

Warm-up

Alexander Golovnev

Outline

Airlines Graph

Knight Transposition

Seven Bridges of Königsberg

Airlines Graph

Consider a small country with five cities: A, B, C, D, E .

There are six flights:

$A - B, A - C, A - E,$
 $B - D, C - D, C - E.$

Is there a direct flight from A to D ?

With one stop?

With two stops?

Airlines Graph

Consider a small country with five cities: A, B, C, D, E .

There are six flights:

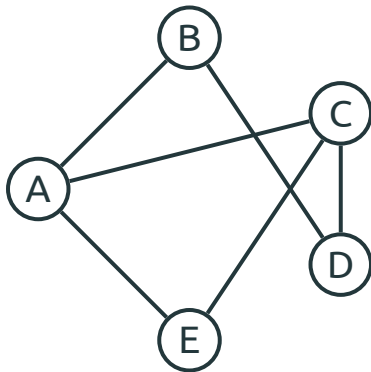
$A - B, A - C, A - E,$

$B - D, C - D, C - E.$

Is there a direct flight from A to D ?

With one stop?

With two stops?



Airlines Graph

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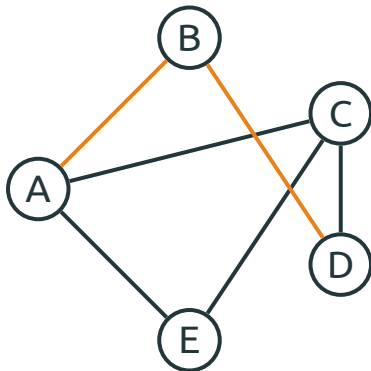
$A - B, A - C, A - E,$

$B - D, C - D, C - E.$

Is there a direct flight from A to D ?

With one stop?

With two stops?



Airlines Graph

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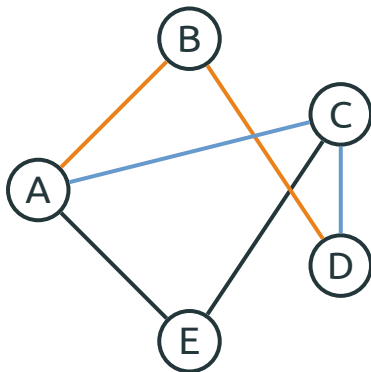
$A - B, A - C, A - E,$

$B - D, C - D, C - E.$

Is there a direct flight from A to D ?

With one stop?

With two stops?



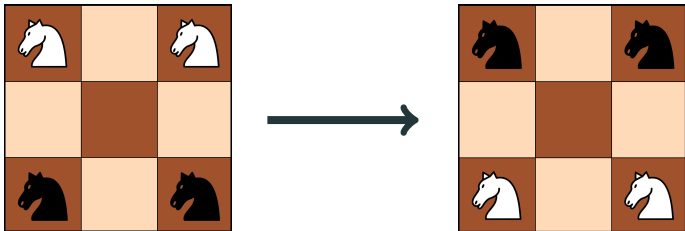
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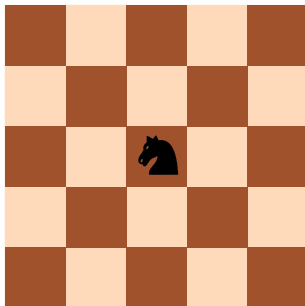
Guarini's Puzzle



Exchange the places of the white knights and the black knights.

