Topics: Normal distribution, Functions of Random Variables

1. The time required for servicing transmissions is normally distributed with μ = 45 minutes and σ = 8 minutes. The service manager plans to have work begin on the transmission of a customer's car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
   A. 0.3875
   B. 0.2676
   C. 0.5
   D. 0.6987

Ans:- The customer is told that the car will be ready within 1 hour from drop-off = **60 minutes**
The service manager plans to have work begin on the transmission of a customer's car **10 minutes** after the car is dropped off.

μ = 45 minutes
σ = 8 minutes
X = The time left to complete work is **50 minutes**
The probability that the service manager cannot meet his commitment =
P(X>50) = 1 – (X<=50) , Convert 50 to Z-Score
Z = (X - μ)/ σ = *(50 – 45)/ 8*
P(X<=50) = P(Z<=(50 – 45)/ 8) = P(Z<=0.625) = 0.7340
The probability that the service manager cannot meet his commitment =
1 – (X<=50) = 1 - 0.7340 = **0.266**

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2. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean μ = 38 and Standard deviation σ=6. For each statement below, please specify True/False. If false, briefly explain why.
   A. More employees at the processing center are older than 44 than between 38 and 44.

Ans:- We have normal distributed with mean **μ = 38** and Standard deviation **σ=6**

a) Probability of employee greater than age 44 = P(X>44)
P(X>44) = 1 – (X<=44)
Z = (X - μ)/ σ = *(44 – 38)/ 6*
P(X<=44) = P(Z<=(44 – 38)/ 6) = P(Z <= 1) = 0.8413
Probability of employee greater than age 44 = 1 – 0.8413 = 0.1587
So the probability of number of employees between 38-44 years of age =
P(X<44)-0.5=0.8413-0.5= 0.341345 = 34.1345%
Therefore the statement that "More employees at the processing center are older than 44 than between 38 and 44" is TRUE

B. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans:- B) Probability of employee less than age 30 = P(X<30)

P(X<30) = 1 – (X>=30)

Z = (X - μ)/ σ = (30 – 38)/ 6

P(X>=30) = P(Z>=(30 – 38)/ 6) = P(Z >= -1.3333) = 0.0918

Probability of employee greater than age 44 = 1 – 0.8413 = 0.1587

So the number of employees with probability 0.912 of them being under age 30 = 0.0912*400=36.48(36 employees).

Therefore the statement B of the question is also TRUE.

3. If $X_1 \sim N(\mu, \sigma^2)$ and $X_2 \sim N(\mu, \sigma^2)$ are *iid* normal random variables, then what is the difference between 2 X_1 and $X_1 + X_2$? Discuss both their distributions and parameters.

Ans:- As we know that if X ~ N(μ1, σ1^2), and Y ~ N(μ2, σ2^2) are two independent random variables then X + Y ~ N(μ1 + μ2, σ1^2 + σ2^2) , and X – Y ~ N(μ1 – μ2, σ1^2 + σ2^2)

Similarly if Z = aX + bY , where X and Y are as defined above, i.e Z is linear combination of X and Y , then Z ~ N(aμ1 + bμ2, a^2σ1^2 + b^2σ2^2).

Therefore in the question

2X1~ N(2 u,4 σ^2) and

X1+X2 ~ N(μ + μ, σ^2 + σ^2) ~ N(2 u, 2σ^2)

2X1-(X1+X2) = N(4μ,6 σ^2)

4. Let X ~ N(100, 20^2). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

A. 90.5, 105.9
B. 80.2, 119.8
C. 22, 78
D. 48.5, 151.5
E. 90.1, 109.9

Ans:- Since we need to find out the values of a and b, which are symmetric about the mean, such that the probability of random variable taking a value between them is 0.99, we have to work out in reverse order.

The Probability of getting value between a and b should be 0.99.

So the Probability of going wrong, or the Probability outside the a and b area is 0.01 (ie. 1-0.99).

The Probability towards left from a = -0.005 (ie. 0.01/2).

The Probability towards right from b = +0.005 (ie. 0.01/2).

So since we have the probabilities of a and b, we need to calculate X, the random variable at a and b which has got these probabilities.

By finding the Standard Normal Variable Z (Z Value), we can calculate the X values.

Z=(X- μ) / σ

For Probability 0.005 the Z Value is -2.57 (from Z Table).
Z * σ + μ = X
Z(-0.005)*20+100 = -(-2.57)*20+100 = 151.4
Z(+0.005)*20+100 = (-2.57)*20+100 = 48.6
So, option D is correct.

5. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions $Profit_1 \sim N(5, 3^2)$ and $Profit_2 \sim N(7, 4^2)$ respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
    A. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
    B. Specify the 5^{th} percentile of profit (in Rupees) for the company
    C. Which of the two divisions has a larger probability of making a loss in a given year?

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Ans:-

- **A.** Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company. => **Range is Rs (99.00810347848784, 980.9918965215122) in Millions**
- B. Specify the 5^{th} percentile of profit (in Rupees) for the company => **5th percentile of profit (in Million Rupees) is 170.0**
- C. Which of the two divisions has a larger probability of making a loss in a given year? **0.040059156863817086**