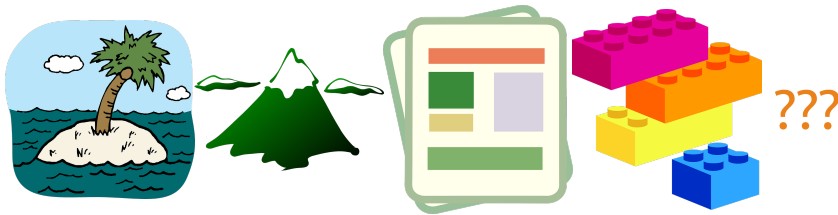# Basic Definitions

Alexander Golovnev

Outline

Definitions
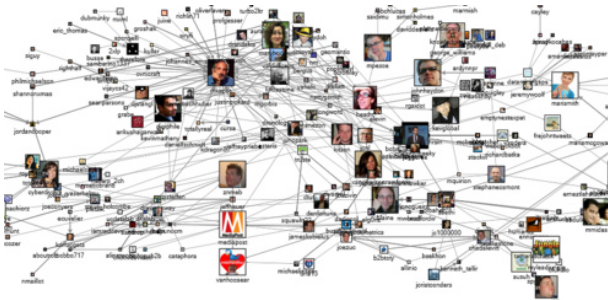
An isolated vertex forms a component

Definitions

An isolated vertex forms a component

The Degree of a Vertex

The number of friends

The Degree of a Vertex

- The Degree of a vertex is the number of its incident edges

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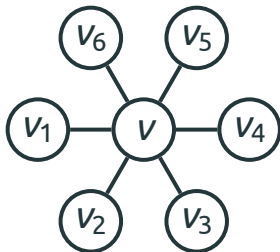
- The degree of a vertex v is denoted by $\deg(v)$

The Degree of a Vertex

- The Degree of a vertex is the number of its incident edges

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- The degree of a vertex v is denoted by $\deg(v)$

- The degree of a graph is the maximum degree of its vertices
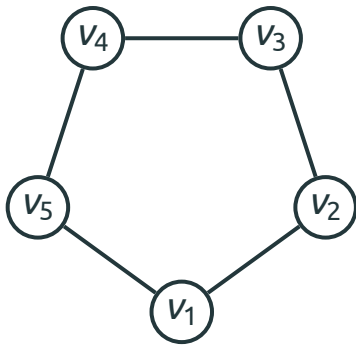
The Degree of a Vertex: Examples

The degree of v is 6: $\deg(v) = 6$

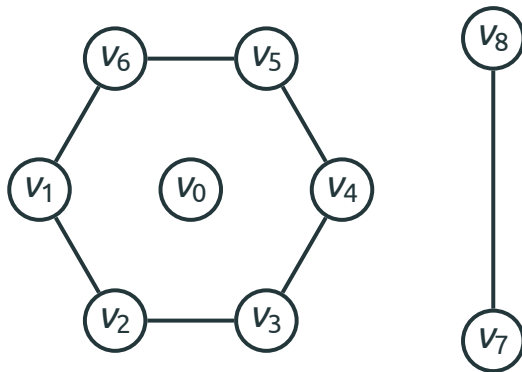The degree of v_6 is 1: $\deg(v_6) = 1$

The Degree of a Vertex: Examples

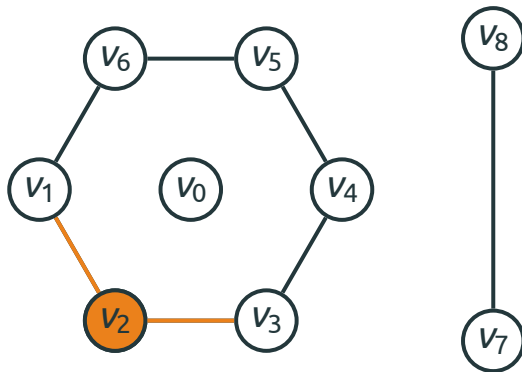The degree of every vertex is 2: $\forall i, \deg(v_i) = 2$
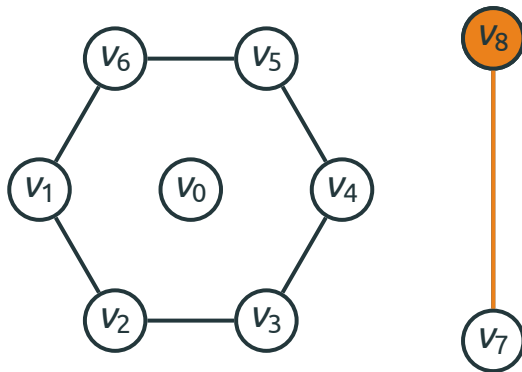
Isolated Vertices

Isolated Vertices

$$\deg(v_2) = 2$$

Isolated Vertices
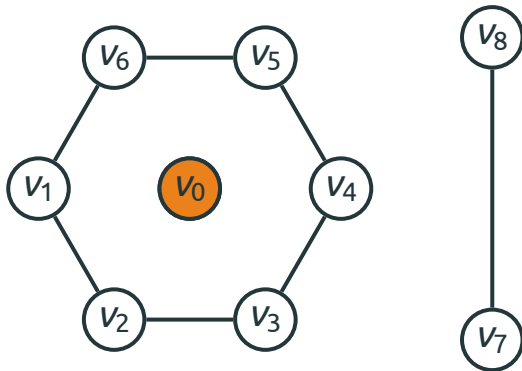
$\deg(v_2) = 2$
$\deg(v_8) = 1$
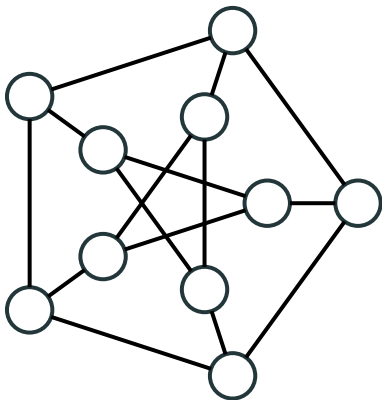
Isolated Vertices

$\deg(v_2) = 2$

$\deg(v_8) = 1$

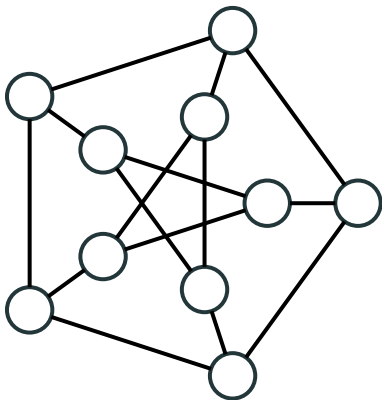$\deg(v_0) = 0$. v_0 is an Isolated Vertex

Regular Graphs

A Regular graph is a graph where each vertex has the same degree

Regular Graphs

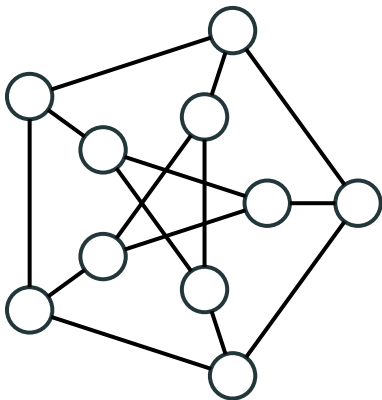A **Regular** graph is a graph where each vertex has the same degree
A regular graph of degree *k* is also called *k*-Regular

