Signal Processing – Lab 2

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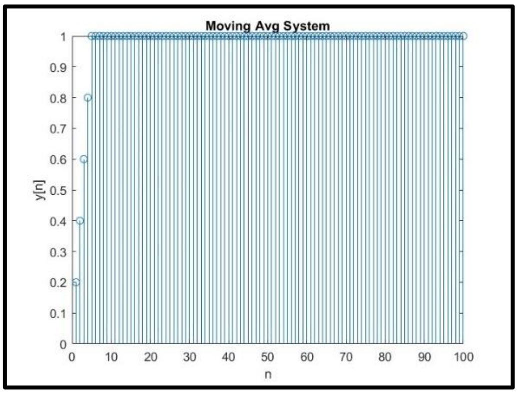
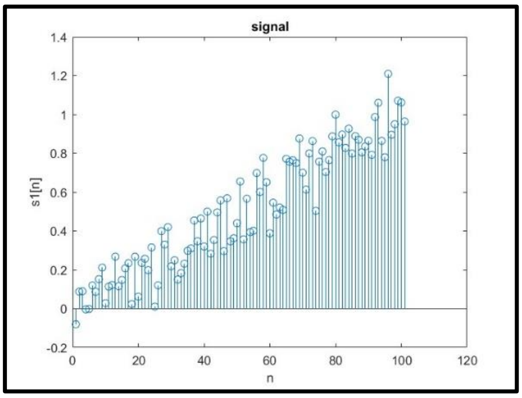
1.

Accumulator: $y[n] = \sum_{k=-\infty}^{n} x[k]$ Moving average system: $y[n] = \frac{1}{N}\sum_{k=n-N}^{n} x[k]$

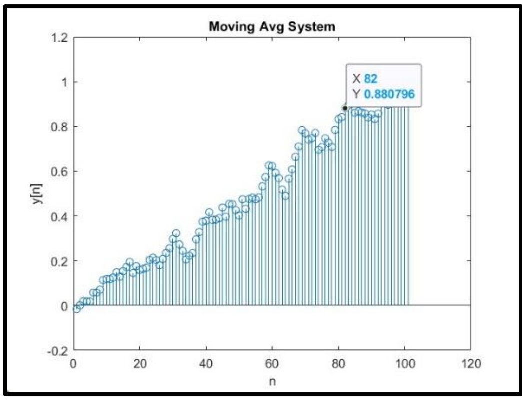
(b) Moving Average System for u[n] where N = 5:



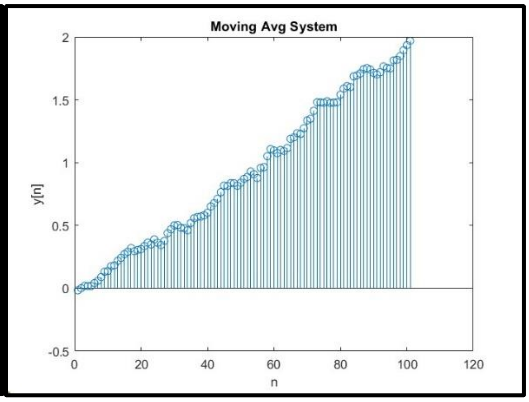
(c) Trend of the test sequence $s_1[n]$:



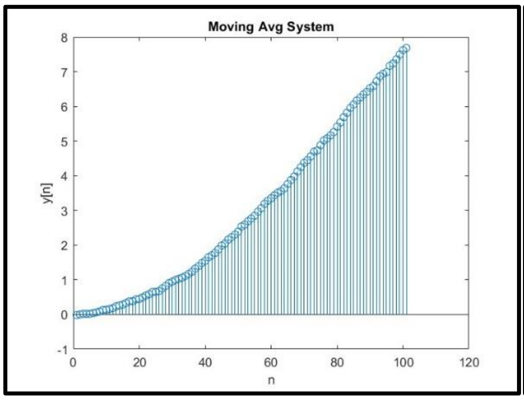
(d) Moving Average System as N is varied for $s_1[n]$:



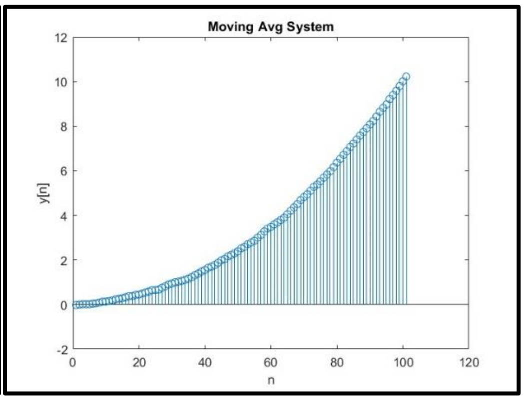
N = 5



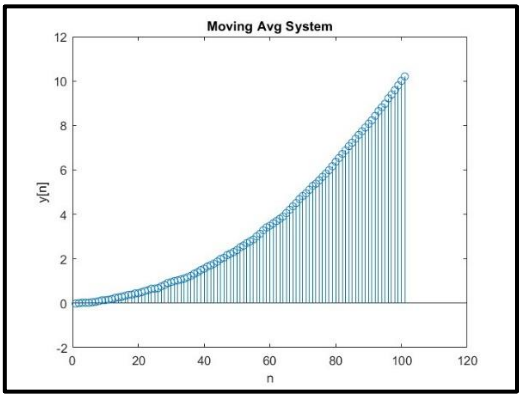
N=10



N=50



N=100

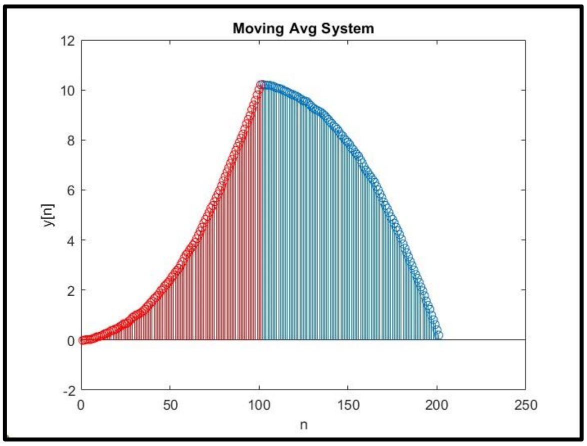


N=101

- The appropriate value of N is 101 for $s_1[n]$. for N = 101, the moving average signal outputs a smooth curve.
- A moving average is a calculation to analyse data points by creating a series of averages of different subsets of the full data set. The reason for calculating the moving average of a signal is to help smooth out the curve by creating a constantly updated average signal. It is a trend-following or lagging, indicator because it is based on past values. The longer the period for the moving average, the greater the lag. This gives a smoother trend.

- The impulse response of the MA system was found and the output for input $s_1[n]$ was found using convolution. When the two signals obtained by the two ways of implementation were overlapped, we could observe that there was no difference in values for n = 1, ……101.

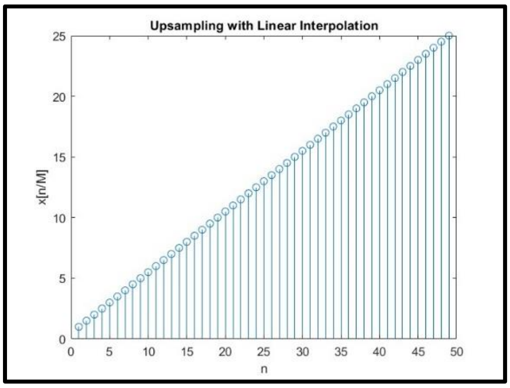
- Convolution gives values for more samples
- By convolution we find the impulse response of the system which can be used to find the output for any other input signal by just convolving the input signal and impulse response. This is more convenient.



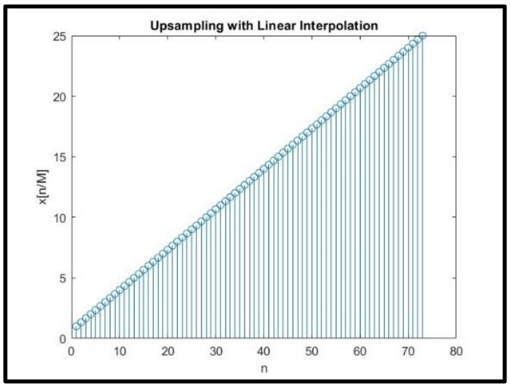
2.

