Why Proofs?

Alexander Shen

LIRMM

Outline

Proofs? Really?

Proof by example

Impossibility proof

Impossibility Proof, II

Conclusion

Do We Need Proofs?

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- programmers: more skeptical
  - correctness proof of a video game? or search engine?
  - but real-time operating system or cryptographic protocol is a different thing

What Is a Proof?

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- not at all: an argument that is so convincing that you are ready to use it to convince other people
- a by-product and a sign of understanding
- believe or not: proofs are fun

Course Objectives

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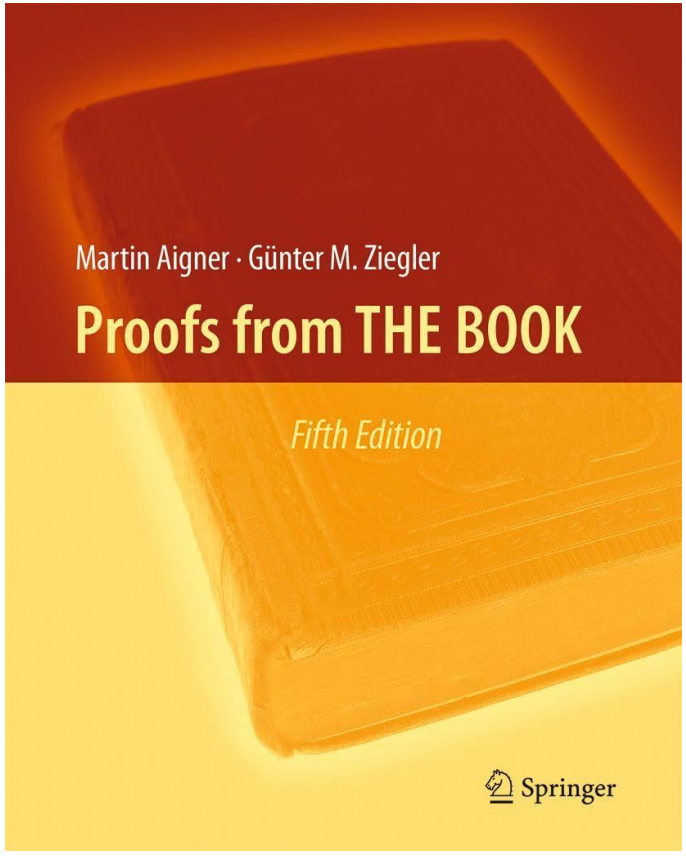
Intended audience: beginners

Prerequsites: curiosity

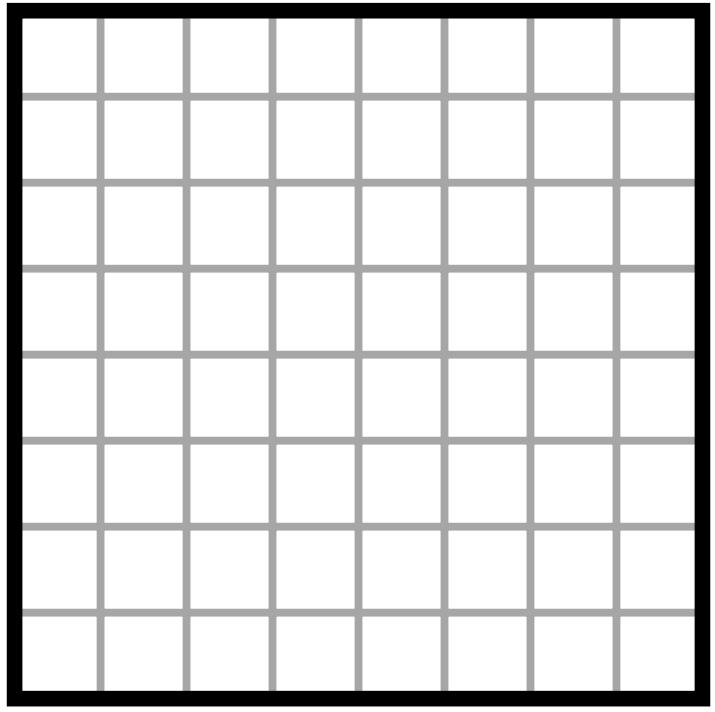
Paul Erdős and The Book



[Google Books and Wikipedia images]

To Begin With…

Can a chessboard 8×8 be tiled (no overlaps or empty space) by domino 1×2-tiles?



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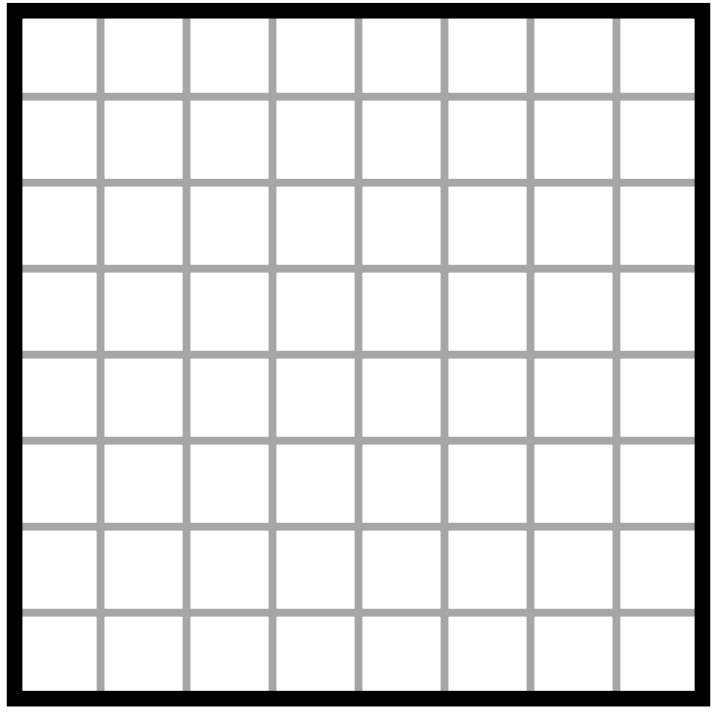

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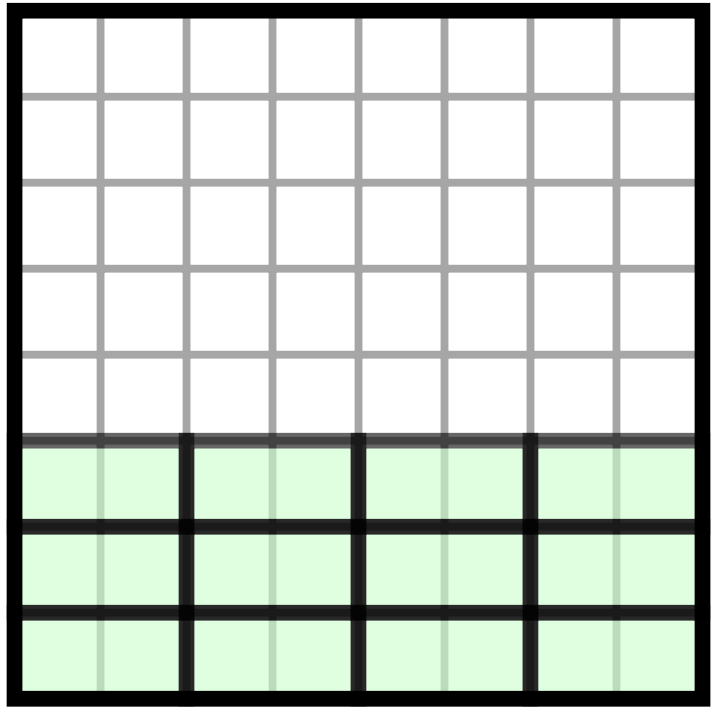

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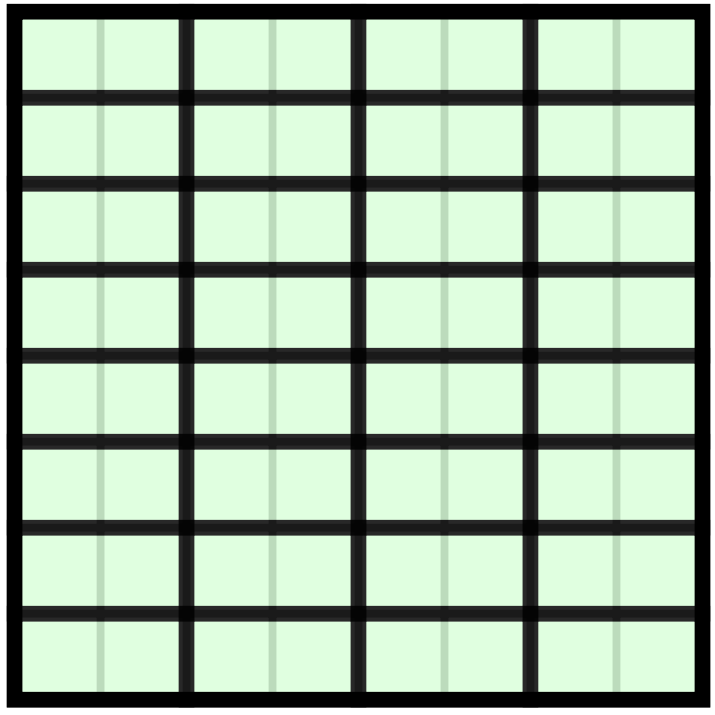
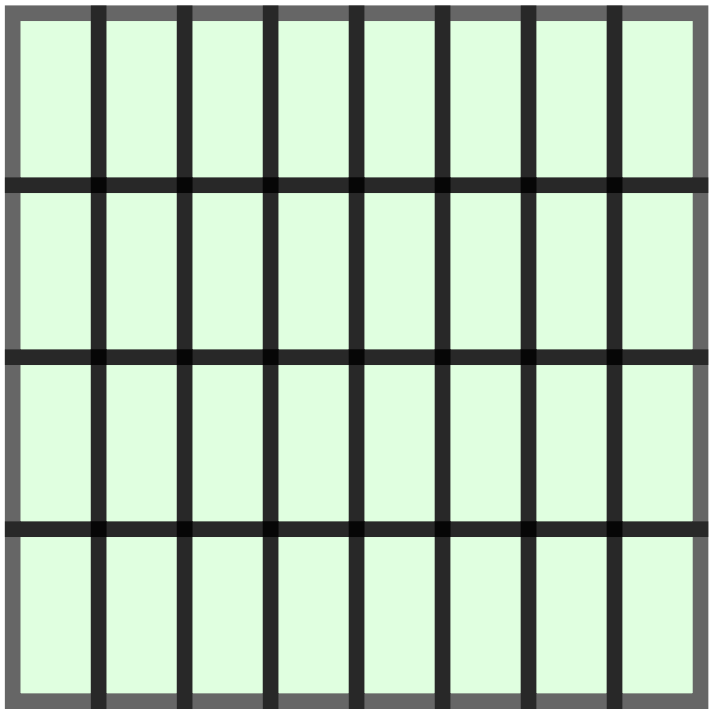
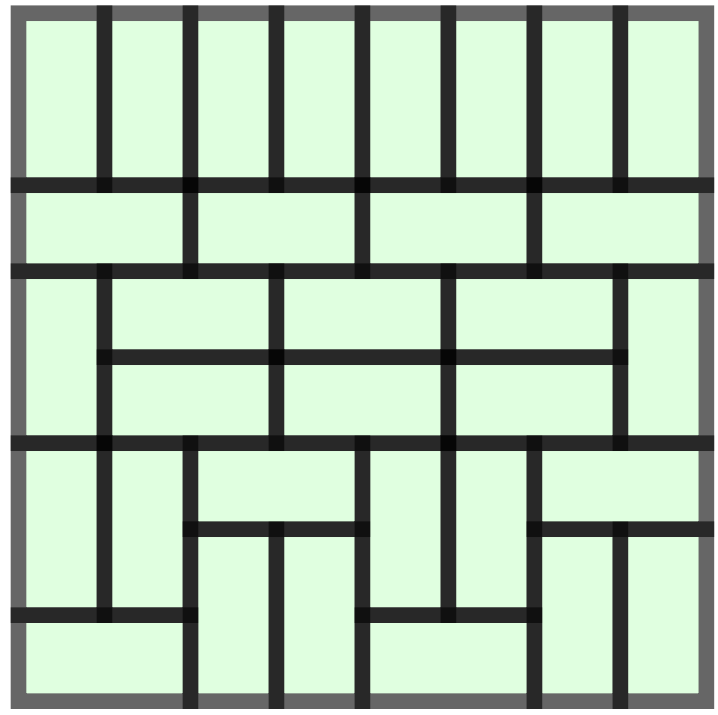


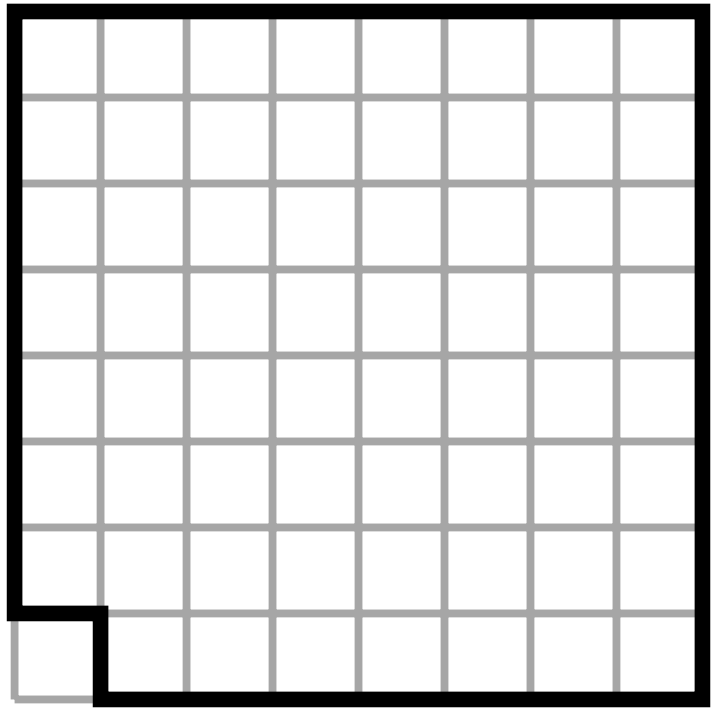
Other Tilings

One tiling is enough for the proof. But there are others:





Can We Tile This Board?



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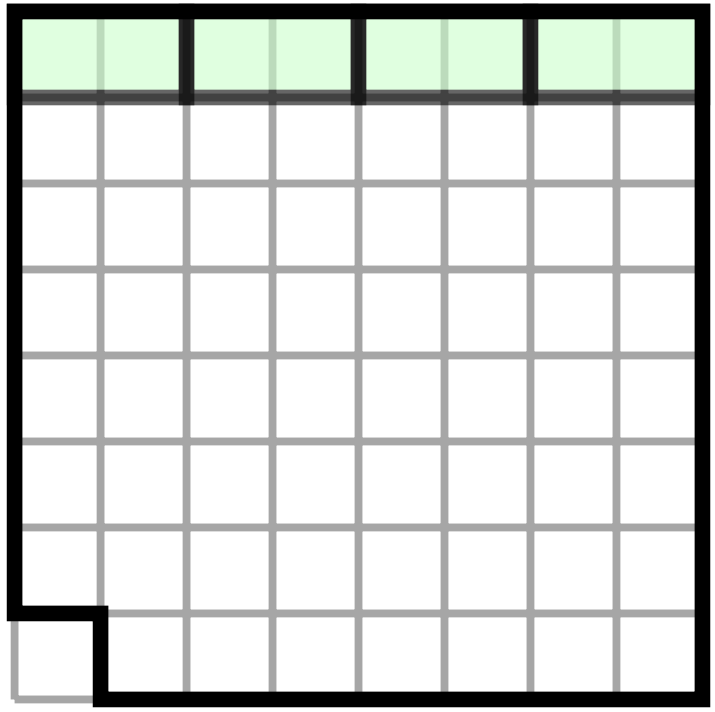

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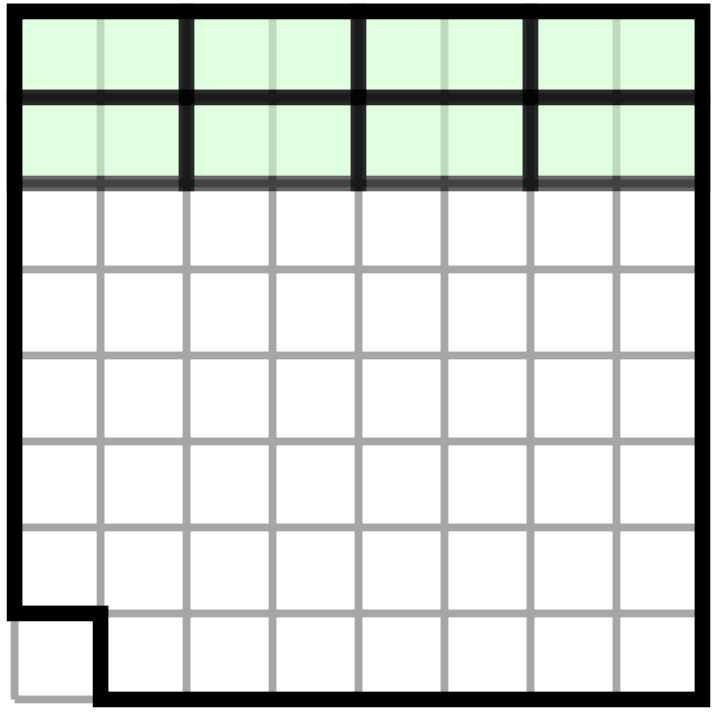

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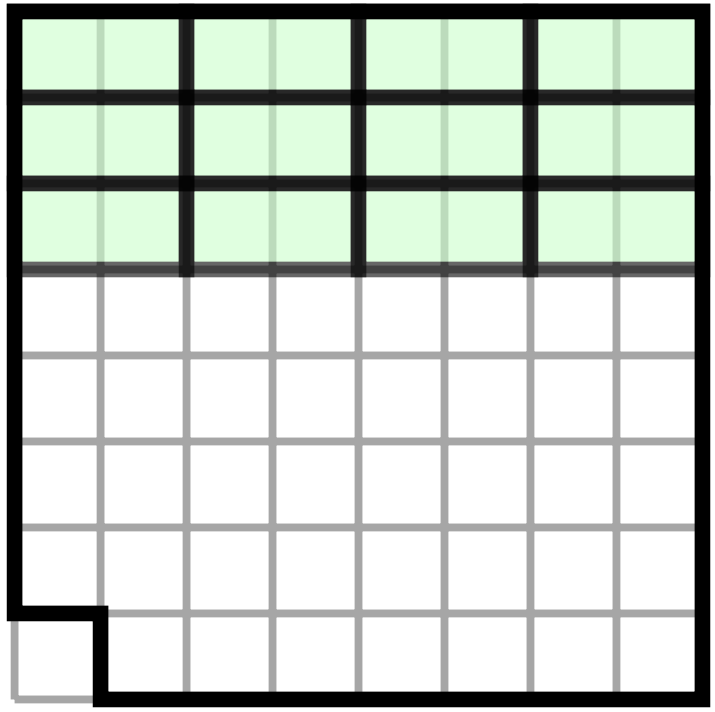

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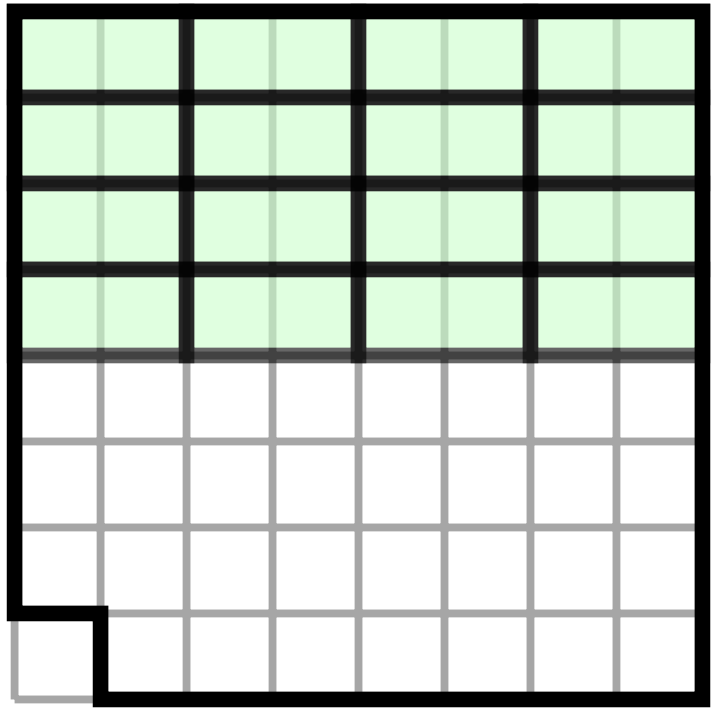

