L_p spaces

- $\|f\|_2 := \sqrt{\int_0^\infty |f(x)|^2 dx}$ $\quad L_2$ norm

$\|\rho_1(x) - \rho_2(x)\|_2 = \sqrt{\int_0^\infty |\rho_1(x) - \rho_2(x)|^2 dx}$

$\|\rho_1(x) - \rho_2(x)\|_1 = \int_0^\infty |\rho_1(x) - \rho_2(x)| dx$

$\|\rho_1(x)\|_1 = \int_0^\infty |\rho_1(x)| dx = \int_0^\infty \rho_1(x) dx = 1$

$\|\rho_1(x) - \rho_2(x)\| \leq \|\rho_1(x)\| + \|\rho_2(x)\|$

$= 1 + 1 = 2$

- Kullback-Leibler divergence

$$D_{p\|q} := \sum_{n=0}^{\infty} p(n) \log\left[\frac{p(n)}{q(n)}\right]$$

$p(n)$: true probability dist.

$q(n)$: approx prob. dist.

Interpretation: 'amount of infomation lost' by using approx prob. dist.

Ex: Birth-death process

$g \xrightarrow{km} m + g$

$m \xrightarrow{dm} \phi$

$$p(m) = \frac{\mu^m e^{-\mu}}{m!}$$

$$q(m) \cong \frac{1}{\sqrt{2\pi\mu}} e^{-\frac{(m-\mu)^2}{2\mu}}$$

$$D_{p\|q} = \sum_{m=0}^{\infty} \frac{\mu^m e^{-\mu}}{m!} \log\left[\frac{\mu^m e^{-\mu}}{m!} \sqrt{2\pi\mu}\, e^{\frac{(m-\mu)^2}{2\mu}}\right]$$

As μ increases, expect $D_{p\|q}$ to <u>decrease</u>