**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. **0.2676**
4. 0.5
5. 0.6987

Ans:- The customer is told that the car will be ready within 1 hour from drop-off = **60 minutes**

The service manager plans to have work begin on the transmission of a customer’s car **10 minutes** after the car is dropped off.

*μ* = 45 minutes

*σ* = 8 minutes

X = The time left to complete work is **50 minutes**

The probability that the service manager cannot meet his commitment =

P(X>50) = 1 – (X<=50) , Convert 50 to Z-Score

Z = (X - *μ* )/ *σ = (50 – 45)/ 8*

P(X<=50) = P(Z<=(50 – 45)/ 8) = P(Z<=0.625) = 0.7340

The probability that the service manager cannot meet his commitment =

1 – (X<=50) = 1 - 0.7340 = **0.266**

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

Ans:- We have normal distributed with mean ***μ* = 38** and Standard deviation ***σ* =6**

1. Probability of employee greater than age 44 = P(X>44)

P(X>44) = 1 – (X<=44)

Z = (X - μ )/σ *= (44 – 38)/ 6*

P(X<=44) = P(Z<=(44 – 38)/ 6) = P(Z <= 1) = 0.8413

Probability of employee greater than age 44 = 1 – 0.8413 = 0.1587

So the probability of number of employees between 38-44 years of age =

P(X<44)-0.5=0.8413-0.5= 0.341345 = 34.1345%

Therefore the statement that “More employees at the processing center are older than 44 than between 38 and 44” is TRUE

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans:- B) Probability of employee less than age 30 = P(X<30)

P(X<30) = 1 – (X>=30)

Z = (X - μ )/σ= (30 – 38)/ 6

P(X>=30) = P(Z>=(30 – 38)/ 6) = P(Z >= -1.3333) = 0.0918

Probability of employee greater than age 44 = 1 – 0.8413 = 0.1587

So the number of employees with probability 0.912 of them being under age 30 = 0.0912\*400=36.48(36 employees).

Therefore the statement B of the question is also TRUE.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans:- As we know that if X ∼ N(µ1, σ1^2 ), and Y ∼ N(µ2, σ2^2 ) are two independent random variables then X + Y ∼ N(µ1 + µ2, σ1^2 + σ2^2 ) , and X − Y ∼ N(µ1 − µ2, σ1^2 + σ2^2 )

Similarly if Z = aX + bY , where X and Y are as defined above, i.e Z is linear combination of X and Y , then Z ∼ N(aµ1 + bµ2, a^2σ1^2 + b^2σ2^2 ).

Therefore in the question

2X1~ N(2 u,4 σ^2) and

X1+X2 ~ N(µ + µ, σ^2 + σ^2 ) ~ N(2 u, 2σ^2 )

2X1-(X1+X2) = N( 4µ,6 σ^2)

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans:- Since we need to find out the values of a and b, which are symmetric about the mean, such that the probability of random variable taking a value between them is 0.99, we have to work out in reverse order.

The Probability of getting value between a and b should be 0.99.

So the Probability of going wrong, or the Probability outside the a and b area is 0.01 (ie. 1-0.99).

The Probability towards left from a = -0.005 (ie. 0.01/2).

The Probability towards right from b = +0.005 (ie. 0.01/2).

So since we have the probabilities of a and b, we need to calculate X, the random variable at a and b which has got these probabilities.

By finding the Standard Normal Variable Z (Z Value), we can calculate the X values.

Z=(X- μ) / σ

For Probability 0.005 the Z Value is -2.57 (from Z Table).

Z \* σ + μ = X

Z(-0.005)\*20+100 = -(-2.57)\*20+100 = 151.4

Z(+0.005)\*20+100 = (-2.57)\*20+100 = 48.6

So, option D is correct.

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

Ans:- 

1. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company. => **Range is Rs (99.00810347848784, 980.9918965215122) in Millions**
2. Specify the 5th percentile of profit (in Rupees) for the company =>

**5th percentile of profit (in Million Rupees) is 170.0**

1. Which of the two divisions has a larger probability of making a loss in a given year? **0.040059156863817086**