Signal Processing

Lab 5

Name: Meghana Tedla

Roll No: 2021102002

1.

1. (a) $p[n] = \cos\left(\frac{2\pi f_0 n}{f_s}\right)$

$$p[n] = \frac{e^{j\frac{2\pi f_0 n}{f_s}} + e^{-j\frac{2\pi f_0 n}{f_s}}}{2}$$

DTFT

$$P(e^{j\omega}) = \sum_{n=-\infty}^{\infty} p[n]\, e^{-j\omega n}$$

$$P(e^{j\omega}) = \sum_{l=-\infty}^{\infty} \frac{2\pi}{2}\,\delta\left(\omega - \frac{2\pi f_0}{f_s} - 2\pi l\right) + \sum_{l=-\infty}^{\infty} \frac{2\pi}{2}\,\delta\left(\omega + \frac{2\pi f_0}{f_s} - 2\pi l\right)$$

(b) The impulses are located at $\omega = \left(\frac{2\pi f_0}{f_s} + 2\pi l\right)$ and $\left(-\frac{2\pi f_0}{f_s} + 2\pi l\right)$ for $l \in (-\infty, \infty)$.

$\rightarrow P(e^{j\omega}) = \pi\delta\left(\omega - \frac{2\pi f_0}{f_s}\right) + \pi\delta\left(\omega + \frac{2\pi f_0}{f_s}\right) \quad -\pi < \omega < \pi$

$P(e^{j\omega})$ repeats periodically with a period of 2π.

(c) $x[n] = p[n] \times w[n]$ where $w[n]$ is the rectangular window function which is 1 for $0 \le n \le L-1$ and 0 otherwise.

$$\text{DTFT}(x[n]) = \text{DTFT}(p[n] \times w[n])$$

$$X(e^{j\omega}) = \frac{1}{2\pi}\int_{2\pi} P(e^{j\omega_0})\, W(e^{j(\omega-\omega_0)})\, d\omega_0$$

$$P(e^{j\omega_0}) = \sum_{l=-\infty}^{\infty} \pi\delta\left(\omega_0 - \frac{2\pi f_0}{f_s} - 2\pi l\right) + \sum_{l=-\infty}^{\infty} \pi\delta\left(\omega_0 + \frac{2\pi f_0}{f_s} - 2\pi l\right)$$

Since $\omega_0 \in [-\pi, \pi]$

$$P(e^{j\omega_0}) = \pi\delta\left(\omega_0 - \frac{2\pi f_0}{f_s}\right) + \pi\delta\left(\omega_0 + \frac{2\pi f_0}{f_s}\right)$$

$$w[n] = 1 \qquad 0 \le n \le L-1$$

$$W(e^{j\omega_0}) = 2\pi \sum_{l=-\infty}^{\infty} \delta(\omega_0 - 2\pi l)$$

$$W(e^{j(\omega-\omega_0)}) = \sum_{l=-\infty}^{\infty} 2\pi\, \delta(\omega - \omega_0 - 2\pi l)$$

Since $\omega_0 \in [-\pi, \pi]$

$$W(e^{j(\omega-\omega_0)}) = 2\pi\, \delta(\omega - \omega_0)$$

$$\therefore \ X(e^{j\omega}) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \pi\left[\delta\left(\omega_0 - \frac{2\pi f_0}{f_s}\right) + \delta\left(\omega_0 + \frac{2\pi f_0}{f_s}\right)\right]\cdot 2\pi \cdot \delta(\omega-\omega_0)\, d\omega_0$$

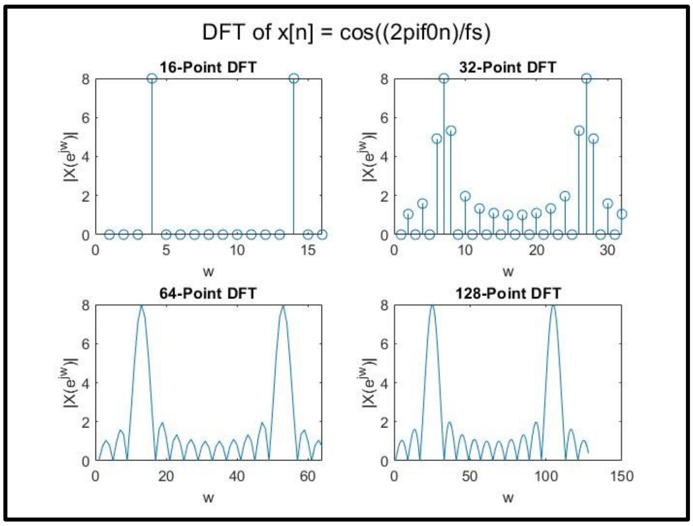
$$X(e^{j\omega}) = \pi\,\delta\left(\omega - \frac{2\pi f_0}{f_s}\right) + \pi\,\delta\left(\omega + \frac{2\pi f_0}{f_s}\right)$$

