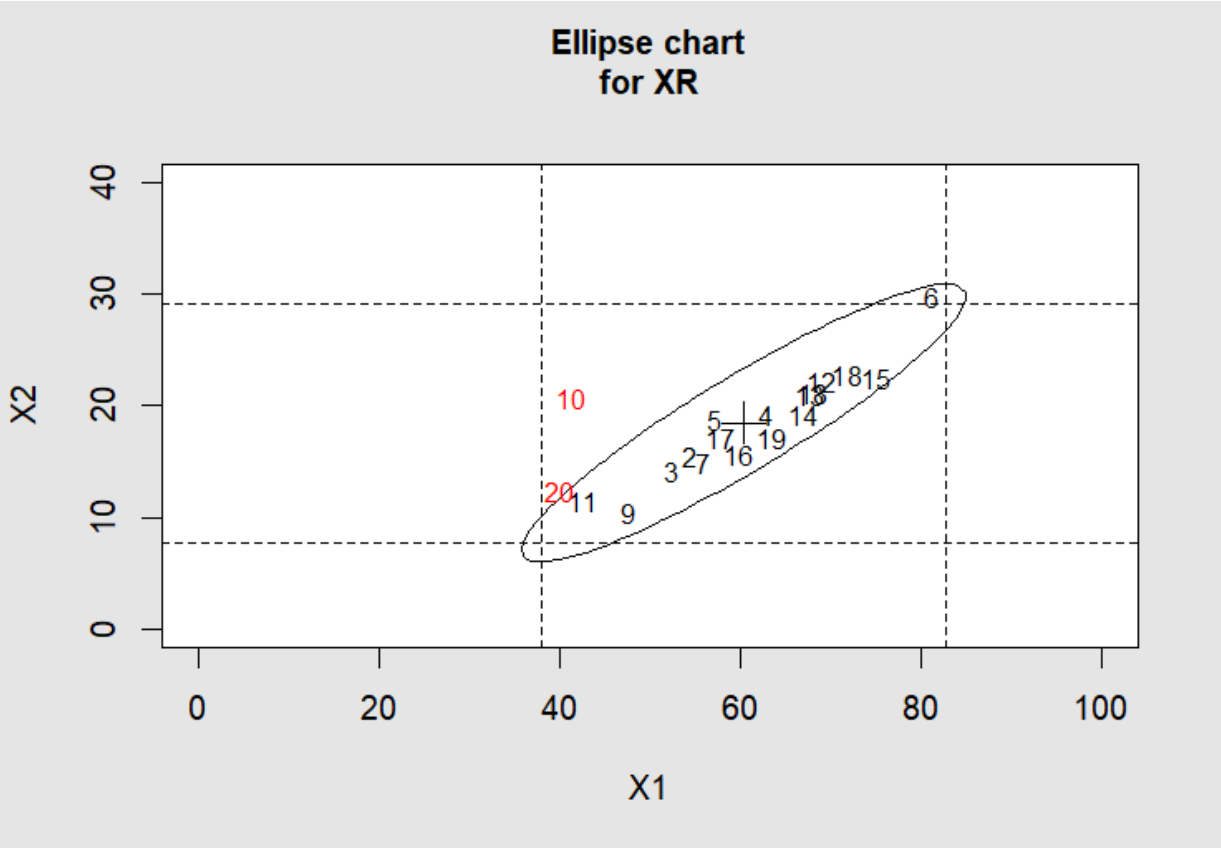
Core Formula

$$T^2 = n(\bar{\mathbf{x}} - \boldsymbol{\mu})' \boldsymbol{\Sigma}^{-1} (\bar{\mathbf{x}} - \boldsymbol{\mu})$$

- $\bar{\mathbf{x}}$: sample mean vector.
- $\boldsymbol{\mu}$: in-control mean vector.
- $\boldsymbol{\Sigma}$: in-control covariance matrix.

