Signal Processing Lab 4

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1.

(b)

(1) The analytical answer for DTFT for d[n] is 1 and we see that the plot matches the analytical answer. The DTFT is a constant, so it is periodic with any period.
(2) The analytical answer for DTFT of d[n+3] is an exponential function e^(3jw) which signifies the shift in time. We see that our plot matches the analytical answer. The real part is a cos wave and imaginary part is a sin wave. The phase keeps shifting between -pi and pi. It is periodic with a period 2pi.
(3) The analytical answer for DTFT for a rectangular wave from -3 to 3 is a sinc function which matches our plot. The result is entirely real, and the phase varies between -pi and pi. It is periodic with a period 2pi.
(4) The analytical answer for DTFT for sin(w0n) is a linear combination or impulses. This matches our plot. It is periodic with a period 2pi.

1. $x[n] = \delta[n]$

$$X(e^{j\omega}) = \sum_{-\infty}^{\infty} x[n]\, e^{-j\omega n} = \sum_{-\infty}^{\infty} \delta[n]\, e^{-j\omega n} = \underline{\underline{1}}$$

2. $x[n] = \delta[n+3]$

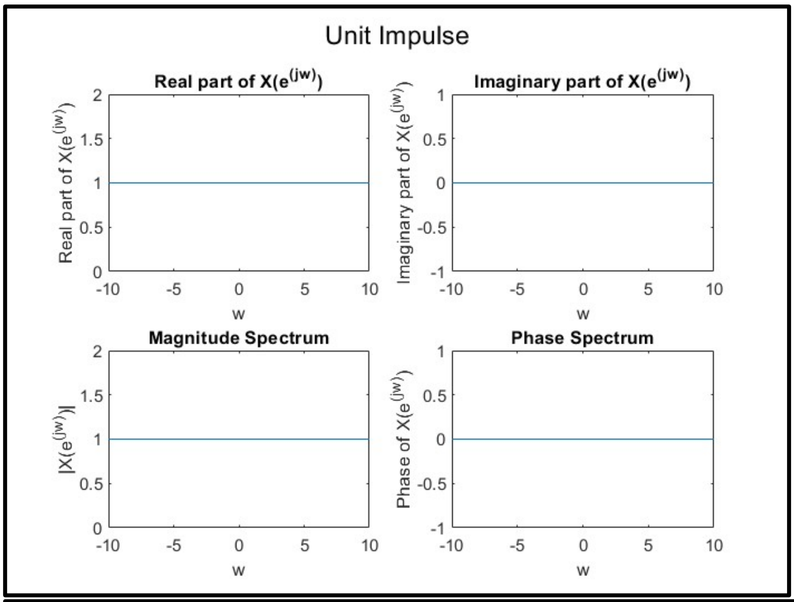
$$X(e^{j\omega}) = \sum_{-\infty}^{\infty} x[n]\, e^{-j\omega n} = \sum_{-\infty}^{\infty} \delta[n+3]\, e^{-j\omega n} = \underline{\underline{e^{3j\omega}}}$$

