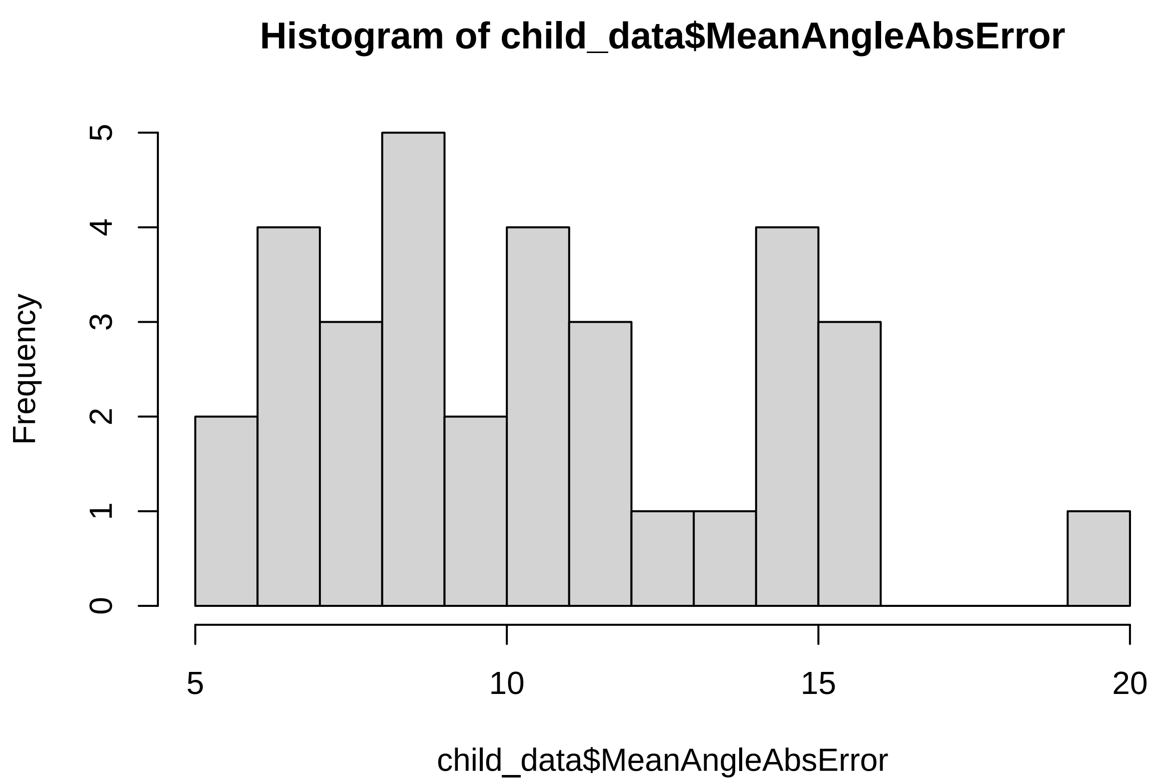
Project

Group 1

2023-12-06

Data Preprocessing

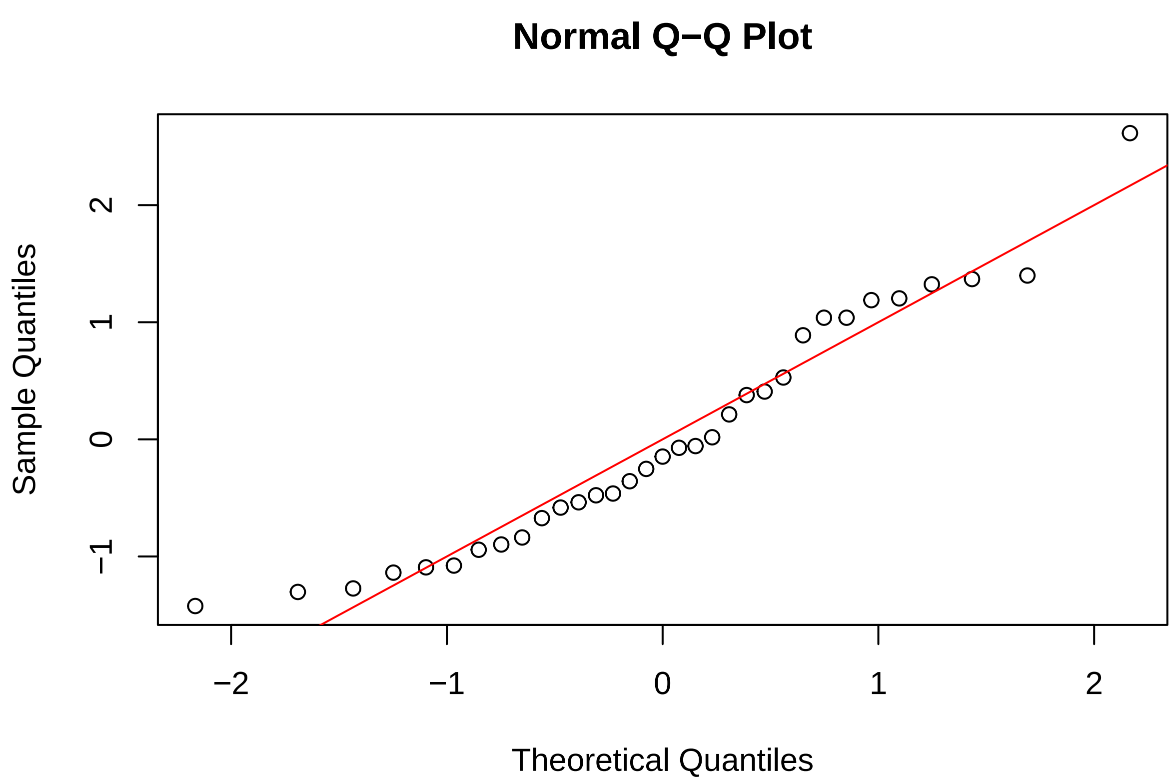
```
child_data =read.csv("child.csv")
#print(child_data)
hist(child_data$MeanAngleAbsError,breaks =20)
```




```
split_and_duplicate <- function(data) {
  values <- eval(parse(text = data$AngleAbsError))
  data_expanded <- data[rep(1:nrow(data), each = 20), ]
  data_expanded$AngleAbsError <- rep(values, length.out = nrow(data_expanded))
  data_expanded
}
child_data2 <- split_and_duplicate(child_data)
```

Data Summary

```
plot_qq<-function(x){
  xx = (x-mean(x))/sd(x)
  qqnorm(xx)
  lines(seq(-5,5,0.01),seq(-5,5,0.01),col="red")
}

plot_qq(child_data$MeanAngleAbsError)
```




```
Y = child_data$MeanAngleAbsError
Z = (Y-mean(Y))/sd(Y)
Z>3
```

