Signal Processing Lab 6

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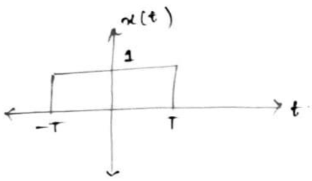
1.

(b)

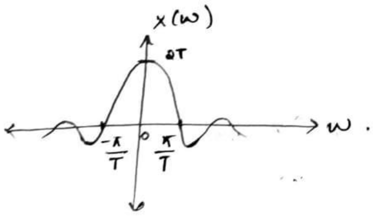
$$x(t) = u(t+T) - u(t-T)$$



The CTFT of $x(t)$ is

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-T}^{T} e^{-j\omega t}\, dt = \left[\frac{e^{-j\omega t}}{-j\omega} \right]_{-T}^{T} = \frac{2\sin(\omega T)}{\omega} = 2T\,\mathrm{sinc}(\omega T)$$



$Re\{X(\omega)\} = 2T\,\mathrm{sinc}(\omega T)$

$Im\{X(\omega)\} = 0$

Magnitude Spectrum :

The magnitude spectrum of the rectangular function is obtained as -

→ At $\omega = 0$

$\mathrm{sinc}(\omega T) = 1$; $|X(\omega)| = 2T$

→ At $\omega T = \pm n\pi$ i.e., at

$\omega = \pm \frac{n\pi}{T}$, $n = 1, 2, \ldots$

$\mathrm{sinc}(\omega T) = 0$, $|X(\omega)| = 0$

Phase Spectrum.

$$\angle X(\omega) = \begin{cases} 0 & \text{if } \mathrm{sinc}(\omega T) > 0 \\ \pm\pi & \text{if } \mathrm{sinc}(\omega T) < 0 \end{cases}$$

(c)

The time scaling property of Fourier Transform supports our observations when T is changed.

This property deals with the effect on the frequency-domain representation of a signal if the time variable is altered. The most important concept to understand for the time scaling property is that signals that are narrow in time will be broad in frequency and vice versa.

$$x(at) \xleftrightarrow{F} \frac{1}{|a|} X\left(j\frac{\omega}{a}\right)$$

Here α = 4, so if CTFT of x(t) is X(jω) (when T = 1) then when T = 4, our signal is x(4t) and it's CTFT is given as 1/|4|X(jω/4).

Therefore at ω = 0; When T = 1, X(ω) = 2 and when T = 4, X(ω) = 8.

(d)

$$x(t) = e^{jt} \qquad [-\pi, \pi]$$

$$X(\omega) = \int_{-\pi}^{\pi} e^{jt} e^{-j\omega t} \, dt .$$

$$= \left[\frac{e^{-jt(\omega-1)}}{-j(\omega-1)} \right]_{-\pi}^{\pi}$$

$$= \frac{+2j \sin \pi(\omega-1)}{j(\omega-1)}$$

$$= \frac{2\sin[\pi(\omega-1)]}{\omega-1}$$

$$Re\{X(\omega)\} = \frac{2\sin[\pi(\omega-1)]}{\omega-1} \qquad Im\{X(\omega)\} = 0$$

$$x(t) = \cos(t) \qquad [-\pi, \pi]$$

$$CTFT\{x(t)\} = \int_{-\pi}^{\pi} \cos(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-\pi}^{\pi} \frac{e^{jt} + e^{-jt}}{2} \cdot e^{-j\omega t}\, dt$$

$$= \frac{1}{2}\int_{-\pi}^{\pi} e^{-jt(\omega-1)} + \frac{1}{2}\int_{-\pi}^{\pi} e^{-jt(\omega+1)}\, dt$$

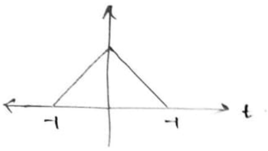
$$= \left[\frac{1}{2} \times \frac{e^{-jt(\omega-1)}}{-j(\omega-1)}\right]_{-\pi}^{\pi} + \left[\frac{1}{2} \times \frac{e^{-jt(\omega+1)}}{-j(\omega+1)}\right]_{-\pi}^{\pi}$$

$$= \frac{\sin[\pi(\omega-1)]}{\omega-1} + \frac{\sin[\pi(\omega+1)]}{\omega+1}$$

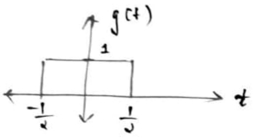
$$Re\{X(\omega)\} = \frac{\sin[\pi(\omega-1)]}{\omega-1} + \frac{\sin[\pi(\omega+1)]}{\omega+1} \qquad Im\{X(\omega)\} = 0$$

(e)

$x(t)$



$g(t) = u(t+\frac{1}{2}) - u(t-\frac{1}{2})$



$x(t) = g(t) * g(t)$

$X(\omega) = G(\omega) \cdot G(\omega)$ [$\because$ convolution in time domain is equal to multiplication in frequency domain].