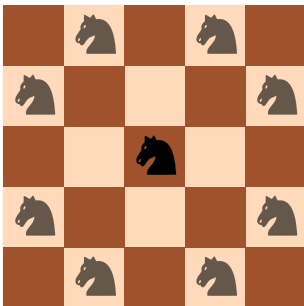
Chess Knight

A chess knight can move in an **L** shape in any direction

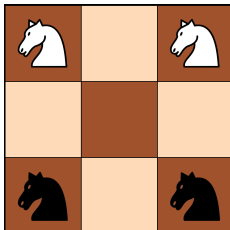


Chess Knight

A chess knight can move in an **L** shape in any direction



Guarini's Puzzle. Solution



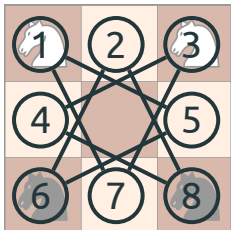
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Guarini's Puzzle. Solution



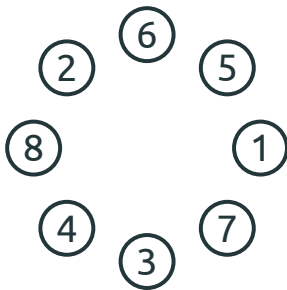
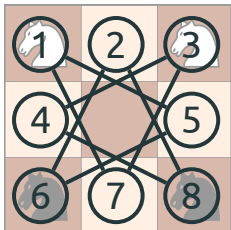
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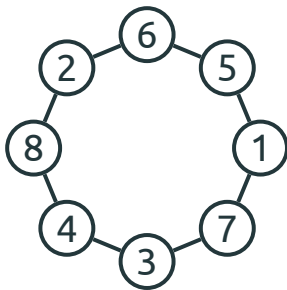
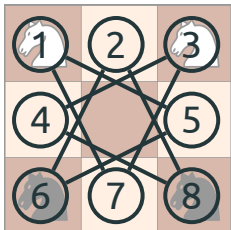
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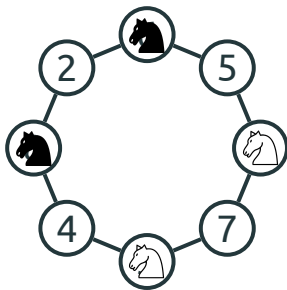
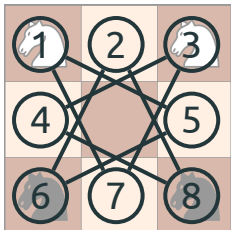
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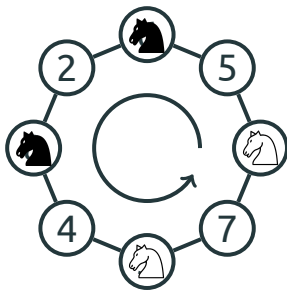
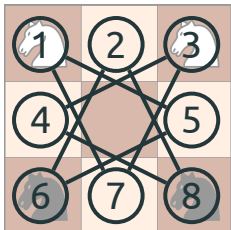
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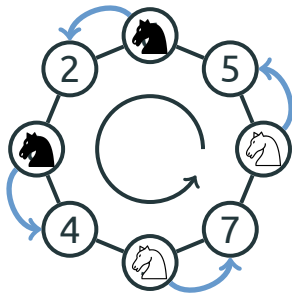
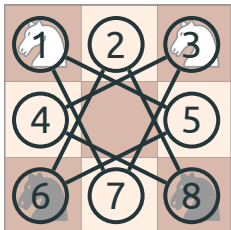
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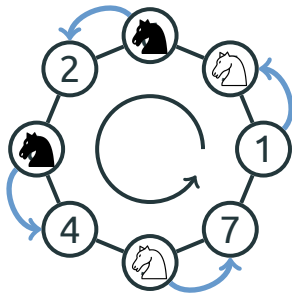
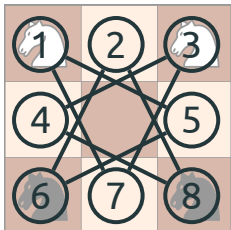
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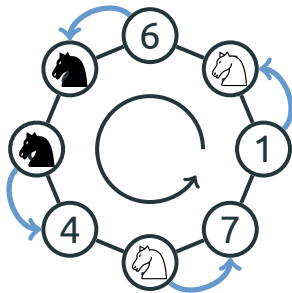
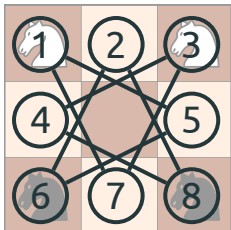