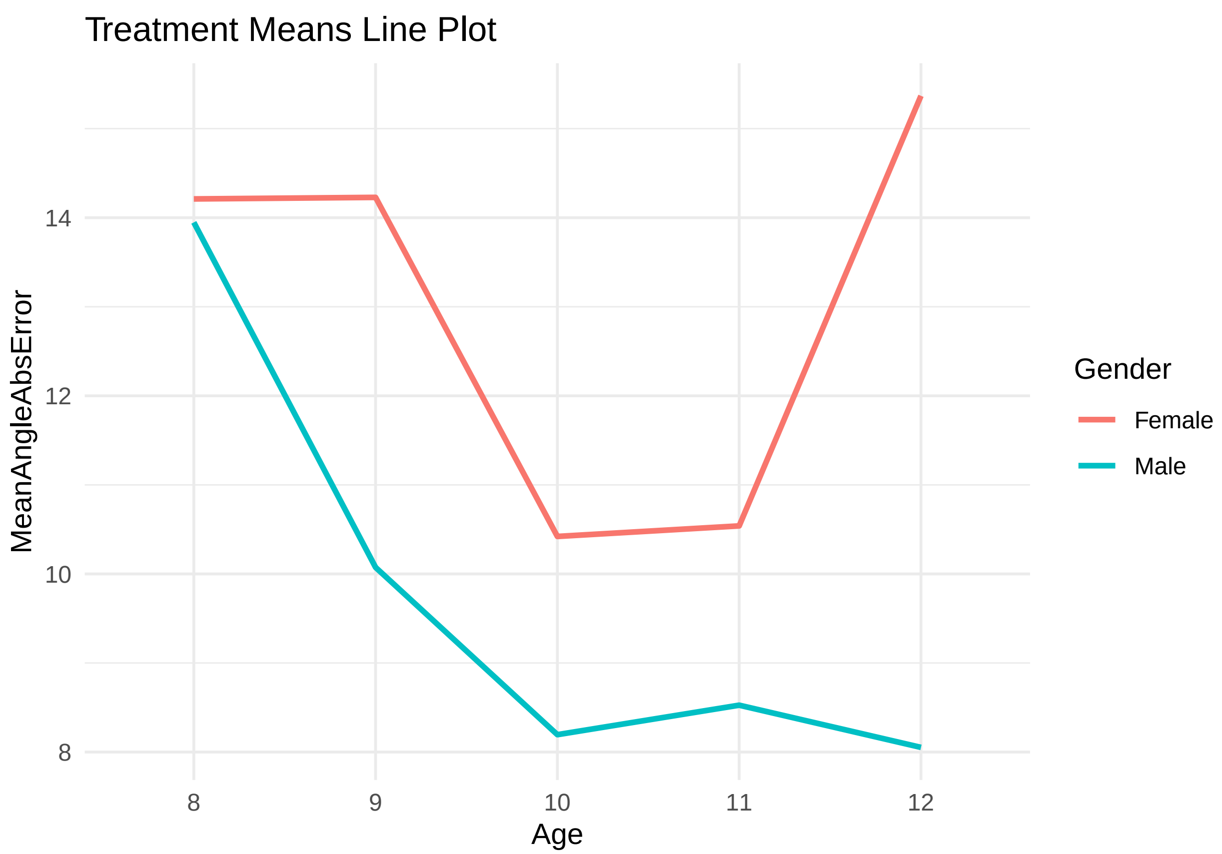
```
##  [1] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
## [13] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
## [25] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
```

```
# collinearity, and we delete grade.
cor(child_data$Age,child_data$Grade)
```

```
## [1] 0.9894427
```

Interpretation: The response variable seems normal. There are severe collinearity between age and grade, which is naturally reasonable. Therefore, we removed 'Grade' and kept 'Age' since this variable is more universal and can be extend to a whole population.

```
library(ggplot2)

ggplot(child_data, aes(x = factor(Age), y = MeanAngleAbsError, group =factor(Gender), color = factor(Ge
  geom_line(stat = "summary",fun="mean", size = 1) +
  labs(title = "Treatment Means Line Plot",
       x = "Age",
       y = "MeanAngleAbsError",
       color = "Gender") +
  theme_minimal()
```



Interpretation: From the visualization, we can observe that older subjects have lower mean angle absolute errors, and male tends to have lower errors. The interaction between 'Age' and 'Gender' is not obvious.

Model Fitting and Selection

1.The first fixed-effect model we fit is the full model, that is the two-factor fixed effect model with interaction. The formula is :

$$Y \sim 1 + factor(Age) + factor(Gender) + factor(Age) : factor(Gender)$$

There are five levels of factor 'Age': 8, 9, 10, 11, and 12 and two levels of factor 'Gender': Male and Female.

```r
c(sum(child_data$Age == 8 & child_data$Gender == 'Male'),
sum(child_data$Age == 9 & child_data$Gender == 'Male'),
sum(child_data$Age == 10 & child_data$Gender == 'Male'),
sum(child_data$Age == 11 & child_data$Gender == 'Male'),
sum(child_data$Age == 12 & child_data$Gender == 'Male'),
sum(child_data$Age == 8 & child_data$Gender == 'Female'),
sum(child_data$Age == 9 & child_data$Gender == 'Female'),
sum(child_data$Age == 10 & child_data$Gender == 'Female'),
sum(child_data$Age == 11 & child_data$Gender == 'Female'),
sum(child_data$Age == 12 & child_data$Gender == 'Female'))
```

```
##  [1] 3 5 7 1 3 1 3 5 4 1
```

It is obvious that this is an unbalanced design.

```r
# Fit the full model
mfixed.1 = lm(child_data$MeanAngleAbsError~factor(Age)*factor(Gender),data = child_data)
summary(mfixed.1)
```

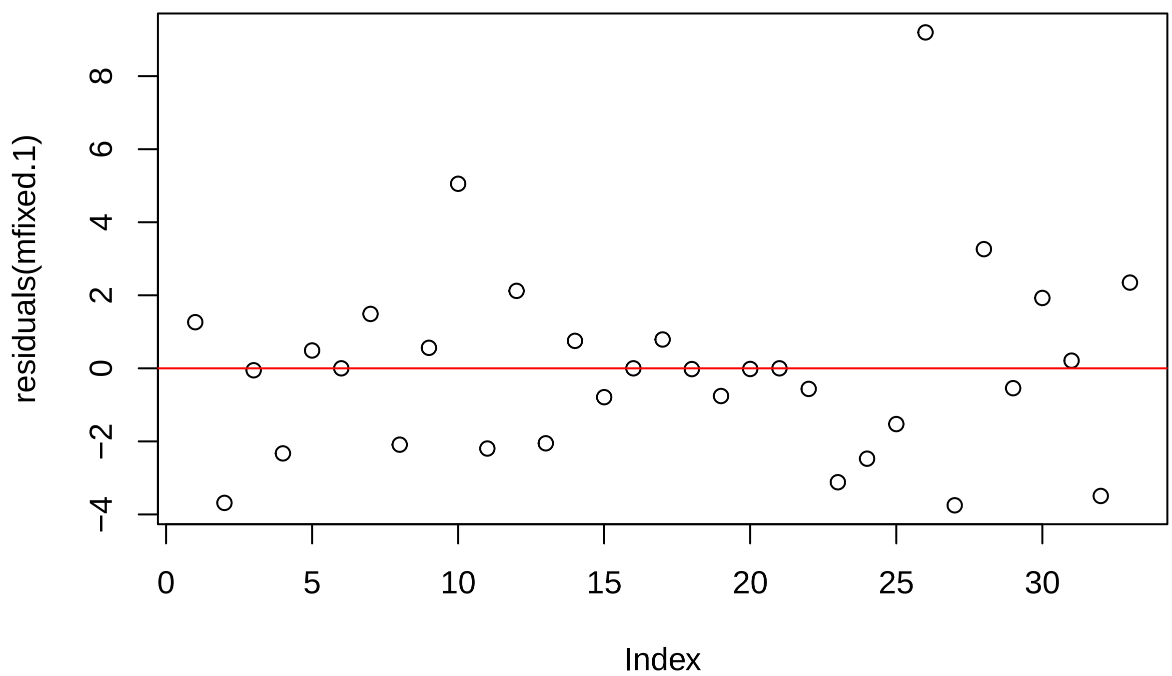
```
##
## Call:
## lm(formula = child_data$MeanAngleAbsError ~ factor(Age) * factor(Gender),
##     data = child_data)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -3.7500 -2.0526 -0.0175  0.7895  9.1974
##
## Coefficients:
##                                Estimate Std. Error t value Pr(>|t|)
## (Intercept)                    14.21053    3.08396   4.608 0.000124 ***
## factor(Age)9                    0.01754    3.56105   0.005 0.996112
## factor(Age)10                  -3.78947    3.37830  -1.122 0.273561
## factor(Age)11                  -3.67105    3.44797  -1.065 0.298062
## factor(Age)12                   1.15789    4.36137   0.265 0.792998
## factor(Gender)Male             -0.26316    3.56105  -0.074 0.941730
## factor(Age)9:factor(Gender)Male  -3.89123    4.21349  -0.924 0.365325
## factor(Age)10:factor(Gender)Male -1.96241    3.99273  -0.491 0.627734
## factor(Age)11:factor(Gender)Male -1.75000    4.95677  -0.353 0.727264
## factor(Age)12:factor(Gender)Male -7.05263    5.03608  -1.400 0.174734
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.084 on 23 degrees of freedom
## Multiple R-squared:  0.4439, Adjusted R-squared:  0.2264
## F-statistic:  2.04 on 9 and 23 DF,  p-value: 0.08098
```

```r
anova(mfixed.1)
```

