

Fundamentals of Robotics

Project Report

Question 1

The joint velocities can be calculated using two methods: inverse Jacobian and Jacobian transpose.

I first computed the Jacobian using the DH parameters that were provided. The detailed computation is in the **direct_kin.m** file. I directly entered the corresponding formulas in the Simulink blocks.

a) Jacobian inverse method to compute the joint positions.

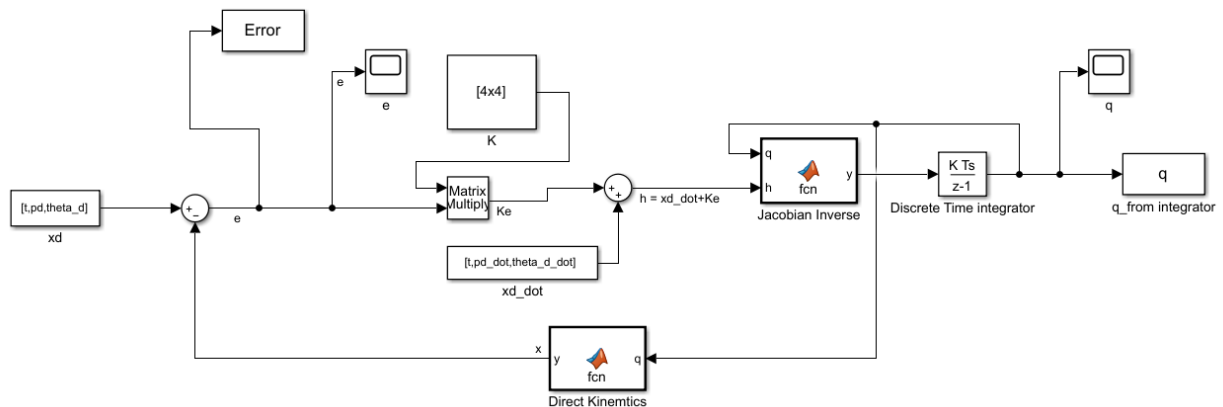


Fig 1: Simulink model for Jacobian Inverse method

The model follows the schematic shown in Fig 2.

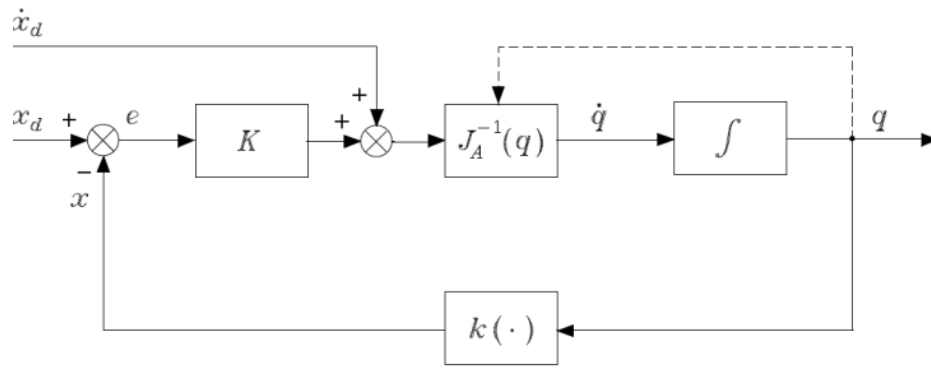


Fig 2: Jacobian Inverse Schematic

- The parameter $x_d = [pd, \theta_d]$ which is obtained from the data provided in the matlab file kinematic_traj.m.
- $\dot{x}_d = [pd_dot, \theta_d_dot]$ which is again information provided in kinematic_traj.m.
- x is obtained from the direct kinematic block. Initially this value is obtained from the initial conditions which is provided in the integrator block as q_0 . This data comes from kinematic_traj.m. Initially, q_0 passes through the direct kinematics block to give x i.e the position and orientation in the operational space.
- The error between the desired and obtained positions i.e $e = x_d - x$,
- The error 'e' is fed to the gain block which is a 4x4 diagonal matrix with 10 as the diagonal elements. The 'K' matrix represents the gain which amplifies the error.
- Following this the amplified gain is summed with \dot{x}_d and fed to the Jacobian Inverse block.
- The Jacobian Inverse block contains formula of the jacobian inverse. The detailed calculation of the Jacobian and its inverse is in direct_kin.m file. The output of the Jacobian Inverse block is the ' \dot{q} ' vector which is a column vector that contains the joint velocities.
- The ' \dot{q} ' vector is fed to the integration block where Euler integration is performed. After integration, ' q ' is obtained which is a column vector containing the joint positions.
- The direct kinematics block takes ' q ' as the input and gives as output the operation space vectors which is given by the ' x ' vector.
- The process repeats for 4 seconds which is the length of the simulation.

