Signal Processing – Lab 2

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1.

A function [N, ROC, C, S] = roc_cs(p) with the input vector p containing location of poles in the Z-plane, was written. It outputs N (number of possible ROC for H(z)), ROC (indicates an ROC in the form $r_1 < |z| < r_2$), C (tells whether the system is causal or not), S (tells about the stability of the system).

(b) Outputs for the given Inputs:

- P =3

$$\mathrm{N} = 2 \quad \mathrm{ROC} = \begin{matrix} 0 & 3 \\ 3 & inf \end{matrix} \quad \mathrm{C} = \begin{matrix} 0 \\ 1 \end{matrix} \quad \mathrm{S} = \begin{matrix} 1 \\ 0 \end{matrix}$$

- P = 0.1

$$\mathrm{N} = 2 \quad \mathrm{ROC} = \begin{matrix} 0 & 0.1 \\ 0.1 & inf \end{matrix} \quad \mathrm{C} = \begin{matrix} 0 \\ 1 \end{matrix} \quad \mathrm{S} = \begin{matrix} 0 \\ 1 \end{matrix}$$

- P = 0

$$\mathrm{N} = 1 \quad \mathrm{ROC} = 0 \quad \mathrm{inf} \quad \mathrm{C} = 1 \quad \mathrm{S} = 1$$

- P = [0, 0.5]

$$\mathrm{N} = 2 \quad \mathrm{ROC} = \begin{matrix} 0 & 0.5 \\ 0.5 & inf \end{matrix} \quad \mathrm{C} = \begin{matrix} 0 \\ 1 \end{matrix} \quad \mathrm{S} = \begin{matrix} 0 \\ 1 \end{matrix}$$

- P = [2. -0.5]

$$\mathrm{N} = 3 \quad \mathrm{ROC} = \begin{matrix} 0 & 0.5 \\ 0.5 & 2 \\ 2 & inf \end{matrix} \quad \mathrm{C} = \begin{matrix} 0 \\ 0 \\ 1 \end{matrix} \quad \mathrm{S} = \begin{matrix} 0 \\ 1 \\ 0 \end{matrix}$$

- P = [0.5, -0.5]

$$\mathrm{N} = 2 \quad \mathrm{ROC} = \begin{matrix} 0 & 0.5 \\ 0.5 & inf \end{matrix} \quad \mathrm{C} = \begin{matrix} 0 \\ 1 \end{matrix} \quad \mathrm{S} = \begin{matrix} 0 \\ 1 \end{matrix}$$

- P = [2, 2, 2]

$$\mathrm{N} = 2 \quad \mathrm{ROC} = \begin{matrix} 0 & 2 \\ 2 & inf \end{matrix} \quad \mathrm{C} = \begin{matrix} 0 \\ 1 \end{matrix} \quad \mathrm{S} = \begin{matrix} 1 \\ 0 \end{matrix}$$

- P = [0, 1 ,2]

$$\mathrm{N} = 3 \quad \mathrm{ROC} = \begin{matrix} 0 & 1 \\ 1 & 2 \\ 2 & inf \end{matrix} \quad \mathrm{C} = \begin{matrix} 0 \\ 0 \\ 1 \end{matrix} \quad \mathrm{S} = \begin{matrix} 0 \\ 0 \\ 0 \end{matrix}$$

- P = [-0.5, j]

$$\mathrm{N} = 3 \quad \mathrm{ROC} = \begin{matrix} 0 & 0.5 \\ 0.5 & 1 \\ 1 & inf \end{matrix} \quad \mathrm{C} = \begin{matrix} 0 \\ 0 \\ 1 \end{matrix} \quad \mathrm{S} = \begin{matrix} 0 \\ 0 \\ 0 \end{matrix}$$

- P = [0, j, -j]

$$\mathrm{N} = 2 \quad \mathrm{ROC} = \begin{matrix} 0 & 1 \\ 1 & inf \end{matrix} \quad \mathrm{C} = \begin{matrix} 0 \\ 1 \end{matrix} \quad \mathrm{S} = \begin{matrix} 0 \\ 0 \end{matrix}$$