Regular Graphs

A Regular graph is a graph where each vertex has the same degree

A regular graph of degree *k* is also called *k*-Regular

E.g., this graph is 3-Regular

Complement Graph

- The Complement of a graph $G = (V, E)$ is a graph $\bar{G} = (V, \bar{E})$ on the same set of vertices V and the following set of edges:
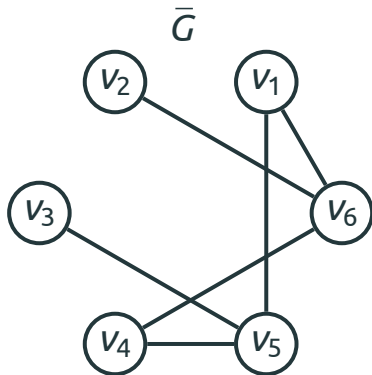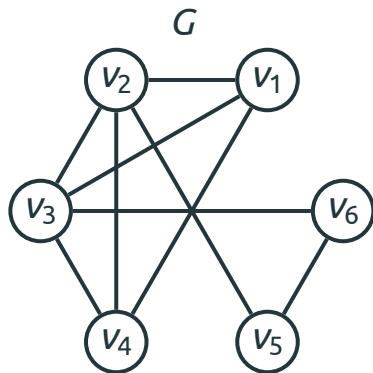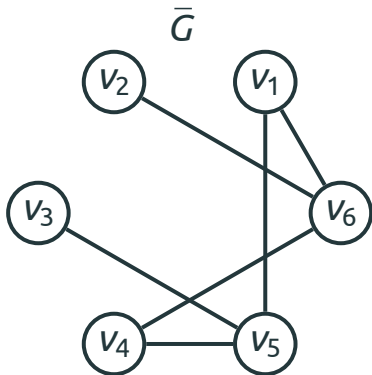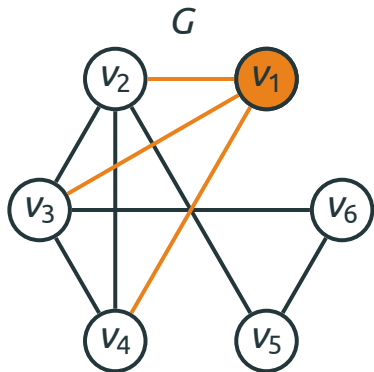
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- Two vertices are connected in $\bar{G}$ if and only if they are not connected in G

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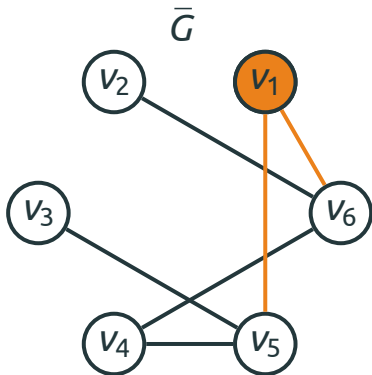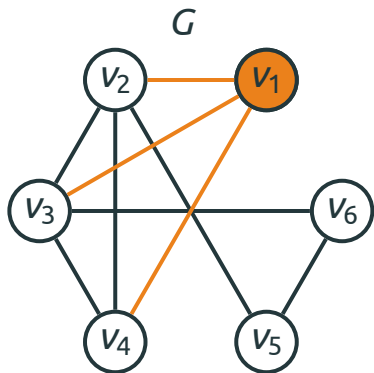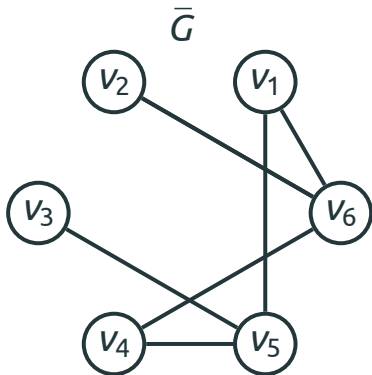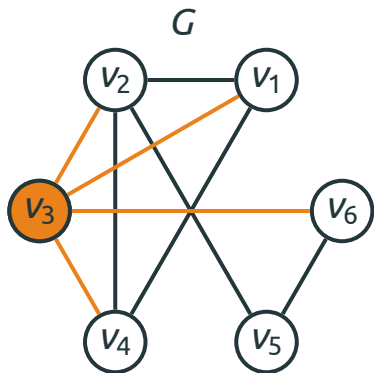- I.e., $(u, v) \in \bar{E}$ if and only if $(u, v) \notin E$

Complement Graph

Complement Graph

Complement Graph

Complement Graph

Complement Graph

Outline

Paths

Is there a path from one point to another?

Walks

- A Walk in a graph is a sequence of edges, such that each edge (except for the first one) starts with a vertex where the previous edge ended

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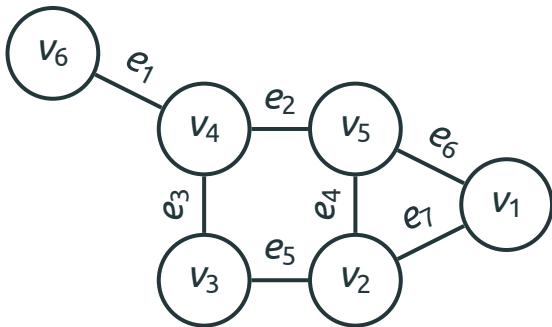- The Length of a walk is the number of edges in it

Walks

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Walks

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- The Length of a walk is the number of edges in it

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- A Simple Path is a walk where all vertices are distinct

Walks: Examples
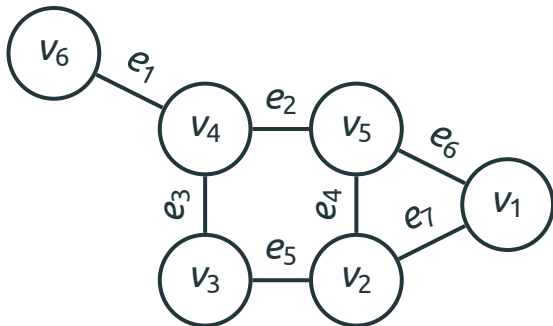
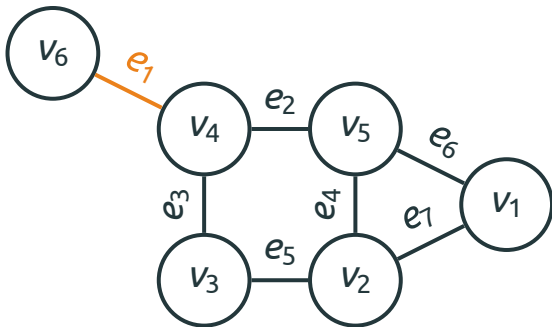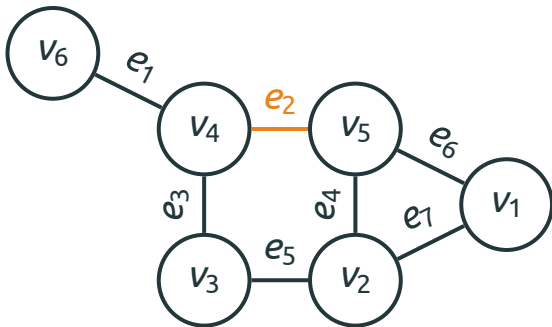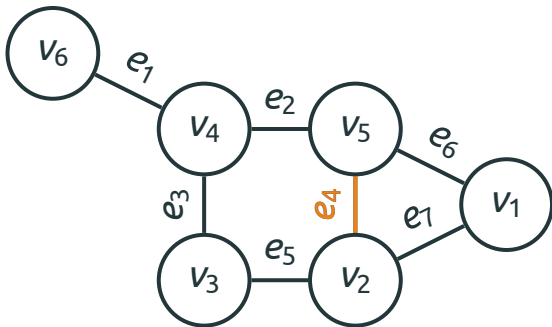# Walks: Examples

