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#### **Playing Dice Game**

You are given a set of dices and you are going to play a dice game with this set. You have to decide who chooses the dice first, you or your opponent. Then you play against the opponent provided by us. Both of you pick your dices. The throws of the chosen dices are simulated many times. Your goal is to win as many games as possible

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- · So, how should one do that?

The following criteria holds

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We actually have to show two statements here:

- If there is a dice that is better than all others, then the first player wins
- If there is no dice that is better than all others, then the first player does not win

#### Case 1: There is the Best Dice



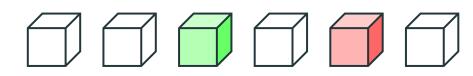
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- Suppose there is the best dice
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- · Whatever the second player picks is worse



Suppose there is no best dice



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- Suppose the first player picks one of the dices
- Than the second player can always pick the one that is not worse!

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- If there is one, you should pick your dice first and you should pick the dice that achieves the highest possible probability against all others
- If there is no such dice, you should find for each dice the one that is the best against it
- In this case you pick your dice second and you choose the dice that is the best against your opponents pick