- It is observed that the error becomes zero as the desired and obtained vectors become equal.

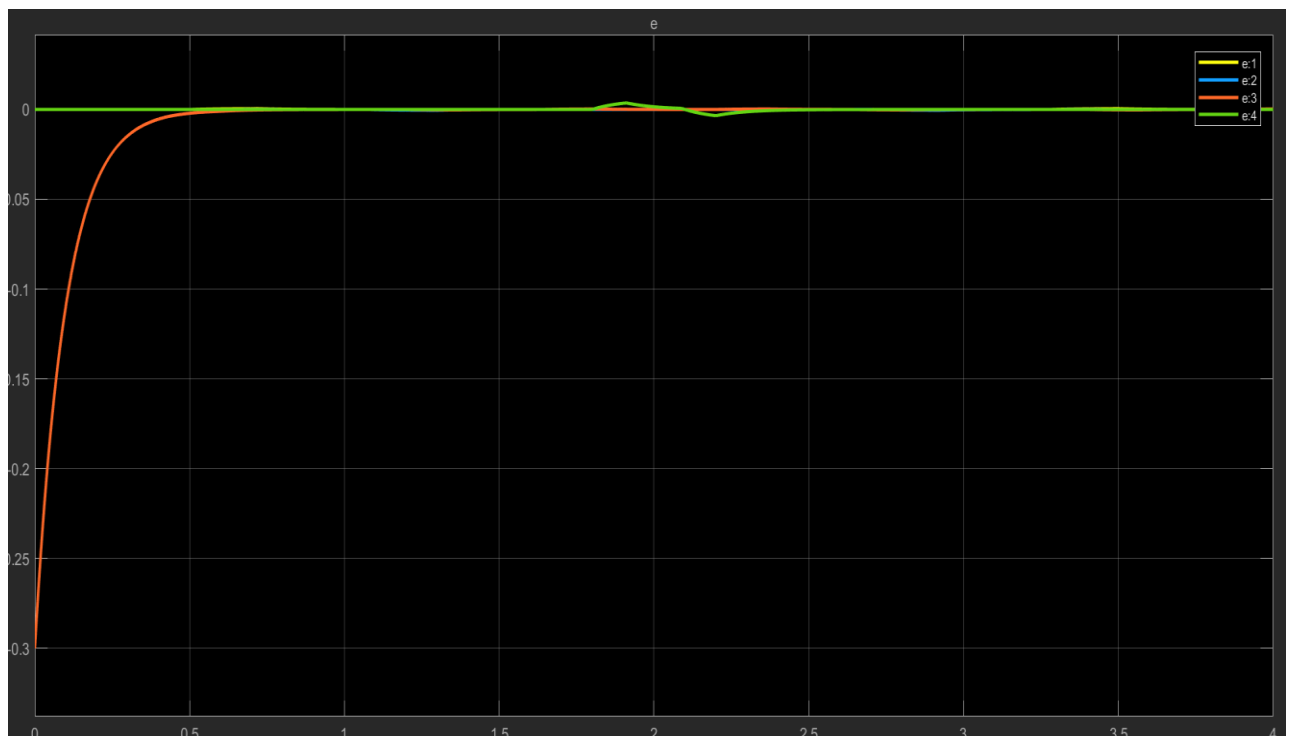


Fig 3: Graph showing Error in operational space parameters ($e = x_d - x$) V/s Time