A walk of length 6: $(e_1, e_2, e_4, e_5, e_3, e_1)$

Walks: Examples
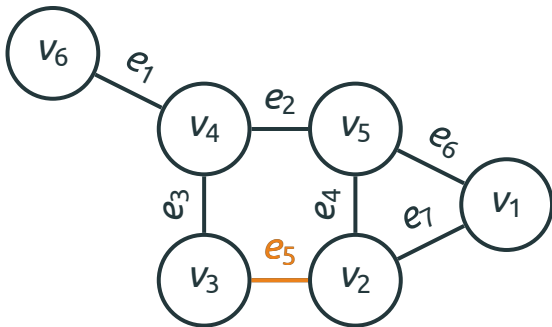
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Walks: Examples

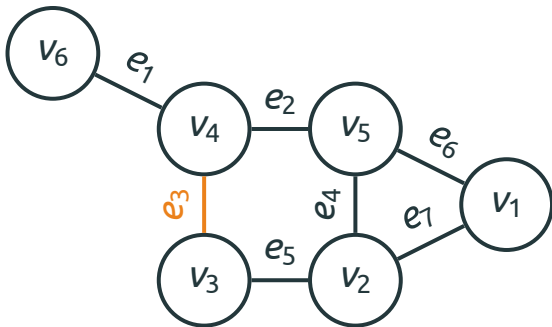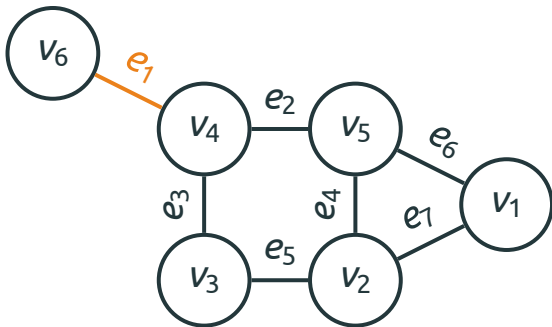A walk of length 6: $(e_1, e_2, e_4, e_5, e_3, e_1)$

Walks: Examples

A **walk** of length 6: $(e_1, e_2, e_4, e_5, e_3, e_1)$

Walks: Examples

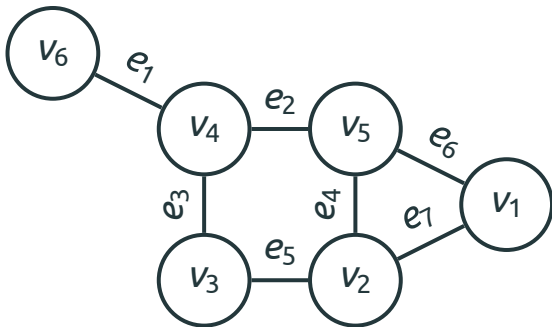A walk of length 6: $(e_1, e_2, e_4, e_5, e_3, e_1)$

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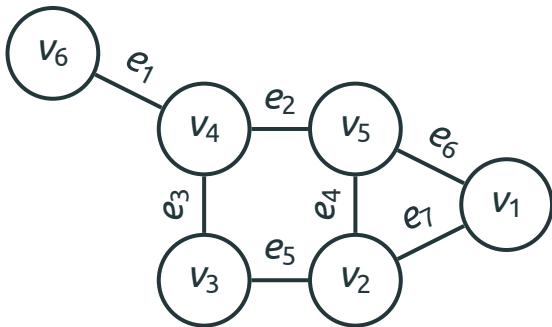A walk of length 6: $(e_1, e_2, e_4, e_5, e_3, e_1)$

Walks: Examples

A walk of length 6: (e_1, e_2, e_4, e_5, e_3, e_1)
Not a path: uses e_1 twice

Walks: Examples

A path of length 4: (e_7, e_6, e_4, e_5)

Walks: Examples
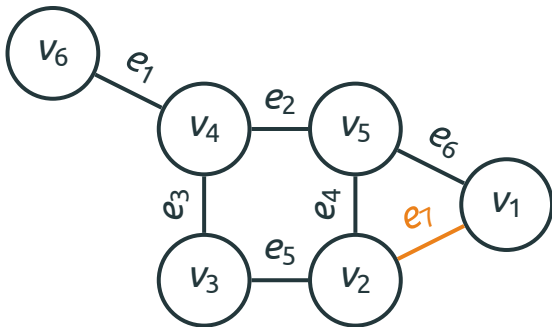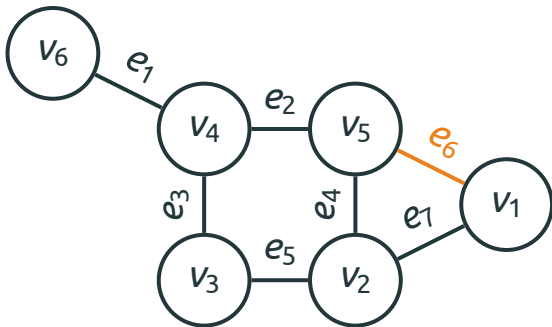
A path of length 4: (e_7, e_6, e_4, e_5)

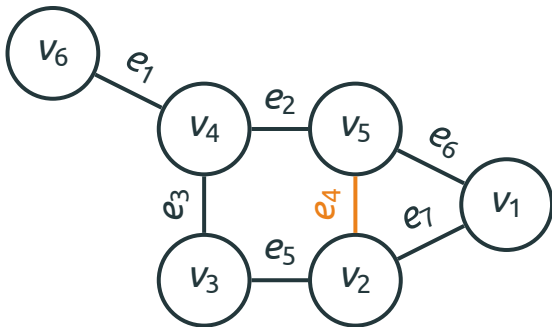# Walks: Examples

A path of length 4: (e_7, e_6, e_4, e_5)

Walks: Examples

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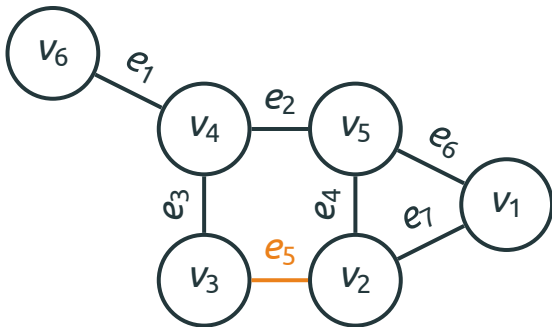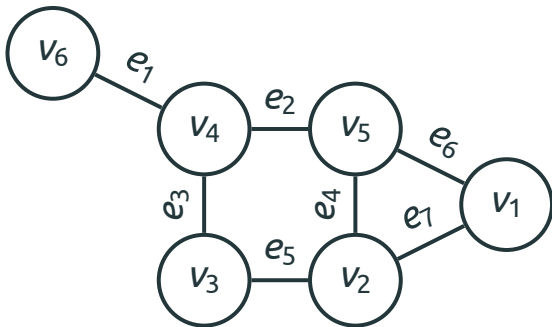
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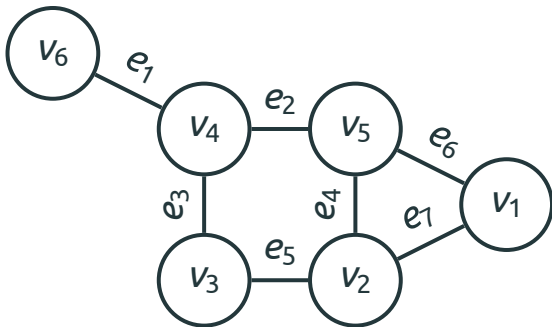
A path of length 4: (e_7, e_6, e_4, e_5)
Not a simple path: visits v_2 twice

Walks: Examples

A simple path of length 4: (e_7, e_6, e_2, e_3)

