Conditional Probability:

Conditional probability is defined as the likelihood of an event or outcome occurring, based on the occurrence of a previous event or outcome. Conditional probability is calculated by multiplying the probability of the preceding event by the updated probability of the succeeding, or conditional, event.

Conditional probability can be contrasted with unconditional probability. Unconditional probability refers to the likelihood that an event will take place irrespective of whether any other events have taken place or any other conditions are present.

- Conditional probability refers to the chances that some outcome occurs given that another event has also occurred.
- It is often stated as the probability of B given A and is written as P(B|A), where the probability of B depends on that of A happening.
- Conditional probability can be contrasted with unconditional probability.
- Probabilities are classified as either conditional, marginal, or joint.
- Bayes' theorem is a mathematical formula used in calculating conditional probability.

Understanding Conditional Probability

Conditional probabilities are contingent on a previous result or event occurring. A conditional probability would look at such events in relationship with one another. Conditional probability is thus the likelihood of an event or outcome occurring *based on* the occurrence of some other event or prior outcome.

Two events are said to be independent if one event occurring does not affect the probability that the other event will occur. However, if one event occurring or not does, in fact, affect the probability that the other event will occur, the two events are said to be dependent. If events are independent, then the probability of some event B is not contingent on what happens with event A. A conditional probability, therefore, relates to those events that are dependent on one another.

Conditional probability is often portrayed as the "probability of A *given* B," notated as P(A|B).

Conditional probability is used in a variety of fields, such as insurance, economics, politics, and many different fields of mathematics.

Conditional Probability Formula

P(B|A) = P(A and B) / P(A)

Or:

$P(B|A) = P(A \cap B) / P(A)$

Where

P(A AND B) = P(B AND A)

$P(A \cap B) = P(B \cap A)$

P = Probability

A = Event A

B = Event B

Unconditional probability is also known as marginal probability and measures the chance of an occurrence ignoring any knowledge gained from previous or external events. Since this probability ignores new information, it remains constant.

Examples of Conditional Probability

As an example, suppose you are drawing three marbles—red, blue, and green—from a bag. Each marble has an equal chance of being drawn. What is the conditional probability of drawing the red marble after already drawing the blue one?

First, the probability of drawing a blue marble is about 33% because it is one possible outcome out of three. Assuming this first event occurs, there will be two marbles remaining, with each having a 50% chance of being drawn. So the chance of drawing a blue marble after already drawing a red marble would be about 16.5% (33% x 50%).

As another example to provide further insight into this concept, consider that a fair die has been rolled and you are asked to give the probability that it was a five. There are six equally likely outcomes, so your answer is 1/6.

But imagine if before you answer, you get extra information that the number rolled was odd. Since there are only three odd numbers that are possible, one of which is five, you would certainly revise your estimate for the likelihood that a five was rolled from 1/6 to 1/3.

This *revised* probability that an event *A* has occurred, considering the additional information that another event *B* has definitely occurred on this trial of the experiment, is called the *conditional probability of A given B* and is denoted by P(A|B).

Another Example of Conditional Probability

As another example, suppose a student is applying for admission to a university and hopes to receive an academic scholarship. The school to which they are applying accepts 100 of every 1,000 applicants (10%) and awards academic scholarships to 10 of every 500 students who are accepted (2%).

Of the scholarship recipients, 50% of them also receive university stipends for books, meals, and housing. For the students, the chance of them being accepted and then receiving a scholarship is .2% (.1 x .02). The chance of them being accepted, receiving the scholarship, then also receiving a stipend for books, etc. is .1% (.1 x .02 x .5).

Conditional Probability vs. Joint Probability and Marginal Probability

- **Conditional probability**: p(A|B) is the probability of event A occurring, **given that** event B occurs. For example, given that you drew a red card, what's the probability that it's a four (p(four|red))=2/26=1/13. So out of the 26 red cards (given a red card), there are two fours so 2/26=1/13.
- **Marginal probability**: the probability of an event occurring (p(A)) in isolation. It may be thought of as an unconditional probability. It is not conditioned on another event. Example: the probability that a card drawn is red (p(red) = 0.5). Another example: the probability that a card drawn is a 4 (p(four)=1/13).
- **Joint probability**: p(A ∩B). Joint probability is that of event A **and** event B occurring. It is the probability of the intersection of two or more events. The probability of the intersection of A and B may be written p(A ∩ B). Example: the probability that a card is a four and red =p(four and red) = 2/52=1/26. (There are two red fours in a deck of 52, the 4 of hearts and the 4 of diamonds).