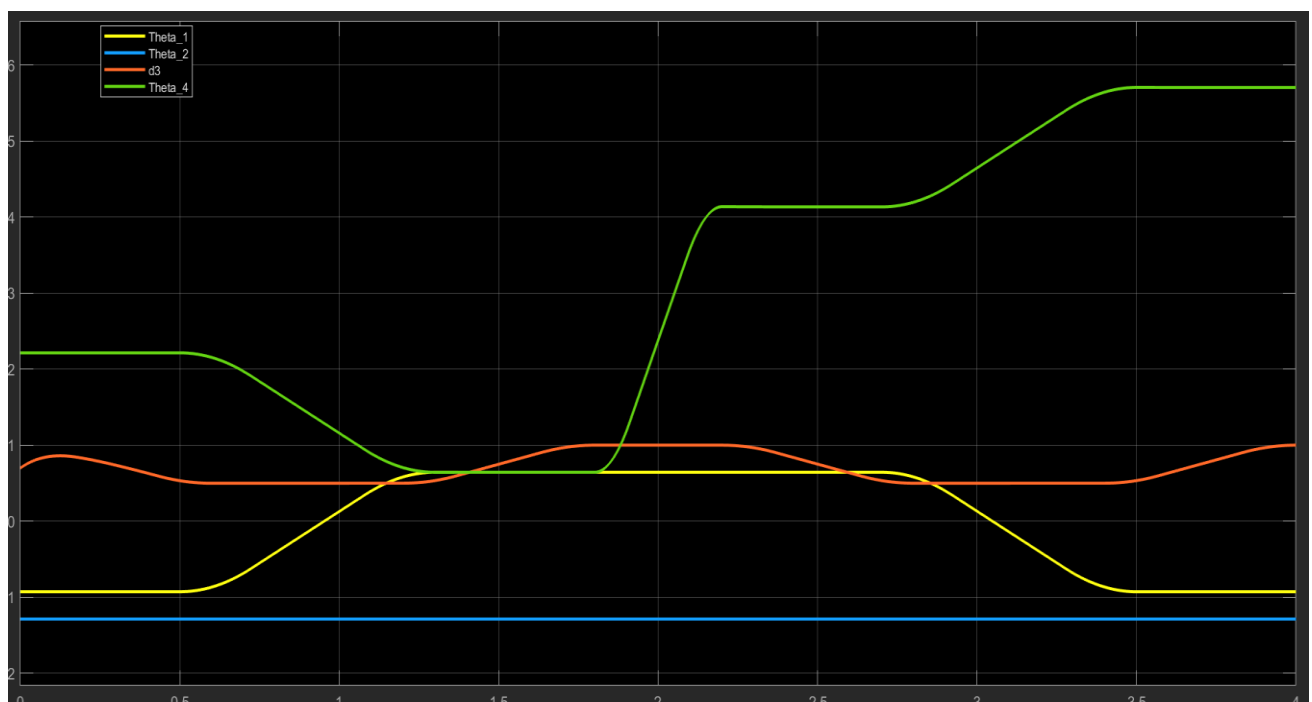


Fig 4: Graph showing the joint space parameters wrt time

b) Jacobian Transpose method to compute joint positions

- The Jacobian transpose method follows the same procedure as the Inverse Jacobian model except that the schematic is implemented as below in figure 4

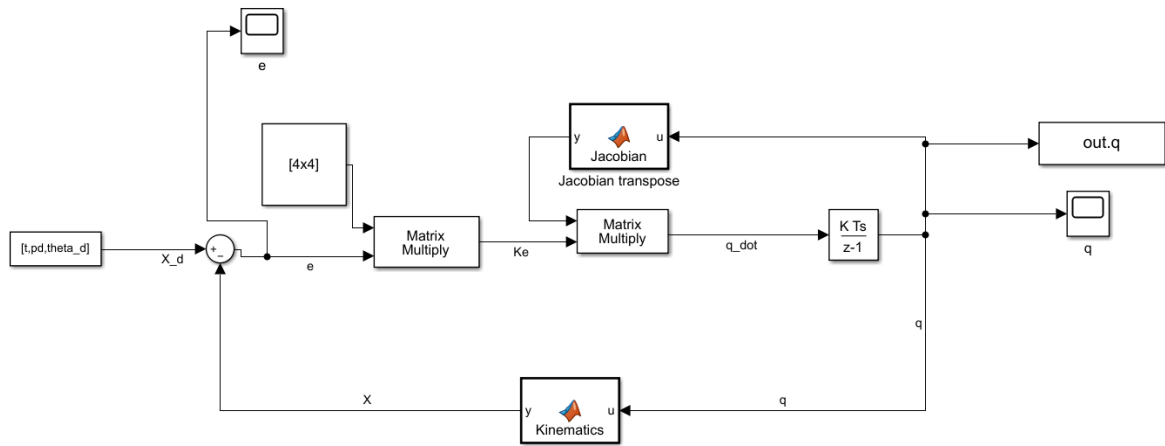


Fig 5: Simulink model for Jacobian Transpose method

The Simulink model for the Jacobian Transpose method follows the schematic shown in Fig 6.