Walks: Examples

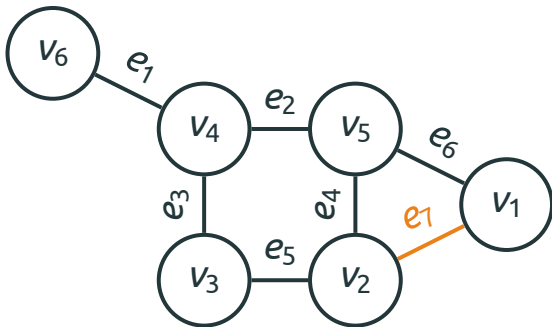A simple path of length 4: (e_7, e_6, e_2, e_3)
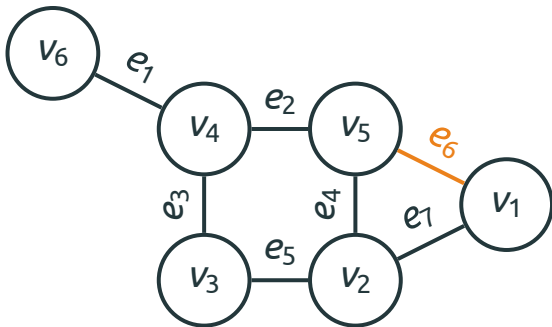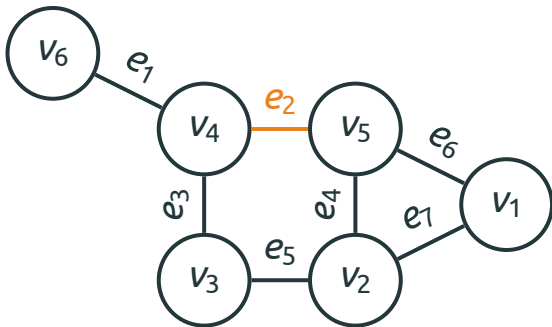
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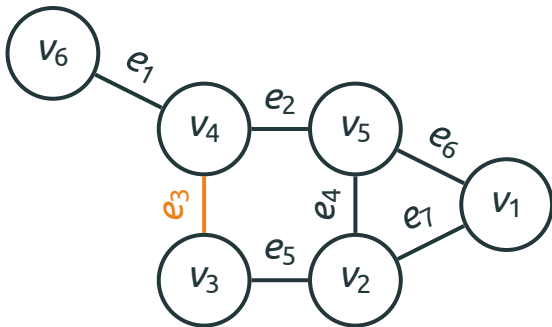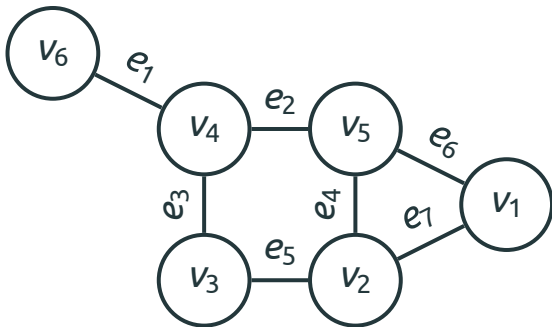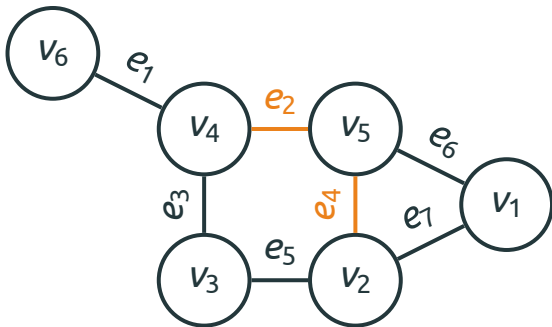A simple path of length 4: (e_7, e_6, e_2, e_3)

Walks: Examples

It is sometimes convenient to specify a path (walk) by a list of its vertices

Walks: Examples

(v_4, v_5, v_2) is a path of length 2

Cycles

- A Cycle in a graph is a path whose first vertex is the same as the last one

Cycles

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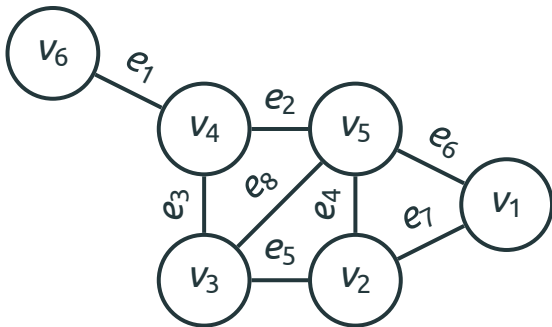
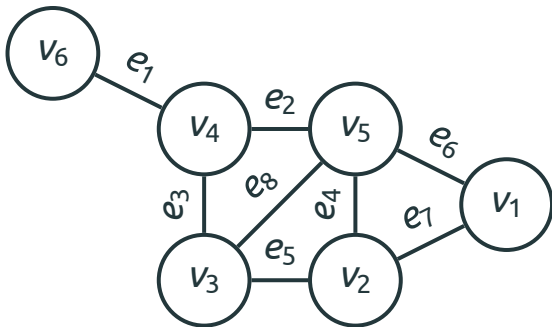- In particular, all the edges in a Cycle are distinct

Cycles

- A Cycle in a graph is a path whose first vertex is the same as the last one

- In particular, all the edges in a Cycle are distinct

- A Simple Cycle is a cycle where all vertices except for the first one are distinct. (And there first vertex is taken twice)
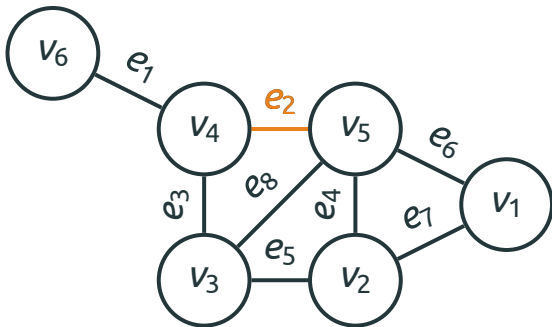
Cycles: Examples

Cycles: Examples

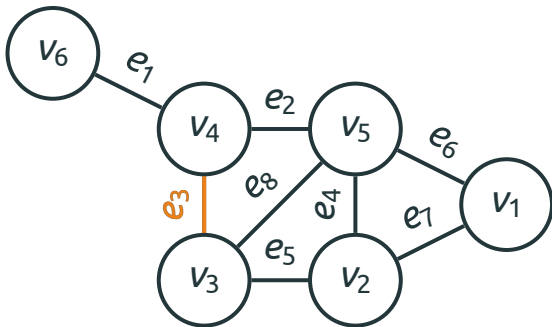A cycle of length 6: $(e_2, e_3, e_8, e_4, e_7, e_6)$

Cycles: Examples

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Cycles: Examples

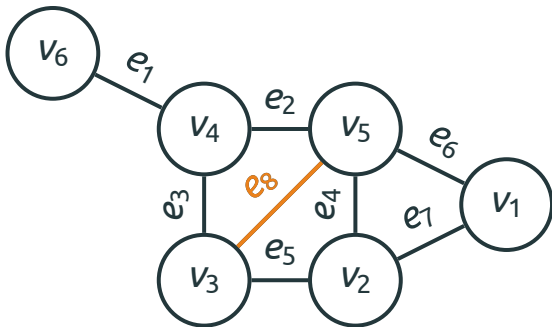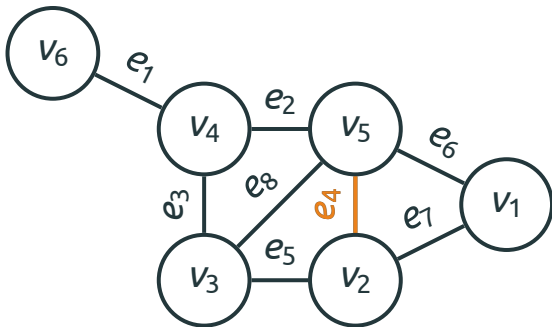A cycle of length 6: (e_2, e_3, e_8, e_4, e_7, e_6)