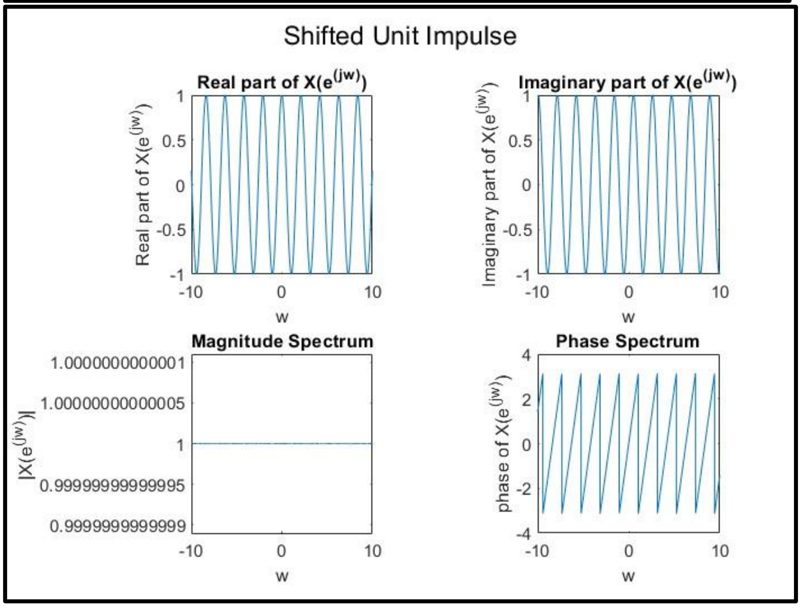
3. $x[n] = u[n+3] - u[n-4]$

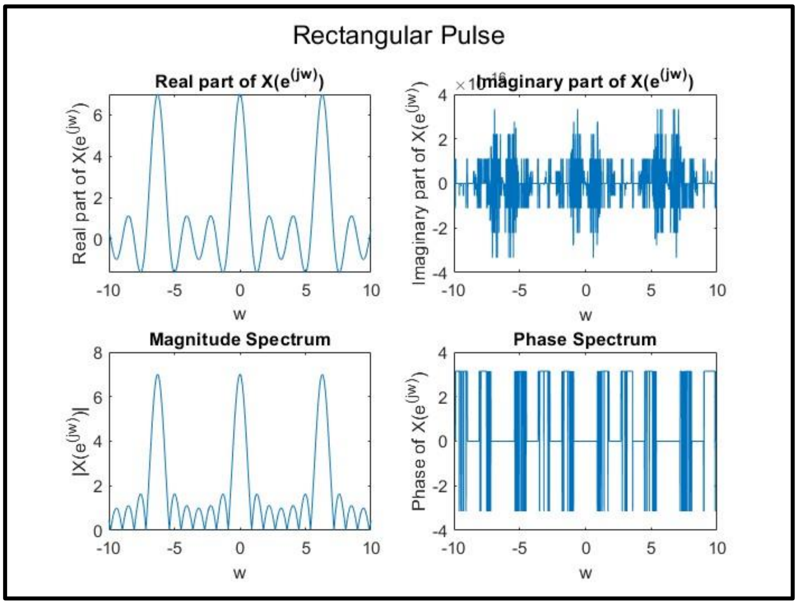
$$X(e^{j\omega}) = \sum_{-\infty}^{\infty} u[n+3]\, e^{-j\omega n} - \sum_{-\infty}^{\infty} u[n-4]\, e^{-j\omega n}$$

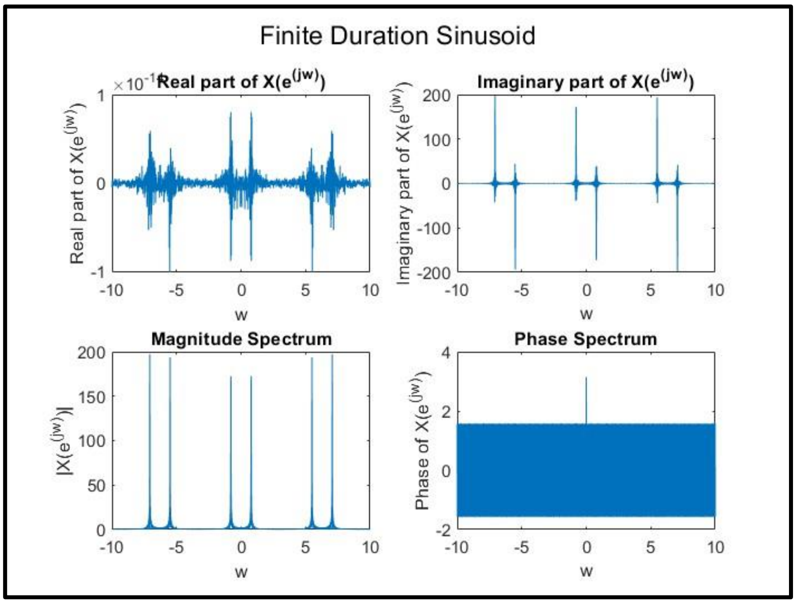
4. $x[n] = \sin(\omega_0 n)$ $\omega_0 = \frac{\pi}{4}$