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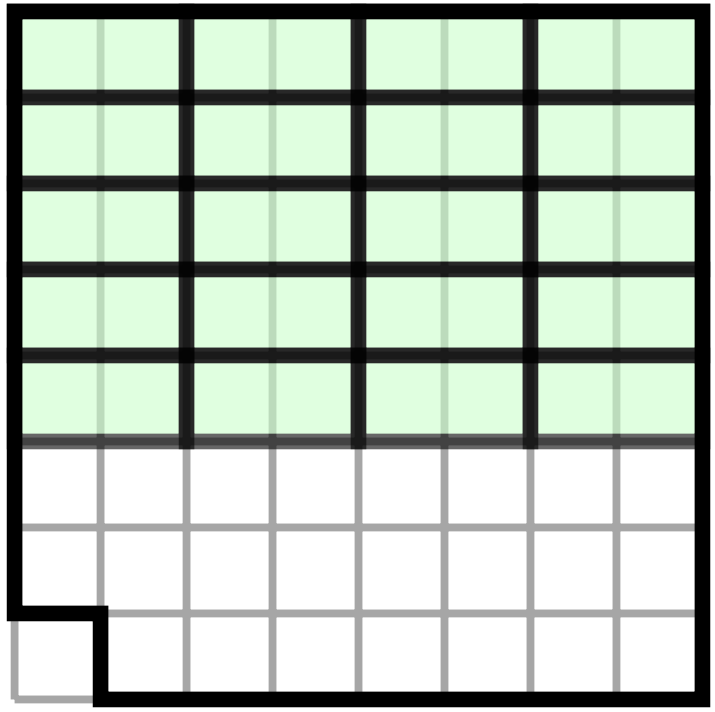

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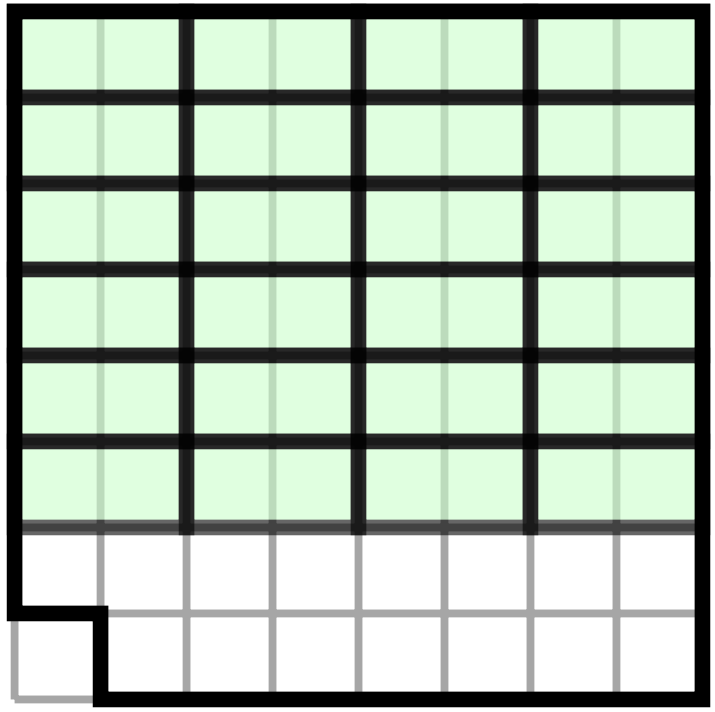

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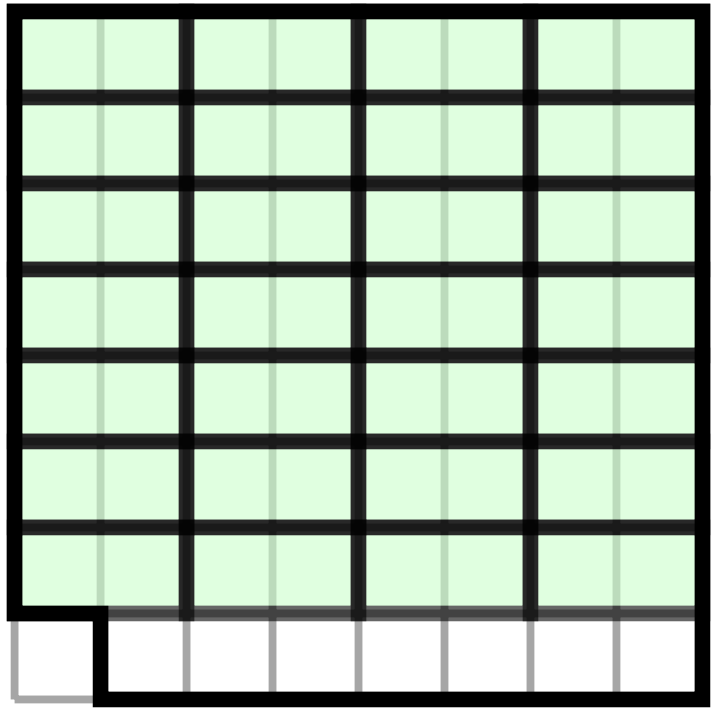

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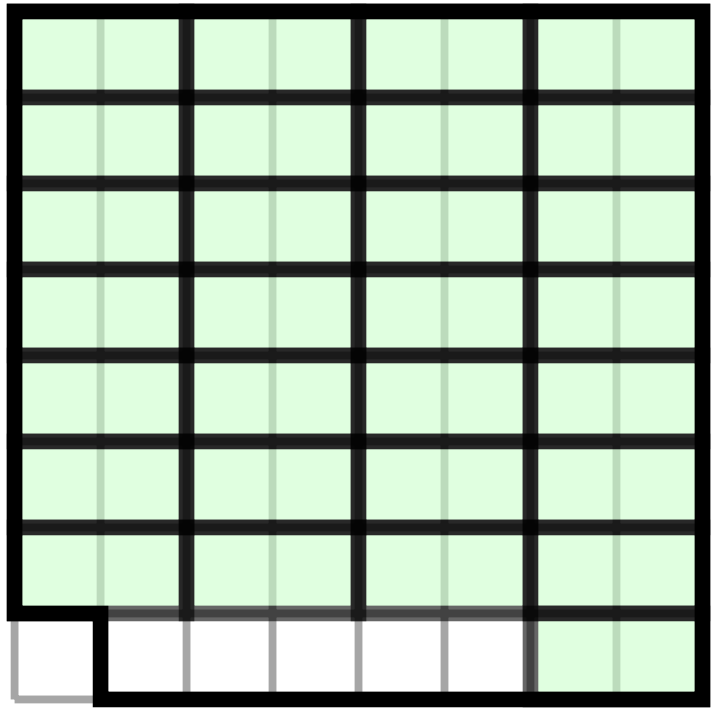

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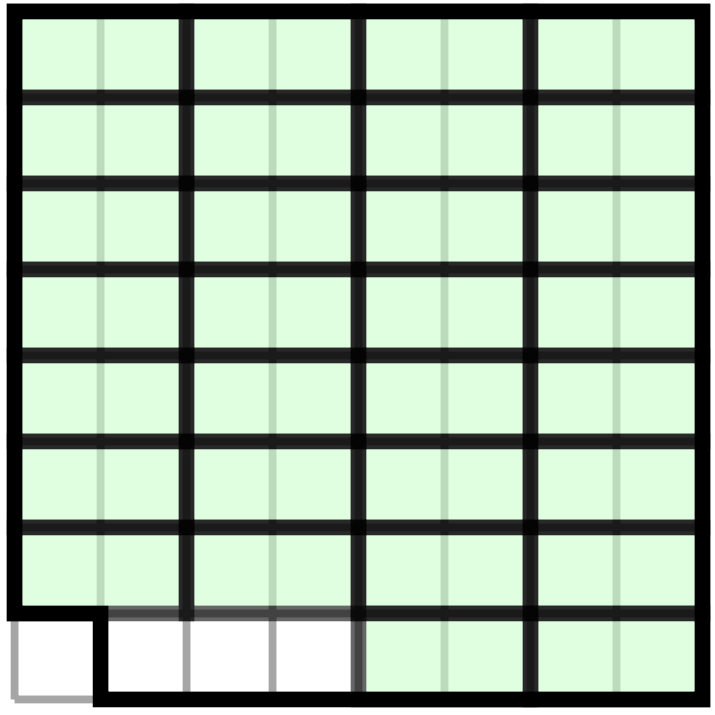

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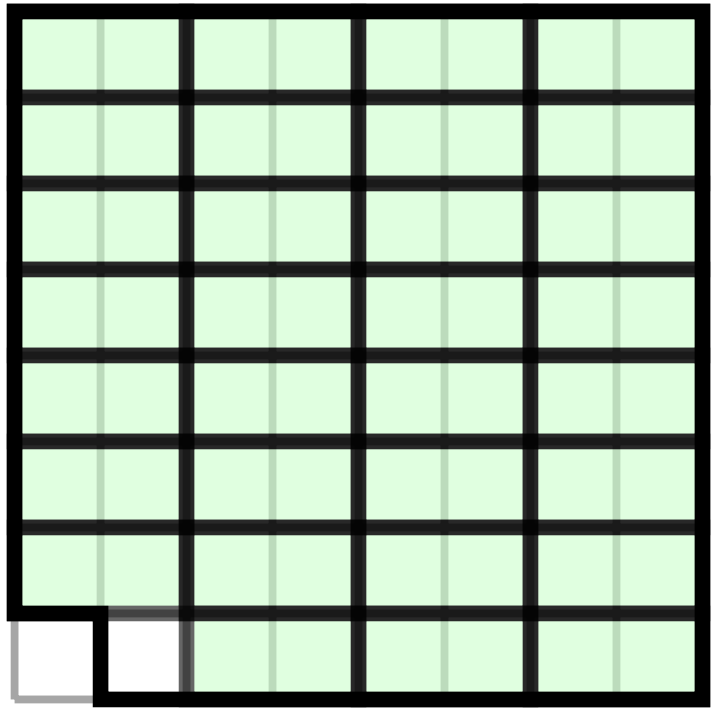

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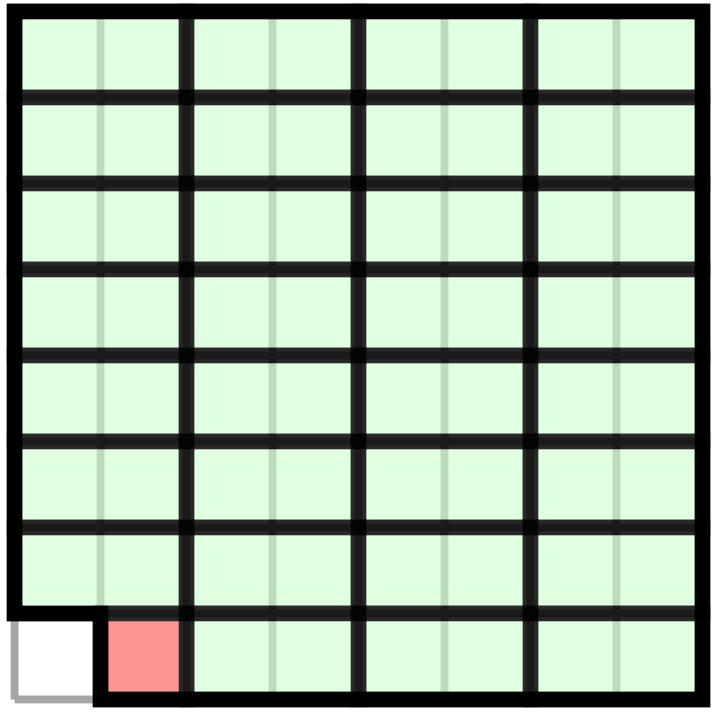

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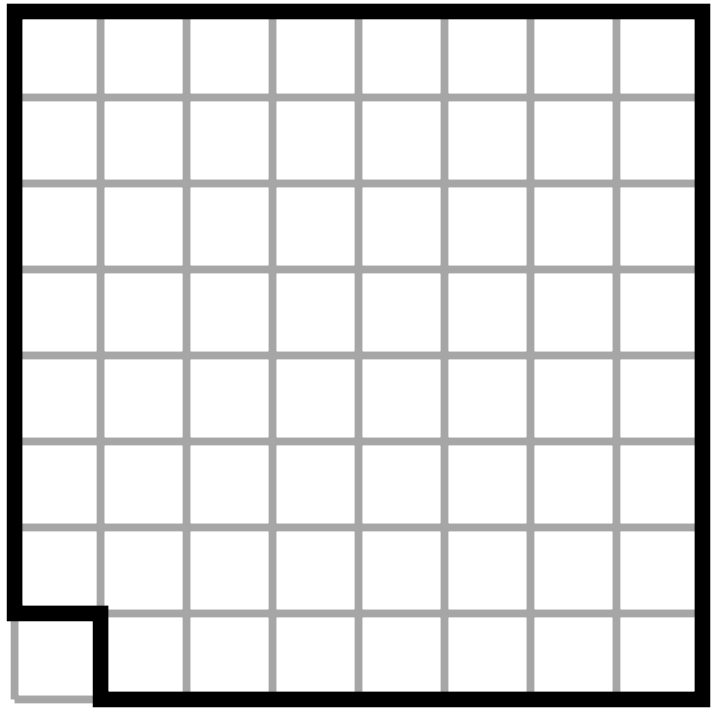

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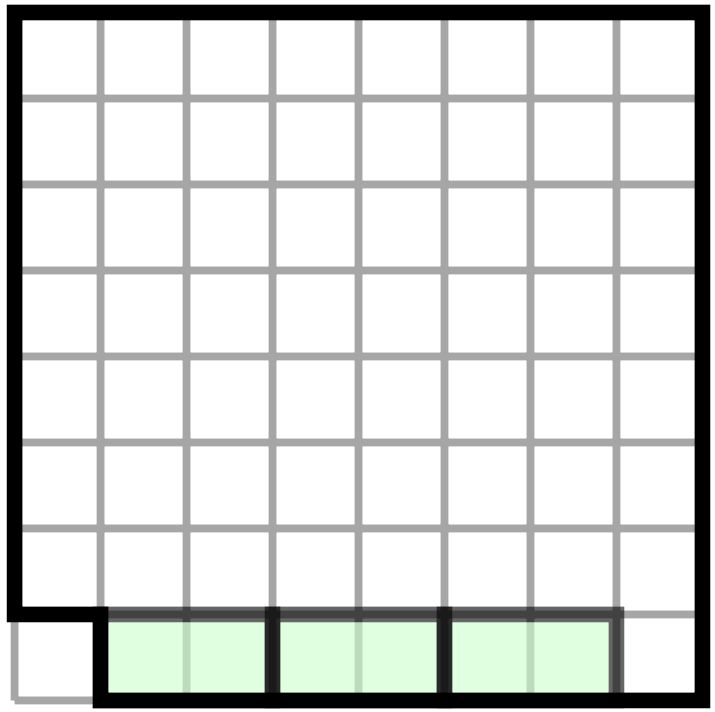

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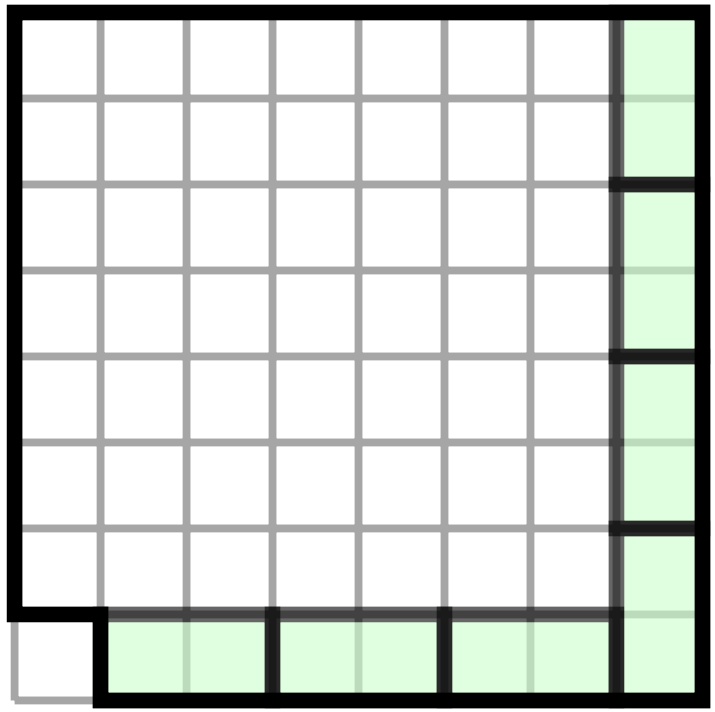

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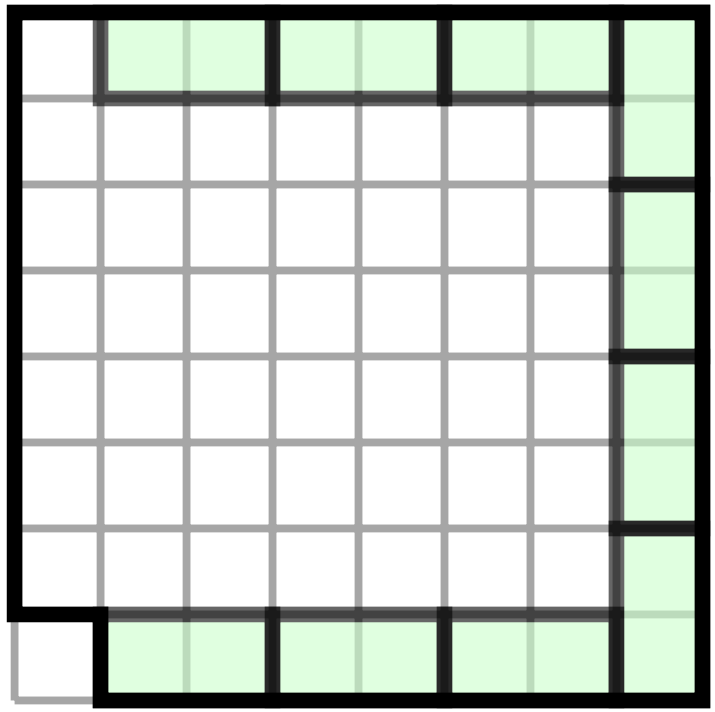

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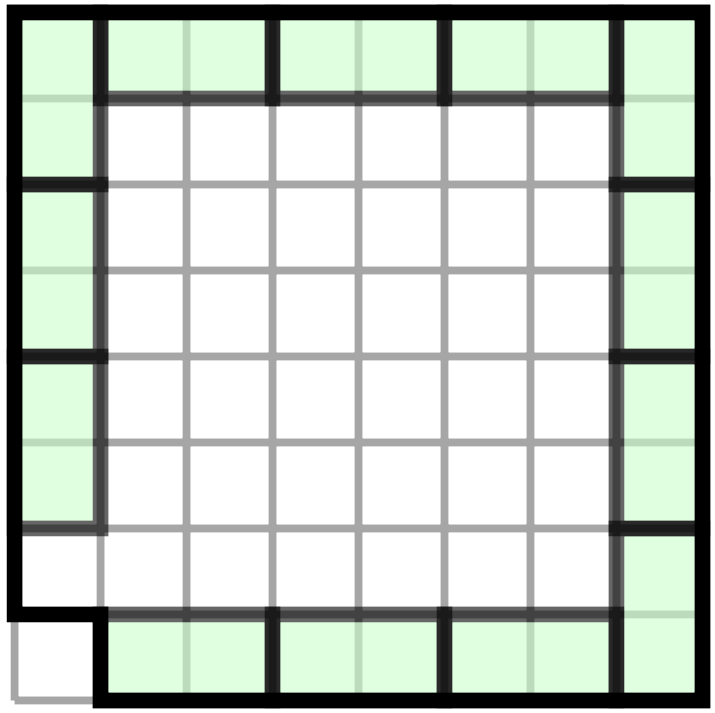

