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PHYS265

Calculating the Invariant Mass of the Z^0- Boson

I. Introduction

The ATLAS (**A** **T**oroidal **L**hc **A**pparatu**S**) experiment at CERN is a general purpose detector investigating a wide array of particle physics topics. At ATLAS, high energy protons are smashed together and the detector measures the particles that come out of the collision. This report describes our calculation of the mass of one such particle, the Z^0- boson, which is the neutral carrier of the weak force. Z^0- bosons decay very quickly, and 10% of the time they decay into an oppositely charged lepton pair. By analyzing the momenta and energies of these leptons, we can deduce the mass, m_0, of the Z^0- boson. We used data from ATLAS to make these calculations, and fitted our calculated mass distribution to a theoretical mass distribution to find the best fit m_0. We used a χ^2analysis to determine the compatibility of the theory to our data, and completed a fitted parameter scan in order to better visualize the fit.

II. The Invariant Mass Distribution

We started by calculating the invariant mass distribution, plotting it, and fitting it to a theoretical distribution. The data from ATLAS we used included the total momentum (p_T), pseudorapidity (η), azimuthal angle (ϕ), and energy (E) of each of the two leptons, for 5,000 different boson decays. The pseudorapidity of a particle is the angle the particle makes with the beamline, and the azimuthal angle is the angle of the particle about the beam. The invariant mass for a given event can be calculated using the following equation:

$$M = \sqrt{E^2 - (p_x^2 + p_y^2 + p_z^2)} \tag{1}$$

where E is the sum of the energies of the two leptons, and p_x, p_y, and p_z are the sums of the x, y, and z components, respectively, of the momenta of the two leptons. p_x, p_y, and p_z can be found using the following equations:

$$p_x = p_T cos(\phi) \qquad p_y = p_T cos(\phi) \qquad p_z = p_T sinh(\eta) \tag{2}$$

Using these formulae, we calculated the 5,000 Z^0 masses. We then plotted the values in a histogram, as shown in Figure 1. Since we can assume each event is independent of the others, we can approximate each bin as a Poisson counting experiment with an average at the center of the bin. This allows us to calculate the error bars of each bin using the following equation:

$$\sigma = \sqrt{N} \tag{3}$$

where N is the count of the bin in question. Because the lepton pair is the product of a decaying Z^0- boson, and energy and mass are conserved, we know that the sum of their energies must be greater than or equal to the energy of the Z^0 particle. Therefore, we expect our histogram to peak at the actual mass of Z^0. The distribution of decays ($\mathcal{D}$) of a reconstructed mass (m) should match what is called a Breit-Wigner distribution, which can be described using the following formula:

$$\mathcal{D}(m; m_0, \Gamma) = \frac{1}{\pi} \frac{\Gamma/2}{(m - m_0)^2 + (\Gamma/2)^2} \tag{4}$$

where m_0 is the true rest mass of the particle in question and Γ is a "width" parameter. We took the central twelve bins of distribution, from 87.0 to 93.0 GeV, which contain roughly 50% of the data. Then we performed a fit of the data to the theory with m_0 and Γ as fitting parameters, using the scipy.optimize.curve_fit function from the scipy library in python, which uses nonlinear least squares. The fit curve is displayed in Figure 1. We calculated a best fit value and found its uncertainty from the covariance matrix. Our resulting value of m_0 was 90.3 $\pm$ 0.1 GeV. Finally, we created a plot of the residuals between the data and the theory.

To evaluate the quality of the fit, we performed a χ^2 analysis between the data and theory. We fitted the twelve data points with two fitting parameters, so we had 10 degrees of freedom. We found a χ^2 value of 10.0, and a p-value of 0.4. This means that if we were to repeat these calculations many times with different data sets, we would get a χ^2 value of our value or higher about 40% of the time, assuming the model is correct. This indicates good agreement between the data and theory.