$$X(e^{j\omega}) = \sum_{-\infty}^{\infty} \sin(\omega_0 n)\, e^{-j\omega n}$$


Unit Impulse
Real part of X(e^(jw))
Imaginary part of X(e^(jw))
Magnitude Spectrum
Phase Spectrum
Real part of X(e^(jw))
Imaginary part of X(e^(jw))
|X(e^(jw))|
Phase of X(e^(jw))
w


Shifted Unit Impulse
Real part of X(e^(jw))
Imaginary part of X(e^(jw))
Magnitude Spectrum
Phase Spectrum
|X(e^(jw))|
phase of X(e^(jw))
1.0000000000001
1.00000000000005
1
0.99999999999995
0.9999999999999
w


Rectangular Pulse
Real part of X(e^(jw))
Imaginary part of X(e^(jw))
Magnitude Spectrum
Phase Spectrum
|X(e^(jw))|
Phase of X(e^(jw))
w



(c) We observe that as b increases the exponential nature of the time domain signal increases. For a=-b we see that the signals also have negative values. In the frequency domain we see that the peaks in the graph become more and more sharper with an increase in the value of b.

2.

(a)

(a) $y[n] = \frac{1}{M}\sum_{m=0}^{M-1} x[n-m]$

To find impulse Response h[n], we replace x[n] with δ[n].

$$\Rightarrow h[n] = \frac{1}{M}\sum_{m=0}^{M-1} \delta[n-m]$$

$$h[n] = \frac{\delta[n] + \delta[n-1] + \cdots \delta[n-(M-1)]}{M}$$

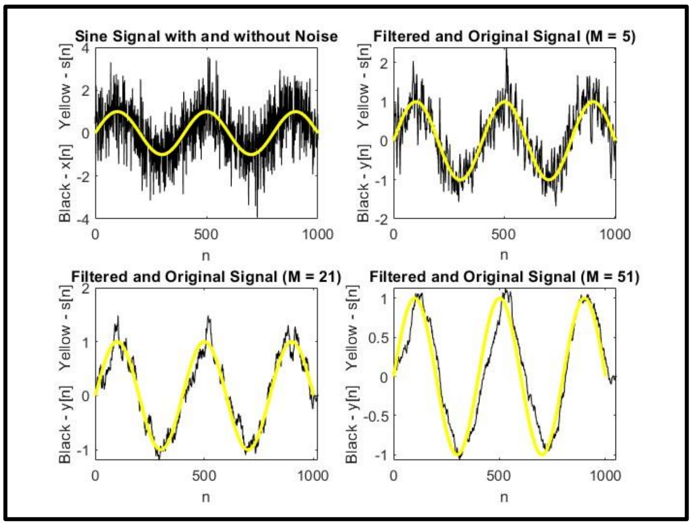
$$\boxed{h[n] = \cancel{ff}\ \frac{u[n] - u[n-M]}{M}}$$

(e) The moving average filter is a simple Low Pass FIR (Finite Impulse Response) filter commonly used for smoothing an array of sampled data/signal. It takes M samples of input at a time and take the average of those M-samples and produces a single output point. It is a very simple LPF (Low Pass Filter) structure that comes handy to filter unwanted noisy component from the intended data.

As the filter length increases (the parameter M) the smoothness of the output increases, whereas the sharp transitions in the data are made increasingly blunt. This implies that this filter has excellent time domain response but a poor frequency response.