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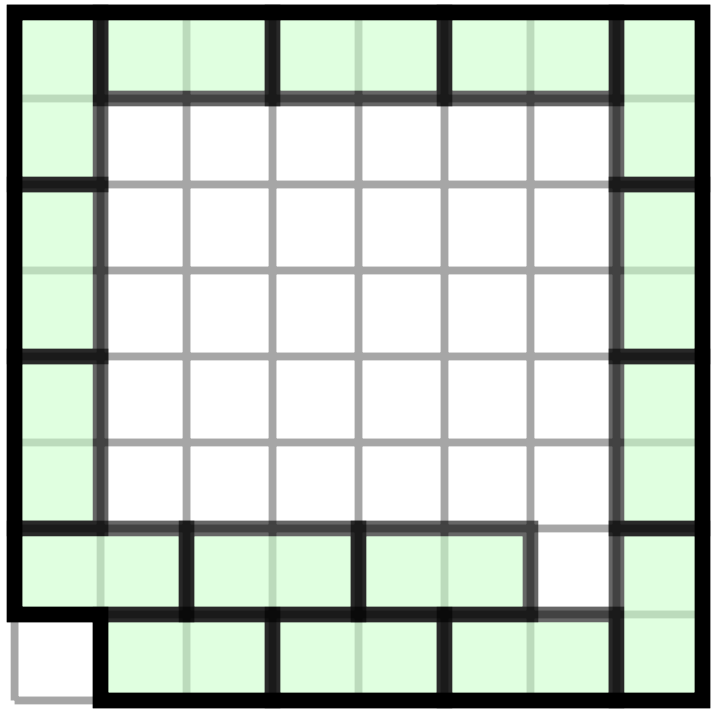

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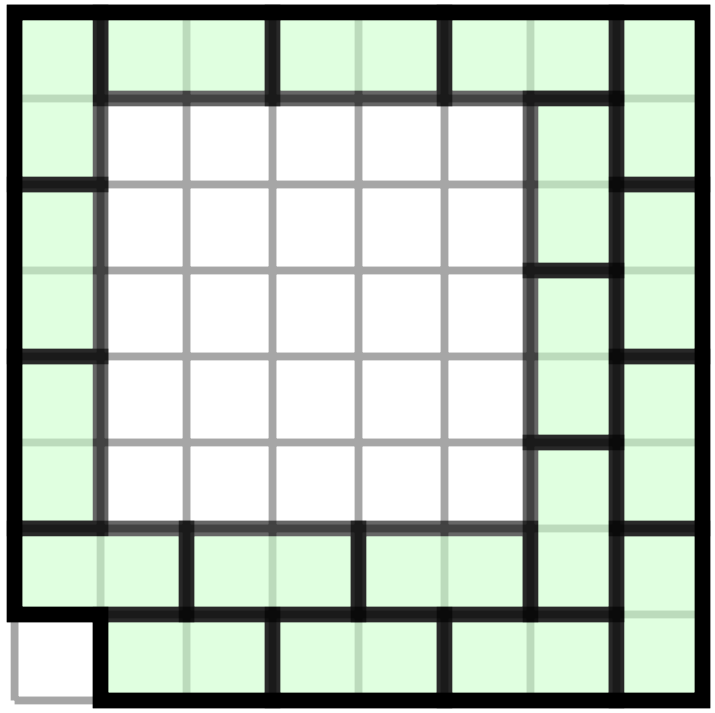

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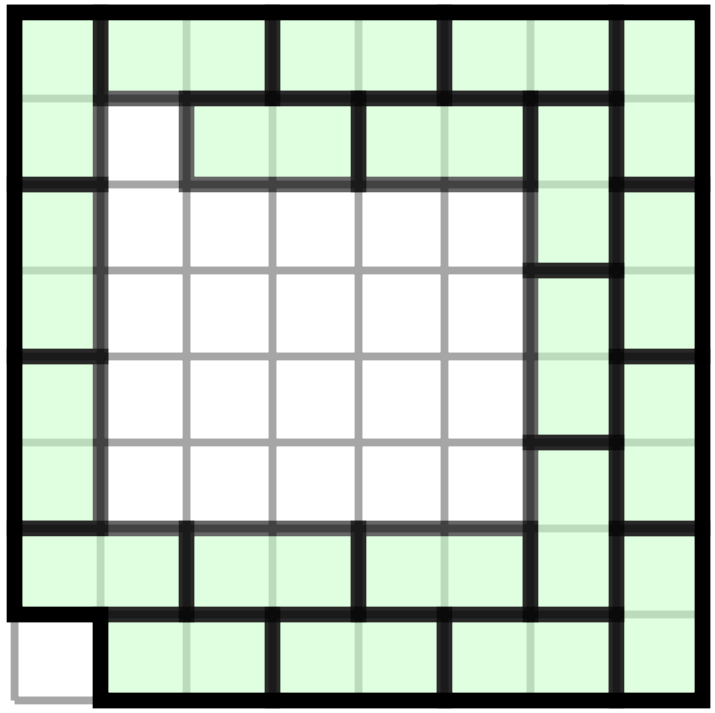

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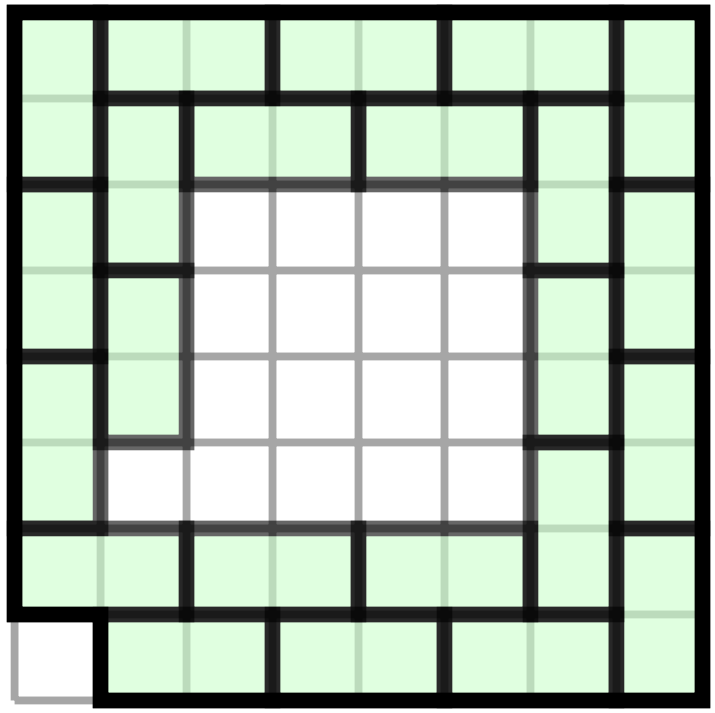

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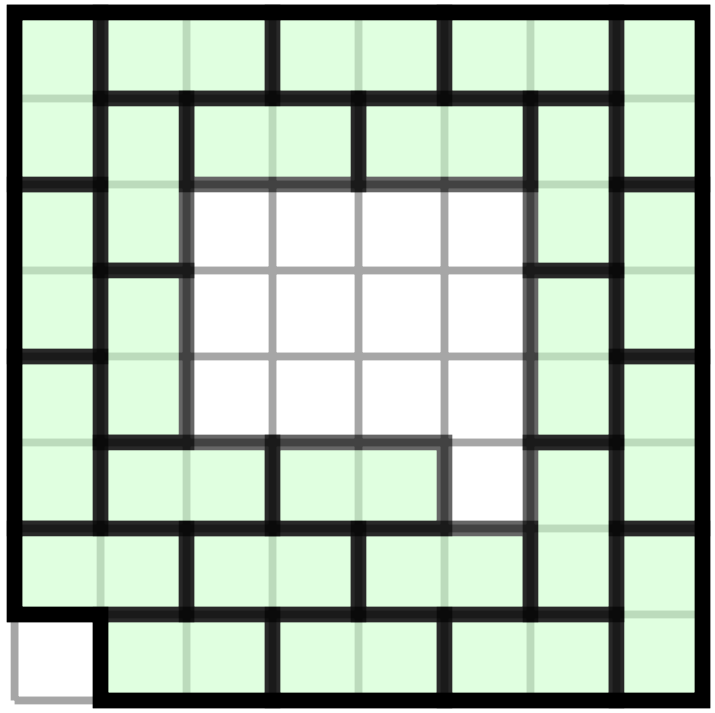

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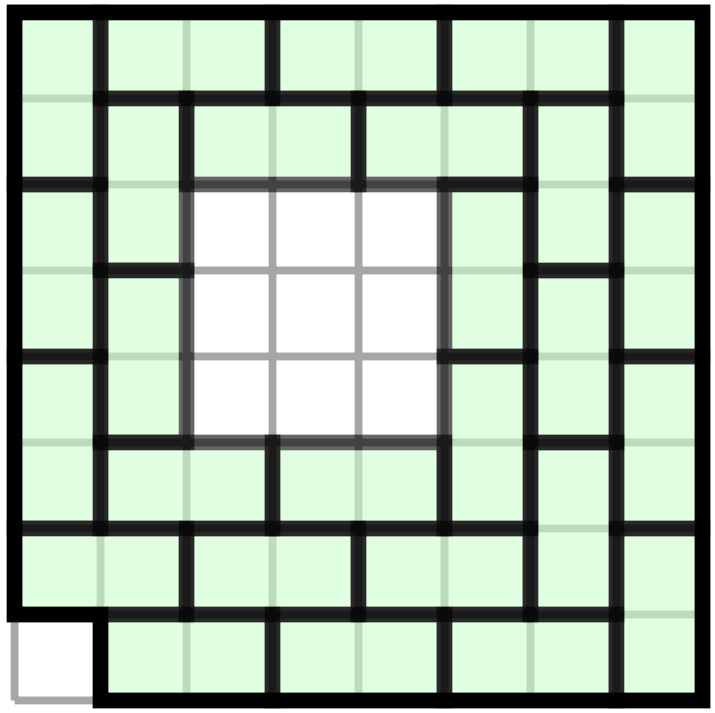

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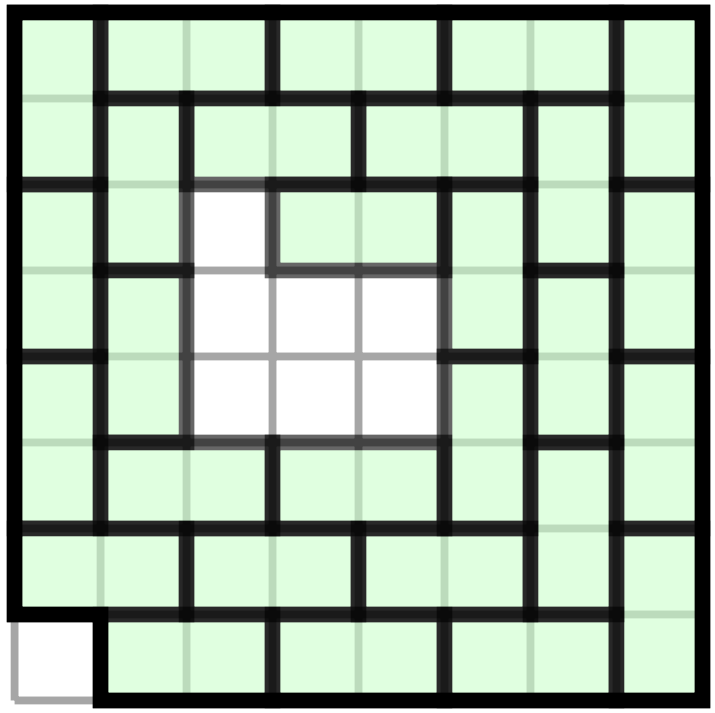

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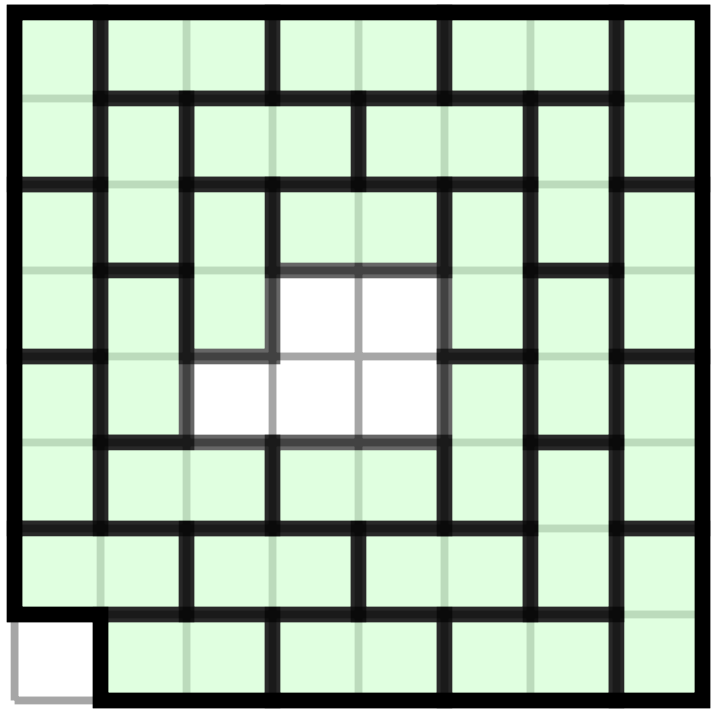

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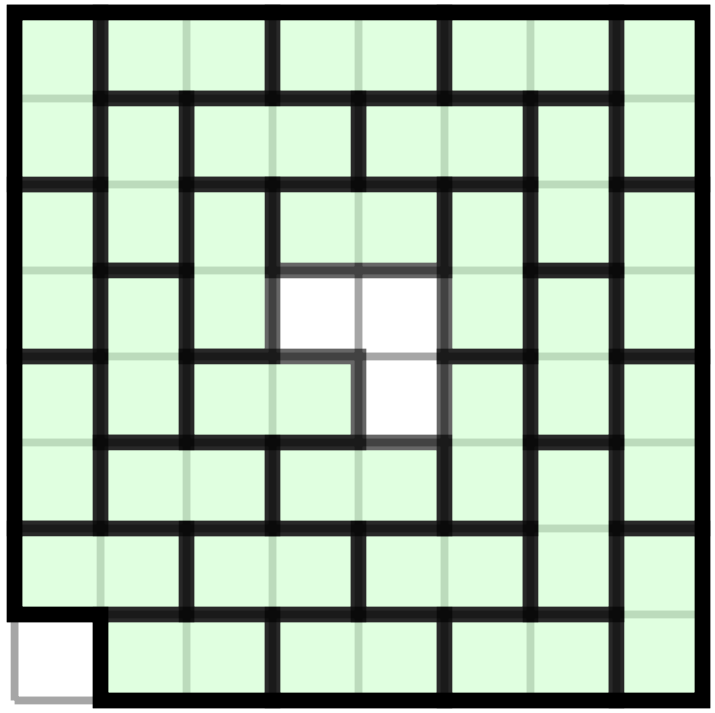

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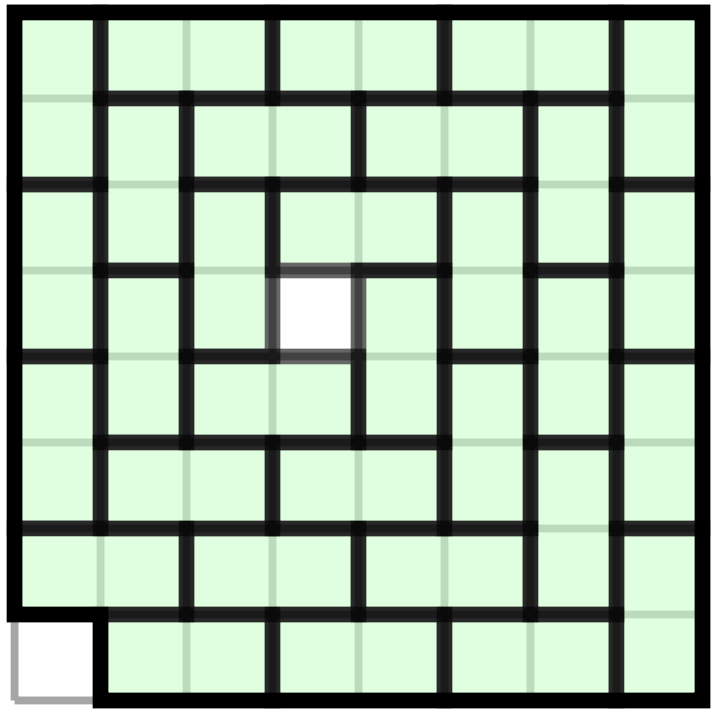

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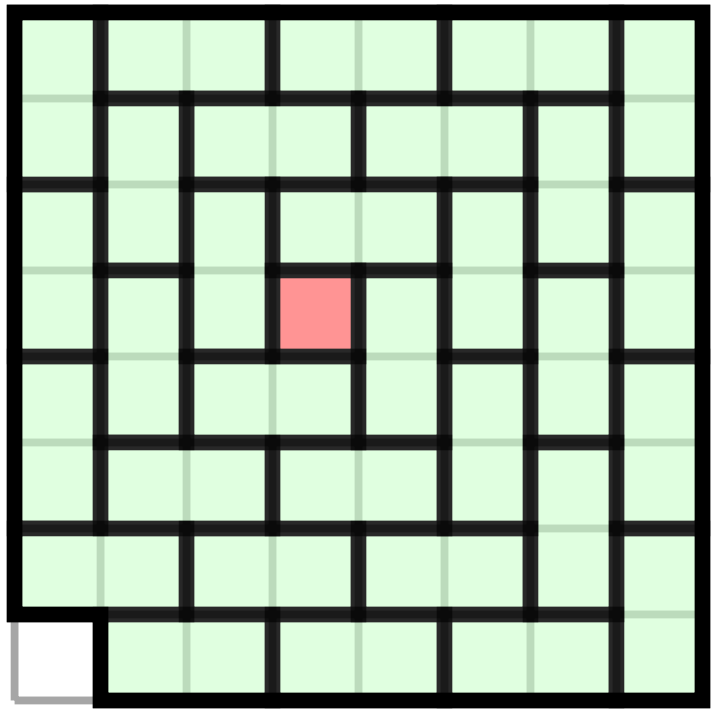

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- did we prove that tiling does not exist? No.
- can we do one of these two proofs?

