Course: AP3121-D Imaging Systems

MATLAB assignment 'Tomography'.

Goal

In this assignment we will reconstruct an object from a set of measured projections.

Background

In computed tomography an object is reconstructed from its projections. We consider the geometry as indicated in Fig. 1 with the coordinates related according to

$$\begin{aligned} x' &= x\cos\theta + y\sin\theta \\ y' &= -x\cos\theta + y\sin\theta\,, \end{aligned} \tag{1}$$

with θ the rotation angle. For a circular object with radius R centered on the origin the

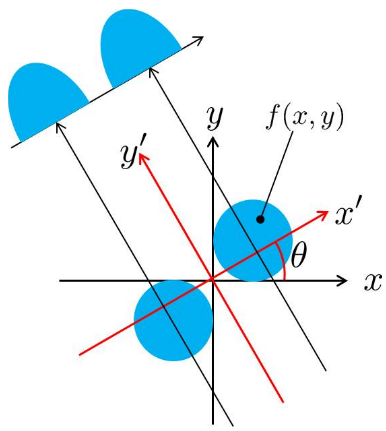

Figure 1: Geometry of computed tomography.

projection of density ρ is given by

$$p(x') = \begin{cases} 2\rho\sqrt{R^2 - x'^2} & |x'| < R \\ 0 & |x'| \geq R \end{cases}\,. \tag{2}$$

An image reconstruction technique is to backproject the projections onto the object. If this is done with the measured projections this in a typical blurred reconstruction of the initial object. Consequently, to create a sharp reconstructed image the projections are filtered. This is done either by multiplying the Fourier transform of the projection in the frequency domain with the ramp filter

$$H(\omega) = |\omega| \tag{3}$$

or by convoluting the projection in real space by the Ram-Lak filter

$$h(n\tau) = \begin{cases} \frac{1}{4\tau^2} & n = 0 \\ -\frac{1}{n^2\tau^2\pi^2} & n\,\text{odd} \\ 0 & n\,\text{even} \end{cases}\,, \tag{4}$$

with τ the detector spacing. Both ways of processing should have a similar result, which is a sharp image of the reconstructed object.

Assignment

In this problem we are going to calculate analytically the projections for a synthetic object. You will first calculate an analytic expression for a circular object. Subsequently you will backproject these projections onto the object. Start with programming the main backprojection algorithm for non filtered data and get that to work. In your backprojection step the normalized estimate for the object is

$$f(x_i, y_i) = \frac{\pi}{N_p} \sum_{p=1}^{N_p} p(\theta_p, n)_{x_i, y_i} \tag{5}$$

with x_i and y_i the coordinates in the to be reconstructed image and N_p the number of projections that are used in the reconstructions (you can start the reconstruction after acquisition of the first projection). Subsequently, implement the filtered backprojection by performing similar steps with the addition of filtering the projection in real space or the Fourier domain. Check that your reconstruction has the correct quantity in the image (i.e. ρ inside the cylinder and 0 outside). Modify the number of angles to see whether you obey the sampling criterion.