Cycles: Examples

A cycle of length 6: $(e_2, e_3, e_8, e_4, e_7, e_6)$

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A cycle of length 6: $(e_2, e_3, e_8, e_4, e_7, e_6)$

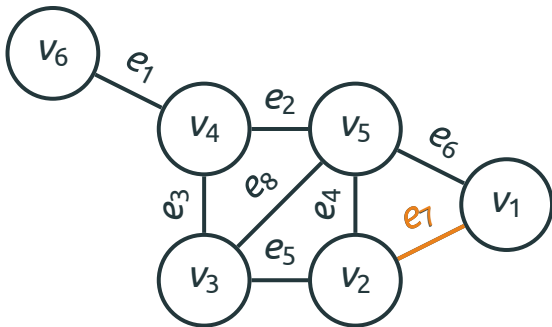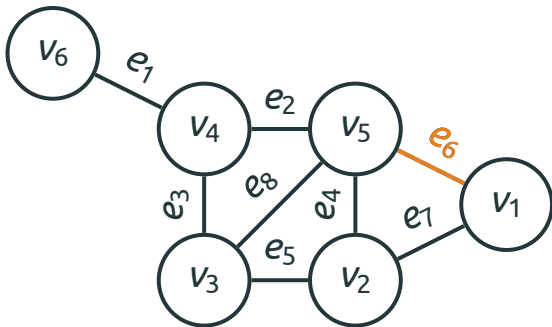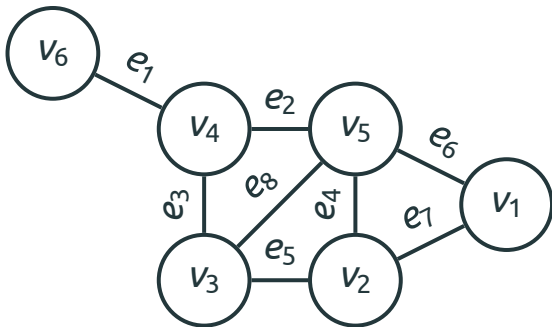# Cycles: Examples

A cycle of length 6: $(e_2, e_3, e_8, e_4, e_7, e_6)$

Cycles: Examples

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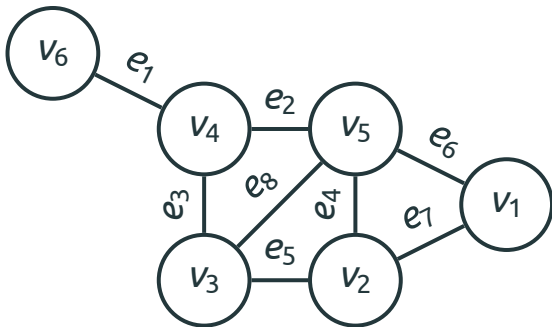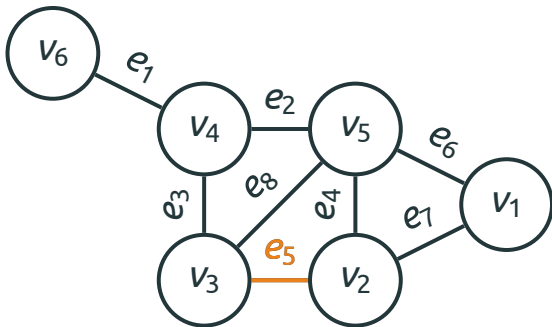Not a simple cycle: visits v_5 three times

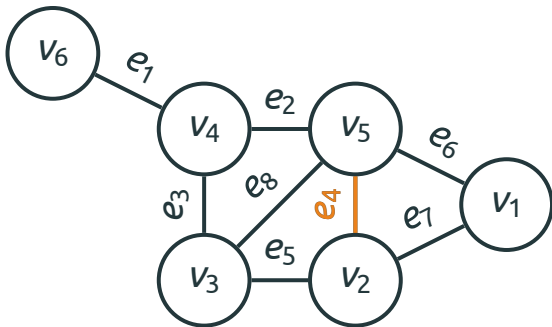# Cycles: Examples

A simple cycle of length 4: (e_5, e_4, e_2, e_3)

Cycles: Examples

A simple cycle of length 4: (e_5, e_4, e_2, e_3)

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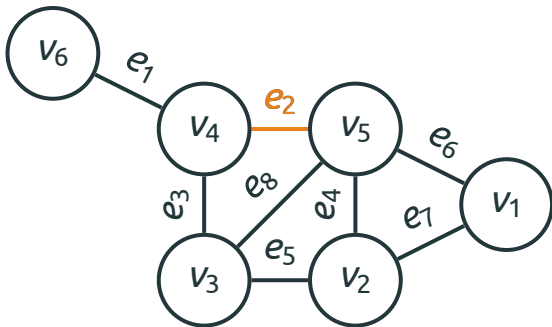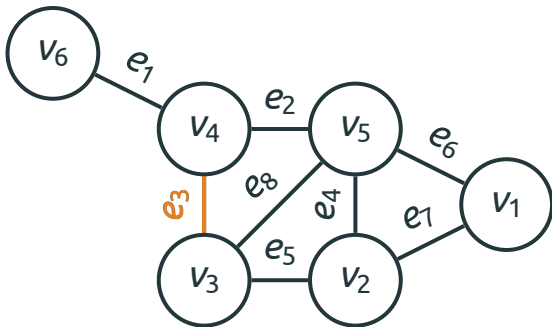# Cycles: Examples

A simple cycle of length 4: (e_5, e_4, e_2, e_3)

Cycles: Examples

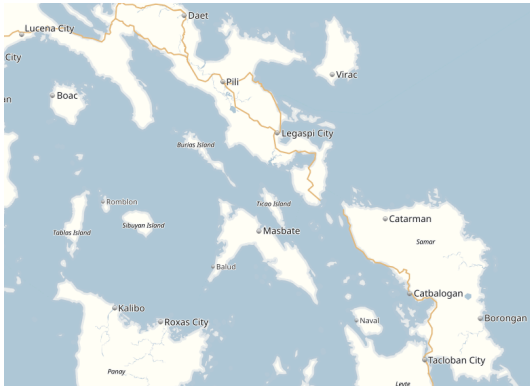A simple cycle of length 4: (e_5, e_4, e_2, e_3)

Outline

Connected Components

The number of islands

Connectivity

- A graph is called Connected if there is a path between every pair of its vertices

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- A Connected Component of a graph G is a maximal connected subgraph of G

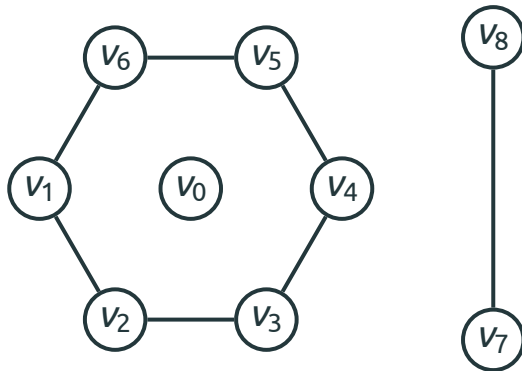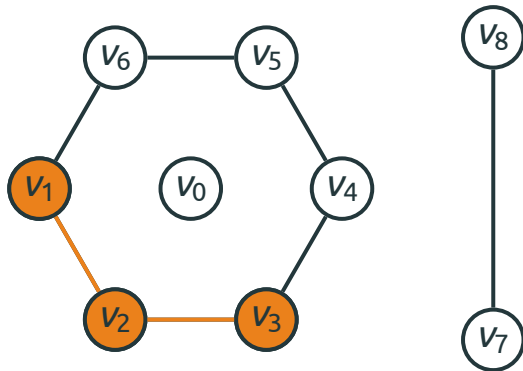# Connectivity