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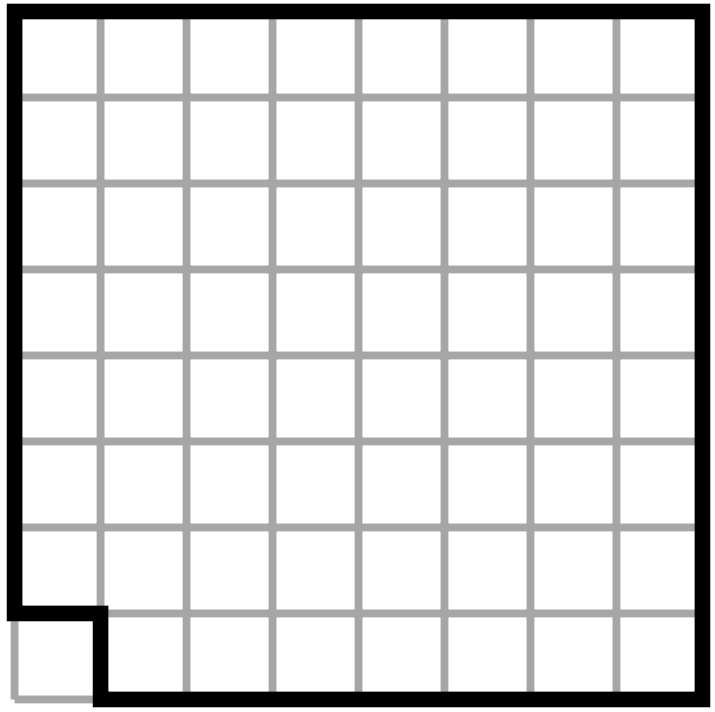

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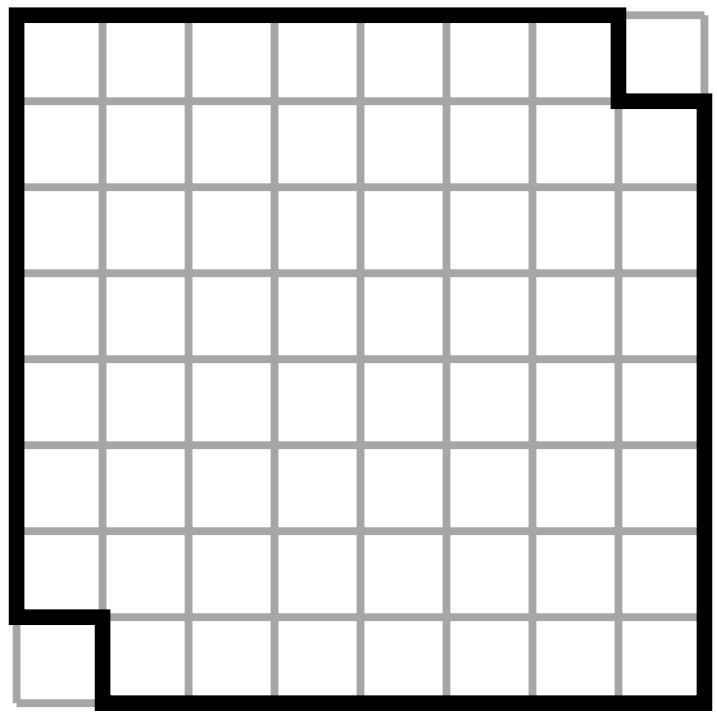
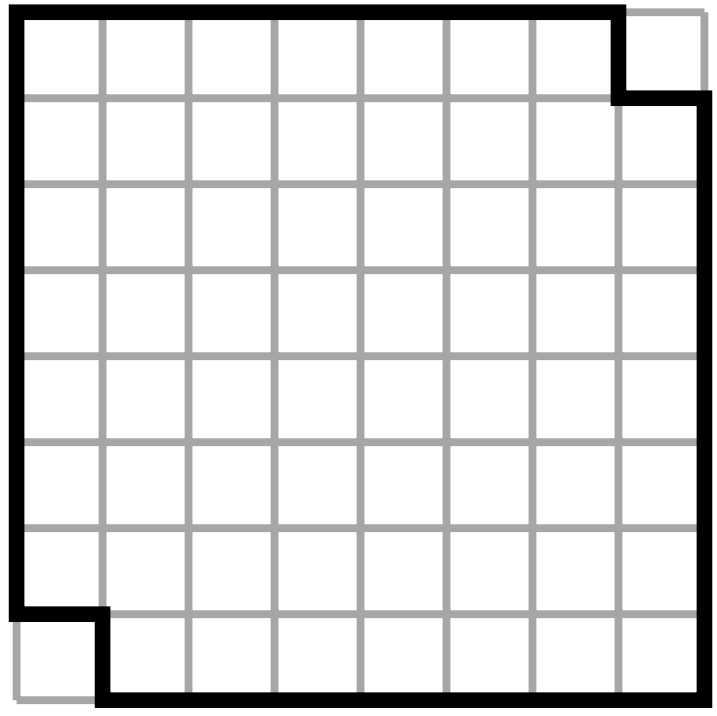
Two Corners Cut



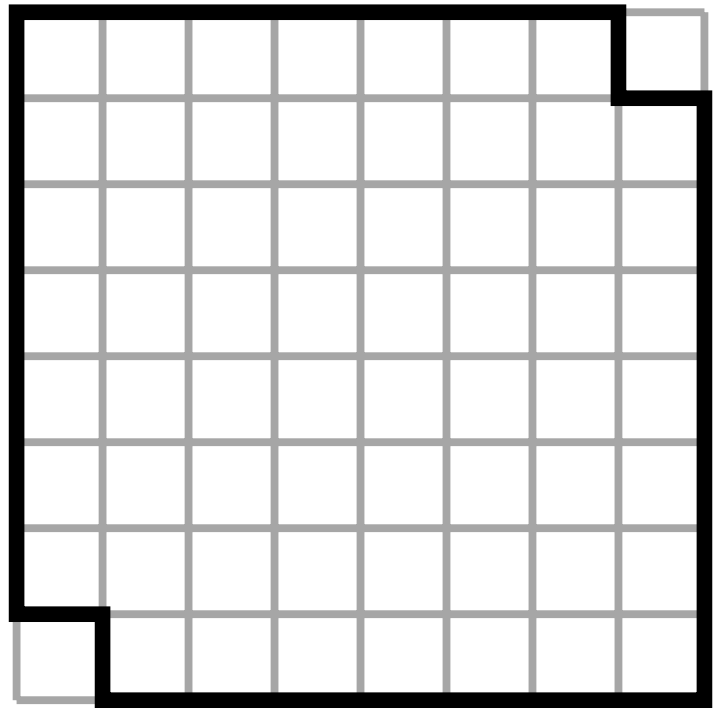
Two Corners Cut

$64 - 2 = 62$ cells, $62/2 = 31$ tiles. So the tiling is possible…



Two Corners Cut

$64 - 2 = 62$ cells, $62/2 = 31$ tiles. So the tiling is possible… is this argument OK?



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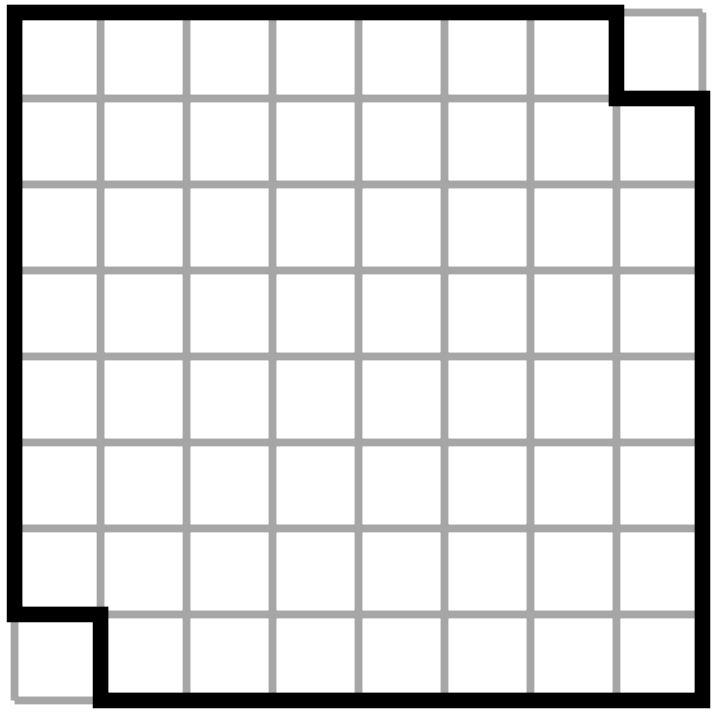

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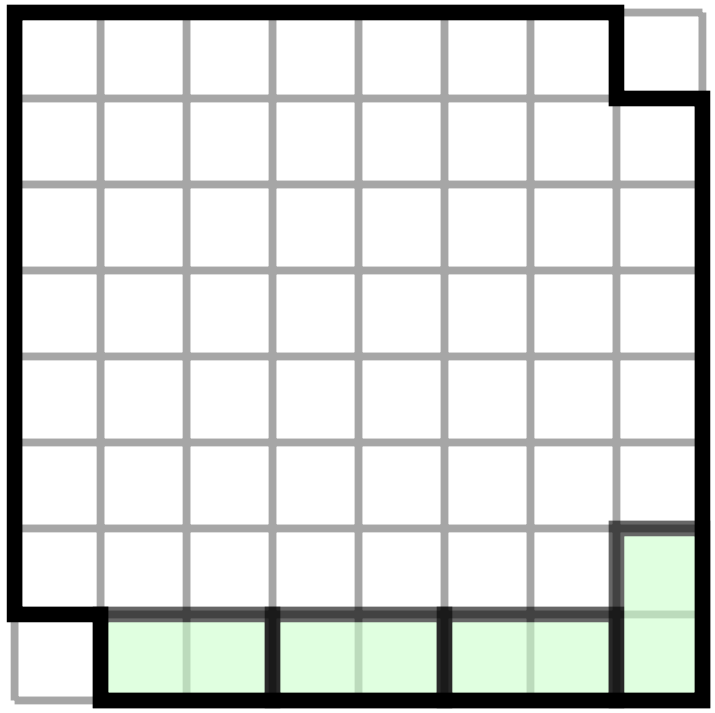

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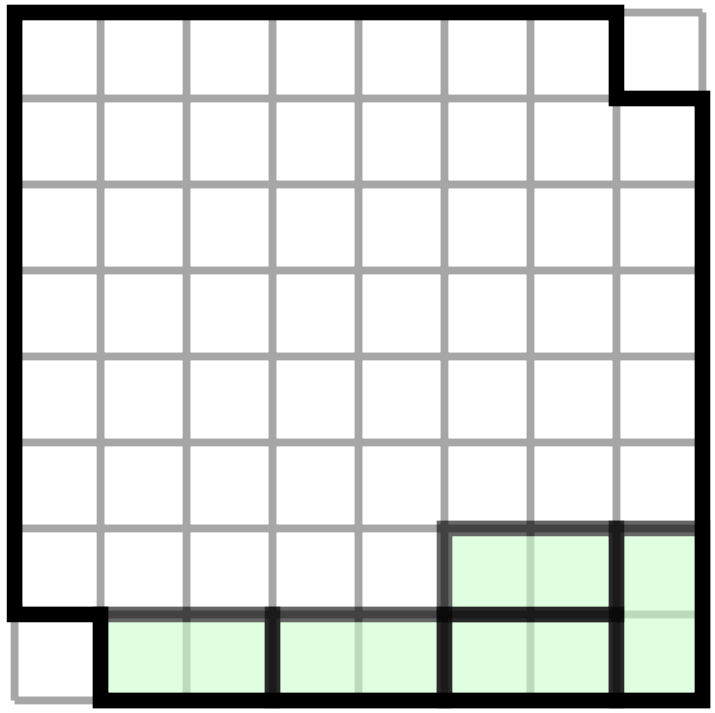

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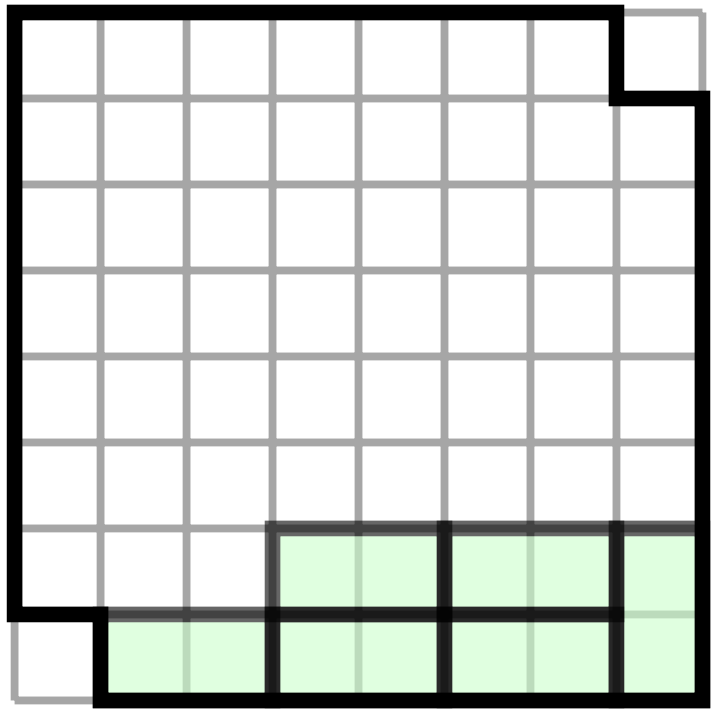

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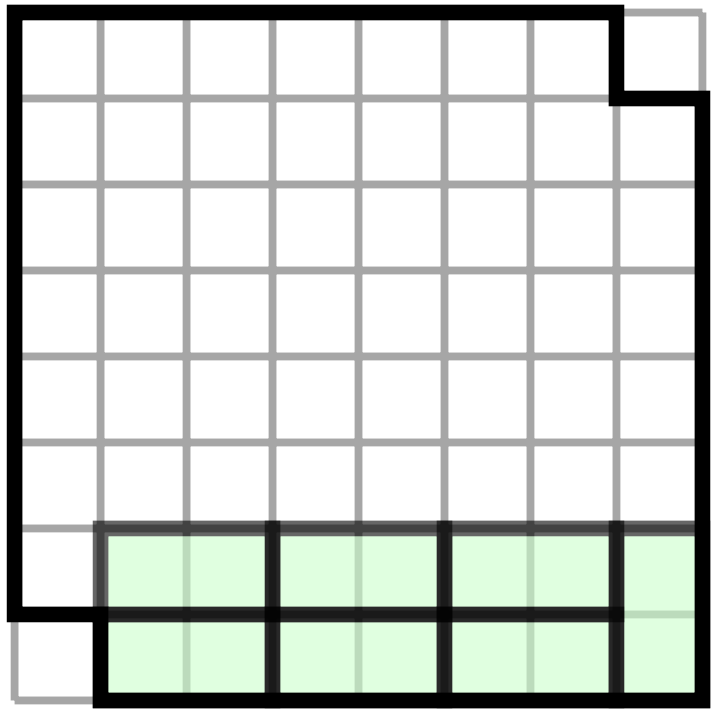

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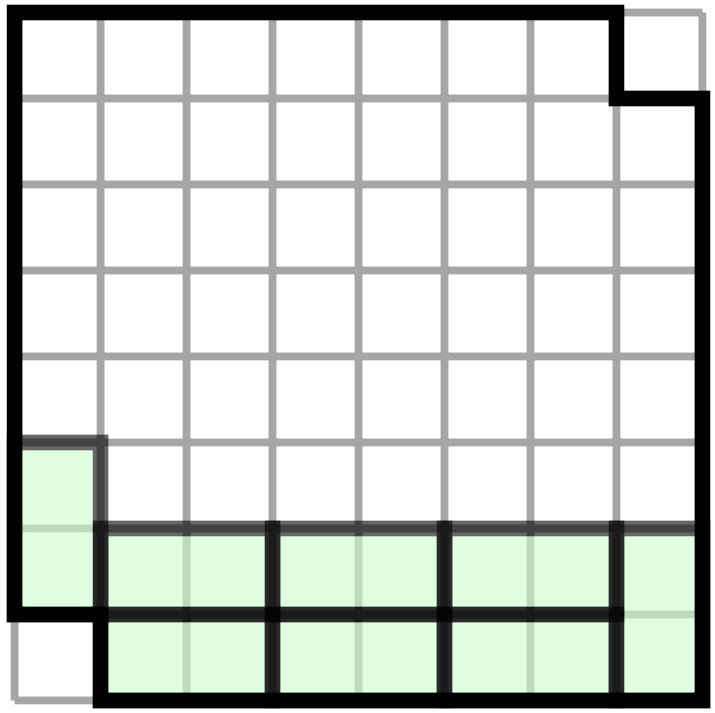

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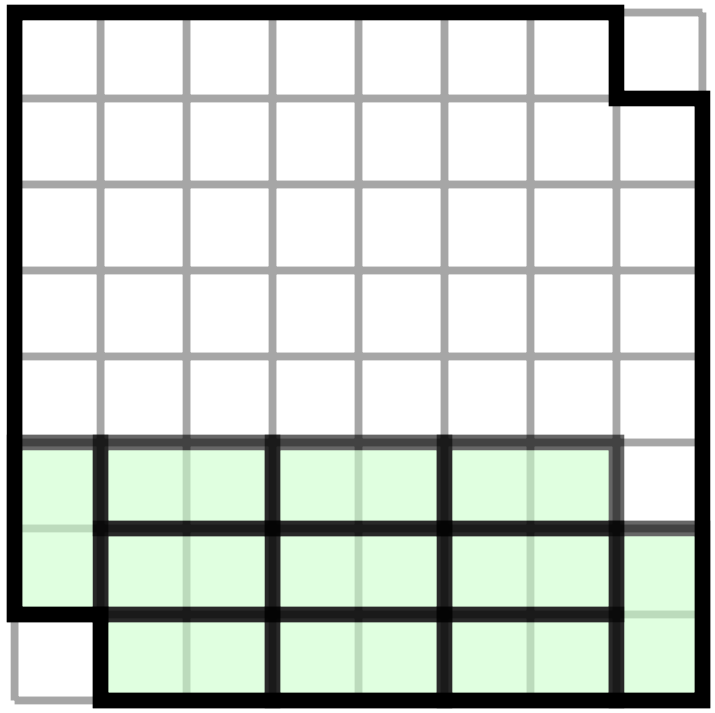

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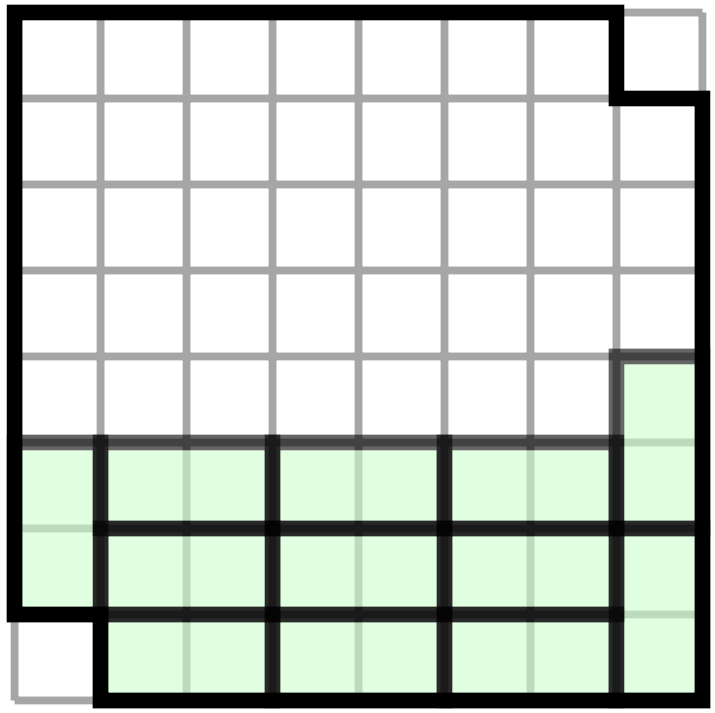

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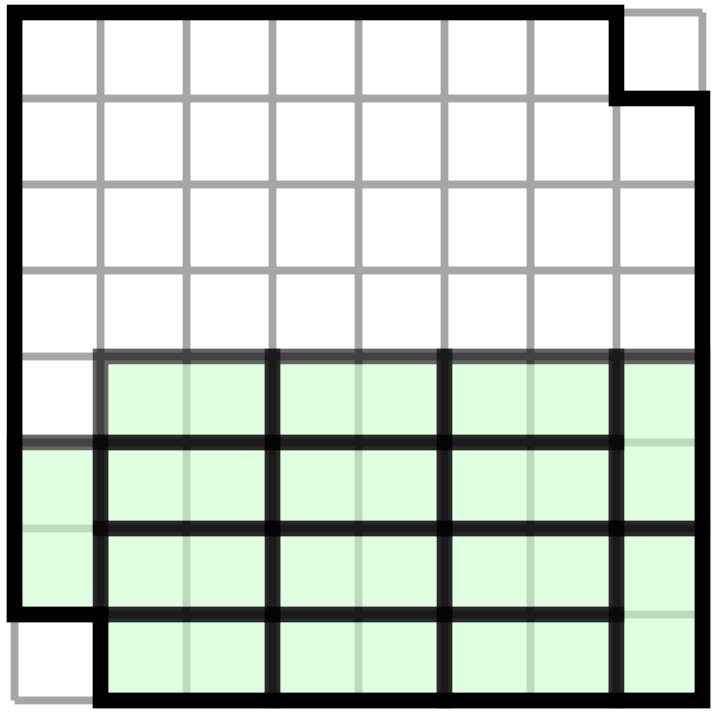