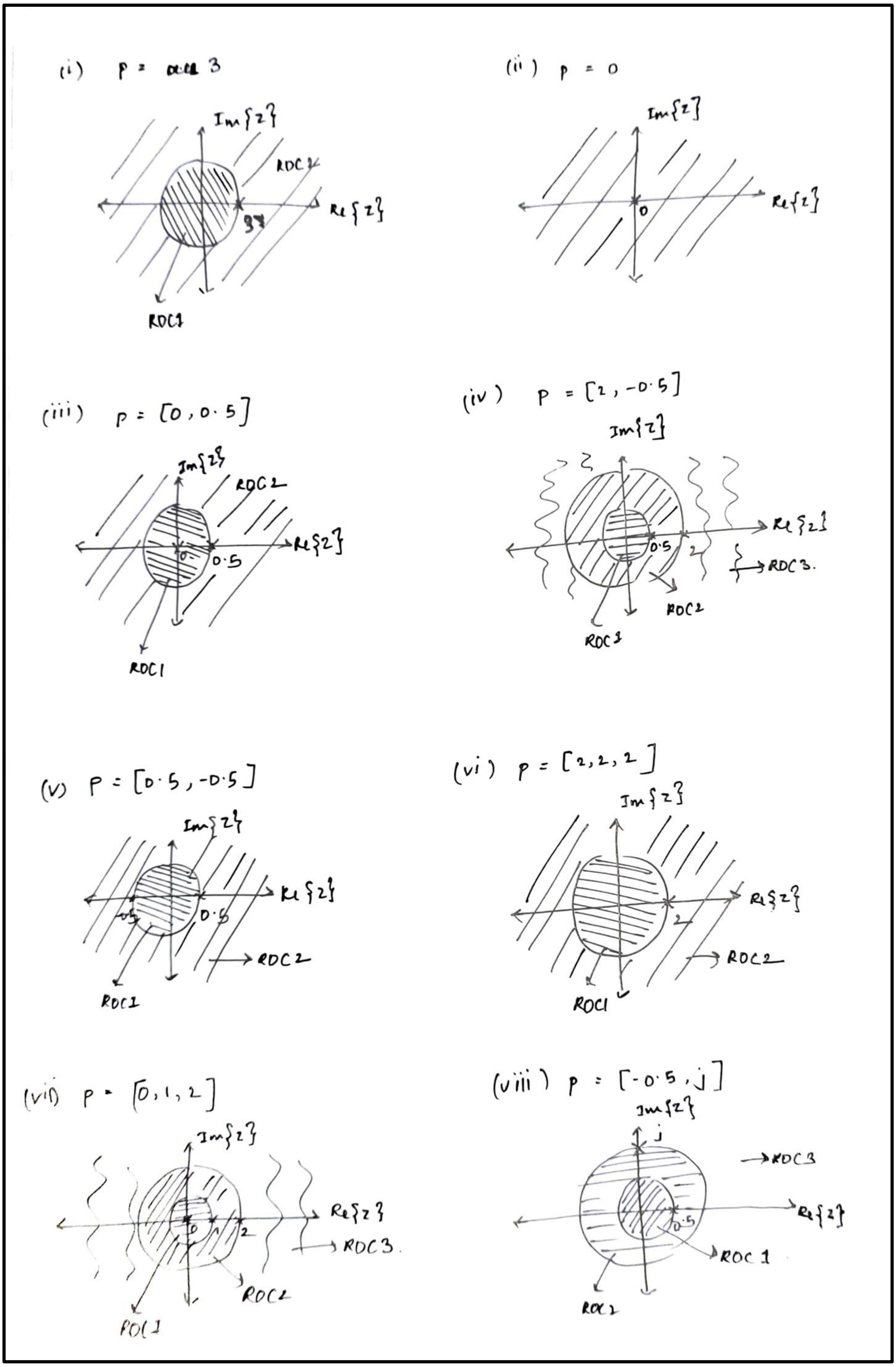
- P = [0.5, -0.5, 2 + j, 2 – j]

