

# Week 1

- A **graph**  $G = (V, E)$  consists of the set of **vertices**  $V$  and the set of edges  $E$ .
- For an edge  $e = \{u, v\}$ , we say:
  - $e$  **connects**  $u$  and  $v$ ;
  - $u$  and  $v$  are **end points** of  $e$ ;
  - $u$  and  $e$  are **incident** ( $v$  and  $e$  are **incident**);
  - $u$  and  $v$  are **adjacent** or **neighbors**.
- The **degree**  $\deg(v)$  of a vertex  $v$  is the number of edges incident to it. A vertex of degree 0 is called **isolated**.
- In a directed graph, the **indegree** (**outdegree**) of a vertex  $v$  is the number of edges ending at  $v$  (leaving  $v$ ).
- The **degree of a graph** is the maximum degree of its vertex. A  $k$ -regular graph is a graph where each vertex has degree  $k$ .
- The **complement** of a graph  $G = (V, E)$  is a graph  $\bar{G} = (V, \bar{E})$  s.t.  $(u, v) \in \bar{E}$  if and only if  $(u, v) \notin E$ .
- A **walk** in a graph is a sequence of edges, where each edge (except for the 1st one) starts with a vertex where the previous edge ended. The **length** of a walk is the number of edges in it.
- A **path** is a walk where all edges are distinct.
- A **simple path** is a walk where all vertices are distinct.
- A **cycle** in a graph is a path whose 1st vertex is the same as the last one.
- A **simple cycle** is a cycle where all vertices except for the 1st one are distinct. (And there 1st vertex is taken twice.)
- A graph is called **connected** if there is a path between every pair of its vertices.
- A **connected component** of a graph  $G$  is a maximal connected subgraph of  $G$ .
- The **path graph**  $P_n$  consists of  $n$  vertices  $v_1, \dots, v_n$  and  $n - 1$  edges  $\{v_1, v_2\}, \dots, \{v_{n-1}, v_n\}$ .
- The **cycle graph**  $C_n$  consists of  $n$  vertices  $v_1, \dots, v_n$  and  $n$  edges  $\{v_1, v_2\}, \dots, \{v_{n-1}, v_n\}, \{v_n, v_1\}$ .
- The **complete graph (clique)**  $K_n$  contains  $n$  vertices  $v_1, \dots, v_n$  and all  $n(n - 1)/2$  edges between them.
- Three equivalent definitions of a **tree**:
  - a connected graph without cycles;
  - a connected graph on  $n$  vertices with  $n - 1$  edges;
  - a graph with a unique simple path between any pair of its vertices.
- A graph  $G$  is **bipartite** if its vertices can be partitioned into two disjoint sets  $L$  and  $R$  s.t. every edge of  $G$  connects a vertex in  $L$  with a vertex in  $R$ .