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Lab session # 6
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Question 1

If Z has more columns than the number of nodes in the graph then the network can simply learn to assign to linearly independent features to every node hence the model should be able to perfectly reconstruct the adjacency matrix. Hence we expect the accuracy to converge to 1. (or something similar to this)

Question 2

We can simply take the feature of node i to be the digits of the binary representation of i. Since $34 < 2^{10}$ this works, is deterministic and unique.

Question 3

To answer this question it suffices to count the number of edges in a complete graph with n nodes. Every edge is defined by its two endpoints, assuming self-loops are not allowed, we have n choices for the first node, $n-1$ choices of the second and we are over-counting every edge twice since the edges are not directed. Hence we have $n(n-1)/2$ edges. Each edge follows a Bernouilly distribution of parameter p. Let E be the random variable corresponding to the number of edges in the graph, then $E \sim \text{Bin}(n(n-1)/2, p)$. Hence we know that

$$\mathbb{E}[E] = \frac{n(n-1)}{2}p \text{ and } \text{Var}[E] = \frac{n(n-1)}{2}p(1-p) \tag{1}$$

Now plugging $n = 15$ and $p = 0.2$ ($p = 0.4$ resp.) we get

$$\mathbb{E}[E(15, 0.2)] = 21 \text{ and } \text{Var}[E(15, 0.2)] = 16.8 \text{ and respectively } \mathbb{E}[E(15, 0.4)] = 42 \text{ and } \text{Var}[E(15, 0.4)] = 25.2 \tag{2}$$

Question 4

We have

$$Z = \begin{pmatrix} 0.21 & -0.95 & 0.4 \\ -0.4 & 0.12 & 0.68 \\ 0.89 & 0.34 & 1.31 \\ 0.68 & 0.08 & -0.5 \\ 0.62 & 0.18 & -0.1 \\ 0.89 & 0.34 & 1.31 \\ 0.81 & -0.2 & 1.29 \\ 0.89 & 0.34 & 1.31 \\ 0.61 & -0.14 & -0.31 \end{pmatrix} \tag{3}$$

Then using the sum readout function we get

$$z_G = \begin{pmatrix} -0.34 \\ 0.4 \\ 2.54 \\ 0.26 \\ 0.7 \\ 2.54 \\ 1.9 \\ 2.54 \\ 0.16 \end{pmatrix} \tag{4}$$

for the mean readout function

$$z_G = \begin{pmatrix} -0.1133 \\ 0.1333 \\ 0.8467 \\ 0.0867 \\ 0.2333 \\ 0.8467 \\ 0.6333 \\ 0.8467 \\ 0.0533 \end{pmatrix} \tag{5}$$

Finally for the max readout function we get

$$z_G = \begin{pmatrix} 0.4 \\ 0.68 \\ 1.31 \\ 0.68 \\ 0.62 \\ 1.31 \\ 1.29 \\ 1.31 \\ 0.61 \end{pmatrix} \tag{6}$$

We see that the sum readout function we get

$$z_{G_1} = 0.8667 \pm 1.5, \quad z_{G_2} = 1.35 \pm 1.053, \text{ and } z_{G_3} = 1.35 \pm 1.683 \tag{7}$$

while for the mean we get

$$z_{G_1} = 0.2822 \pm 0.5018, \quad z_{G_2} = 0.45 \pm 0.3511, \text{ and } z_{G_3} = 0.45 \pm 0.561 \tag{8}$$

and for the max

$$z_{G_1} = 0.8 \pm 0.4660, \quad z_{G_2} = 0.975 \pm 0.3761, \text{ and } z_{G_3} = 0.96 \pm 0.5 \tag{9}$$

We see that in all cases graphs 2 and 3 are indistinguishable. The best separation between graphs 1 and 2 is achieved for the mean readout function.