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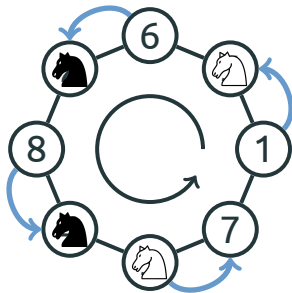
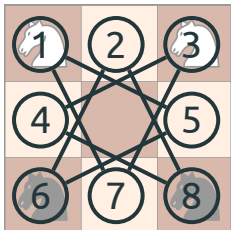
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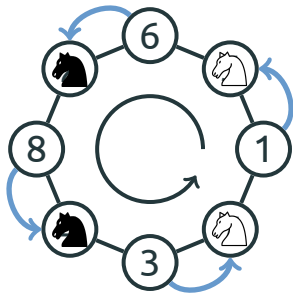
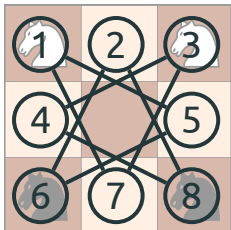
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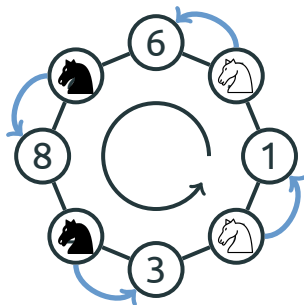
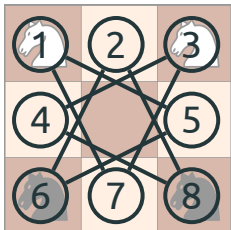
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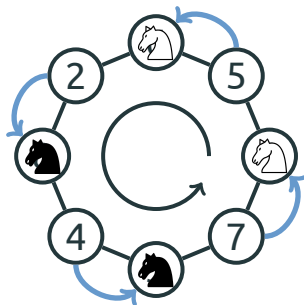
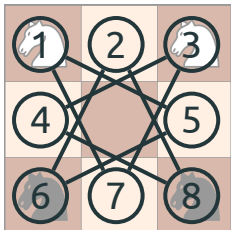
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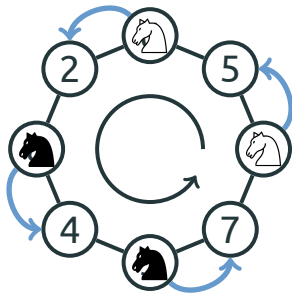
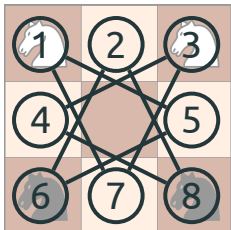
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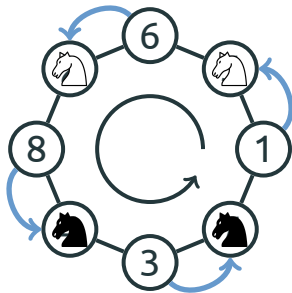
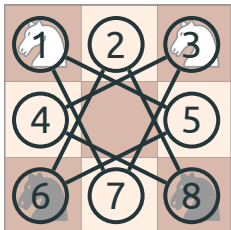
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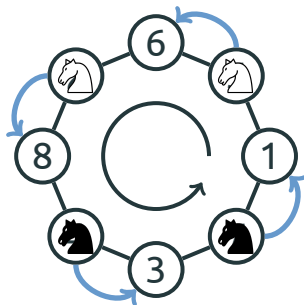
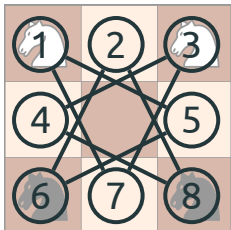
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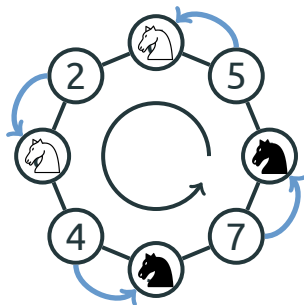
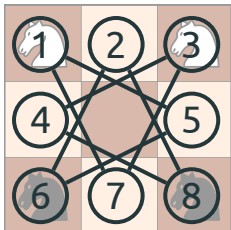
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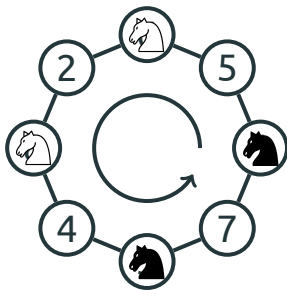
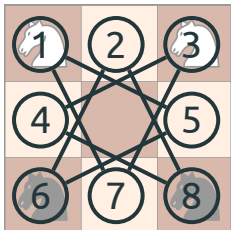
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Guarini's Puzzle. Solution



