Signal Processing – Lab 1

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2.

(c)

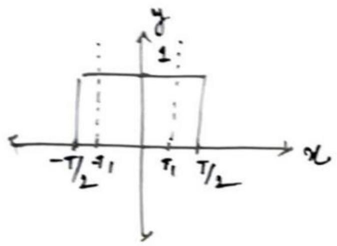
For a fixed set of input parameters, from the outputs of the above function, the following two types of errors were computed:

i. MAE (Maximum Absolute Error): 4.4496e-16

ii. RMS (Root Mean Squared Error): 7.9937e-17

3.

(a)



$$f(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

$$a_k = \frac{1}{T}\int_{-T_1}^{T_1} (1) e^{-jk\omega_0 t}\, dt.$$

$$a_k = \frac{1}{T} \times \frac{-1}{jk\omega_0}\left[e^{-jk\omega_0 t}\right]_{-T_1}^{T_1}$$

$$a_k = \frac{+1}{\cancel{j}k\omega_0 T} \times 2\sin(k\omega_0 T_1)\cancel{j} = \frac{\sin\left(\frac{2\pi k}{T} T_1\right)}{2\pi k}$$

$T = 1$ and $T_1 = 0.1$

$$a_k = \frac{\sin(0.2\pi k)}{2\pi k}$$ for $k = -N, \ldots\ldots 0, \ldots N$ where $N = 10^* T$.

(b)

Since the duration of the signal does not affect the highest frequency, it will not affect the range of frequencies. However, it does affect the resolution of the signal in frequency domain. the shortest non-zero frequency is 1/T. This shortest non-zero frequency is also the resolution of the frequency domain, meaning it is the distance on the frequency axis between two successive coefficients. So, as we increase T, the resolution in the frequency domain will increase.

(c)

For a periodic signal with discontinuities, if the signal is reconstructed by adding the Fourier series, then overshoots appear around the edges. These overshoots decay outwards in a damped oscillatory manner away from the edges. This is known as Gibbs phenomenon.

The function's Nth partial Fourier series (formed by summing its N lowest constitute sinusoids) produces large peaks around the jump which overshoots and undershoots the function's actual values. This approximation error approaches a limit of about 9% of the jump as more sinusoids are used (N increases), though the infinite Fourier series sum does eventually converge almost everywhere except the point of discontinuity.

4.

(c)

Every function can be represented as a sum of even and odd signals. In the Fourier representation of a signal, the summation of sine terms represents the odd part while the summation of the cosine terms represents the even part of the signal.

$$\boxed{f(t) = a_0 + \sum_{m=1}^{\infty} a_m \cos\left(\frac{2\pi mt}{T}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{2\pi nt}{T}\right)}$$

$$\boxed{f(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}}$$

If we have an even signal then the odd part is zero i.e., there won't be any sine terms or in complex Fourier representation, the imaginary part will be zero. While if we have an odd signal then even part will be zero i.e., cosine terms will be zero or in the complex Fourier representation the real part will be zero.

- x1(t) is symmetric about the y axis. This means that x1(t) is an even function. Therefore, imaginary part of Fourier coefficients will be zero.
- x2(t) is symmetric about the origin. This means that x2(t) is an odd function. Therefore, real part of its Fourier coefficients will be zero.