- A graph is called Connected if there is a path between every pair of its vertices

- A Connected Component of a graph G is a maximal connected subgraph of G

- I.e., a connected subgraph of G which is not contained in a larger connected subgraph of G

Connected Components: Examples

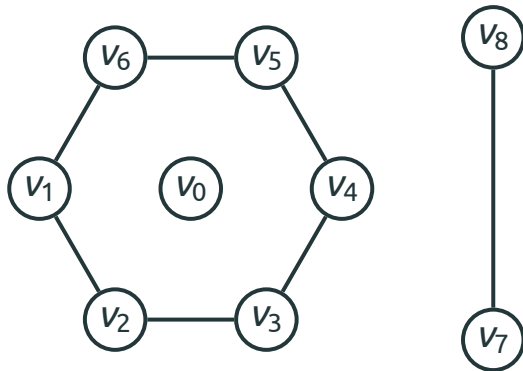# Connected Components: Examples

v_1, v_2, v_3 form a connected subgraph

Connected Components: Examples

v_1, v_2, v_3 form a connected subgraph
But not a connected component
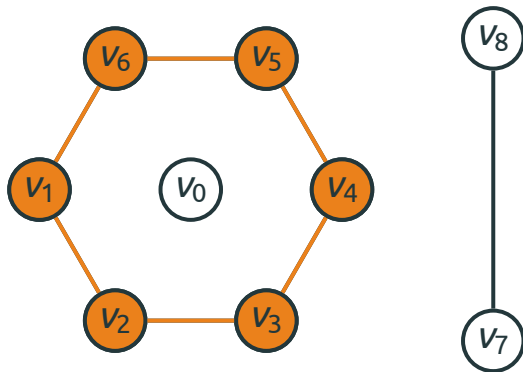
Connected Components: Examples

v_1, v_2, v_3 form a connected subgraph
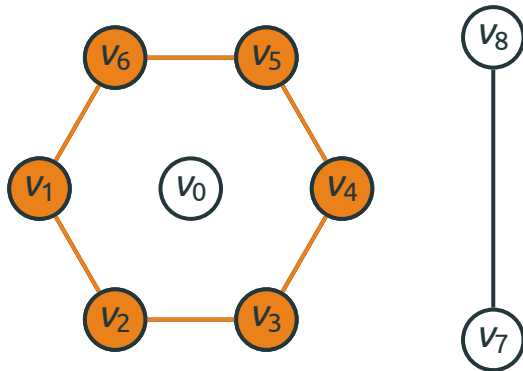But not a connected component

Connected Components: Examples

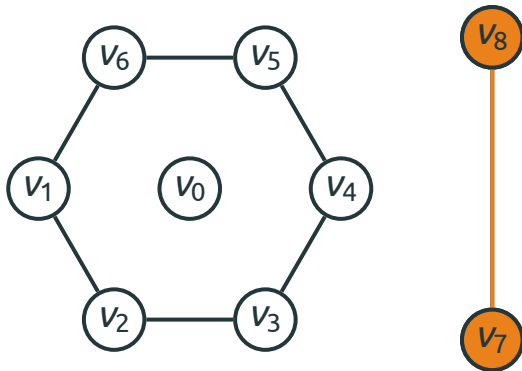v_1, v_2, v_3 form a connected subgraph
But not a connected component
$v_1, v_2, v_3, v_4, v_5, v_6$ form a Connected Component

Connected Components: Examples

v_7, v_8 form a Connected Component

v_1, v_2, v_3, v_4, v_5, v_6 form a Connected Component
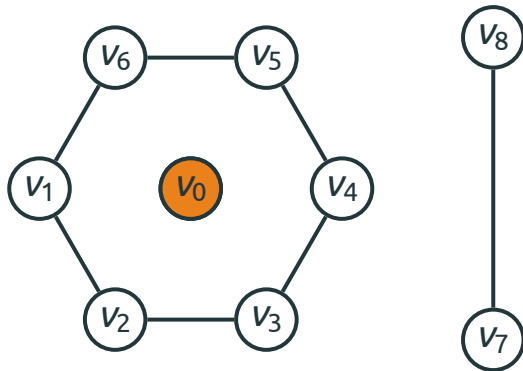
Connected Components: Examples

v_0 forms a Connected Component
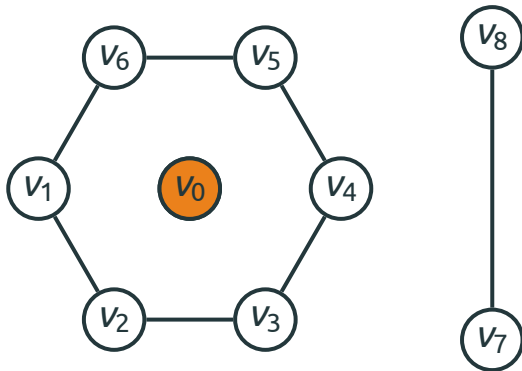
v_7, v_8 form a Connected Component

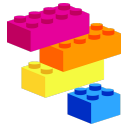v_1, v_2, v_3, v_4, v_5, v_6 form a Connected Component
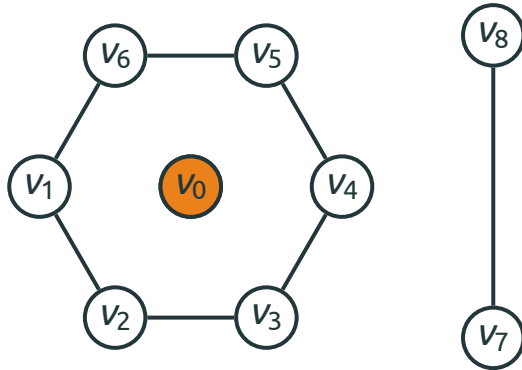
Connected Components: Examples

Each isolated vertex forms a Connected Component

Connected Components: Examples

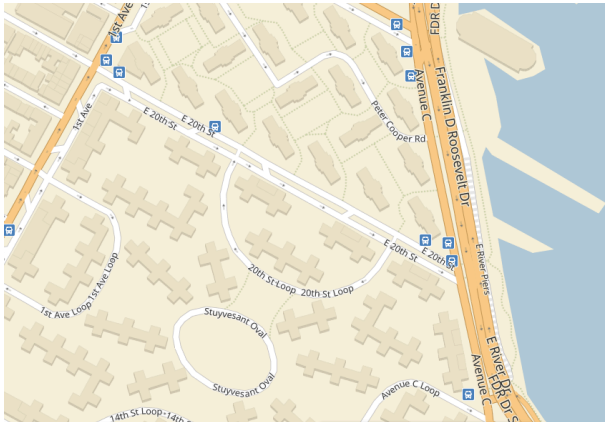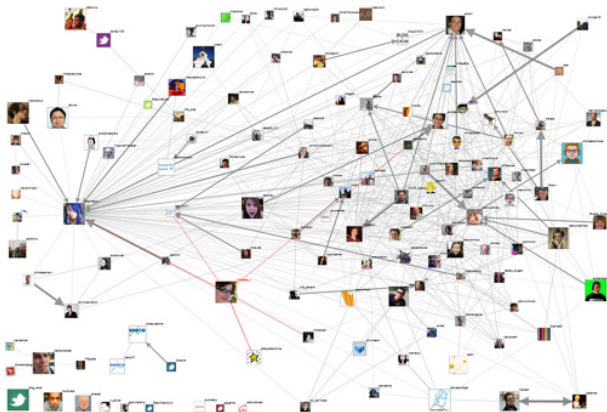Each isolated vertex forms a Connected Component

Outline

Directed Graphs

One-way Streets

Directed Graphs

Followers

Undirected Edge (Edge)

Edge $\{u, v\}$

Directed Edge (Arc)

Arc (u, v)

Directed Edge (Arc)