$X(\omega) = [G(\omega)]^2$

$$G(\omega) = \int_{-\infty}^{\infty} g(t)\, e^{-j\omega t}\, dt.$$

$$= \int_{-1/2}^{1/2} e^{-j\omega t}\, dt.$$

$$= \left[\frac{e^{-j\omega t}}{-j\omega}\right]_{-1/2}^{1/2}$$

$$= \frac{-e^{-j\omega/2} + e^{j\omega/2}}{j\omega}$$

$$= \frac{2\sin \omega/2}{\omega} = \operatorname{Sinc}(\omega/2)$$

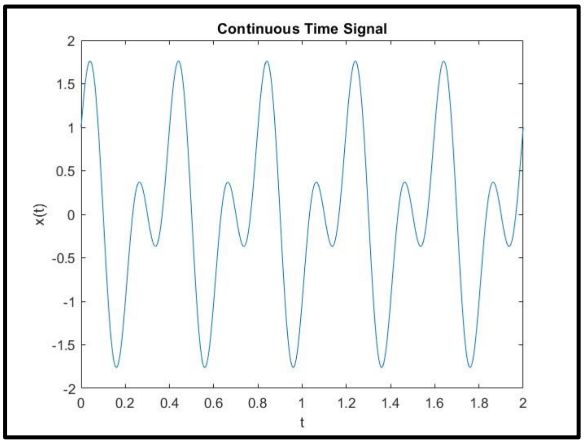
$$X(\omega) = [\operatorname{Sinc}(\omega/2)]^2$$

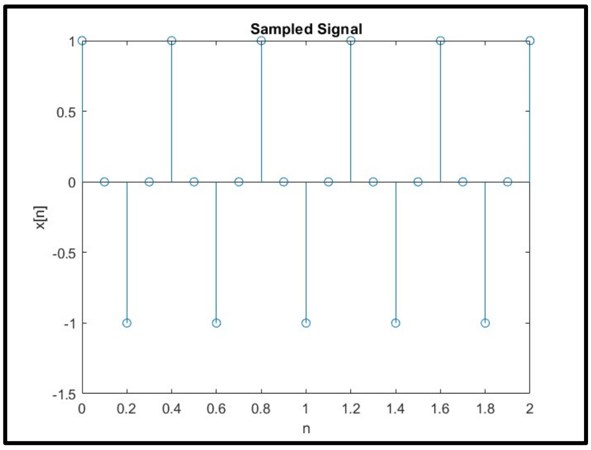
2.

$$x(t) = \cos(5\pi t) + \sin(10\pi t)$$

$$x[n] = x(nT_s)$$

Where x[n] is the sampled signal with sampling frequency Ts = 0.1s.

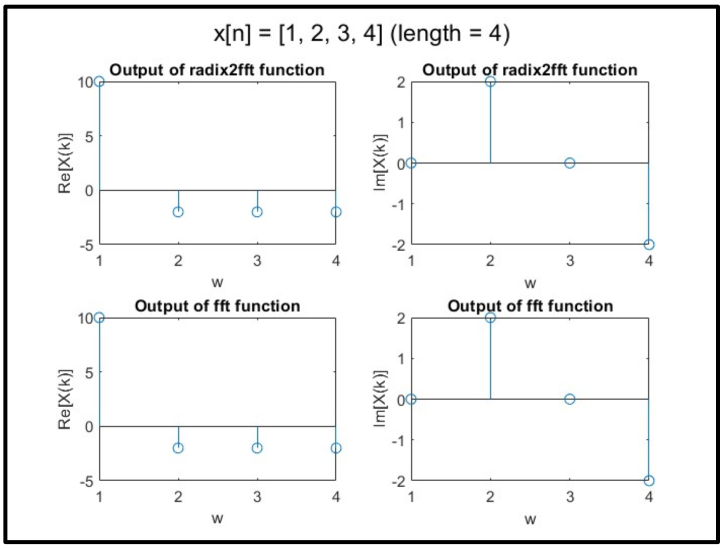




3.