(c)

Any practical signal processing will involve a window function w[n] of some kind to get a finite length sequence. The signal changes from periodic (with period 2π) to a finite duration signal because of the window. Therefore, the spectrum is also not periodic anymore due to the window. As the length of window function increases, the number of impulses in the spectrum increases.

(d)

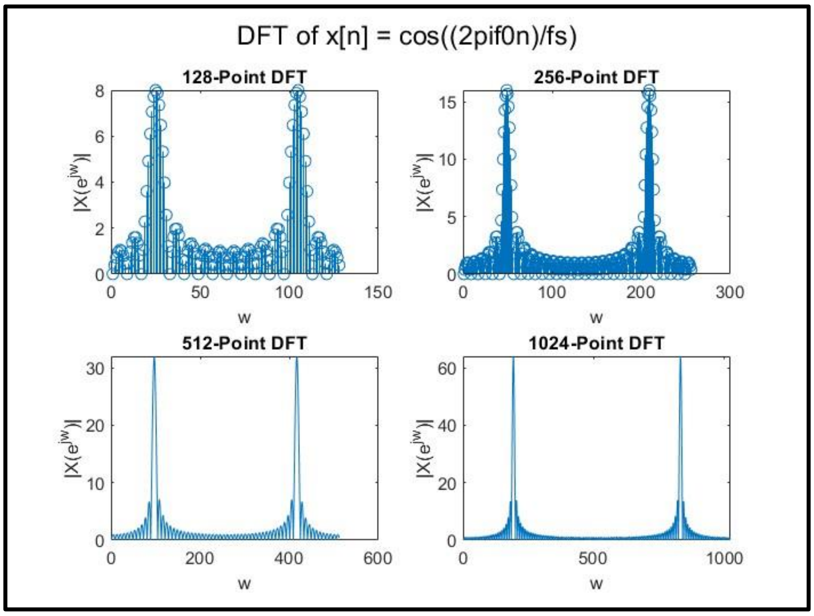
A Rectangular window has a value of 1.0 across the entire measurement time.
A Uniform window creates no frequency or amplitude distortion when the measured signal is periodic.
The plots are consistent with the answers in part (c).



The rectangular window has a rectangular shape and uniform height. The multiplication of the input signal in the time domain by the rectangular window is equivalent to convolving the spectrum of the signal with the spectrum of the rectangular window in the frequency domain, which has a sinc function characteristic.

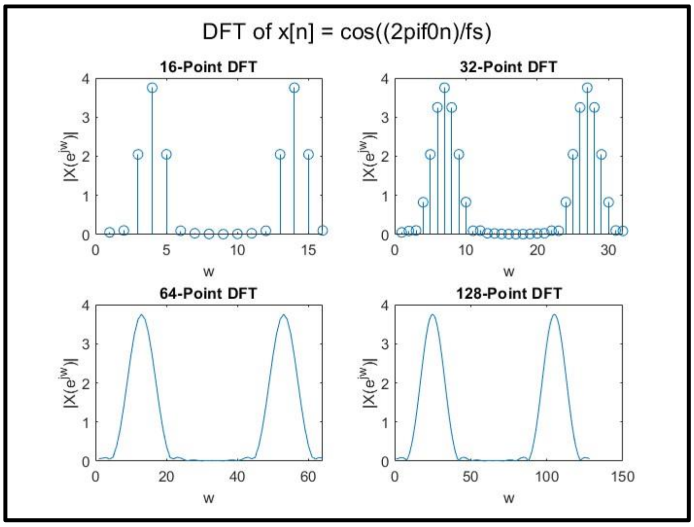
Using N = L gives a proper samples in DFT.

(e)



The magnitude spectrum can only change in discrete jumps as new samples are included when the window duration is increased in the time domain. Frequency resolution increases.