(a)

- Linear Interpolation for y[n] = n/M:

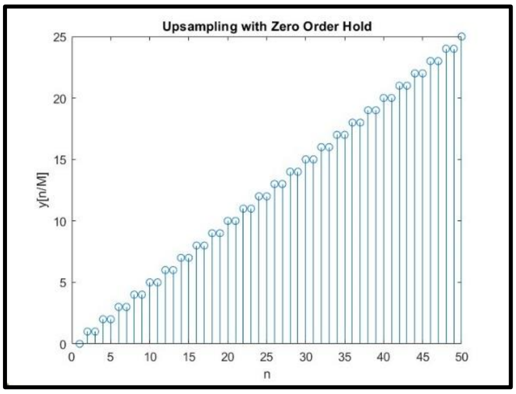


M=2

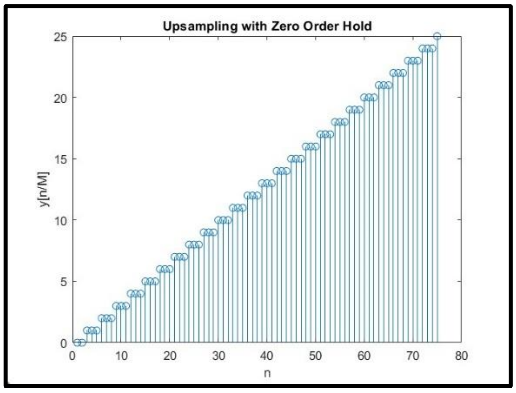


M=3

- Zero order Hold for y[n] = n/M:



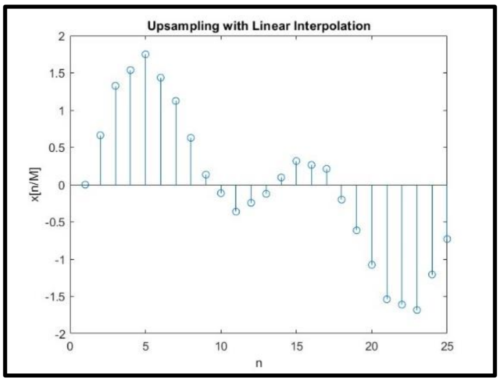
M=2



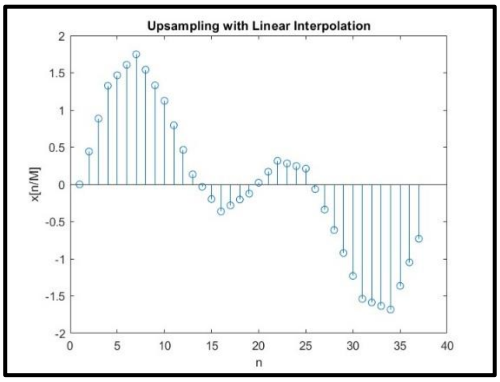
M=3

(b)

- Linear Interpolation for test sequence in q2_1.mat file:

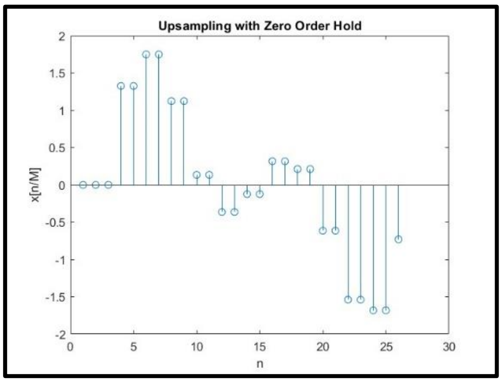


M=2

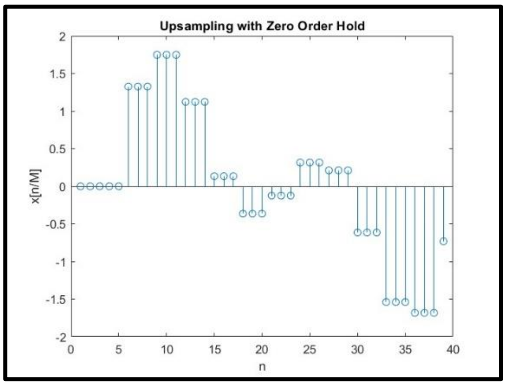


M=3

- Zero order Hold for test sequence in q2_1.mat file:

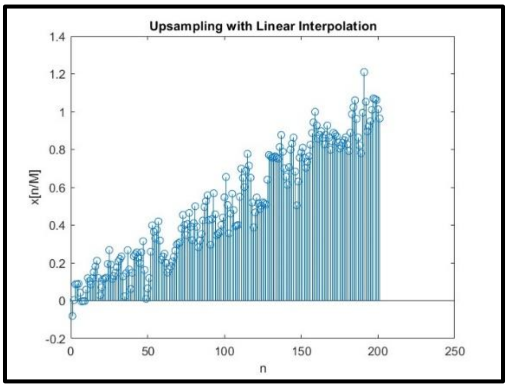


M=2

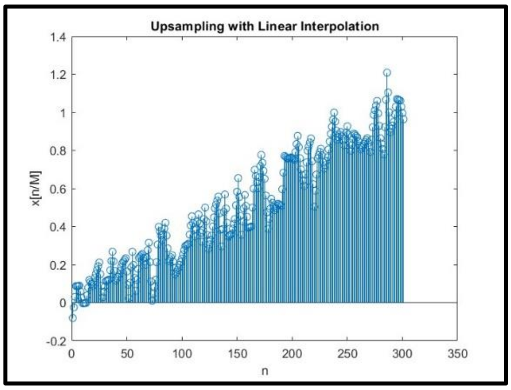


M=3

- Linear Interpolation for test sequence in q2_2.mat file:

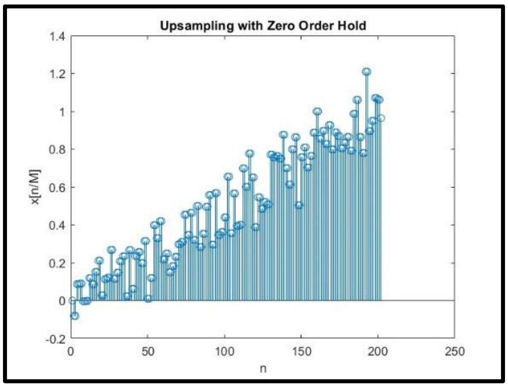


M=2

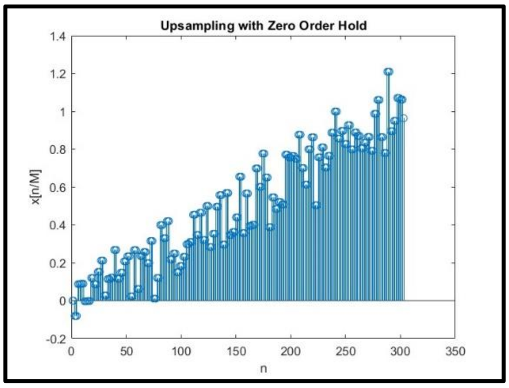


M=3

- Zero order Hold for test sequence in q2_2.mat file:



M=2



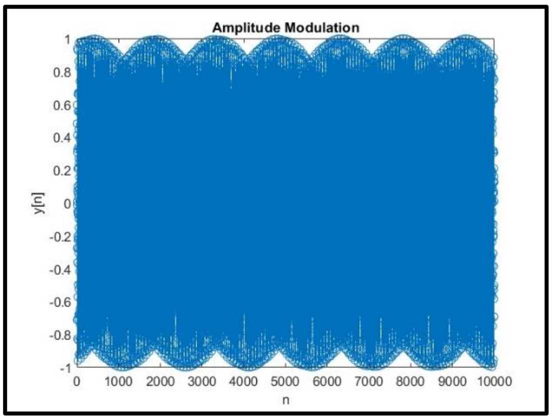
M=3

- ➢ The Zero-Order Hold circuit generates a continuous input signal u(t) by holding each sample value u[k] constant over one sample period, a First-Order Hold circuit uses linear interpolation between samples.
- ➢ These mathematical models of the practical signal reconstruction done by a conventional digital-to-analog converter (DAC), describes the effect of converting a discrete-time signal to a continuous-time signal by holding each sample value or linearly interpolating for one sample interval.

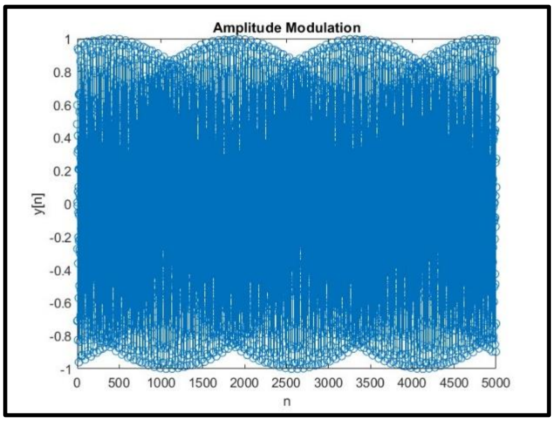
3.

$$\boxed{y|n| = (\cos\omega_0 n)x[n]}$$

(a) Amplitude Modulated wave:



n = 10000



n = 5000

- We observe amplitude modulation.

