Basic Graphs

Alexander Golovnev

Outline

Paths, Cycles and Complete Graphs

Trees

Bipartite Graphs

Path Graphs

The Path Graph P_n, $n \geq 2$, consists of n vertices $v_1, \ldots, v_n$ and $n - 1$ edges $\{v_1, v_2\}, \ldots, \{v_{n-1}, v_n\}$

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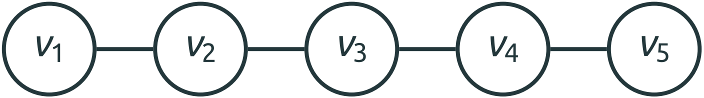
The Graph P_5



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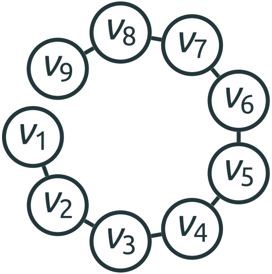
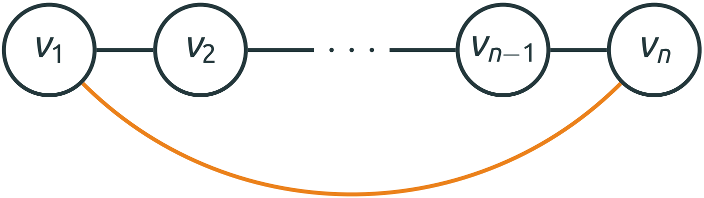


Cycle Graphs

The Cycle Graph C_n, $n \geq 3$, consists of n vertices $v_1, \ldots, v_n$ and n edges $\{v_1, v_2\}, \ldots, \{v_{n-1}, v_n\}$, $\{v_n, v_1\}$



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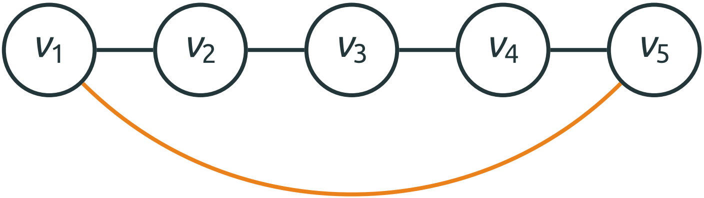

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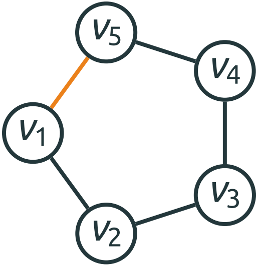

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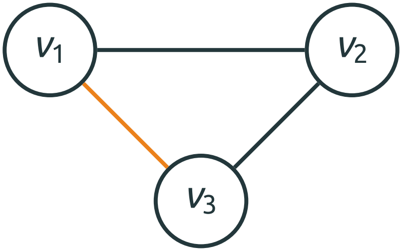
The Graph C_3



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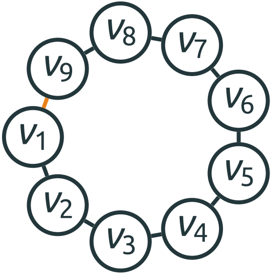
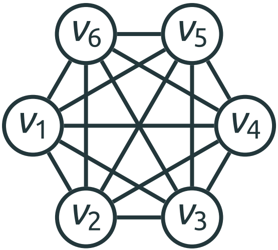


Complete Graphs

The Complete Graph (Clique) K_n, $n \geq 2$, contains n vertices $v_1, \ldots, v_n$ and all edges between them ($n(n-1)/2$ edges)



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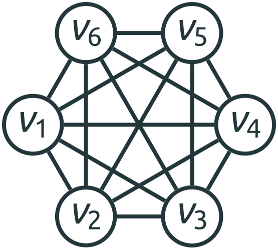

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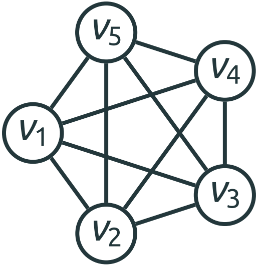

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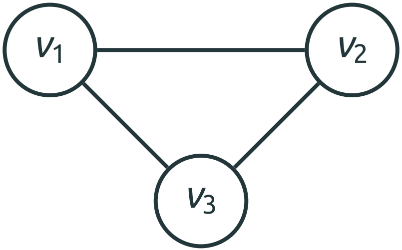

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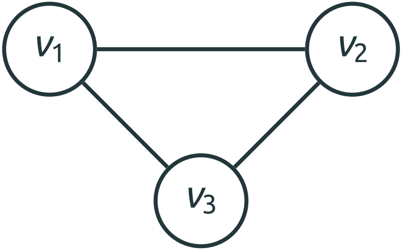
The Graph $K_3 = C_3$



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The Graph K_2

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The Complete Graph (Clique) K_n, $n \geq 2$, contains n vertices $v_1, \ldots, v_n$ and all edges between them ($n(n-1)/2$ edges)

The Graph $K_2 = P_2$

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Trees

Bipartite Graphs

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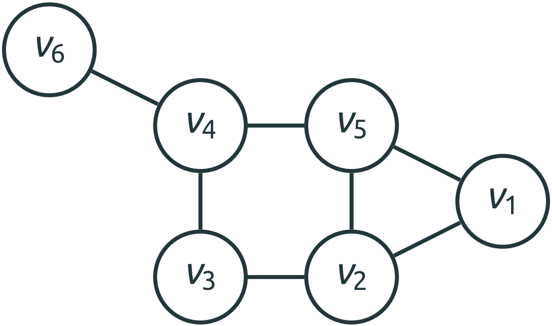

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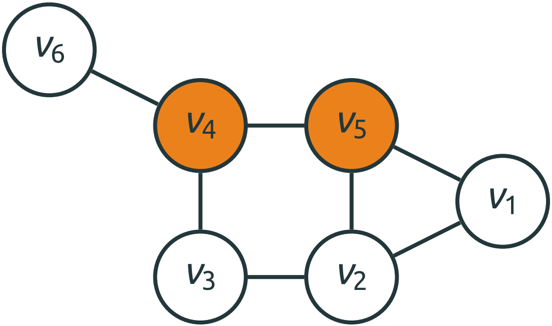