Radix-2 algorithm is a member of the family of Fast Fourier transform (FFT) algorithms. It computes separately the DFTs of the even-indexed inputs (x0,x2,...,xN−2) and of the odd-indexed inputs (x1,x3,...,xN−1), and then combines those two results to produce the DFT of the whole sequence.

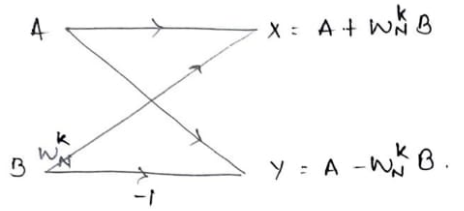
Radix-2 algorithm involves $M=\log_2(N)$ stages, each with N/2 butterflies per stage. Each butterfly requires 1 complex multiply and 2 adds per butterfly.



$x[n] = \{1, 2, 3, 4\}$ length = 4

computing $X(\omega)$ using Radix 2 Algorithm.

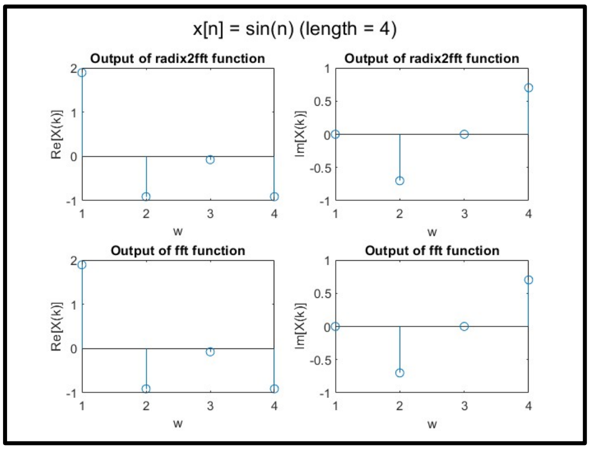
Basic Butterfly Structure:

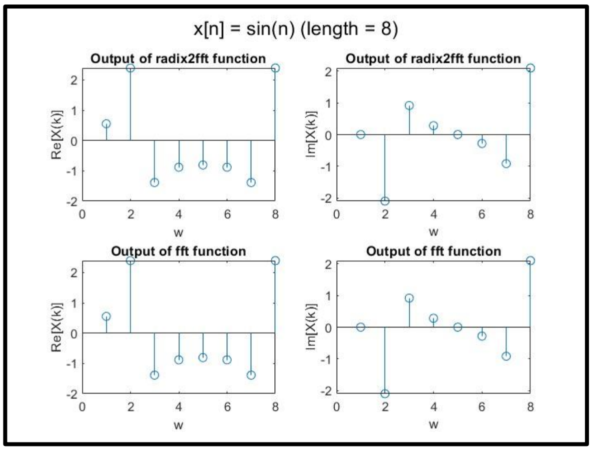


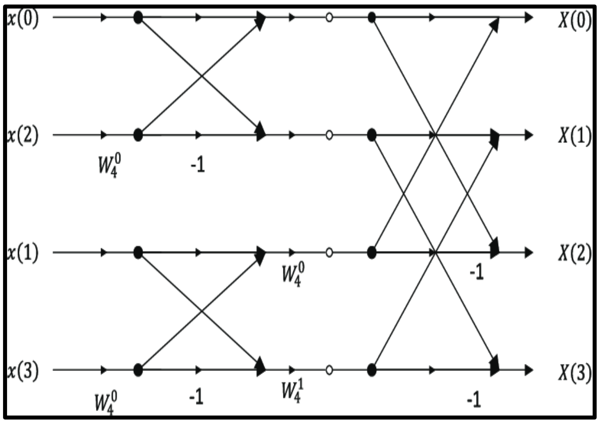
4-point DFT using Radix-2 FFT.

