## I. Introduction

The purpose of this report is to simulate the gravitational potential energy and force on the Apollo 11 capsule, and the performance of the Saturn V Stage 1 rocket. We used the numpy, SciPy, and matplotlib libraries in python, which include tools for complex calculations and plotting. For the gravitational potential in the area of interest, and gravitational force on the capsule, we defined functions that calculated these values for the Earth-Moon system, and then plotted them over a distance of -1.5 to 1.5 times the distance from the Earth to the Moon in both the x and y directions. For these plots, we placed the Earth at the origin, and the Moon at coordinates

$$\left(\frac{dist_{Earth,Moon}}{\sqrt{2}}\right), \left(\frac{dist_{Earth,Moon}}{\sqrt{2}}\right)$$

We then calculated and plotted the value in question over the entire grid. The following sections detail the important components of our calculations.

## II. The Gravitational Potential of the Earth-Moon System

Part two of the report describes the gravitational potential of the Earth-Moon system. Gravitational potential is the measure of gravitational potential energy per unit mass of an object due to a source object, like the Earth or Moon. Gravitational potential is important in orbital mechanics to describe the motion of satellites and rockets. To create the figure, we defined a function in python which takes the masses, radii and positions of the Earth and Moon, and the point you wish to calculate the potential at. It then calculates and returns the total gravitational potential at that point in the system. The calculation is based on the formula for gravitational potential,

$$\Phi(r) = \frac{-GM}{r}$$

where G is the gravitational constant, M is the mass of the Earth or Moon, and r is the distance from the Earth or Moon to the point of interest.

We used matplotlib to plot the gravitational potential energy in a color-mesh plot and a contour plot, using the procedure described above. The color-mesh plot (Fig. 1) shows a continuous distribution of

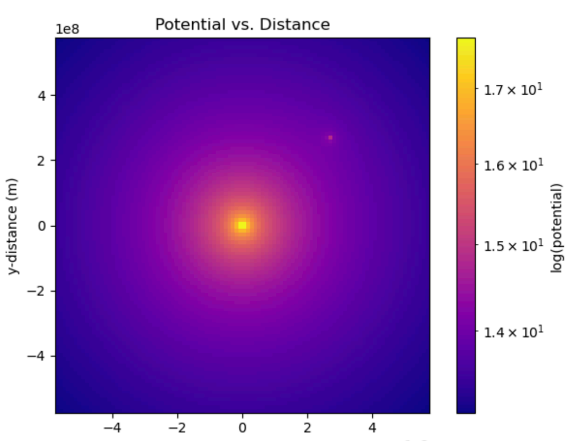


Fig. 1: Color-mesh plot of the gravitational potential from the Earth-Moon system over a distance of three times the distance from the Earth to the moon in the x and y directions. Yellow indicates a larger magnitude.

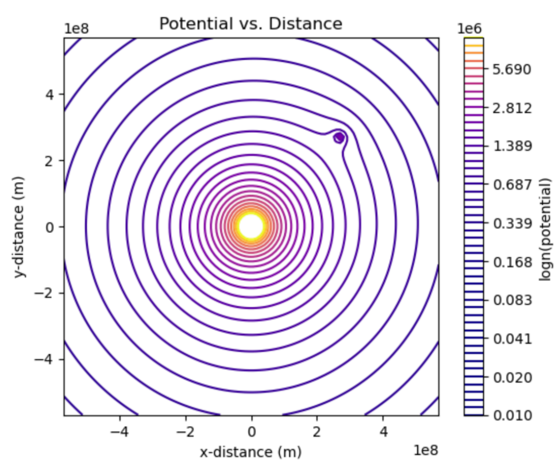


Fig. 2: Contour plot of the gravitational potential from the Earth-Moon system over a distance of three times the distance from the Earth to the moon in the x and y directions. Yellower, denser lines indicate a larger magnitude.

the gravitational potential energy, where the color yellow represents a higher magnitude, and purple represents a lower magnitude. The contour plot (Fig. 2) uses the same color scale, but uses discrete lines. A larger density of lines also indicates a greater magnitude of gravitational potential.

As we expect, both plots show a larger, yellow region near the Earth, and a smaller yellow region near the Moon, indicating stronger potential near the two bodies.