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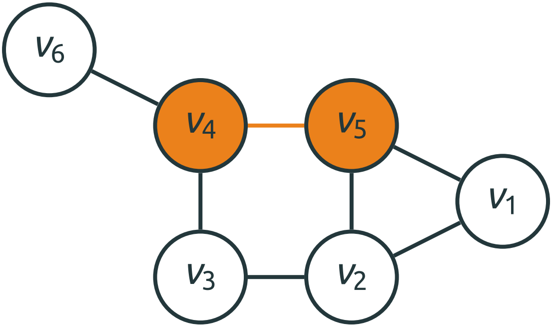

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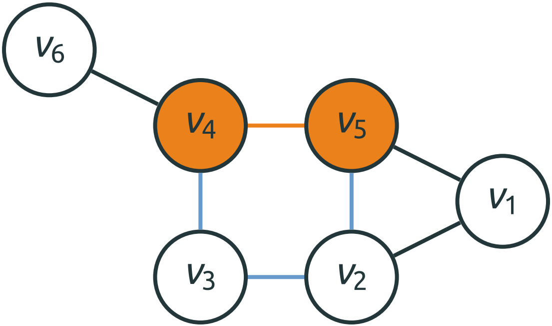

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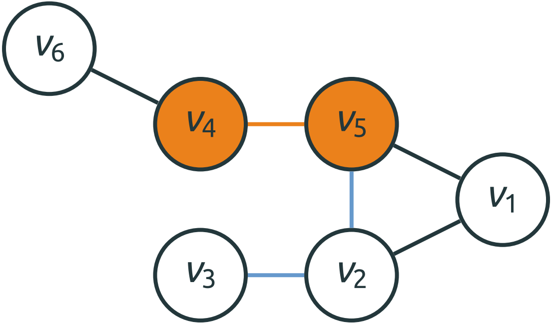

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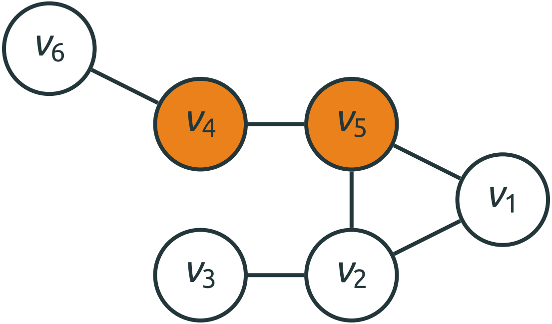

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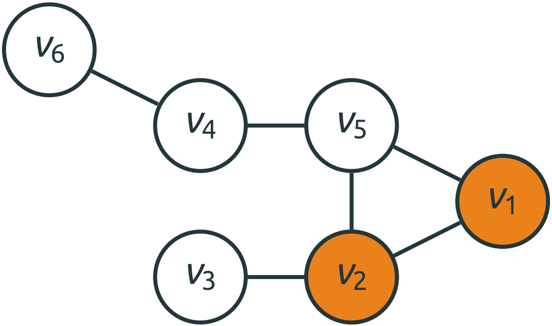

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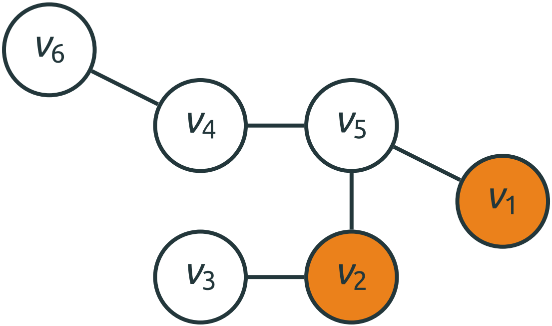

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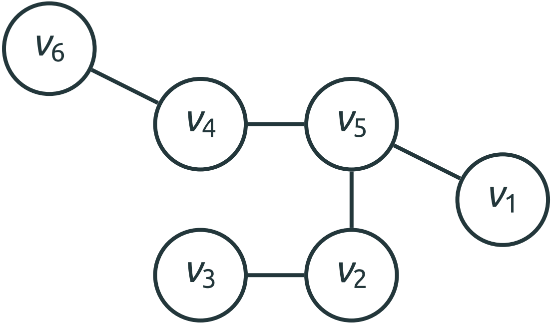

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- A tree is a connected graph on n vertices with $n - 1$ edges

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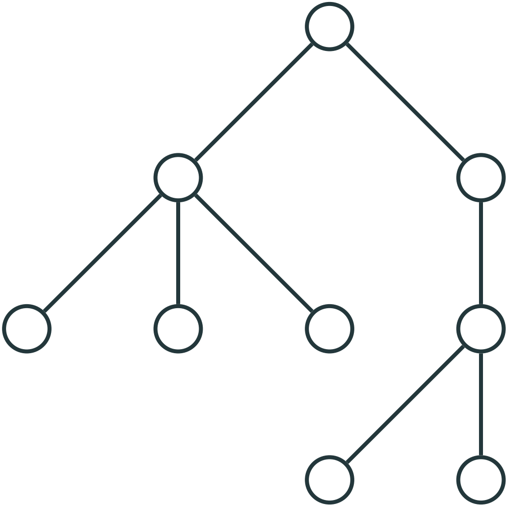
- A tree is a connected graph on n vertices with $n - 1$ edges
- A graph is a tree if and only if there is a unique simple path between any pair of its vertices

Trees: Examples



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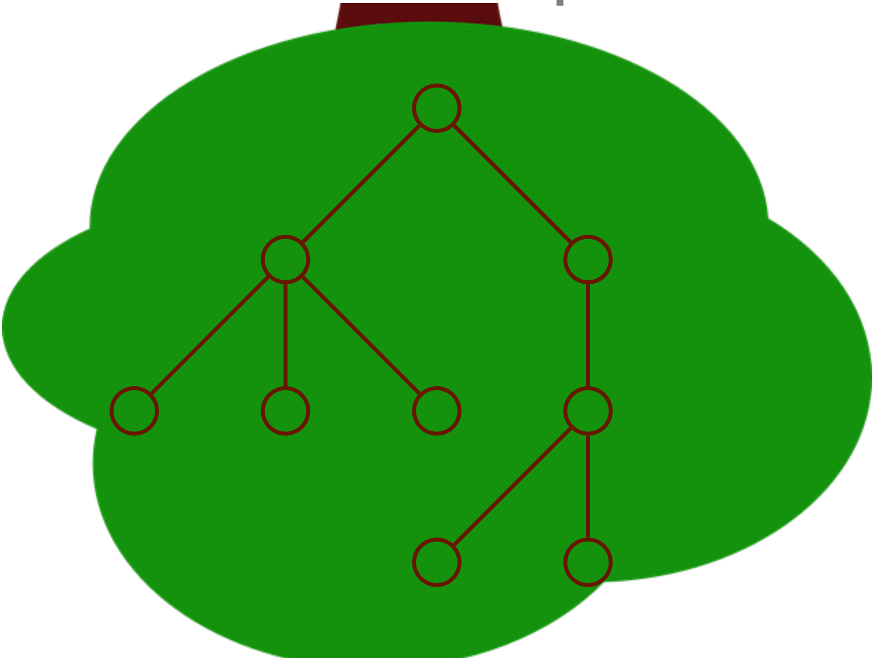