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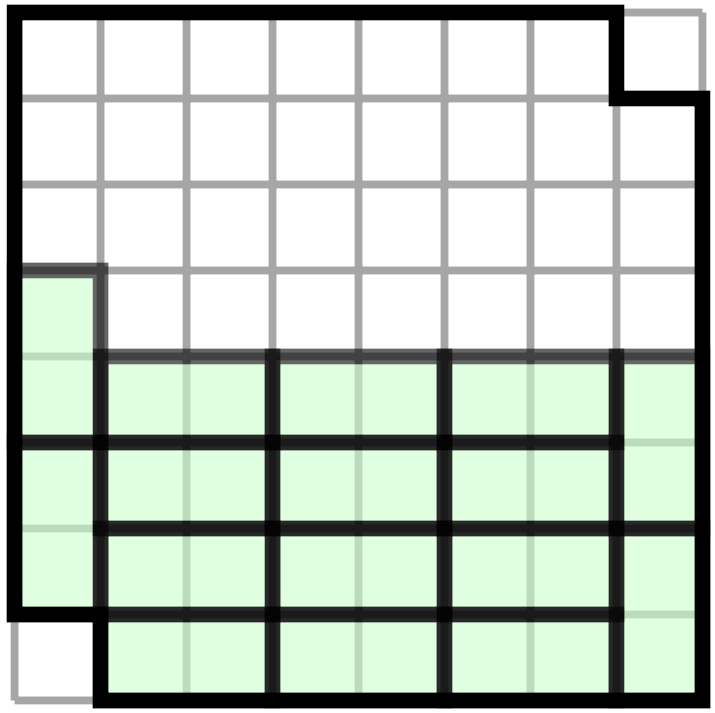

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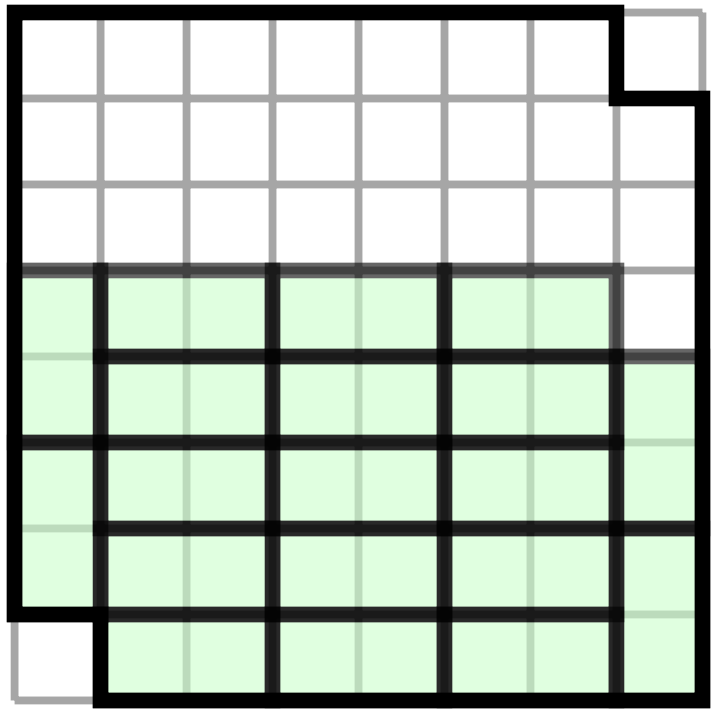

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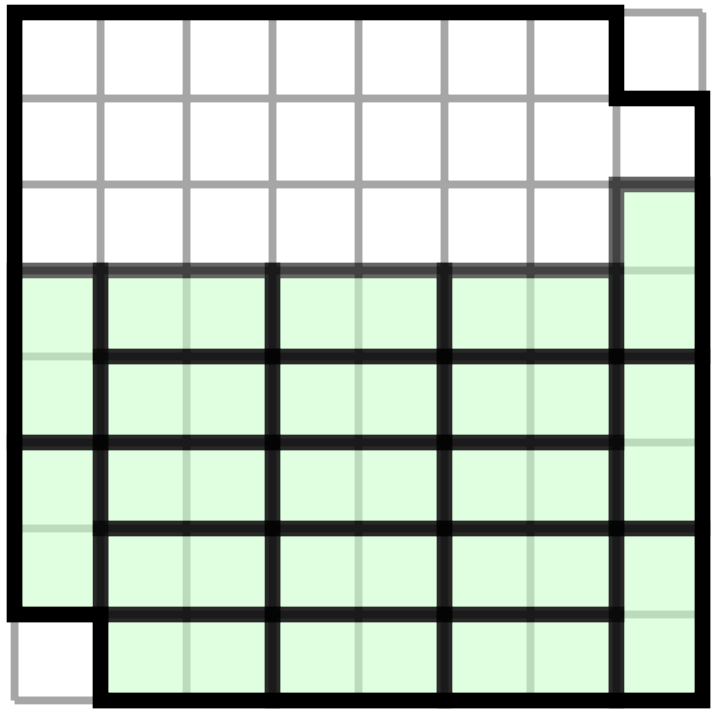

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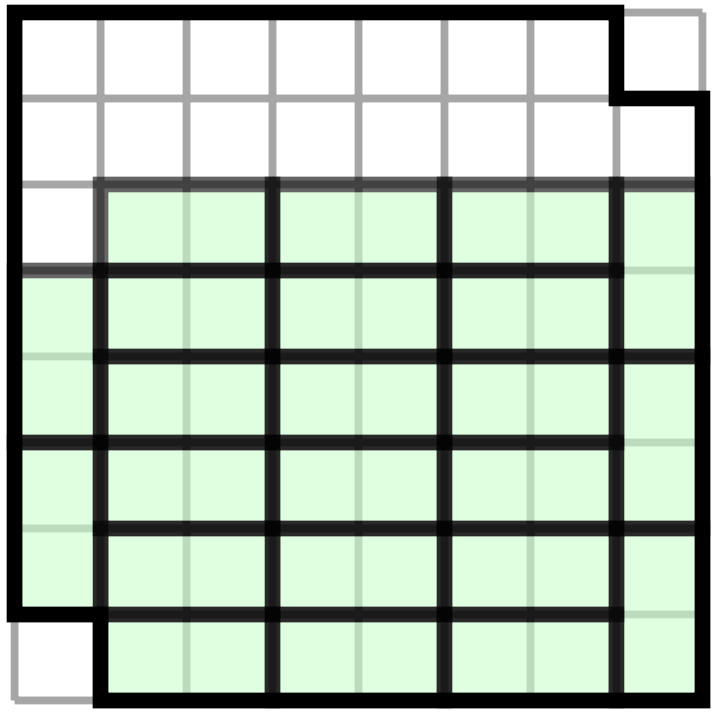

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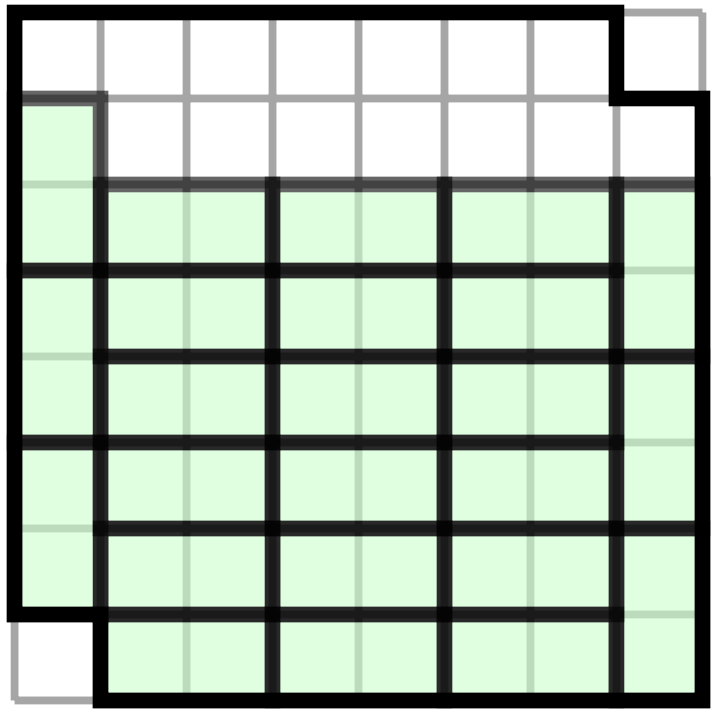

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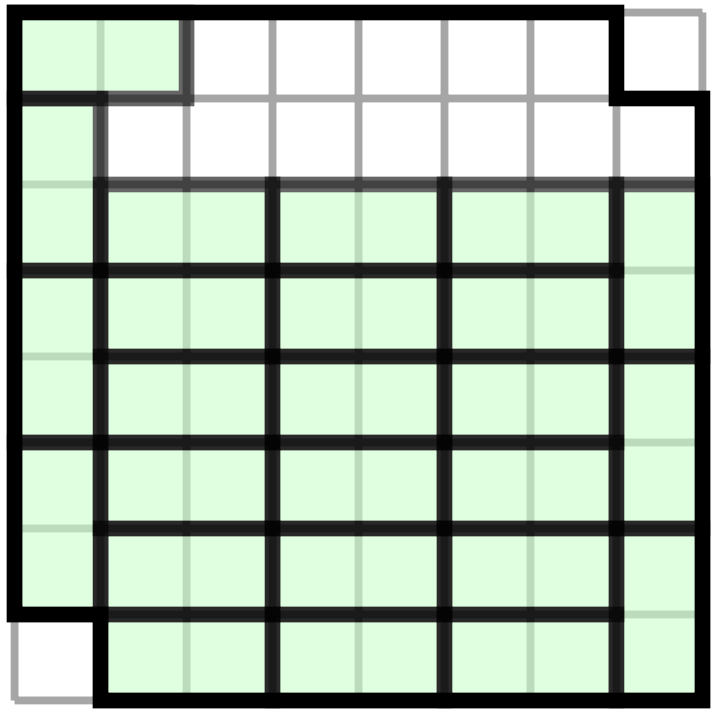

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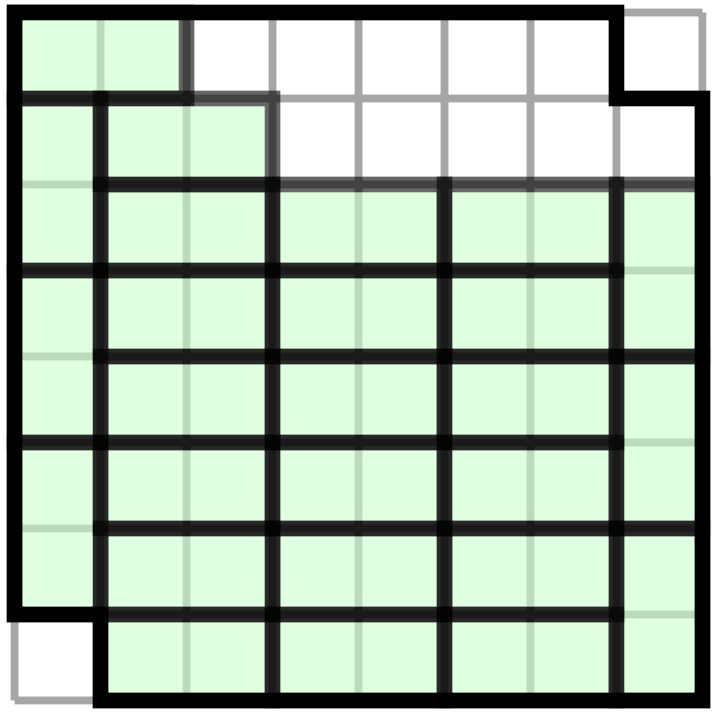

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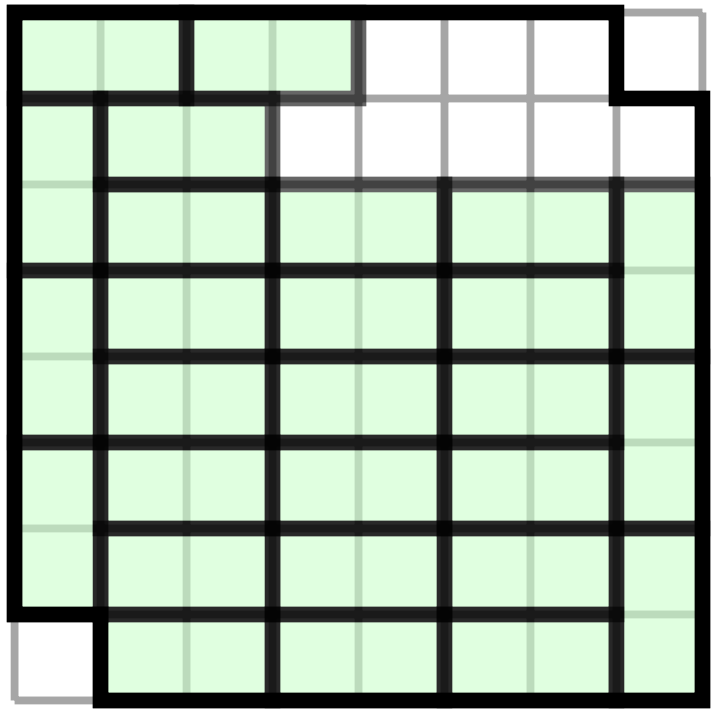

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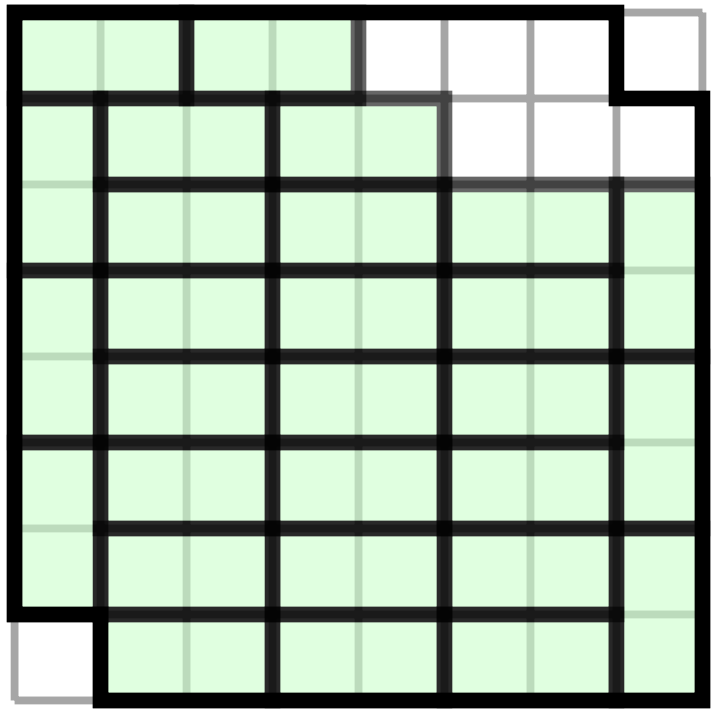

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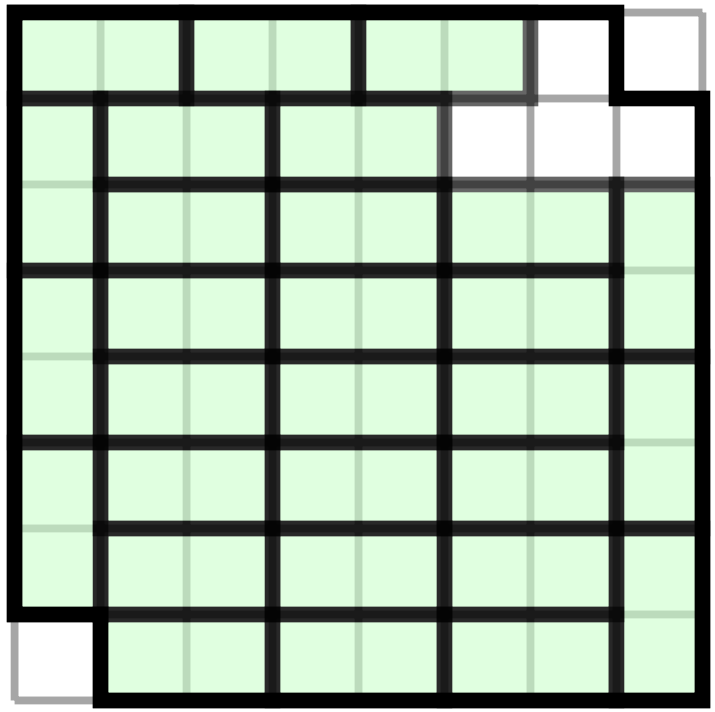

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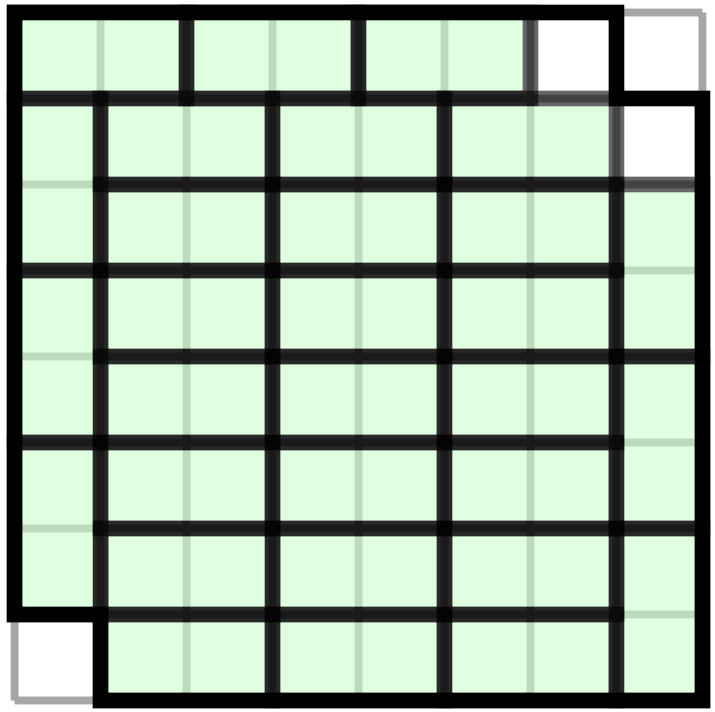

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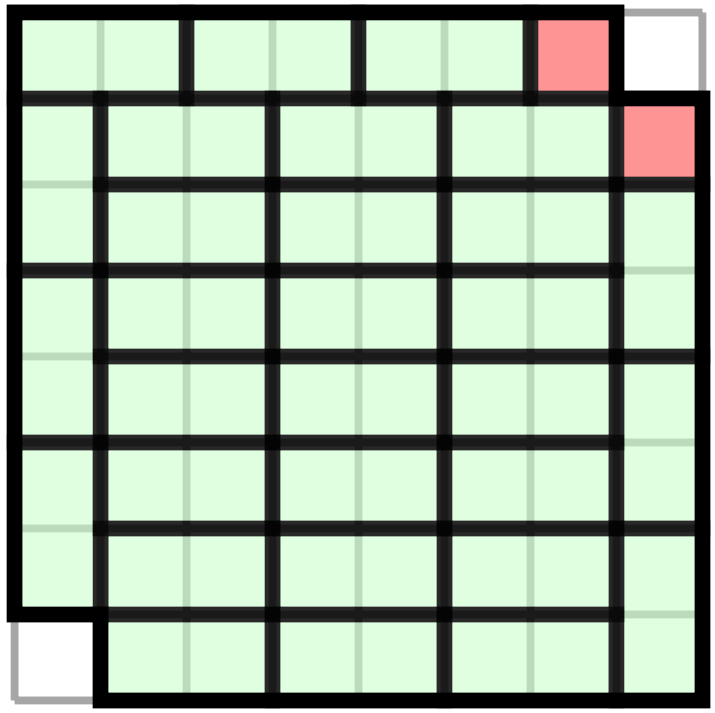

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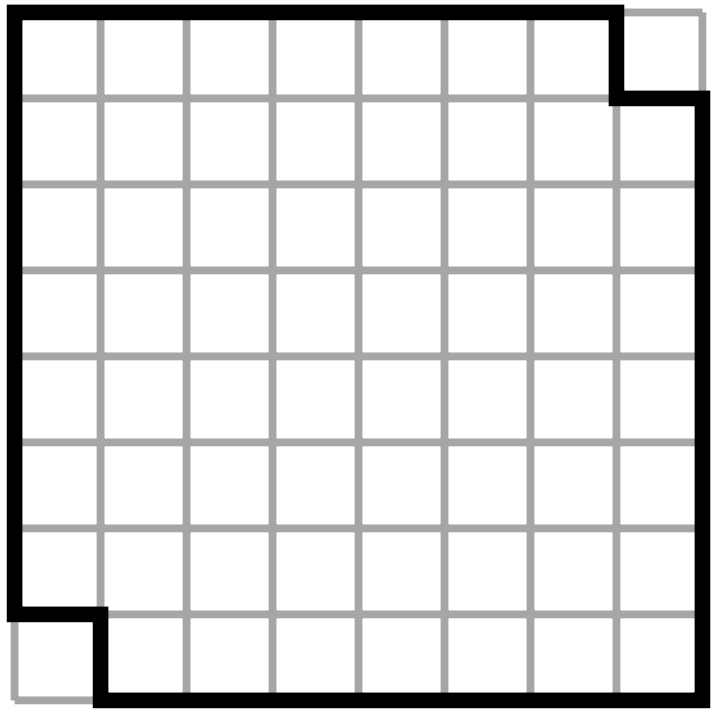