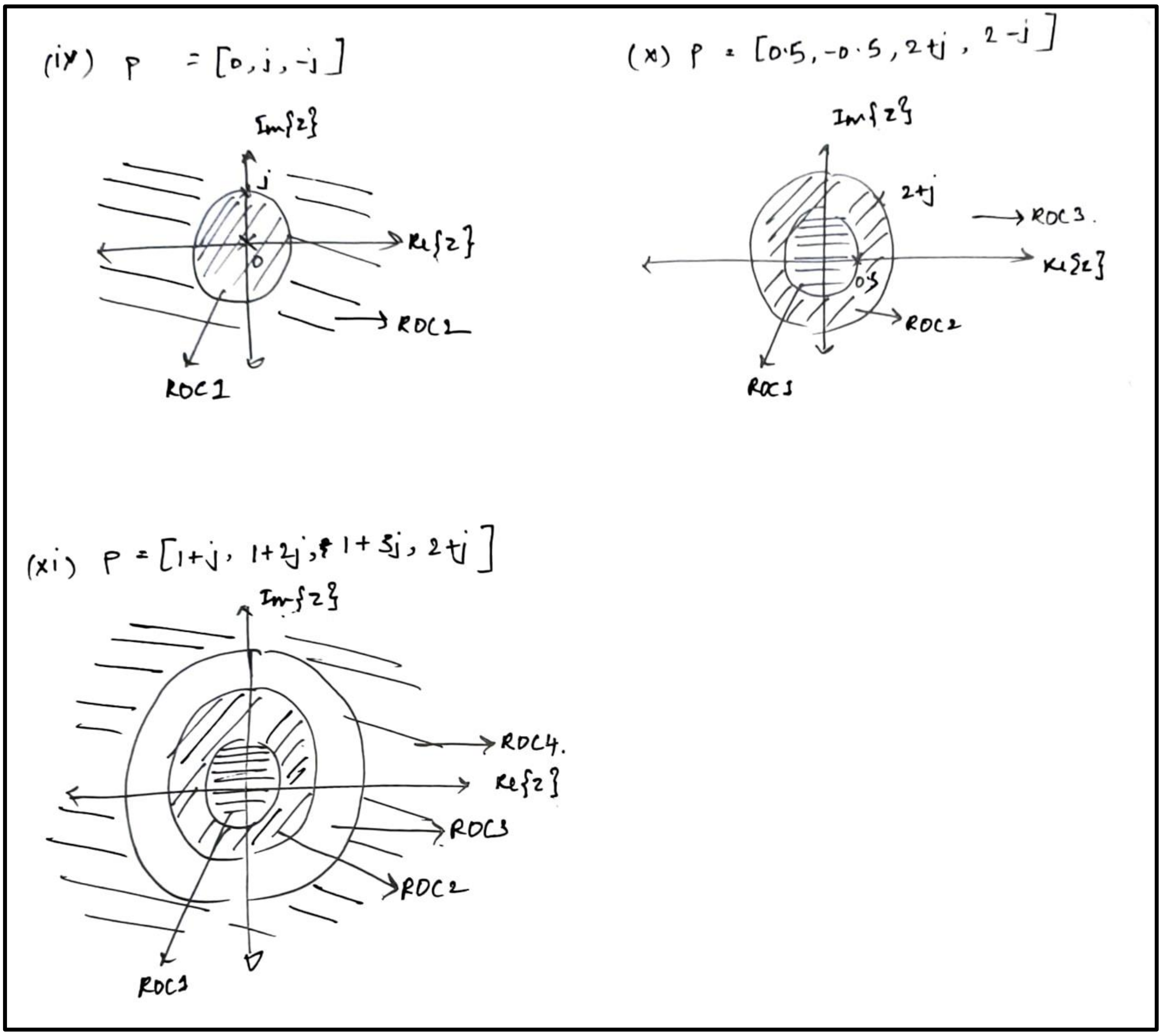
$$\mathrm{N} = 3 \quad \mathrm{ROC} = \begin{matrix} 0 & 0.5 \\ 0.5 & 2.236 \\ 2.236 & inf \end{matrix} \quad \mathrm{C} = \begin{matrix} 0 \\ 0 \\ 1 \end{matrix} \quad \mathrm{S} = \begin{matrix} 0 \\ 1 \\ 0 \end{matrix}$$