(b)

$$y[n] = \cos(w_0 n)\, x[n]$$

$$x[n] = \sin(w_m n)\, u[n] = \frac{1}{2}\left(e^{jw_m n} - e^{-jw_m n}\right) u[n]$$

$$X(z) = \frac{1}{2}\left(\frac{1}{1-e^{jw_m}z^{-1}} - \frac{1}{1-e^{-jw_m}z^{-1}}\right) \quad \text{with ROC } |z|>1$$

$$y[n] = \frac{1}{2}\left(e^{jw_0 n} + e^{-jw_0 n}\right) x[n]$$

$$Y(z) = \frac{1}{2}\left(X\left(z/e^{jw_0}\right) + X\left(z/e^{-jw_0}\right)\right) \quad \text{with ROC } |z|>1$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\frac{1}{2}\left(X(z/e^{jw_0}) + X(z/e^{-jw_0})\right)}{X(z)} \quad \text{with ROC the entire z plane.}$$

$$H(z) = \frac{\sqrt{z}(z-1)}{z^2-2z+1} = \frac{\sqrt{z}\,(z-1)}{(z-1)^2}$$

zeros : $z = 0, 1$

poles : $z = 1$

(c) Pole: z = 1

Zero: z= 0, 1.

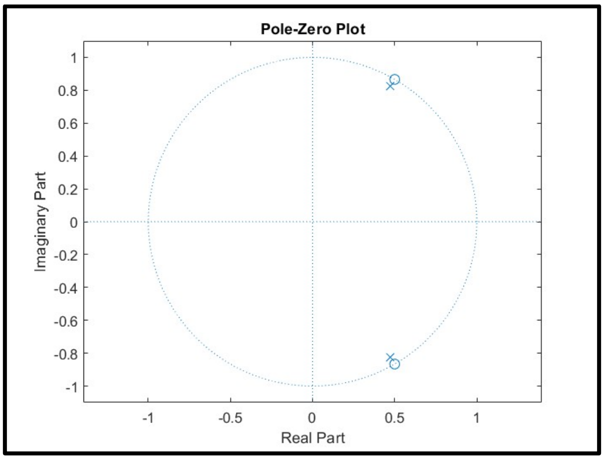
(d) ROC is the entire z plane. Since the poles and zeros cancel out.

4.

$$H(z) = \frac{z^2 - (2\cos\theta)\, z + 1}{z^2 - (2r\cos\theta)z + r^2},$$

$$r = 0.95 \text{ and } \theta = \frac{\pi}{3}.$$

(a) Pole-Zero Plot:

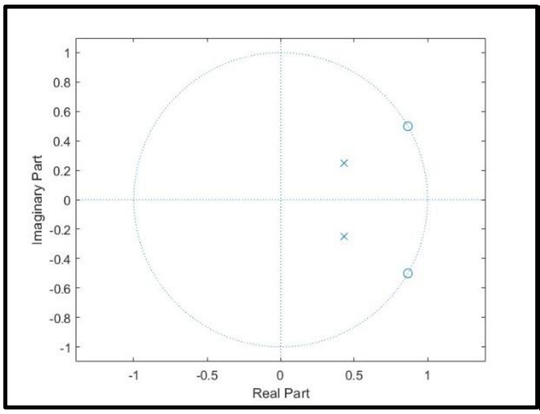


The poles of the transfer function lie on a circle with radius r and at an angle theta on either side of the x axis. While the zeros of the transfer function lie on a unit circle at angle theta from the x axis.

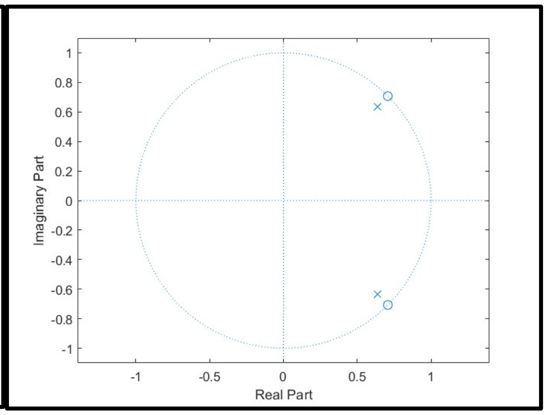
Zeros of the transfer function are: 0.5 + 0.86j, 0.5 – 0.86j