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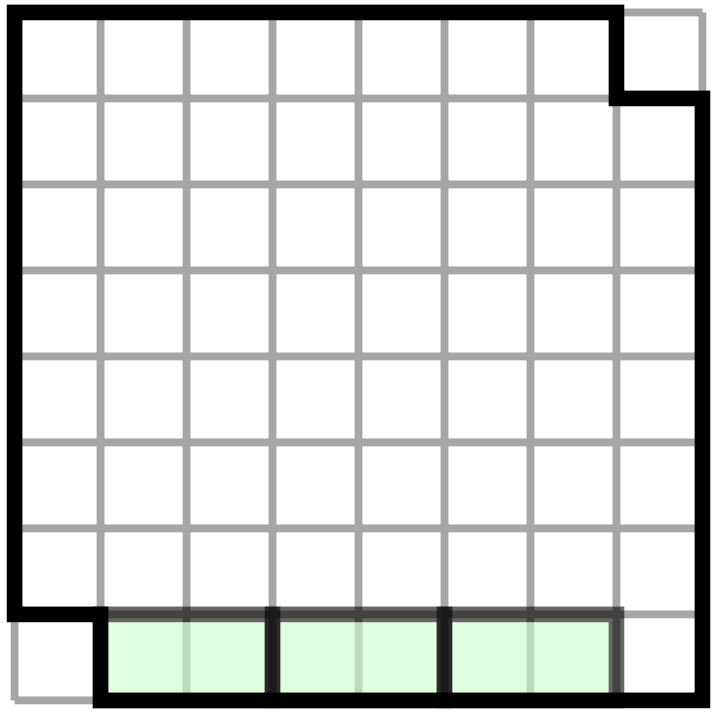

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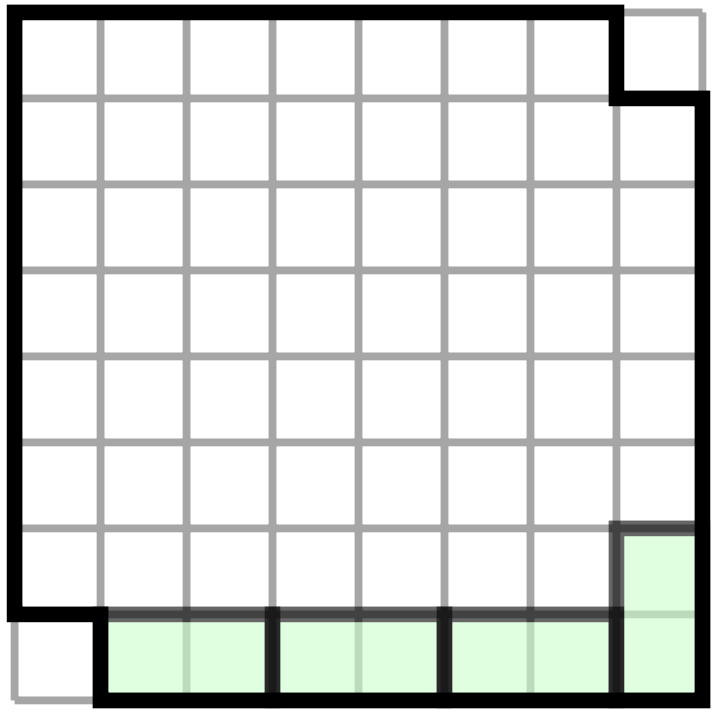

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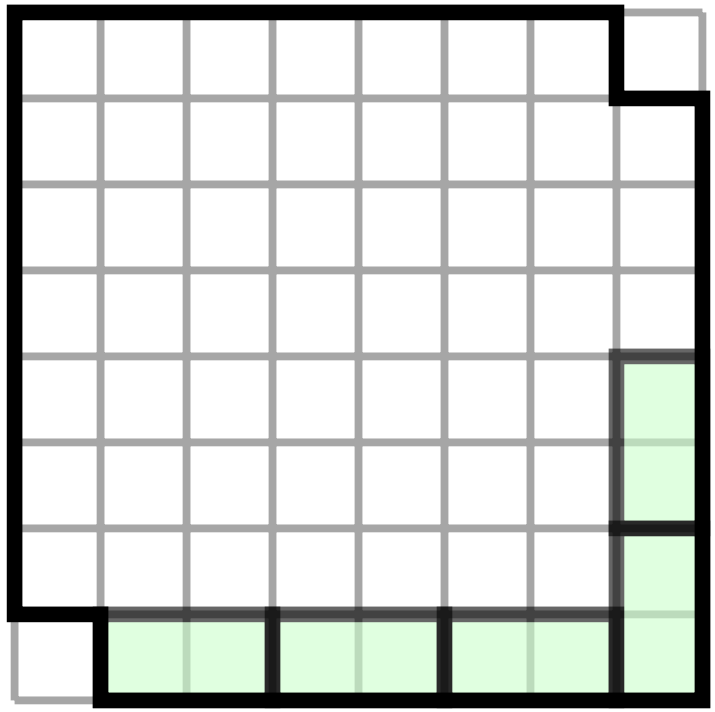

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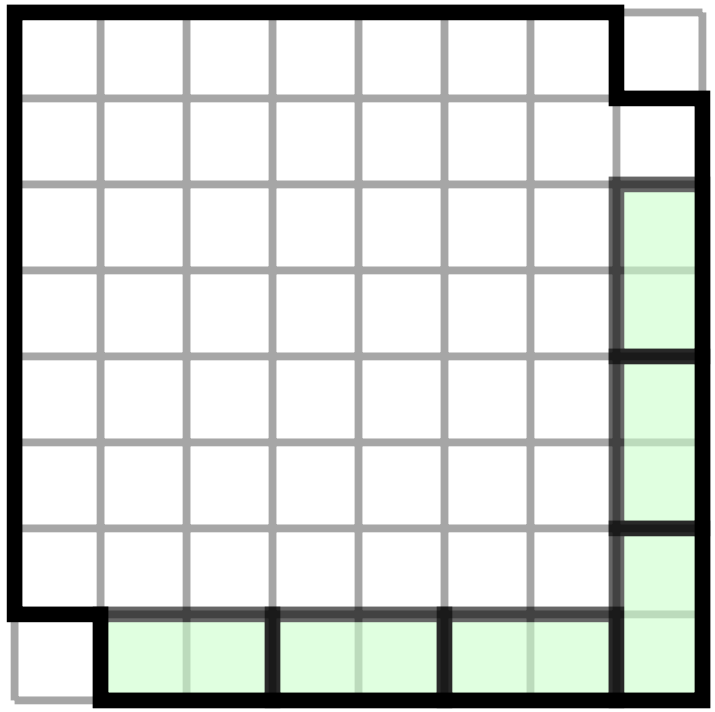

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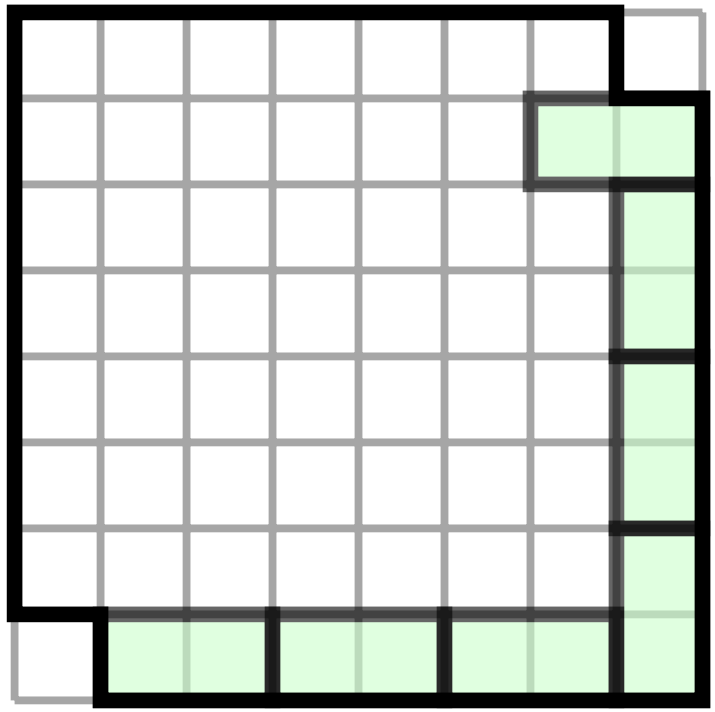

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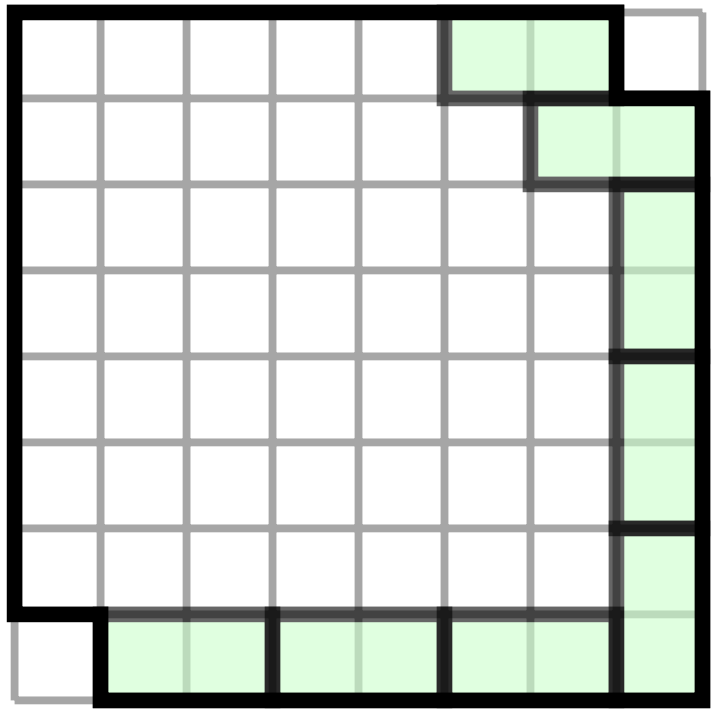

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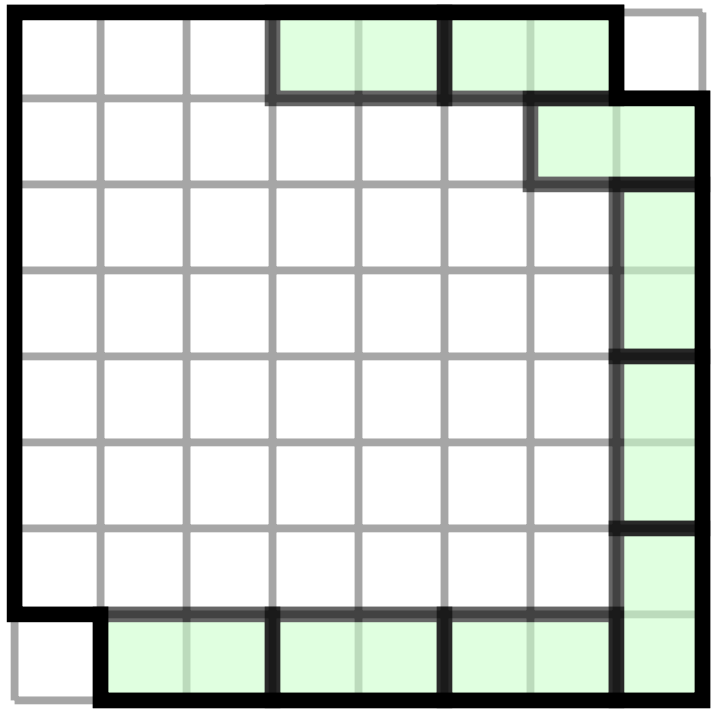

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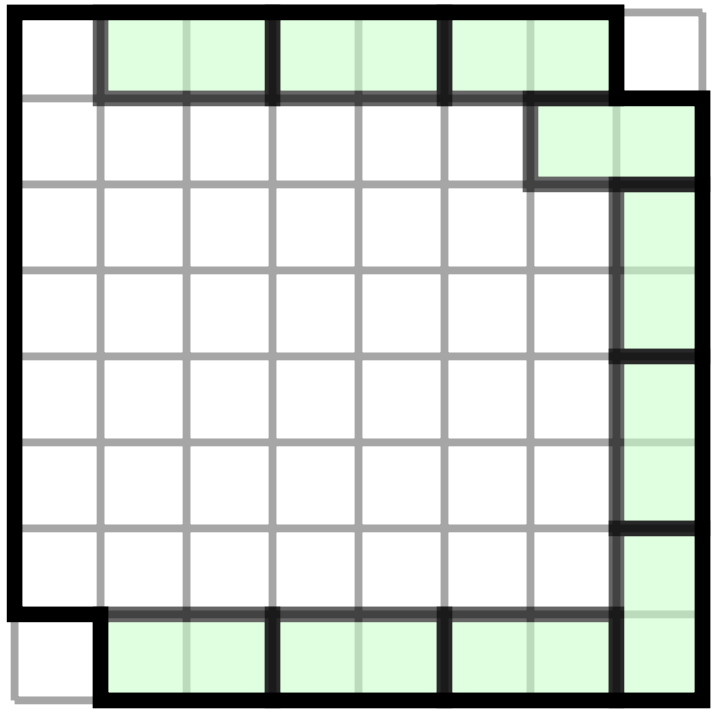

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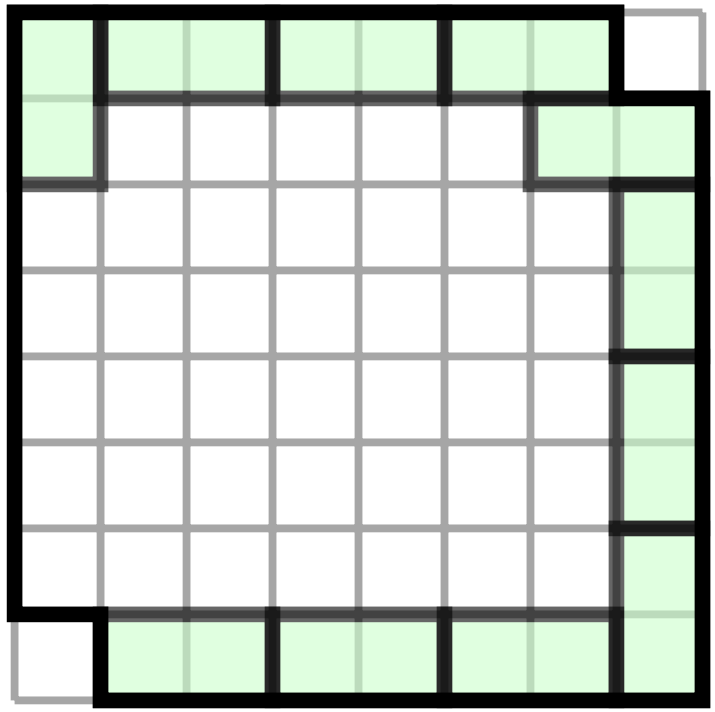

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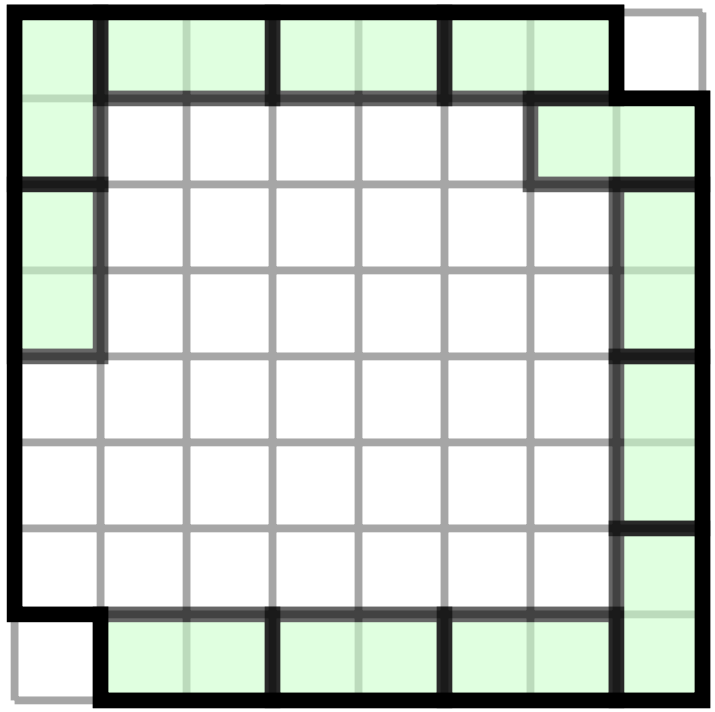

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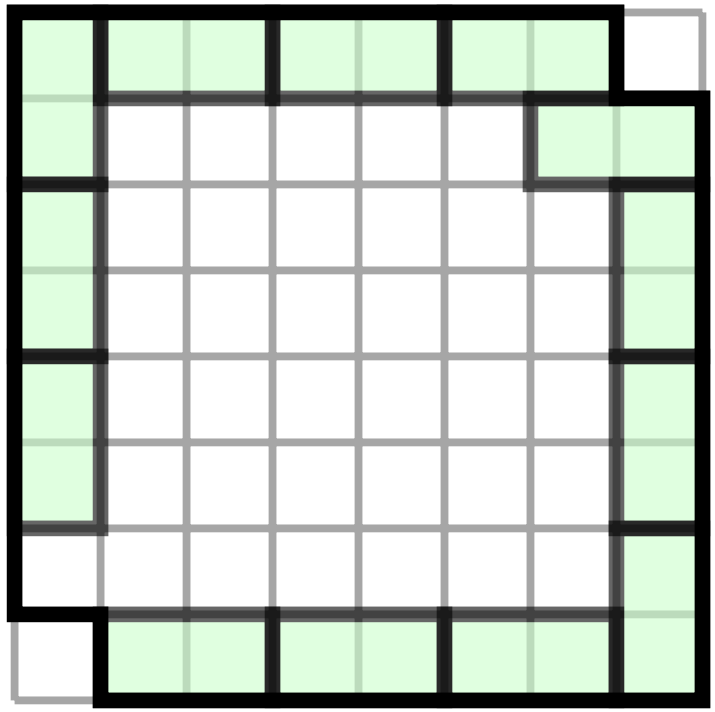

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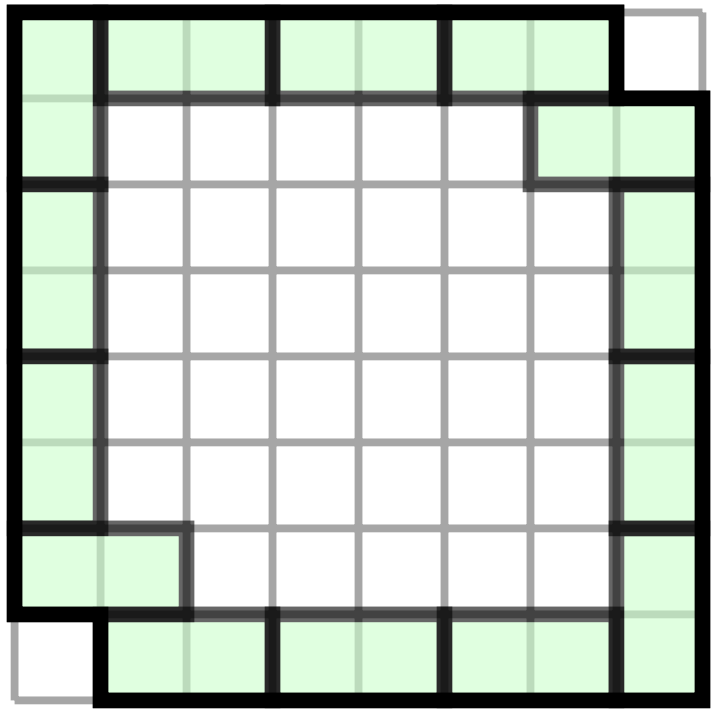