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Guarini's Puzzle. Solution



Exchange the places of the white knights and the black knights.

Outline

Airlines Graph

Knight Transposition

Seven Bridges of Königsberg

Seven Bridges of Königsberg

Königsberg, Prussia, 1735



Seven Bridges of Königsberg

Königsberg, Prussia, 1735

Walk through Königsberg

Cross each bridge
exactly once



Seven Bridges of Königsberg

Königsberg, Prussia, 1735

Walk through Königsberg

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Leonhard Euler



Seven Bridges of Königsberg

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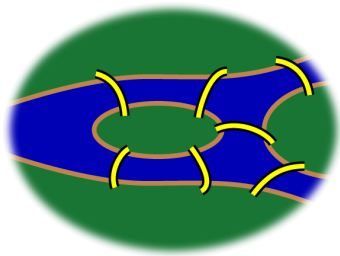
Impossible!



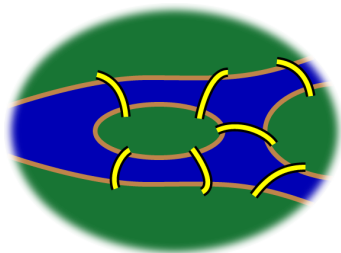
Bridges of Königsberg. Graph



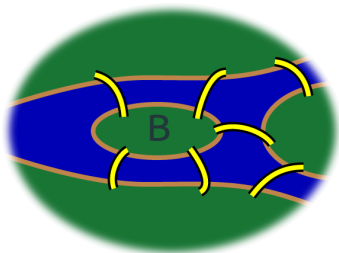
Bridges of Königsberg. Graph



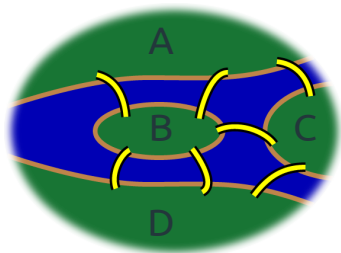
Bridges of Königsberg. Graph



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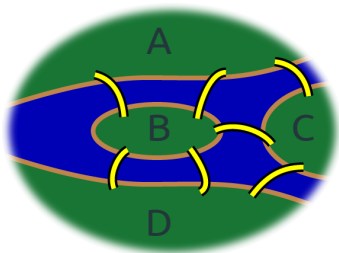
Bridges of Königsberg. Graph

A

B

D

C

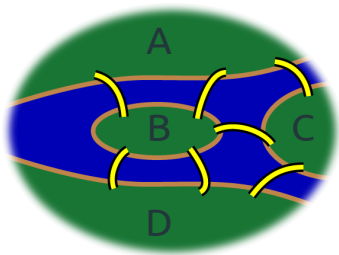


Bridges of Königsberg. Graph

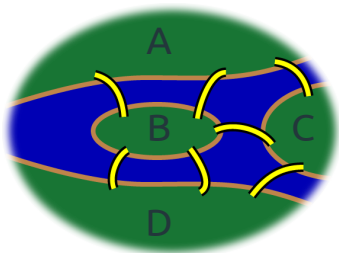
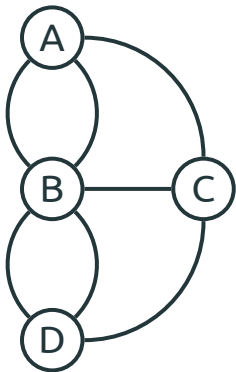
A