- P = [1 + j, 1 + 2j, 1 + 3j, 2 + j]

$$\mathrm{N} = 4 \quad \mathrm{ROC} = \begin{matrix} 0 & 1.414 \\ 1.414 & 2.236 \\ 2.236 & 3.162 \\ 3.162 & inf \end{matrix} \quad \mathrm{C} = \begin{matrix} 0 \\ 0 \\ 0 \\ 1 \end{matrix} \quad \mathrm{S} = \begin{matrix} 1 \\ 0 \\ 0 \\ 0 \end{matrix}$$

Verification:





2.

(c)

There are infinite impulse responses for p =0.8

(d)

The graph is monotonically decreasing for p = -0.8 and flipping alternatively for p = 0.1 and p = 0.8.

3.

(a)

$$H(z) = \frac{z^2 - (2\cos\theta) z + 1}{z^2 - (2r\cos\theta) z + r^2} \quad , \quad r \in (0, 1), \ \theta \in [0, \pi]$$

- $r = 0.1$ and $\theta = \pi/2$

$$H(z) = \frac{z^2 + 1}{z^2 + \frac{1}{100}}$$

Poles $\rightarrow$ $0.1j$, $-0.1j$ zeros $\rightarrow$ j, $-j$

- $r = 0.2$ and $\theta = \pi/6$

$$H(z) = \frac{z^2 - \sqrt{3} z + 1}{z^2 - 0.2\sqrt{3} z + 0.04}$$

Poles $\rightarrow$ $0.24 - 0.42j$, $0.24 + 0.42j$ zeros $\rightarrow$ $\frac{\sqrt{3}}{2} + -j/2$, $\frac{\sqrt{3}}{2} + j/2$

- $r = 0.9$ and $\theta = \pi/4$

$$H(z) = \frac{z^2 - \sqrt{2} z + 1}{z^2 - 0.9\sqrt{2} z + 0.81}$$

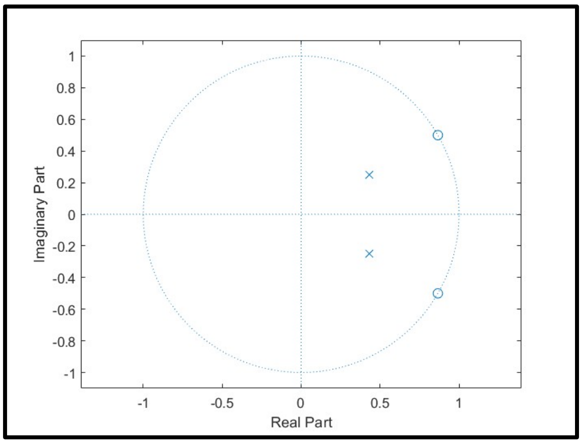
Poles $\rightarrow$ $-0.63 - 0.63j$, $-0.63 + 0.63j$ zeros $\rightarrow$ $\frac{-1}{\sqrt{2}} + \frac{j}{\sqrt{2}}$, $\frac{-1}{\sqrt{2}} - \frac{j}{\sqrt{2}}$