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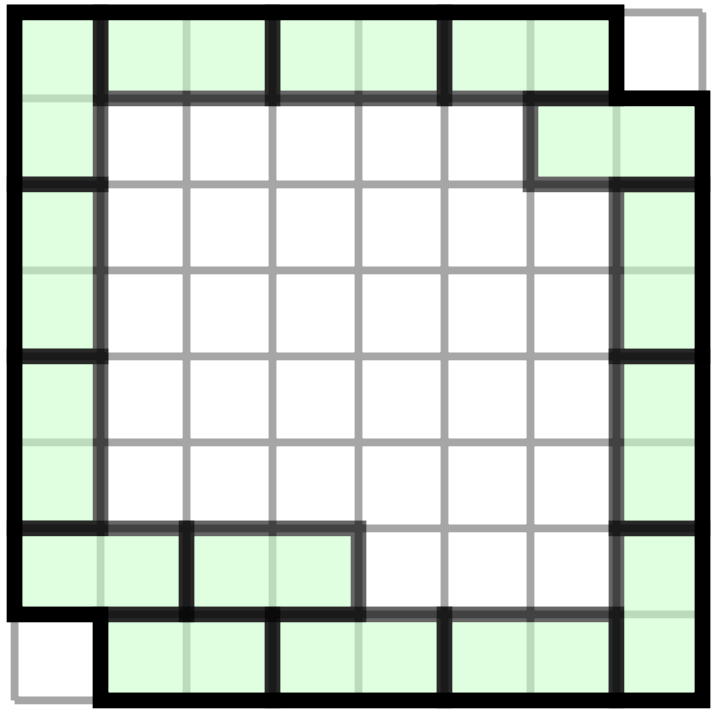

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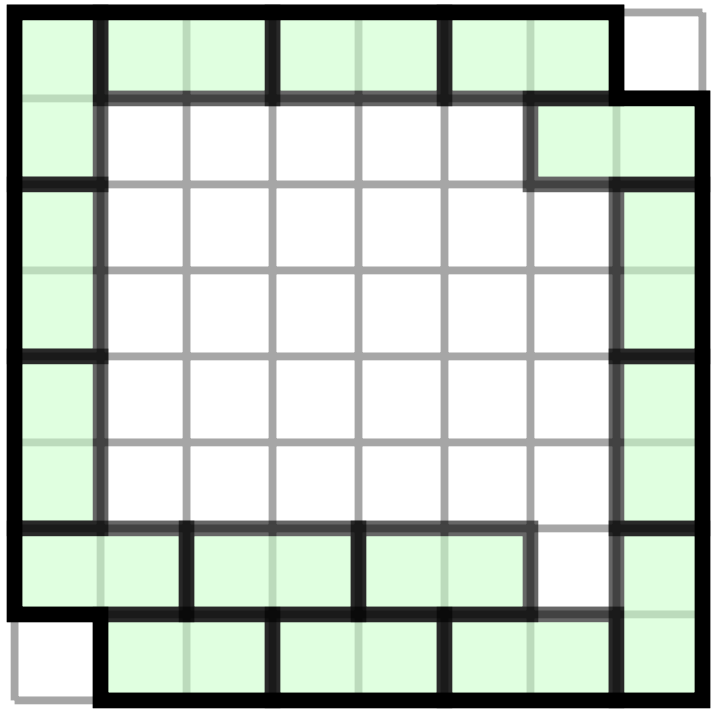

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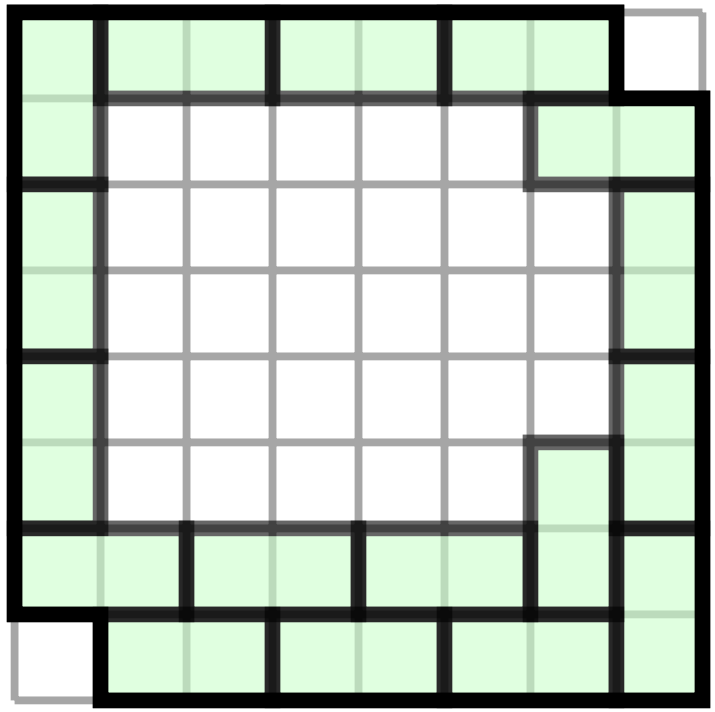

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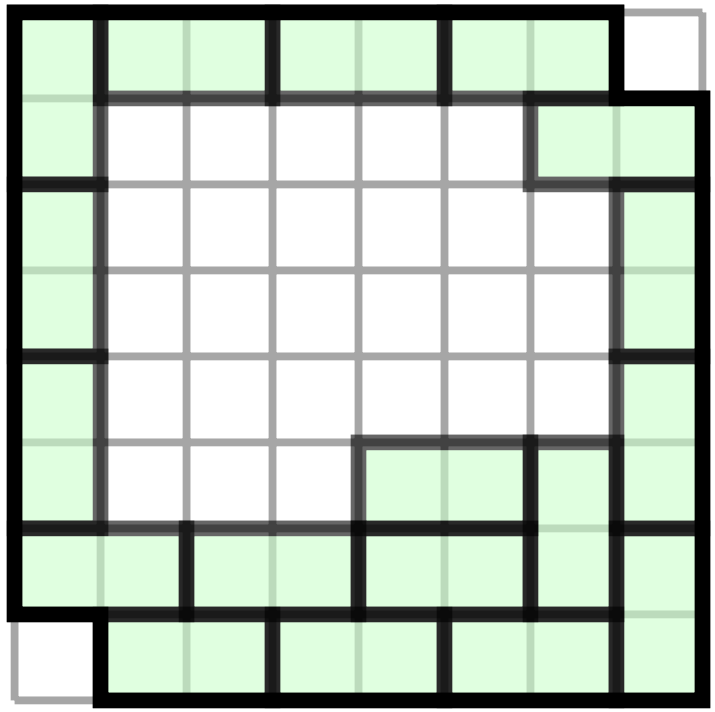

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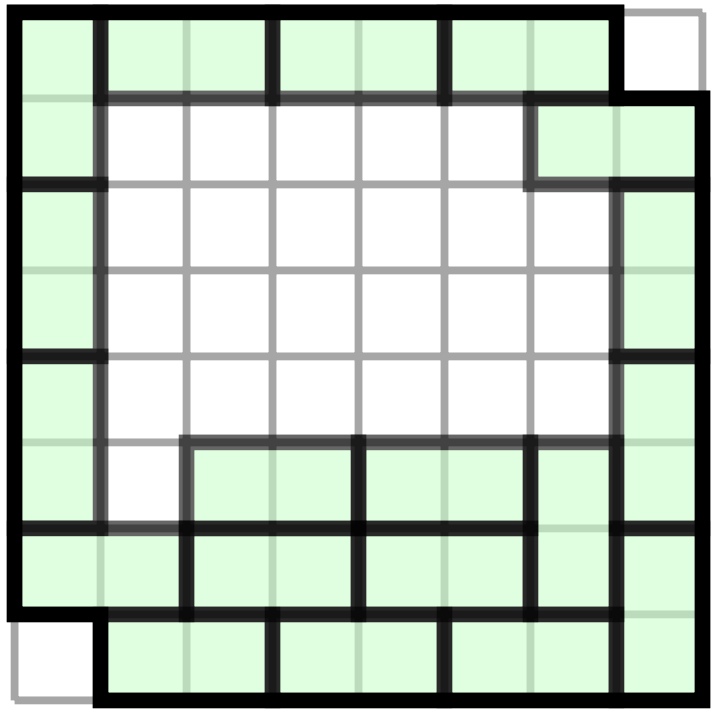

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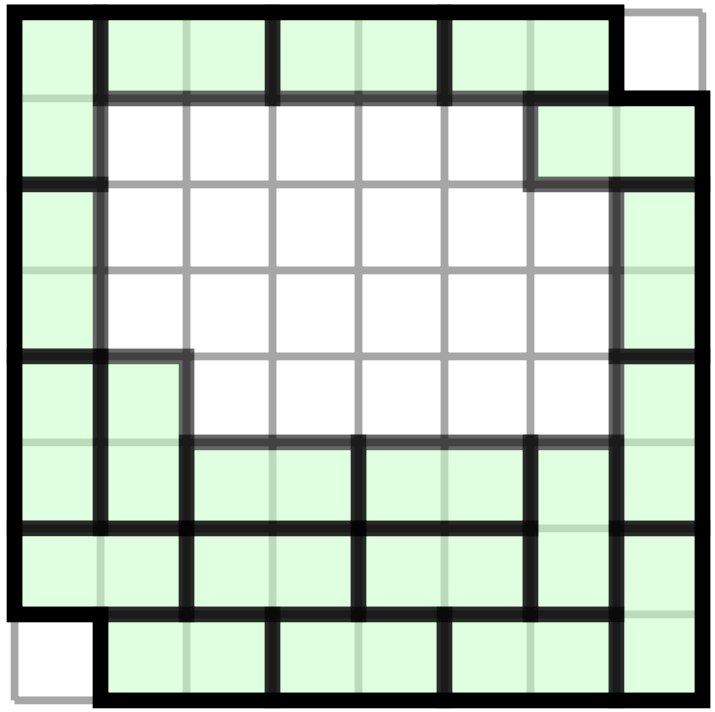

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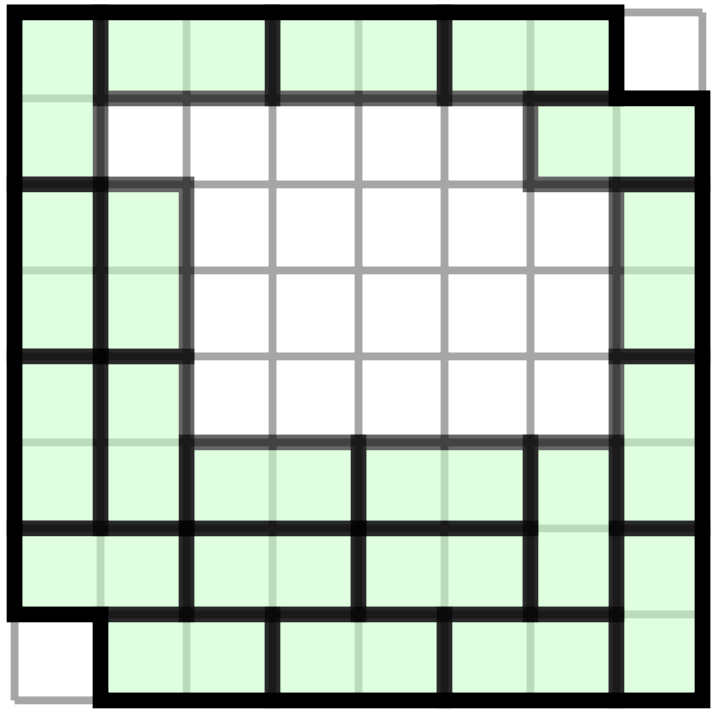

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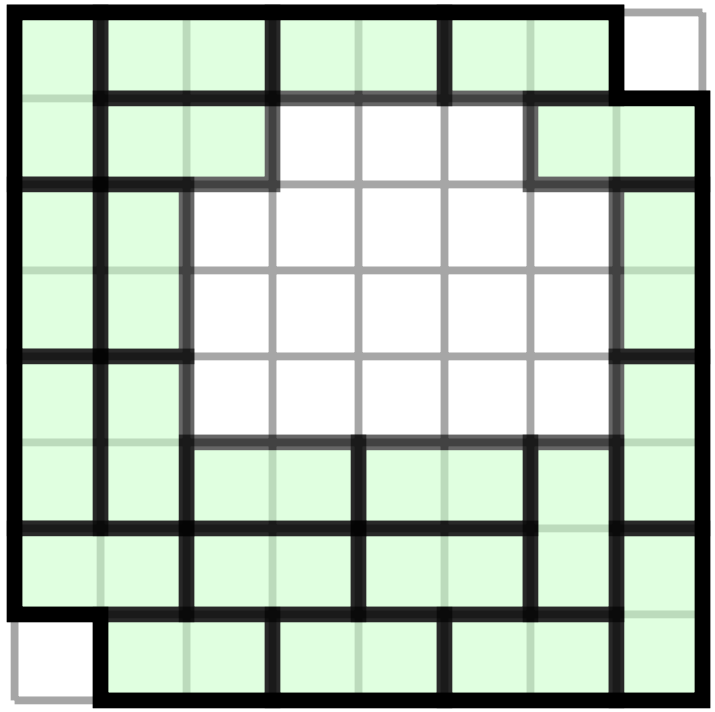

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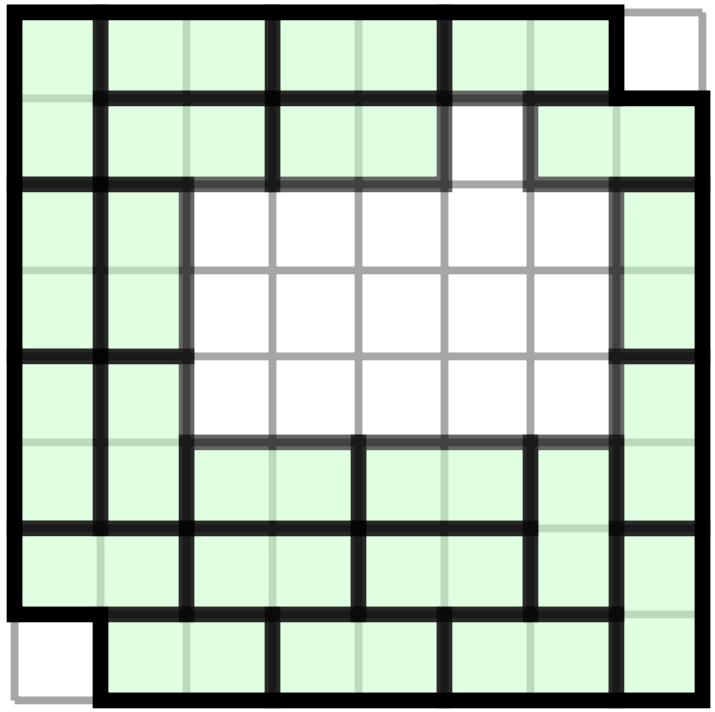

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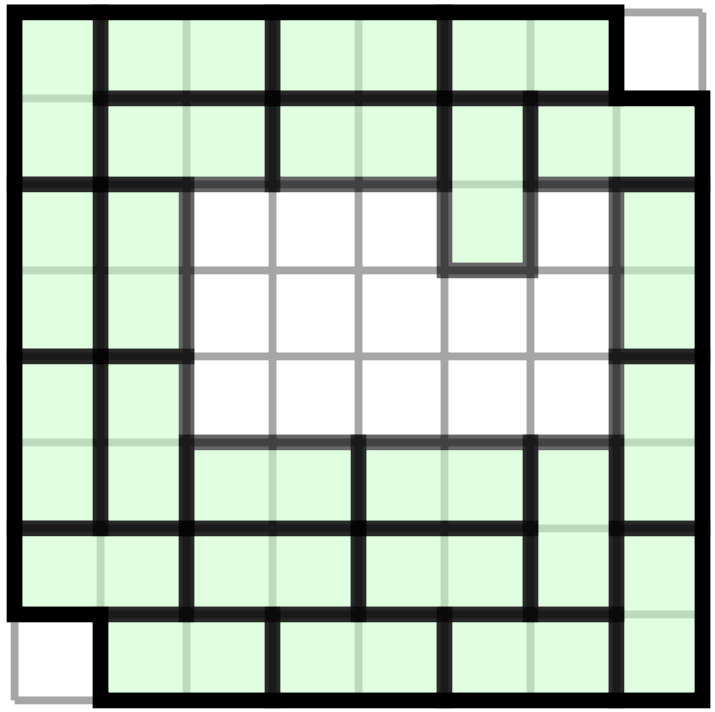

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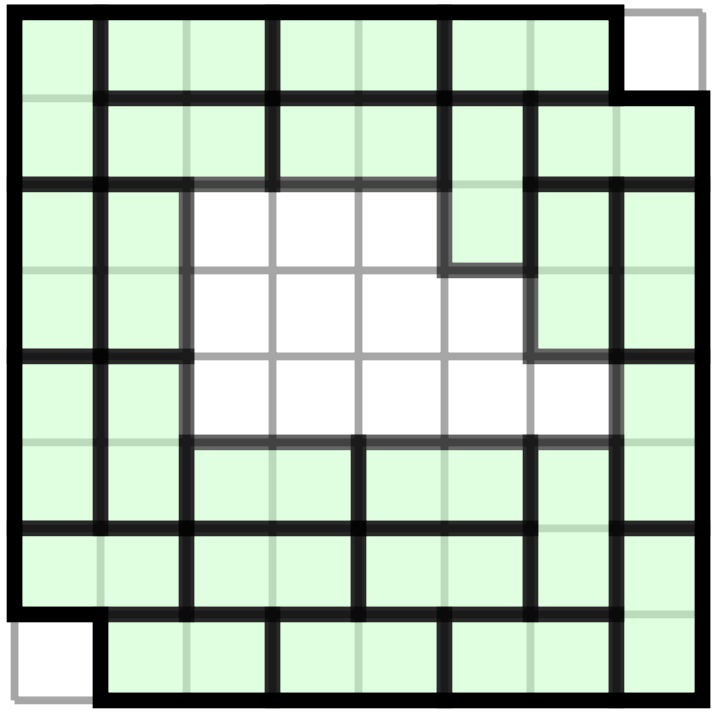

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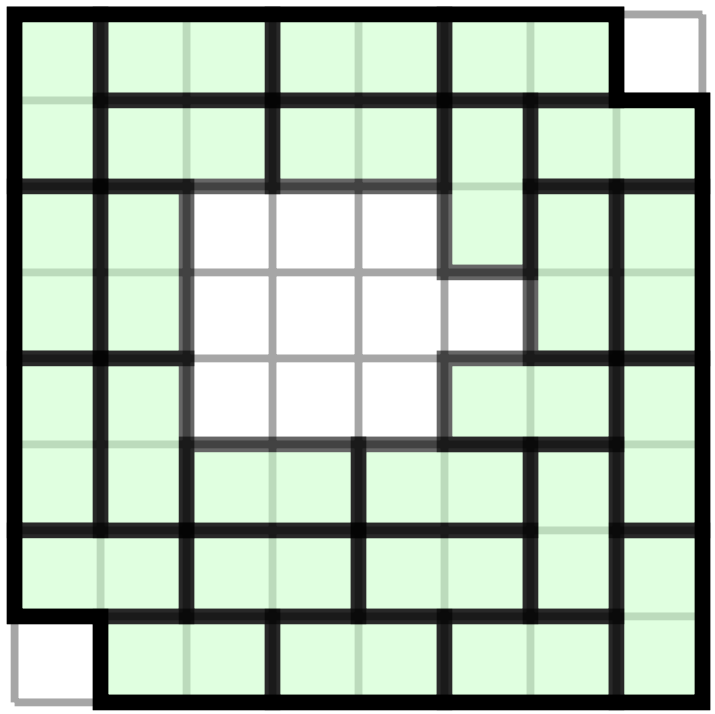