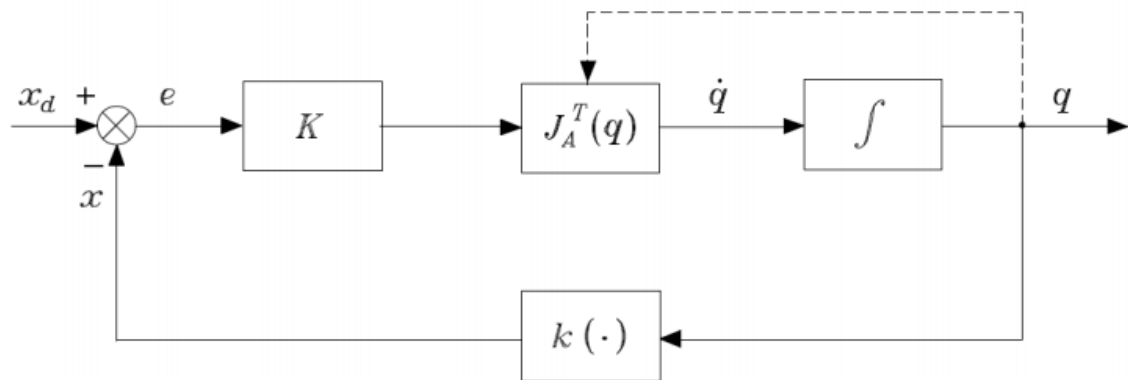


Fig 6: Jacobian Transpose Algorithm Schematic

The output of the simulation is as follows:

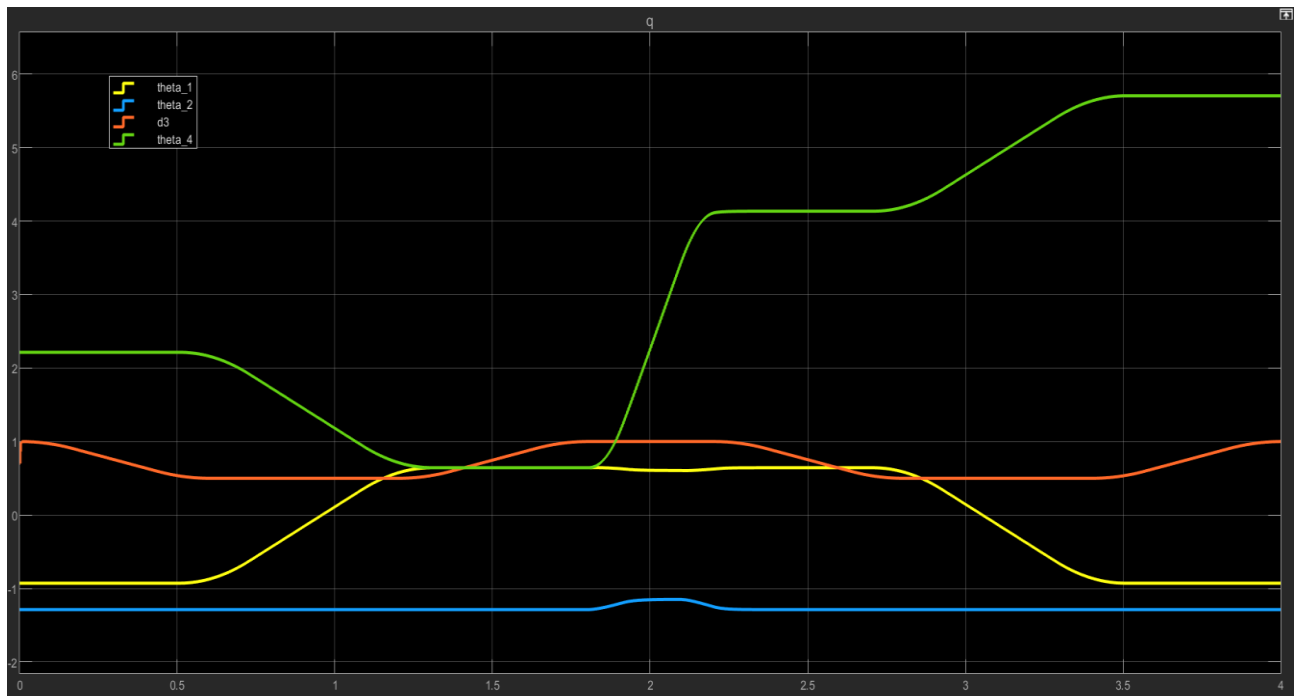


Fig 7: Variation of joint space parameters with time

- It is observed that the error in the desired and observed trajectories converges to zero, this is proof of the manipulator following the trajectory.

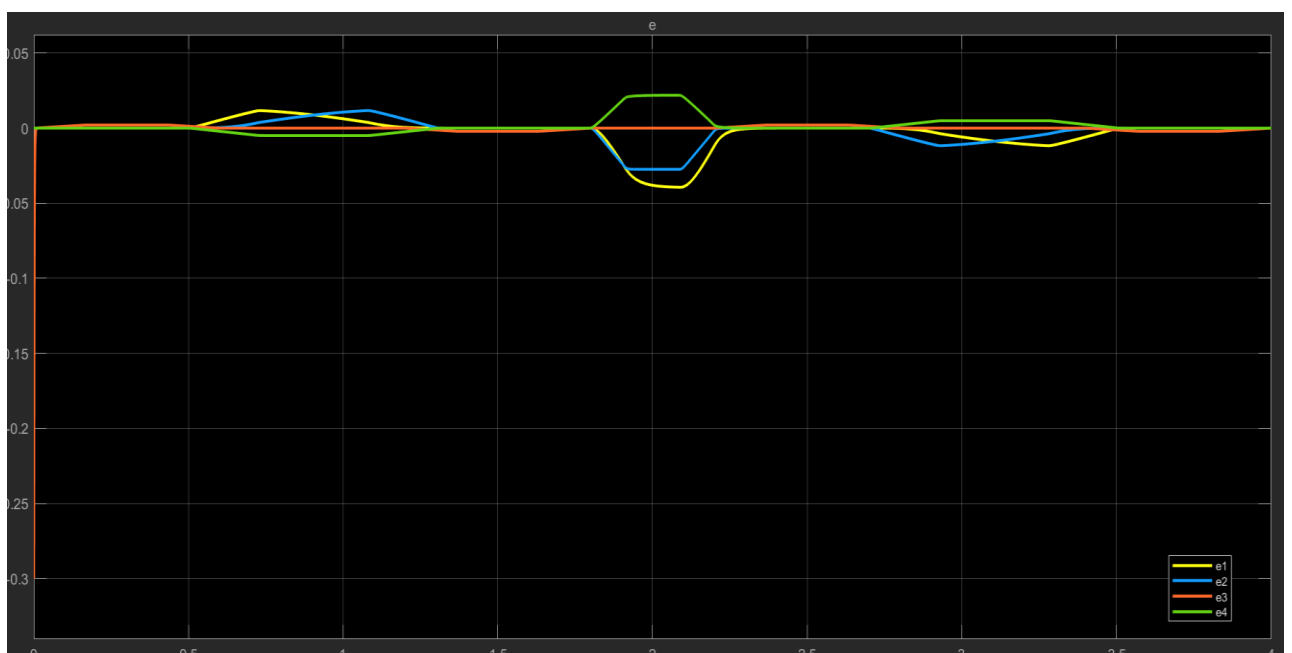


Fig 8: Graph showing Error in operational space parameters ($e = x_d - x$) V/s Time

