Signal Processing
Lab 8

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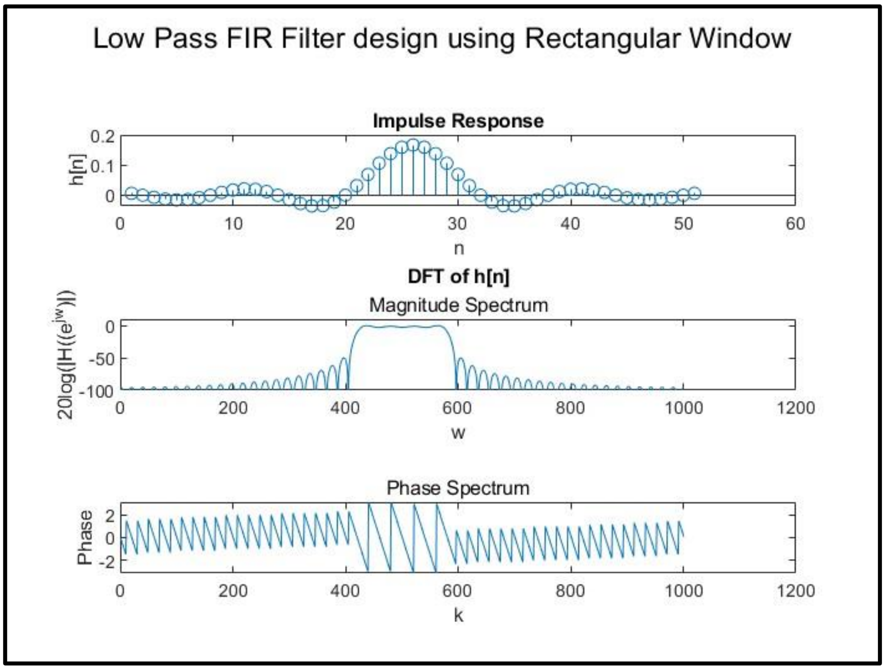
Q1.

$$H_{\mathrm{LPF}}(e^{j\omega}) = \begin{cases} 1, & |\omega| \le \frac{\pi}{6} \\ 0, & \frac{\pi}{6} \le |\omega| < \pi \end{cases} \qquad H_d(e^{j\omega}) = \begin{cases} e^{-j\omega n_c}, & |\omega| \le \frac{\pi}{6} \\ 0, & \frac{\pi}{6} \le |\omega| < \pi \end{cases}$$

$$h_{LPF}[n] = \frac{\sin\left(\frac{\pi}{6}n\right)}{\pi n}$$

$$h_d[n] = \frac{\sin(\frac{\pi}{6}(n - n_c))}{\pi(n - n_c)}$$

(b)

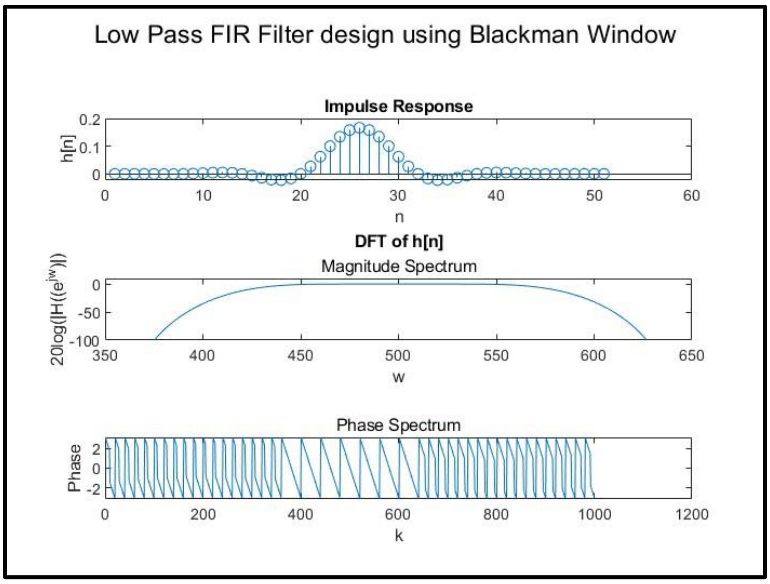


- We observe that the filter has a linear phase.
- Linear phase is a property of a filter where the phase response of the filter is a linear function of frequency. The result is that all frequency components of the input signal

are shifted in time by the same constant amount. Linear Phase filters are desirable for certain applications because they maintain the shape of the input signal.