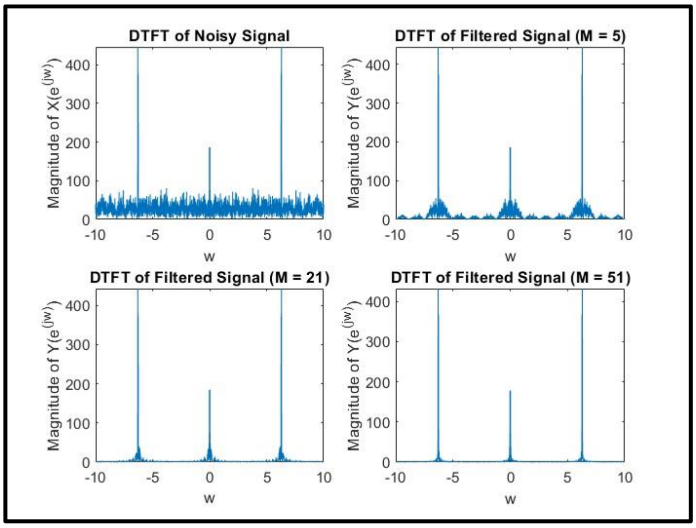
It takes M input points, computes the average of those M-points and produces a single output point. Due to the computation/calculations involved, the filter introduces a definite amount of delay.

We observe that as we increase the filter length to 51-points, the noise in the output has reduced a lot.



Output of filter as M value increases

(f)

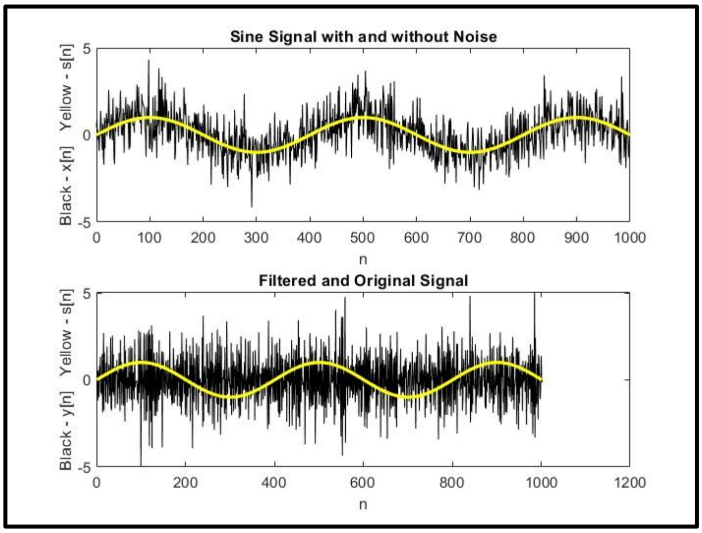


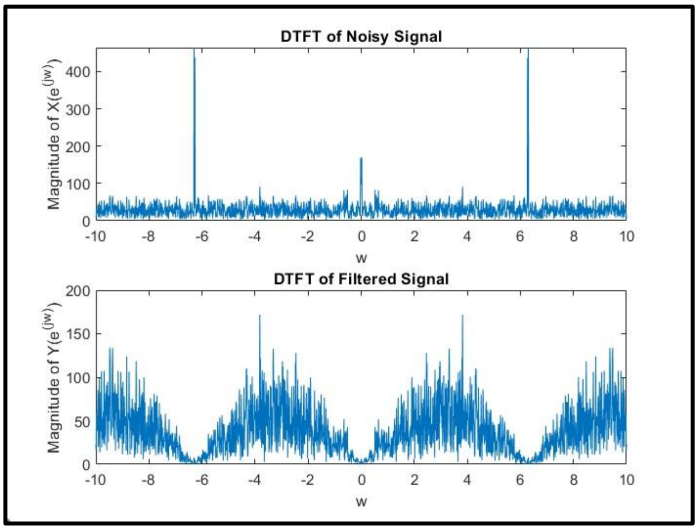
DTFT of the Output Signal as M value increases

We observe that as M is increased the output becomes smoother, that is, the sharp transitions in the data are made blunt resulting in a poor frequency domain response.