- $r = 0.4$ and $\theta = 2\pi/3$

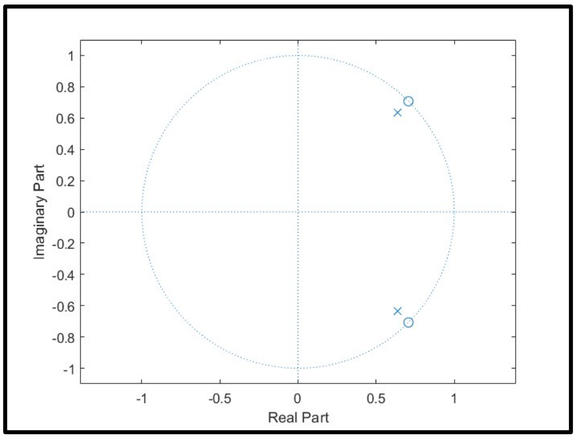
$$H(z) = \frac{z^2 + z + 1}{z^2 + 0.4z + 0.16}$$

Poles $\rightarrow$ $-0.2 - 0.34j$, $-0.2 + 0.34j$ zeros $\rightarrow$ $\frac{-1}{2} - j\frac{\sqrt{3}}{2}$, $\frac{-1}{2} + j\frac{\sqrt{3}}{2}$

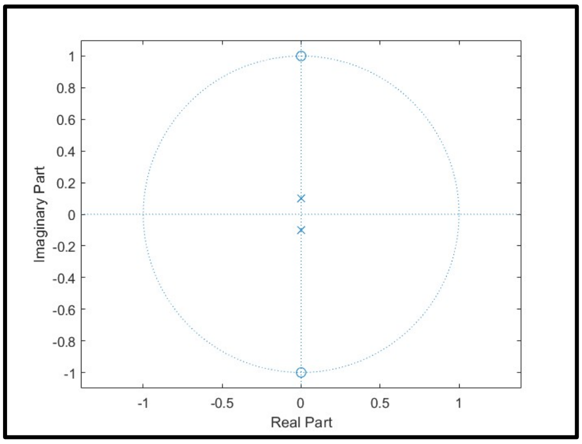
Plots for different values of r and theta:



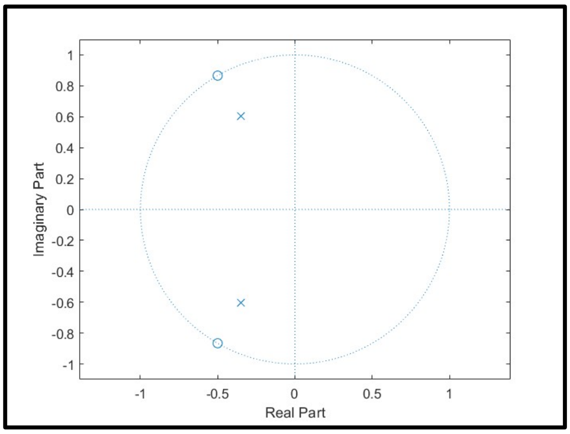
R = 0.2 theta = 30°



R = 0.9 theta = 45°



R = 0.1 theta = 90°



R = 0.4 theta = 120°

Pole-zero plots are in correspondence to the roots that we got.

The poles of the transfer function lie on a circle with radius r and at an angle theta on either side of the x axis. While the zeros of the transfer function lie on a unit circle at angle theta from the x axis.

(b)

Yes, the system can be both causal and stable simultaneously.

For a system to be causal, the ROC must be outside the outermost circle and also include infinity. While for a system to be stable the ROC must include the unit circle.

Since the pole lies between 0 and 1, and the ROC extends till infinity, the system is both causal and stable simultaneously.

(c)

When theta value is changed from 0 to pi, we observe that the rate at which the magnitude tends to infinity (phase tends to 90 degrees) increases.

Increasing theta decreases the bandwidth of the filter because the zeros shift by an angle theta wrt the origin. When theta is increased from 0 to pi, its nature changes from a low pass filter to a high pass filter.

(d)

When r is changed from 0 to 1 gradually, the variation in magnitude decreases. This does not affect where the pole occurs, and the filter becomes more ideal.

4.

(a)

From the frequency response we can say that there is a pole at points where we get a peak. Furthermore, we can say that there will be zeros at troughs of the frequency response.

(b)

When we substitute the values of z in the transfer function, the denominator equals to zero since there is a pole at that value of z.