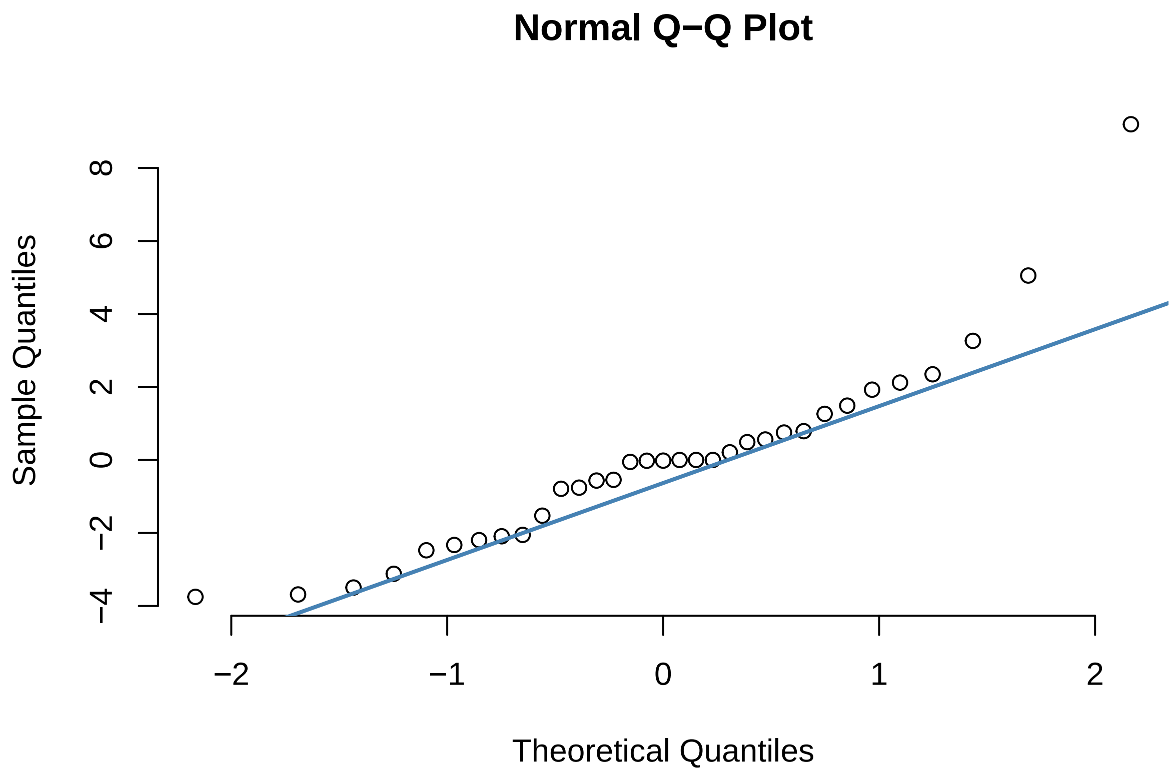
```
## Analysis of Variance Table
##
## Response: child_data$MeanAngleAbsError
```

```
##                            Df  Sum Sq Mean Sq F value  Pr(>F)
## factor(Age)                 4  84.403  21.101  2.2186 0.09856 .
## factor(Gender)              1  65.640  65.640  6.9017 0.01507 *
## factor(Age):factor(Gender)  4  24.602   6.150  0.6467 0.63487
## Residuals                  23 218.748   9.511
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
# do residual plot/ histogram/ qqplot to check the assumptions
plot(residuals(mfixed.1))
abline(h = 0, col="red")
```




```r
qqnorm(residuals(mfixed.1), pch = 1, frame = FALSE)
qqline(residuals(mfixed.1), col = "steelblue", lwd = 2)
```




```
# we found that the interaction term is not significant since p-value = 0.63>0.05,
```

Interpretation: By investigating the significance of the interaction effect, we found that the interaction term is not significant since the p-value of all interaction terms are greater than 0.05. And from the ANOVA table, we notice that the interaction term is not significant since the p-value $= 0.63487 > 0.05$. From the qqplot and residual plot, we found that the redisuals are not normally distributed. Therefore, we rerun the model without interaction to see if there is an improvement.

pairwise comparison between ages with results averaged over gender

```
library(emmeans)
model <- lm(MeanAngleAbsError ~ factor(Age) * factor(Gender), data = child_data)
# Create pairwise comparisons for Age with Tukey adjustment
comparisons <- emmeans(model, pairwise ~ factor(Age), adjust = "tukey")
```

```
## NOTE: Results may be misleading due to involvement in interactions
```

```
summary(comparisons)
```

```
## $emmeans
##  Age emmean    SE df lower.CL upper.CL
##    8  14.08 1.781 23    10.40     17.8
```

```
##     9  12.15 1.126 23     9.82     14.5
##    10   9.31 0.903 23     7.44     11.2
##    11   9.53 1.724 23     5.97     13.1
##    12  11.71 1.781 23     8.03     15.4
##
## Results are averaged over the levels of: Gender
## Confidence level used: 0.95
##
## $contrasts
##  contrast      estimate   SE df t.ratio p.value
##  Age8 - Age9      1.928 2.11 23   0.915  0.8881
##  Age8 - Age10     4.771 2.00 23   2.390  0.1540
##  Age8 - Age11     4.546 2.48 23   1.834  0.3792
##  Age8 - Age12     2.368 2.52 23   0.941  0.8780
##  Age9 - Age10     2.843 1.44 23   1.969  0.3115
##  Age9 - Age11     2.618 2.06 23   1.271  0.7106
##  Age9 - Age12     0.440 2.11 23   0.209  0.9995
##  Age10 - Age11   -0.225 1.95 23  -0.115  1.0000
##  Age10 - Age12   -2.402 2.00 23  -1.203  0.7495
##  Age11 - Age12   -2.178 2.48 23  -0.879  0.9018
##
## Results are averaged over the levels of: Gender
## P value adjustment: tukey method for comparing a family of 5 estimates
```

Interpretation: Pairwise comparisons with Tukey adjustment test whether the differences between age groups are statistically significant. In this case, none of the pairwise differences are statistically significant (all p-values > 0.05). The lack of significance suggests that there's no strong evidence of a difference in MeanAngleAbsError between different age groups, after accounting for the 'Gender' factor.

pairwise comparison between gender with results averaged over age

```r
library(emmeans)
model <- lm(MeanAngleAbsError ~ factor(Age) * factor(Gender), data = child_data)
# Create pairwise comparisons for Age with Tukey adjustment
comparisons <- emmeans(model, pairwise ~ factor(Gender), adjust = "tukey")
```

```
## NOTE: Results may be misleading due to involvement in interactions
```

```r
summary(comparisons)
```

```
## $emmeans
##  Gender emmean    SE df lower.CL upper.CL
##  Female  12.95 1.029 23    10.82     15.1
##  Male     9.76 0.874 23     7.95     11.6
##
## Results are averaged over the levels of: Age
## Confidence level used: 0.95
##
## $contrasts
##  contrast      estimate   SE df t.ratio p.value
##  Female - Male     3.19 1.35 23   2.366  0.0268
```

```
##
## Results are averaged over the levels of: Age
```

Interpretation: The pairwise comparison tests whether there is a statistically significant difference in MeanAngleAbsError between females and males. In this case, the p-value of 0.0268 is less than the conventional significance level of 0.05. Therefore, the result suggests that there is a statistically significant difference in MeanAngleAbsError between females and males. The positive estimate (3.19) indicates that, on average, females have higher MeanAngleAbsError compared to males.