Bayes' Theorem and Conditional Probability

Bayes' theorem, named after 18th-century British mathematician Thomas Bayes, is a mathematical formula for determining conditional probability.1 The theorem provides a way to revise existing predictions or theories (update probabilities) given new or additional evidence. In finance, Bayes' theorem can be used to rate the risk of lending money to potential borrowers.

How Do You Calculate Conditional Probability?

Conditional probability is calculated by multiplying the probability of the preceding event by the probability of the succeeding or conditional event. Conditional probability looks at the probability of one event happening based on the probability of a preceding event happening.

What Is the Difference Between Probability and Conditional Probability?

Probability looks at the likelihood of one event occurring. Conditional probability looks at two events occurring in relation to one another. It looks at the probability of a second event occurring based on the probability of the first event occurring.

What Is Prior Probability?

Prior probability is the probability of an event occurring before any data has been gathered to determine the probability. It is the probability as determined by a prior belief. Prior probability is a component of Bayesian statistical inference.

What Is Compound Probability?

Compound probability looks to determine the likelihood of two independent events occurring. Compound probability multiplies the probability of the first event by the probability of the second event. The most common example is that of a coin flipped twice and the determination if the second result will be the same or different than the first.

The Bottom Line

Conditional probability examines the likelihood of an event occurring based on the likelihood of a preceding event occurring. The second event is dependent on the first event. It is calculated by multiplying the probability of the first event by the probability of the second event.

Conditional probability and Bayes theorem questions

Conditional probability formula gives the measure of the probability of an event given that another event has occurred. If the event of interest is A and the event B is known or assumed to have occurred, "the conditional probability of A given B", or "the probability of A under the condition B". The events are usually written as P(A|B), or sometimes P B(A). The formula for conditional probability for both the conditions i.e. "the probability of A under the condition B" and "the probability of B under the condition A" are stated below.

Formula for Conditional Probability

Conditional Probability of A given B	$P(A\|B) = P(A \cap B)/P(B)$
Conditional Probability of B given A	$P(B\|A) = P(B \cap A)/P(A)$

Solved Examples Using Conditional Probability Formula

Question 1:

The probability that it is Friday and that a student is absent is 0.03. Since there are 5 school days in a week, the probability that it is Friday is 0.2. What is the probability that a student is absent given that today is Friday?

Solution:

The formula of Conditional probability Formula is:

$P(B|A) = P(A \cap B)/P(A)$

P(Absent | Friday)= P (Absent and Friday)/P(Friday)

= 0.03/0.2

= 0.15

= 15 %

Question 2: A teacher gave her students of the class two tests namely maths and science. 25% of the students passed both the tests and 40% of the students passed the maths test. What percent of those who passed the maths test also passed the science test?

Solution:
Given,
Percentage of students who passed the maths test = 40%
Percentage of students who passed both the tests = 25%
Let A and B be the events of the number of students who passed maths and science tests.
According to the given,
P(A) = 40% = 0.40
P(A ∩ B) = 25% = 0.25
Percent of students who passed the maths test also passed the science test
= Condition probability of B given A
= P(B|A)
= P(A ∩ B)/P(A)
= 0.25/0.40
= 0.625
= 62.5%

Question 3: A bag contains green and yellow balls. Two balls are drawn without replacement. The probability of selecting a green ball and then a yellow ball is 0.28. The probability of selecting a green ball on the first draw is 0.5. Find the probability of selecting a yellow ball on the second draw, given that the first ball drawn was green.

Solution:
Let A and B be the events of drawing a green in the first draw and yellow ball in the second draw respectively.
From the given,
P(A) = 0.5
P(A ∩ B) = 0.28
Probability of selecting a yellow ball on the second draw, given that the first ball drawn was green = Conditional of B given A
= P(B|A)
= P(A ∩ B)/P(A)

= 0.28/0.5

= 0.56

Example 1:

Suppose a biscuits factory in Karachi wants to fill the post of Programmer for their Computer Division. After interviewing many applicants, the firm has identified 20 applicants catergorized as in the following contingency table

	Certificate Course	Diploma Course	
Male	10	2	12
Female	5	3	8
	15	5	20

If a Diploma holder is selected, what is the probability that the person is a Male i.e. we have to find P(M/D)

Now this is the problem of Bayes' theorem.

therefore $P(M/D) = \frac{P(D \cap M)}{P(D)}$ by the definition of conditional probability.