**III. The Gravitational force of the Earth-Moon system**

Part three of the report describes the gravitational force on the Apollo 11 Command Module from the Earth-Moon system. Gravitational force is the force due to gravity on an object or system, and in this case we considered the Earth and the Moon. Again, we started by defining a function which takes the mass of the command module, mass of the Earth or Moon, and the locations of both masses as inputs, and returns the force on the object. The calculation is based on the formula for gravitational force,

$$\vec{F}_{21} = -G\frac{M_1 m_2}{|\vec{r}_{21}|^2}\hat{r}_{21}$$

where $M_1$ is the mass of the Earth or moon, $m_2$ is the mass of the command module, r is the displacement between the objects, and the subscript '21' represents the direction of r, which points from $m_2$ to $M_1$. To make a plot of the force we made a streamplot with the same methods described above. We calculated the force from the Earth and the force from the Moon for many points over the grid. We then added the forces from the Earth and Moon together and plotted the total on the grid to create the streamplot (Fig. 3). On the plot, the arrows indicate the direction of the force, and the colors indicate the magnitude. Again, yellow represents a larger magnitude, purple a smaller magnitude. As expected, there is a larger magnitude nearby the Earth and Moon, and all arrows point towards the two bodies.

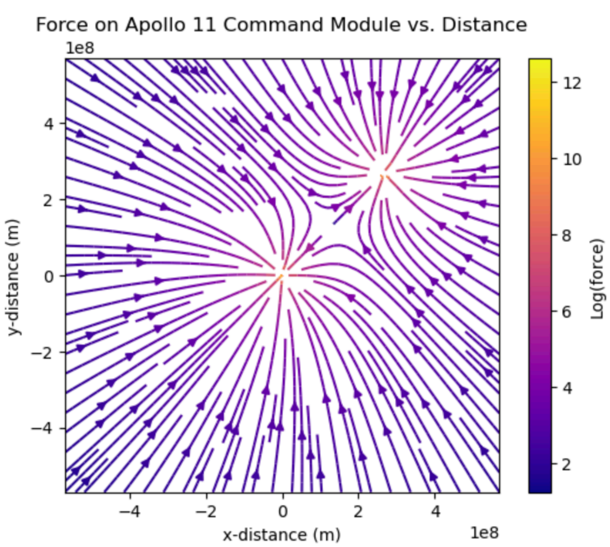


Fig. 3: Stream plot of the gravitational force from the Earth-Moon system over a distance of three times the distance from the Earth to the moon in the x and y directions. Yellow indicates a larger magnitude.

**IV. Projected performance of the Saturn V Stage 1**

Part four of the report calculated the projected performance of the first stage of the Saturn V rocket by finding the burn time, or the length of time the fuel will burn for, and the altitude the rocket would reach by the end of the burn. To find the burn time, we used the formula,

$$T = \frac{m_0 - m_f}{\dot{m}}$$

where T is the total burn time, $m_0$ is the wet mass, or the initial mass including the fuel, rocket, and payload, $m_f$ is the dry mass, or the mass of the rocket without the fuel, and $\dot{m}$ is the fuel burn rate. Based on this formula, the total burn time is about 158 seconds.

Next we calculated the projected altitude of the Saturn V rocket at the end of the burn. We started with the formula for the change in the rocket's velocity at a given time,

$$\Delta v(t) = v_e \ln\left(\frac{m_0}{m(t)}\right) - gt$$

where $v_e$ is the fuel exhaust velocity, which is the velocity at which the fuel is ejected from the rocket, $m(t) = m_0 - \dot{m}t$ is the mass of the rocket at time t, and g is acceleration due to gravity. By integrating this function from 0 to T, you can find the final altitude of the rocket when it finished burning.

To perform the integration, we defined a function that returns $\Delta v(t)$ and takes time, wet mass, dry mass, burn rate, exhaust speed, and acceleration due to gravity as inputs. We defined it such that once the time reaches the burn time T, the function goes to zero, since the rocket will no longer be accelerating once the fuel is used up. We used the scipy.integrate.quad() function in python to find a numerical approximation of the value of the integral. The estimated final altitude was about 74 km.

**V. Discussion and Future Work**

We made several approximations and assumptions to perform these calculations. For all constants, including the masses and radii of the Earth and Moon, the wet mass, dry mass, burn rate, exhaust speed of the Saturn V rocket, and acceleration due to gravity and the gravitational constant, we only used between two and four significant figures. Additionally, in calculating the projected burn time and altitude of the Saturn V rocket, we assumed the fuel burn rate and fuel exhaust speed are constant, which in practice they are not.

NASA's recent test results from the Saturn V prototype found the burn time to be about 160 seconds, and the altitude to be about 70 km. In our calculations, we found the burn time to be about 158 seconds. The small disagreement could be due to the incorrect assumption that the fuel burn rate is constant. We found the altitude to be about 74 km, which is larger than NASA's value. This difference could be because we did not consider factors such as atmospheric drag or weather conditions that could impact the path of the rocket.

In future calculations, we should use more significant figures to improve accuracy, and we should use a more accurate model of the fuel burn rate and fuel exhaust speed. We should also include more details in the projection of the rocket burn time and altitude to create a more accurate simulation of the environment.