- Linear-phase filters have a symmetric impulse response, and the symmetric-impulse-response constraint means that linear-phase filters must be FIR filters, because a causal recursive filter cannot have a symmetric impulse response. Therefore, an ideal low pass filter can be designed to be a low pass FIR filter using a rectangular window.

(c)



(d)

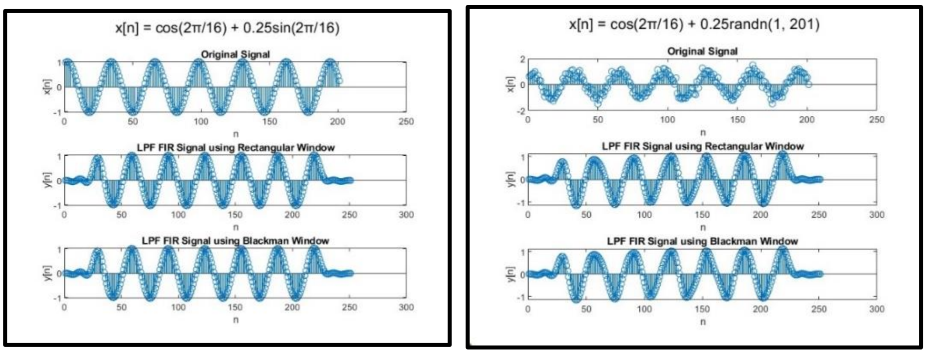
FIR filter designed using Blackman window has a wider main lobe as compared to the FIR filter designed using rectangular window.

Blackman window results in a larger transition bandwidth. The transition bandwidth of a filter response is smaller for a window with a narrower main lobe.

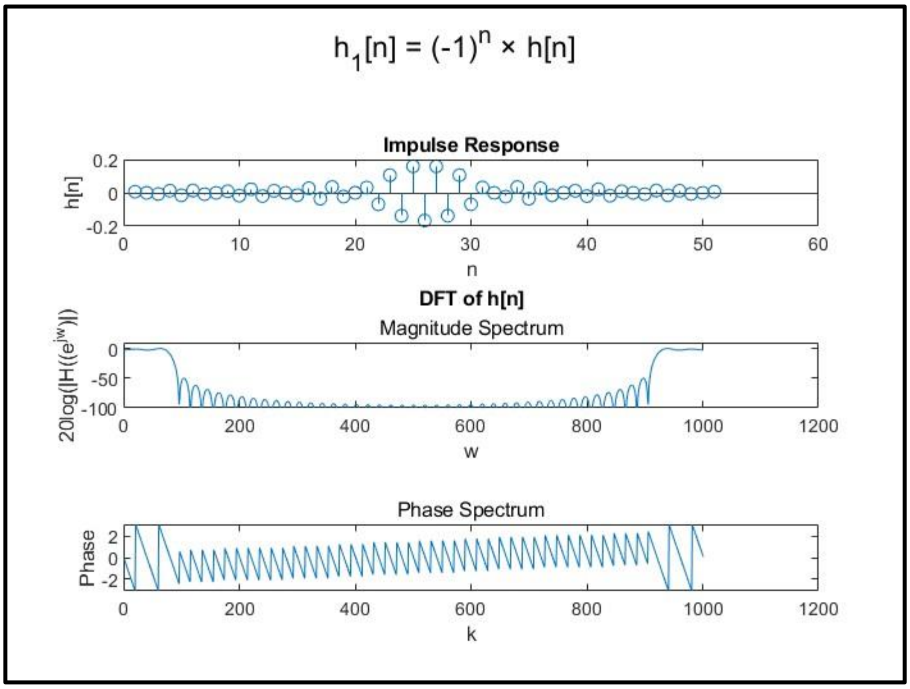
Also, a Blackman window can eliminate ripple in the FIR filter.