But $P(D) = P(D \cap M) + P(D \cap F)$

since $P(A \cap M) = 0.10$

and $P(D \cap F) = 0.15$

threfore $P(D) = 0.10 + 0.15$

$= 0.25$

and therefore

$P(M/D) = \frac{P(M \cap D)}{P(D)} = \frac{P(M \cap B)}{P(M \cap D)+P(F \cap D)} = \frac{0.10}{0.25} = 0.4$

shown by the Venn Diagram

Conditional probability: Bayes' Theorem

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

Bayes Theorem Statement

Let $E_1, E_2, \ldots, E_n$ be a set of events associated with a sample space S, where all the events $E_1, E_2, \ldots, E_n$ have nonzero probability of occurrence and they form a partition of S. Let A be any event associated with S, then according to Bayes theorem,

$$P(E_i|A) = \frac{P(E_i)P(A|E_i)}{\sum_{k=1}^{n} P(E_k)P(A|E_k)}$$

for any k = 1, 2, 3, …., n

Bayes Theorem Proof

According to the conditional probability formula,

$$P(E_i|A) = \frac{P(E_i \cap A)}{P(A)} \ldots (1)$$

Using the multiplication rule of probability,

$$P(E_i \cap A) = P(E_i)P(A|E_i) \ldots (2)$$

Using total probability theorem,

$$P(A) = \sum_{k=1}^{n} P(E_k)P(A|E_k) \ldots (3)$$

Putting the values from equations (2) and (3) in equation 1, we get

$$P(E_i|A) = \frac{P(E_i)P(A|E_i)}{\sum_{k=1}^{n} P(E_k)P(A|E_k)}$$

Note:

The following terminologies are also used when the Bayes theorem is applied:

Hypotheses: The events $E_1, E_2, \ldots E_n$ is called the hypotheses

Priori Probability: The probability $P(E_i)$ is considered as the priori probability of hypothesis E_i

Posteriori Probability: The probability $P(E_i|A)$ is considered as the posteriori probability of hypothesis E_i

Bayes' theorem is also called the formula for the probability of "causes". Since the E_i's are a partition of the sample space S, one and only one of the events E_i occurs (i.e. one of the events

E_i must occur and the only one can occur). Hence, the above formula gives us the probability of a particular E_i (i.e. a "Cause"), given that the event A has occurred.

Bayes Theorem Formula

If A and B are two events, then the **formula for the Bayes theorem** is given by:

P(A|B)=P(B|A)P(A)P(B)whereP(B)≠0

Where P(A|B) is the probability of condition when event A is occurring while event B has already occurred.

Bayes Theorem Derivation

Bayes Theorem can be derived for events and random variables separately using the definition of conditional probability and density.

From the definition of conditional probability, Bayes theorem can be derived for events as given below:

$P(A|B) = P(A \cap B)/ P(B)$, where $P(B) \neq 0$ (CONDITIONAL)

$P(B|A) = P(B \cap A)/ P(A)$, where $P(A) \neq 0$ (CONDITIONAL)

Here, the joint probability $P(A \cap B)$ of both events A and B being true such that,

$P(B \cap A) = P(A \cap B)$

$P(A \cap B) = P(A | B) P(B) = P(B | A) P(A)$

$P(A|B) = [P(B|A) P(A)]/ P(B)$, where $P(B) \neq 0$ (BAYE'S THEOREM)

Bayes' theorem is a way to figure out conditional probability. Conditional probability is the probability of an event happening, given that it has some relationship to one or more other events. For example, your probability of getting a parking space is connected to the time of day you park, where you park, and what conventions are going on at any time. Bayes' theorem is slightly more nuanced. In a nutshell, it gives you the actual probability of an **event** given information about **tests**.

- "Events" Are different from "tests." For example, there is a **test** for liver disease, but that's separate from the **event** of actually having liver disease.
- **Tests are flawed**: just because you have a positive test does not mean you actually have the disease. Many tests have a high false positive rate. **Rare events tend to have higher false positive rates** than more common events. We're not just talking about medical tests here. For example, spam filtering can have high false positive rates. Bayes' theorem takes the test results and calculates your *real probability* that the test has identified the event.

