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Assignment-5

Roll No. : FWC22045

0.1 Problem Statement:

If the lines 2x+3y+1 =0 and 3x-y-4 = 0 lie along diameter of a circle of circumference 10π, then the equation of the circle is.

0.2 SOLUTION:

Given:

Two line equations are

$$\mathbf{n_1}^\top \mathbf{x} = c_1 \tag{1}$$
$$\mathbf{n_2}^\top \mathbf{x} = c_2 \tag{2}$$

Above two equations are diameters of the circle.

We know that the diameters intersect at the **centre** of the circle.
So solving those two equations, we get the centre of the circle.
Let $\mathbf{x}$ be the centre of the circle.

$$\mathbf{x} = \begin{pmatrix} \mathbf{n_1} & \mathbf{n_2} \end{pmatrix}^{-\top} \mathbf{c} \tag{3}$$

where,

$$\mathbf{n_1} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}, \mathbf{n_2} = \begin{pmatrix} 3 \\ -1 \end{pmatrix} \mathbf{c} = \begin{pmatrix} -1 \\ 4 \end{pmatrix} \tag{4}$$
$$\mathbf{x} = \begin{pmatrix} 2 & 3 \\ 3 & -1 \end{pmatrix}^{-\top} \begin{pmatrix} -1 \\ 4 \end{pmatrix} \tag{5}$$

To Find

We can find the centre of the circle by solving the above equation through finding the inverse

From the above equation we get the centre of the circle i.e,

$$\mathbf{x} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{6}$$

STEP-1

Given that the Circumference of the circle is 10π.

$$2\pi r = 10\pi \tag{7}$$
$$\tag{8}$$
$$r = 5 \tag{9}$$

STEP-2

The general equation of the circle is given by,

$$\mathbf{X}^\top \mathbf{V}\mathbf{X} + 2\mathbf{u}^\top \mathbf{X} + f = 0 \tag{10}$$

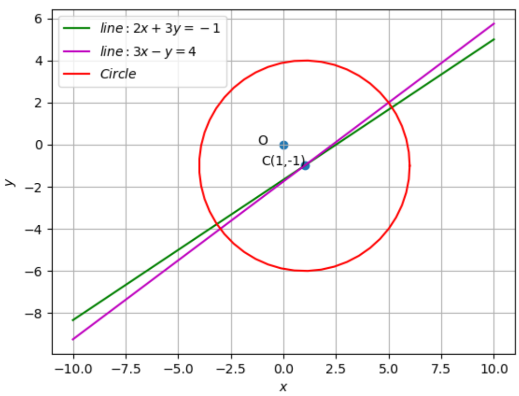
where,

$$f = \|\mathbf{u}\|^2 - r^2 \tag{11}$$
$$\mathbf{V} = \mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{12}$$
$$\mathbf{u} = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \tag{13}$$

Substituting all the values in the above equation, we get

$$\mathbf{X}^\top \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} + 2 \begin{pmatrix} -1 & 1 \end{pmatrix} \mathbf{X} - 23 = 0 \tag{14}$$

0.3 Construction



Download the code

Github link:https://github.com/Meghana9121/FWC.