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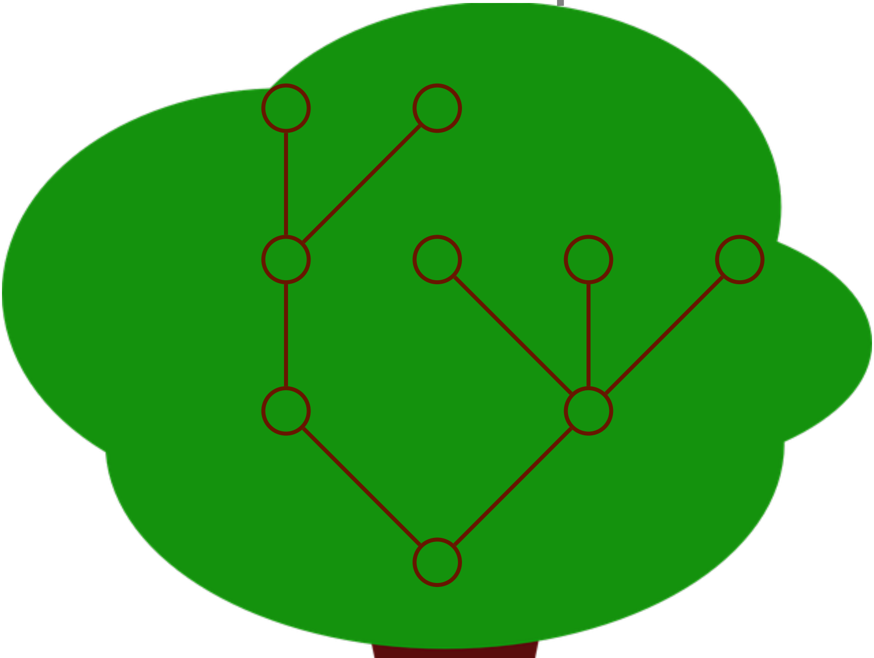

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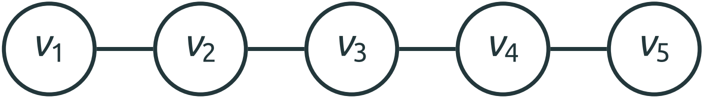

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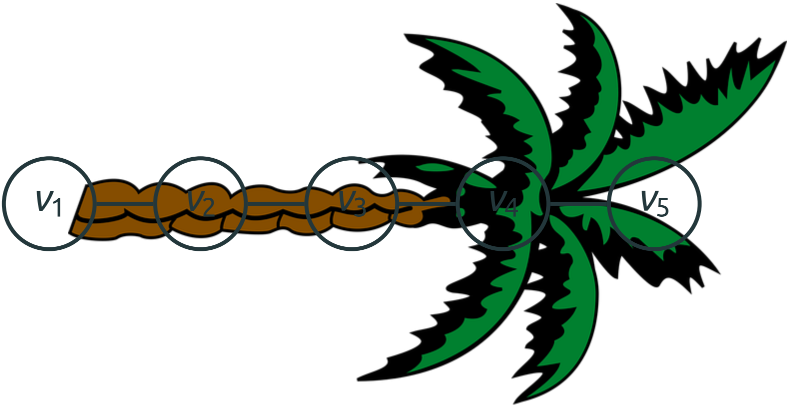

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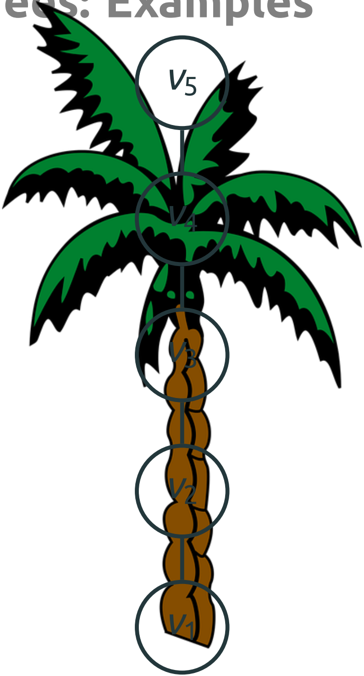
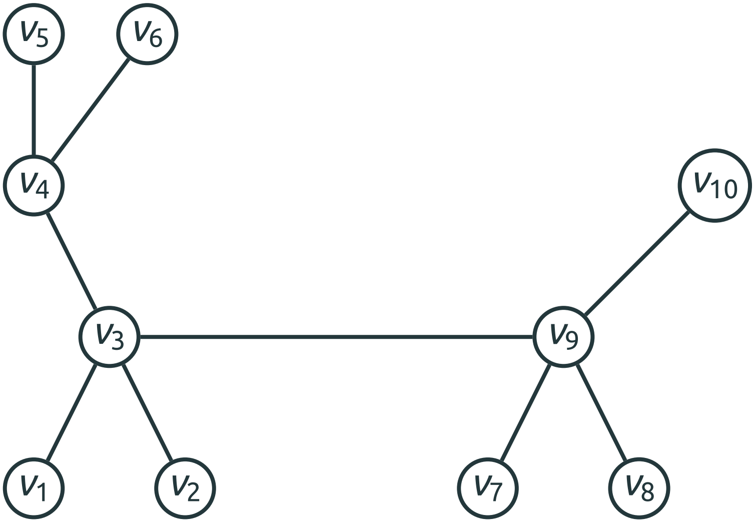