2.The second model is the additive model of two fixed-effect factors.

The model formula is :

$$Y \sim 1 + factor(Age) + factor(Gender)$$

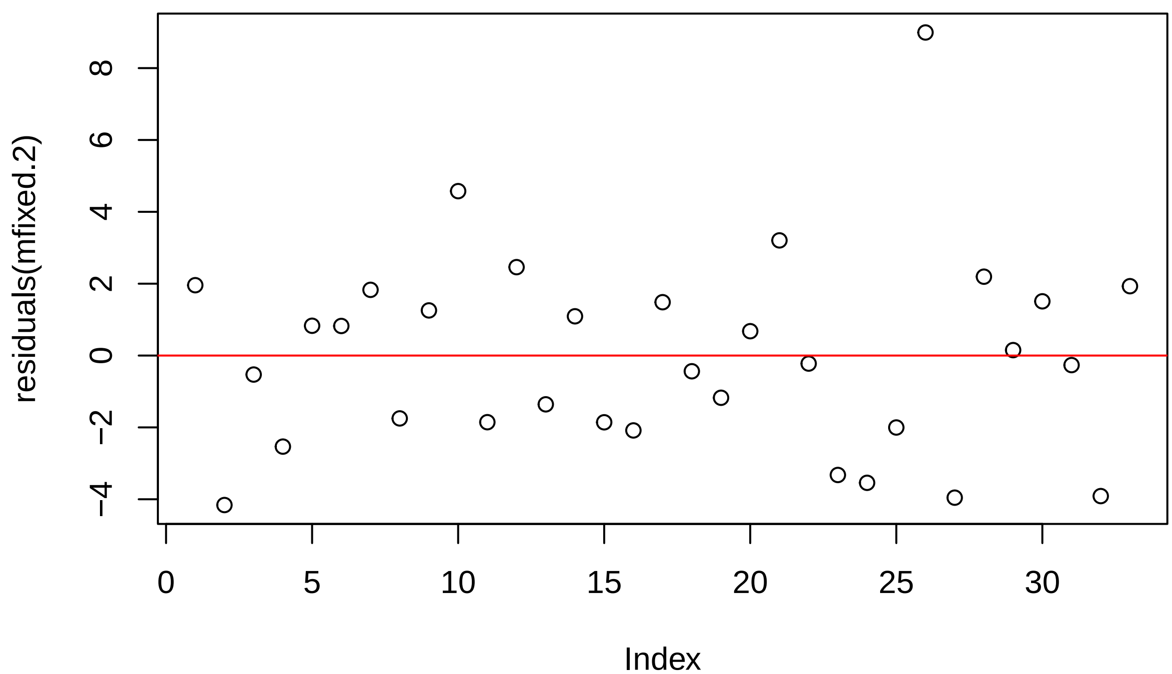
```
mfixed.2 = lm(child_data$MeanAngleAbsError~factor(Age)+factor(Gender),data = child_data)
summary(mfixed.2)
```

```
##
## Call:
## lm(formula = child_data$MeanAngleAbsError ~ factor(Age) + factor(Gender),
##     data = child_data)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -4.1607 -1.8578 -0.2236  1.5093  8.9915
##
## Coefficients:
##                    Estimate Std. Error t value Pr(>|t|)
## (Intercept)          16.295      1.723   9.458 4.63e-10 ***
## factor(Age)9         -2.762      1.844  -1.498  0.14576
## factor(Age)10        -5.397      1.743  -3.096  0.00454 **
## factor(Age)11        -5.550      2.107  -2.634  0.01381 *
## factor(Age)12        -4.132      2.123  -1.946  0.06210 .
## factor(Gender)Male   -3.042      1.127  -2.699  0.01186 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.002 on 27 degrees of freedom
## Multiple R-squared:  0.3814, Adjusted R-squared:  0.2669
## F-statistic:  3.33 on 5 and 27 DF,  p-value: 0.01803
```

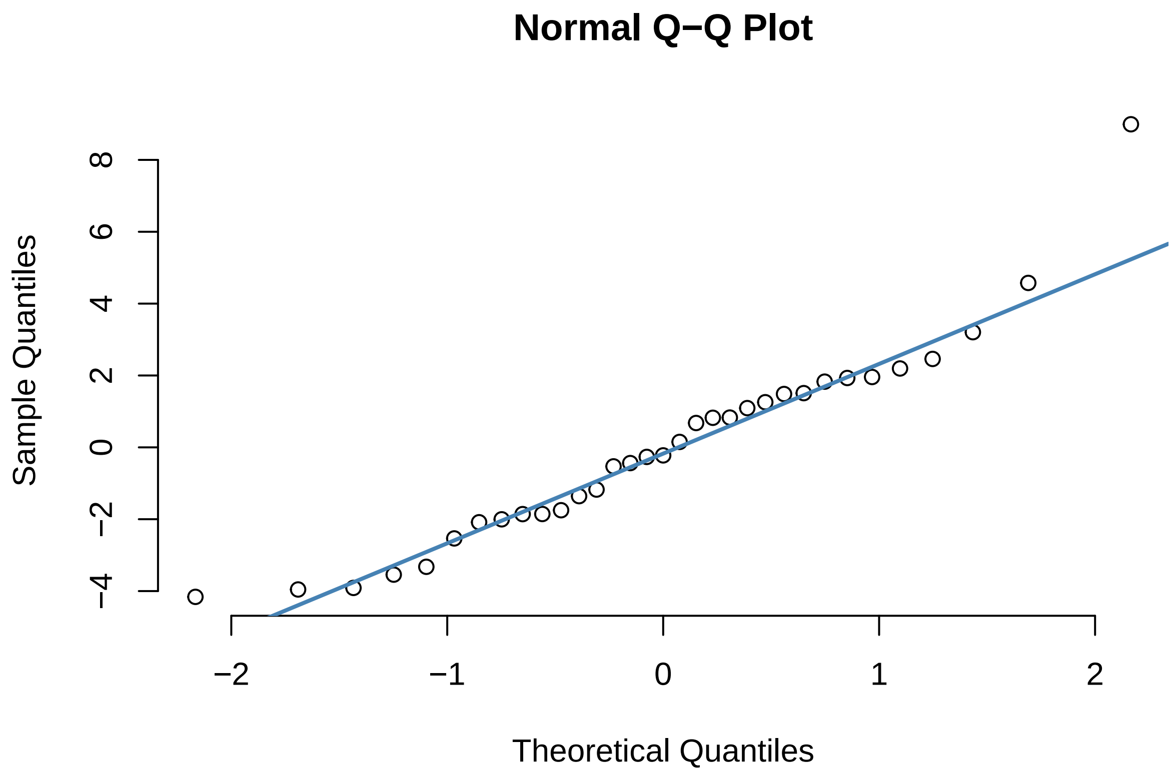
```
anova(mfixed.2)
```

```
## Analysis of Variance Table
##
## Response: child_data$MeanAngleAbsError
##                Df  Sum Sq Mean Sq F value  Pr(>F)
## factor(Age)     4  84.403  21.101  2.3412 0.08039 .
## factor(Gender)  1  65.640  65.640  7.2829 0.01186 *
## Residuals      27 243.350   9.013
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
# After pooling, the factors become significant.
# do residual plot/ histogram/ qqplot to check the assumptions
plot(residuals(mfixed.2))
abline(h = 0, col="red")
```




```r
qqnorm(residuals(mfixed.2), pch = 1, frame = FALSE)
qqline(residuals(mfixed.2), col = "steelblue", lwd = 2)
```



Interpretations: From the output, we found that both of the Age and Gender are significant. From the qqplot, we found that the residuals are approximately Normally distributed. From the residual plot, we found that the residual has zero mean and constant variance. Therefore, we keep this as the final fixed-effect model.