Poles of the transfer function are: -0.475 + 0.82j, -0.475 – 0.82j

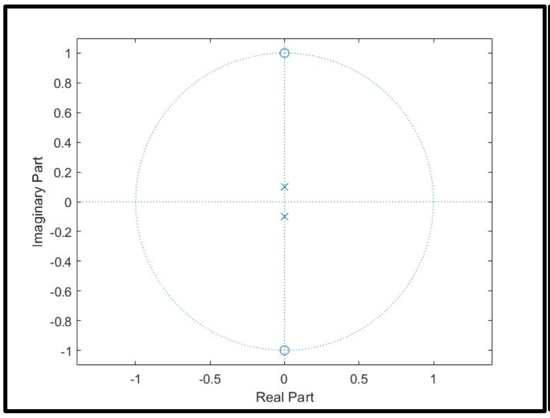
- Plots for different values of r and theta:



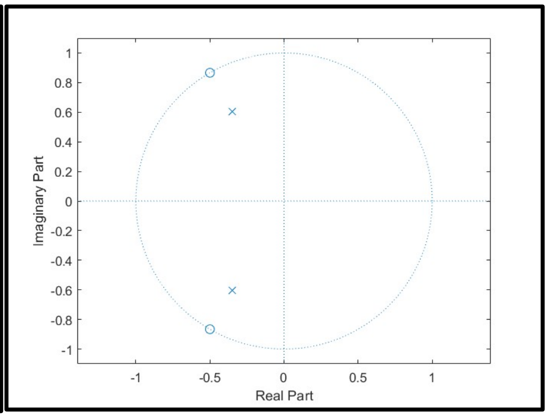
R = 0.2 theta = 30°



R = 0.9 theta = 45°



R = 0.1 theta = 90°



R = 0.4 theta = 120°

- Increasing theta decreases the bandwidth of the filter because the zeros shift by an angle theta wrt the origin.
- When r is changed from 0 to 1 gradually, the variation in magnitude decreases. This does not affect where the pole occurs.

(b) A MATLAB code written to find the inverse z-transform of H(z):

```
kroneckerDelta(n, 0) - ((r^2 - 1)*kroneckerDelta(n, 0))/r^2 - (cos(n*theta)*(2*cos(theta) - 2*r*cos(theta))*(r^2)^n)/(r^(n + 1)*cos(theta)) + (r^(n - 1)*cos(n*theta)*(r^2*(2*cos(theta) - 2*r*cos(theta)) + r*cos(theta)*(r^2 - 1)))/(r^2*cos(theta)) - (r^(n - 1)*sin(n*theta)*(r^2*(2*cos(theta) - 2*r*cos(theta)) + r*cos(theta)*(r^2 - 1)))/(r^2*sin(theta))
```

$$\frac{1}{2r^2}(r-1)\,\theta(n-2)\Big(r\,(r\,(\cos(a)-i\sin(a)))^n + (r\,(\cos(a)-i\sin(a)))^n + r\,(r\,(\cos(a)+i\sin(a)))^n + (r\,(\cos(a)+i\sin(a)))^n + i\,r\cot(a)\,(r\,(\cos(a)-i\sin(a)))^n - i\cot(a)\,(r\,(\cos(a)-i\sin(a)))^n - i\,r\cot(a)\,(r\,(\cos(a)+i\sin(a)))^n + i\cot(a)\,(r\,(\cos(a)+i\sin(a)))^n\Big) + 2\,(r-1)\cos(a)\,\theta(1-n)\,\theta(n-1) + \theta(-n)$$

$\cot(x)$ is the cotangent function

$\theta(x)$ is the Heaviside step function

Taken from an online inverse z-transform tool

As value of theta and r are varied:

$$H(z) = \frac{z^2 - 2\cos\theta\, z + 1}{z^2 - 2r\cos\theta + r^2}$$

$\rightarrow$ $r = 1$, $\theta = \pi/3$

$$H(z) = 1$$

$$\boxed{h[n] = \delta[n]}$$

$\rightarrow$ $r = 0.5$, $\theta = \pi/2$

$$H(z) = \frac{z^2 + 1}{z^2 + (0.5)^2}$$

$$H(z) = 1 + \frac{0.75}{j}\left[\frac{1}{z - 0.5j} - \frac{1}{z + 0.5j}\right]$$

$$\boxed{h[n] = \delta[n] + \frac{0.75}{j}\left((0.5j)^n u[n] - (-0.5j)^n u[n]\right)}$$