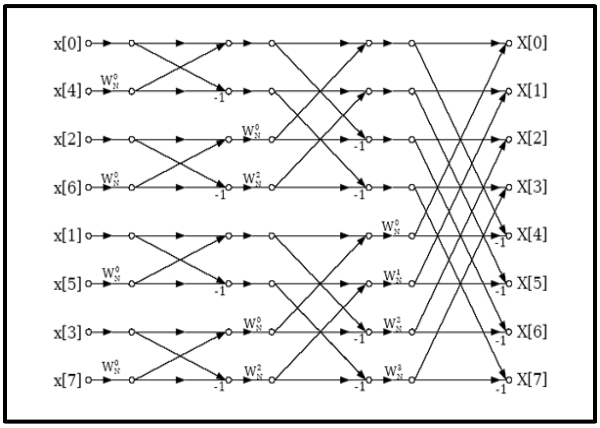
<u>Input</u>	Stage 1 output	Final output.
$x[0]$ 1	$S_0 = x[0] + W_4^0 x[2]$ $= 1 + 1(3) = 4$	$X(0) = S_0 + W_4^0 S_2$ $= 4 + 1(6) = 10$
$x[2]$ 3	$S_1 = x[0] - W_4^0 x[2]$ $1 - 1(3) = -2$	$X(1) = S_1 + W_4^1 S_3$ $= -2 + (-j)(-2) = -2+2j$
$x[1]$ 2	$S_2 = x[1] + W_4^0 x[3]$ $2 + 1(4) = 6$	$X(2) = S_0 - W_4^0 S_2$ $4 - (1)6 = -2$
$x[3]$ 4	$S_3 = x[1] - W_4^0 x[3]$ $2 - 1(4) = -2$	$X(3) = S_1 - W_4^1 S_3$ $-2 - (j)(-2) = -2-2j$

$\therefore X(\omega) = \{10, -2+2j, -2, -2-2j\}$









DFT when N = 2:

DFT when N=2

$$X(k) = x[0]\, W_1^0 + W_2^k\, x[1]\, W_1^0$$

$$X(k+1) = x[0]\, W_1^0 - W_2^k\, x[1]\, W_1^0$$

4.

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jkt}$$

$$c_k = \frac{1}{2\pi} \int_{-\infty}^{\infty} x(t)\, e^{-jkt}\, dt$$

$$x(t) = \cos(t)$$

$$c_k = \frac{1}{2\pi} \int_{-\infty}^{\infty} \cos t \cdot e^{-jkt}\, dt$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{e^{jt} + e^{-jt}}{2} \cdot e^{-jkt}\, dt$$

$$= \frac{1}{2\pi} \times \pi \left[\delta(k-1) + \delta(k+1) \right]$$

$$= \frac{1}{2} \left[\delta(k-1) + \delta(k+1) \right]$$

$$c_k = \begin{cases} \frac{1}{2} & k = -1, 1 \\ 0 & \text{otherwise} \end{cases}$$

(b)

A **low-pass filter** is a filter that passes signals with a frequency lower than a selected cut-off frequency and attenuates signals with frequencies higher.

When $\omega_c = 2$, the low pass filter passes signals with a frequency lower than 2. We get Fourier coefficients at k = 1 and -1. Since $\omega < \omega_c$ (ω at which we observe nonzero Fourier coefficients, that is, at $\omega = 1$ and -1), the Fourier coefficients of the output signal are the same. Therefore, reconstruction of the signal from its Fourier coefficients results in the same signal given as input. The output of Low pass filter is equal to the input signal.

When the cut-off frequency is changed to $\omega_c = 0.5$, $\omega > \omega_c$ (ω at which we observe nonzero Fourier coefficients, that is, at $\omega = 1$ and -1) therefore the output of the low pass filter is zero.

(c)

A **high-pass filter** (**HPF**) is a filter that passes signals with a frequency higher than a certain cut-off frequency and attenuates signals with frequencies lower.

When $\omega_c = 2$, the high pass filter passes signals with a frequency higher than 2. We get Fourier coefficients at k = 1 and -1. Since $\omega < \omega_c$ (ω at which we observe nonzero Fourier coefficients, that is, at $\omega = 1$ and -1), therefore the output of the low pass filter is zero.

When the cut-off frequency is changed to $\omega_c = 0.5$, $\omega > \omega_c$ (ω at which we observe nonzero Fourier coefficients, that is, at $\omega = 1$ and -1), the Fourier coefficients of the output signal are the same. Therefore, reconstruction of the signal from its Fourier coefficients results in the same signal given as input. The output of Low pass filter is equal to the input signal.

(d)

This is a **non-ideal low pass filter**.

Let a_k be the Fourier coefficients of input

Let b_k be the Fourier coefficients of output

The relation between the Fourier coefficients of input and output signal are given as:

$$b_k = a_k H(k\omega_0)$$

The LTI system is complex valued in nature, therefore the output is a scaled version of the input.