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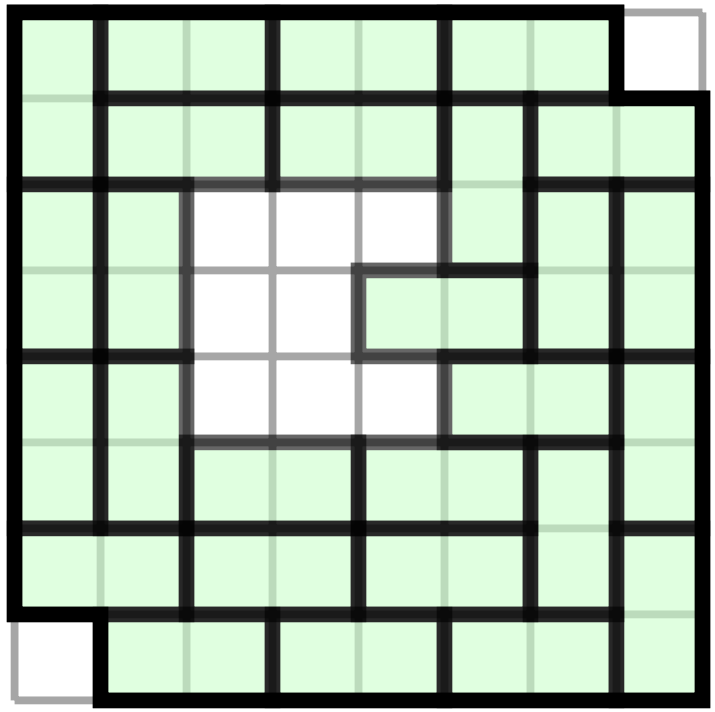

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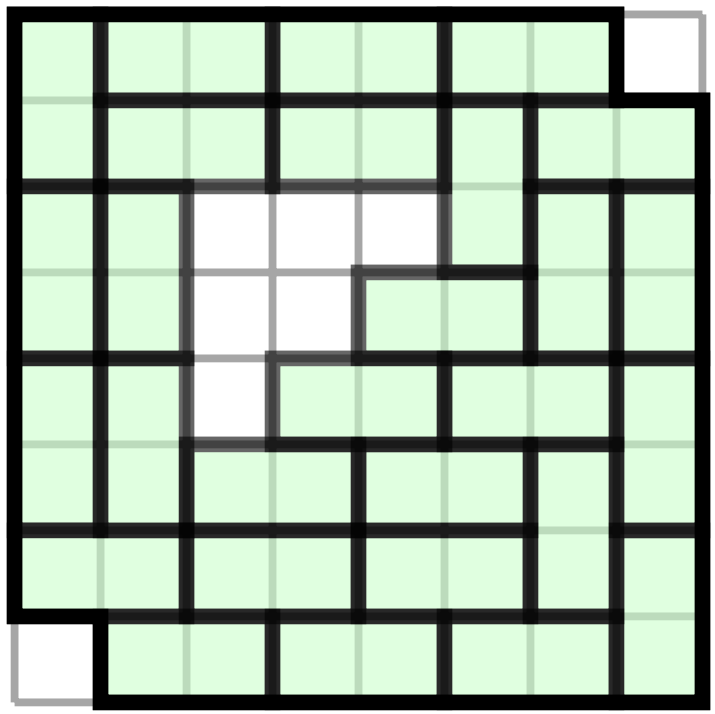

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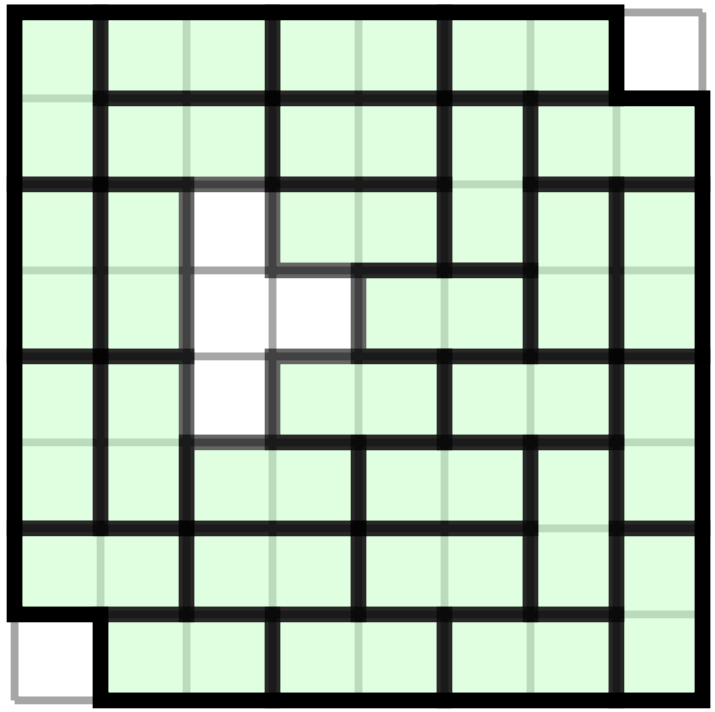

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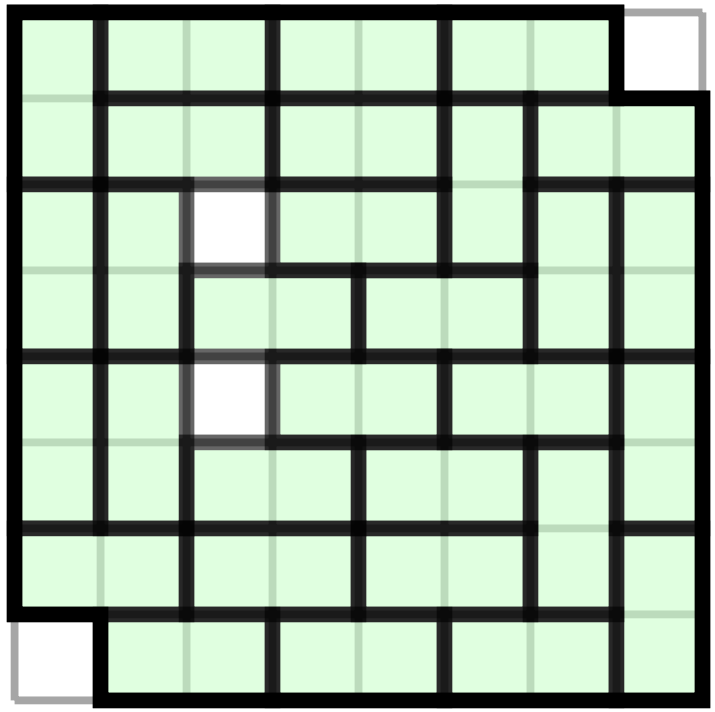

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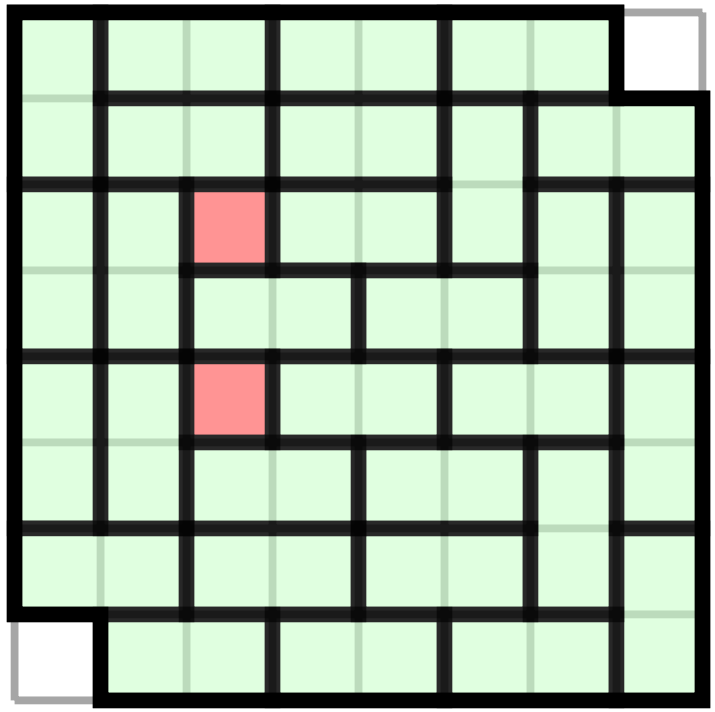
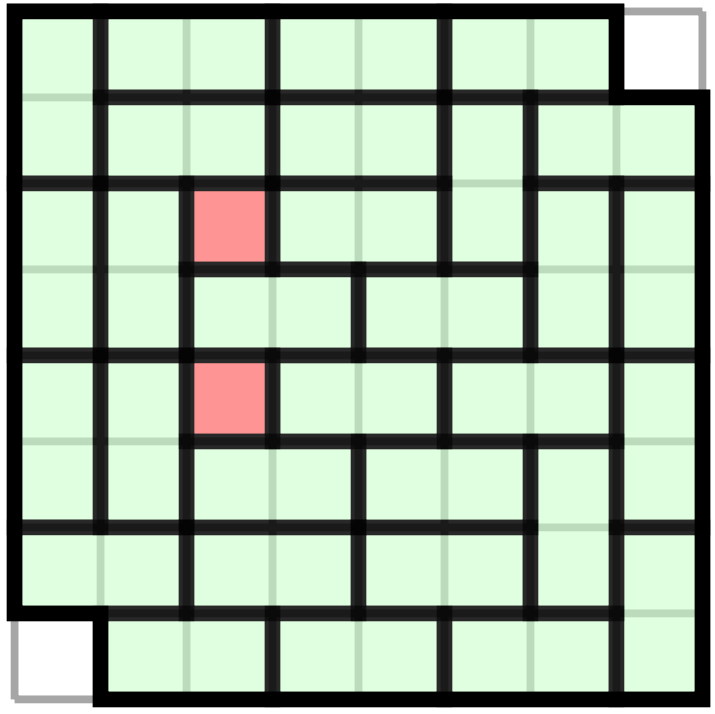


One More Attempt

should we try harder?



Outline

Proofs? Really?

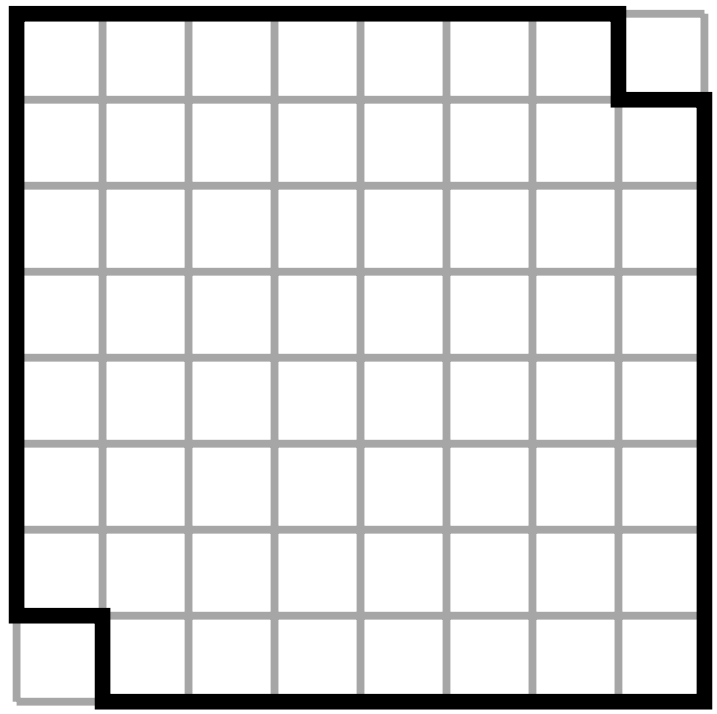
Proof by example

Impossibility proof

Impossibility Proof, II

Conclusion

Can We Tile This Board?



Spoiler

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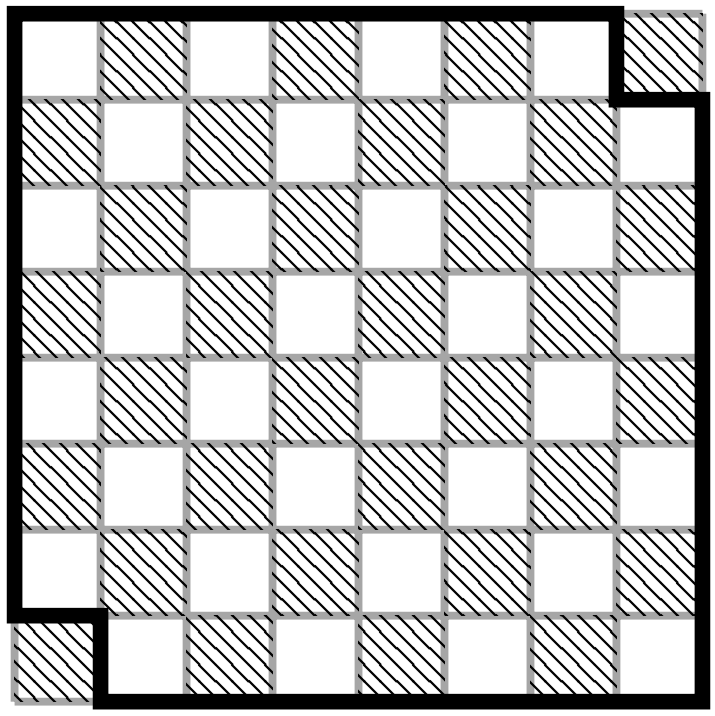

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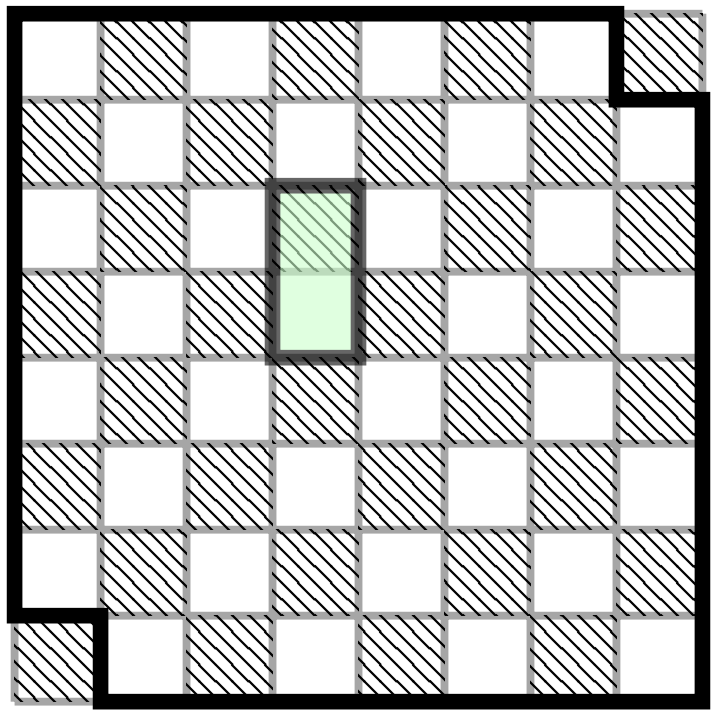

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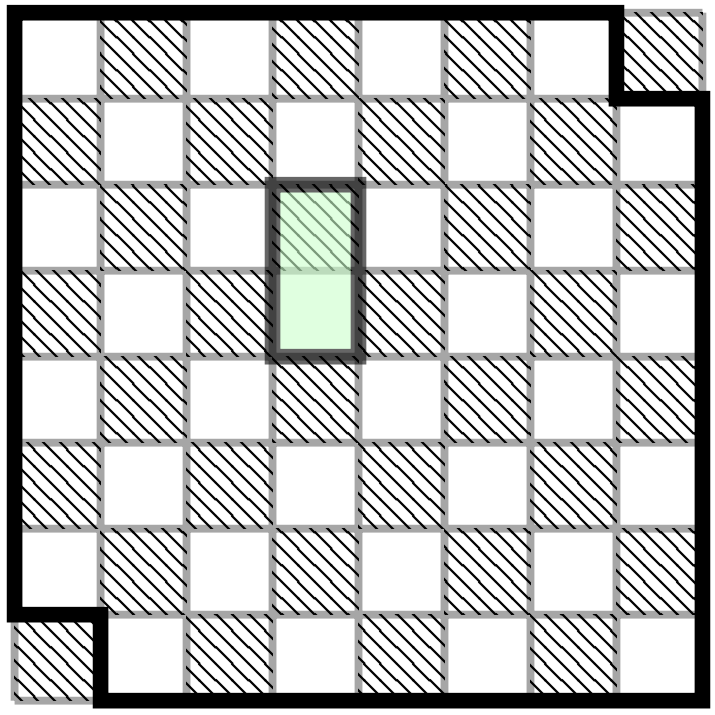
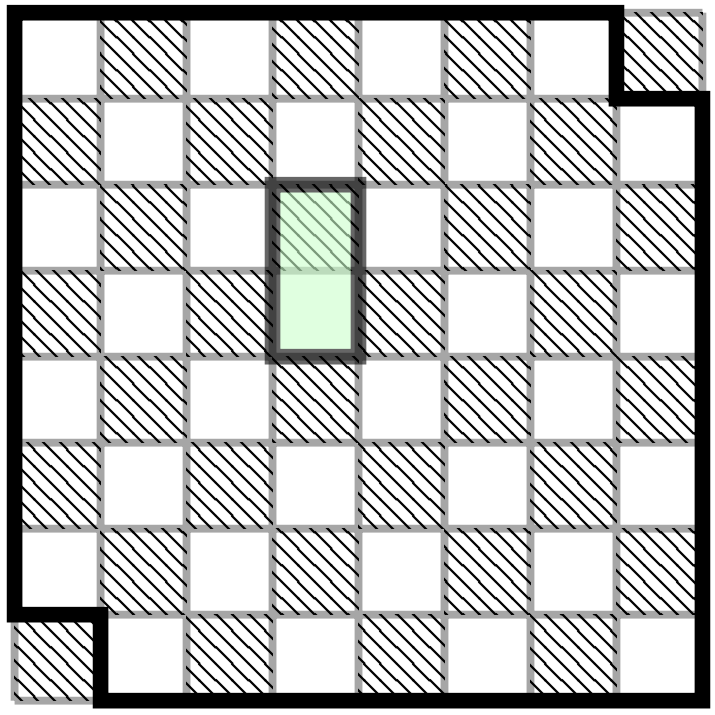


32 white, 30 black

Spoiler



32 white, 30 black $\Rightarrow$ two whites remain

Recap

Theorem. A chess board 8×8 without two opposite corners cannot be tiled by 1×2 dominos.

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- opposite corners are (say) black
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- a domino: two different colors
- (at least) two white remain
- quod erat demonstrandum

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Take-home Message and Questions

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- Proofs can be really convincing

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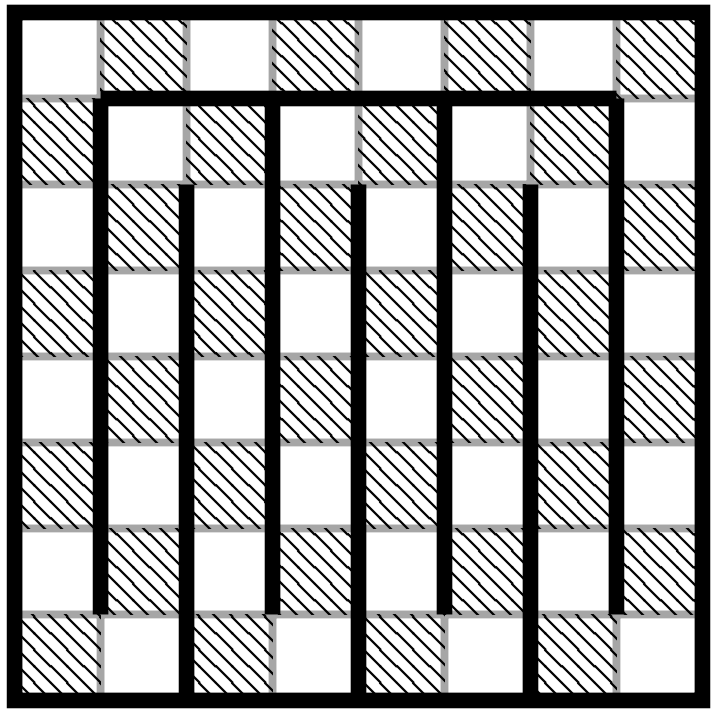
- What if we cut two non-opposite corners? Can we tile the rest?

Take-home Message and Questions

- Proofs can be really convincing
- What if we cut two non-opposite corners? Can we tile the rest?
- Bonus problem: what if we cut any two cells of different colors?

Spoiler

Spoiler