Blackman has more side lobe attenuation (which is desirable) while the rectangular window has minimal side lobe attenuation.

(e)



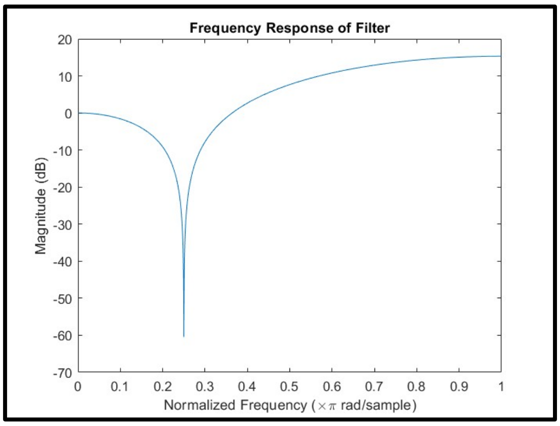
(f)



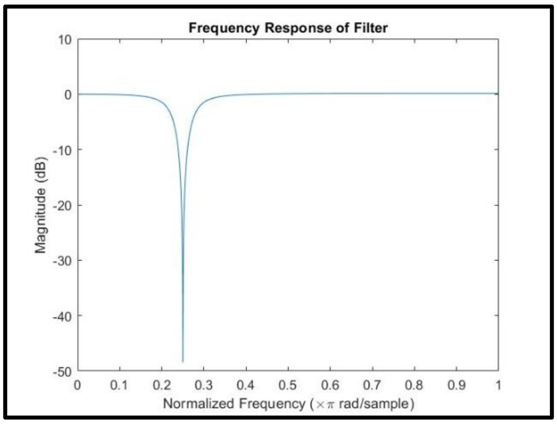
- This is a high pass filter.

Q2.

(a)



(b)



(c)

The first filter has a system function given as

$$H(z) = b_0(1 - e^{jw_0}z^{-1})(1 - e^{-jw_0}z^{-1})$$

This is valid for all values of z therefore the filter is both causal and stable. Hence this is an FIR filter.

The second filter has a system function given as

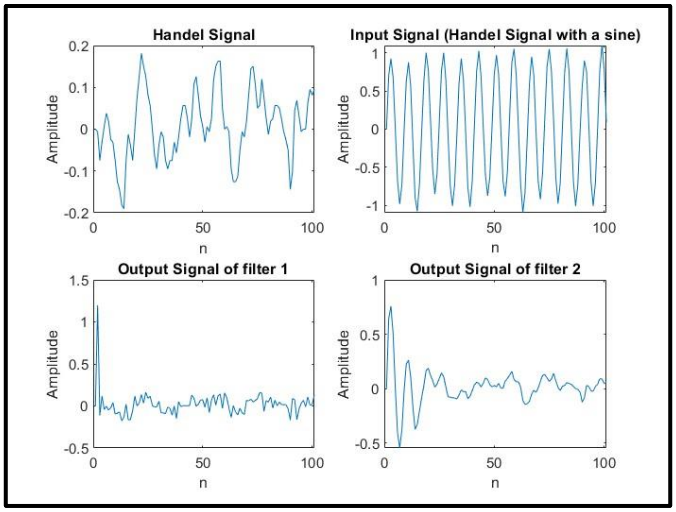
$$H(z) = b_0 \frac{(1 - e^{jw_0} z^{-1})(1 - e^{-jw_0} z^{-1})}{(1 - r_0 e^{jw_0} z^{-1})(1 - r_0 e^{-jw_0} z^{-1})}$$

The system function has a pole at $|z| = r_0$ where $r_0 < 1$.

If the ROC is $|z| > r_0$, then the ROC includes the unit circle and extends till infinity. Therefore, the system will be both stable and causal.