5.

(a)

Interp1() is the function used to interpolate the samples to reconstruct the original signal. For Zero Order Hold "previous" method of reconstructing a signal is used.

(b)

Interp1() is the function used to interpolate the samples to reconstruct the original signal. For Linear Interpolation "linear" method of reconstructing a signal is used.

(c)

Comparing the three ways of interpolation we can say that Sinc reconstruction results in a signal with better accuracy as compared to the other two methods.

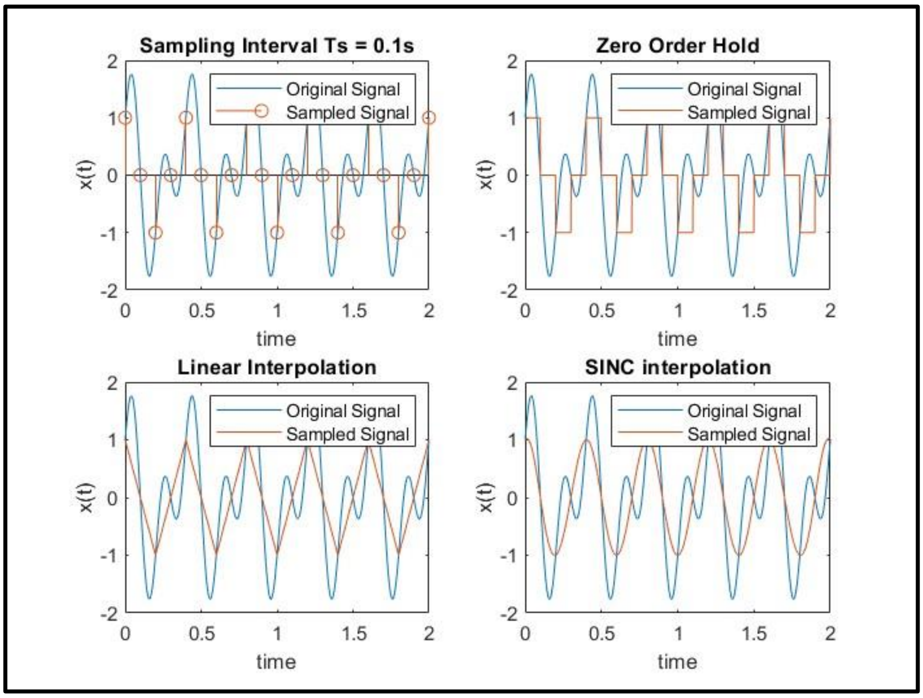
The accuracy in the reconstruction is of the order: Sinc Reconstruction, Linear interpolation, Zero Order Hold.

Zero Order Hold is not accurate for non-linear data. If the points in the data set to change by some value, then Zero Order Hold may not give a good estimate. It is the simplest interpolation method where the nearest data value is assigned.

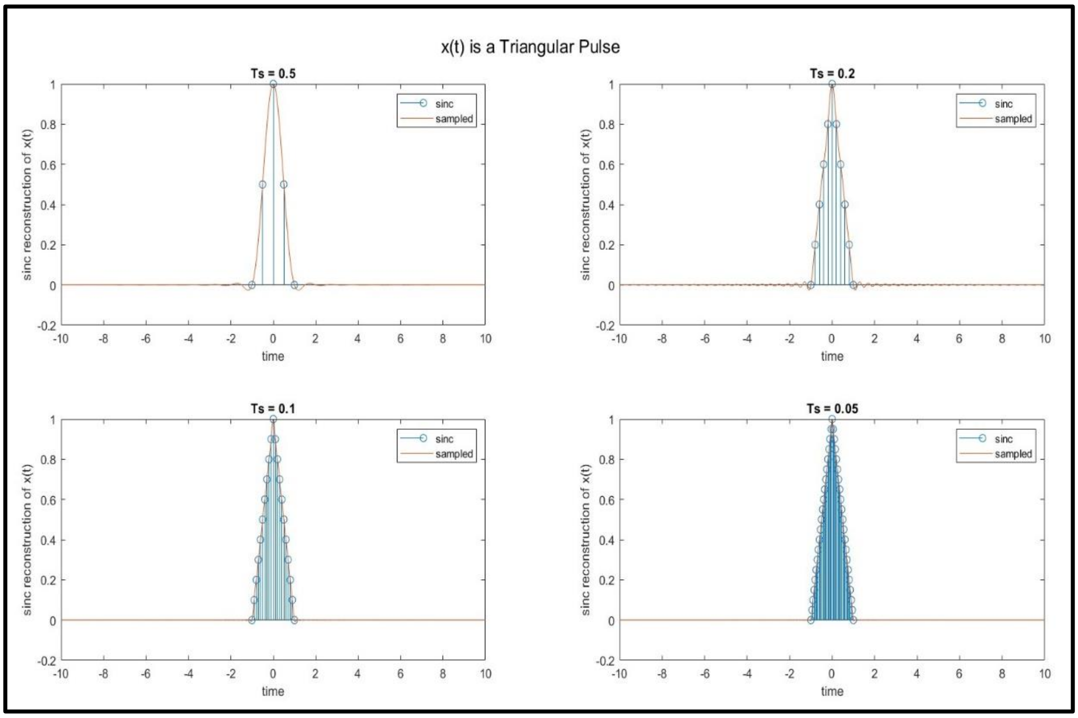
Linear Interpolation has better accuracy than zero order hold but it is still not ideal method of reconstruction of non-ideal data. If the points in the data set change by a large amount, linear interpolation may not give a good estimate. Linear extrapolation can help us estimate values that are either higher or lower than the values in the data set.