(g)



When doing operational noise and vibration measurements, the Hanning window is commonly used. The Hanning window starts at a value of zero and ends at a value of zero. In the centre of the window, it has a value of one. This gradual transition between 0 and 1 ensures a smooth change in amplitudes when multiplying the measured signal by the window, **which helps reduce the spectral leakage.** Hanning windows are often used with random data because they have moderate impact on the frequency resolution and amplitude accuracy of the resulting frequency spectrum. **The maximum amplitude error of a Hanning window is 15%**, while the **frequency leakage is typically confined to 1.5 spectral lines** to each side of the original sine wave signal**. If the Hanning window is used on sine wave that is captured periodically, the error would be 0%.**

main-lobe width is the minimum distance about the centre such that the window-transform magnitude does not exceed the specified side-lobe level anywhere outside this interval.

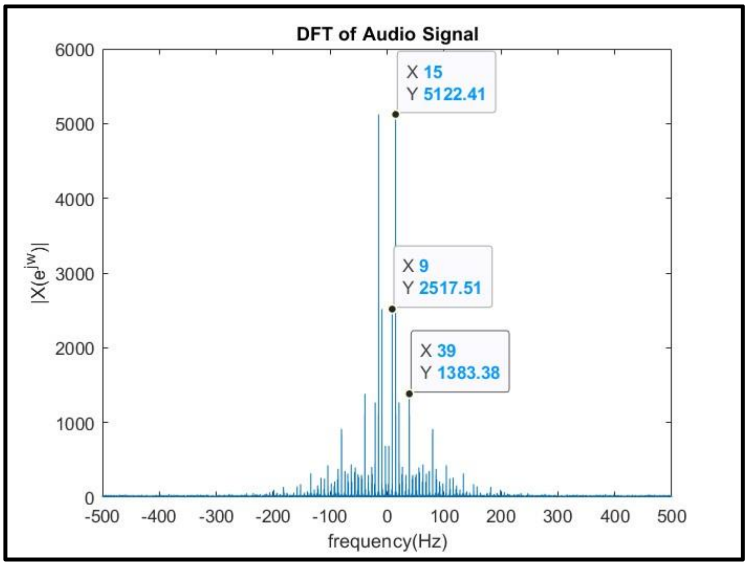
The main-lobe width is of course double that of the rectangular window.

Spectral leakage is caused by discontinuities in the original, nonintegral number of periods in a signal and can be improved using windowing. Windowing reduces the amplitude of the discontinuities at the boundaries of each finite sequence acquired by the digitizer.

Windowing of a simple waveform like sin(ωt) causes its Fourier transform to develop non-zero values (commonly called spectral leakage) at frequencies other than ω. The leakage tends to be worst (highest) near ω and least at frequencies farthest from ω.

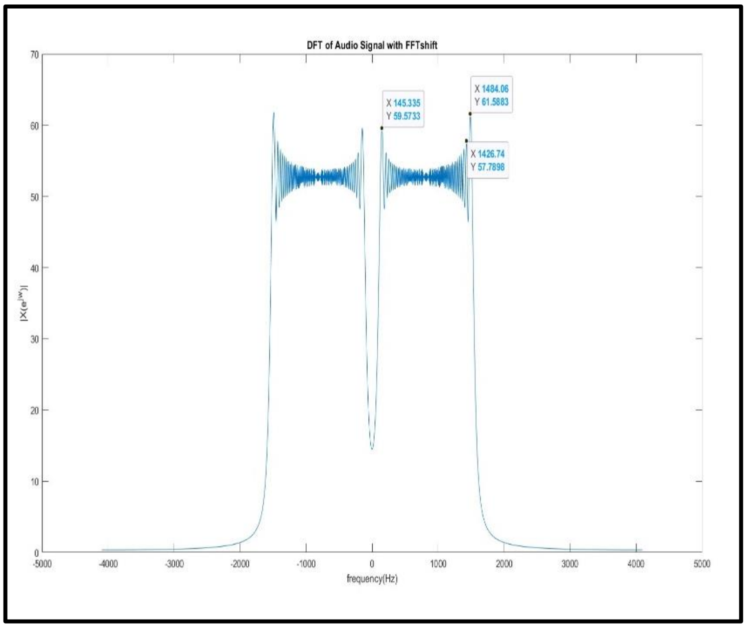
The Hanning window has fewer side lobes, and the leakage is less in this windowing technique.