Question 2

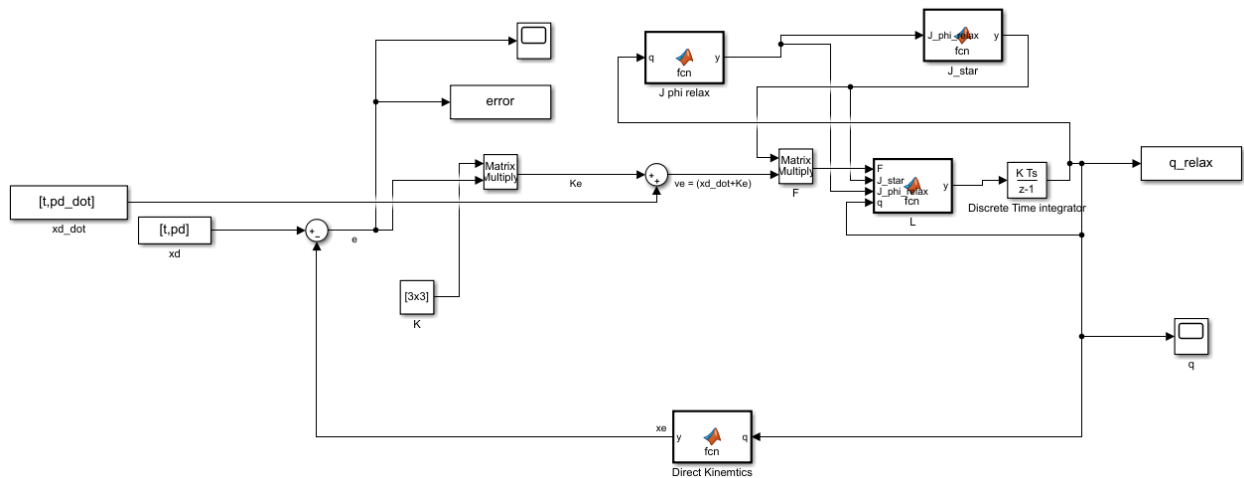


Fig 9: Simulink model for computing joint positions with relaxation of phi using Inverse Jacobian algorithm

- This question requires the relaxation of the phi component of the joint space. This creates a redundancy for the robot.
- To relax phi from the operational space, we simply remove the section of the Jacobian that is related to the movement of phi. The new Jacobian is now a 3 x 4 matrix.
- I then computed the Jacobian pseudo inverse using the new Jacobian this done in matlab function block J_star .
- In order to obtain the maximum distance from end joints. We compute joint velocities with respect to joint limits. This is given by partially differentiating $w(q)$ w.r.t joint space variables (q) i.e $\frac{\partial w(q)}{\partial q}$.
- In block F of the matlab function, I compute the \dot{q}_0 vector by multiplying with vector $\frac{\partial w(q)}{\partial q}$ a constant gain k_0 .
- Finally, I compute \dot{q} by multiplying J_star with J , subtracting it from identity matrix I , then I multiply this with \dot{q}_0 and then add to this the F matrix which is a product of J_star and v_e .
- Using the redundant manipulator in the system (i.e., theta 4), we use Jacobian inverse algorithm to compute joint positions for the given trajectory. The same schematic as Fig 2 is followed.