Moreover, the error of male subjects is lower than female subjects by 3.042, and subjects with age 11 has the lowest error among all ages.

pairwise comparison between ages with results averaged over gender

```
library(emmeans)
model <- lm(MeanAngleAbsError ~ factor(Age) + factor(Gender), data = child_data)
# Create pairwise comparisons for Age with Tukey adjustment
comparisons <- emmeans(model, pairwise ~ factor(Age), adjust = "tukey")
summary(comparisons)
```

```
## $emmeans
##  Age emmean    SE df lower.CL upper.CL
##    8  14.77 1.527 27    11.64     17.9
##    9  12.01 1.071 27     9.81     14.2
##   10   9.38 0.872 27     7.59     11.2
##   11   9.22 1.385 27     6.38     12.1
##   12  10.64 1.527 27     7.51     13.8
##
## Results are averaged over the levels of: Gender
```

```
## Confidence level used: 0.95
##
## $contrasts
##  contrast      estimate   SE df t.ratio p.value
##  Age8 - Age9      2.762 1.84 27   1.498  0.5726
##  Age8 - Age10     5.397 1.74 27   3.096  0.0337
##  Age8 - Age11     5.550 2.11 27   2.634  0.0921
##  Age8 - Age12     4.132 2.12 27   1.946  0.3185
##  Age9 - Age10     2.636 1.37 27   1.922  0.3303
##  Age9 - Age11     2.788 1.78 27   1.569  0.5292
##  Age9 - Age12     1.370 1.84 27   0.743  0.9443
##  Age10 - Age11    0.152 1.66 27   0.092  1.0000
##  Age10 - Age12   -1.266 1.74 27  -0.726  0.9486
##  Age11 - Age12   -1.418 2.11 27  -0.673  0.9606
##
## Results are averaged over the levels of: Gender
## P value adjustment: tukey method for comparing a family of 5 estimates
```

Interpretation: Age 8 has a significantly higher MeanAngleAbsError compared to Age 10 (p = 0.0337). No other pairwise comparisons reach statistical significance after adjusting for multiple testing using the Tukey method.

pairwise comparison between gender with results averaged over age

```r
library(emmeans)
model <- lm(MeanAngleAbsError ~ factor(Age) + factor(Gender), data = child_data)
# Create pairwise comparisons for Age with Tukey adjustment
comparisons <- emmeans(model, pairwise ~ factor(Gender), adjust = "tukey")
summary(comparisons)
```

```
## $emmeans
##  Gender emmean    SE df lower.CL upper.CL
##  Female  12.73 0.870 27    10.94     14.5
##  Male     9.68 0.742 27     8.16     11.2
##
## Results are averaged over the levels of: Age
## Confidence level used: 0.95
##
## $contrasts
##  contrast      estimate   SE df t.ratio p.value
##  Female - Male     3.04 1.13 27   2.699  0.0119
##
## Results are averaged over the levels of: Age
```

Interpretation: the p-value of 0.0119 is less than the conventional significance level of 0.05. Therefore, the result suggests that there is a statistically significant difference in MeanAngleAbsError between females and males. The positive estimate (3.04) indicates that, on average, females have higher MeanAngleAbsError compared to males.

3. The third model is the random effect model with interaction.

Since here the participant id variable can be viewed as samples from a large population, we can treat it as a random effect. Moreover, the cardinality of the sample space of Gender is 2 and that of age is 5, hence we can only treat it as a fixed effect.

We assume the random effect of factor is distributed as $N(0, \sigma^2_{age})$.

We fit the models using both MLE and REML methods.

```r
library(lme4)
```

```
## Loading required package: Matrix
```

```r
child_data2$Participant.IDF = factor(child_data2$Participant.ID)
child_data2$AgeF = factor(child_data2$Age)
child_data2$GenderF = factor(child_data2$Gender)
mrandom.1 = lmer(AngleAbsError ~ factor(Age) + factor(Gender) + (1|Participant.IDF)+ (1|Participant.IDF
```

```
## boundary (singular) fit: see help('isSingular')
```

```r
summary(mrandom.1)
```

```
## Linear mixed model fit by REML ['lmerMod']
## Formula:
## AngleAbsError ~ factor(Age) + factor(Gender) + (1 | Participant.IDF) +
##     (1 | Participant.IDF:AgeF) + (1 | Participant.IDF:GenderF)
##    Data: child_data2
##
## REML criterion at convergence: 5157.8
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -1.0521 -0.6258 -0.2194  0.2185  2.7175
##
## Random effects:
##  Groups                  Name        Variance Std.Dev.
##  Participant.IDF         (Intercept)   0.0     0.00
##  Participant.IDF:AgeF    (Intercept)   0.0     0.00
##  Participant.IDF:GenderF (Intercept)   0.0     0.00
##  Residual                            149.1    12.21
## Number of obs: 660, groups:
## Participant.IDF, 33; Participant.IDF:AgeF, 33; Participant.IDF:GenderF, 33
##
## Fixed effects:
##                    Estimate Std. Error t value
## (Intercept)         11.8186     1.5668   7.543
## factor(Age)9         0.5136     1.6769   0.306
## factor(Age)10        0.5514     1.5856   0.348
## factor(Age)11        0.7197     1.9164   0.376
## factor(Age)12        0.0625     1.9306   0.032
## factor(Gender)Male   0.3085     1.0253   0.301
##
## Correlation of Fixed Effects:
##             (Intr) fc(A)9 f(A)10 f(A)11 f(A)12
```

```
## factor(Ag)9 -0.747
## factr(Ag)10 -0.803  0.709
## factr(Ag)11 -0.765  0.602  0.645
## factr(Ag)12 -0.616  0.576  0.609  0.504
## fctr(Gndr)M -0.491  0.076  0.108  0.294  0.000
## optimizer (nloptwrap) convergence code: 0 (OK)
## boundary (singular) fit: see help('isSingular')
```

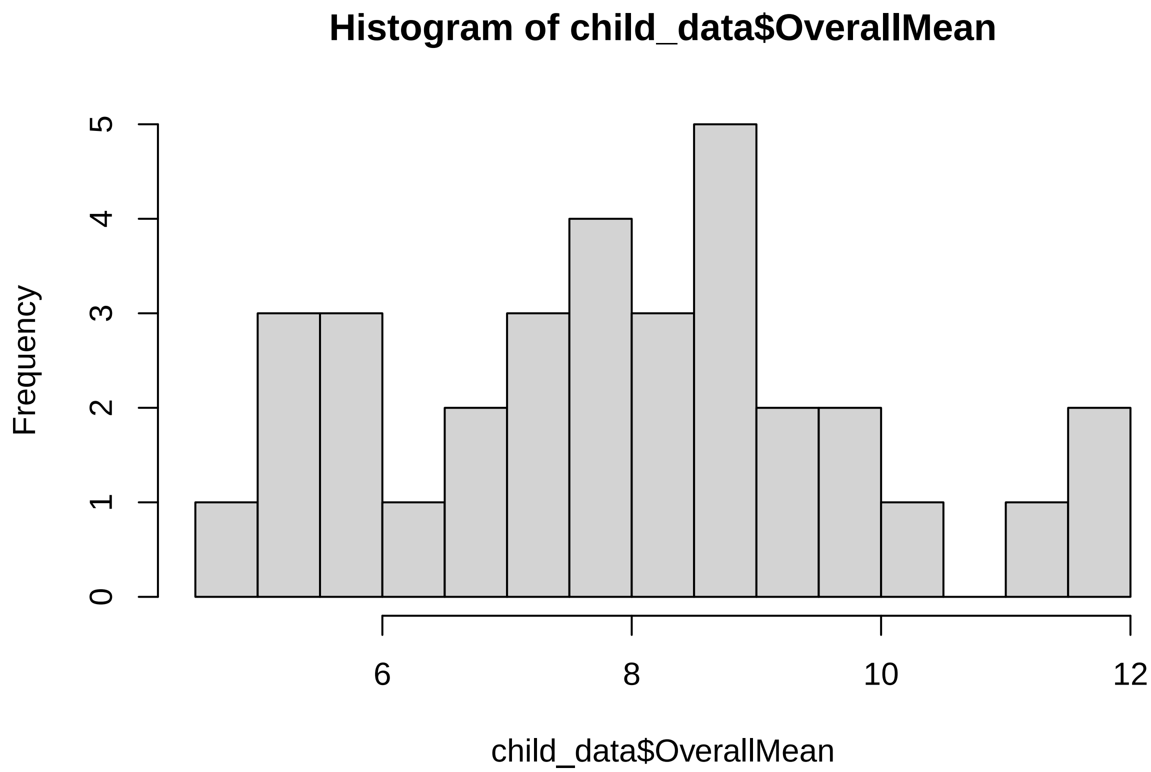
From the output, we can see that the random effects do not exist.

