PYTHON PROGRAMMING ON MATRICES

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FWC22045 IITH Future Wireless Communication (FWC) Matrix:Lines

1 Problem

Three distinct points A, B and C are given in the 2-dimensional coordinates plane such that the ratio of the distance of any one of them from the point (1, 0) to the distance from the point (-1, 0) is equal to 1:3 . Then the circumcentre of the triangle ABC is at the point:

2 Solution

$$\mathbf{P} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} ...(1)$$

$$\mathbf{Q} = \begin{pmatrix} -1 \\ 0 \end{pmatrix} ...(2)$$

Given that:
Distance between any vertex of a triangle to points P and Q is

$$\frac{\|\mathbf{A-P}\|}{\|\mathbf{A-Q}\|}=\frac{\|\mathbf{B-P}\|}{\|\mathbf{B-Q}\|}=\frac{\|\mathbf{C-P}\|}{\|\mathbf{C-Q}\|}=\frac{1}{3}...(3)$$

Circumcentre:

The circumcenter of a triangle is defined as the point where the perpendicular bisectors of the sides of that particular triangle intersect.
Circumcentre interms of vector is

$$\|\mathbf{A}-\mathbf{O}\|=\|\mathbf{B}-\mathbf{O}\|=\|\mathbf{C}-\mathbf{O}\| = r...(4)$$

$$\frac{\|\mathbf{A-P}\|^2}{\|\mathbf{A-Q}\|^2}=\frac{\|\mathbf{B-P}\|^2}{\|\mathbf{B-Q}\|^2}=\frac{\|\mathbf{C-P}\|^2}{\|\mathbf{C-Q}\|^2}=\frac{1^2}{3^2}...(5)$$

$$9(A-P)^T.(A-P) = (A-Q)^T.(A-Q)...(6)$$

$$9A^T.A - 18P^T.A + \|\mathbf{P}\|^2 = A^T.A - 2Q^T.A + \|\mathbf{Q}\|^2...(7)$$

$$8A^T.A - 2(9P-Q)^T.A + \|\mathbf{P}\|^2 - \|\mathbf{Q}\|^2 = 0...(8)$$

$$8.\|\mathbf{A}\|^2 + 2(Q-9P)^T.A + \|\mathbf{P}\|^2 - \|\mathbf{Q}\|^2 = 0...(9)$$

Similarly, we can do this for B and C

$$8.\|\mathbf{B}\|^2 + 2(Q-9P)^T.B + \|\mathbf{P}\|^2 - \|\mathbf{Q}\|^2 = 0...(10)$$

$$8.\|\mathbf{C}\|^2 + 2(Q-9P)^T.C + \|\mathbf{P}\|^2 - \|\mathbf{Q}\|^2 = 0...(11)$$

By squarring the circumcentre equation on both sides,we get

$$\left\|\mathbf{A}-\mathbf{O}^2\right\| = \left\|\mathbf{B}-\mathbf{O}^2\right\| = \left\|\mathbf{C}-\mathbf{O}^2\right\| = r^2...(12)$$

By expanding the terms,

$$\|\mathbf{A}\|^2 - 2(A)^T.O + \|\mathbf{O}\|^2 = r^2...(13)$$

$$\|\mathbf{B}\|^2 - 2(B)^T.O + \|\mathbf{O}\|^2 = r^2...(14)$$

$$\|\mathbf{C}\|^2 - 2(C)^T.O + \|\mathbf{O}\|^2 = r^2...(15)$$

Subtract the eqns (13),(14) and (14),(15) then we get

$$O^T.(A-B) = \frac{\|\mathbf{A}\|^2 - \|\mathbf{B}\|^2}{2}...(16)$$

$$O^T.(B-C) = \frac{\|\mathbf{B}\|^2 - \|\mathbf{C}\|^2}{2}...(17)$$

Subtract the eqns (9),(10) and (10),(11) then we get

$$\frac{1}{8}.[(Q-9P)^T.(A-B)] = \frac{\|\mathbf{A}\|^2 - \|\mathbf{B}\|^2}{2}...(18)$$

$$\frac{1}{8}.[(Q-9P)^T.(B-C)] = \frac{\|\mathbf{B}\|^2 - \|\mathbf{C}\|^2}{2}...(19)$$

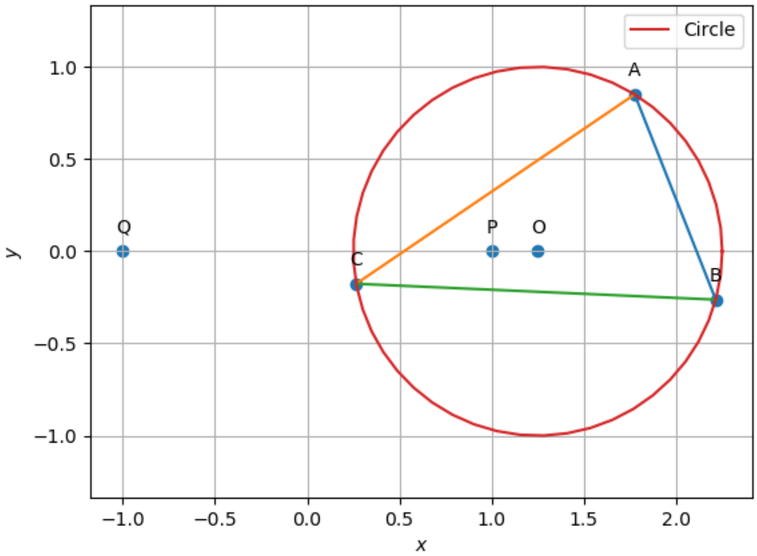
By comparing equations (16),(18) and (17),(19) we get

$$O = \frac{1}{8}.(9P-Q)...(20)$$

By substituting the P and Q values we get the Circumcentre,

$$\mathbf{O} = \begin{pmatrix} \frac{5}{4} \\ 0 \end{pmatrix}$$

3 Construction



4 Execution

*Verify the above proofs in the following code.

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https://github.com/pavan170850/Fwciith2022/blob/main/
    Matrix_Lines/codes/para.py
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