Drawing a Tree



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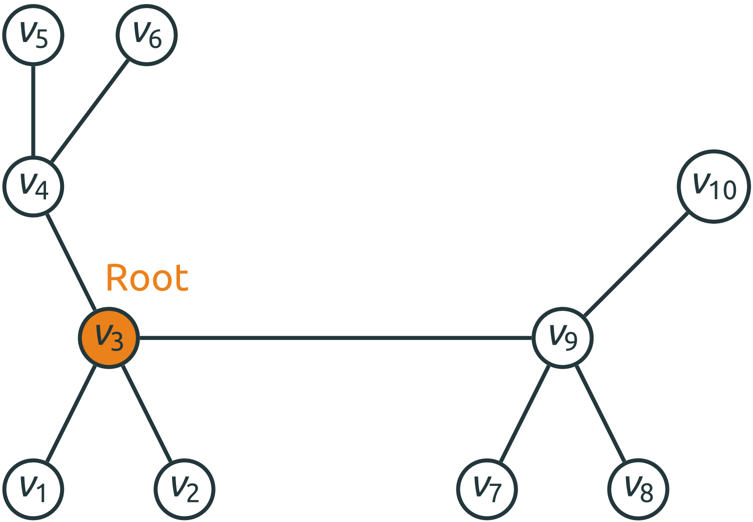

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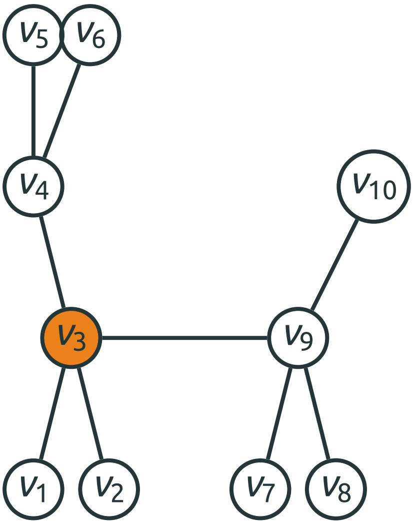

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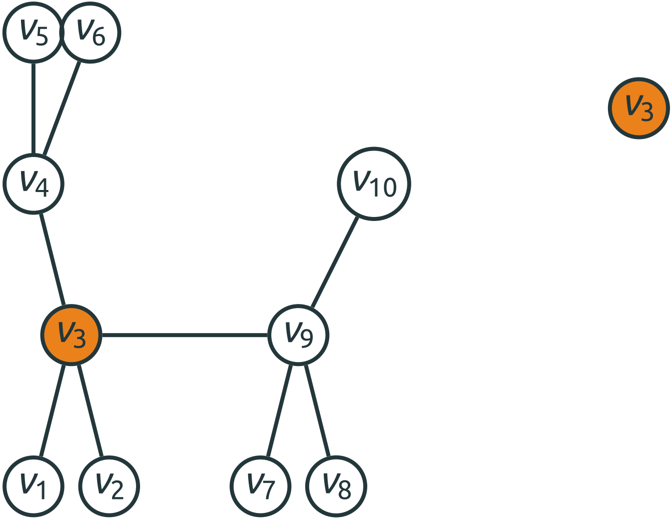

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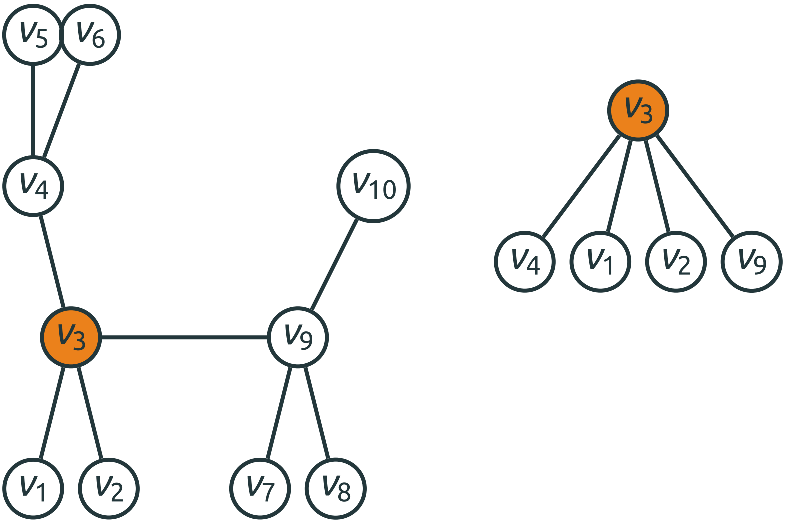

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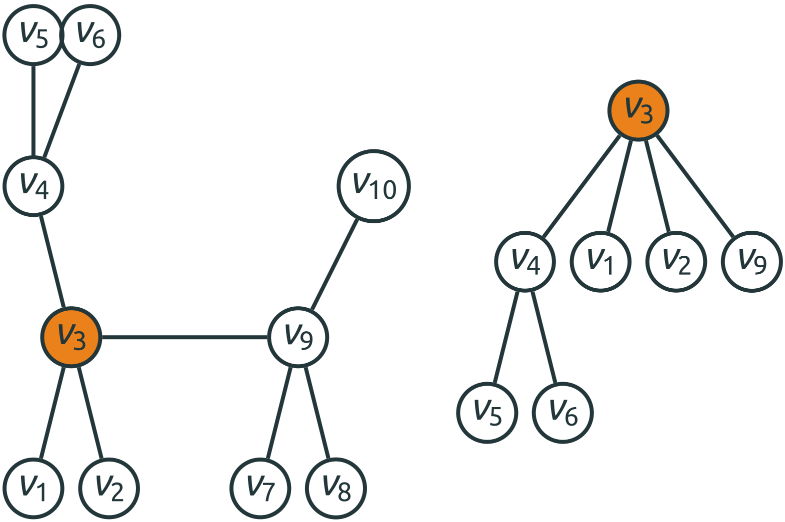