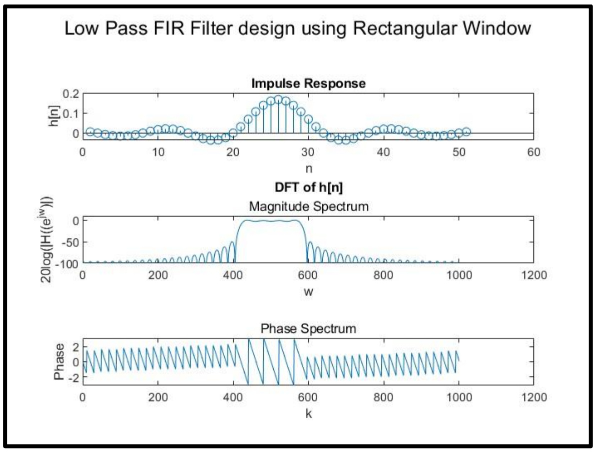
If the ROC is $|z| < r_0$, then the ROC doesn't include the unit circle. Therefore, the system is neither stable nor causal.

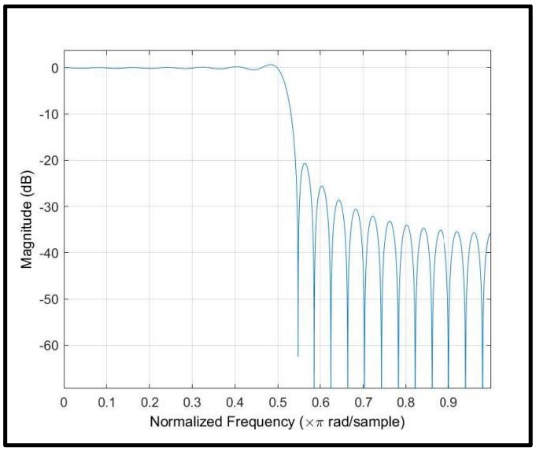
(e)



- We observe that the output of filter 1 gives a sound that resembles the original sound but of lower amplitude and higher frequency. While the output of filter 2 resembles more with the original signal.

Q3.

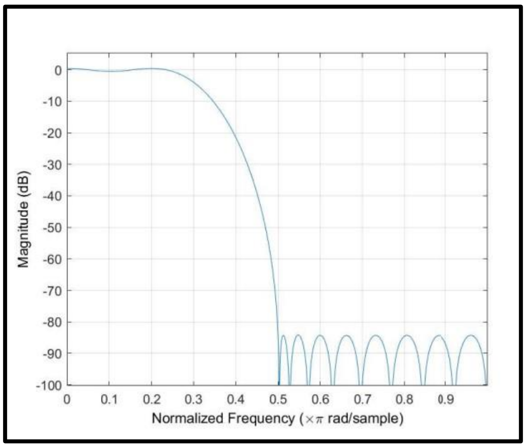




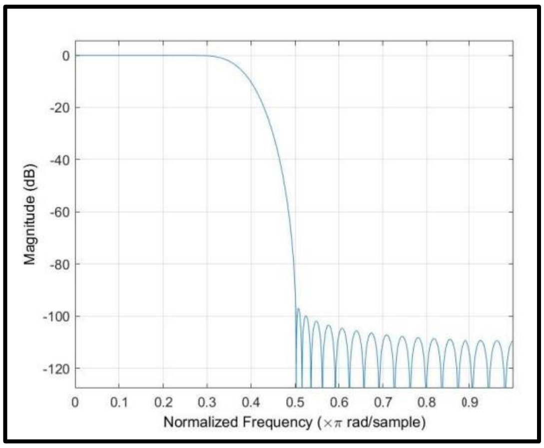
- The filter design using filterDesign gives the same filter characteristics as observed in Q1.

(d)

Equiripple Filter



Least Square Filter



- Equiripple filters seek to minimize the maximum error between the desired filter response and the designed approximation. Equiripple designs achieve optimality by distributing the deviation from the ideal response uniformly. This results in ripples of equal height.
- However, the overall deviation, measured in terms of its energy tends to be large. This may not always be desirable. When this is a concern, least-squares methods provide optimal designs that minimize the energy in the stopband.
- Least-squares filters seek to minimize the total squared error between the desired filter response and the designed approximation. We observe that the height of the ripples decreases gradually.
- For a least-squares designs, the ripple in the passband region, close to the passband edge tends to be large. For low pass filters in general, it is desirable that passband frequencies of a signal to be filtered are affected as little as possible.