

Problem 1: Ch. 7 - Q. 7.10

- a) $\text{Smoke} \rightarrow \text{Smoke}$ VALID
 $\neg \text{Smoke} \vee \text{Smoke}$ (Definition of implication)
 True (Valid definition of \vee)
- b) $\text{Smoke} \rightarrow \text{Fire}$ INVALID
 $\neg \text{Smoke} \vee \text{Fire}$ (Definition of implication)
 False (Unsatisfiable)
- c) $(\text{Smoke} \rightarrow \text{Fire}) \rightarrow (\neg \text{Smoke} \rightarrow \neg \text{Fire})$ INVALID
 $(\neg \text{Smoke} \vee \text{Fire}) \rightarrow (\text{Smoke} \vee \neg \text{Fire})$ (Definition of implication)
 $\neg(\neg \text{Smoke} \vee \text{Fire}) \vee (\text{Smoke} \vee \neg \text{Fire})$ (Definition of implication)
 $(\neg(\neg \text{Smoke}) \wedge \neg \text{Fire}) \vee (\text{Smoke} \vee \neg \text{Fire})$ (De Morgan's laws)
 $(\text{Smoke} \wedge \neg \text{Fire}) \vee (\text{Smoke} \vee \neg \text{Fire})$ (Remove double negation)
 $(\text{Smoke} \wedge \neg \text{Fire}) \vee \text{Smoke} \vee \neg \text{Fire}$ (Associativity of \vee)
 False (Unsatisfiable)
- d) $\text{Smoke} \vee \text{Fire} \vee \neg \text{Fire}$ VALID
 $\text{Smoke} \vee \text{True}$ (Definition of \vee)
 True (Definition of \vee)
- e) $((\text{Smoke} \wedge \text{Heat}) \rightarrow \text{Fire}) \leftrightarrow ((\text{Smoke} \rightarrow \text{Fire}) \vee (\text{Heat} \rightarrow \text{Fire}))$ VALID

Smoke	Heat	Fire	$(\text{Smoke} \wedge \text{Heat})$	$((\text{Smoke} \wedge \text{Heat}) \rightarrow \text{Fire})$	$(\text{Smoke} \rightarrow \text{Fire})$	$(\text{Heat} \rightarrow \text{Fire})$	$((\text{Smoke} \rightarrow \text{Fire}) \vee (\text{Heat} \rightarrow \text{Fire}))$	Result
0	0	0	0	1	1	1	1	1
0	0	1	0	1	1	1	1	1
0	1	0	0	1	1	0	1	1
0	1	1	0	1	1	1	1	1
1	0	0	0	1	0	1	1	1
1	0	1	0	1	1	1	1	1
1	1	0	1	0	0	0	0	1
1	1	1	1	1	1	1	1	1

- f) $(\text{Smoke} \rightarrow \text{Fire}) \rightarrow ((\text{Smoke} \wedge \text{Heat}) \rightarrow \text{Fire})$ VALID
 $(\neg \text{Smoke} \vee \text{Fire}) \rightarrow (\neg(\text{Smoke} \wedge \text{Heat}) \vee \text{Fire})$ (Definition of implication)
 $\neg(\neg \text{Smoke} \vee \text{Fire}) \vee (\neg(\text{Smoke} \wedge \text{Heat}) \vee \text{Fire})$ (Definition of implication)
 $(\text{Smoke} \wedge \neg \text{Fire}) \vee (\neg \text{Smoke} \vee \neg \text{Heat}) \vee \text{Fire}$ (De Morgan's laws)

$(\text{Smoke} \wedge \neg \text{Fire}) \vee \neg \text{Smoke} \vee \neg \text{Heat} \vee \text{Fire}$
True

(Associativity of \vee)
(Definition of \vee)

g) $\text{Big} \vee \text{Dumb} \vee (\text{Big} \rightarrow \text{Dumb})$
 $\text{Big} \vee \text{Dumb} \vee (\neg \text{Big} \vee \text{Dumb})$
 $\text{Big} \vee \text{Dumb} \vee \neg \text{Big} \vee \text{Dumb}$
 $\text{Big} \vee \neg \text{Big} \vee \text{Dumb} \vee \text{Dumb}$
True \vee Dumb
True

VALID
(Definition of implication)
(Associativity of \vee)
(Commutativity of \vee)
(Definition of \vee)
(Definition of \vee)