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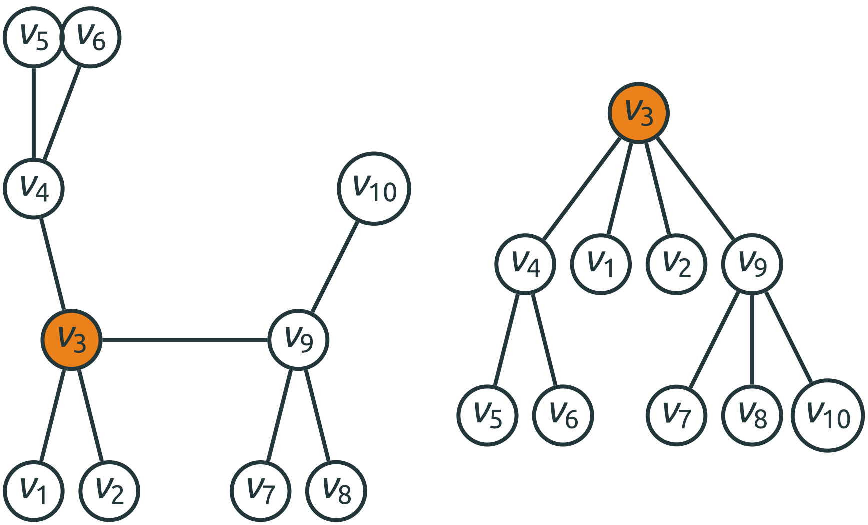

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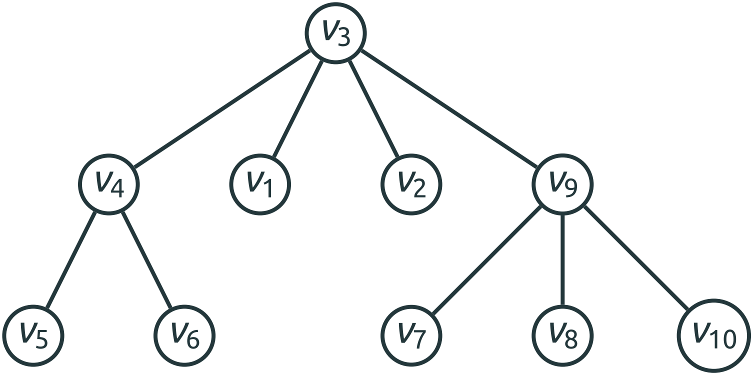
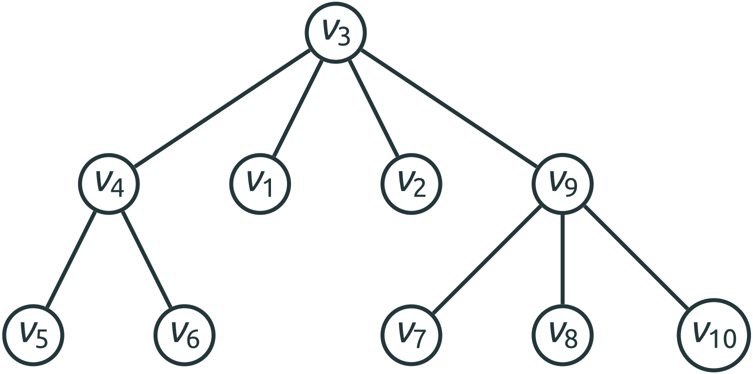
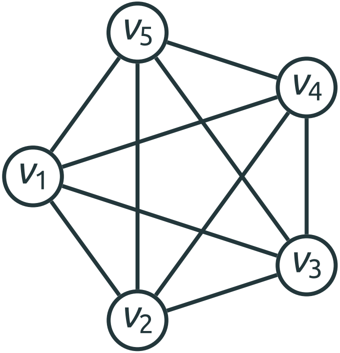


Drawing a Tree

Connected; the number of edges is $n - 1$

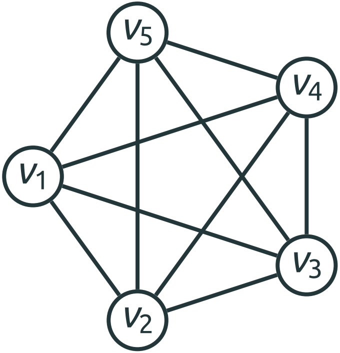


Make a Tree



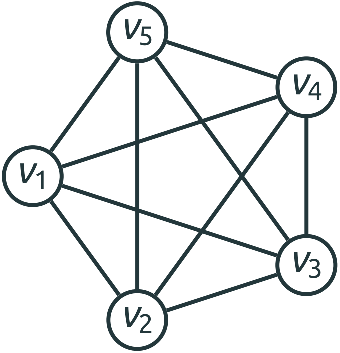
Make a Tree

Remove any edge, keeping the graph connected



Make a Tree

Remove any edge, keeping the graph connected
Stop when only $n - 1$ edges left



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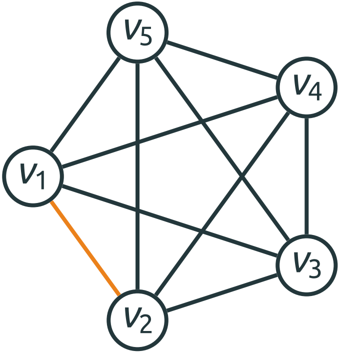

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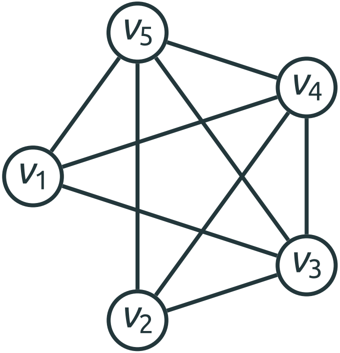

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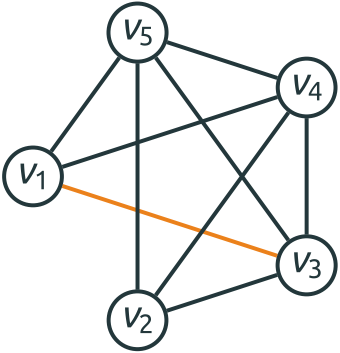

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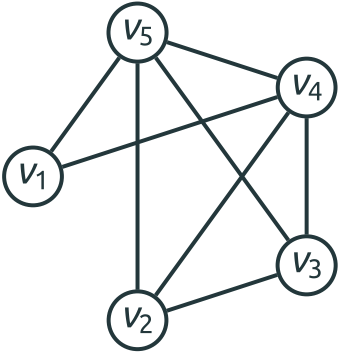

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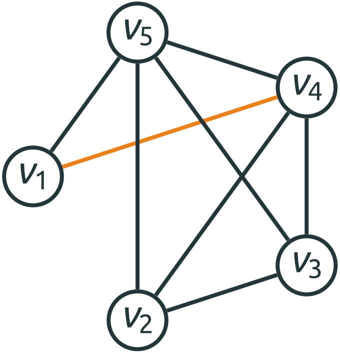