Change the response variable to the overall mean error and redo all the analysis

```r
library(dplyr)
```

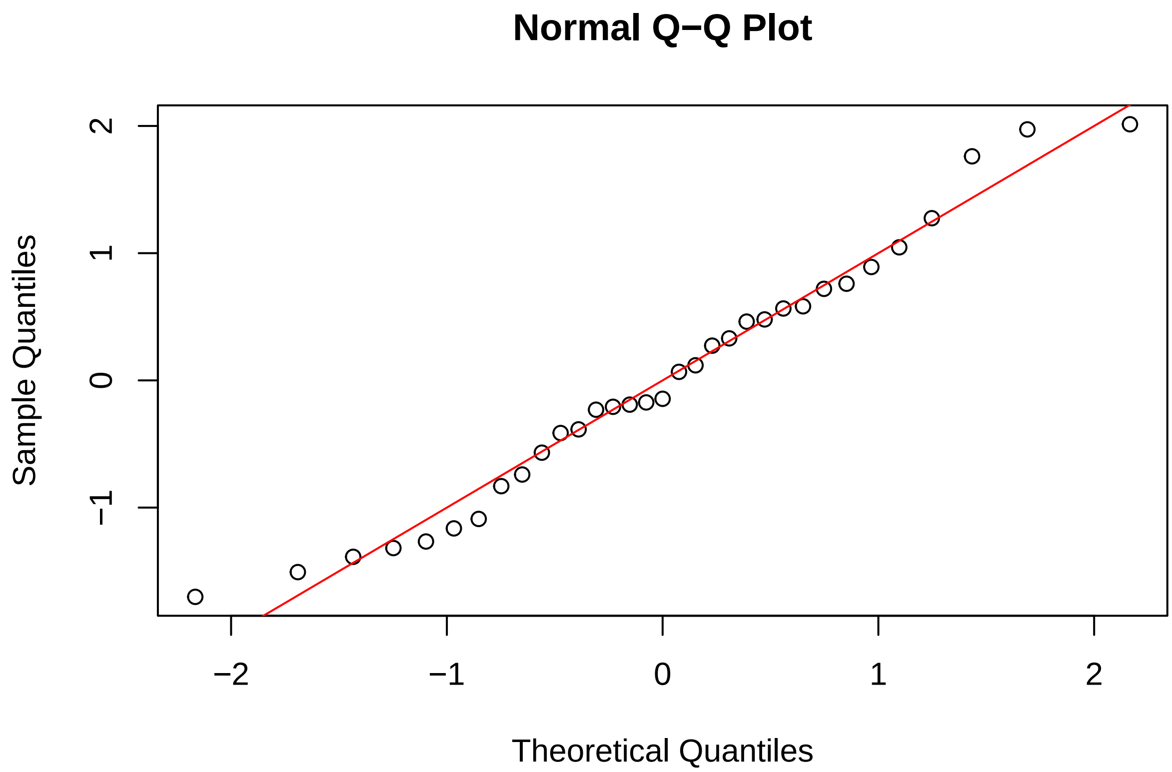
```
##
## Attaching package: 'dplyr'

## The following objects are masked from 'package:stats':
##
##     filter, lag

## The following objects are masked from 'package:base':
##
##     intersect, setdiff, setequal, union
```

```r
child_data <- child_data %>%
  mutate(OverallMean = rowMeans(select(., starts_with("Mean")), na.rm = TRUE))

#print(child_data)
hist(child_data$OverallMean,breaks =20)
```



Data Summary

```
plot_qq<-function(x){
  xx = (x-mean(x))/sd(x)
  qqnorm(xx)
  lines(seq(-5,5,0.01),seq(-5,5,0.01),col="red")
}

plot_qq(child_data$OverallMean)
```




```r
Y = child_data$OverallMean
Z = (Y-mean(Y))/sd(Y)
Z>3
```

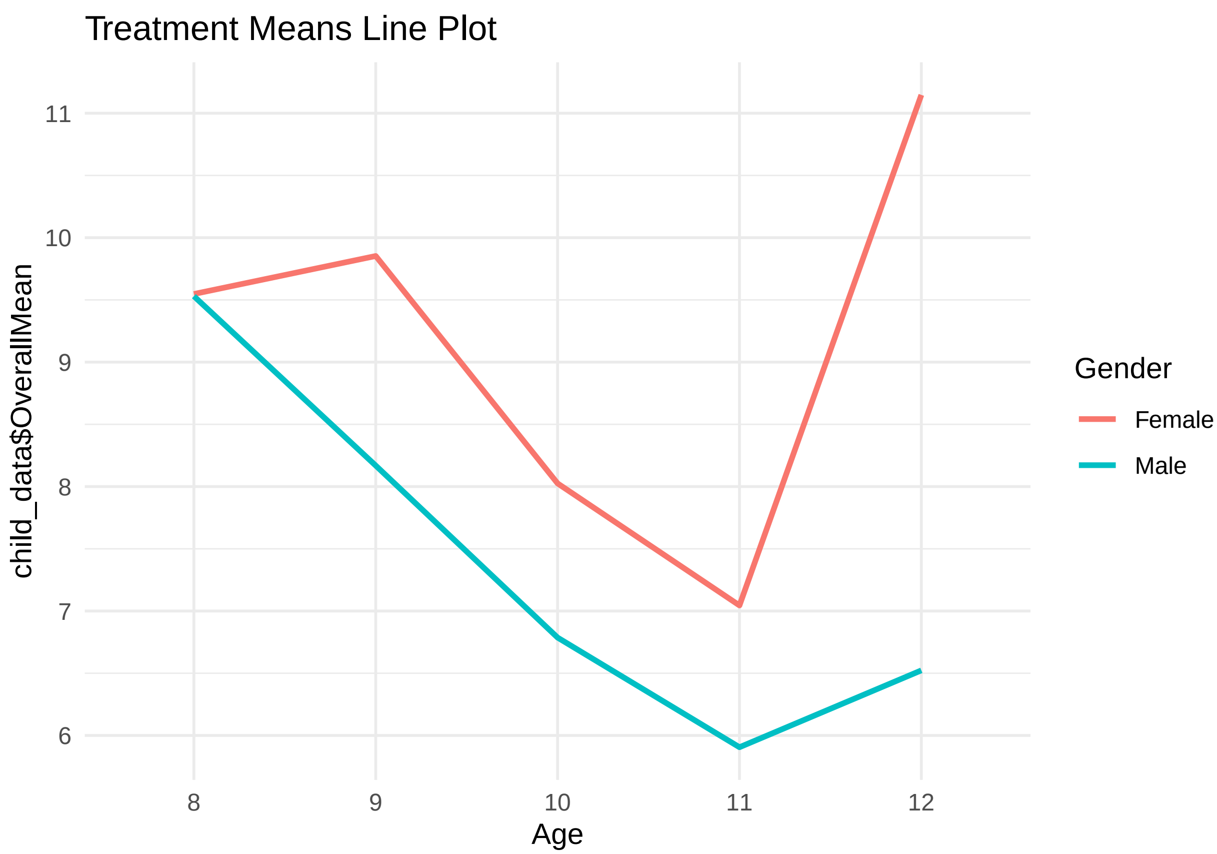
```
##  [1] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
## [13] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
## [25] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
```

```r
# collinearity, and we delete grade.
cor(child_data$Age,child_data$Grade)
```

```
## [1] 0.9894427
```

