KEITH DEVLIN: Introduction to Mathematical Thinking PROBLEM SET 1 (for Lectures 1 and 2)

This problem set focuses on material covered in Lectures 1 and 2, so I recommend you to watch both lectures and attempt Assignments 1 and 2 before submitting your answers.

1. Is it possible for one of $(\phi \wedge \psi) \wedge \theta$ and $\phi \wedge (\psi \wedge \theta)$ to be true and the other false? (If not, then the associative property holds for conjunction.)
2. Is it possible for one of $(\phi \vee \psi) \vee \theta$ and $\phi \vee (\psi \vee \theta)$ to be true and the other false? (If not, then the associative property holds for disjunction.)
3. Is it possible for one of $\phi \wedge (\psi \vee \theta)$ and $(\phi \wedge \psi) \vee (\phi \wedge \theta)$ to be true and the other false? (If not, then the distributive property holds for conjunction across disjunction.)
4. Is it possible for one of $\phi \vee (\psi \wedge \theta)$ and $(\phi \vee \psi) \wedge (\phi \vee \theta)$ to be true and the other false? (If not, then the distributive property holds for disjunction across conjunction.)
5. Is showing that the negation $\neg\phi$ is true equivalent to showing that ϕ is false?
6. Assuming you know nothing more about Alice, which of (a) – (e) is most likely? (Or does (f) hold?)
   (a) Alice is a rock star and works in a bank.
   (b) Alice is quiet and works in a bank.
   (c) Alice is quiet and reserved and works in a bank.
   (d) Alice is honest and works in a bank.
   (e) Alice works in a bank.
   (f) None of these is more or less likely.
7. Assuming you know nothing more about Alice, which of (a) – (e) is most likely? (Or does (f) hold?)
   (a) Alice is a rock star or she works in a bank.
   (b) Alice is quiet and works in a bank.
   (c) Alice is a rock star.
   (d) Alice is honest and works in a bank.
   (e) Alice works in a bank.
   (f) None of these is more or less likely.
8. Identify which of the following are true (where x denotes an arbitrary real number). If you do not select a particular statement, the system will assume you think it is false.
   (a) $(x > 0) \wedge (x \leq 10)$ means $0 \leq x \leq 10$
   (b) $(x \geq 0) \wedge (x^2 < 9)$ means $0 \leq x < 3$
   (c) $(x \geq 0) \wedge (x \leq 0)$ means $x = 0$
   (d) There is no x for which $(x < 4) \wedge (x > 4)$
   (e) $-5 \leq x \leq 5$ means x is at most 5 units from 0.
   (f) $-5 < x < 5$ implies that x cannot be exactly 5 units from 0.
   (g) $(x \geq 0) \vee (x < 0)$
   (h) $(0 = 1) \vee (x^2 \geq 0)$
   (i) If $(x > 0 \vee x < 0)$ then $x \neq 0$.
   (j) If $x^2 = 9$ then $(x = 3 \vee x = -3)$.