B — C

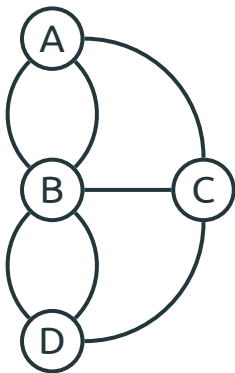
D



Bridges of Königsberg. Graph



Bridges of Königsberg. Graph



Is there a path which visits every edge exactly once?

Eulerian Path

Eulerian Path

START

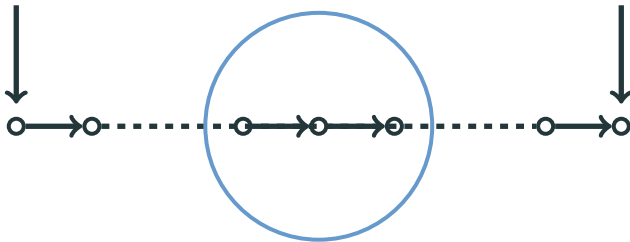
FINISH



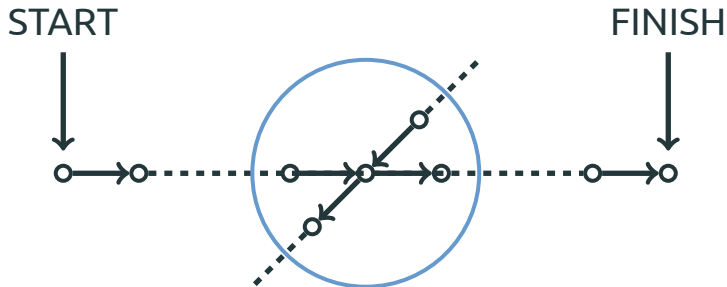
Eulerian Path

START

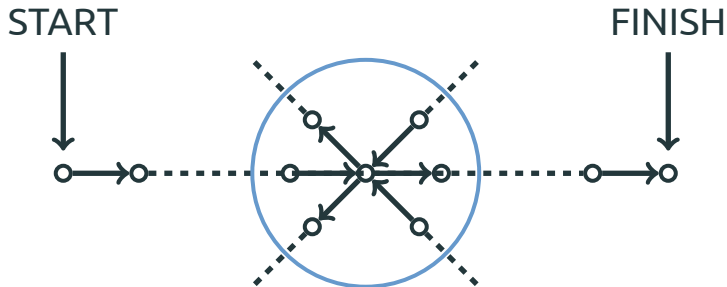
FINISH



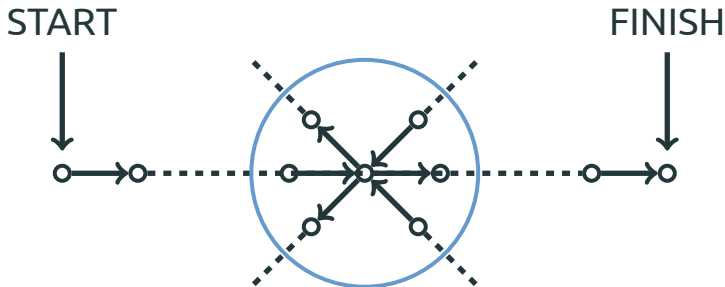
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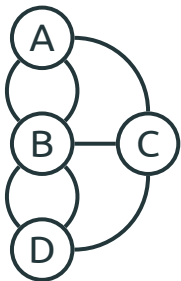