From the frequency response it can be asserted that the roll-off is very slow and the stop band attenuation is not good. Given this stop band attenuation, clearly, the moving average filter cannot separate one band of frequencies from another. In short, the moving average is an exceptionally good smoothing filter (the action in the time domain), but an exceptionally bad low-pass filter (the action in the frequency domain)

(h) The differentiator circuit is essentially **a high-pass filter**. It can generate a square wave from a triangle wave input and produce alternating-direction voltage spikes when a square wave is applied. In ideal cases, a differentiator reverses the effects of an integrator on a waveform, and conversely.





Ideally a differentiator is supposed to give a cos wave.

The moving average filter is a low pass filter while the digital differentiator is a high pass filter.

3.

(a) Finding inverse DTFT for the frequency domain rectangular wave which in the interval is given by:

$$X(e^{j\omega}) = \begin{cases} 1, & |\omega| \leq \omega_c \\ 0, & \omega_c < |\omega| < \pi \end{cases}$$

x[n] is expected to be real.

$$x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{jw})\, e^{jwn}\, dw$$

$$X(e^{jw}) = \begin{cases} 1 & w \leq |w_c| \\ 0 & w_c < |w| < \pi \end{cases}$$

when $w_c \leq \pi$

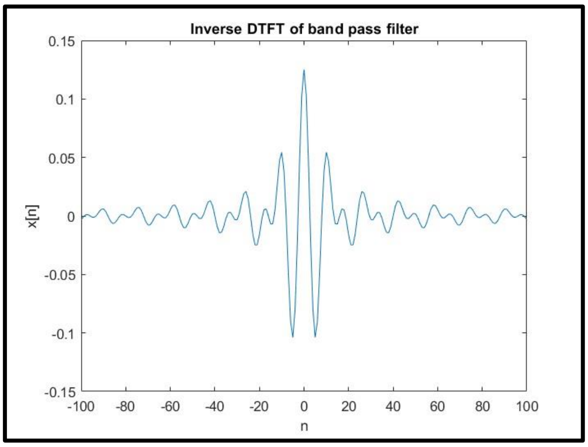
$$x[n] = \frac{1}{2\pi} \int_{-w_c}^{w_c} e^{jwn}\, dw.$$

$$x[n] = \frac{1}{2\pi} \times \frac{1}{jn} \left[e^{jwn} \right]_{-w_c}^{w_c}$$

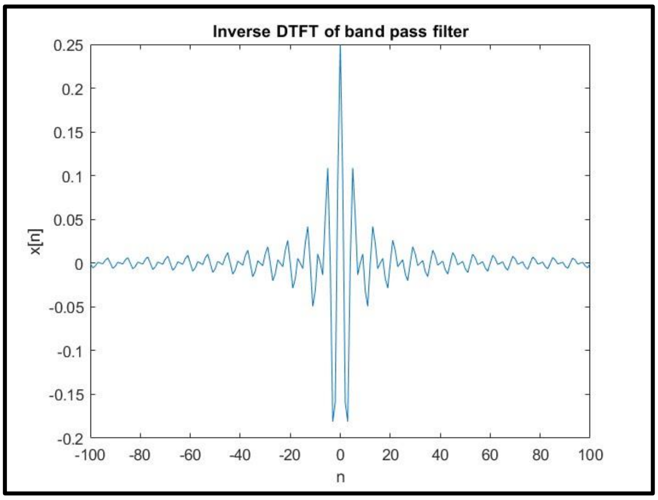
$$x[n] = \frac{\sin(w_c n)}{\pi n}$$

(b) As value of w_c increases, the frequency of x[n] increases. As w_c is equal to pi, x[n] is equal to 1 at only n= 0 while it is 0 for all other values of n.

(c)



W_1 = pi/8 and W_2 = pi/4



W_1 = pi/4 and W_2 = pi/2

As we increase the value of w_1 and w_2 the frequency of x[n] increases and the peaks become sharper. If the difference between w_1 and w_2 is increased further, then x[n] tends to become a delta function.