visualization

```r
library(ggplot2)

ggplot(child_data, aes(x = factor(Age), y = child_data$OverallMean, group =factor(Gender), color = facto
  geom_line(stat = "summary",fun="mean", size = 1) +
  labs(title = "Treatment Means Line Plot",
       x = "Age",
       y = "child_data$OverallMean",
       color = "Gender") +
  theme_minimal()
```




```r
library(emmeans)
model <- lm(child_data$OverallMean ~ factor(Age) * factor(Gender), data = child_data)
# Create pairwise comparisons for Age with Tukey adjustment
anova(model)
```

```
## Analysis of Variance Table
##
## Response: child_data$OverallMean
##                            Df Sum Sq Mean Sq F value  Pr(>F)
## factor(Age)                 4 27.504  6.8760  2.9376 0.04246 *
## factor(Gender)              1 17.576 17.5755  7.5087 0.01166 *
## factor(Age):factor(Gender)  4  9.298  2.3245  0.9931 0.43118
## Residuals                  23 53.836  2.3407
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
comparisons <- emmeans(model, pairwise ~ factor(Age), adjust = "tukey")
```

```
## NOTE: Results may be misleading due to involvement in interactions
```

```r
summary(comparisons)
```

```
## $emmeans
##  Age emmean    SE df lower.CL upper.CL
```

```
##    8   9.54 0.883 23     7.71    11.37
##    9   9.01 0.559 23     7.85    10.17
##   10   7.41 0.448 23     6.48     8.33
##   11   6.47 0.855 23     4.71     8.24
##   12   8.84 0.883 23     7.01    10.66
##
## Results are averaged over the levels of: Gender
## Confidence level used: 0.95
##
## $contrasts
##  contrast      estimate    SE df t.ratio p.value
##  Age8 - Age9      0.528 1.045 23   0.505  0.9860
##  Age8 - Age10     2.133 0.990 23   2.153  0.2326
##  Age8 - Age11     3.064 1.230 23   2.492  0.1274
##  Age8 - Age12     0.704 1.249 23   0.563  0.9791
##  Age9 - Age10     1.605 0.716 23   2.241  0.2005
##  Age9 - Age11     2.536 1.022 23   2.482  0.1297
##  Age9 - Age12     0.175 1.045 23   0.168  0.9998
##  Age10 - Age11    0.931 0.965 23   0.964  0.8682
##  Age10 - Age12   -1.429 0.990 23  -1.443  0.6075
##  Age11 - Age12   -2.360 1.230 23  -1.920  0.3356
##
## Results are averaged over the levels of: Gender
## P value adjustment: tukey method for comparing a family of 5 estimates
```

Interpretation: Pairwise comparisons with Tukey adjustment test whether the differences between age groups are statistically significant. In this case, none of the pairwise differences are statistically significant (all p-values > 0.05). The lack of significance suggests that there's no strong evidence of a difference in OverallMean between different age groups, after accounting for the 'Gender' factor.

pairwise comparison between gender with results averaged over age

```r
library(emmeans)
model <- lm(child_data$OverallMean ~ factor(Age) * factor(Gender), data = child_data)
# Create pairwise comparisons for Age with Tukey adjustment
comparisons <- emmeans(model, pairwise ~ factor(Gender), adjust = "tukey")
```

```
## NOTE: Results may be misleading due to involvement in interactions
```

```r
summary(comparisons)
```

```
## $emmeans
##  Gender emmean    SE df lower.CL upper.CL
##  Female   9.12 0.510 23     8.07    10.18
##  Male     7.38 0.434 23     6.49     8.28
##
## Results are averaged over the levels of: Age
## Confidence level used: 0.95
##
## $contrasts
##  contrast      estimate   SE df t.ratio p.value
```

```
##  Female - Male     1.74 0.67 23   2.599  0.0161
##
## Results are averaged over the levels of: Age
```

Interpretation: The pairwise comparison tests whether there is a statistically significant difference in OverallMean between females and males. In this case, the p-value of 0.0161 is less than the conventional significance level of 0.05. Therefore, the result suggests that there is a statistically significant difference in OverallMean between females and males. The positive estimate (1.74) indicates that, on average, females have higher OverallMean compared to males.

2.The second model is the additive model of two fixed-effect factors.

The model formula is :

$$Y \sim 1 + factor(Age) + factor(Gender)$$

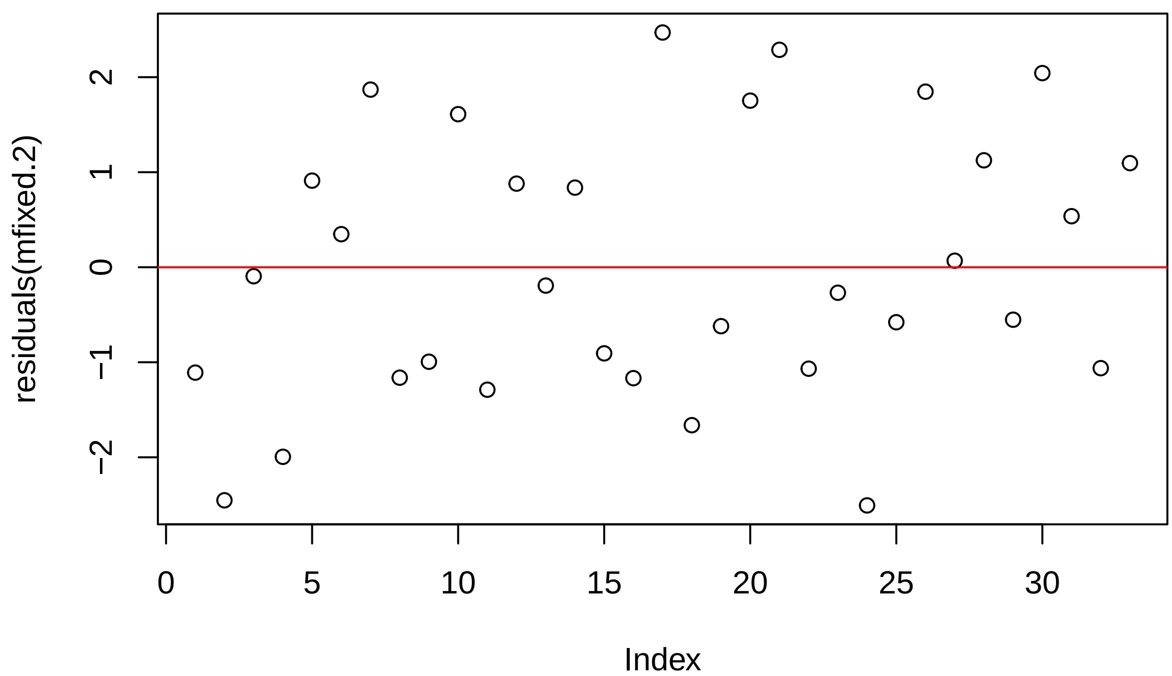
```r
mfixed.2 = lm(child_data$OverallMean~factor(Age)+factor(Gender),data = child_data)
summary(mfixed.2)
```

```
##
## Call:
## lm(formula = child_data$OverallMean ~ factor(Age) + factor(Gender),
##     data = child_data)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -2.5064 -1.0677 -0.1933  1.0956  2.4699
##
## Coefficients:
##                      Estimate Std. Error t value Pr(>|t|)
## (Intercept)           10.7149     0.8775  12.210 1.67e-12 ***
## factor(Age)9          -0.9310     0.9392  -0.991  0.33033
## factor(Age)10         -2.4940     0.8880  -2.808  0.00914 **
## factor(Age)11         -3.5832     1.0733  -3.339  0.00247 **
## factor(Age)12         -1.8553     1.0813  -1.716  0.09765 .
## factor(Gender)Male    -1.5743     0.5742  -2.742  0.01071 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.529 on 27 degrees of freedom
## Multiple R-squared:  0.4166, Adjusted R-squared:  0.3085
## F-statistic: 3.856 on 5 and 27 DF,  p-value: 0.009137
```