Maximum Absolute Error:

i. Zero Order Hold: 1.7601
ii. Linear Interpolation:1.2076
iii. Sinc Reconstruction:1.0341



6.



As Ts decreases the number of samples considered increases and therefore the reconstructed signals come closer to the triangular pulse. The accuracy increases.

7.

"file_example_WAV_1MG.wav":

Bit rate: 256kbpps

Sampling Frequency: 8000

Duration: 33.5296s

Number of Bits: 32

Levels of Quantization: 4.3×10^{9}

"file_example_WAV_2MG.wav":

Bit rate: 512kbps

Sampling Frequency: 16000

Duration:33.529s

Number of Bits: 32

Levels of Quantization: 4.3×10^{9}

"file_example_WAV_5MG.wav":

Bit rate: 1411kbps

Sampling Frequency: 44100

Duration: 29.6287s

Number of Bits: 31.9955

Levels of Quantization: 4.3×10^9

"file_example_WAV_10MG.wav":

Bit rate: 1411kbps

Sampling Frequency: 44100

Duration: 58.9936s

Number of Bits: 31.9955

Levels of Quantization: 4.3×10^9

In principle the audio quality goes down as you lower the sample rate and as the sample rate was reduced the duration of the audio signal increased (there is a delay in the signal) while when the sample rate was increased the duration of the audio signal decreased and quality increases.

These observations can be explained using the time scaling property of Fourier transform.

$$x(t) \leftrightarrow X(\omega)$$

$$x(at) \leftrightarrow \frac{1}{|a|} X\left(\frac{\omega}{a}\right)$$

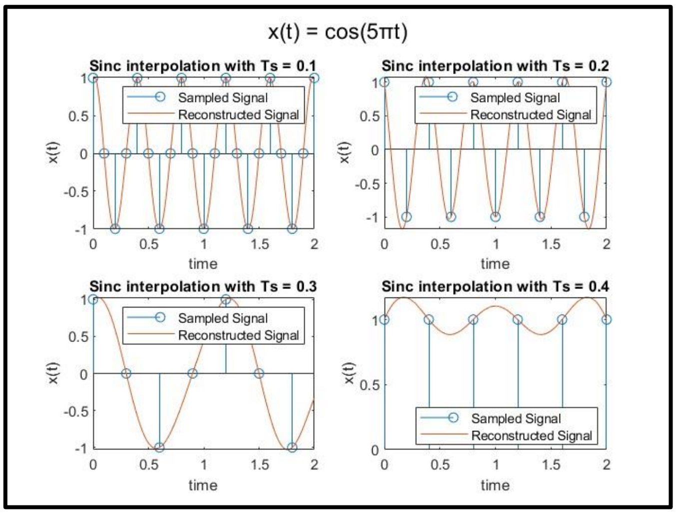
8.

In signal processing, the Nyquist rate is a value (in units of samples per second or hertz, Hz) equal to twice the highest frequency (bandwidth) of a given function or signal.

$x(t) = \cos(5\pi t)$

Highest frequency = 2.5Hz

Nyquist rate = 5Hz



- As Ts increases, the number of samples considered decreases and as the number of samples considered decreases, the reconstructed signal represents the original signal lesser and lesser.