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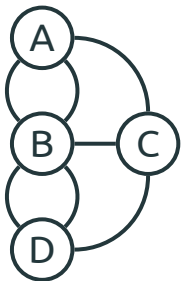
All but START and FINISH vertices
have **even** number of neighbors

Eulerian Path



All but START and FINISH vertices
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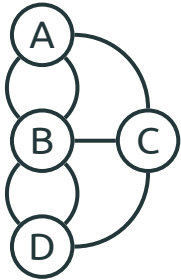
Eulerian Path



All four vertices have
odd number of neighbors

All but START and FINISH vertices
have **even** number of neighbors

Eulerian Path

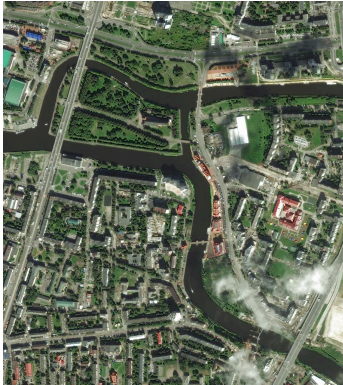


All four vertices have
odd number of neighbors
Impossible!

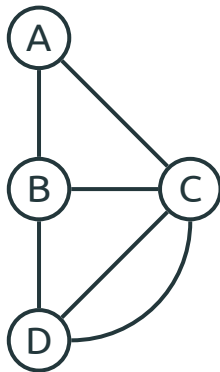
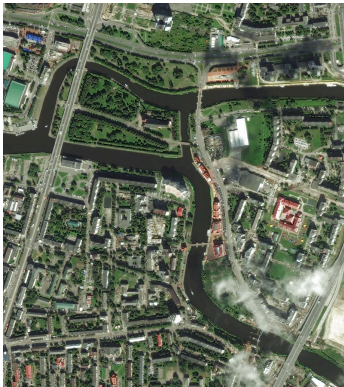


All but START and FINISH vertices
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Six Bridges of Kaliningrad



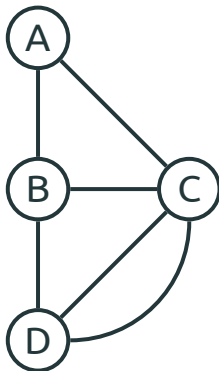
Six Bridges of Kaliningrad



Six Bridges of Kaliningrad

B and D have **odd**
number of neighbors

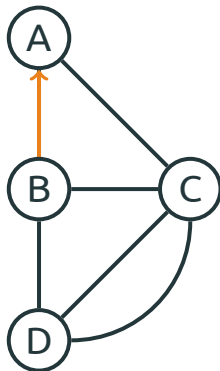
If there exists an Eulerian path, B and D must be START and FINISH



Six Bridges of Kaliningrad

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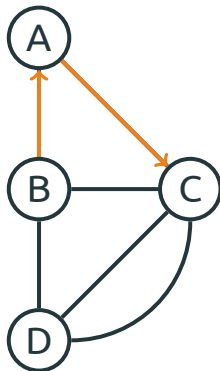
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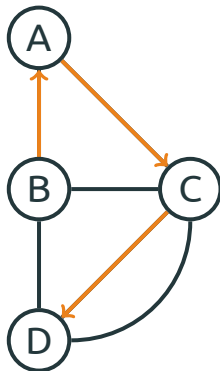
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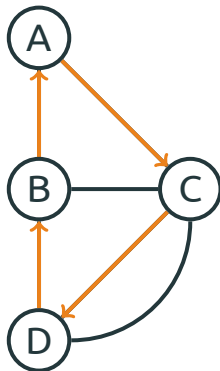
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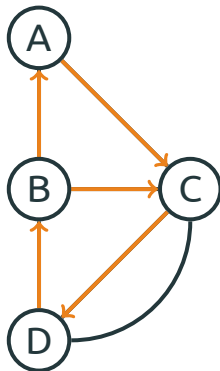
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