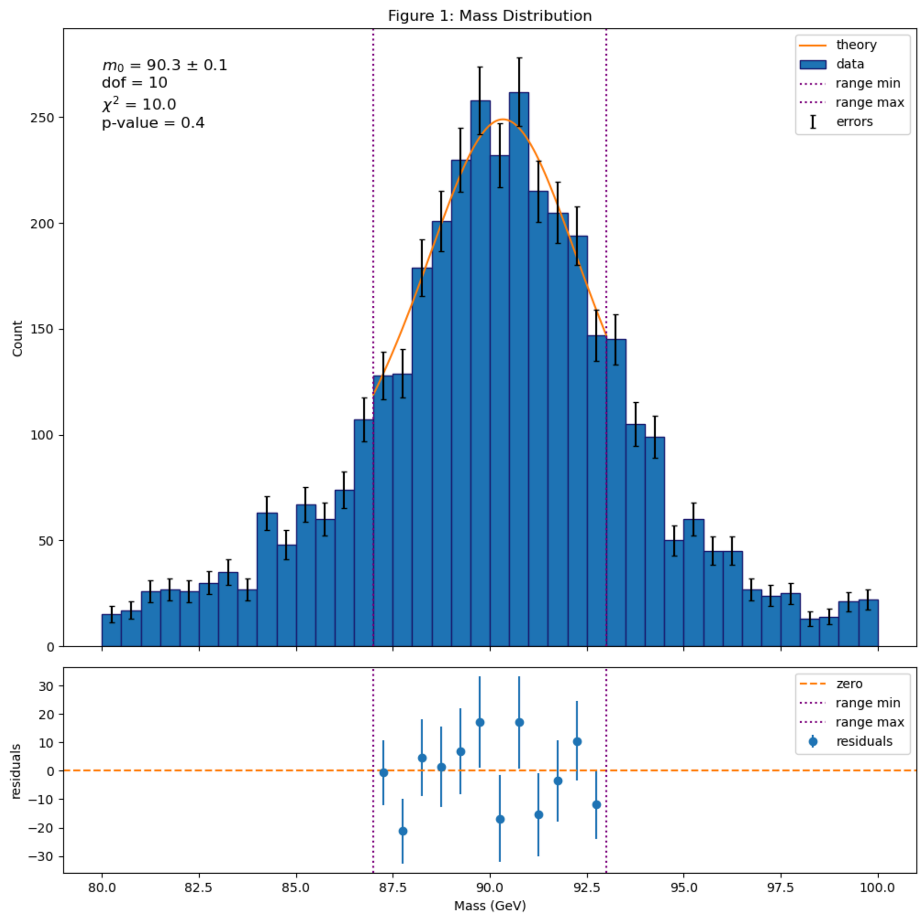

Figure 1: Top: A histogram of the reconstructed mass distribution and theoretical distribution. For the histogram, we used 40 bins ranging from 80 to 100 GeV, since about 75% of the data fell in this range. Important values are included in the top left. Bottom: A residual plot between the data and theory. The zero line indicates total agreement.

III. The 2D Parameter Scan

In order to better visualize the fit, we performed a χ^2 parameter scan of the two fitting parameters, and created a 2D contour plot of the scan. To do this, we created a χ^2 map by looping through a range of values for each of the fitting parameters, such that a point on the map represents a unique combination of fitting parameter values. We used each combination as parameters in the Breit-Wigner distribution, then found the theoretical values and calculated the χ^2. We then used the following equation to create the contour plot:

$$\Delta\chi^2 = \chi^2 - \chi^2_{min} \tag{5}$$

where χ^2_{min} is the minimum χ^2 on the whole map. We clipped the $\Delta\chi^2$ at 35 to improve readability. The plot is shown in Figure 2. Additionally, the figure shows the 1σ and 3σ lines, or where $\Delta\chi^2$ equals 2.30 and 9.21 respectively. These lines represent where the fitting parameters are 1 and 3 standard deviations away from their best fit values. If we were to repeat these calculations many times with different data sets, we would expect the best fit parameters to fall within the 1σ line about 68% of the time, and within the 3σ line about 99.7% of the time.

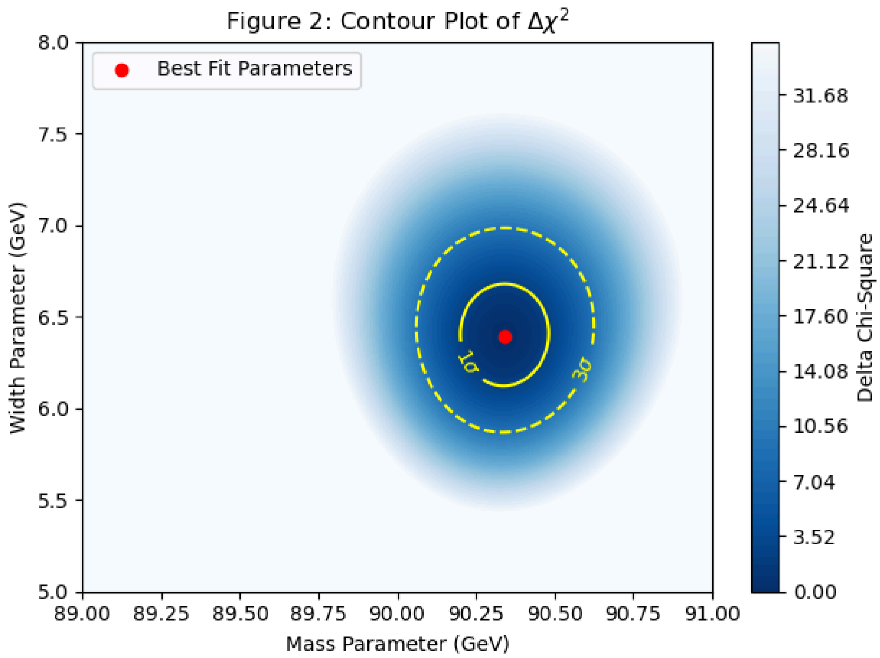

Figure 2: Contour plot of $\Delta\chi^2$ over a range of 5 - 8 GeV for the width parameter, and 89 - 91 GeV for the mass parameter. The yellow solid and dashed lines represent the 1σ and 3σ, respectively. The best fit parameters we marked are the pair we generated in our analysis.

IV. Discussion and Future Work

Through the process described in this report, we have determined the rest mass of the Z^0- boson to be 90.3 $\pm$ 0.1 GeV from analyzing ATLAS data from 5,000 independent events. From our p-value of 0.4, we can conclude that this theory and our data are consistent.

According to the Particle Data Group (PDG), the mass of a Z^0- boson is 91.1880 ± 0.0020 GeV. The difference between this value and our calculated value is 0.8 ± 0.1 GeV. Since the difference is more than two standard deviations away from zero, we can conclude that these values are not consistent with each other. Over the course of our investigation, we made several assumptions and simplifications which could explain this discrepancy. Firstly, we did not consider any specific uncertainties in the ATLAS data due to the instrumentation used to make the original measurements. If we had considered these, it would have led to a larger uncertainty in our final value. Additionally, we only performed the fit to the theory using about 50% of the data, which resulted in different values of our fitting parameters than if we had used all of the data.

More work is needed to understand the significance of these simplifications in order to explain and reduce the difference in our calculated value to the literature value. In future studies, calculations which include larger amounts of data, as well as take into consideration specific uncertainties of each measurement, should be used.