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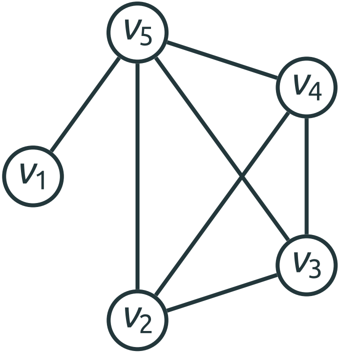

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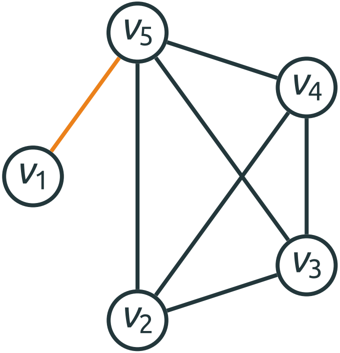

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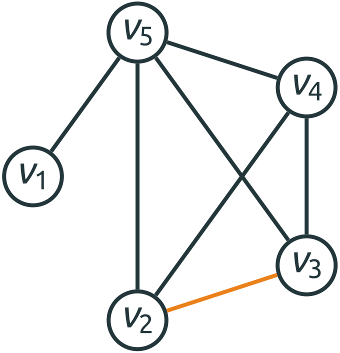

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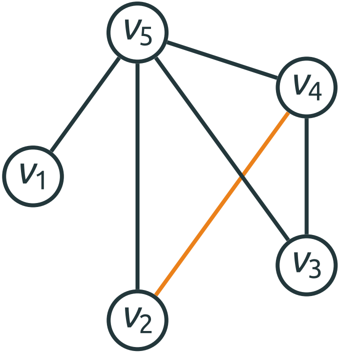

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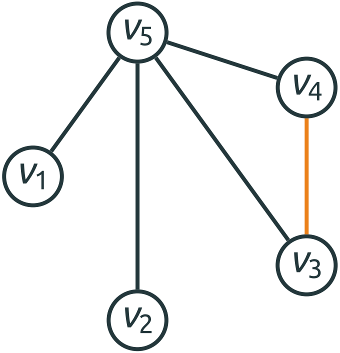

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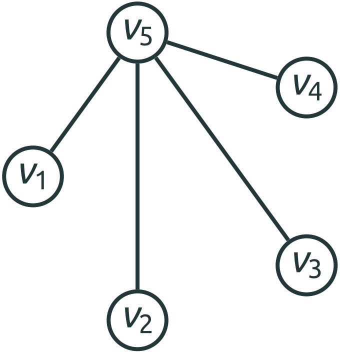

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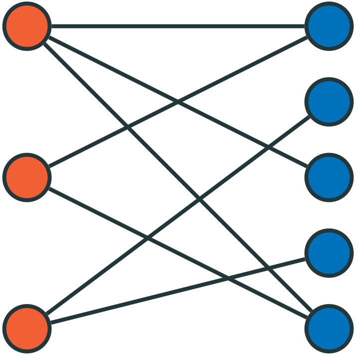
  - I.e., no edge connects two vertices from the same part
- L and R are called the parts of G

Bipartite Graphs: Examples



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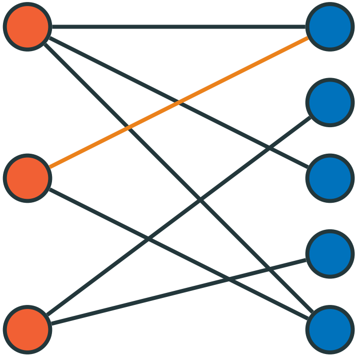

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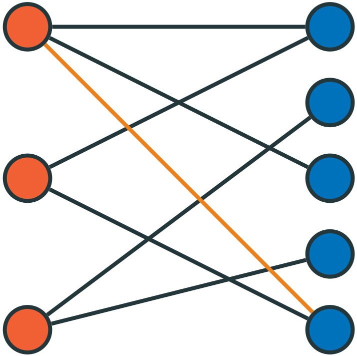

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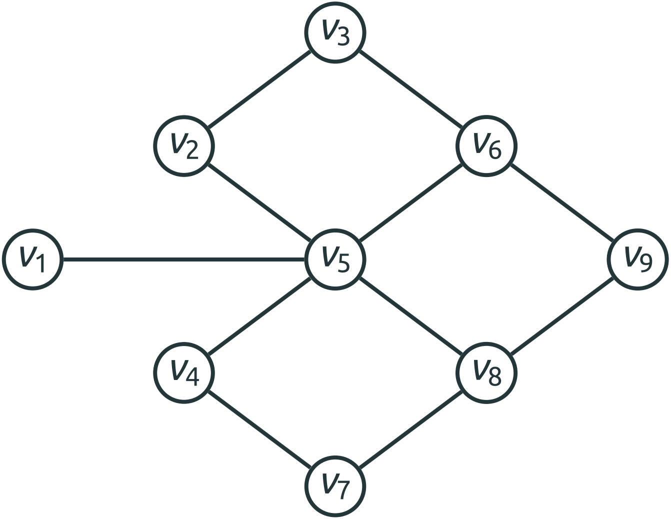

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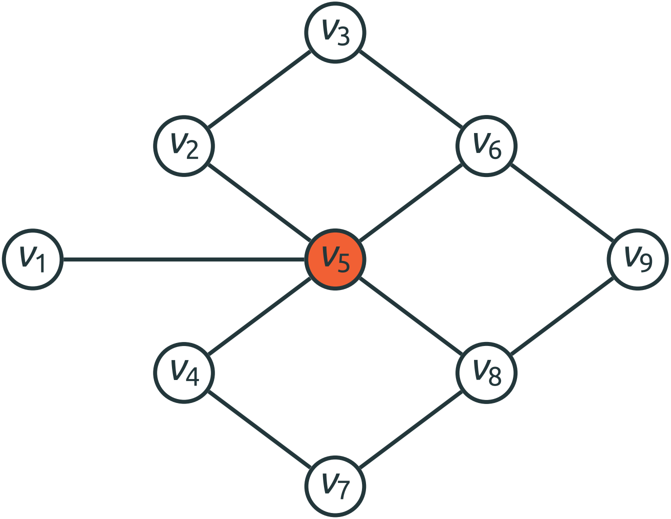