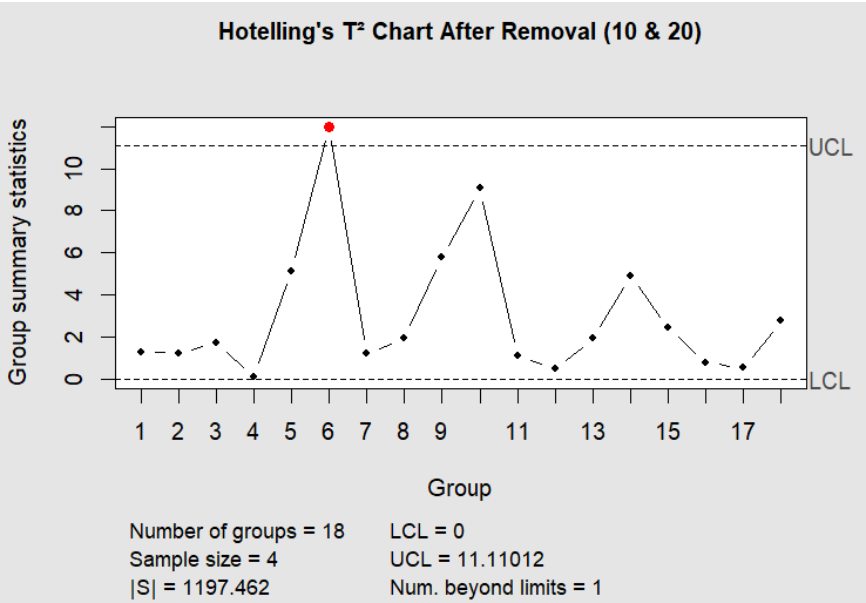
Visualizing T^2: The Control Ellipse

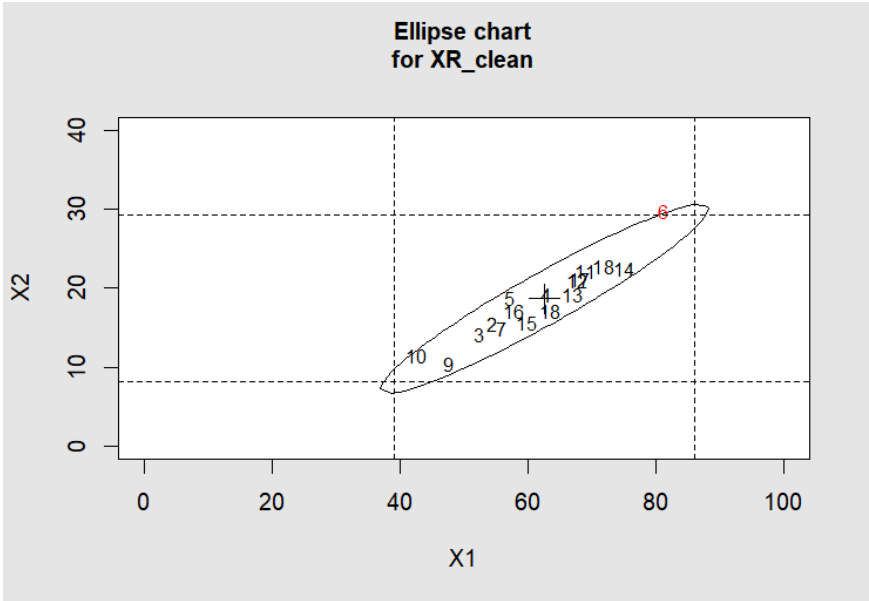
- The ellipse shows the joint control region for X1 and X2.
- Points 10 and 20 lie outside the 95% confidence ellipse.
- These would be hard to detect using univariate monitoring.



After Removing Outliers

- Subgroups 10 and 20 were removed from the Phase I analysis.
- New T^2 chart shows most points in control — only subgroup 6 remains a concern.





Limitations of Hotelling's T^2

- Less sensitive to **small or gradual shifts**.
- Hard to identify **which variable triggered the alarm**.
- Need complementary methods for early detection.