Arc (u, v)

tail   head

Directed Edge (Arc)

Arc (u, v)
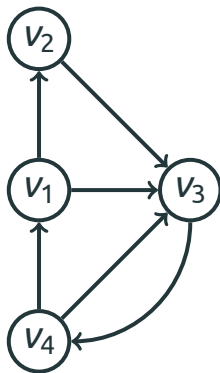
Directed Edge (Arc)

Arc (u, v)

The Degree of a Vertex

- The Indegree of a vertex *v* is the number of edges ending at *v*
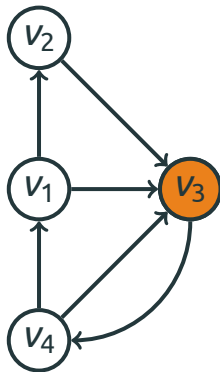
The Degree of a Vertex

- The Indegree of a vertex v is the number of edges ending at v

- The Outdegree of a vertex v is the number of edges leaving v
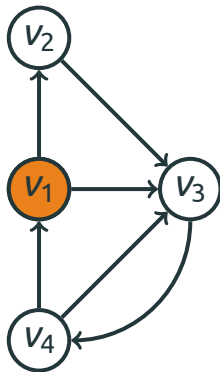
The Degree of a Vertex: Examples

The Degree of a Vertex: Examples

The Indegree of v_3 is 3,
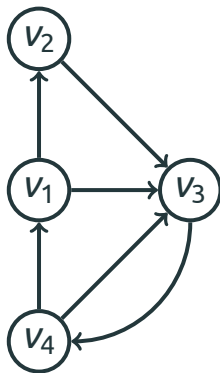the Outdegree of v_3 is 1

The Degree of a Vertex: Examples
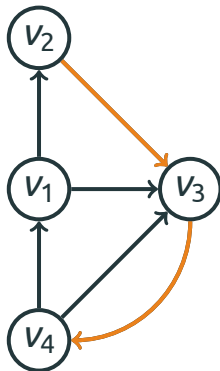
The Indegree of v_1 is 1,
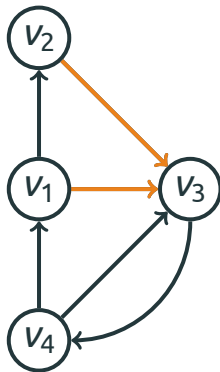the Outdegree of v_1 is 2

Directed Paths

Directed Paths

(v_2, v_3, v_4) is a
Path of length 2

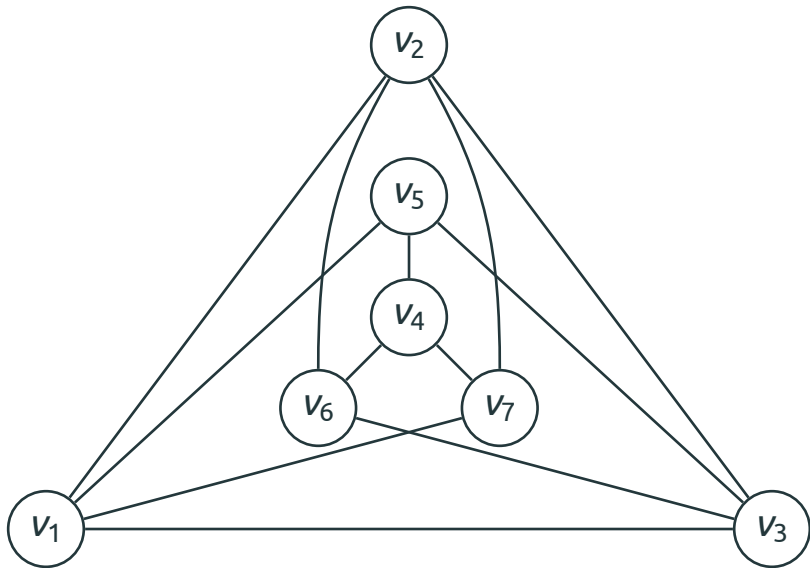# Directed Paths

(v_1, v_3, v_2) is not a Path

Outline
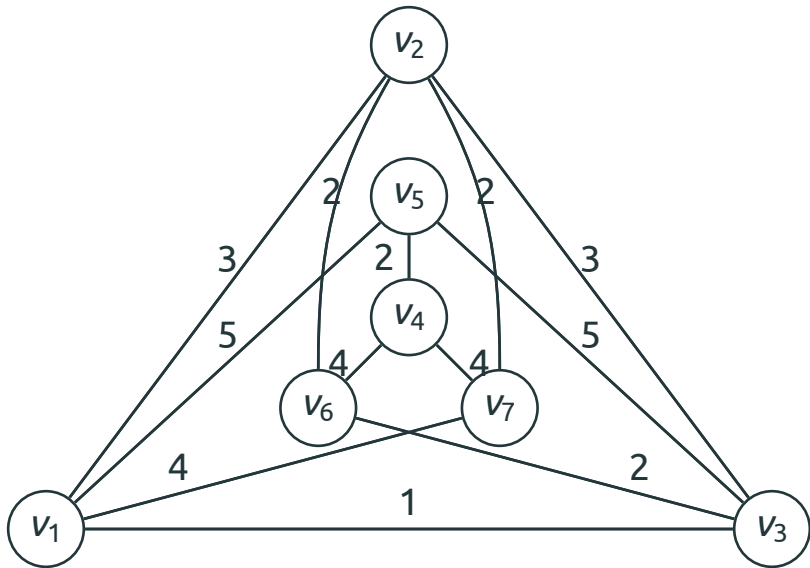
Weighted Graphs

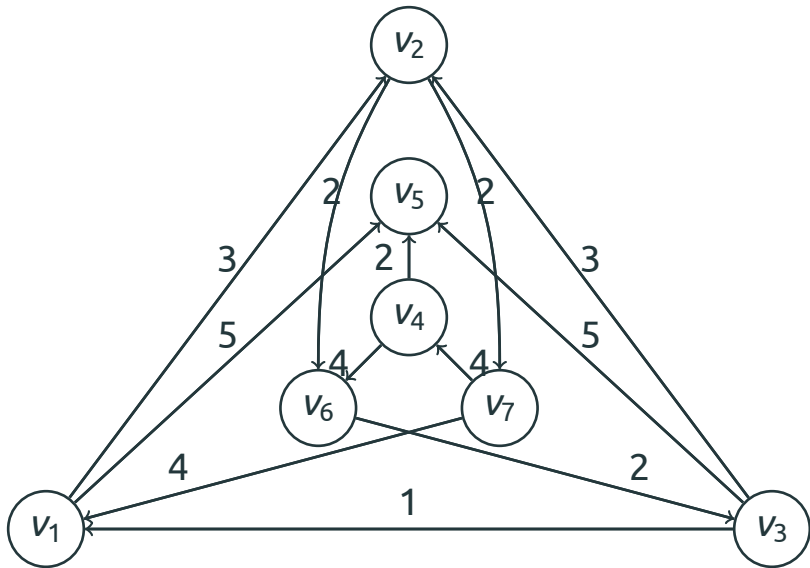## Distance, Driving Time, etc.