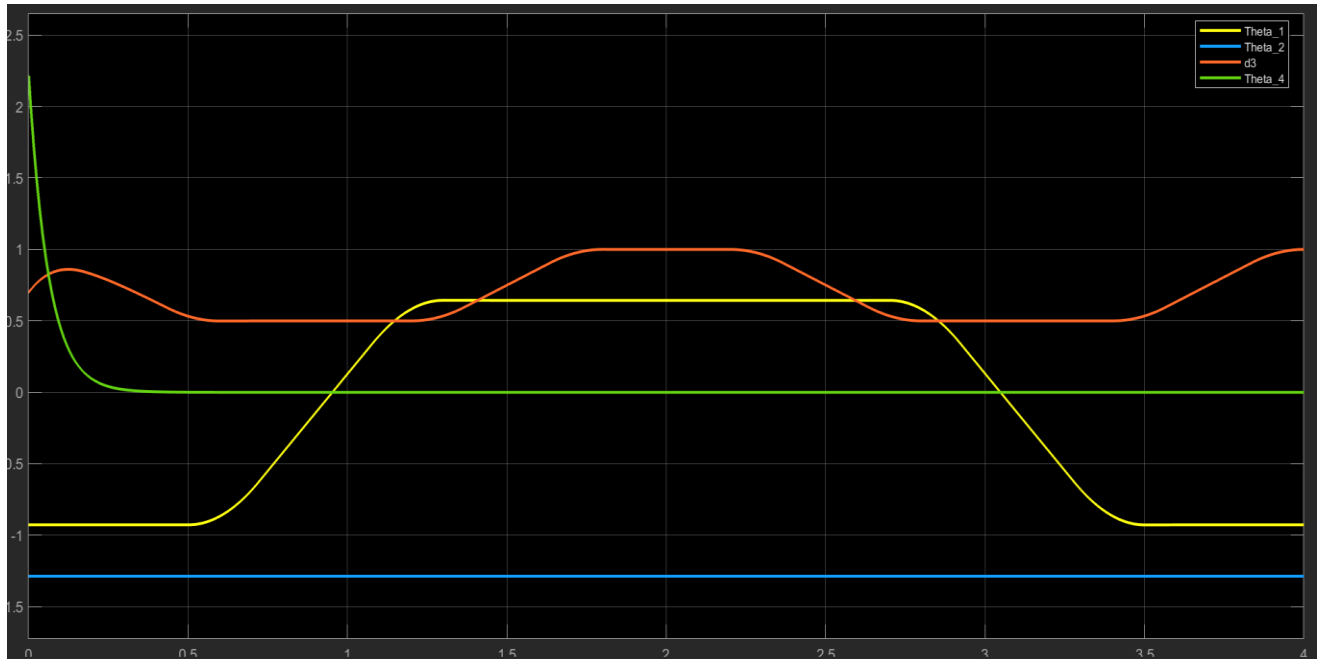


Fig 10: Graph showing the evolution of joint space parameters wrt time

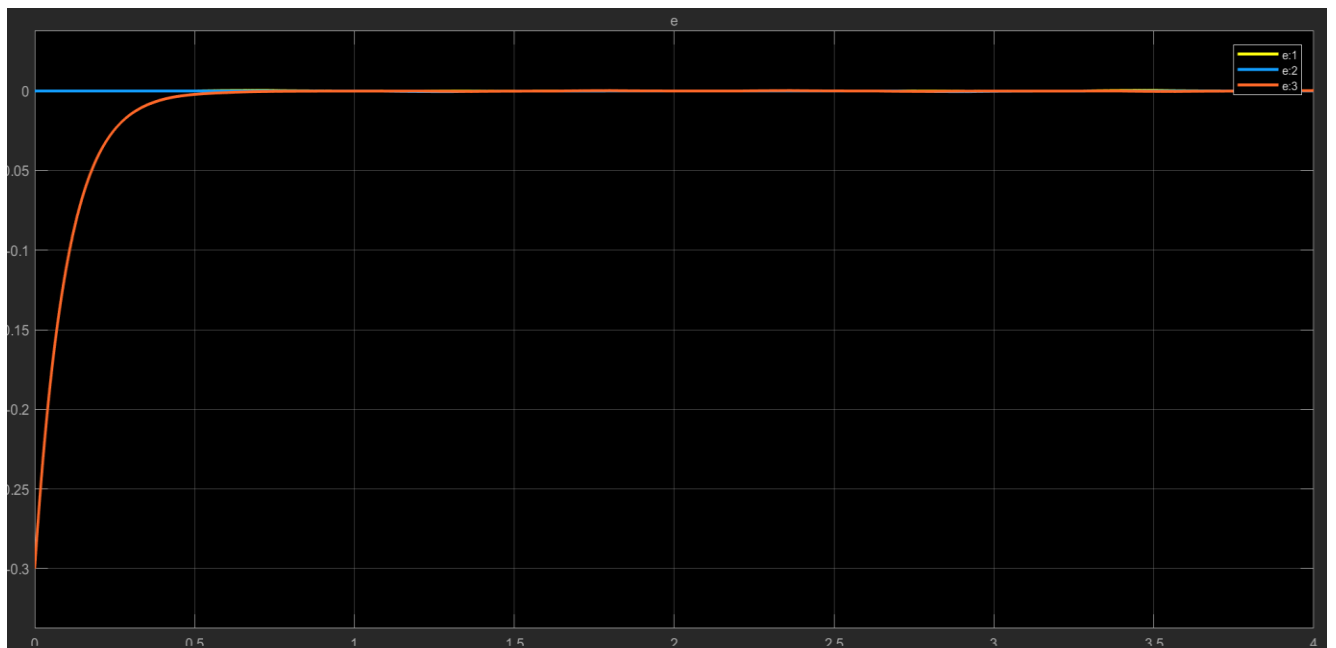


Fig 11: Graph showing Error in operational space parameters ($e = \dot{x}_d - \dot{x}$) V/s Time