Solutions to Limitations

- Bonferroni-adjusted confidence intervals.
- Principal Component Control Charts.
- **T^2 Decomposition:**

$$d_i = T^2 - T^2_{(-i)}$$

MEWMA: Detecting Small Shifts

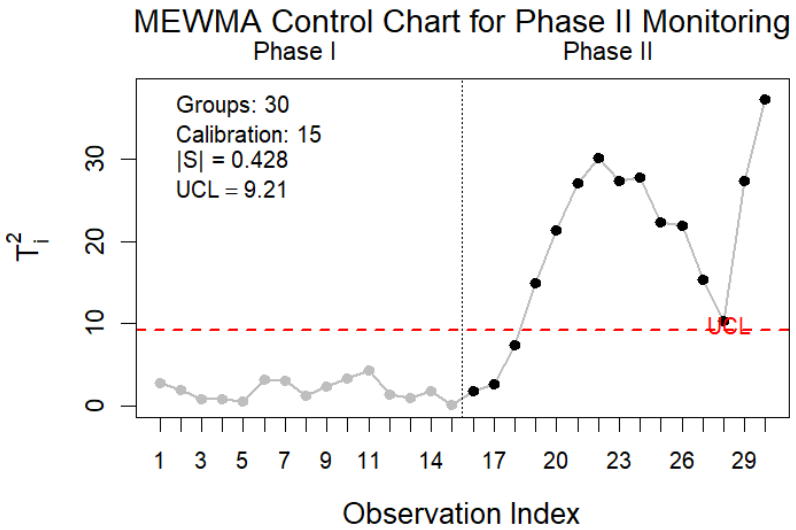
- **Multivariate Exponentially Weighted Moving Average (MEWMA)**:
- Uses weighted history: $\mathbf{Z}_i = \lambda \mathbf{X}_i + (1 - \lambda)\mathbf{Z}_{i-1}$
- Sensitive to small drifts, ideal for Phase II monitoring.

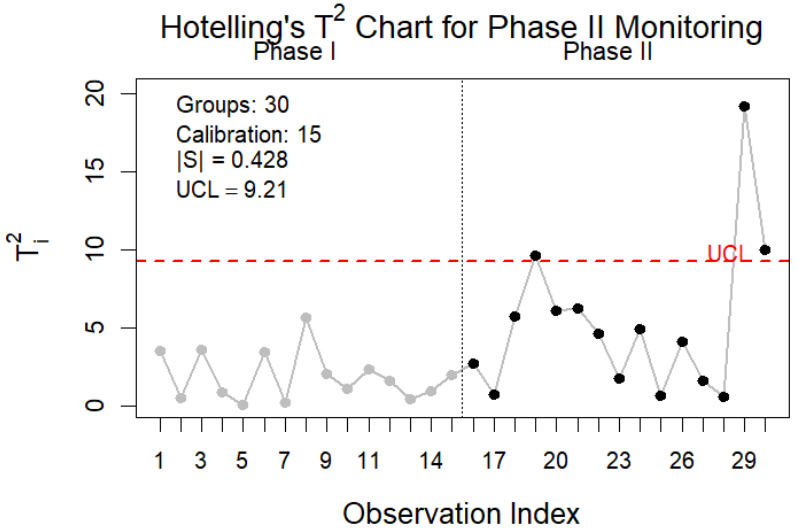
Control Limit

$$UCL = \chi^2_{1-\alpha,p} \approx 9.21 \ (p = 2, \alpha = 0.01)$$

MEWMA vs. T^2: Example

- Simulated 30 observations, small mean shift after observation 15.
- Hotelling's T^2: detects large shifts but misses small drifts.
- MEWMA: detects the shift promptly.





Quick Quiz:

Which chart would you choose to detect small drifts in a process?

- A) Hotelling's T^2
- B) MEWMA

Conclusion

- Multivariate control charts extend SPC to correlated variables.
- Hotelling's T^2 for large shifts.
- MEWMA for small, gradual shifts.
- Together, they form a **robust monitoring toolkit** for modern processes.

Questions

Thank you for your attention!
Questions or clarifications?