The Formula

Bayes' Theorem (also known as Bayes' rule) is a deceptively simple formula used to calculate conditional probability. The Theorem was named after English mathematician Thomas Bayes (1701-1761). The formal definition for the rule is:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

In most cases, you can't just plug numbers into an equation; You have to figure out what your "tests" and "events" are first. For two events, A and B, Bayes' theorem allows you to figure out p(A|B) (the probability that event A happened, given that test B was positive) from p(B|A) (the probability that test B happened, given that event A happened). It can be a little tricky to wrap your head around as technically you're working backwards; you may have to switch your tests and events around, which can get confusing. An example should clarify what I mean by "switch the tests and events around."

Bayes' Theorem Example #1

You might be interested in finding out a patient's probability of having liver disease if they are an alcoholic. "Being an alcoholic" is the **test** (kind of like a litmus test) for liver disease.

- **A** could mean the event "Patient has liver disease." Past data tells you that 10% of patients entering your clinic have liver disease. P(A) = 0.10.
- **B** could mean the litmus test that "Patient is an alcoholic." Five percent of the clinic's patients are alcoholics. P(B) = 0.05.
- You might also know that among those patients diagnosed with liver disease, 7% are alcoholics. This is your **B|A:** the probability that a patient is alcoholic, given that they have liver disease, is 7%.

Bayes' theorem tells you:

P(A|B) = (0.07 * 0.1)/0.05 = 0.14

In other words, if the patient is an alcoholic, their chances of having liver disease is 0.14 (14%). This is a large increase from the 10% suggested by past data. But it's still unlikely that any particular patient has liver disease.

Bayes' Theorem Problems Example #2

Another way to look at the theorem is to say that one event follows another. Above I said "tests" and "events", but it's also legitimate to think of it as the "first event" that leads to the "second event." There's no one right way to do this: use the terminology that makes most sense to you.

In a particular pain clinic, 10% of patients are prescribed narcotic pain killers. Overall, five percent of the clinic's patients are addicted to narcotics (including pain killers and illegal substances). Out of all the people prescribed pain pills, 8% are addicts. *If a patient is an addict, what is the probability that they will be prescribed pain pills?*

Step 1: **Figure out what your event "A" is from the question.** That information is in the italicized part of this particular question. The event that happens first (A) is being prescribed pain pills. That's given as 10%.

Step 2: **Figure out what your event "B" is from the question.** That information is also in the italicized part of this particular question. Event B is being an addict. That's given as 5%.

Step 3: **Figure out what the probability of event B (Step 2) given event A (Step 1)**. In other words, find what (B|A) is. We want to know "Given that people are prescribed pain pills, what's the probability they are an addict?" That is given in the question as 8%, or .8.

Step 4: **Insert your answers from Steps 1, 2 and 3 into the formula and solve.**
$P(A|B) = P(B|A) * P(A) / P(B) = (0.08 * 0.1)/0.05 = 0.16$

The probability of an addict being prescribed pain pills is 0.16 (16%).

Example #3: the Medical Test

A slightly more complicated example involves a medical test (in this case, a genetic test):

There are **several forms of Bayes' Theorem** out there, and they are all equivalent (they are just written in slightly different ways). In this next equation, "X" is used in place of "B." In addition, you'll see some changes in the denominator. The proof of why we can rearrange the equation like this is beyond the scope of this article (otherwise it would be 5,000 words instead of 2,000!). However, if you come across a question involving medical tests, you'll likely be using this alternative formula to find the answer:

$$\Pr(\mathrm{A}|\mathrm{X}) = \frac{\Pr(\mathrm{X}|\mathrm{A})\Pr(\mathrm{A})}{\Pr(\mathrm{X}|\mathrm{A})\Pr(A) + \Pr(\mathrm{X}|\sim\mathrm{A})\Pr(\sim A)}$$

1% of people have a certain genetic defect.
90% of tests for the gene detect the defect (true positives).
9.6% of the tests are false positives.
If a person gets a positive test result, **what are the odds they actually have the genetic defect?**

The first step into solving Bayes' theorem problems is to assign letters to events:

- A = chance of having the faulty gene. That was given in the question as 1%. That also means the probability of *not* having the gene (~A) is 99%.
- X = A positive test result.

So:

1. P(A|X) = Probability of having the gene given a positive test result.
2. P(X|A) = Chance of a positive test result given that the person actually has the gene. That was given in the question as 90%.
3. p(X|~A) = Chance of a positive test if the person *doesn't* have the gene. That was given in the question as 9.6%

Now we have all of the information we need to put into the equation:
P(A|X) = (.9 * .01) / (.9 * .01 + .096 * .99) = 0.0865 (8.65%).

The probability of having the faulty gene on the test is 8.65%.