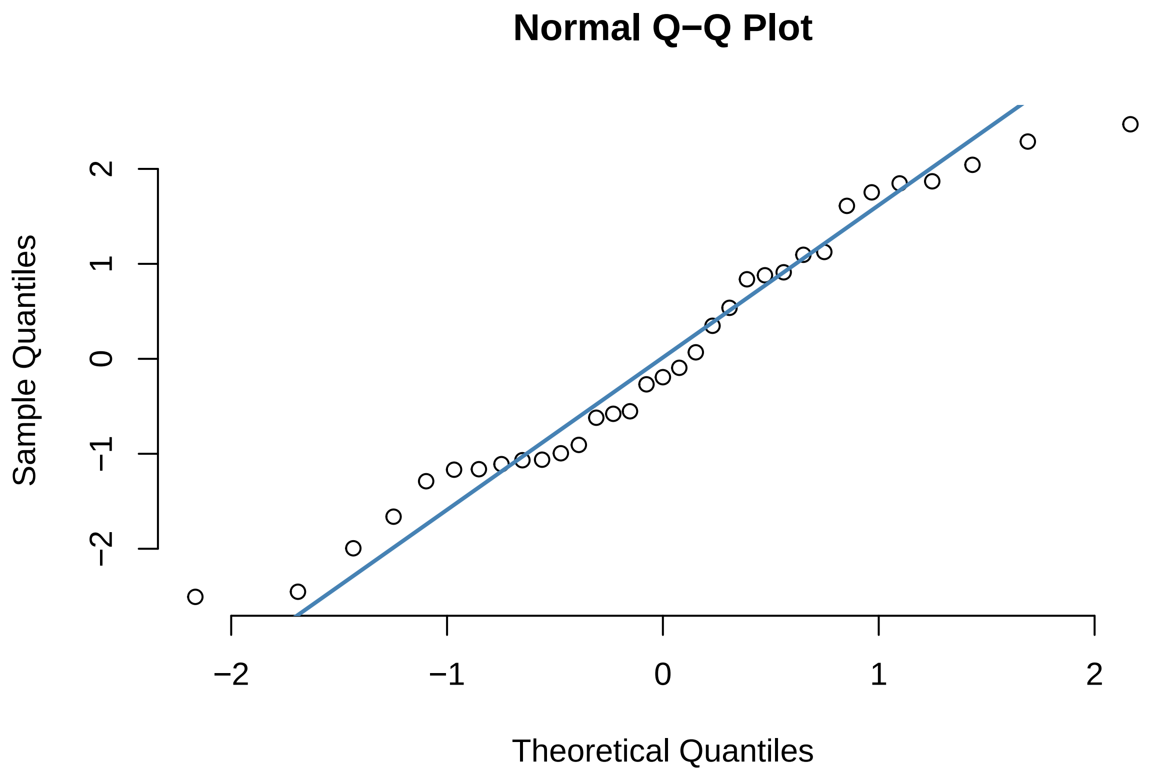
```r
anova(mfixed.2)
```

```
## Analysis of Variance Table
##
## Response: child_data$OverallMean
##                Df Sum Sq Mean Sq F value  Pr(>F)
## factor(Age)     4 27.504  6.8760  2.9406 0.03865 *
## factor(Gender)  1 17.576 17.5755  7.5164 0.01071 *
## Residuals      27 63.134  2.3383
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
# After pooling, the factors become significant.
# do residual plot/ histogram/ qqplot to check the assumptions
plot(residuals(mfixed.2))
abline(h = 0, col="red")
```




```r
qqnorm(residuals(mfixed.2), pch = 1, frame = FALSE)
qqline(residuals(mfixed.2), col = "steelblue", lwd = 2)
```



Interpretations: From the output, we found that both of the Age and Gender are significant. From the qqplot, we found that the residuals are approximately Normally distributed. From the residual plot, we found that the residual has zero mean and constant variance. Therefore, we keep this as the final fixed-effect model.

Moreover, the error of male subjects is lower than female subjects by 1.5743, and subjects with age 11 has the lowest error among all ages.

pairwise comparison between ages with results averaged over gender

```r
library(emmeans)
model <- lm(child_data$OverallMean ~ factor(Age) + factor(Gender), data = child_data)
# Create pairwise comparisons for Age with Tukey adjustment
comparisons <- emmeans(model, pairwise ~ factor(Age), adjust = "tukey")
summary(comparisons)
```

```
## $emmeans
##  Age emmean    SE df lower.CL upper.CL
##    8   9.93 0.778 27     8.33    11.52
##    9   9.00 0.545 27     7.88    10.12
##   10   7.43 0.444 27     6.52     8.34
##   11   6.34 0.705 27     4.90     7.79
##   12   8.07 0.778 27     6.48     9.67
##
## Results are averaged over the levels of: Gender
```

```
## Confidence level used: 0.95
##
## $contrasts
##  contrast      estimate    SE df t.ratio p.value
##  Age8 - Age9      0.931 0.939 27   0.991  0.8569
##  Age8 - Age10     2.494 0.888 27   2.808  0.0638
##  Age8 - Age11     3.583 1.073 27   3.339  0.0192
##  Age8 - Age12     1.855 1.081 27   1.716  0.4416
##  Age9 - Age10     1.563 0.698 27   2.238  0.1966
##  Age9 - Age11     2.652 0.905 27   2.930  0.0490
##  Age9 - Age12     0.924 0.939 27   0.984  0.8601
##  Age10 - Age11    1.089 0.843 27   1.292  0.6983
##  Age10 - Age12   -0.639 0.888 27  -0.719  0.9502
##  Age11 - Age12   -1.728 1.073 27  -1.610  0.5041
##
## Results are averaged over the levels of: Gender
## P value adjustment: tukey method for comparing a family of 5 estimates
```

Interpretation: Age 8 has a significantly higher OverallMean compared to Age 11 ($p = 0.0192$). No other pairwise comparisons reach statistical significance after adjusting for multiple testing using the Tukey method.

pairwise comparison between gender with results averaged over age

```r
library(emmeans)
model <- lm(child_data$OverallMean ~ factor(Age) + factor(Gender), data = child_data)
# Create pairwise comparisons for Age with Tukey adjustment
comparisons <- emmeans(model, pairwise ~ factor(Gender), adjust = "tukey")
summary(comparisons)
```

```
## $emmeans
##  Gender emmean    SE df lower.CL upper.CL
##  Female   8.94 0.443 27     8.03     9.85
##  Male     7.37 0.378 27     6.59     8.14
##
## Results are averaged over the levels of: Age
## Confidence level used: 0.95
##
## $contrasts
##  contrast      estimate    SE df t.ratio p.value
##  Female - Male     1.57 0.574 27   2.742  0.0107
##
## Results are averaged over the levels of: Age
```

Interpretation: the p-value of 0.0107 is less than the conventional significance level of 0.05. Therefore, the result suggests that there is a statistically significant difference in OverallMean between females and males. The positive estimate (1.57) indicates that, on average, females have higher OverallMean compared to males.