(i)



The three strongest frequencies are:

$$\pm 15Hz, \qquad \pm 9\text{Hz}, \qquad \pm\, 39\text{Hz}$$

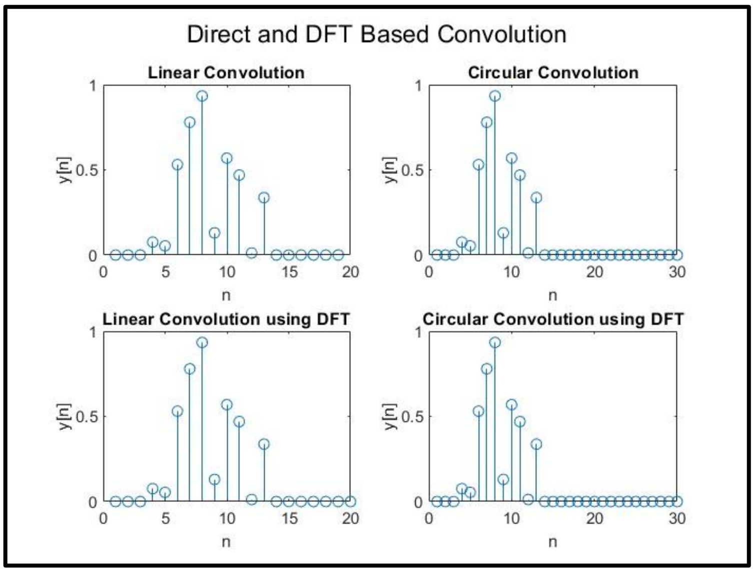


The three strongest frequencies are:

$$\pm 1484.06Hz, \qquad \pm 145.335\text{Hz}, \qquad \pm\, 1426.74\text{Hz}$$

2.

(d)

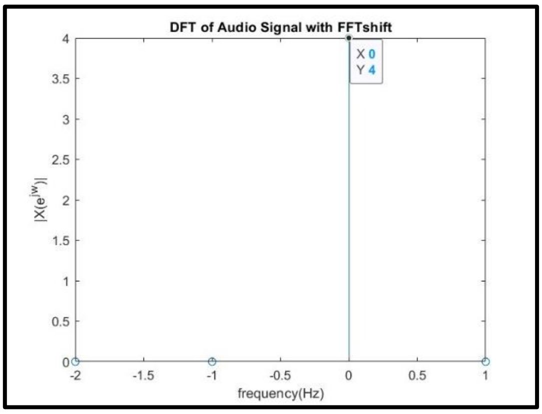


Direct and DFT based convolution give the same results for linear and circular convolution.

3.

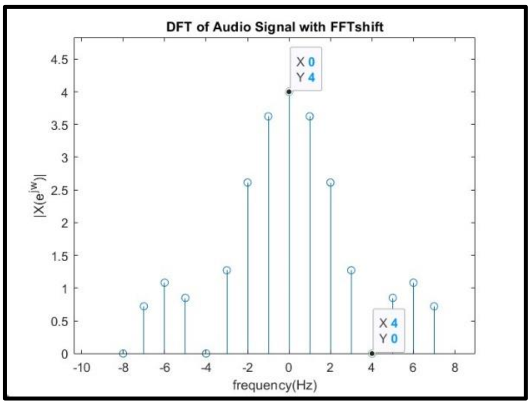
Yes, we can identify the low and high frequencies from the spectrum.
The high frequencies contribute to the fast-varying parts of the signal (the sharp transitions), while the low frequencies contribute to the slow variations of the signal in the time domain.
If you are plotting something like amplitude (voltage, pressure, height, etc.) against time, and you see lots of up and down action in a relatively short distance, those may be due to higher frequency spectral content. If you see some up and down trends over a much larger span those may represent some lower frequencies.

(a) 4-Point DFT



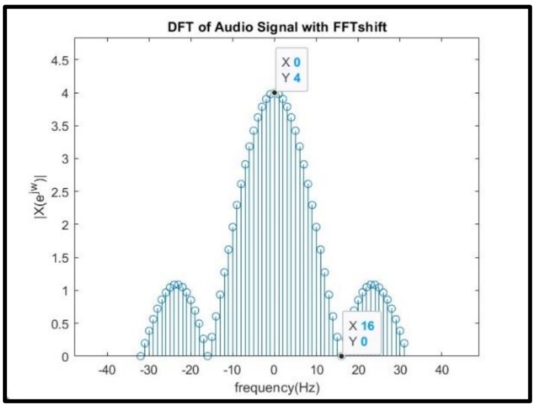
The Highest magnitude corresponds to 0Hz while rest all have the lowest magnitude