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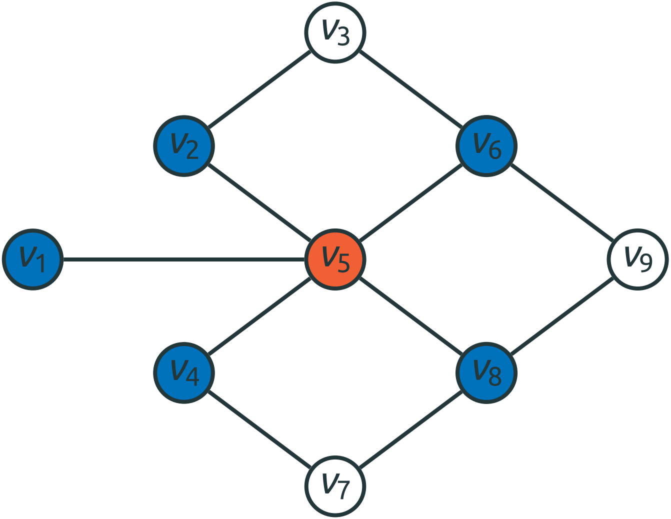

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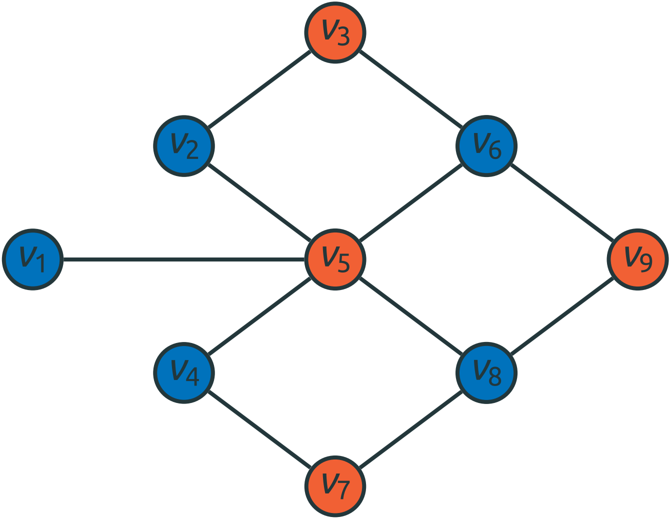
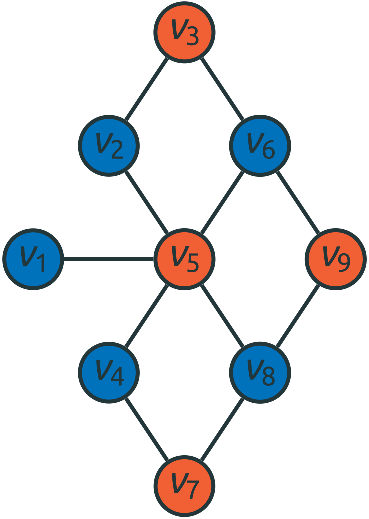
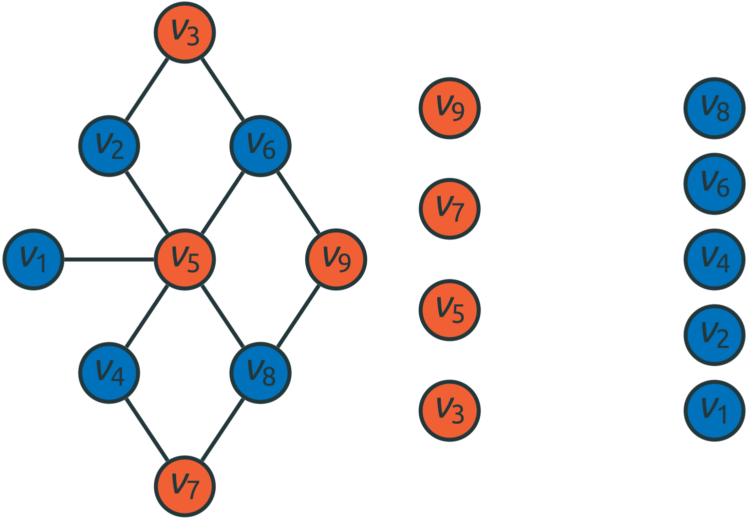