Weighted Graphs: Examples

Weighted Graphs: Examples

Weighted Graphs: Examples

Weighted Paths

- A Weighted Graph associates a weight with every edge

Weighted Paths

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- The Weight of a path is the sum of the weights of its edges

Weighted Paths

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Weighted Paths
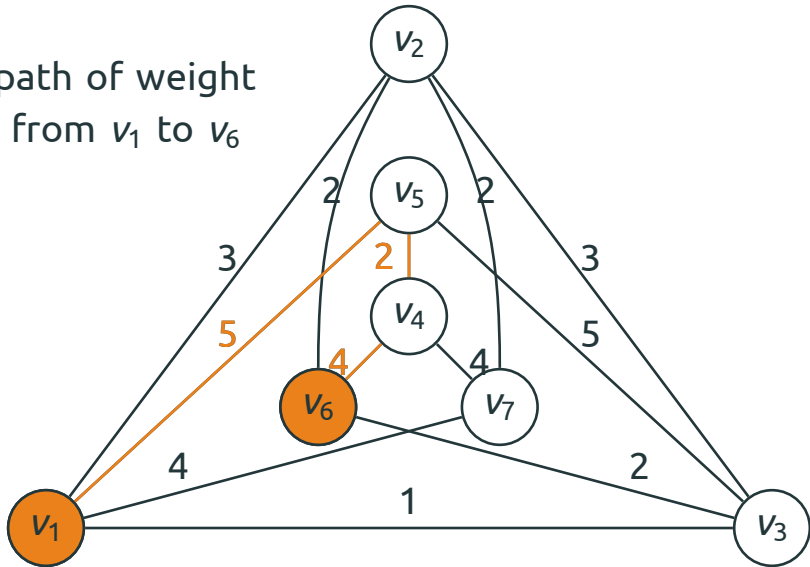
- A Weighted Graph associates a weight with every edge

- The Weight of a path is the sum of the weights of its edges

- A Shortest Path between two vertices is a path of the minimum weight

- The Distance between two vertices is the length of a shortest path between them
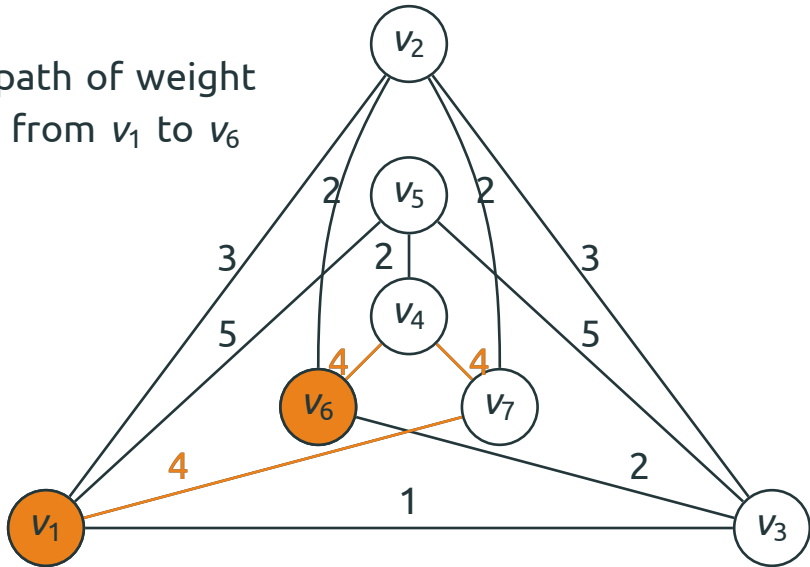
Weighted Paths: Examples
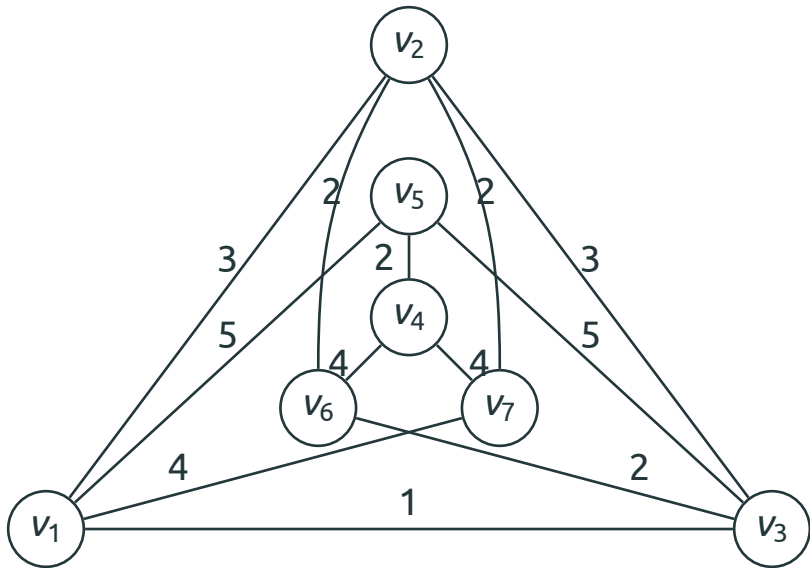
Weighted Paths: Examples

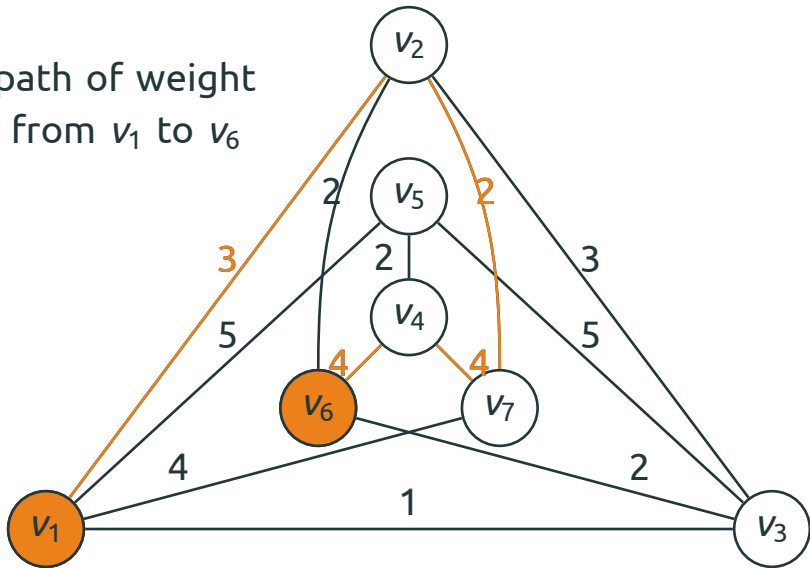A path of weight 11 from v_1 to v_6

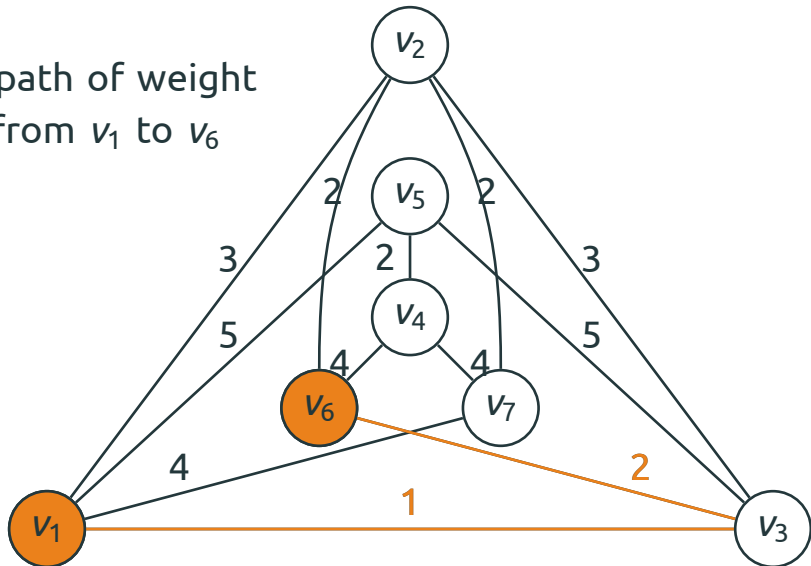# Weighted Paths: Examples

A path of weight 12 from v_1 to v_6

Weighted Paths: Examples

A path of weight 13 from v_1 to v_6

Weighted Paths: Examples

A path of weight 3 from v_1 to v_6

Weighted Paths: Examples

The distance between v_1 and v_6 is 3