16-Point DFT



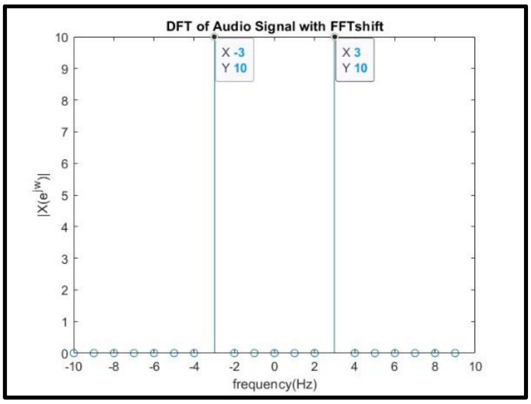
The Highest magnitude corresponds to 0Hz while the least magnitude corresponds to ± 4Hz

64-Point DFT



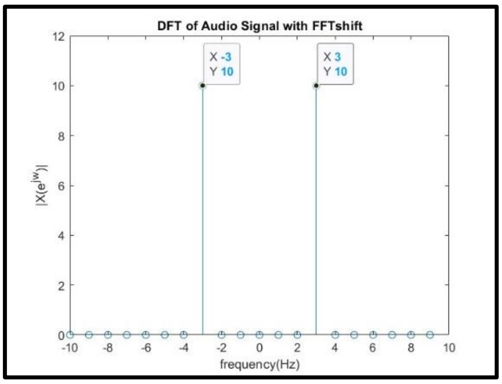
The Highest magnitude corresponds to 0Hz while the least magnitude corresponds to ± 16Hz

(b)



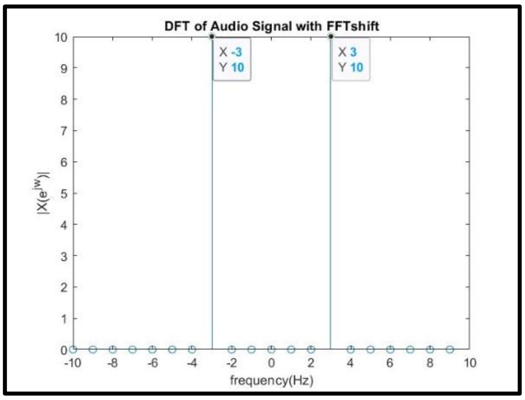
The Highest magnitude corresponds to ± 3Hz while rest all have the least magnitude

(c)



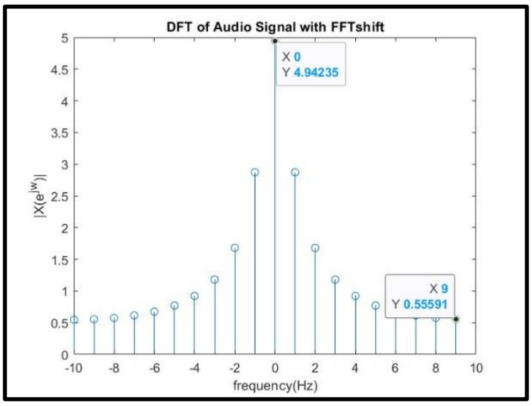
The Highest magnitude corresponds to ±3Hz while rest all have the least magnitude

(d)



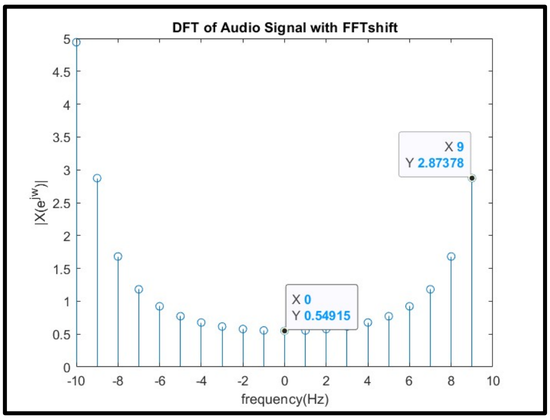
The Highest magnitude corresponds to ±3Hz while rest all have the least magnitude

(e)



The Highest magnitude corresponds to 0Hz while the least magnitude corresponds to ±9Hz

(f)



The Highest magnitude corresponds to ± 9Hz while the least magnitude corresponds to 0Hz