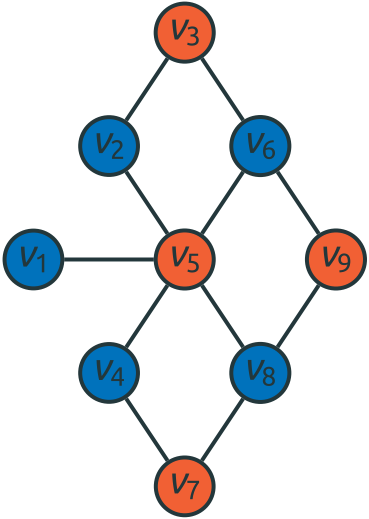
Bipartite Graphs: Examples

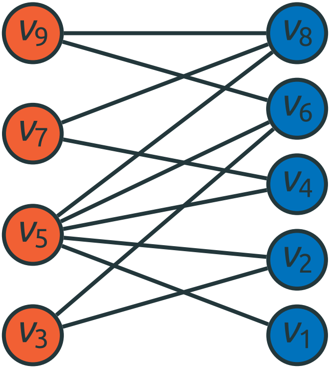


Bipartite Graphs: Examples

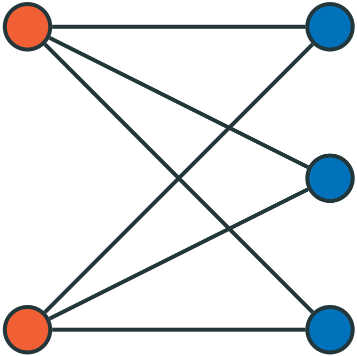


Bipartite Graphs: Examples



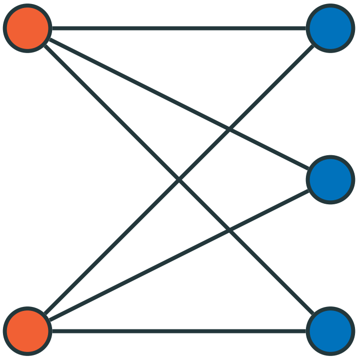


Complete Bipartite Graphs



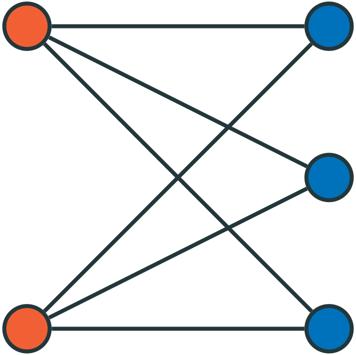
Complete Bipartite Graphs

Complete bipartite graph

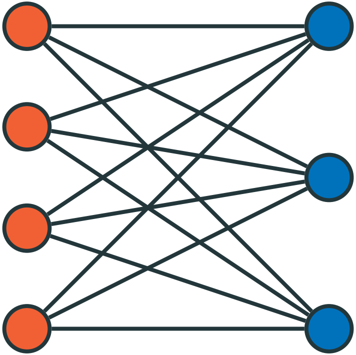


Complete Bipartite Graphs

Complete bipartite graph $K_{2,3}$

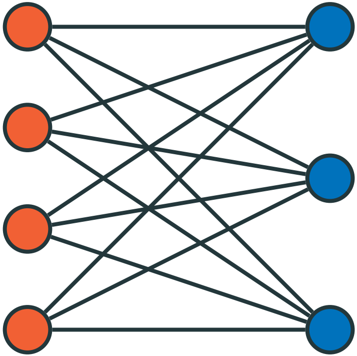


Complete Bipartite Graphs



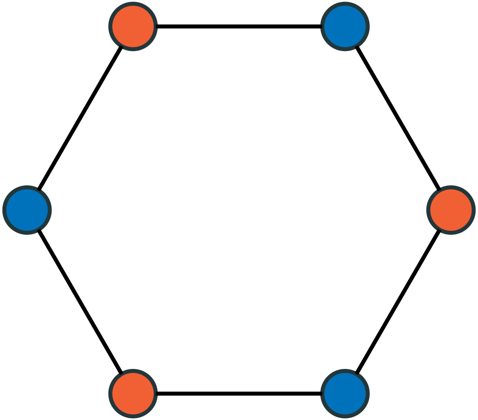
Complete Bipartite Graphs

Complete bipartite graph $K_{4,3}$



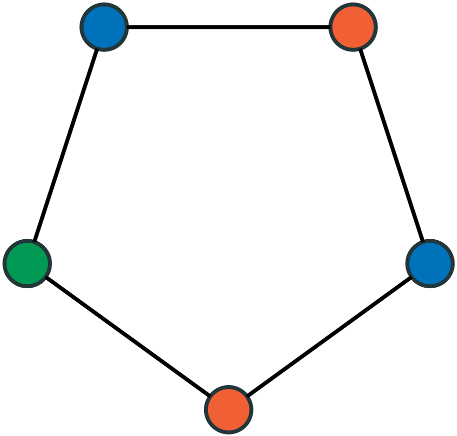
Cycle Graphs

For even n, C_n is bipartite

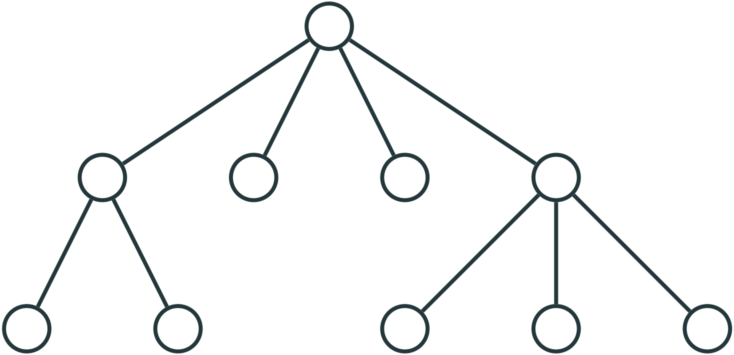


Cycle Graphs

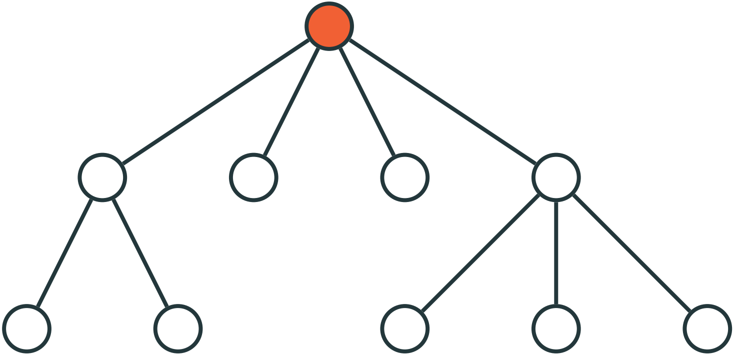
For odd $n > 2$, C_n is not bipartite



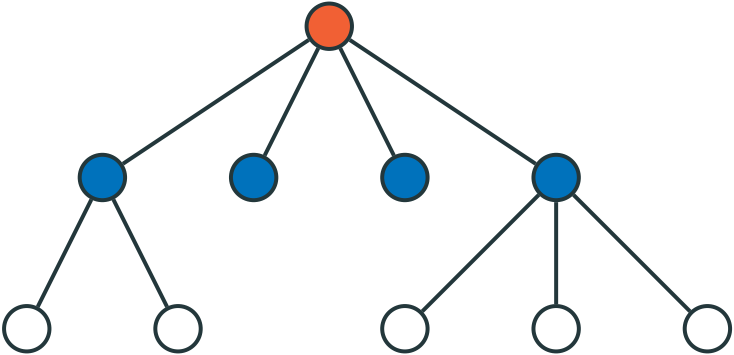
Trees are bipartite



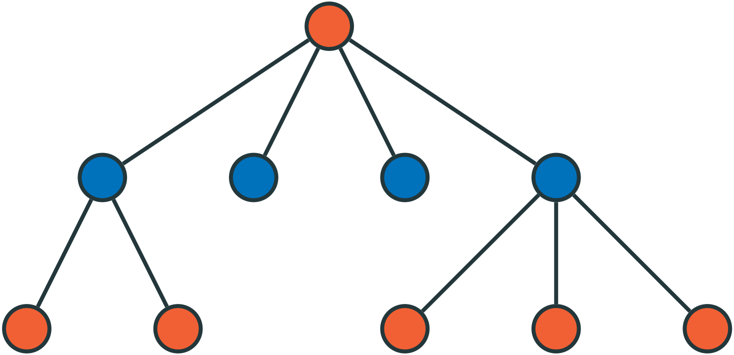
Trees are bipartite



Trees are bipartite



Trees are bipartite



Trees are bipartite